General covariant gauge fixing for massless spin-two fields

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The most general covariant gauge fixing Lagrangian is considered for a spin-two gauge theory in the context of the Faddeev-Popov procedure. In general, five parameters characterize this gauge fixing. Certain limiting values for these parameters give rise to a spin-two propagator that is either traceless or transverse, but for no values of these parameters is this propagator simultaneously traceless and transverse. Having a traceless-transverse propagator ensures that only the physical degrees of freedom associated with the tensor field propagate, and hence it is analogous to the Landau gauge in electrodynamics. To obtain such a traceless-transverse propagator, a gauge fixing Lagrangian which is not quadratic must be employed; this sort of gauge fixing Lagrangian is not encountered in the usual Faddeev-Popov procedure. It is shown that when this nonquadratic gauge fixing Lagrangian is used, two fermionic and one bosonic ghosts arise. As a simple application we discuss the energy-momentum tensor of the gravitational field at finite temperature.

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I. INTRODUCTION

The quantum mechanical path integral provides a useful way of quantizing gauge field theories as the contributions of superfluous gauge degrees of freedom to physical process can be canceled by the contribution of "ghost" fields without breaking general covariance [1–4]. A degree of arbitrariness in this procedure occurs, as one must at the outset choose a particular "gauge fixing" Lagrangian, although physical quantities are necessarily independent of this choice.

A spin-one field A_μ, even when it is not a gauge field (i.e. it is a "Proca field"), satisfies the transversality condition

$$\partial \cdot A = 0, \tag{1}$$

so that it has only the three degrees of freedom normally associated with spin-one. In order to restrict the propagating degrees of freedom to those that are physical, it is often convenient that the propagator for a spin-one gauge field $D_{\mu\nu}(k)$ is also taken to be transverse so that

$$k^\mu D_{\mu\nu}(k) = 0. \tag{2}$$

This condition is satisfied in the so-called "Landau gauge" in which the quadratic gauge fixing Lagrangian

$$\mathcal{L}_{\rm gf} = -\frac{1}{2\alpha}(\partial \cdot A)^2 \tag{3}$$

is used and the limit $\alpha \to 0$ is taken.

A spin-two field is associated with a symmetric tensor field $h_{\mu\nu}$; in order for it to have five independent degrees of freedom it must be both traceless and transverse

$$\eta^{\mu\nu} h_{\mu\nu} = 0 \quad (\eta^{\mu\nu} \equiv {\rm diag}(+---)), \tag{4}$$

$$\partial^\mu h_{\mu\nu} = 0. \tag{5}$$

When this field $h_{\mu\nu}$ becomes a gauge field and self-coupled to its own energy-momentum tensor, it is identified with the graviton [5]. It is often convenient to use a "traceless-transverse (TT) propagator" $D^{\rm TT}_{\mu\nu,\lambda\sigma}(k)$ which satisfies

$$\eta^{\mu\nu} D^{\rm TT}_{\mu\nu,\lambda\sigma}(k) = 0, \tag{6}$$

$$k^\mu D^{\rm TT}_{\mu\nu,\lambda\sigma}(k) = 0. \tag{7}$$

In Refs. [6–8], such a gauge proves to be quite useful when dealing with the thermal properties of the gravitational field. In this paper we explain how such a propagator arises when using the path integral quantization.

We begin by examining the Faddeev-Popov procedure for quantizing gauge theories using a more transparent matrix analogue for illustrative purpose. We then apply this procedure to a spin-two gauge field, using the most general covariant quadratic gauge fixing Lagrangian possible. We show how a traceless propagator [satisfying (6)] and a transverse propagator [satisfying (7)] can occur, while it is impossible to obtain a propagator that satisfies both Eqs. (6) and (7).

Next, the Faddeev-Popov procedure is generalized to accommodate a nonquadratic gauge fixing Lagrangian. It is shown how such a Lagrangian can be used to give rise to $D^{\rm TT}_{\mu\nu,\lambda\sigma}$ satisfying Eqs. (6) and (7). Three ghost fields occur in this procedure, two fermionic and one bosonic. In the last section, we calculate the leading temperature corrections to the energy-momentum tensor and confirm that the result, which has been previously obtained, is gauge invariant.

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II. THE FADDEEV-POPOV PROCEDURE AND COVARIANT GAUGE FIXING FOR SPIN-TWO

If we consider the standard integral

$$Z = \int \mathrm{d}\vec{h} \exp(-\vec{h}^T \underset{\sim}{M} \vec{h}) = \frac{\pi^{n/2}}{\det^{1/2} \underset{\sim}{M}}, \tag{8}$$

where $\vec{h}$ is an n-dimensional vector, it is understood that all eigenvalues of the matrix $\underset{\sim}{M}$ are positive definite. If there exists a matrix $\underset{\sim}{A}^{(0)}$ such that

$$\underset{\sim}{M}\underset{\sim}{A}^{(0)}\vec{\theta} = 0 \tag{9}$$

for any given vector $\vec{\theta}$, then $\underset{\sim}{M}$ has vanishing eigenvalues and Eq. (8) is ill defined. The Faddeev-Popov [3,9] procedure for ascribing a meaning to Eq. (8) when this problem arises involves first inserting

$$1 = \int \mathrm{d}\vec{\theta} \delta(\underset{\sim}{F}(\vec{h} + \underset{\sim}{A}^{(0)}\vec{\theta}) - \vec{p}) \det(\underset{\sim}{F}\underset{\sim}{A}^{(0)}) \tag{10}$$

into Eq. (8), and then making the change of variable

$$\vec{h} \to \vec{h} - \underset{\sim}{A}^{(0)}\vec{\theta}, \tag{11}$$

leaving us with

$$Z = \int \mathrm{d}\vec{\theta} \int \mathrm{d}\vec{h} \delta(\underset{\sim}{F}\vec{h} - \vec{p}) \det(\underset{\sim}{F}\underset{\sim}{A}^{(0)}) \exp(-\vec{h}^T \underset{\sim}{M} \vec{h}), \tag{12}$$

where we have used Eq. (9). If now a factor of

$$1 = \pi^{-n/2} \int \mathrm{d}\vec{p} e^{-\vec{p}^T \underset{\sim}{N} \vec{p}} \det^{1/2}(\underset{\sim}{N}) \tag{13}$$

were inserted into Eq. (12), then we would be left with

$$Z = \pi^{-n/2} \int \mathrm{d}\vec{\theta} \int \mathrm{d}\vec{h} \det(\underset{\sim}{F}\underset{\sim}{A}^{(0)}) \det^{1/2}(\underset{\sim}{N}) \times \exp[-\vec{h}^T(\underset{\sim}{M} + \underset{\sim}{F}^T \underset{\sim}{N}\,\underset{\sim}{F})\vec{h}]. \tag{14}$$

