

Large- N_c relations for the electromagnetic nucleon-to- Δ form factors

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We examine the large- N_c relations which express the electromagnetic N -to- Δ transition quantities in terms of the electromagnetic properties of the nucleon. These relations are based on the known large- N_c relation between the $N \rightarrow \Delta$ electric quadrupole moment and the neutron charge radius, and a newly derived large- N_c relation between the electric quadrupole ($E2$) and Coulomb quadrupole ($C2$) transitions. Extending these relations to finite, but small, momentum transfer, we find that the description of the electromagnetic $N \rightarrow \Delta$ ratios (R_{EM} and R_{SM}) in terms of the nucleon form factors predicts a structure which may be ascribed to the effect of the “pion cloud.” These relations also provide useful constraints for the $N \rightarrow \Delta$ generalized parton distributions.

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I. INTRODUCTION

The electromagnetic properties of the nucleon, such as magnetic moments and charge radii, provide benchmark information of the nucleon structure. In recent years the nucleon electromagnetic form factors (FFs) have been charted very precisely, thanks to the new generation of experiments that make use of the target- and recoil-polarization techniques; see [1–3] for recent reviews. These precise data allow, e.g., to map out the spatial densities in the nucleon, address the role of the meson cloud, and study the transition to the asymptotic regime.

The $N \rightarrow \Delta$ FFs, describing the electromagnetic transition of the nucleon to its first excited state, contain complementary information, such as the sensitivity on the nucleon shape; see, e.g., [4,5]. It is therefore interesting to see that, in the limit of a large number of colors (N_c) [6,7], QCD provides relations between the nucleon and $N \rightarrow \Delta$ properties. For example, the relation between the isovector nucleon magnetic moment and the $N \rightarrow \Delta$ transition magnetic moment ($M1$) is well known [8]. In this paper we establish a large- N_c relation between the small electric quadrupole ($E2$) and Coulomb quadrupole ($C2$) $N \rightarrow \Delta$ amplitudes, and relate them to the neutron charge radius; see Sec. II. Furthermore, we extend these relations to finite momentum transfer (Q^2) and hence express the $N \rightarrow \Delta$ FFs in terms of the nucleon FFs; see Sec. III. We shall then use a recent empirical parametrization of the nucleon FFs to test the large- N_c relations on the ratios $E2/M1$ and $C2/M1$, for which precise experimental data are available as well. The main points of this study are summarized in Sec. IV.

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II. LARGE- N_c RELATIONS

To introduce the electromagnetic $N \rightarrow \Delta$ transition we start with an effective $\gamma N \Delta$ Lagrangian (see, e.g., [9,10]):

$$\begin{aligned} \mathcal{L}_{\gamma N \Delta} = & \frac{3ie}{2M_N(M_N + M_\Delta)} \bar{N} T^3 \\ & \times \left[g_M \partial_\mu \Delta_\nu \tilde{F}^{\mu\nu} + i g_E \gamma_5 \partial_\mu \Delta_\nu F^{\mu\nu} \right. \\ & \left. - \frac{g_C}{M_\Delta} \gamma_5 \gamma^\alpha (\partial_\alpha \Delta_\nu - \partial_\nu \Delta_\alpha) \partial_\mu F^{\mu\nu} \right] + \text{H.c.}, \quad (1) \end{aligned}$$

where N denotes the nucleon (spinor) and Δ_μ the Δ -isobar (vector-spinor) fields, M_N and M_Δ are, respectively, their masses, $F^{\mu\nu}$ and $\tilde{F}^{\mu\nu}$ are the electromagnetic field strength and its dual, and T^3 is the isospin-1/2-to-3/2 transition operator. An important observation here is that the couplings g_M , g_E , and g_C appear with the same structure of spin-isospin and field operators, and hence we expect them to scale with the same power of N_c , for large N_c .

It is customary to characterize the three different types of the $\gamma N \Delta$ transition in terms of the Jones-Scadron FFs [11]: G_M^* , G_E^* , G_C^* . The contribution of the effective couplings entering Eq. (1) to these FFs can straightforwardly be computed, with the following result:

$$\begin{aligned} G_M^*(Q^2) &= g_M + (-M_\Delta \omega g_E + Q^2 g_C)/Q_+^2, \\ G_E^*(Q^2) &= (-M_\Delta \omega g_E + Q^2 g_C)/Q_+^2, \\ G_C^*(Q^2) &= -2M_\Delta(\omega g_C + M_\Delta g_E)/Q_+^2, \end{aligned} \quad (2)$$

