

Renormalization group analysis of a Gürsey-model-inspired field theory. IIB. C. Lütfüoğlu^{1,*} and F. Taşkın^{1,2,†}¹*Department of Physics, Istanbul Technical University, Istanbul, Turkey*²*Department of Physics, Erciyes University, Kayseri, Turkey*

(Received 18 July 2007; published 12 November 2007)

Recently a model, which is equivalent to the scalar form of a Gürsey model, is shown to be a nontrivial field theoretical model when it is gauged with a $SU(N)$ field. In this paper we study another model that is equivalent to the vector form of the Gürsey model. We get a trivial theory when it is coupled with a scalar field. This result changes drastically when it is coupled with an additional $SU(N)$ field. We find a nontrivial field theoretical model under certain conditions.

DOI: [10.1103/PhysRevD.76.105010](https://doi.org/10.1103/PhysRevD.76.105010)

PACS numbers: 11.10.Hi, 12.60.Rc

I. INTRODUCTION

Historically, there has always been a continuing interest in building nontrivial field theoretical models. A while ago it was shown that perturbative expansions are not adequate in deciding whether a model is nontrivial or not. Baker *et al.* showed that the ϕ^4 theory, although perturbatively nontrivial, went to a free theory as the cutoff was lifted in four dimensions [1,2]. Continuing research is going on this subject [3]. Alternative methods become popular. Renormalization group (RG) methods are the most commonly used ones. They were first introduced by Wilson *et al.* [4]. Another method is using the exact RG algorithm which was proposed by Polchinski [5]. Recent studies gave important insights on both methods [6–8].

Another endeavor is building a model of nature using only fermions. Here all the observed bosons are constructed as composites of these ingredient spinors. In solid state physics, electrons come together to form bosonic particles [9,10]. Historically, the first work on models with only spinors goes back to the work of Heisenberg [11]. Two years later Gürsey proposed his model as a substitute for the Heisenberg model [12]. This Gürsey spinor model is important since it is conformally invariant classically and has classical solutions [13] which may be interpreted as instantons and merons [14], similar to the solutions of pure Yang-Mills theories in four dimensions [15]. This original model can be generalized to include vector, pseudovector, and pseudoscalar interactions.

We have worked on different forms of the Gürsey model [16–18] using the earlier works [19–23] as a starting point. In those references it was claimed that a polynomial Lagrangian could be written equivalently to Gürsey's non-polynomial Lagrangian. Recently it is shown that they are equivalent only in a naive sense [16,17]. In [16], using perturbative methods, we showed that only composite particles took part in physical processes whereas the constituent fields did not interact with each other. Recently in [18], we showed that, when this model is coupled to a

constituent $U(1)$ gauge field, we were mimicking a gauge Higgs-Yukawa (gHY) system, which had the known problems of the Landau pole, with all of its connotations of triviality. There, our motivation was the famous Nambu-Jona-Lasinio model [24], which was written only in terms of spinor fields. This model was shown to be trivial [25,26]. Recent attempts to gauge this model to obtain a nontrivial theory are given in Refs. [27–31].

The essential point in our analysis is the factor of ϵ in the composite propagator [16,17]. This main difference makes many of the diagrams convergent when the cutoff is removed. Consequently, we find that we can construct a nontrivial model from the scalar Gürsey model when a non-Abelian gauge field is coupled to the fermions [32]. In this paper we will investigate the vector form of the Gürsey model. Here we will closely follow the line of discussion followed in Refs. [27,32].

This article is organized as follows. In the next section we describe the vector form of the Gürsey-like model. There we derive the composite vector field propagator. In Sec. III, we couple a constituent scalar field to our model and discuss the new results. Then we solve the renormalization group equations (RGE's) and find a Landau pole in the solution. In Sec. IV, we introduce another field, a non-Abelian gauge field to the model. In the subsections we write the new RGE's and derive the solutions by using some RG invariants. We discuss some limiting cases of the coupling constant solutions before giving the criteria of the nontriviality condition in Sec. V. Then we find the fixed point solutions. In the following subsections we analyze the solutions of the coupled equations and find their asymptotic behaviors. The final section is devoted to conclusions.

II. THE MODEL

The vector form of the pure spinor Gürsey model [21] is given as

$$L = \bar{\psi}(i\not{\partial} - ig\not{a}g^{-1} - m)\psi + \alpha[(\bar{\psi}\gamma_{\mu}\psi)(\bar{\psi}\gamma^{\mu}\psi)]^{2/3}. \quad (1)$$

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Here only the spinor fields have a kinetic part. The g field is a pure gauge term to restore the local gauge symmetry, when the spinor field is transformed. This nonpolynomial Lagrangian has been converted to an equivalent polynomial form by introducing auxiliary fields λ_μ and G_μ in [21]. The constrained Lagrangian in the polynomial form is given as

$$L_c = \bar{\psi}[i\not{\partial} - ig\not{\partial}g^{-1} + e(\not{G} + \not{\lambda}) - m]\psi - e^4\lambda_\mu G^\mu G^2 + \text{ghost terms.} \quad (2)$$