Exponentiating the determinants occurring in Eq. (14) using Grassmann "ghost" fields leads to

$$Z = \pi^{-n/2} \int \mathrm{d}\vec{\theta} \int \mathrm{d}\vec{h} \int \mathrm{d}\vec{\bar{c}} \int \mathrm{d}\vec{c} \int \mathrm{d}\vec{k} \exp[-\vec{\bar{c}}\,\underset{\sim}{F}\,\underset{\sim}{A}^{(0)}\vec{c} - \vec{k}^T \underset{\sim}{N}\,\vec{k} - \vec{h}^T(\underset{\sim}{M} + \underset{\sim}{F}^T \underset{\sim}{N}\,\underset{\sim}{F})\vec{h}]. \tag{15}$$

The Faddeev-Popov ghosts are $\vec{c}$ and $\vec{\bar{c}}$; $\vec{k}$ is a Nielsen-Kallosh ghost [2,10,11]. The "infinity" occurring in Eq. (8) as a result of $\det \underset{\sim}{M}$ vanishing now is parametrized by the integral over the "gauge function" $\vec{\theta}$ which can be absorbed into a normalization factor.

For a spin-two gauge field, we take the second order term in the Einstein-Hilbert action to be the classical action so that

$$S = -\int \mathrm{d}^d x (h^{\lambda\sigma} M_{\lambda\sigma,\mu\nu} h^{\mu\nu}), \tag{16}$$

where in momentum space

$$M_{\lambda\sigma,\mu\nu} = \frac{k^2}{2}\left[\frac{1}{2}(\eta_{\mu\lambda}\eta_{\nu\sigma} + \eta_{\nu\lambda}\eta_{\mu\sigma}) - \eta_{\mu\nu}\eta_{\lambda\sigma}\right] - \frac{1}{4}[k_\mu k_\lambda \eta_{\nu\sigma} + k_\nu k_\lambda \eta_{\mu\sigma} + k_\mu k_\sigma \eta_{\nu\lambda} + k_\nu k_\sigma \eta_{\mu\lambda}] + \frac{1}{2}[k_\mu k_\nu \eta_{\lambda\sigma} + k_\lambda k_\sigma \eta_{\mu\nu}]. \tag{17}$$

This is invariant under the gauge transformation

$$\delta h_{\mu\nu} = \partial_\mu \theta_\nu + \partial_\nu \theta_\mu \equiv A^{(0)}_{\mu\nu\lambda}\theta^\lambda, \tag{18}$$

where $A^{(0)}_{\mu\nu\lambda} = \eta_{\nu\lambda}\partial_\mu + \eta_{\mu\lambda}\partial_\nu$. The most general covariant "gauge fixing" condition is

$$\underset{\sim}{F}\vec{h} = F_\alpha{}^{\lambda\sigma} h_{\lambda\sigma} = \left[\frac{1}{\alpha}k_\alpha \eta^{\lambda\sigma} + \frac{1}{\beta}(k^\lambda \delta^\sigma_\alpha + k^\sigma \delta^\lambda_\alpha) + \frac{1}{\gamma}\frac{k_\alpha k^\lambda k^\sigma}{k^2}\right] h_{\lambda\sigma}, \tag{19}$$

so that the "gauge fixing" Lagrangian is

$$\mathcal{L}_{\rm gf} = -h_{\lambda\sigma} F_\alpha{}^{\lambda\sigma} N^{\alpha\beta} F_\beta{}^{\mu\nu} h_{\mu\nu}, \tag{20}$$

where the "Nielsen-Kallosh" factor is

$$N^{\alpha\beta} = \xi \eta^{\alpha\beta} + \zeta \frac{k^\alpha k^\beta}{k^2}. \tag{21}$$

In the special case when $\gamma \to \infty$, $\xi = 1$, and $\zeta = 0$ this general class of gauges reduces to the one considered in [12] where the spin-two propagator was considered in various limits of the gauge parameters.

Upon introducing

$$T^1_{\lambda\sigma,\mu\nu} = \eta_{\mu\lambda}\eta_{\nu\sigma} + \eta_{\nu\lambda}\eta_{\mu\sigma}, \tag{22a}$$

$$T^2_{\lambda\sigma,\mu\nu} = \eta_{\mu\nu}\eta_{\lambda\sigma}, \tag{22b}$$

$$T^3_{\lambda\sigma,\mu\nu} = \frac{1}{k^2}(k_\mu k_\lambda \eta_{\nu\sigma} + k_\mu k_\sigma \eta_{\nu\lambda}) + (\mu \leftrightarrow \nu), \tag{22c}$$

$$T^4_{\lambda\sigma,\mu\nu} = \frac{1}{k^2}(k_\mu k_\nu \eta_{\lambda\sigma} + k_\lambda k_\sigma \eta_{\mu\nu}), \tag{22d}$$

$$T^5_{\lambda\sigma,\mu\nu} = \frac{1}{k^4}(k_\mu k_\nu k_\lambda k_\sigma), \tag{22e}$$

then Eq. (20) becomes

$$L_{\rm gf} = -h^{\lambda\sigma}\left\{\frac{\xi + \zeta}{\alpha^2} T^2_{\lambda\sigma,\mu\nu} + \frac{\xi}{\beta^2} T^3_{\lambda\sigma,\mu\nu} + \frac{\xi + \zeta}{\alpha}\left(\frac{2}{\beta} + \frac{1}{\gamma}\right) T^4_{\lambda\sigma,\mu\nu} + \left[\frac{\xi + \zeta}{\gamma}\left(\frac{4}{\beta} + \frac{1}{\gamma}\right) + \frac{4\zeta}{\beta^2}\right] T^5_{\lambda\sigma,\mu\nu}\right\} k^2 h^{\mu\nu}. \tag{23}$$

The propagator for the spin-two field with this gauge fixing

Lagrangian is given by $D_{\lambda\sigma,\alpha\beta}$:

$$D^{\lambda\sigma,\alpha\beta}(M_{\alpha\beta,\mu\nu} + F_{\rho,\alpha\beta}N^{\rho\delta}F_{\delta,\mu\nu}) = \tfrac{1}{2}(\delta^\lambda_\mu\delta^\sigma_\nu + \delta^\lambda_\nu\delta^\sigma_\mu) \equiv \bar{\Delta}^{\lambda\sigma}_{\mu\nu}. \tag{24}$$

