Wilson-'t Hooft loops in finite-temperature noncommutative dipole field theory from dual supergravity

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We first study the temporal Wilson loop in the finite-temperature noncommutative dipole field theory from the string/gauge correspondence. The associated dual supergravity background is constructed from the near-horizon geometry of near-extremal D branes, after applying T duality and smeared twist. We investigate the string configuration therein and find that while the temperature produces a maximum distance $L_{\max}$ in the interquark distance the dipole in there could produce a minimum distance $L_{\min}$. The quark boundary pair therefore could be found only if their distance is between $L_{\min}$ and $L_{\max}$. We also show that, beyond a critical temperature the quark pair becomes totally free due to screening by thermal bath. We next study the spatial Wilson loop and find the confining nature in the zero temperature 3D and 4D nonsupersymmetry dipole gauge theory. The string tension of the linear confinement potential is obtained and found to be a decreasing function of the dipole field. We also investigate the associated t'Hooft loop and determine the corresponding monopole antimonopole potential. The conventional screening of magnetic charge which indicates the confinement of the electric charge is replaced by a strong repulsive however. Finally, we show that the dual string which is rotating along the dipole deformed S^5 will behave as a static one without dipole field, which has no minimum distance and has larger energy than a static one with dipole field. We discuss the phase transition between these string solutions.

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I. INTRODUCTION

The expectation value of Wilson loop is one of the most important observations in the gauge theory. In the AdS/CFT duality [1–3] it becomes tractable to understand this highly nontrivial quantum field theory effect through a classical description of the string configuration in the AdS background. Using the AdS/CFT duality Maldacena [4] derived for the first time the expectation value of the rectangular Wilson loop operator from the Nambu-Goto action in which the string world sheet is bounded by a loop and along a geodesic on the $\mathrm{AdS}_5 \times S^5$ with end points on AdS_5. It is found that the interquark potential exhibits the Coulomb type behavior expected from conformal invariance of the gauge theory.

In order to make contact with nature many investigations had gone beyond the initial conjectured duality and generalized the method to investigate the theories breaking conformality and (partially) supersymmetry [5–8]; for example, the Klebanov-Witten solution [9], the Klebanov-Tseytlin solution [10], the Klebanov-Strassler solution [11], and Maldacena-Núñez (MN) solution [12] which dual to the $\mathcal{N} = 1$ gauge theory.

Historically, the first literature to find a confining theory was discussed by Witten [13] in which the finite-temperature system which breaks the conformal symmetry of the theory was considered. The Maldacena's computational technique was then extended to the finite-temperature case by replacing the AdS metric by a Schwarzschild-AdS metric [13–17]. Note that in considering the finite-temperature theory one direction (the Euclidean time) is compactified along a circular of radius $1/2\pi T$. Thus, at high temperature the original 4D (5D) theory corresponds essentially to the 3D (4D) theory. As both the fermions and scalars get a mass of the order of temperature this theory reduces to a pure gauge field theory. Also, the bosons have periodic and fermions antiperiodic boundary condition; in going to a finite-temperature theory we also break the supersymmetry. Thus, in the high-temperature limit the associated spatial Wilson loop describes the zero-temperature nonsupersymmetric gauge theory in 3D (4D) dimension, which then reveals the nature of quark confinement.

The Wilson loop in the noncommutative dipole field theory from the string/gauge correspondence had been investigated. First, the dual supergravity background of the noncommutative dipole theory had been found in [18–20]. Alishahiha and Yavartanoo [20] had also found the general dual supergravity of Dp branes in the presence of a nonzero B field with one leg along the brane world volume and the other transverse to it, which duals to a noncommutative dipole theory. They had investigated the associated Wilson loop and found that when the distance between quark and antiquark is much bigger then their dipole size the energy will show a Coulomb type behavior with a small correction from the noncommutativity. In the previous papers [21] we reexamined the problem and found that it exists a minimum distance between the quarks. We also find that the dual string which is rotating along the dipole deformed S^5 will behave as a static one without dipole and has higher energy than the static one with dipole field. In this paper, we will follow the method

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in [14–16] to extend our investigation to the finite-temperature noncommutative dipole field theory from the string/gauge correspondence.

In Sec. II we first construct the dual supergravity background of the finite-temperature noncommutative dipole theory by considering the near-horizon geometry of near-extremal D branes, after applying T duality and smeared as that described in [18–20]. We study the temporal Wilson loop in the string/gauge correspondence by investigating the associated string configuration. We find that while the temperature produces a maximum distance $L_{\max}$ the dipole could produce a minimum distance $L_{\min}$. The quark boundary pair therefore could be found only if their distance is between $L_{\min}$ and $L_{\max}$. Especially, we show that, beyond a critical temperature the quark pair becomes totally free due to screening by thermal bath.

In Sec. III we study the spatial Wilson loop and see that the confining nature could be shown in the zero temperature 3D and 4D nonsupersymmetry dipole gauge theory. We obtain the string tension which is found to be a decreasing function of the dipole field.

In Sec. IV we follow the method in [16] to investigate the associated t'Hooft loop in the 4D nonsupersymmetry gauge theory which shows the nature of quark confinement. The t'Hooft loop is the ''electric-magnetic'' dual of the Wilson loop and describes the monopole antimonopole potential. The string theory realized of the monopole is the D2 brane ending on the D4 brane. The D2 brane is wrapped along x_0 so from the point of view of the 4D theory it is a pointlike object. We find that the expectation value of the t'Hooft loop shows strong repulsive force between the monopole and antimonopole, in contrast to the conventional of screening of magnetic charge.

In Sec. V we study the dual string which is rotating along the dipole deformed S^5 and see that it will behave as a static one without dipole field. We find that it has no minimum distance and has larger energy than a static one with dipole field. Collecting the above analysis we discuss the phase transition between these string solutions. The last section is devoted to a summary.