where ω is the photon energy in the Δ rest frame: $\omega = (M_\Delta^2 - M_N^2 - Q^2)/(2M_\Delta)$ and we use the notation

$$Q_\pm = \sqrt{(M_\Delta \pm M_N)^2 + Q^2}. \quad (3)$$

At $Q^2 = 0$, we immediately find

$$G_E^*(0) = -\frac{\Delta}{2(M_N + M_\Delta)} g_E, \quad (4a)$$

$$G_C^*(0) = -\frac{2M_\Delta^2}{(M_N + M_\Delta)^2} \left[\frac{M_\Delta^2 - M_N^2}{2M_\Delta^2} g_C + g_E \right] \quad (4b)$$

where $\Delta \equiv M_\Delta - M_N$ is the Δ -nucleon mass difference. In the large- N_c limit this mass difference goes as $1/N_c$, whereas the baryon masses increase proportionally to N_c :

$$M_{N(\Delta)} = \mathcal{O}(N_c), \quad \Delta = \mathcal{O}(N_c^{-1}). \quad (5)$$

Given the fact that, for large N_c , g_E and g_C scale with the same power of N_c , the first term in Eq. (4b) is suppressed by $1/N_c^2$, and we obtain the following relation:

$$G_C^*(0) = \frac{2M_\Delta}{M_N + M_\Delta} \frac{2M_\Delta}{\Delta} G_E^*(0). \quad (6)$$

Of special interest are the multipole ratios: $R_{EM} = E2/M1$ and $R_{SM} = C2/M1$, which in terms of the Jones-Scadron FFs are given by

$$R_{EM} = -\frac{G_E^*}{G_M^*}, \quad R_{SM} = -\frac{Q_+ Q_-}{4M_\Delta^2} \frac{G_C^*}{G_M^*}. \quad (7)$$

It is easy to see that the relation (6) tells us that these ratios are equal ($R_{SM} = R_{EM}$) for large N_c and $Q^2 = 0$. Note also that using Eqs. (4a) and (7) one easily recovers the result of Ref. [12]: $R_{EM} = \mathcal{O}(1/N_c^2)$.

Let us now recall the other relevant large- N_c results. Namely, the magnetic $N \rightarrow \Delta$ transition FF is related to the isovector anomalous magnetic moment of the nucleon, $\kappa_V \approx 3.7$, as [8]

$$G_M^*(0) = \frac{1}{\sqrt{2}} \kappa_V, \quad (8)$$

whereas the $N \rightarrow \Delta$ quadrupole moment $Q_{p \rightarrow \Delta^+}$ can be related to the neutron charge radius r_n as [13]

$$Q_{p \rightarrow \Delta^+} = \frac{1}{\sqrt{2}} r_n^2. \quad (9)$$

The latter relation can directly be expressed in terms of the $E2$ Jones-Scadron FF¹:

$$G_E^*(0) = -\frac{M_\Delta^2 - M_N^2}{12\sqrt{2}} \left(\frac{M_N}{M_\Delta} \right)^{3/2} r_n^2. \quad (10)$$

Moreover, using the new relation (6), we can express the $C2$ $N \rightarrow \Delta$ FF in terms of the neutron charge radius as well:

$$G_C^*(0) = -\sqrt{2M_N M_\Delta} M_N r_n^2 / 6. \quad (11)$$

¹The exact relation between the two $E2$ quantities is

$$G_E^*(0) = -\frac{1}{12} (M_N/M_\Delta)^{3/2} (M_\Delta^2 - M_N^2) Q_{p \rightarrow \Delta^+}.$$

Therefore, the $\gamma N \rightarrow \Delta$ transition ratios in the large- N_c limit can be expressed entirely in terms of the nucleon electromagnetic properties:

$$R_{EM} = R_{SM} = \frac{1}{12} \left(\frac{M_N}{M_\Delta} \right)^{3/2} (M_\Delta^2 - M_N^2) \frac{r_n^2}{\kappa_V}. \quad (12)$$

This is one of the central results of this work.²

Empirically, the large- N_c limit value for $M1$ is off by about 15%; compare $G_M^*(0) = \kappa_V/\sqrt{2} \approx 2.62$ with the empirical value [15]: $G_M^*(0) \approx 3.02$. The value for $E2$, $G_E^*(0) \approx 0.07$, obtained from Eq. (10) using the experimental neutron charge radius [16] ($r_n^2 = -0.113 \pm 0.003$ fm²), is in better agreement with the empirical value [15]: $G_E^*(0) \approx 0.075$. The strength of $C2$, $G_C^*(0) \approx 0.7$ is also somewhat smaller than the empirical value [5,10]: $G_C^*(0) \approx 1.0$.