Explicit calculation leads in d dimensions to

$$D_{\mu\nu,\lambda\sigma}(k) = \frac{1}{k^2}\sum_{i=1}^{5}\mathbf{C}^i T^i_{\mu\nu,\lambda\sigma}, \tag{25}$$

where

$$\mathbf{C}^1 = 1, \tag{26a}$$

$$\mathbf{C}^2 = -\frac{2}{d-2}, \tag{26b}$$

$$\mathbf{C}^3 = \left(\frac{\beta^2}{4\xi} - 1\right), \tag{26c}$$

$$\mathbf{C}^4 = \frac{2}{d-2}\left[1 + \frac{\beta\gamma}{\alpha(\beta+\gamma) + \gamma(\alpha+\beta)}\right], \tag{26d}$$

$$\mathbf{C}^5 = -\frac{\beta^2}{\xi} + \frac{1}{\xi+\zeta}\frac{(\alpha\beta\gamma)^2}{[\alpha(\beta+\gamma)+\gamma(\alpha+\beta)]^2} + \frac{2}{d-2}\frac{(d-3)\alpha(\beta+2\gamma) - 2\beta\gamma}{\alpha(\beta+\gamma)+\gamma(\alpha+\beta)}. \tag{26e}$$

For comparison, we note that the analogous propagator for a spin-one gauge field when using the gauge fixing Lagrangian $\bar{\mathcal{L}}_{gf} = -\frac{1}{2\alpha}(\partial\cdot A)^2$ is

$$\left[-k^2\eta^{\mu\nu} + \left(1-\frac{1}{\alpha}\right)k^\mu k^\nu\right]^{-1} = \left(-\frac{\eta_{\mu\nu}}{k^2} + (1-\alpha)\frac{k_\mu k_\nu}{k^4}\right) \equiv D_{\mu\nu}. \tag{27}$$

This inverse $D_{\mu\nu}$ is transverse (i.e. it satisfies $k^\mu D_{\mu\nu} = 0$) in the limit $\alpha \to 0$, even though $\bar{\mathcal{L}}_{gf}$ is ill defined in this limit. It is interesting to consider the possibility of $D_{\mu\nu,\lambda\sigma}$ being transverse. From Eq. (25) it follows that

$$k^\mu D_{\mu\nu,\lambda\sigma} = \frac{1}{k^2}\left[(k_\lambda\eta_{\nu\sigma} + k_\sigma\eta_{\nu\lambda})(\mathbf{C}^1 + \mathbf{C}^3) + k_\nu\eta_{\lambda\sigma}(\mathbf{C}^2 + \mathbf{C}^4) + \frac{k_\nu k_\sigma k_\lambda}{k^2}(2\mathbf{C}^3 + \mathbf{C}^4 + \mathbf{C}^5)\right]. \tag{28}$$

From Eqs. (26) we find that

$$\mathbf{C}^1 + \mathbf{C}^3 = \frac{\beta^2}{4\xi}, \tag{29a}$$

$$\mathbf{C}^2 + \mathbf{C}^4 = \frac{2}{d-2}\frac{\beta\gamma}{\alpha(\beta+\gamma)+\gamma(\alpha+\beta)}, \tag{29b}$$

$$2\mathbf{C}^3 + \mathbf{C}^4 + \mathbf{C}^5 = -\frac{\beta^2}{2\xi} + \frac{\beta\gamma}{[\alpha(\beta+\gamma)+\gamma(\alpha+\beta)]^2}\times\left(\frac{\alpha^2\beta\gamma}{\xi+\zeta} - \frac{2[\alpha(\beta+\gamma)+\gamma(\alpha+d\beta)]}{d-2}\right); \tag{29c}$$

these all vanish if $\beta = 0$ for all values of α, γ, ξ, and ζ. If $\beta = 0$, then

$$\eta^{\mu\nu}D_{\mu\nu,\lambda\sigma}(k)|_{\beta=0} = -\frac{2}{d-2}\frac{1}{k^2}\left(\eta^{\lambda\sigma} - \frac{k^\lambda k^\sigma}{k^2}\right) \tag{30}$$

showing that $D_{\mu\nu,\lambda\sigma}$ cannot be simultaneously traceless and transverse with $\mathcal{L}_{gf}$ given by (20), irrespective of the values of α, γ, ξ, and ζ.

In general, from Eq. (25) it follows that

$$\eta^{\mu\nu}D_{\mu\nu,\lambda\sigma}(k) = \frac{1}{k^2}\left[(2\mathbf{C}^1 + d\mathbf{C}^2 + \mathbf{C}^4)\eta_{\lambda\sigma} + (4\mathbf{C}^3 + d\mathbf{C}^4 + \mathbf{C}^5)\frac{k_\lambda k_\sigma}{k^2}\right]; \tag{31}$$

from Eqs. (26) it follows that

$$2\mathbf{C}^1 + d\mathbf{C}^2 + \mathbf{C}^4 = -\frac{1}{k^2}\frac{2}{d-2}\frac{\alpha(\beta+2\gamma)}{\alpha(\beta+\gamma)+\gamma(\alpha+\beta)}, \tag{32a}$$

$$4\mathbf{C}^3 + d\mathbf{C}^4 + \mathbf{C}^5 = \frac{1}{k^2}\frac{1}{[\alpha(\beta+\gamma)+\gamma(\alpha+\beta)]^2}\times\left[\frac{(\alpha\beta\gamma)^2}{\xi+\zeta} + \frac{2\alpha}{d-2}(\beta+2\gamma)\times(\alpha(\beta+\gamma)+\gamma(\alpha+d\beta))\right]. \tag{32b}$$

Thus if $\alpha = 0$, we find that $D_{\mu\nu,\lambda\sigma}$ satisfies the traceless condition of Eq. (6) for all values of β, γ, ξ, and ζ; if $\alpha = 0$ then

$$D_{\mu\nu,\lambda\sigma}(k)|_{\alpha=0} = \frac{1}{k^2}(P_{\mu\lambda}P_{\nu\sigma} + P_{\mu\sigma}P_{\nu\lambda}) + \frac{\beta^2}{4k^4\xi}\times\Big[k_\mu k_\lambda\eta_{\nu\sigma} + k_\mu k_\sigma\eta_{\nu\lambda} + k_\nu k_\lambda\eta_{\mu\sigma} + k_\nu k_\sigma\eta_{\mu\lambda} - \frac{4}{k^2}k_\mu k_\nu k_\lambda k_\sigma\Big] - \frac{2}{(d-2)k^2}\times\left(\eta_{\mu\nu} - 2\frac{k_\mu k_\nu}{k^2}\right)\left(\eta_{\lambda\sigma} - 2\frac{k_\lambda k_\sigma}{k^2}\right) - \frac{2}{k^6}k_\mu k_\nu k_\lambda k_\sigma, \tag{33}$$

where

$$P_{\mu\nu} \equiv \eta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}. \tag{34}$$

We note that in Eqs. (26) the limits $\alpha \to 0$ and $\beta \to 0$ do not commute and we have found that the former limit leads to a traceless propagator that is not transverse while the latter limit leads to a transverse propagator that is not traceless.

