

Production of a pion in association with a high- Q^2 dilepton pair in antiproton-proton annihilation at GSI-FAIR

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We evaluate the cross section for $\bar{p}p \rightarrow \ell^+ \ell^- \pi^0$ in the forward direction and for a large lepton pair invariant mass. In this kinematical region, the leading-twist amplitude factorizes into a short-distance matrix element, long-distance antiproton-distribution amplitudes, and proton to pion transition distribution amplitudes (TDAs). Using a modeling inspired from the chiral limit for these TDAs, we obtain a first estimate of this cross section, which demonstrates that this process can be measured at GSI-FAIR.

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Transition distribution amplitudes (TDAs) [1] are universal nonperturbative objects describing the transitions between two different particles (e.g. $p \rightarrow \pi$, $\pi \rightarrow \gamma$, $\pi \rightarrow \rho$). They appear in the study of backward electroproduction of a pion [2], of $\gamma^* \gamma \rightarrow \rho \pi$ and $\gamma^* \gamma \rightarrow \pi \pi$ reactions [3] as well as in hard exclusive production of a $\gamma^* \pi$ pair in $\bar{p}p$ annihilation:

$$\bar{p}(p_{\bar{p}})p(p_p) \rightarrow \gamma^* \pi(p_\pi) \rightarrow \ell^+(p_{\ell^+})\ell^-(p_{\ell^-})\pi(p_\pi) \quad (1)$$

at small $t = (p_\pi - p_p)^2$ [or at small $u = (p_\pi - p_{\bar{p}})^2$], which is the purpose of the present work. The TDAs are an extension of the concept of generalized parton distributions (GPDs), as already advocated in [4]. The proton to meson TDAs are defined from the Fourier transform of a matrix element of a three-quark-light-cone operator between a proton and a meson state. They obey QCD evolution equations which follow from the renormalization-group equation of the three-quark operator. Their Q^2 dependence is thus completely under control.

Whereas in the pion to photon case models used for GPDs [5–7] could be applied to TDAs since they are defined from matrix elements of the same quark-antiquark operators, the situation is clearly different for the nucleon to meson TDAs. Before estimates based on models such as the meson-cloud model [8] become available, it is important to use as much model-independent information as possible. In [2], we derived constraints from the chiral limit on the TDAs $p \rightarrow \pi$ and made a first evaluation of the cross section for the backward electroproduction of a pion in the large- ξ (or small E_π) region. Related processes were also recently studied in [9] similarly to what was proposed in [10]. In this work, we apply the same setting to evaluate the cross sections for $\bar{p}p \rightarrow \ell^+ \ell^- \pi^0$ in the kinematical region accessible by GSI-FAIR [11] in the forward limit and at moderate energy of the meson. For a discussion of a related process with fermionic exchange in the t -channel in a different framework, see [12].

In the scaling regime where $Q^2 = q^2$ is of the order of $W^2 = (p_{\bar{p}} + p_p)^2$, the amplitude for the process (1) at small t —or c.m. angle of the pion θ_π^* close to 0—involves the $p \rightarrow \pi$ TDAs $V^{p\pi}(x_i, \xi, \Delta^2)$, $A^{p\pi}(x_i, \xi, \Delta^2)$, $T^{p\pi}(x_i, \xi, \Delta^2)$, where x_i ($i = 1, 2, 3$) denote the light-cone-momentum fractions carried by the participant quarks and ξ is the skewedness parameter such that $2\xi = x_1 + x_2 + x_3$. The amplitude is a convolution of the antiproton DAs, a perturbatively calculable-hard-scattering amplitude and the $p \rightarrow \pi$ TDAs.

The momenta of the subprocess $\bar{p}p \rightarrow \gamma^* \pi$ are defined as shown in Fig. 1. The z -axis is chosen along the colliding proton and antiproton and the x - z plane is identified with the collision or hadronic plane. We define the light-cone vectors p and n such that $2p \cdot n = 1$, as well as $P = (p_p + p_{\bar{p}})/2$, $\Delta = p_\pi - p_p$, and its transverse component Δ_T ($\Delta_T^2 < 0$). ξ is defined as $\xi = -\frac{\Delta \cdot n}{2P \cdot n}$. We express the particle momenta through a Sudakov decomposition:

$$p_p = (1 + \xi)p + \frac{M^2}{1 + \xi}n \quad (2)$$

$$p_{\bar{p}} = \frac{2M^2(1 + \xi)}{\alpha}p + \frac{\alpha}{2(1 + \xi)}n$$

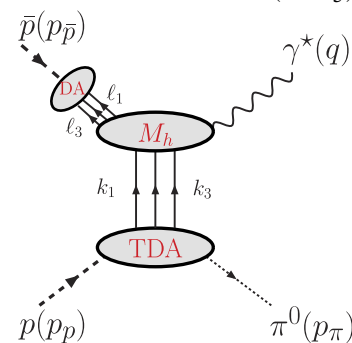


FIG. 1 (color online). The factorization of the process $p\bar{p} \rightarrow \gamma^* \pi$ into antiproton-distribution amplitudes (DA), the hard-subprocess amplitude (M_h) and proton \rightarrow pion TDA.