Recently it was shown that this equivalence should be taken only ‘‘naively’’ [17]. This expression contains two constraint equations, obtained from writing the Euler-Lagrange equations for the auxiliary fields. Hence it should be quantized by using the constraint analysis *à la* Dirac [33]. This calculation is performed using the path integral method. We find out that one can write the effective Lagrangian as

$$L_{\text{eff}} = \bar{\psi}[i\not{\partial} - ig\not{\partial}g^{-1} + e(\not{G} + \not{\lambda}) - m]\psi - e^4\lambda_\mu G^\mu G^2 + \bar{w}^\mu(g_{\mu\nu}G^2 + 2G_\mu G_\nu)w^\nu. \quad (3)$$

Here \bar{w}^μ and w^ν are the ghost fields. With a suitable redefinition of the fields the effective action can be given as

$$S_{\text{eff}} = \text{Tr} \ln(i\not{\partial} + e\not{J} + m) + \int dx^4 [e^4(J_\mu J^\mu J_\lambda J^\lambda) + \text{other terms}], \quad (4)$$

where $J_\mu = -ig\partial_\mu g^{-1} + G_\mu + \lambda_\mu$. The second derivative of the effective action with respect to the J_μ field gives us the induced inverse propagator as

$$\left. \frac{\partial^2 S_{\text{eff}}}{\partial J_\mu \partial J_\nu} \right|_{J_\mu=0} = -\frac{g^2}{3\pi^2} (q_\mu q_\nu - g_{\mu\nu} q^2) \times \left[\frac{1}{\epsilon} + \text{finite part} \right]. \quad (5)$$

Here dimensional regularization is used for the momentum integral and $\epsilon = 4 - n$. All the other fields not shown in this expression, including ghost fields arising from the constrained equations, decouple from the model. The only remaining fields are the spinors and the J_μ field. This procedure is explicitly carried out in [17]. In the Feynman gauge the propagator of the composite vector field can be written as $\epsilon \frac{g^{\mu\nu}}{p^2}$ where the spinor propagator is the usual Dirac propagator in the lowest order.

Although the original Lagrangian does not have a kinetic term for the vector field, one loop corrections generate this term and make this composite field as a dynamical entity like it is done in [16], where the composite vector field is replaced by composite scalar field. In the literature there are also other similar models with differential operators in the interaction Lagrangian [34].

In Ref. [17], the contributions to the fermion propagator at higher orders were investigated by studying the Dyson-Schwinger equations for the two point function. We found that there is a phase which has no additions to the existing fermion mass.

III. COUPLING WITH A SCALAR FIELD

We may add a constituent complex scalar field to the model and investigate the consequences of this addition. Our motivation is the work of Bardeen *et al.* [35,36]. When they added a vector field to the Nambu-Jona-Lasinio model, a complementary procedure to our work, they got interesting results. Since we already have a composite vector field, we can couple a massless scalar field which has its kinetic term, a self-interacting term with coupling constant a and an interaction term with new coupling constant y in the Lagrangian. Then the effective Lagrangian becomes

$$L_{\text{eff}} = \bar{\psi}[i\not{\partial} - ig\not{\partial}g^{-1} + e(\not{G} + \not{\lambda}) - m]\psi - e^4\lambda_\mu G^\mu G^2 + \bar{w}^\mu(g_{\mu\nu}G^2 + 2G_\mu G_\nu)w^\nu + \frac{\partial_\mu \phi \partial^\mu \phi}{2} - \frac{a}{4}\phi^4 - y\bar{\psi}\phi\psi. \quad (6)$$

Since the G_μ , λ_μ , and ghost fields decouple, this Lagrangian reduces to the effective expression given below:

$$L_{\text{eff}} = \bar{\psi}(i\not{\partial} + e\not{J} - y\phi - m)\psi - e^4 J^4 + \frac{\partial_\mu \phi \partial^\mu \phi}{2} - \frac{a}{4}\phi^4. \quad (7)$$

If our fermion field had a color index i where $i = 1 \dots N$, we could perform an $1/N$ expansion to justify the use of only ladder diagrams for higher orders for the scattering processes. Although in our model the spinor has only one color, we still consider only ladder diagrams anticipating that one can construct a variation of the model with N colors. In the following subsection we summarize the changes in our results for this new model.

A. New results and higher orders

In the model described in Ref. [17], it is shown that only composites can scatter from each other with a finite expression, due to the presence of ϵ in the composite vector propagator. There is also a tree-diagram process where the spinor scatters from a composite particle, a Compton-like scattering, with a finite cross section. This diagram can be written in the other channel, which can be interpreted as spinor production out of vector particles. Note that in the original model the four spinor kernel was of order ϵ . The lowest order diagram vanishes due to the presence of the composite vector propagator. In higher orders this expression can be written in the quenched ladder approximation

[10], where the kernel is separated into a vector propagator with two spinor legs joining the proper kernel. If the proper kernel is of order ϵ , the loop involving two spinors and a vector propagator can be at most finite that makes the whole diagram in first order in ϵ . This fact shows that there is no nontrivial spinor-spinor scattering in the original model.