For the ratios we then obtain

$$R_{EM} = R_{SM} = -2.77\%. \quad (13)$$

For R_{EM} this large- N_c prediction is in excellent agreement with experiment [17]: $R_{EM} = -2.5 \pm 0.5\%$. For R_{SM} , a direct measurement at the real-photon point is of course not possible. In the following section we will examine an extension of the large- N_c prediction to finite Q^2 , which will, in particular, allow for a direct comparison with the experimental data for R_{SM} .

III. EXTENSION TO FINITE MOMENTUM TRANSFER

It would be desirable to extend the above relations to finite Q^2 . For example, the most straightforward generalization of Eq. (8) gives [18]:

$$G_M^*(Q^2) = \frac{1}{\sqrt{2}} \{F_{2p}(Q^2) - F_{2n}(Q^2)\}, \quad (14)$$

where $F_{2p} - F_{2n}$ is the (isovector) combination of the proton (p)–neutron (n) Pauli FFs.

Analogously, extending the newly derived relation, Eq. (6), to finite, but small Q^2 , we have

$$G_C^*(Q^2) = \frac{4M_\Delta^2}{M_\Delta^2 - M_N^2} G_E^*(Q^2), \quad (15)$$

or, equivalently, for the ratios, Eq. (7),

$$R_{SM}(Q^2) = \frac{Q_+ Q_-}{M_\Delta^2 - M_N^2} R_{EM}(Q^2). \quad (16)$$

Finally, we may use the fact that, for small Q^2 , the neutron electric FF is expressed as $G_{En}(Q^2) \approx -r_n^2 Q^2/6$, and hence an extension of Eq. (10) is given by

²A similar set of relations for the $\gamma N \Delta$ ratios was derived earlier by Buchmann [14] using a combination of the large- N_c expansion and the spin-flavor symmetry. Our results rely solely on the large- N_c limit.

$$G_E^*(Q^2) = \left(\frac{M_N}{M_\Delta}\right)^{3/2} \frac{M_\Delta^2 - M_N^2}{2\sqrt{2}Q^2} G_{En}(Q^2). \quad (17)$$

Bringing these results together we obtain the following expression for the $\gamma N\Delta$ ratios in terms of the nucleon form factors:

$$R_{EM} = -\left(\frac{M_N}{M_\Delta}\right)^{3/2} \frac{M_\Delta^2 - M_N^2}{2Q^2} \frac{G_{En}}{F_{2p} - F_{2n}}, \quad (18a)$$

$$R_{SM} = -\left(\frac{M_N}{M_\Delta}\right)^{3/2} \frac{Q_+ Q_-}{2Q^2} \frac{G_{En}}{F_{2p} - F_{2n}}. \quad (18b)$$

The latter relations are tested in Fig. 1, where we show the Q^2 dependence of the neutron electric FF and the resulting $N \rightarrow \Delta$ transition ratios, compared to experimental data. The curves are obtained by using the empirical parametrization of the nucleon FFs by Bradford *et al.* [19], together with the relations of Eq. (18) for R_{EM} and R_{SM} .

The arrows on the y axis indicate the large N_c prediction, Eq. (13). The numerical values at $Q^2 = 0$ are slightly different from the ones obtained through Eq. (18) (solid curves) only because the form-factor parametrization of