$$\begin{aligned}
p_\pi &= (1 - \xi)p + \frac{m_\pi^2 - \Delta_T^2}{1 - \xi}n + \Delta_T \\
\Delta &= -2\xi p + \left[\frac{m_\pi^2 - \Delta_T^2}{1 - \xi} - \frac{M^2}{1 + \xi} \right]n + \Delta_T \\
q &\simeq 2\xi p + \frac{M^2}{W^2}(1 + \xi) + \left[\frac{W^2}{1 + \xi} - \frac{m_\pi^2 - \Delta_T^2}{1 - \xi} \right]n - \Delta_T \\
\Delta_T^2 &= \frac{1 - \xi}{1 + \xi} \left(t - 2\xi \left[\frac{M^2}{1 + \xi} - \frac{m_\pi^2}{1 - \xi} \right] \right), \quad (3)
\end{aligned}$$

where $\alpha = W^2 - 2M^2 + W\sqrt{W^2 - 4M^2} \simeq 2W^2$; the approximate expression for q is obtained with $M \ll W$.

For $\Delta_T = 0$, $M \ll W$, and $m_\pi = 0$, one gets

$$\begin{aligned}
p_p &= (1 + \xi)p, & p_{\bar{p}} &= \frac{W^2}{1 + \xi}n, & p_\pi &= (1 - \xi)p, \\
t &= \frac{2\xi M^2}{1 + \xi}, & \xi &= \frac{Q^2}{2W^2 - Q^2}. \quad (4)
\end{aligned}$$

In the fixed-target mode, the maximal reachable value for $W^2 = 2M^2 + 2ME_{\bar{p}}$ at GSI will be $\simeq 30 \text{ GeV}^2$ (for $E_{\bar{p}} = 15 \text{ GeV}$). The highest invariant mass of the photon could be $Q_{\text{max}}^2 \simeq 30 \text{ GeV}^2$. We refer to Ref. [13] for a complete discussion of the kinematically allowed domain. In terms of our notations, in the proton rest frame, we have $p = \frac{M}{1+\xi}(1, 0, 0, -1)$ and $n = \frac{1+\xi}{4M}(1, 0, 0, 1)$. Thus $\xi \in [0.5, 1]$ corresponds to $|p_\pi^z| < M/3 \simeq 310 \text{ MeV}$ in the laboratory frame at $\Delta_T = 0$.

Let us now turn to the kinematics of $\bar{p}p \rightarrow \ell^+\ell^-\pi^0$. We have for the unpolarized differential cross section

$$d\sigma = \frac{1}{2\sqrt{\lambda(W^2, M^2, M^2)}(2\pi)^5} |\bar{\mathcal{M}}|^2 d_3(PS). \quad (5)$$

The 3-particle differential Lorentz invariant phase space (dLIPS), $d_3(PS)$, can be decomposed into two 2-particle dLIPS (where q is the momentum of the γ^*):

$$\begin{aligned}
d_3(PS) &= \delta^4(p_p + p_{\bar{p}} - p_{\ell^+} - p_{\ell^-} - p_\pi) \frac{d^3\vec{p}_{\ell^+}}{2p_{\ell^+}^0} \frac{d^3\vec{p}_{\ell^-}}{2p_{\ell^-}^0} \frac{d^3\vec{p}_\pi}{2p_\pi^0} \\
&= \delta^4(p_p + p_{\bar{p}} - q - p_\pi) \frac{d^3\vec{p}_\pi}{2p_\pi^0} \frac{d^3\vec{q}}{2q^0} \\
&\quad \times dQ^2 \delta^4(q - p_{\ell^+} - p_{\ell^-}) \frac{d^3\vec{p}_{\ell^+}}{2p_{\ell^+}^0} \frac{d^3\vec{p}_{\ell^-}}{2p_{\ell^-}^0}.
\end{aligned}$$