Note that it is a long belief that in quantum theories including gravity, spacetime must change its nature at distances comparable to the Planck scale. Quantum gravity has an uncertainty principle which prevents one from measuring positions to better accuracies than the Planck length. Thus, the quantum effects could be modeled by a noncommutation relation and nonlocal properties. String theory is not local and it was discovered in [22,23] that simple limits of M theory and string theory lead directly to noncommutative gauge theories. As we now know, the noncommutative gauge theories can be realized in string theory as the world volume of D branes in a constant background B field. It was found by [18] that when the B field has one leg along the brane and the other transverse to it the noncommutative dipole field theory (NCDFT) appears. NCDFT's are also interesting by themselves. It has a chance of finding a CP (and even CPT) violating theory [19]. It is also an appropriate candidate to study the interaction of neutral particles with finite dipole moments, like neutrinos, with gauge particles like photons. There are some experimental evidences of such interactions, which cannot be described by the commutative version of the standard model of particles [24].

II. TEMPORAL WILSON LOOP IN FINITE-TEMPERATURE DIPOLE THEORY

A. Supergravity solution

To find the explicit supergravity solution of the D3 brane describing the finite-temperature dipole theory we could start with the following type II supergravity solution describing N coincident near-extremal D3 brane [25]

$$\begin{aligned} ds^2 &= f(r)^{-1/2}[-h(r)dt^2 + dx_1^2 + dx_2^2 + dx_3^2] \\ &\quad + f(r)^{1/2}[h(r)^{-1}dr^2 + r^2 d\Omega_5^2], \\ f(r) &= 1 + \frac{N^4}{r^4}, \qquad h(r) = 1 - \frac{r_0^4}{r^4}, \end{aligned} \tag{2.1}$$

in which dr and $d\Omega$ constitute $x_4, \ldots, x_9$ coordinates. The horizon is located at $r = r_0$ and extremality is achieved in the limit $r_0 \to 0$. A solution with $r_0 \ll N$ is called near extremal.

Now, as described in [18–20], we first apply the T-duality transformation on the x_3 axis, then smeared twist along $x_4, \ldots, x_9$, and finally apply the T duality on the x_3 axis. In the large N limit the geometry is described by[1]

$$\begin{aligned} ds_{10}^2 &= U^2\bigg[-\Big(1 - \frac{U_T^4}{U^4}\Big)dt^2 + dx^2 + dy^2 \\ &\quad + \frac{dz^2}{1 + B^2U^2\sin^2\theta_1\sin^2\theta_2}\bigg] \\ &\quad + \frac{1}{U^2}\bigg[\Big(1 - \frac{U_T^4}{U^4}\Big)^{-1} dU^2 + U^2 d\Omega_5^2 \\ &\quad - U^4B^2\sin^4\theta_1\sin^4\theta_2 \frac{(a_3 d\theta_3 + a_4 d\theta_4 + a_5 d\theta_5)^2}{1 + U^2B^2\sin^2\theta_1\sin^2\theta_2}\bigg], \end{aligned} \tag{2.2}$$

$$\begin{aligned} e^{2\Phi} &= \frac{1}{1 + U^2B^2\sin^2\theta_1\sin^2\theta_2}, \\ B_{z\theta_i}d\theta_i &= -\frac{U^2B\sin^2\theta_1\sin^2\theta_2}{1 + U^2B^2\sin^2\theta_1\sin^2\theta_2} a_i d\theta_i, \end{aligned} \tag{2.3}$$

[1] A simple way to derive the metric (2.2) is that we first rewrite the part of metric of (2.1) as $h(r)^{-1}dr^2 + r^2d\Omega_5^2 = (h(r)^{-1} - 1)dr^2 + (dr^2 + r^2d\Omega_5^2) = (h(r)^{-1} - 1)dr^2 + (dx_4^2 + \ldots + +x_9^2)$. As the smeared twist could only add some terms propositional to $d\theta_i$ it does not change the value of dr. Thus, after the twist along $x_4, \ldots, x_9$ we could combine the $(h(r)^{-1} - 1)dr^2$ with dr^2 to the term $(1 - \frac{U_T^4}{U^4})^{-1}dU^2$ shown in (2.2).

in which $a_3 \equiv \cos\theta_4$, $a_4 \equiv -\sin\theta_3\cos\theta_3\sin\theta_4$, and $a_5 \equiv \sin^2\theta_3\sin^2\theta_4$, where θ_i are the angular coordinates parametrizing the sphere S^5 transverse to the D3 brane. Thus there is a nonzero B field with one leg along the brane world volume and the others transverse to it. The value B in (2) is proportional to the dipole length ℓ defined in the "noncommutative dipole product": $\Phi_a(x) * \Phi_a(x) = \Phi_a(x - \ell_b/2)\Phi_b(x + \ell_a/2)$ for the dipole field $\Phi(x)$ [18].

B. Temporal Wilson loop

To investigate the Wilson loop on the finite-temperature noncommutative dipole field theory in the dual string description we parametrize the string configuration by

$$\tau = t, \qquad U = \sigma, \qquad z = z(\sigma), \tag{2.4}$$

the Nambu-Goto action becomes

$$S = \frac{1}{2\pi}\int d\sigma d\tau(\sqrt{-\det g} + B_{\mu\nu}\partial_\tau X^\mu \partial_\sigma X^\nu) = \frac{T_0}{2\pi}\int d\sigma\sqrt{1 + \frac{(U^4 - U_T^4)(\partial_\sigma z)^2}{1 + B^2U^2}}, \tag{2.5}$$

in which T_0 denotes the time interval we are considering and we have set $\alpha' = 1$. In the above calculation we have let $\theta_1 = \theta_2 = \pi/2$. Note that the second term in the Nambu-Goto action (2.5) does not contribute in the static case while it will play an important role in the rotating case which is investigated in Sec. V.