The DeDonder propagator [5,13]

$$D_{\mu\nu\lambda\sigma}(k) = \frac{1}{k^2}\left[\eta_{\mu\lambda}\eta_{\nu\sigma} + \eta_{\nu\lambda}\eta_{\mu\sigma} - \frac{2}{d-2}\eta_{\mu\nu}\eta_{\lambda\sigma}\right] \tag{35}$$

is recovered if $\xi = 1$, $\zeta = 0$, $\alpha = \beta = -4\gamma = 2$, or $\xi = 1$, $\zeta = 0$, $\alpha = -\beta = -2$, $\gamma = \infty$.

III. NONQUADRATIC GAUGE FIXING AND THE TRANSVERSE-TRACELESS GAUGE

We start by observing that with

$$\mathcal{L}_{\rm gf} = -\frac{1}{\rho}(\partial_\mu h^{\mu\nu})(\partial^\nu h_{\nu\lambda} - \partial_\lambda h^\nu_\nu) \tag{36}$$

then the sum of the classical and gauge fixing Lagrangian contains the operator

$$W_{\mu\nu,\lambda\sigma} = B_{\mu\lambda}B_{\nu\sigma} + B_{\mu\sigma}B_{\nu\lambda} - 2B_{\mu\nu}B_{\lambda\sigma}; \quad B_{\mu\nu} = \eta_{\mu\nu}k^2 - \left(1 - \frac{1}{\rho}\right)k_\mu k_\nu, \tag{37}$$

so that if $W_{\mu\nu,\alpha\beta}D^{(\rho)\alpha\beta}{}_{,\lambda\sigma} = \bar{\Delta}_{\mu\nu,\lambda\sigma}$ then

$$D^{(\rho)}_{\mu\nu,\lambda\sigma} = \frac{1}{2k^2}\left(P^\rho_{\mu\lambda}P^\rho_{\nu\sigma} + P^\rho_{\nu\lambda}P^\rho_{\mu\sigma} - \frac{2}{d-1}P^\rho_{\mu\nu}P^\rho_{\lambda\sigma}\right), \tag{38}$$

where $P^\rho_{\mu\nu} = \eta_{\mu\nu} - (1-\rho)k_\mu k_\nu/k^2$. As $\rho \to 0$, from Eq. (38) it follows that

$$k^\mu D^{(\rho=0)}_{\mu\nu\lambda\sigma} = \eta^{\mu\nu}D^{(\rho=0)}_{\mu\nu\lambda\sigma} = 0. \tag{39}$$

However, Eq. (36) is not of the form of Eq. (20) and hence the Faddeev-Popov procedure must be modified to accommodate such a nonquadratic gauge fixing Lagrangian, which is needed if a transverse-traceless propagator is to arise.

We begin by inserting two factors of "1" into Eq. (8); these are

$$1 = \int {\rm d}\vec{\theta}_1 \delta(\underset{\sim}{F}(\vec{h} + \alpha\underset{\sim}{A}\vec{\theta}_1) - \vec{p})\det(\alpha\underset{\sim}{F}\underset{\sim}{A}^{(0)}), \tag{40a}$$

$$1 = \int {\rm d}\vec{\theta}_2 \delta(\underset{\sim}{G}(\vec{h} + \alpha\underset{\sim}{A}\vec{\theta}_2) - \vec{q})\det(\alpha\underset{\sim}{G}\underset{\sim}{A}^{(0)}), \tag{40b}$$

as well as another "1" of the form

$$1 = \pi^{-n}\int {\rm d}\vec{p}{\rm d}\vec{q}{\rm e}^{-(1/\alpha)\vec{p}^T\underset{\sim}{N}\vec{q}}\det(\underset{\sim}{N}/\alpha). \tag{41}$$

This leads to

$$Z = \pi^{-n}\int {\rm d}\vec{\theta}_1{\rm d}\vec{\theta}_2 \int {\rm d}\vec{h}\det(\alpha\underset{\sim}{F}\underset{\sim}{A}^{(0)})\det(\alpha\underset{\sim}{G}\underset{\sim}{A}^{(0)})\det\left(\frac{\underset{\sim}{N}}{\alpha}\right) \times \exp\left\{-\vec{h}^T\underset{\sim}{M}\vec{h} - \frac{1}{\alpha}[\underset{\sim}{F}(\vec{h} + \alpha\underset{\sim}{A}^{(0)}\vec{\theta}_1)]^T \times \underset{\sim}{N}[\underset{\sim}{G}(\vec{h} + \alpha\underset{\sim}{A}^{(0)}\vec{\theta}_2)]\right\}. \tag{42}$$

We now make the shift $\vec{h} \to \vec{h} - \alpha\underset{\sim}{A}\vec{\theta}_1$ in Eq. (42) and let $\vec{\theta} = \vec{\theta}_2 - \vec{\theta}_1$ so that by Eq. (9)

$$Z = \left(\frac{\alpha}{\pi}\right)^n \int {\rm d}\vec{\theta}_1 \int {\rm d}\vec{\theta} \int {\rm d}\vec{h}\det(\underset{\sim}{F}\underset{\sim}{A}^{(0)})\det(\underset{\sim}{G}\underset{\sim}{A}^{(0)})\det(\underset{\sim}{N}) \times \exp\left\{-\vec{h}^T\left(\underset{\sim}{M} + \frac{1}{\alpha}\underset{\sim}{F}^T\underset{\sim}{N}\underset{\sim}{G}\right)\vec{h} - \vec{h}^T\underset{\sim}{F}^T\underset{\sim}{N}\underset{\sim}{G}\underset{\sim}{A}^{(0)}\vec{\theta}\right\}. \tag{43}$$