In the $\bar{p}p$ c.m., we have

$$\delta^4(p_p + p_{\bar{p}} - q - p_\pi) \frac{d^3\vec{p}_\pi}{2p_\pi^0} \frac{d^3\vec{q}}{2q^0} = \frac{d\Omega_\pi^*}{8W^2} \sqrt{\lambda(W^2, Q^2, m_\pi^2)}$$

and in the $\ell^+\ell^-$ c.m., we have ($m_\ell \simeq 0$)

$$\delta^4(q - p_{\ell^+} - p_{\ell^-}) \frac{d^3\vec{p}_{\ell^+}}{2p_{\ell^+}^0} \frac{d^3\vec{p}_{\ell^-}}{2p_{\ell^-}^0} = \frac{d\Omega_\ell}{8} = \frac{d\cos\theta_\ell d\varphi_\ell}{8}. \quad (6)$$

Expressing $t = (p_\pi - p_p)^2$ in terms of $\cos\theta_\pi^*$ [14], we get

$$dt = \frac{d\cos\theta_\pi^*}{2W^2} \sqrt{\lambda(W^2, M^2, M^2)} \sqrt{\lambda(W^2, Q^2, m_\pi^2)}. \quad (7)$$

Altogether, by integrating on φ_π^* and on φ_ℓ ,

$$\frac{d\sigma}{dtdQ^2 d\cos\theta_\ell} = \frac{\int d\varphi_\ell |\overline{\mathcal{M}}^{\bar{p}p \rightarrow \ell^+\ell^-\pi^0}|^2}{64W^2(W^2 - 4M^2)(2\pi)^4}, \quad (8)$$

to be compared with the cross section for $\bar{p}p \rightarrow \gamma^*\pi^0$:

$$\frac{d\sigma}{dt} = \frac{|\overline{\mathcal{M}}^{\bar{p}p \rightarrow \gamma^*\pi^0}|^2}{16\pi W^2(W^2 - 4M^2)}. \quad (9)$$

At $\Delta_T = 0$, the leading-twist TDAs for the $p \rightarrow \pi^0$ transition, $V_i^{p\pi^0}(x_i, \xi, \Delta^2)$, $A_i^{p\pi^0}(x_i, \xi, \Delta^2)$, and $T_i^{p\pi^0}(x_i, \xi, \Delta^2)$ are defined as (see Appendix for details)

$$\begin{aligned}
&\mathcal{F}(\langle \pi^0(p_\pi) | \epsilon^{ijk} u_\alpha^i(z_1 n) u_\beta^j(z_2 n) d_\gamma^k(z_3 n) | P(p_p, s_p) \rangle \rangle) \\
&= \frac{i}{4} \frac{f_N}{f_\pi} [V_1^{p\pi^0} (\not{p}C)_{\alpha\beta} (u^+(p_p, s_p))_\gamma \\
&\quad + A_1^{p\pi^0} (\not{p}\gamma^5 C)_{\alpha\beta} (\gamma^5 u^+(p_p, s_p))_\gamma \\
&\quad + T_1^{p\pi^0} (\sigma_{p\mu} C)_{\alpha\beta} (\gamma^\mu u^+(p_p, s_p))_\gamma], \quad (10)
\end{aligned}$$

where $\sigma^{\mu\nu} = 1/2[\gamma^\mu, \gamma^\nu]$, C is the charge conjugation matrix, $f_\pi = 131 \text{ MeV}$ is the pion decay constant, and $f_N \sim 5.2 \times 10^{-3} \text{ GeV}^2$. u^+ is the large component of the nucleon spinor: $u(p_p, s_p) = (\not{p} + \not{p})u(p_p, s_p) = u^-(p_p, s_p) + u^+(p_p, s_p)$ with $u^+(p_p, s_p) \sim \sqrt{p_p^+}$ and $u^-(p_p, s_p) \sim \sqrt{1/p_p^+}$.

For the three TDAs $V_1^{p\pi^0}$, $A_1^{p\pi^0}$, and $T_1^{p\pi^0}$, contributing in the limit $\Delta_T \rightarrow 0$, we use the following expressions for $\Delta_T = 0$ and large ξ (see the Appendix):

$$\begin{aligned}
&\{V_1^{p\pi^0}, A_1^{p\pi^0}, T_1^{p\pi^0}\}(x_1, x_2, x_3, \xi, \Delta^2) \\
&= \frac{1}{4\xi} \{V^p, A^p, 3T^p\} \left(\frac{x_1}{2\xi}, \frac{x_2}{2\xi}, \frac{x_3}{2\xi} \right), \quad (11)
\end{aligned}$$

where V^p , A^p , and T^p are the proton DAs [15].