These results change drastically with scalar field coupling. Two fermion scattering is now possible due to the presence of the scalar field instead of vector field channel. In the lowest order this process goes through the tree diagram given in Fig. 1(a). At the next higher order the box diagram with two spinors and two scalar particles, Fig. 1(b), is finite from dimensional analysis. If the scalar particles are used as intermediaries, the spinor production from scattering of composite vector particles becomes possible as shown in Fig. 1(c) where the dotted, straight, and wiggly lines represent scalar, spinor, and composite vector particles, respectively.

B. Renormalization group equations and solutions

In Ref. [17], it is widely discussed that the $\langle \bar{\psi}\psi J_\mu \rangle$ vertex and the spinor box diagram give finite results. The higher diagrams do not change this result, since each momentum integration is accompanied by an ϵ term in the composite vector propagator. Therefore, there is no need for infinite coupling constant renormalization.

In the new model where a massless scalar field is added, all the three coupling constants are renormalized. One can write the first order RGE's for these coupling constants, similar to the analysis in [27]. We take μ_0 as a reference scale at low energies, $t = \ln(\mu/\mu_0)$, where μ is the renormalization point:

$$16\pi^2 \frac{d}{dt} y(t) = Ay^3(t), \quad (8)$$

$$16\pi^2 \frac{d}{dt} e(t) = Be(t)y^2(t), \quad (9)$$

$$16\pi^2 \frac{d}{dt} a(t) = Ca^2(t) - Dy^4(t). \quad (10)$$

Here A , B , C , and D are positive numerical constants. We find out that Yukawa and $\langle \bar{\psi}\psi J_\mu \rangle$ vertices have only scalar

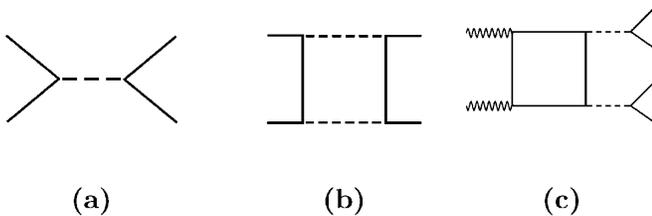


FIG. 1. (a) two spinor scattering through the scalar particle channel, (b) higher order diagram for two spinor scattering, (c) spinor production from scattering of composite vectors.

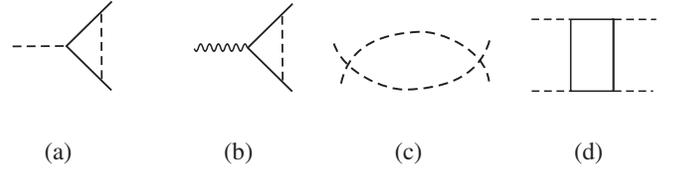


FIG. 2. The three coupling constant corrections in one loop.

correction. The composite vector correction to these vertices are finite due to the ϵ in the propagator. Therefore, our equations differ from those in Refs. [27,32]. These processes are illustrated in diagrams shown in Fig. 2.

The RGE's have the immediate solutions

$$y^2(t) = \frac{y_0^2}{Z(t)}, \quad (11)$$

$$e(t) = e_0 Z(t)^{-B/2A}, \quad (12)$$

$$a(t) = \frac{A \pm \sqrt{A^2 + CD}}{C} \frac{y_0^2}{Z(t)}, \quad (13)$$

where $Z(t) = 1 - \frac{Ay_0^2}{8\pi^2} t$.

The main problem of models with $U(1)$ coupling, namely, the Landau pole, is expected to make our new model a trivial one. We expect that coupling to a non-Abelian gauge theory will remedy this defect by new contributions to the RGE's. Thus, obtaining a nontrivial model will be possible. Coupling to a non-Abelian gauge field will also give us more degrees of freedom in studying the behavior of the beta function. This may allow us to find the critical number of gauge and fermion fields to obtain a zero of this function at nontrivial values of the coupling constants of the model.

IV. COUPLING WITH A NON-ABELIAN FIELD

In this section we consider our model with $SU(N_C)$ gauge field interaction, where the spinors have N_f different flavors. Although we study in the leading order of $\frac{1}{N_C}$ expansion, where all the planar diagrams contribute to the RGE's, we are interested in the high-energy asymptotic region where the gauge coupling is perturbatively small; $\frac{g^2 N_C}{4\pi} \ll 1$. However, the number of fermions is in the same order as N_C . Only n_f fermions have a degenerate large Yukawa coupling. We start with the effective Lagrangian

$$L_{\text{eff}} = \sum_{i=1}^{N_f} \bar{\psi}_i (i\not{D} + e\not{J} - m)\psi_i - e^4 J^4 + \frac{(\partial_\mu \phi)^2}{2} - \frac{a}{4} \phi^4 - \sum_{i=1}^{n_f} \bar{\psi}_i \gamma \phi \psi_i - \frac{1}{4} \text{Tr}(F_{\mu\nu})^2 + L_{\text{ghost}} + L_{\text{g.f.}} \quad (14)$$

The gauge field belongs to the adjoint representation of the color group $SU(N_C)$ where D_μ is the color covariant

derivative. y , a , e , and g are the Yukawa, quartic scalar, composite vector, and gauge coupling constants, respectively.