Dropping the infinite normalization factors in Eq. (39) and making the shift

$$\vec{h} \to \vec{h} - \frac{1}{2}\left(\underset{\sim}{M} + \frac{1}{\alpha}\underset{\sim}{F}^T\underset{\sim}{N}\underset{\sim}{G}\right)^{-1}(\underset{\sim}{F}^T\underset{\sim}{N}\underset{\sim}{G}\underset{\sim}{A}^{(0)})\vec{\theta} \tag{44}$$

to diagonalize the exponential in Eq. (43) in $\vec{h}$ and $\vec{\theta}$, we obtain

$$Z = \int {\rm d}\vec{\theta}\int {\rm d}\vec{h}\det(\underset{\sim}{F}\underset{\sim}{A}^{(0)})\det(\underset{\sim}{G}\underset{\sim}{A}^{(0)})\det(\underset{\sim}{N}) \times \exp\left\{-\vec{h}^T\left(\underset{\sim}{M} + \frac{1}{\alpha}\underset{\sim}{F}^T\underset{\sim}{N}\underset{\sim}{G}\right)\vec{h} + \frac{1}{4}\vec{\theta}^T(\underset{\sim}{A}^{(0)^T}\underset{\sim}{G}^T\underset{\sim}{N}^T\underset{\sim}{F})\left(\underset{\sim}{M} + \frac{1}{\alpha}\underset{\sim}{F}^T\underset{\sim}{N}\underset{\sim}{G}\right)^{-1} \times (\underset{\sim}{F}^T\underset{\sim}{N}\underset{\sim}{G}\underset{\sim}{A}^{(0)})\vec{\theta}\right\}. \tag{45}$$

[We are assuming that $\underset{\sim}{F}$, $\underset{\sim}{G}$, $\underset{\sim}{N}$, and $\underset{\sim}{A}^{(0)}$ are all independent of $\vec{h}$ so that no Jacobian arises as a result of the change of variable in Eq. (44).]

If now we take

$$\vec{h}^T\underset{\sim}{F}^T\underset{\sim}{N}\underset{\sim}{G}\vec{h} = h^{\mu\nu}F^T_{\mu\nu,\alpha}N^{\alpha\beta}G_{\beta,\lambda\sigma}h^{\lambda\sigma} \tag{46}$$

with

$$F^T_{\mu\nu,\alpha} = g_1\eta_{\mu\nu}\partial_\alpha + \eta_{\mu\alpha}\partial_\nu, \tag{47a}$$

$$G_{\beta,\lambda\sigma} = g_2\eta_{\lambda\sigma}\partial_\beta + \eta_{\lambda\beta}\partial_\sigma, \tag{47b}$$

$$N^{\alpha\beta} = \eta^{\alpha\beta}, \tag{47c}$$

then inverting the quadratic form $\underset{\sim}{M} + \frac{1}{\alpha}\underset{\sim}{F}^T\underset{\sim}{N}\underset{\sim}{G}$ leaves us with the coefficients in Eq. (25) being

$$\mathbf{C}^1 = 1, \tag{48a}$$

$$\mathbf{C}^2 = -2\frac{(g_2-g_1)^2 + 2(g_1+1)(g_2+1)\alpha}{(d-1)(g_2-g_1)^2 + 2(d-2)(g_1+1)(g_2+1)\alpha}, \tag{48b}$$

$$\mathbf{C}^3 = \alpha - 1, \tag{48c}$$

$$\mathbf{C}^4 = 2\frac{(g_2-g_1)^2 + [4(g_1+1)(g_2+1) - g_1 - g_2 - 2]\alpha}{(d-1)(g_2-g_1)^2 + 2(d-2)(g_1+1)(g_2+1)\alpha}, \tag{48d}$$

$$\begin{aligned}\mathbf{C}^5 = {} & [(d-1)(g_2-g_1)^2 + 2(d-2)(g_1+1)(g_2+1)\alpha]^{-1} \\ & \times \{4\alpha[(g_1+g_2)(d-4) + (2g_1g_2+1)(d-3) \\ & - (g_1^2+g_2^2)(d-1)] + 2(d-2)[(g_1-g_2)^2 \\ & - \alpha^2(4(g_1+1)(g_2+1)-1)]\}.\end{aligned} \tag{48e}$$

From these expressions we see that the limits $g_2 \to g_1$ and $\alpha \to 0$ do not commute. If we take the limit $\alpha \to 0$, with $g_2 \neq g_1$, the propagator becomes independent of g_1 and g_2, and we obtain the transverse and traceless propagator. On the other hand, if we set $g_2 = g_1$, the resulting propagator is not transverse and traceless even for $\alpha = 0$. This is another verification of the impossibility of obtaining the transverse and traceless propagator using the quadratic gauge fixing where $g_1 = g_2$. This general gauge fixing also can be used to find the DeDonder propagator of Eq. (35) by taking $g_1 = g_2 = -1/2$ and $\alpha = 1$. It is also interesting to note that for $d = 2$, and arbitrary values of g_1, g_2, and α, Eqs. (48) are well defined while the DeDonder propagator of Eq. (35) is not.

The determinants in Eq. (45) can all be exponentiated using Grassmann quantities $\vec{c}$, $\vec{\bar{c}}$, $\vec{b}$, $\vec{\bar{b}}$, $\vec{k}$, and $\vec{\bar{k}}$, so that

$$\begin{aligned} Z = {} & \int \mathrm{d}\vec{\theta} \int \mathrm{d}\vec{h} \int \mathrm{d}\vec{c}\mathrm{d}\vec{\bar{c}} \int \mathrm{d}\vec{b}\mathrm{d}\vec{\bar{b}} \int \mathrm{d}\vec{k}\mathrm{d}\vec{\bar{k}} \\ & \times \exp\Big\{-\vec{h}^T\Big(\underset{\sim}{M} + \frac{1}{\alpha}\underset{\sim}{F}^T\underset{\sim}{N}\,\underset{\sim}{G}\Big)\vec{h} - \vec{\bar{b}}(\underset{\sim}{F}\underset{\sim}{A}^{(0)})\vec{b} \\ & - \vec{\bar{c}}(\underset{\sim}{G}\underset{\sim}{A}^{(0)})\vec{c} - \vec{\bar{k}}\underset{\sim}{N}\,\vec{k} + \frac{1}{4}\vec{\theta}^T(\underset{\sim}{A}^{(0)^T}\underset{\sim}{G}^T\underset{\sim}{N}^T\underset{\sim}{F}) \\ & \times \Big(\underset{\sim}{M} + \frac{1}{\alpha}\underset{\sim}{F}^T\underset{\sim}{N}\,\underset{\sim}{G}\Big)^{-1}(\underset{\sim}{F}^T\underset{\sim}{N}\,\underset{\sim}{G}\,\underset{\sim}{A}^{(0)})\vec{\theta}\Big\} \end{aligned} \tag{49}$$

up to a normalization factor.