At the leading order in α_s and at $\Delta_T = 0$, the amplitude $\mathcal{M}_\lambda^{s_p s_{\bar{p}}}$ for $\bar{p}(p_{\bar{p}}, s_{\bar{p}})p(p_p, s_p) \rightarrow \gamma^*(q, \lambda)\pi^0(p_\pi)$ reads

$$\mathcal{M}_\lambda^{s_p s_{\bar{p}}} = -i \frac{(4\pi\alpha_s)^2 \sqrt{4\pi\alpha_{em}} f_N^2}{54f_\pi Q^4} S_\lambda^{s_p s_{\bar{p}}} I \quad (12)$$

with $S_\lambda^{s_p s_{\bar{p}}} = \bar{v}^+(p_{\bar{p}}, s_{\bar{p}}) \not{\epsilon}^*(\lambda) \gamma^5 u^+(p_p, s_p)$ and

$$I = \int_{-1+\xi}^{1+\xi} [dx] \int_0^1 [dy] \left(2 \sum_{\alpha=1}^7 R_\alpha + \sum_{\alpha=8}^{14} R_\alpha \right), \quad (13)$$

where $[dx] = dx_1 dx_2 dx_3 \delta(2\xi - \sum_k x_k)$ and $[dy] = dx_1 dx_2 dx_3 \delta(1 - \sum_k y_k)$; the coefficients R_α ($\alpha = 1, \dots, 14$) exactly correspond to T_α in [2] after the replacement $-i\epsilon \rightarrow i\epsilon$ due to the presence of the γ^* in the final instead of initial state. Even though the TDA formalism can be applied at any value of ξ (or E_π), we have for now at our disposal estimates for the $p \rightarrow \pi$ TDAs only at large ξ .

In the following, we shall therefore limit ourselves to the computation of the cross section for reaction (1) in this region. At large ξ , the ERBL regime ($x_i > 0$) covers most of the integration domain. Therefore it is legitimate to approximate the cross section only from the ERBL contribution, i.e. when the integration range of the momentum fractions is restricted to $[0, 2\xi]$.

The differential cross section for unpolarized protons and antiprotons is calculated as usual using Eq. (9) from the averaged-squared amplitudes,

$$|\overline{\mathcal{M}}_{\lambda\lambda'}|^2 = \frac{1}{4} \sum_{s_p, s_{\bar{p}}} \mathcal{M}_{s_p s_{\bar{p}}}^\lambda (\mathcal{M}_{s_p s_{\bar{p}}}^{\lambda'})^*. \quad (14)$$

$|\overline{\mathcal{M}}_{00}|^2$ vanishes at the leading-twist accuracy, as in the nucleon-form-factor case. The same is true for $|\overline{\mathcal{M}}_{+-}|^2$ and $|\overline{\mathcal{M}}_{0+}|^2$, etc., since the x and y directions are not distinguishable when Δ_T^z is vanishing. We then define $|\overline{\mathcal{M}}_T|^2 \equiv |\overline{\mathcal{M}}_{++}|^2 + |\overline{\mathcal{M}}_{--}|^2$.

To compute I , we need to choose models for the DAs and the deduced TDAs. For the sake of coherence with experimental data, we shall use reasonable parametrizations of Chernyak-Zhitnitsky (CZ) [15] and King-Sachrajda (KS) [16], which are both based on an analysis of QCD sum rules. For CZ, they are

$$V^P(x_i) = \varphi_{as}[11.35(x_1^2 + x_2^2) + 8.82x_3^2 - 1.68x_3 - 2.94],$$

$$A^P(x_i) = \varphi_{as}[6.72(x_2^2 - x_1^2)],$$

$$T^P(x_i) = \varphi_{as}[13.44(x_1^2 + x_2^2) + 4.62x_3^2 + 0.84x_3 - 3.78],$$

and for KS (which we used in Fig. 2)

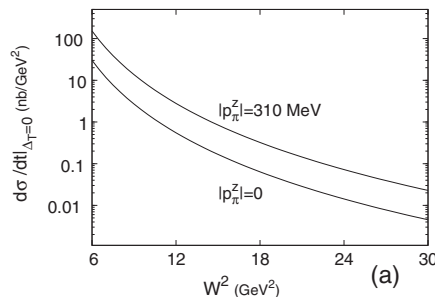
$$V^P = \varphi_{as}[17.64(x_1^2 + x_2^2) + 22.68x_3^2 - 6.72x_3 - 5.04],$$

$$A^P = \varphi_{as}[2.52(x_2^2 - x_1^2) + 1.68(x_2 - x_1)],$$

$$T^P = \varphi_{as}[21.42(x_1^2 + x_2^2) + 15.12x_3^2 + 0.84x_3 - 7.56],$$

and we evaluate our model TDAs from Eq. (11). This gives $I \simeq 1.28 \times 10^5$ for CZ and $I \simeq 2.15 \times 10^5$ for KS; this yields an induced uncertainty of order 3 for our estimates of the cross section. In the following, we use $\alpha_s = 0.3$ as suggested in [15].