There are two kinds of ghost fields in the model. The first one, which comes from the composite constraints, decouples from our model [20,21]. The second one, coming from the gauge condition on the vector field, does not decouple and contributes to the RGE's in the usual way.

A. Renormalization group equations and solutions

In this subsection we will analyze the RGE's in the leading order of the approximation given above. In the one loop approximation the RGE's are

$$16\pi^2 \frac{d}{dt} g(t) = -Ag^3(t), \quad (15)$$

$$16\pi^2 \frac{d}{dt} y(t) = By^3(t) - Cy(t)g^2(t), \quad (16)$$

$$16\pi^2 \frac{d}{dt} e(t) = De(t)y^2(t) - Ee(t)g^2(t), \quad (17)$$

$$16\pi^2 \frac{d}{dt} a(t) = Fa(t)y^2(t) - Gy^4(t). \quad (18)$$

Here A , B , C , D , E , F , and G are positive constants.

In the RGE's we see that the diagrams, where the composite vector field takes part, are down by the order of ϵ . Therefore we do not have contributions proportional to $e^2(t)$, $e^3(t)$, $y(t)e^2(t)$, and $g(t)e^2(t)$. Also we neglect the scalar loop contribution to the gauge coupling $g(t)$, similar to the work of [27].

The solutions of the first RG equation (15) can be given as

$$g^2(t) = g_0^2 \left(1 + \frac{A\alpha_0}{2\pi} t \right)^{-1}, \quad (19)$$

where $\alpha_0 = \frac{g_0^2}{4\pi}$. We define

$$\eta(t) \equiv \frac{\alpha(t)}{\alpha_0} \equiv \frac{g^2(t)}{g_0^2}, \quad (20)$$

where $g_0 = g(t=0)$ which is the initial value at the reference scale μ_0 . For the solution of the second RG equation (16), we can propose a RG invariant $H(t)$ as

$$H(t) = -\eta^{-1+C/A}(t) \left[1 - \frac{C-A}{B} \frac{g^2(t)}{y^2(t)} \right]. \quad (21)$$

Since $H(t)$ is a constant, we call it H_0 . Then, the solution of the Yukawa coupling constant can be written as

$$y^2(t) = \frac{C-A}{B} g^2(t) [1 + H_0 \eta^{1-(C/A)}(t)]^{-1}. \quad (22)$$

The solution of the third RG equation (17) can be defined by another RG invariant $P(t)$ if and only if the constant B

equals to D and C equals to E . Then the invariant becomes

$$P(t) = -\eta^{-1+C/A}(t) \left[1 - \frac{B}{C-A} \frac{y^2(t)}{g^2(t)} \right] \frac{e^2(t)}{y^2(t)} \frac{g^2(t)}{y^2(t)}. \quad (23)$$

The solution of the composite vector coupling $e(t)$ can be written as

$$e^2(t) = -\frac{P_0}{H_0} \left(\frac{C-A}{B} \right)^2 g^2(t) [1 + H_0 \eta^{1-(C/A)}(t)]^{-1}, \quad (24)$$

where P_0 denotes the value of the invariant $P(t)$. The solution of the last RG equation (18) can be defined by another RG invariant $K(t)$, given as

$$K(t) = -\eta^{-1+(2C/A)}(t) \left[1 - \frac{2C-A}{2B} \frac{a(t)}{y^2(t)} \frac{g^2(t)}{y^2(t)} \right]. \quad (25)$$

We can rewrite the solution with a value of the invariant $K(t)$ as K_0

$$a(t) = \frac{2(C-A)^2}{(2C-A)B} g^2(t) \frac{1 + K_0 \eta^{1-(2C/A)}(t)}{[1 + H_0 \eta^{1-(C/A)}(t)]^2}. \quad (26)$$

Here we notice that the RG constants H_0 , P_0 , and K_0 play important roles on the behavior of the solutions of the coupling equations (19), (22), (24), and (26). Similar works have been studied in [27,32]. The values of the constants are given in these equations as

$$A = \frac{11N_C - 4T(R)N_f}{3}, \quad B = D = \frac{G}{4} = 2n_f N_C, \\ C = E = 6C_2(R), \quad F = G. \quad (27)$$

Here $C_2(R)$ is a second Casimir, $C_2(R) = \frac{(N_C^2 - 1)}{2N_C}$, R is the fundamental representation with $T(R) = \frac{1}{2}$.

Before entering the analysis of the fixed point, we briefly investigate the results of some limits.