As the associated Lagrangian ($\mathcal{L}$) does not explicitly depend on z the function $\frac{\partial\mathcal{L}}{\partial(\partial_\sigma z)}$ will be proportional to an integration constant, i.e.

$$\frac{\partial\mathcal{L}}{\partial(\partial_\sigma z)} = \frac{\frac{(U^4 - U_T^4)(\partial_\sigma z)}{1+B^2U^2}}{\sqrt{1 + \frac{(U^4-U_T^4)(\partial_\sigma z)^2}{1+B^2U^2}}} = \frac{\sqrt{U_0^4 - U_T^4}}{\sqrt{1 + B^2U_0^2}}, \tag{2.6}$$

as at $U = U_0$ we have the property of $(\partial_\sigma z) \to \infty$. From the above relation we can find the function $(\partial_\sigma z)^2$

$$(\partial_\sigma z)^2 = \frac{\frac{1+B^2U^2}{U^4-U_T^4}}{\frac{U^4-U_T^4}{U_0^4-U_T^4}\frac{1+B^2U_0^2}{1+B^2U^2} - 1}. \tag{2.7}$$

Now, we put a quark at place $z = \sigma = -L/2$ and an antiquark at $z = \sigma = L/2$, thus

$$L = 2\int_0^{L/2} dz = 2\int_{U_0}^\infty dU(\partial_\sigma z) = \frac{2}{U_0}\int_1^\infty dy\frac{\sqrt{\frac{1+y^2B^2U_0^2}{y^4-(U_T^4/U_0^4)}}}{\sqrt{\frac{y^4-(U_T^4/U_0^4)}{1-(U_T^4/U_0^4)}\frac{1+B^2U_0^2}{1+B^2U_0^2y^2} - 1}}. \tag{2.8}$$

The above relation implies that

$$L \approx \begin{cases} 0 & \text{as } U_0 \to U_T, \\ B\pi - \frac{\pi}{4BU_0^2} & \text{as } U_0 \to \infty. \end{cases} \tag{2.9}$$

Thus the interquark distant L will asymptotically approach to a constant $L_0 \equiv B\pi$ as $U_0 \to \infty$. This indicates that it exists a minimum distance between the quark and antiquark, in contrast to that without dipole in which the interquark distant L could approach zero [14,15].

Note that on the near-extremal D-brane background a string shall end at the horizon, $U_0 = U_T$, and not at $U_0 = 0$ [26]. Thus the minimum value of U_0 adopted in (2.9) is at $U_0 = U_T$.

We can evaluate the interquark potential H form the Nambu-Goto action (2.5) with a help of (2.7). The formula is

$$H = \frac{1}{\pi}\left[U_0\int_1^\infty dy\left(\sqrt{\frac{(y^4 - (U_T^4/U_0^4))(1 + B^2U_0^2)}{(y^4 - (U_T^4/U_0^4))(1 + B^2U_0^2) - (1 - (U_T^4/U_0^4))(1 + B^2U_0^2y^2)}} - 1\right) - U_0 + U_T\right]. \tag{2.10}$$

Here we have subtracted the infinity coming from the mass of the W boson which corresponds to the string stretching from $U = U_T$ to $U = \infty$ [14–16]. The above relation implies that

$$H \approx \begin{cases} 0 & \text{as } U_0 \to U_T, \\ \frac{U_T}{\pi} - \frac{1}{4\pi BU_0} & \text{as } U_0 \to \infty. \end{cases} \tag{2.11}$$

Thus the interquark potential H will asymptotically approach a constant $\frac{U_T}{\pi}$ as $U_0 \to \infty$, in contrast to that without dipole in which H will asymptotically approach $-\infty$. The appearances of terms $\frac{\pi}{4BU_0^2}$ in (2.9) and $\frac{1}{4\pi BU_0}$ in (2.11) reveal the fact of nonperturbative behavior at $B \to 0$. Thus, there shall be a qualitative difference between the theories with and without dipole.

For a clear illustration we first show in Fig. 1 the functions $L(U_0)$, $H(U_0)$, and $H(L)$ at $(B, U_T) = (0, 1)$,

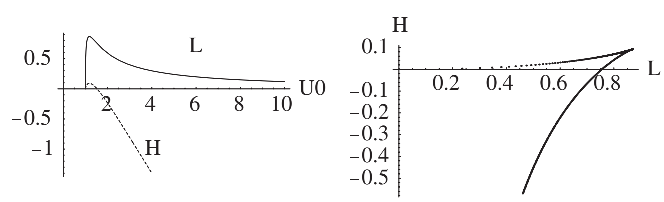

FIG. 1. The functions $L(U_0)$, $H(U_0)$, and $H(L)$ at $(B, U_T) = (0, 1)$. There exists a maximum distance and beyond which the quark pair will become free due to screening by thermal bath. The local extremity on $L(U_0)$ and $H(U_0)$ corresponds to the spike on diagram $H(L)$.

which are obtained by performing the numerical evaluation of (2.8) and (2.10).

Let us make the following comments about Fig. 1.

(i) Figure 1 is just the case of $B = 0$ which was studied in [14,15] while plotted in a coordinate different from them.

(ii) From Fig. 1 we see that increasing the turn point U_0 from U_T the interquark distance L will be increasing and in this case the interquark potential is also a positive increasing function. However, after the distance reaches its maximum value it will turn to a decreasing function of U_0. In this situation the positive interquark potential is a decreasing function. Finally, the interquark potential becomes negative. Thus, the dual string with two different point values of U_0 may correspond to the same interquark distance L which, however, has different interquark potential. The dual string configuration which has a small potential is more stable and dual to the physical quark system.

(iii) Figure 1 shows that there exists a maximum distance $L_{\max} \approx 0.8$ and we encounter two regions with different behavior. For $L < 0.7$ we observe a Coulomb like behavior. When $0.7 < L < 0.8$ the dual string configuration has a positive energy which, however, does not correspond to the lowest energy configuration. In this case it is energetically favorable for the system to be in a configuration of two parallel strings ending on the horizon, which corresponds to zero energy (after the subtraction). Thus, the dual quarks have zero energy and become free due to screening by thermal bath. The property was found in [14–16].

(iv) It is interesting to see that the dual string does not exist after $L > L_{\max}$. Of course, two parallel strings ending on the horizon with displacement $L > L_{\max}$ could be formed, which, as we know, has zero energy (after the subtraction). Thus the quarks will become totally free if the interquark distance is too large. In this case the physical picture is, as mentioned above, the quarks become free due to screening by thermal bath.

In Fig. 2 we show the functions $L(U_0)$, $H(U_0)$, and $H(L)$ at $(B, U_T) = (0.2, 1)$.