The gauge fixing of Eq. (36) corresponds to $g_2 = -1$, $g_1 \to 0$, and $\alpha = \rho$. In this case, the determinant $\det(\underset{\sim}{G}\underset{\sim}{A}^{(0)}) = 0$ and hence the ghost Lagrangian $\vec{\bar{b}}(\underset{\sim}{G}\underset{\sim}{A}^{(0)})\vec{b}$ itself possess a gauge invariance $\vec{b} \to \vec{b} + \underset{\sim}{B}^{(0)}\vec{\omega}$, where $\vec{\omega}$ is a Grassmann gauge function. Following the Faddeev-Popov procedure, we find that

$$\begin{aligned} \int \mathrm{d}\vec{b}\mathrm{d}\vec{\bar{b}} \exp[-\vec{\bar{b}}(\underset{\sim}{G}\underset{\sim}{A}^{(0)})\vec{b}] = {} & \int \mathrm{d}\vec{b}\mathrm{d}\vec{\bar{b}} \int \mathrm{d}\vec{\beta}\mathrm{d}\vec{\bar{\beta}} \int \mathrm{d}\vec{\mathcal{H}} \\ & \times \exp[-\vec{\bar{b}}(\underset{\sim}{G}\underset{\sim}{A}^{(0)} + \underset{\sim}{\Gamma}^T\underset{\sim}{\eta}\,\underset{\sim}{\Gamma})\vec{b} \\ & - \vec{\bar{\beta}}^T(\underset{\sim}{\Gamma}\underset{\sim}{B}^{(0)})\vec{\beta} - \vec{\mathcal{H}}^T\underset{\sim}{\eta}\,\vec{\mathcal{H}}], \end{aligned} \tag{50}$$

where $\vec{\beta}$ and $\vec{\bar{\beta}}$ are complex Faddeev-Popov ghosts and $\vec{\mathcal{H}}$ is a real Nielsen-Kallosh ghost (with neither of these being Grassmann). We will not consider this gauge fixing further in order to avoid having to introduce these "ghosts of ghosts."

The field $\vec{\theta}$ appearing in Eq. (49) is a nontrivial propagating field that has no analogue in the usual Faddeev-Popov procedure. The propagator for $\vec{\theta}$ with the gauge fixing chosen to be Eqs. (47) and the gauge transformation given by (18) is

$$\begin{aligned} D_\theta^{\mu\nu}(k) = {} & \frac{1}{\alpha}\frac{1}{k^4}\Big\{\eta^{\mu\nu} - \Big[\Big(1 - \frac{1}{4(g_1+1)(g_2+1)}\Big) \\ & - \frac{1}{8\alpha}\frac{d-1}{d-2}\Big(\frac{1}{g_1+1} + \frac{1}{g_2+1}\Big)^2\Big]\frac{k^\mu k^\nu}{k^2}\Big\}. \end{aligned} \tag{51}$$

Upon performing the functional integrals over the fields $\vec{\theta}$, $\vec{h}$, $\vec{\bar{c}}$, $\vec{c}$, and $\vec{\bar{b}}$, $\vec{b}$ (taking $\underset{\sim}{N} = 1$) in (49) we find that

$$\begin{aligned} Z = {} & \det{}^{-1/2}\Big(\underset{\sim}{M} + \frac{1}{\alpha}\underset{\sim}{F}^T\underset{\sim}{G}\Big)\det(\underset{\sim}{F}\underset{\sim}{A}^{(0)})\det(\underset{\sim}{G}\underset{\sim}{A}^{(0)}) \\ & \times \det{}^{-1/2}\Big[(\underset{\sim}{A}^{(0)^T}\underset{\sim}{G}^T\underset{\sim}{F})\Big(\underset{\sim}{M} + \frac{1}{\alpha}\underset{\sim}{F}^T\underset{\sim}{G}\Big)^{-1}(\underset{\sim}{F}^T\underset{\sim}{G}\underset{\sim}{A}^{(0)})\Big]. \end{aligned} \tag{52}$$

With Eqs. (18) and (47) these determinants become

$$\begin{aligned} Z = {} & \Big[-\frac{3(g_1-g_2)^2 + 4\alpha(g_1+1)(g_2+1)}{(4\alpha)^5}(\det\partial^2)^{10}\Big]^{-1/2} \\ & \times\Big[2(g_1+1)(\det\partial^2)^4][2(g_1+1)(\det\partial^2)^4\Big] \\ & \times\Big[16\frac{\alpha^5(g_1+1)^2(g_2+1)^2}{3(g_1-g_2)^2 + 4\alpha(g_1+1)(g_2+1)}(\det\partial^2)^8\Big]^{-1/2}, \end{aligned} \tag{53}$$

which reduces to

$$Z = (\det\partial^2)^{-1}. \tag{54}$$

This indicates that there are in fact just two bosonic degrees of freedom, as the contribution of a single scalar degree of freedom is

$$\int \mathrm{d}\phi \mathrm{e}^{\phi\partial^2\phi} = (\det\partial^2)^{-1/2}. \tag{55}$$

These two degrees of freedom are of course the transverse polarizations of the free graviton. The free energy is thus given by [14]

$$
\begin{aligned}
-TV\log Z^{(0)} &= \Omega(T) \\
&= 2V\int \frac{d^3k}{(2\pi)^3}\left[\frac{|\vec{k}|}{2} + T\log(1-e^{-(k/T)})\right],
\end{aligned}
\tag{56}
$$

the factor of two coming from the two degrees of freedom.

We first note that in Eq. (54) all dependence on the gauge parameters has vanished. We also see that from Eq. (53) all determinants in Eq. (52) are nonzero.