Altogether, we have the following analytic results for the dominant ERBL contribution:



$$|\overline{\mathcal{M}}_T|^2 = \frac{(4\pi\alpha_s)^4 (4\pi\alpha_{em}) f_N^4}{54^2 f_\pi^2} \frac{2(1+\xi)|I|^2}{\xi Q^6}. \quad (15)$$

From this, we straightforwardly obtain $\frac{d\sigma}{dt}$, whose W^2 evolution is displayed on Fig. 2(a) at $\Delta_T = 0$ for the two extreme values of meson longitudinal momentum where one may trust the soft-pion limit, corresponding to $p_\pi^z = 0$ or $|p_\pi^z| = M/3$ in the laboratory frame ($\xi = 1$ or $1/2$).

For the process (1), the averaged-squared amplitude is

$$|\overline{\mathcal{M}}^{\bar{p}p \rightarrow \ell^+ \ell^- \pi^0}|^2 = \frac{1}{4} \sum_{s_p, s_{\bar{p}}, \lambda, \lambda'} \mathcal{M}_{s_p s_{\bar{p}}}^\lambda \frac{1}{Q^2} \mathcal{L}^{\lambda\lambda'} \frac{1}{Q^2} (\mathcal{M}_{s_p s_{\bar{p}}}^{\lambda'})^*,$$

with $\mathcal{L}^{\lambda\lambda'} = e^2 \text{Tr}(\not{p}_\ell - \not{\epsilon}(\lambda) \not{p}_{\ell'} - \not{\epsilon}'(\lambda'))$. Integrating on the lepton azimuthal angle φ_ℓ , we have

$$\int d\varphi_\ell |\overline{\mathcal{M}}^{\bar{p}p \rightarrow \ell^+ \ell^- \pi^0}|^2 = |\overline{\mathcal{M}}_T|^2 \frac{2\pi e^2 (1 + \cos^2 \theta_\ell)}{Q^2}, \quad (16)$$

from which we get, via Eq. (8) and integrating over θ_ℓ , the differential cross section displayed in Fig. 2(b).

Although the cross sections are evaluated at $\Delta_T = 0$, we do not anticipate any dramatic Δ_T -dependence of the TDAs below a few hundred MeV, so that our estimates are likely to be valid in a not-too-narrow Δ_T region. To evaluate a magnitude of the integrated cross section, we take as an example the kinematical region with $W^2 = 10 \text{ GeV}^2$ in Fig. 2(b) accompanied by the Q^2 window $7 \text{ GeV}^2 < Q^2 < 8 \text{ GeV}^2$, which corresponds to $\xi \approx 1/2$ or to the pion momentum of the order 310 MeV. Integrating over this Q^2 -bin and in a t -bin corresponding to $\Delta_T < 500 \text{ MeV}$ leads then to a cross section around 100 fb. With an integrated luminosity of about 2 fb^{-1} for a 100-day experiment, this yields a few hundreds of events above $Q^2 = 7 \text{ GeV}^2$, which is more than sufficient to get a first sight through this new physics window.

The calculations done until now and which involve the proton $\rightarrow \pi^0$ TDA are valid for the small t region. Let us however stress that—due to the charge symmetry—an identical result will be obtained in the small u region but with the $\bar{p} \rightarrow \pi^0$ TDA. In the laboratory frame at GSI-FAIR, this second region is quite different from the previous one since the π^0 meson is boosted in the forward direction. A precise detection of the particles of the final state, either in the proton or the antiproton “fragmenta-

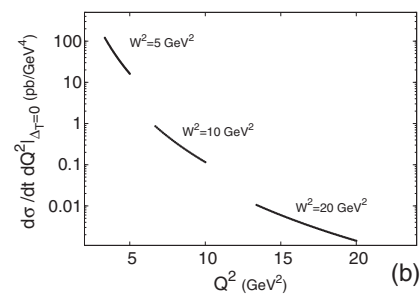


FIG. 2. (a) Differential cross section $d\sigma/dt$ for $\bar{p}p \rightarrow \gamma^* \pi^0$ as a function of W^2 for $|p_\pi^z| = 0$ (lower curve) and $|p_\pi^z| = M/3$. (b) Differential cross section $d\sigma/(dt dQ^2)$ for $\bar{p}p \rightarrow \ell^+ \ell^- \pi^0$ as a function of Q^2 for various beam energies.

tion" kinematics, will depend on the detector performances in the respective regions.