Figure 2 shows that, like that in the theory without dipole, there exists a maximum distance $L_{\max}$ and beyond which the quark boundary pair will become free due to screening by thermal bath, as that in the nondipole field system. However, in the presence of dipole field there will exist a minimum distance $L_{\min}$, which is an increasing function of dipole field B. Thus, the quark boundary pair could be formed only if their distance is between $L_{\min}$ and $L_{\max}$.

Note that in the dipole theory there will exist a minimum distance $L_{\min}$. However, as the maximum distance $L_{\max}$ is a decreasing function of temperature the value of $L_{\min}$ may be coincident with $L_{\max}$ at sufficiently high temperature. This means that in the dipole theory the quark boundary pair will become totally free due to screening by thermal bath at a sufficiently high temperature. In this situation we could not find any quark boundary state, in contrast to the nondipole theory in which the quark pair could be formed in the Coulomb phase at short distance, as shown in Fig. 1.

III. SPATIAL WILSON LOOP IN FINITE-TEMPERATURE DIPOLE THEORY

The nonsupersymmetric dipole theories at zero temperature could be investigated by considering the spatial Wilson loop in the background of Euclidean near-extremal Dp-brane solutions [13,16]. This is because when the spatial size is much larger than $1/T$ (T is the Hawking temperature of the near-extremal solution and Euclidean time is compactified along a circular of radius $1/2\pi T$), the effective low energy theory reduces effectively to a p-dimensional nonsupersymmetric theory. Therefore, the spatial Wilson loop gives us the energy between a quark and an antiquark of the p-dimensional nonsupersymmetric theory at zero temperature.

We first use metric (2.2) to study 3D nonsupersymmetric theory in III A and then construct 4d metric to study 4D nonsupersymmetric theory in III B.

A. 3D nonsupersymmetric theory

To investigate the spatial Wilson loop we parametrize the string configuration by

$$\tau = x \quad (\text{or } y), \qquad U = \sigma, \qquad z = z(\sigma), \tag{3.1}$$

the Nambu-Goto action calculated from the metric (2.2) becomes

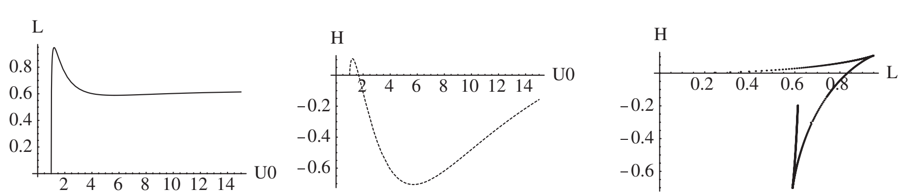

FIG. 2. The functions $L(U_0)$, $H(U_0)$, and $H(L)$ at $(B, U_T) = (0.2, 1)$. There exists a maximum and a minimum distance and the quark boundary pair could be found only if their distance is between the values. The local extremities on $L(U_0)$ and $H(U_0)$ correspond to the spikes on diagram $H(L)$.

$$S = \frac{T_0}{2\pi}\int d\sigma\sqrt{\frac{1}{1-(U_T^4/U_0^4)} + \frac{U^4(\partial_\sigma z)^2}{1+B^2U^2}}. \tag{3.2}$$

As the associated Lagrangian ($\mathcal{L}$) does not explicitly depend on z the function $\frac{\partial\mathcal{L}}{\partial(\partial_\sigma z)}$ will be proportional to an integration constant, i.e.

$$\frac{\partial\mathcal{L}}{\partial(\partial_\sigma z)} = \frac{\frac{U^4(\partial_\sigma z)}{1+B^2U^2}}{\sqrt{\frac{1}{1-(U_T^4/U_0^4)} + \frac{U^4(\partial_\sigma z)^2}{1+B^2U^2}}} = \frac{U_0^2}{\sqrt{1+B^2U_0^2}}, \tag{3.3}$$

as at $U = U_0$ we have the property of $(\partial_\sigma z) \to \infty$. From the above relation we can find the function $(\partial_\sigma z)^2$

$$(\partial_\sigma z)^2 = \frac{\frac{1+B^2U^2}{U^4-U_T^4}}{\frac{U^4}{U_0^4}\frac{1+B^2U^2}{1+B^2U_0^2} - 1}. \tag{3.4}$$

Put a quark at place $z = \sigma = -L/2$ and an antiquark at $z = \sigma = L/2$, then

$$L = 2\int_{U_0}^{\infty} dU(\partial_\sigma z) = \frac{2}{U_0\sqrt{1+B^2U_0^2}} \times \int_1^\infty dy \frac{\sqrt{\frac{y^4}{y^4-(U_T^4/U_0^4)}}}{\sqrt{(\frac{y^4}{1+B^2U_0^2y^2})^2 - \frac{y^4}{1+B^2U_0^2y^2}\frac{1}{1+B^2U_0^2}}}. \tag{3.5}$$

The interquark potential H calculated form the Nambu-Goto action (2.5) with the help of (3.4) becomes

$$H = \frac{1}{\pi}\left[U_0\int_1^\infty dy\left(\frac{\sqrt{\frac{y^4}{y^4-(U_T^4/U_0^4)}\frac{y^4}{1+B^2U_0^2y^2}}}{\sqrt{\frac{y^4}{1+B^2U_0^2y^2} - \frac{1}{1+B^2U_0^2}}} - 1\right) - U_0 + U_T\right]. \tag{3.6}$$

Equation (3.5) implies that

$$L \approx \begin{cases} \infty & \text{as } U_0 \to U_T, \\ B\pi - \frac{\pi}{4BU_0^2} & \text{as } U_0 \to \infty. \end{cases} \tag{3.7}$$

Thus the interquark distant L will asymptotically approach a constant $L_0 \equiv B\pi$ as $U_0 \to \infty$. This indicates that a minimum distance between the quark and antiquark exists, in contrast to that without dipole in which interquark distant L could approach zero [16].

Equation (3.6) implies that

$$H \approx \begin{cases} \infty & \text{as } U_0 \to U_T, \\ \frac{U_T}{\pi} - \frac{1}{4\pi BU_0} & \text{as } U_0 \to \infty. \end{cases} \tag{3.8}$$

Thus the interquark potential H will asymptotically approach a constant $\frac{U_T}{\pi}$ as $U_0 \to \infty$, in contrast to that without dipole in which H will asymptotically approach $-\infty$. For a clear illustration we first show in Fig. 3 the functions $L(U_0)$, $H(U_0)$, and $H(L)$ at $(B, U_T) = (0, 1)$.