We now consider the situation in which the spin-two field is no longer a free-field due to the self-interactions. The path integral to be considered then is not in the form of Eq. (8); we now must examine

$$
Z_I = \int d\vec{h} \exp[-\vec{h}^T \underset{\sim}{M}\, \vec{h} - S_I(\vec{h})], \tag{57}
$$

where $S_I(\vec{h})$ is at least cubic in $\vec{h}$. The argument of the exponential in Eq. (57) is now invariant under a transformation

$$
\vec{h} \to (\vec{h})_{\vec{\omega}} = \vec{h} + \alpha \underset{\sim}{A}(\vec{h})\vec{\omega} + \mathcal{O}(\vec{\omega}^2), \tag{58}
$$

where $\vec{\omega}$ is arbitrary and $\underset{\sim}{A}(\vec{h})$ now depends on $\vec{h}$, with $\underset{\sim}{A}(\vec{h}) = \underset{\sim}{A}^{(0)} + \mathcal{O}(\vec{h})$. Factors of "1" are now inserted into Eq. (57), using Eqs. (40) with $(\vec{h})_{\vec{\omega}}$ replacing $\vec{h} + \alpha \underset{\sim}{A}^{(0)}\vec{\theta}$, and keeping Eq. (41). Thus in place of Eq. (43) we obtain (up to a normalization factor)

$$
\begin{aligned}
Z_I = \int d\vec{\theta}\int d\vec{h} \det(\underset{\sim}{F}\,\underset{\sim}{A}(\vec{h}))\det(\underset{\sim}{G}\,\underset{\sim}{A}(\vec{h}))\det(\underset{\sim}{N}) \\
\times \exp\left[-\vec{h}^T\underset{\sim}{M}\,\vec{h} - S_I(\vec{h}) - \frac{1}{\alpha}\vec{h}^T\underset{\sim}{F}^T\underset{\sim}{N}\,\underset{\sim}{G}(\vec{h})_\theta\right],
\end{aligned}
\tag{59}
$$

where $(\vec{h})_\theta = ((\vec{h})_{\theta_2})_{\theta_1^{-1}} \approx \vec{h} + \alpha\underset{\sim}{A}^{(0)}(\vec{\theta}_2 - \vec{\theta}_1)$. The shift of Eq. (44) can again be used to diagonalize the terms appearing in the argument of the exponential in (59) that are quadratic in $\vec{h}$ and $\vec{\theta}$, but this shift also induces extra vertices involving the field $\vec{\theta}$, as Eq. (44) is not of the form of a gauge transformation. However, as has been noted above, the gauge fixing of Eq. (47) results in $(\underset{\sim}{M} + \frac{1}{\alpha}\underset{\sim}{F}^T\underset{\sim}{N}\,\underset{\sim}{G})^{-1}$ being traceless and transverse as $\alpha \to 0$, so that the shift $-\frac{1}{2}(\underset{\sim}{M} + \frac{1}{\alpha}\underset{\sim}{F}^T\underset{\sim}{N}\,\underset{\sim}{G})^{-1}(\underset{\sim}{F}^T\underset{\sim}{N}\,\underset{\sim}{G}\,\underset{\sim}{A}^{(0)})\vec{\theta}$ appearing in Eq. (47a) is formally of order α. Keeping in mind that the propagator for the field $\vec{\theta}$ in Eq. (51) has contributions of order $1/\alpha$ and $1/\alpha^2$ (though the latter disappears if $1/(g_1+1) + 1/(g_2+1) = 0$), we see that as $\alpha \to 0$ the contribution of these extra vertices is reduced.

IV. DISCUSSION

We have examined the most general covariant quadratic gauge fixing Lagrangians for a spin-two gauge field and have shown that none of them can be used to obtain the transverse-traceless propagator for this field. Nonquadratic gauge fixing Lagrangians can, however, be used to obtain this propagator, and we have shown that their systematic introduction results in an unconventional ghost contribution to the effective action. In a different context Drummond and Shore have also considered nonquadratic gauge fixing Lagrangians [15,16].

It would be worth to derive the Ward-Takahashi-Slavnov-Taylor [17–21] and Becchi-Rouet-Stora-Tyutin [22] identities when these nonquadratic gauge fixing Lagrangians are used and to verify them by explicit calculation of loop diagrams. As a first step towards the calculation of more involved perturbative quantities, one may consider the one-loop contributions to the thermal energy-momentum tensor. Since this result is known in the usual formulation of thermal gravity [6,7], one can verify the consistence of the nonquadratic gauge fixing approach in a specific scenario such that the interactions cannot be neglected.

The general relation between the one-graviton function $\Gamma^{\mu\nu}$ and the energy-momentum tensor $T^{\mu\nu}$ is such that

$$
\Gamma^{\mu\nu} = \frac{\delta\Gamma}{\delta h_{\mu\nu}} = -\frac{1}{2}\sqrt{-g}\,T^{\mu\nu}, \tag{60}
$$

where Γ is the one-loop thermal effective action. In Fig. 1 we show the lowest order diagrams which contribute to $\Gamma^{\mu\nu}$. In order to compute these diagrams we need the

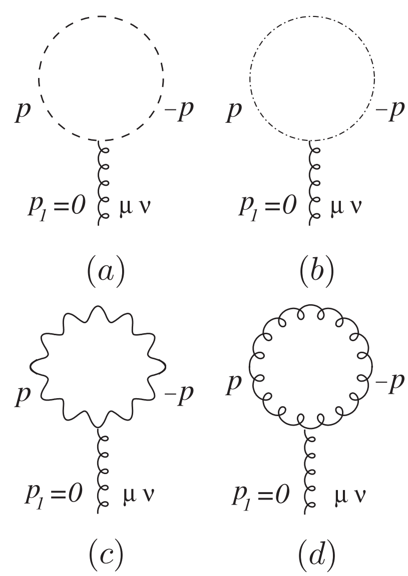

FIG. 1. Diagrams which contribute to the thermal one-graviton function. The dashed and dash-dotted lines represent the ghost fields b and c, respectively. The wavy lines represent the θ field and the curly lines represent gravitons.

propagators, derived in the previous sections, as well as the interaction vertices. Let us first consider the diagram (a) in Fig. 1. The ghost-graviton vertices can be obtained from the gauge-invariant completion of the quantity $-\vec{\bar{b}}(\underset{\sim}{F}\underset{\sim}{A}^{(0)})\vec{b}$ [see Eq. (49)] so that $\underset{\sim}{A}^{(0)}$ is replaced by

$$\begin{aligned} A_{\mu\nu\rho} &= g_{\mu\rho}\partial_\nu + g_{\nu\rho}\partial_\mu + (\partial_\rho g_{\mu\nu}) \\ &= \eta_{\mu\rho}\partial_\nu + \eta_{\nu\rho}\partial_\mu + h_{\mu\rho}\partial_\nu + h_{\nu\rho}\partial_\mu + (\partial_\rho h_{\mu\nu}). \end{aligned} \tag{61}$$