In conclusion, we have demonstrated that the study of proton-antiproton exclusive annihilation into a lepton pair and a pion is feasible at large values of the lepton pair invariant mass in the forthcoming PANDA or PAX experiments at GSI-FAIR. We believe that such a study will bring unique information about the inner structure of the proton, and particularly about the pion content of a proton, provided that the predictions (scaling behavior, angular dependence of the lepton pair) of the factorized framework used here are shown to be valid. Expressing the cross section of this process as the convolution of a hard-scattering amplitude for quarks with a photon with DAs and proton to pion TDAs will allow one to evaluate these new hadronic matrix elements which contain much information about the confinement dynamics. It has to be emphasized that the same hadronic matrix elements appear also in a similar description of backward electroproduction of a pion [2]. Note that other channels are also of much interest, such as $\bar{p}p \rightarrow l^+l^- \eta$ or $\bar{p}p \rightarrow l^+l^- \rho^0$. The theoretical treatment of the η is identical to the one for the π^0 , but for the isosinglet nature of η . In the ρ^0 case, one should distinguish between the longitudinally polarized meson where the TDA has the same structure as for the π^0 and the transversally polarized case which leads to more TDAs. Needless to say, we are strongly lacking model estimates for these $p \rightarrow \eta$ and $p \rightarrow \rho$ TDAs but their experimental determination (or at least the measurement of their ratios to the $p \rightarrow \pi$ TDAs) opens a fascinating window on the properties of the sea quarks in the proton wave function.

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APPENDIX

We now derive the general limit of the three contributing TDAs at $\Delta_T = 0$ in the soft-pion limit, when ξ gets close to 1. In that limit, the soft-meson theorem [17] derived from current algebra applies [10], which allows us to express these 3 TDAs in terms of the 3 distribution amplitudes (DAs) of the corresponding baryon. Conventionally [15], the three proton DAs are defined through the decomposition of the following matrix element of the 3-quark operator in terms of three invariant functions of the scalar product of the lightlike separation $z_i n \equiv \tilde{z}_i$ with the proton momentum p_p , $V^p(\tilde{z}_i \cdot p_p)$, $A^p(\tilde{z}_i \cdot p_p)$, and $T^p(\tilde{z}_i \cdot p_p)$,

$$\begin{aligned} & \langle 0 | u_\alpha(z_1 n) u_\beta(z_2 n) d_\gamma(z_3 n) | p_p \rangle \\ &= \frac{1}{4} f_N [V^p(\tilde{z}_i \cdot p_p) (\not{p}_p C)_{\alpha\beta} (\gamma^5 u^+(p_p, s_p))_\gamma \\ &+ A^p(\tilde{z}_i \cdot p_p) (\not{p}_p \gamma^5 C)_{\alpha\beta} u^+(p_p, s_p)_\gamma \\ &+ T^p(\tilde{z}_i \cdot p_p) (\sigma_{p_p \mu} C)_{\alpha\beta} (\gamma^\mu \gamma^5 u^+(p_p, s_p))_\gamma]. \quad (\text{A1}) \end{aligned}$$

The latter functions satisfy $V^p(\tilde{z}_i \cdot p_p = 0) = T^p(\tilde{z}_i \cdot p_p = 0) = 1$ and $A^p(\tilde{z}_i \cdot p_p = 0) = 0$, which provides the interpretation of f_N as the value of the proton wave function at the origin. To go to momentum space one writes a Fourier transform [16] which enables one to define functions of momentum fractions x_i ($F = V^p, A^p, T^p$) [$[d\tilde{z} \cdot (\cdot)] = d(\tilde{z}_1 \cdot (\cdot)) d(\tilde{z}_2 \cdot (\cdot)) d(\tilde{z}_3 \cdot (\cdot))$]:

$$\tilde{F}(x_i) \equiv \int_{-\infty}^{\infty} \frac{[d\tilde{z} \cdot p_p]}{(2\pi)^3} e^{i\sum_k x_k \tilde{z}_k \cdot p_p} F(\tilde{z}_i \cdot p_p). \quad (\text{A2})$$

Inspired by [10], which considered the related case of the distribution amplitude of the proton-meson system, we use the soft-pion theorems [17] to write

$$\langle \pi^a(p_\pi) | \mathcal{O} | P(p_1, s_1) \rangle = -\frac{i}{f_\pi} \langle 0 | [Q_5^a, \mathcal{O}] | P(p_1, s_1) \rangle, \quad (\text{A3})$$

where we neglected the nucleon pole term, which does not contribute at threshold.