Figure 3 shows the linear potential at large distance L. The quark pair could be formed at any distance.

In Fig. 4 we show the functions $L(U_0)$, $H(U_0)$, and $H(L)$ at $(B, U_T) = (0.4, 1)$.

Figure 4 shows the linear potential at large distance L. However, the quark pair could be formed only at distance $L > L_{\text{min}}$.

Note that, in the limit $U_0 \to U_T$ both L and H will approach ∞ and we have the following simple relations

$$L \to \frac{2\sqrt{1+B^2U_T^2}}{U_T}\int_1^\infty \frac{dy}{y^4-1}, \tag{3.9}$$

$$H \to \frac{U_T}{\pi}\int_1^\infty \frac{dy}{y^4-1}. \tag{3.10}$$

Thus we find a linear confined potential at large distance

$$H = \frac{U_T^2}{2\pi\sqrt{1+B^2U_T^2}}L = \frac{\pi T^2}{2\sqrt{1+B^2\pi^2T^2}}L, \tag{3.11}$$

in which we have used the relation $U_T = \pi T$. The tension of the QCD string is

$$\sigma = \frac{\pi T^2}{2\sqrt{1+B^2\pi^2T^2}}, \tag{3.12}$$

which is a decreasing function of dipole field B. This means that although in the limit $LT \gg 1$ the nature of

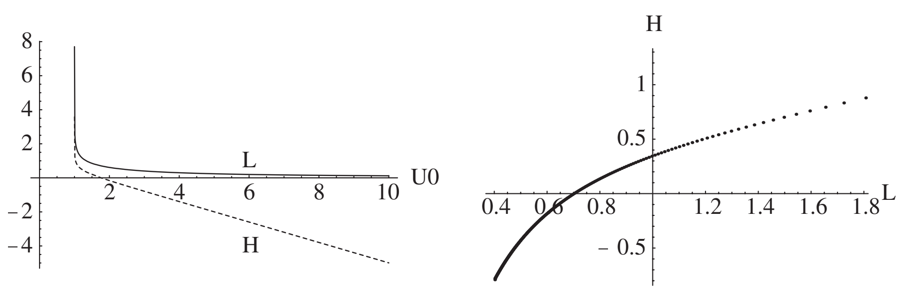

FIG. 3. The functions $L(U_0)$, $H(U_0)$, and $H(L)$ at $(B, U_T) = (0, 1)$. The linear potential is shown at large distance L.

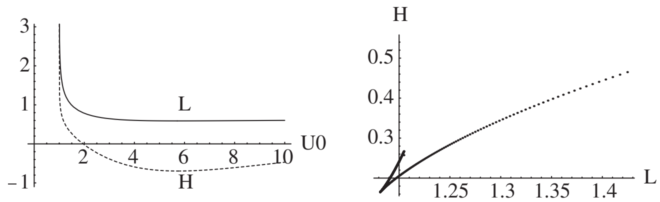

FIG. 4. The functions $L(U_0)$, $H(U_0)$, and $H(L)$ at $(B, U_T) = (0.4, 1)$. While the linear potential is shown at large distance L there exists a minimum distance between quarks. The local extremity on $L(U_0)$ and $H(U_0)$ corresponds to the spike on diagram $H(L)$.

confinement could be shown in the theory with and without dipole, the dipole field will decrease the string tensor.

B. 4D nonsupersymmetric theory

To consider the nonsupersymmetric 4D dipole theory at zero temperature we shall consider the supergravity background which is constructed form near-extremal D4-brane solutions, instead of a D3 brane. We could follow the prescription of II A to find the proper background which is described by

$$\begin{aligned} ds_{10}^2 = U^{3/2}\Big[& -\Big(1-\frac{U_T^3}{U^3}\Big)dt^2 + dw^2 + dx^2 + dy^2 \\ & + \frac{dz^2}{1+B^2U^2\sin^2\theta_1}\Big] + \frac{1}{U^{3/2}}\Big[\Big(1-\frac{U_T^3}{U^3}\Big)^{-1}dU^2 \\ & + U^2 d\Omega_4^2 \\ & - U^4B^2\sin^4\theta_1 \frac{(a_2 d\theta_2 + a_3 d\theta_3 + a_4 d\theta_4)^2}{1+U^2B^2\sin^2\theta_1}\Big], \end{aligned} \tag{3.13}$$

$$e^{2\Phi} = \frac{U^{3/2}}{1+U^2B^2\sin^2\theta_1}, \qquad B_{z\theta_i} = -\frac{a_i U^2 B\sin^4\theta_1}{1+U^2B^2\sin^2\theta_1}, \tag{3.14}$$

in which $a_2 \equiv \cos\theta_3$, $a_3 \equiv -\sin\theta_2\cos\theta_2\sin\theta_3$, and $a_4 \equiv \sin^2\theta_2\sin^2\theta_3$, where θ_i are the angular coordinates parametrizing the sphere S^4 transverse to the D4 brane.

To investigate the spatial Wilson loop we parametrize the string configuration by

$$\tau = w \quad (\text{or } x, y), \qquad U = \sigma, \qquad z = z(\sigma), \tag{3.15}$$

the Nambu-Goto action calculated from the metric (3.13) becomes

$$S = \frac{T_0}{2\pi}\int d\sigma\sqrt{\frac{1}{1-(U_T^3/U_0^3)} + \frac{U^3(\partial_\sigma z)^2}{1+B^2U^2}}, \tag{3.16}$$

and the function $(\partial_\sigma z)^2$ calculated as before becomes

$$(\partial_\sigma z)^2 = \frac{\frac{1+B^2U^2}{U^3-U_T^3}}{\frac{U^3}{U_0^3}\frac{1+B^2U^2}{1+B^2U_0^2} - 1}. \tag{3.17}$$