In this way, both the propagator and the interaction vertex can be read from the Lagrangian density

$$-\bar{b}_\lambda (FA)^\lambda_\rho b^\rho. \tag{62}$$

Using Eq. (47a), we obtain

$$\begin{aligned} \bar{b}_\lambda(\underset{\sim}{F}\underset{\sim}{A})^\lambda_\rho b^\rho &= \bar{b}_\lambda[(2g_1+1)\partial^\lambda\partial_\rho + \delta^\lambda_\rho\partial^2]b^\rho \\ &\quad + \bar{b}_\lambda[g_1(2\partial^\lambda h^\nu_\rho\partial_\nu + \partial^\lambda(\partial_\rho h^\nu_\nu)) + \partial^\nu h^\lambda_\rho\partial_\nu \\ &\quad + \partial_\nu h^\nu_\rho\partial^\lambda + \partial_\nu(\partial_\rho h^{\nu\lambda})]b^\rho. \end{aligned} \tag{63}$$

Let us now associate momenta p_1, p_2, and p_3, in momentum space, respectively, to the graviton field $h_{\mu\nu}$, to $\bar{b}_\lambda$, and to b_ρ. This yields the following interaction vertex for the b ghost:

$$\begin{aligned} V_b^{\mu\nu;\lambda\rho}(p_1,p_2,p_3) &= \frac{g_1}{2}(2p_2^\lambda p_3^\mu\eta^{\nu\rho} + p_2^\lambda p_1^\rho\eta^{\mu\nu}) \\ &\quad + \frac{1}{2}(p_2\cdot p_3\eta^{\mu\lambda}\eta^{\nu\rho} + p_2^\mu p_3^\lambda\eta^{\nu\rho} \\ &\quad + p_2^\mu p_1^\rho\eta^{\nu\lambda}) + (\mu\leftrightarrow\nu). \end{aligned} \tag{64}$$

In the diagrams of Fig. 1 the momenta are such that $p_1 = 0$ and $p_2 = -p_3 = p$ so that

$$\begin{aligned} V_b^{\mu\nu;\lambda\rho}(0,p,-p) &= -\frac{\eta^{\nu\rho}}{2}[(2g_1+1)p^\mu p^\lambda + p^2\eta^{\mu\lambda}] \\ &\quad - \frac{\eta^{\mu\rho}}{2}[(2g_1+1)p^\nu p^\lambda + p^2\eta^{\nu\lambda}]. \end{aligned} \tag{65}$$

When we contract with the ghost propagator, which is given by the inverse of the first term in Eq. (63), we obtain

$$\begin{aligned} &V_b^{\mu\nu;\lambda\rho}(0,p,-p)[(2g_1+1)p^\lambda p^\rho + \eta^{\lambda\rho}p^2]^{-1} \\ &\quad = -\tfrac{1}{2}(\eta^{\nu\rho}\delta^\mu_\rho + \eta^{\mu\rho}\delta^\nu_\rho) = -\eta^{\mu\nu}. \end{aligned} \tag{66}$$

The same can be done with the diagram (b) in Fig. 1, which is associated with the ghost field c, so that

$$V_c^{\mu\nu;\lambda\rho}(0,p,-p)[(2g_2+1)p^\lambda p^\rho + \eta^{\lambda\rho}p^2]^{-1} = -\eta^{\mu\nu}. \tag{67}$$

We now have to integrate these expressions over $\mathrm{d}^{d-1}p$ and sum over the Matsubara frequencies $p_0 = 2\pi nT$. Then, the dimensionally regularized integral will yield a zero result for both ghost loops.

Let us now consider the contribution of the θ field. The vertex in the diagram (c) in Fig. 1 is the sum of two types of contributions. The first contribution comes from the order h terms when we replace $\tilde{A}^{(0)}$ in Eq. (49) by the Eq. (61). This type of contribution will also yield an expression for the integrand which is proportional to $\eta_{\mu\nu}$. Indeed, as in the case of the ghost fields b and c, the only relevant part of the interaction vertex is the one which has a zero momentum external graviton, so that the order h terms in A will yield a contribution proportional to the inverse of the propagator. Therefore, this part of the interaction will not contribute to the energy-momentum tensor.

The second part of the interaction between the θ and h fields arises when the cubic term in the interaction Lagrangian is modified by the shift given by (44). In order to compute this contribution we employ the known expression for the three graviton vertex [23] and contract two of its external legs with the operator on the right-hand side of (44). Finally, contracting the resulting expression with the θ propagator in (51) we have obtained

$$\frac{(d-3)(g_1-g_2)^2}{(d-1)(g_1-g_2)^2 + 2\mu(d-2)(g_1+1)(g_2+1)}\frac{p^\mu p^\nu}{p^2}. \tag{68}$$

The last diagram in Fig. 1 has the usual interaction vertex contracted with the general propagator given by Eqs. (25) and (48). A straightforward calculation yields (we have employed the symbolic computer package HIP [24])

$$\frac{(d-3)(d-2)[(d+1)(g_1-g_2)^2 + 2\alpha d(g_1+1)(g_2+1)]}{2[(d-1)(g_1-g_2)^2 + 2\mu(d-2)(g_1+1)(g_2+1)]} \times\frac{p^\mu p^\nu}{p^2}. \tag{69}$$

Adding the two previous expressions, we obtain

$$\frac{d(d-3)}{2}\frac{p^\mu p^\nu}{p^2}. \tag{70}$$

All the gauge parameter dependence has been canceled in the final expression for the integrand of the one point function and the result agrees with the known result in the DeDonder gauge. Of course this gauge independent result is expected for a physical quantity like the energy-momentum tensor. This rather simple calculation indicates that the interactions can be taken into account consistently in the double gauge fixing formulation. Since this calculation has been done without restricting the values of the gauge parameters α, g_1, and g_2, it also holds in the particular case of the TT graviton propagator. The combination of the expressions (68) and (69) yielding the gauge-invariant result shows how the modes associated with the θ and h fields combine to produce the correct result.

Another interesting application of the TT gauge can be made to study the thermal loop-corrections to the free-energy in quantum gravity. This would allow for a simple and physical analysis of the Jeans-like instabilities which develop at nonzero temperature. Work on this topic is in progress.

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