1. The limiting case $A \rightarrow +0$ for finite t

In this case the coupling constant solutions can be written as

$$g^2(t) = g_0^2, \quad (28)$$

$$y^2(t) = \frac{8\pi^2}{B} \frac{\alpha}{\alpha_c} \left[1 + H_0 \exp\left(\frac{\alpha}{\alpha_c} t\right) \right]^{-1}, \quad (29)$$

$$e^2(t) = -\frac{16\pi^3}{B^2} \frac{P_0}{H_0} \frac{\alpha}{\alpha_c^2} \left[1 + H_0 \exp\left(\frac{\alpha}{\alpha_c} t\right) \right]^{-1}, \quad (30)$$

$$a(t) = \frac{8\pi^2}{B} \frac{\alpha}{\alpha_c} \frac{[1 + K_0 \exp(\frac{2\alpha}{\alpha_c} t)]}{[1 + H_0 \exp(\frac{\alpha}{\alpha_c} t)]^2}. \quad (31)$$

Here $\alpha_0 = \alpha$ and $\frac{C}{2\pi} = \frac{1}{\alpha_c}$.

2. The limiting case $A \rightarrow C$ for finite t

In this limit case the solutions of the couplings (22), (24), and (26) seem to vanish. If we suggest new RG invariant H_1 , instead of H_0 , as $H_0 = -1 + \frac{C-A}{A}H_1$, we find that two of the coupling solutions do not vanish, whereas the composite vector coupling goes to zero. These behaviors are given below:

$$y^2(t) = \frac{A}{B} g^2(t) [H_1 + \ln \eta(t)]^{-1}, \quad (32)$$

$$e^2(t) = P_0 \left(\frac{C-A}{B} \right) \left(\frac{A}{B} \right) g^2(t) [H_1 + \ln \eta(t)]^{-1}, \quad (33)$$

$$a(t) = \frac{2A}{B} g^2(t) \frac{1 + K_0 \eta^{-1}(t)}{[H_1 + \ln \eta(t)]^2}. \quad (34)$$

It is amusing to see that the added interactions nullify the original vector-spinor coupling.

3. The limiting case $A \rightarrow 2C$ for finite t

In this limit case only the quartic coupling constant solution (26) behaves critically. Similarly we can redefine RG invariant K_1 instead of K_0 as $K_0 = -1 + \frac{2C-A}{A}K_1$, then the quartic coupling solution takes the form

$$a(t) = \frac{C}{B} g^2(t) \frac{K_1 + \ln \eta(t)}{[1 + H_0 \eta^{1/2}(t)]^2}. \quad (35)$$

This limit is not allowed because it does not give asymptotic freedom.

In the next section we will mention which criteria are needed to define a nontrivial theory.

V. NONTRIVIALITY OF THE SYSTEM

To have a nontrivial theory all the running coupling constants should not diverge at any finite energy, which means the absence of Landau poles of the system. For a consistent theory these solutions should not vanish identically and must have real and positive values. These conditions make the model unitary and satisfy the vacuum stability criterion. Note that if we decouple the scalar and composite vector field from the system, we have a nontrivial theory, similar to QCD. Therefore, $e(t) \equiv g(t) \equiv a(t) \equiv 0$ solution will not be named as the nontriviality of our composite model. The mass parameter can be renormalized in the \overline{MS} scheme and the mass can be chosen as zero.

Remember that we are restricted by neglecting the scalar loop contributions to the gauge coupling where the composite vector contributions are not neglected but down due to the presence of ϵ in its propagator. If the Yukawa and/or quartic scalar couplings become so large and break the $1/N_C$ expansion then the behavior of the gauge coupling might be affected.

These restriction conditions are the same as the ones in the gHY system which was discussed widely in [27]. A while ago, one of us, B. C. L., with a collaborator, studied the scalar form of the Gürsey model in this fashion [32]. In that model, there is a composite scalar field with a propagator completely different from a constituent scalar field used in Ref. [27]. There, we showed that a restriction is not needed between the scalar and the gauge field coupling since the contribution of the scalar field to the gauge field is down by the factor of ϵ in the scalar propagator. In this work, the vector form of the Gürsey model, we have a constituent scalar field and a composite vector field which is missing in the gHY system. This composite field adds a new RGE to the system but does not contribute to the former ones in the gHY system with a totally different reason.

After these remarks we will discuss the nontriviality conditions of our model in the following subsections.