The interquark distance is

$$\begin{aligned} L &= 2\int_{U_0}^{\infty} dU(\partial_\sigma z) \\ &= \frac{2}{U_0\sqrt{1+B^2U_0^2}} \\ &\quad \times \int_1^\infty dy \frac{\sqrt{\frac{y^3}{y^3-(U_T^3/U_0^3)}}}{\sqrt{(\frac{y^3}{1+B^2U_0^2y^2})^2 - \frac{y^3}{1+B^2U_0^2y^2}\frac{1}{1+B^2U_0^2}}}. \end{aligned} \tag{3.18}$$

The interquark potential H calculated form the Nambu-Goto action (2.5) with the help of (3.17) becomes

$$\begin{aligned} H = \frac{1}{\pi}\Bigg[& U_0\int_1^\infty dy\left(\frac{\sqrt{\frac{y^3}{y^3-(U_T^3/U_0^3)}\frac{y^3}{1+B^2U_0^2y^2}}}{\sqrt{\frac{y^3}{1+B^2U_0^2y^2} - \frac{1}{1+B^2U_0^2}}} - 1\right) \\ & - U_0 + U_T\Bigg]. \end{aligned} \tag{3.19}$$

Using (3.18) and (3.19) we could plot the diagrams of $L(U_0)$, $H(U_0)$, and $H(L)$ which are qualitatively like Fig. 4. Thus it will show the linear potential at large distance L and the quark pair could be formed only at distance $L > L_{\min}$.

As that in 3D theory, in the limit $U_0 \to U_T$ both L and H will approach ∞ and we have the following simple relations

$$L \to \frac{2\sqrt{1+B^2U_T^2}}{\sqrt{U_T}}\int_1^\infty \frac{dy}{y^3-1}, \tag{3.20}$$

$$H \to \frac{U_T}{\pi}\int_1^\infty \frac{dy}{y^3-1}. \tag{3.21}$$

Thus we find a linear confined potential at large distance

$$H = \frac{U_T^{3/2}}{2\pi\sqrt{1+B^2U_T^2}}L = \frac{(4\pi T/3)^3}{2\pi\sqrt{1+B^2(4\pi T/3)^4}}L, \tag{3.22}$$

in which we have used the relation $U_T = (4\pi T/3)^2$ (which is obtained from the relation $T \equiv \frac{1}{4\pi}\frac{dg_{tt}}{dU}|_{U=U_T}$). The tension of the QCD string is therefore

$$\sigma = \frac{(4\pi T/3)^3}{2\pi\sqrt{1+B^2(4\pi T/3)^4}}, \tag{3.23}$$

which is a decreasing function of dipole field B.

In conclusion, we have studied the spatial Wilson loop in the high-temperature limit of the D3-brane and D4-brane background and find the confining nature in the zero-temperature 3D and 4D nonsupersymmetry dipole gauge theory. The string tension of the linear confinement potential we obtained is found to be a decreasing function of the dipole field.

IV. T'HOOFT LOOP IN 4D NONSUPERSYMMETRIC DIPOLE THEORY

In this section we follow the method in [16] to investigate the associated t'Hooft loop in the 4D nonsupersymmetry dipole theory which shows the nature of quark confinement as proved in (3.21).

The string theory realized of the monopole is the D2 brane ending on the D4 brane. For the metric (3.13) the D2 brane is wrapped along x_0 and the action becomes

$$S = \frac{1}{(2\pi)^{3/2}} \int d\sigma_1 d\sigma_2 d\tau e^{-\Phi}\sqrt{-\det g} = \frac{T_0\sqrt{1+B^2U^2}}{(2\pi)^{3/2}} \int d\sigma \sqrt{1 + (U^3 - U_T^3)\frac{(\partial_\sigma z)^2}{1+B^2U^2}}, \tag{4.1}$$

and the function $(\partial_\sigma z)^2$ calculated as before becomes

$$(\partial_\sigma z)^2 = \frac{\frac{1+B^2U^2}{U^3-U_T^3}}{\frac{U^3-U_T^3}{U_0^3-U_T^3} - 1}. \tag{4.2}$$

The interquark distance is

$$L = 2\int_{U_0}^{\infty} dU(\partial_\sigma z) = \frac{2\sqrt{1-(U_T^3/U_0^3)}}{\sqrt{U_0}} \int_1^\infty dy \frac{\sqrt{\frac{1+B^2U_0^2y^2}{y^3-(U_T^3/U_0^3)}}}{\sqrt{y^3-1}}. \tag{4.3}$$

The interquark potential H calculated form the action (4.1) with the help of (4.2) becomes

$$H = \frac{2}{(2\pi)^{3/2}}\left[U_0\int_1^\infty dy\sqrt{1+B^2U^2} \times\left(\sqrt{\frac{y^3-(U_T^3/U_0^3)}{y^3-1}} - 1\right) + \int_{U_0}^{U_T} dU\sqrt{1+B^2U^2}\right]. \tag{4.4}$$

Let us first consider the case of $B = 0$. In this case Eq. (4.3) implies that

$$L \approx \begin{cases} 0 & \text{as } U_0 \to U_T, \\ 0 & \text{as } U_0 \to \infty. \end{cases} \tag{4.5}$$

Thus there will exist a maximum distance between the monopole antimonopole pair configuration, as shown in Fig. 5. As the monopole antimonopole pair configuration

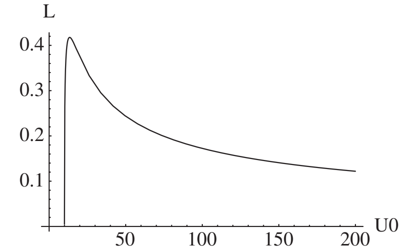

FIG. 5. The functions $L(U_0)$ for the case of $(B, U_T) = (0, 10)$. Note that there exists a minimum distance between the monopole antimonopole pair configuration.

could only exist if their distance is less than the minimum distance in the case of nondipole theory, in the region $LU_T \gg 1$ the system would become the free monopole antimonopole pair configuration. This is the trivial configuration of two parallel D2 branes ending on the horizon and wrapping along x_0. The screening of magnetic charge thus indicates the confinement of the electric charge and the quark confinement. The property was first shown in [16]. Note that as our arguments are based on the existence of a maximum distance, in contrast to the positive energy result used in [16], it is more easy to see the screening property.