For the transition $p \rightarrow \pi^0$, $Q_5^a = Q_5^3$, and $\mathcal{O} = u_\alpha u_\beta d_\gamma$. Since the commutator of the chiral charge Q_5 with the quark field ψ (τ^a being the Pauli matrix)

$$[Q_5^a, \psi] = -\frac{\tau^a}{2} \gamma^5 \psi, \quad (\text{A4})$$

the right-hand side (rhs) of Eq. (A3) gives three terms from $(\gamma^5 u)_\alpha u_\beta d_\gamma$, $u_\alpha (\gamma^5 u)_\beta d_\gamma$, and $u_\alpha u_\beta (\gamma^5 d)_\gamma$. The corresponding multiplication by γ^5 [or $(\gamma^5)^T$ when it acts on the index β] on the vector and axial-vector structures of the DA [Eq. (A1)] gives two terms which cancel and the third one, which remains, is the same as the one for the TDA up to the modification that on the DA decomposition, p_p is the proton momentum, whereas for the TDA one, p is the light-cone projection of P , i.e. half the proton momentum if one neglects the pion one. This introduces a factor 2ξ in the relations between the 2 DAs A^p and V^p and the 2 TDAs $V_1^{p\pi^0}$ and $A_1^{p\pi^0}$.

To what concerns the tensorial structure multiplying T^p , the three terms are identical at leading-twist accuracy and correspond to the structure multiplying $T_1^{p\pi^0}$, this gives a factor 3. We eventually have the soft limit for our three TDAs at $\Delta_T = 0$:

$$(V_1^{p\pi^0}, A_1^{p\pi^0}, T_1^{p\pi^0})(\tilde{z}_i \cdot p) = \xi(V^p, A^p, 3T^p)(\tilde{z}_i \cdot p_p). \quad (\text{A5})$$

We will derive now this relation in the momentum representation of DAs and TDAs. To do so, we start from translational invariance which implies

$$\begin{aligned} & \langle 0 | u_\alpha(\tilde{z}_1 + a) u_\beta(\tilde{z}_2 + a) d_\gamma(\tilde{z}_3 + a) | p_p \rangle \\ &= e^{-ia \cdot p_p} \langle 0 | u_\alpha(\tilde{z}_1) u_\beta(\tilde{z}_2) d_\gamma(\tilde{z}_3) | p_p \rangle, \quad (\text{A6}) \end{aligned}$$

and thus $F((\tilde{z}_i + a) \cdot p_p) = e^{-ia \cdot p_p} F(\tilde{z}_i \cdot p_p)$. In momentum space, we correspondingly get

$$\begin{aligned} \tilde{F}(x_i) &= \int_{-\infty}^{\infty} \frac{[d(\tilde{z} + a) \cdot p_p]}{(2\pi)^3} e^{i\sum_k x_k (\tilde{z}_k + a) \cdot p_p} F((\tilde{z}_i + a) \cdot p_p) \\ &= e^{i(\sum_k x_k - 1) a \cdot p_p} \tilde{F}(x_i). \quad (\text{A7}) \end{aligned}$$

This condition is conveniently expressed by the following redefinition: $\tilde{F}(x_i) = \delta(\sum_k x_k - 1)F(x_i)$. The inverse Fourier transform is then written as ($[dx] = dx_1 dx_2 dx_3$):

$$F(\tilde{z}_i \cdot p_p) = \int_0^1 [dx] e^{-i\sum_k x_k \tilde{z}_k \cdot p_p} \delta\left(\sum_k x_k - 1\right) F(x_i). \quad (\text{A8})$$

The normalization conditions then read

$$\int_0^1 [dx] \delta\left(\sum_k x_k - 1\right) (V^p, A^p, T^p)(x_i) = (1, 0, 1). \quad (\text{A9})$$

Note that the delta function ensuring translational invariance is exactly the one expected from the interpretation that x_i be the momentum fraction carried by the quark i off the proton of momentum p_p . This shows that the natural conjugate variable to the momentum fractions x_i 's are the $\tilde{z}_i \cdot p_p$'s, i.e. the spatial separation dotted by the proton momentum. Indeed, p_p enters in the exponential of the rhs of Eq. (A7), via the conjugate variable to x_i , $\tilde{z}_i \cdot p_p$, and as the initial-state momentum of the matrix element. Another choice than $\tilde{z}_i \cdot p_p$ would not have provided the correct support for the x_i 's.