A. Fixed point solution

The RGE's can be rewritten as

$$8\pi^2 \frac{d}{dt} \left[\frac{y^2(t)}{g^2(t)} \right] = B g^2(t) \left[\frac{y^2(t)}{g^2(t)} \right] \left[\frac{y^2(t)}{g^2(t)} - \frac{C-A}{B} \right], \quad (36)$$

$$8\pi^2 \frac{d}{dt} \left[\frac{e^2(t)}{y^2(t)} \frac{g^2(t)}{y^2(t)} \right] = (C-A) g^2(t) \left[\frac{e^2(t)}{y^2(t)} \right] \times \left[\frac{g^2(t)}{y^2(t)} - \frac{B}{C-A} \right], \quad (37)$$

$$8\pi^2 \frac{d}{dt} \left[\frac{a(t)}{y^2(t)} \frac{g^2(t)}{y^2(t)} \right] = (2C-A) g^2(t) \left[\frac{a(t)}{y^2(t)} \frac{g^2(t)}{y^2(t)} - \frac{2B}{2C-A} \right]. \quad (38)$$

The fixed point solutions can be given as

$$\frac{y^2(t)}{g^2(t)} = \frac{C-A}{B}, \quad (39)$$

$$\frac{e^2(t)}{y^2(t)} = \text{Arbitrary constant}, \quad (40)$$

$$\frac{a(t)}{y^2(t)} = \frac{2(C-A)^2}{B(2C-A)}. \quad (41)$$

These are also the solutions of Eqs. (22), (24), and (26) where the RG invariants are $P_0 = H_0 = K_0 = 0$ as $P_0 = \zeta H_0$. Here ζ is a constant. It is clear that the behavior of all the coupling constants are determined by the gauge coupling which means the Kubo, Sibold, and Zimmermann's "coupling constant reduction" [37]. This corresponds to the Pendleton-Ross fixed point [38] in the context of the RGE. Remark that only the case, $C > A$, prevents the violation of the unitarity and keeps the stability of the

vacuum. This gives rise to nontriviality of the model when the RG invariants are set to zero. In the following subsections we will analyze the coupling constant solutions only in this case with nonzero RG invariants.

B. Yukawa coupling

The Yukawa coupling solution is given in Eq. (22). It is obvious that the sign of the RG invariant, H_0 , plays an important role in the behavior of the solution where B is positive. The ultraviolet (UV) limit of $\eta(t)$ is needed before continuing the analysis in the $C > A$ case.

$$\eta^{1-(C/A)}(t \rightarrow \infty) \rightarrow +\infty. \quad (42)$$

The UV behavior of the Yukawa coupling with a nonzero RG invariant H_0 is

$$y^2(t \rightarrow \infty) \rightarrow \begin{cases} +0, & 0 < H_0 < \infty; \\ \text{Landau Pole}, & -1 < H_0 < 0; \\ -0, & -\infty < H_0 \leq -1. \end{cases} \quad (43)$$

For the $-1 < H_0 < 0$ case, there exists a finite value of t before it goes to infinity

$$1 + \frac{A\alpha_0}{2\pi}t = \left(\frac{-1}{H_0}\right)^{A/(C-A)}. \quad (44)$$

In this t value the Yukawa coupling diverges and changes its sign. These asymptotic behaviors show that the theory is nontrivial if and only if the RG invariant H_0 is positive.

The RG flows in the $(g^2(t), y^2(t))$ plane are shown in Fig. 3. The upper bound of the figure denotes the ‘‘Landau Pole.’’

C. Composite vector field coupling

The composite vector coupling solution is given in Eq. (24). In this case not only the sign of H_0 but also the sign of P_0 is crucial for nontriviality. Since H_0 is positive,

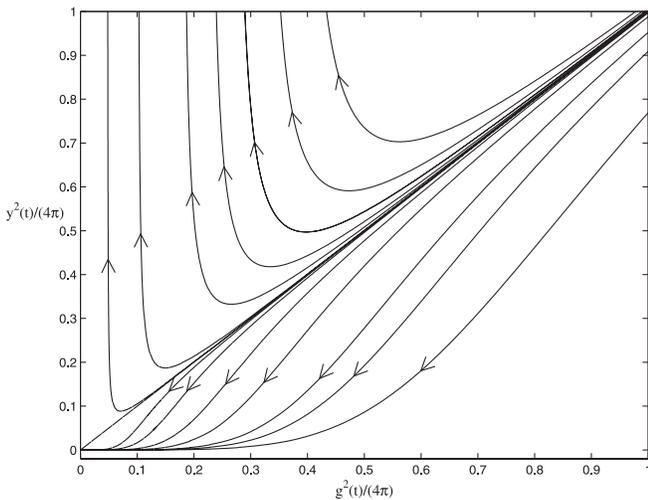


FIG. 3. Plot of $g^2(t)$ vs $y^2(t)$ for different values of H_0 . The arrows denote the flow directions toward the UV region.