In the case of dipole theory Eqs. (4.3) and (4.4) imply that

$$\begin{aligned} L &\to \frac{B\sqrt{\pi U_0}\Gamma[1/6]}{\Gamma[5/3]} && \text{as } U_0 \to \infty, \\ H &\to -\frac{BU_0^2}{(2\pi)^{3/2}} && \text{as } U_0 \to \infty. \end{aligned} \tag{4.6}$$

Thus there is a repulsive force between the monopole and antimonopole. In Fig. 6 we plot a diagram of $L(U_0)$, $H(U_0)$, and $H(L)$ for the case of $(B, U_T) = (0.2, 10)$.

We see that, in the case of nonsupersymmetric dipole theory the monopole and antimonopole show strong repulsive force at $LU_T \gg 1$.

Let us make the following comments to conclude this section.

(i) The Wilson loop studied in III B shows that, for the 4D nonsupersymmetric nondipole theory, the quarks are in confinement phase which is extended to all distance (shown in Fig. 3). The t'Hooft loop studied in [16] and this section shows that, for the 4D nonsupersymmetric nondipole theory, the monopoles are in free phase which have a finite maximum distance (shown in Fig. 5).

(ii) The Wilson loop studied in III B shows that, for the 4D nonsupersymmetric dipole theory, the quarks are in confinement phase which could be found only if their distance is larger than a critical value (shown in Fig. 4). The t'Hooft Loop studied in this section shows that, for the 4D nonsupersymmetric dipole theory, the monopoles are in strong repulsive force phase which is extended to all distance (shown in Fig. 6).

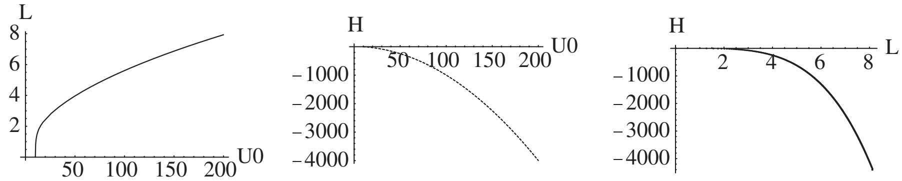

FIG. 6. The functions $L(U_0)$, $H(U_0)$, and $H(L)$ at $(B, U_T) = (0.2, 10)$. The monopole and antimonopole shows strong repulsive force at $LU_T \gg 1$.

(iii) It is important to mention that, as discussed in [16], we can trust the supergravity description only if $TL < N^{2/3}$ (N is the number of D4 brane which is used to construct the metric of (3.13)). Thus, although the behavior of the monopole pair shows a repulsive force for long distance it is useful only at finite L. This means that it does not have to show a divergent in the energy. The property at large distance, however, could not be understood form the dual supergravity. The problem needs further investigations.

V. ROTATING STRING CONFIGURATIONS

As the NS-NS B field appears in the background we shall consider the effect of it on the dual string. In this situation we shall turn to investigate the dual string configuration which is moving with an angular velocity ω along the angular θ_3.

A. Temporal Wilson loop

Let us first study the rotating string which corresponds to the static one investigated in II B. We can now parametrize the string configuration by

$$t = \tau, \qquad U = \sigma, \qquad z = z(\sigma), \qquad \theta_3 = \omega\tau. \quad (5.1)$$

The action becomes

$$S = \frac{T_0}{2\pi}\int d\sigma \times \sqrt{\left(1 - \frac{\omega^2}{U^2(1-\frac{U^4}{U_T^4})(1+B^2U^2)}\right)\left(1 + \frac{(U^4-U_T^4)(\partial_\sigma z)^2}{1+B^2U^2}\right)} + \frac{BU^2\omega\partial_\sigma z}{1+B^2U^2}. \quad (5.2)$$

As the associated Lagrangian ($\mathcal{L}$) does not explicitly depend on z the function $\frac{\partial\mathcal{L}}{\partial(\partial_\sigma z)}$ will be proportional to an integration constant, i.e.

$$\frac{\partial\mathcal{L}}{\partial(\partial_\sigma z)} = \sqrt{1 - \frac{\omega^2}{U^2(1-\frac{U^4}{U_T^4})(1+B^2U^2)}}\frac{\frac{(U^4-U_T^4)(\partial_\sigma z)}{1+B^2U^2}}{\sqrt{1+\frac{(U^4-U_T^4)(\partial_\sigma z)^2}{1+B^2U^2}}} + \frac{BU^2\omega}{1+B^2U^2} = \frac{\omega}{B}, \quad (5.3)$$

as at $U \to \infty$ we have the property of $(\partial_\sigma z) = 0$ and $U \cdot \partial_\sigma z = 0$ to ensure that the end points of the string on the boundary have a finite distance. From the above relation we can find the function $(\partial_\sigma z)^2$

$$(\partial_\sigma z)^2 = \frac{\omega^2(1+B^2U^2)}{B^2(U^4-U_T^4)} \times \frac{1}{(1+B^2U^2)(U^4-U_T^4) - \omega^2U^2 - \frac{\omega^2}{B^2}}. \quad (5.4)$$

Using the property that $(\partial_\sigma z) \to \infty$ at $U = U_0$ we find that

$$\omega = \pm B\sqrt{U_0^4 - U_T^4}. \quad (5.5)$$

Substituting this relation into (5.4) we have a simple relation

$$(\partial_\sigma z)^2 = \frac{U_0^4 - U_T^4}{U^4 - U_T^4}\frac{1}{U^4 - U_0^4}. \quad (5.6)$$

Using (5.5) and (5.6) the action (5.2) could be calculated and the corresponding Hamiltonian is

$$H = \sqrt{\frac{U^4 - U_T^4}{U^4 - U_0^4}}. \quad (5.7)$$

As (5.6) is just (2.7), (5.7) is just (2.10) in the case without a dipole field. Thus the rotating string configuration will correspond to the static case without a dipole field.

In conclusion, comparing the quark pair energy function $H(L)$ in Fig. 1 to that in Fig. 2 we thus see that, while beyond $L_{\max}$ the quark pair is free it will become the boundary state of the dipole system as $L < L_{\max}$, and below the $L_{\min}$ it will transit to the rotating configuration. As the energy is discontinuous at $L_{\min}$ the transition from the static to the rotating configuration is the first order phase transition. For clarity we present the phase structure of 4D finite-temperature noncommutative dipole theory in Table I.