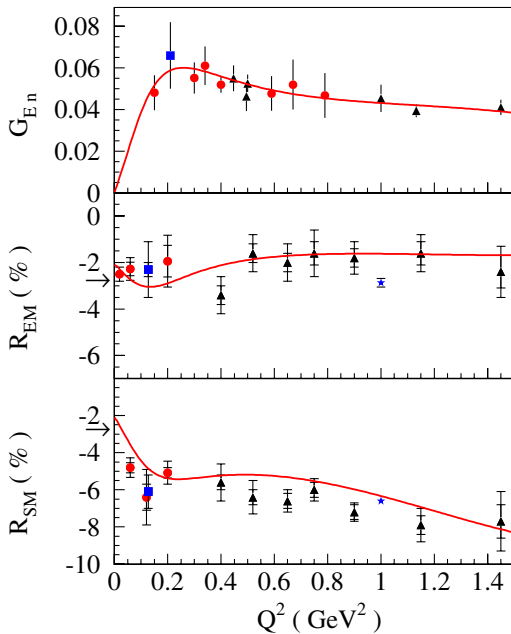


FIG. 1 (color online). Neutron electric FF, G_{En} (upper panel), in comparison with the $N \rightarrow \Delta$ R_{EM} (middle panel) and R_{SM} (lower panel) ratios. The curve for G_{En} shows the empirical parametrization of Bradford *et al.* [19]. The curves for R_{EM} and R_{SM} are obtained using the relations of Eq. (18) with the empirical nucleon FFs [19]. The data for G_{En} are from double-polarization experiments at MAMI [24–28] (red circles), NIKHEF [29] (blue squares), and JLab [30–32] (black triangles). The data for R_{EM} and R_{SM} are from BATES [33] (blue squares), MAMI [34–36] (red circles), JLab/CLAS [37] (black triangles), and JLab/HALLA [38] (blue stars).

Bradford *et al.*, which is used here, does not exactly reproduce the neutron charge radius.

We can see that the prediction of the Q^2 dependence for both ratios is in very good agreement with the experimental data, even at higher momentum transfers. At low Q^2 , it is very interesting to see that the slight bump in G_{En} around $Q^2 \simeq 0.2$ GeV² results in a shoulder structure in both of the ratios, which seems to be consistent with the experimental data. Friedrich and Walcher, in their model analysis of the nucleon FF data [20], observe such bump structures in all four nucleon electromagnetic FFs and attribute them to the effects of the “pion cloud.” The present large- N_c relations show that analogous effects must then arise in R_{EM} and R_{SM} at low Q^2 . Although the available data for R_{EM} and R_{SM} seem to support this finding, it would certainly be necessary to improve on the accuracy of the data in the region of $Q^2 \simeq 0.10$ – 0.25 GeV², to provide convincing evidence for such structures.

We emphasize again that the relations of Eq. (18) are derived assuming that the momentum transfer is small, $Q^2 \ll 1$ GeV². Nevertheless, it is intriguing to see that their phenomenological success extends into the region of intermediate Q^2 , as is explored in Fig. 1.

It is perhaps useful to point out that the excellent agreement of the large- N_c relations with experimental data for these quantities makes it interesting to study the quark-loop effects (which are suppressed in the large- N_c limit [7]) by comparing the quenched versus full QCD calculations for these quantities. Presently, quenched lattice QCD calculations for R_{EM} and R_{SM} are done at larger (than physical) pion masses [21], but indeed they compare favorably with chiral perturbation theory predictions [22]. It would be interesting to see if this agreement persists for pion masses down to the physical point.

IV. CONCLUSION

In the large- N_c limit of QCD, the properties of the N -to- Δ transition can be related to the properties of the nucleon. We have shown here how the $E2$ and $C2$ $\gamma N\Delta$ transitions are related to each other and to the neutron charge radius [Eq. (12)]. We have extended these relations to low momentum transfers, thus relating the R_{EM} and R_{SM} ratios to the ratio of the neutron electric form factor over the nucleon isovector Pauli form factor [Eq. (18)].

Using an empirical representation of the nucleon form factors, we have tested the prediction of the large- N_c relations for R_{EM} and R_{SM} ratios versus the available experimental data. The predictions for the ratios show a remarkable consistency with experiment. We note however that the predictions for the absolute strength of the transitions are less successful phenomenologically; the large- N_c relations underestimate the $M1$ and $C2$ strength by about 20%. Evidently, the relations for the ratios work much better.

It is particularly interesting to see that the large- N_c relations translate the structures in the nucleon form factors, which arise due to the long-range effects, into a dip structure in R_{EM} and a shoulder structure in R_{SM} around $Q^2 \simeq 0.15\text{--}0.25 \text{ GeV}^2$. This prediction calls for more precise measurements in that Q^2 range to confirm that such structures are indeed present. Finally, we remark that even at higher momentum transfers the large- N_c relations for the ratios are in surprisingly good agreement with experiment, while the perturbative QCD prediction [23] (i.e., $R_{EM} \rightarrow +100\%$ and $R_{SM} \rightarrow \text{const}$, as $Q^2 \rightarrow \infty$) is nowhere in sight.

The large- N_c relations between the nucleon and $N \rightarrow \Delta$ electromagnetic form factors can also be used to constrain the first moment of the $N \rightarrow \Delta$ generalized parton distri-

butions (GPDs); see [5] for more details. The relations examined here are relevant for the $E2$ and $C2$ $N \rightarrow \Delta$ GPDs, and may shed light on the quark distributions inducing the Δ -resonance excitation.

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