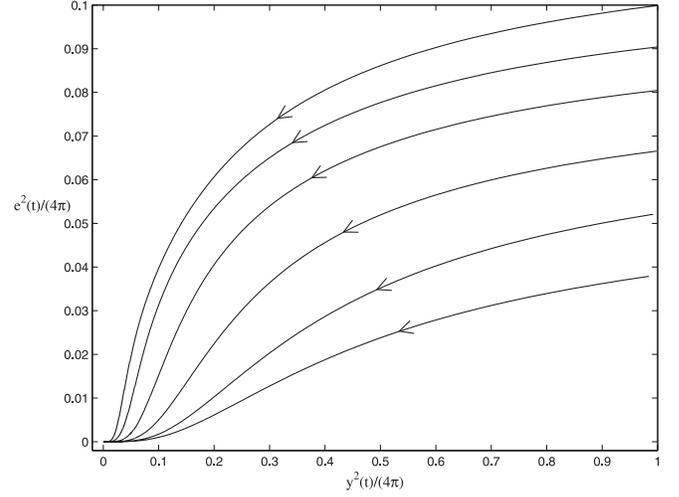


FIG. 4. Plot of $y^2(t)$ vs $e^2(t)$ for the values of $H_0 > 0$ and $P_0 < 0$.

P_0 must be negative. The composite vector field coupling behaves similarly to the Yukawa coupling up to a constant multiplier. In Fig. 4 we plot $e^2(t)$ vs $y^2(t)$ where $P_0 < 0$, $H_0 > 0$. Both coupling constants approach the origin as t goes to infinity. Thus, our model fulfills the condition required by the asymptotic freedom criterion.

D. Quartic scalar field coupling

Finally the quartic scalar coupling solution given in Eq. (26) can be analyzed. We have already restricted ourselves with $C > A$, $H_0 > 0$, and $P_0 < 0$ for nontriviality. In the limit where $t \gg 1$, the η terms in the last fraction of Eq. (26) become dominant therefore 1 can be neglected. Hence we can express the solution as

$$a(t) \approx \frac{2(C-A)^2}{(2C-A)B} g_0^2 \eta(t) \frac{K_0 \eta^{1-2C/A}(t)}{[H_0 \eta^{1-C/A}(t)]^2}, \quad (45)$$

which is equal to

$$a(t \rightarrow \infty) = \frac{2(C-A)^2}{(2C-A)B} g_0^2 \frac{K_0}{H_0^2}. \quad (46)$$

This asymptotic behavior shows that to have a nontrivial model the RG invariant K_0 should be equal to zero. The other possibilities for a nonzero solution for K_0 have been widely discussed in Ref. [27]. In Fig. 5, we plot the RG flows in $(a(t), y^2(t))$ plane for different values of H_0 higher than zero while the gauge coupling $\alpha(t=0)$ is fixed to one. The origin is the limit where t goes to infinity, there both coupling constants approach zero when $K_0 = 0$.

VI. CONCLUSION

A while ago, one of us, F. T., with a collaborator, showed that the scattering of composite vector particles gives nontrivial results while the constituent spinors do not. In that work [17], a polynomial Lagrangian model inspired by the

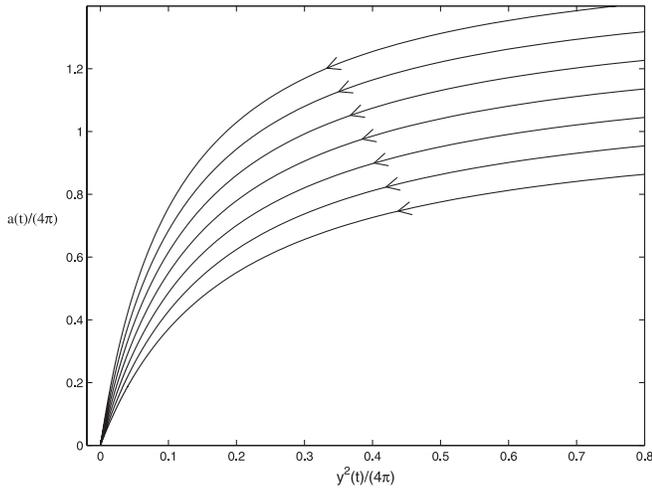


FIG. 5. Plot of $a(t)$ vs $y^2(t)$ for the values of $H_0 > 0$ and $K_0 = 0$.

vector form of the Gürsey model was used. Here we couple a constituent massless scalar field to our previous model. We find out that many of the features, related to the creating and scattering of the spinor particles of the original model, are not true anymore. In the one loop approxi-

mation we find the RGE's whose solutions have all the problems associated with the Landau pole, like the case in Ref. [18]. To remedy this defect we couple a $SU(N_C)$ non-Abelian gauge field to the new model. We solve the new RGE's and conclude that if the conditions $C > A$, $H_0 > 0$, $P_0 \leq 0$, and $K_0 = 0$ are satisfied, the model gives a result which can be interpreted as a nontrivial field theoretical model. We find fixed point solutions where the coupling constants are not equal to zero. In Sec. V we plot the UV region behavior of the coupling constants. There, they all go to zero asymptotically which means asymptotic freedom, which is another feature of a nontrivial model.

Our calculation shows that one can construct a nontrivial field theory starting from constrained Lagrangians.

ACKNOWLEDGMENTS

We thank Mahmut Hortaçsu for discussions and both scientific and technical assistance while preparing this manuscript. We also thank Nazmi Postacıoğlu for technical discussions. This work is supported by the ITU BAP Project No. 31595. This work is also supported by TUBITAK, the Scientific and Technological Council of Turkey.