The case of TDAs is similar except for the choice of the natural conjugate variable. We start with translational invariance:

$$\begin{aligned} \langle B(p_B) | \epsilon^{ijk} u_\alpha^i(\tilde{z}_1 + a) u_\beta^j(\tilde{z}_2 + a) d_\gamma^k(\tilde{z}_3 + a) | A(p_A) \rangle \\ = e^{-ia \cdot (p_A - p_B)} \langle B(p_B) | \epsilon^{ijk} u_\alpha^i(\tilde{z}_1) u_\beta^j(\tilde{z}_2) d_\gamma^k(\tilde{z}_3) | A(p_A) \rangle. \end{aligned}$$

Now let us define a Fourier transform without specifying the momentum p_0 to which we dot the spatial separation:

$$\tilde{F}_{A \rightarrow B}(x_i) = \int_{-\infty}^{\infty} \frac{d((\tilde{z} + a) \cdot p_0)}{(2\pi)^3} e^{i\sum_k x_k (\tilde{z}_k + a) \cdot p_0}$$

$$F_{A \rightarrow B}((\tilde{z}_i + a) \cdot p_0) = e^{i(\sum_k x_k a \cdot p_0 - a \cdot (p_A - p_B))} \tilde{F}_{A \rightarrow B}(x_i).$$

The condition derived from translational invariance would

then be satisfied by $\delta(\sum_k x_k a \cdot p_0 - a \cdot (p_A - p_B))$. In order to get the correct support, i.e. $\delta(2\xi - \sum_k x_k)$, for a translation a along n , we have to choose p_0 such that $2\xi = \frac{n \cdot (p_A - p_B)}{n \cdot p_0} = -\frac{n \cdot \Delta}{n \cdot p_0}$, which is satisfied by $p_0 = (p_A + p_B)/2 = P$ or a light-cone vector p ($p^2 = 0$) such that $P \cdot n = p \cdot n$. We choose p and have

$$\begin{aligned} \delta\left(2\xi - \sum_k x_k\right) F_{A \rightarrow B}(x_i) &\equiv \mathcal{F}(F(z_i(n \cdot p))) \\ &\equiv (n \cdot p)^3 \int_{-\infty}^{\infty} \frac{[dz]}{(2\pi)^3} \\ &\quad \times e^{i\sum_k x_k z_k (n \cdot p)} F(z_i(n \cdot p)). \end{aligned}$$

Let us now use the Fourier transform Eq. (A2) on both sides of Eq. (A5). Defining α such that $p_p \cdot n = \alpha P \cdot n = \alpha p \cdot n$, we get for instance for the V 's,

$$\begin{aligned} \delta\left(\sum_k x_k - 1\right) V^p(x_i) &= \int_{-\infty}^{\infty} \frac{[d\tilde{z} \cdot p_p]}{(2\pi)^3} e^{i\sum_k x_k \tilde{z}_k \cdot p_p} V_1^{p\pi^0}(\tilde{z}_i \cdot p) \\ &= \alpha^3 (n \cdot p)^3 \int_{-\infty}^{\infty} \frac{[dz]}{(2\pi)^3} e^{i\sum_k x_k z_k \alpha (n \cdot p)} \\ &\quad \times V_1^{p\pi^0}(z_i(n \cdot p)) \\ &= \alpha^3 \delta\left(2\xi - \sum_k (\alpha x_k)\right) V_1^{p\pi^0}(\alpha x_i). \end{aligned}$$

We conclude that translational invariance imposes naturally, through the delta function, that $\alpha = 2\xi$; the change of variable $x_i' = \alpha x_i$ then yields

$$\begin{aligned} \{V_1^{p\pi^0}, A_1^{p\pi^0}, T_1^{p\pi^0}\}(x_1, x_2, x_3, \xi, \Delta^2) \\ = \frac{1}{4\xi} \{V^p, A^p, 3T^p\}\left(\frac{x_1}{2\xi}, \frac{x_2}{2\xi}, \frac{x_3}{2\xi}\right), \quad (\text{A10}) \end{aligned}$$

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