TABLE I. The phase structure of the 4D finite-temperature noncommutative dipole theory.

4D finite-temperature noncommutative dipole theory
$LU_T \gg 1$: free phase
$LU_T \approx 1$: static Coulomb phase
$LU_T \ll 1$: rotating Coulomb phase

B. Spatial Wilson loop

Let us next study the rotating string which corresponds to the static one investigated in III A. We can now parametrize the string configuration by

$$\tau = x \quad (\text{or } y), \qquad U = \sigma, \qquad z = z(\sigma), \qquad \theta_3 = \omega\tau, \tag{5.8}$$

the action calculated form the ansatz becomes

$$S = \frac{T_0}{2\pi}\int d\sigma \sqrt{\frac{1}{(1-\frac{U^4}{U_T^4})}\left(1-\frac{\omega^2}{U^2(1+B^2U^2)}\right)\left(1+\frac{(U^4-U_T^4)(\partial_\sigma z)^2}{1+B^2U^2}\right)} + \frac{BU^2\omega\partial_\sigma z}{1+B^2U^2}. \tag{5.9}$$

As before, we can find the function $(\partial_\sigma z)^2$

$$(\partial_\sigma z)^2 = \frac{\omega^2(1+B^2U^2)}{B^2(U^4-U_T^4)}\frac{1}{U^4(1+B^2U^2)-\omega^2U^2-\frac{\omega^2}{B^2}}. \tag{5.10}$$

Using the property that $(\partial_\sigma z)\to\infty$ at $U=U_0$ we find that

$$\omega = \pm BU_0^2. \tag{5.11}$$

It is interesting to see that the above angular velocity ω in the zero-temperature system does not depend on temperature, in contrast to that in the finite-temperature system shown in (5.5), which depends on the temperature U_T.

Substituting this relation into (5.10) we have a simple relation

$$(\partial_\sigma z)^2 = \frac{U_0^4}{U^4-U_T^4}\frac{1}{U^4-U_0^4}. \tag{5.12}$$

Using (5.11) and (5.12) the action (5.9) could be calculated and the corresponding Hamiltonian is

$$H = \frac{U^4}{\sqrt{(U^4-U_0^4)(U^4-U_T^4)}}. \tag{5.13}$$

As (5.12) is just (3.4), (5.13) is just (3.6) in the case without a dipole field. Thus the rotating string configuration will correspond to the static case without a dipole field.

Now, from Eq. (3.11) we see that the quark pair of the dipole system has lower energy than that without the dipole (which corresponds to the rotating configuration), thus at long distance the quarks will be in the confinement phase with dipole field. However, below the $L_{\min}$ it will transit to the rotating configuration. As the energy is discontinued at $L_{\min}$ the transition from the static to the rotating configuration is the first order phase transition. For clarity we present the phase structure of the zero-temperature 3D nonsupersymmetric noncommutative dipole theory in Table II. (Note that the zero-temperature 4D nonsupersymmetric noncommutative dipole theory has a similar phase structure after the similar analysis.)

TABLE II. The phase structure of the zero-temperature nonsupersymmetric noncommutative dipole theory.

Zero-temperature nonsupersymmetric noncommutative dipole theory
$LU_T \gg 1$: static confinement phase
$LU_T \approx 1$: static phase
$LU_T \ll 1$: rotating phase

Finally, let us make the following comments about the above result.

(i) It is surprising that the moving string has the same result as that without dipole field. The reason behind it may be argued as follows. The dual string in a background with $B_{z\theta_3}$ field is somewhat analogous to the situation when a charged particle enters a region with a magnetic field. Thus, the string will be rotating along θ_3 with a constant angular momentum ω which is proportional to the strength of the NS-NS B field, as shown in (5.5) and (5.11). The rotating configuration therefore will have the extra binding energy (it is negative) from the B field which just can be canceled by the kinetic energy (it is positive) from the moving. Thus the rotating dual string does not depend on the value of the dipole field and we have the same result as that without a dipole field.

(ii) As the angular velocity of the rotating string configuration is along θ_3 it does not show the angular momentum in the real world spacetime (t, x, y, z). Thus, the system associated to the rotating string may have an intrinsic dynamic (something likes isospin) arising from the dipole. Our investigations have shown that the rotating dual string dynamics could dramatically change the string behavior and thus the quark system.

VI. CONCLUSION

In this paper, we have investigated the Wilson loop in the finite-temperature noncommutative dipole field theory from the string/gauge correspondence. We first construct the dual supergravity background of the finite-temperature noncommutative dipole theory by considering the near-horizon geometry of near-extremal D3 branes, after applying T duality and smeared. We study the temporal Wilson loop and find that while the temperature produces a maximum distance $L_{\max}$ between the quarks the dipole field could produce a minimum distance $L_{\min}$. The quark boundary pair therefore could be found only if their distance is between $L_{\min}$ and $L_{\max}$. We also show that, beyond a critical temperature the quark pair becomes totally free due to screening by thermal bath.

We next study the spatial Wilson loop in the corresponding D3- and D4-branes background, which is dual to the zero-temperature 3D and 4D nonsupersymmetry dipole gauge theory. We find that the interquark potential shows the nature of confinement and the string tension is a decreasing function of the dipole field. We also investigate the associated t'Hooft loop in the 4D nonsupersymmetry gauge theory to see the nature of quark confinement. As the t'Hooft loop is the ''electric-magnetic'' dual of the Wilson loop it will describe the monopole antimonopole potential. We find that the expectation value of the t'Hooft loop shows strong repulsive force between the monopole and antimonopole, in contrast to the conventional of screening of magnetic charge.

We finally study the dual string which is rotating along the dipole deformed S^5 and see that it will behave as a static one without a dipole field. We find that it has no minimum distance and has larger energy than a static one with a dipole field. We compare the energy between the static and rotating configurations and find the phase transition between them. Our results are plotted in Tables I and II which show clearly the phase structure of the finite-temperature noncommutative dipole theory and zero-temperature nonsupersymmetric noncommutative dipole theory.

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