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- [1] G. A. Baker and J. M. Kincaid, Phys. Rev. Lett. **42**, 1431 (1979).
 - [2] G. A. Baker and J. M. Kincaid, J. Stat. Phys. **24**, 469 (1981).
 - [3] J. R. Klauder, Ann. Phys. (N.Y.) **322**, 2569 (2007).
 - [4] K. G. Wilson and J. B. Kogut, Phys. Rep. **12**, 75 (1974).
 - [5] J. Polchinski, Nucl. Phys. **B231**, 269 (1984).
 - [6] H. Sonoda, J. Phys. A: Math. Theor. **40**, 5733 (2007).
 - [7] H. Sonoda, J. Phys. A: Math. Theor. **40**, 9675 (2007).
 - [8] Y. Igarashi, K. Itoh, and H. Sonoda, Prog. Theor. Phys. **118**, 121 (2007).
 - [9] J. Bardeen, L. N. Cooper, and J. R. Schrieffer, Phys. Rev. **108**, 1175 (1957).
 - [10] V. A. Miransky, *Dynamical Symmetry Breaking in Quantum Field Theories* (World Scientific, Singapore, 1993).
 - [11] W. Heisenberg, Z. Naturforsch. **9A**, 292 (1954).
 - [12] F. Gürsey, Nuovo Cimento **3**, 988 (1956).
 - [13] F. Kortel, Nuovo Cimento **4**, 210 (1956).
 - [14] K. G. Akdeniz, Lett. Nuovo Cimento **33**, 40 (1982).
 - [15] W. I. Fushchich and W. M. Shtelen, J. Phys. A: Math. Gen. **23**, L517 (1990).
 - [16] M. Hortaçsu and B. C. Lütfüoğlu, Mod. Phys. Lett. A **21**, 653 (2006).
 - [17] M. Hortaçsu and F. Taşkın, Int. J. Mod. Phys. A **22**, 83 (2007).
 - [18] M. Hortaçsu, B. C. Lütfüoğlu, and F. Taşkın, Mod. Phys. Lett. A **22**, 2521 (2007).
 - [19] K. G. Akdeniz, M. Arık, M. Durgut, M. Hortaçsu, S. Kaptanoğlu, and N. K. Pak, Phys. Lett. B **116**, 34 (1982).
 - [20] K. G. Akdeniz, M. Arık, M. Durgut, M. Hortaçsu, S. Kaptanoğlu, and N. K. Pak, Phys. Lett. **116B**, 41 (1982).
 - [21] K. G. Akdeniz, M. Arık, M. Hortaçsu, and N. K. Pak, Phys. Lett. **124B**, 79 (1983).
 - [22] M. Arık and M. Hortaçsu, J. Phys. G **9**, L119 (1983).
 - [23] M. Arık, M. Hortaçsu, and J. Kalaycı, J. Phys. G **11**, 1 (1985).
 - [24] Y. Nambu and G. Jona-Lasinio, Phys. Rev. **122**, 345 (1961).
 - [25] A. Kocic and J. B. Kogut, Nucl. Phys. **B422**, 593 (1994).
 - [26] J. Zinn-Justin, *Quantum Field Theory and Critical Phenomena* (Clarendon, Oxford, 1989).
 - [27] M. Harada, Y. Kikukawa, T. Kugo, and H. Nakano, Prog. Theor. Phys. **92**, 1161 (1994).
 - [28] K. Aoki, K. Morikawa, Jun Sumi, H. Terao, and M. Tomoyose, Prog. Theor. Phys. **102**, 1151 (1999).
 - [29] K. I. Aoki, K. Morikawa, J. I. Sumi, H. Terao, and M. Tomoyose, Phys. Rev. D **61**, 045008 (2000).
 - [30] K. Kubota and H. Terao, Prog. Theor. Phys. **102**, 1163 (1999).
 - [31] K. Kondo, M. Tanabashi, and K. Yamasaki, Prog. Theor. Phys. **89**, 1249 (1993).
 - [32] M. Hortaçsu and B. C. Lütfüoğlu, Phys. Rev. D **76**, 025013 (2007).
 - [33] P. A. M. Dirac, *Lectures on Quantum Mechanics* (Belfer Graduate School of Science, Yeshiva University, New York, 1964).
 - [34] D. Amati, R. Barbieri, A. C. Davis, and G. Veneziano,

- Phys. Lett. **102B**, 408 (1981).
- [35] W. A. Bardeen, C. N. Leung, and S. T. Love, Phys. Rev. Lett. **56**, 1230 (1986).
- [36] C. N. Leung, S. T. Love, and W. A. Bardeen, Nucl. Phys. **B273**, 649 (1986).
- [37] J. Kubo, K. Sibold, and W. Zimmermann, Phys. Lett. B **220**, 191 (1989).
- [38] B. Pendleton and G. G. Ross, Phys. Lett. **98B**, 291 (1981).