

Loop quantum dynamics of the Schwarzschild interiorChristian G. Böhrer^{1,2,*} and Kevin Vandersloot^{2,3,†}¹*Department of Mathematics, University College London, Gower Street, London, WC1E 6BT, United Kingdom*²*Institute of Cosmology & Gravitation, University of Portsmouth, Portsmouth PO1 2EG, United Kingdom*³*Institute for Gravitational Physics and Geometry, Physics Department, Pennsylvania State University, University Park, Pennsylvania 16802, USA*

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We examine the Schwarzschild interior of a black hole, incorporating quantum gravitational modifications due to loop quantum gravity. We consider an improved loop quantization using techniques that have proven successful in loop quantum cosmology. The central Schwarzschild singularity is resolved and the implications for the fate of an in-falling test particle in the interior region is discussed. The singularity is replaced by a Nariai type universe. We discuss the resulting conformal diagram, providing a clear geometrical interpretation of the quantum effects.

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I. INTRODUCTION

A long-held expectation is that the singularities predicted from general relativity signal a breakdown of the classical theory requiring a more proper accounting of the quantum effects of gravity. The situation is analogous to the hydrogen atom in ordinary mechanics, where classically the electron is expected to in-spiral towards the proton leading to a singularity. This behavior is cured when quantum mechanics is taken into account leading to a stable nonsingular hydrogen atom. In general relativity, two of the most relevant forms of singularities are the big-bang cosmological singularity, as well as black-hole singularities. A central question to be answered by any quantum theory of gravity is whether these singularities are regulated by incorporating quantum effects. Furthermore, if a quantum theory of gravity were to resolve the classical singularities, it would be of paramount interest as to what replaces them dynamically.

A leading candidate theory of quantum gravity is known as loop quantum gravity (LQG) (for reviews see [1–3]). The application of loop quantum gravity techniques to cosmological models is known as loop quantum cosmology [4] which has led to a resolution of the big-bang singularity, replacing it with a big-bounce for homogeneous and isotropic models [5–7]. These results provide tantalizing first hints that loop quantum gravity may indeed provide the singularity resolution hoped for in a quantum theory of gravity. Thus the next step is to consider loop quantum gravity for the black-hole scenario.

The simplest first step in considering loop quantum black holes consists of examining the interior of a Schwarzschild black hole. There, the temporal and radial coordinates flip roles, and the interior becomes a Kantowski-Sachs cosmological model whereby the metric components are homogeneous and only time dependent.

The interior therefore can be quantized in a similar fashion as loop quantum cosmology leading to the possibility that the singularity is resolved as in the cosmological case. The loop quantization of the Schwarzschild interior has been initially developed in [8,9]. There, the quantization indicates that the quantum Einstein equations are nonsingular; in a similar way loop quantum cosmology was originally shown to be nonsingular [10]. However, the question of what replaces the black-hole singularity is not answered.

Therefore, we attempt to provide an answer as to what dynamics are predicted from the loop quantization of the Schwarzschild interior. Initial attempts in this direction are given in [11] which is based on the quantum Einstein equations of the original quantization of [8]. However, this quantization is the direct analog of the original formulations of loop quantum cosmology which have been shown to not have a good semiclassical limit [5]. In particular, a crucial parameter in the construction of the Hamiltonian constraint operator is taken to be a constant in the early constructions, and it is this assumption that can spoil the semiclassical limit in loop quantum cosmology. An improved quantization of loop quantum cosmology whereby the parameter is assumed to explicitly depend on the scale factor has been applied in [5] and leads to the correct semiclassical behavior as well as more physically sensible quantum corrections to the standard cosmological dynamics.

In this paper, we consider the improved quantization scheme of loop quantum cosmology applied to the loop quantization of the Schwarzschild interior. We will consider the resulting modifications to the classical interior by examining effective semiclassical equations that incorporate quantum effects. While we do not have a complete handling of the physical sector of the quantum theory owing to its complexity, the effective semiclassical description is intended to be an approximate shortcut to the physical predictions of the quantum theory. In the framework of the effective description, we will show that the singularity is resolved and discuss the implications for an

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in-falling test particle as well as discuss the resulting conformal diagram for the nonsingular interior region.

II. CLASSICAL FRAMEWORK

A. Classical Hamiltonian

The interior of the Schwarzschild geometry can be described by a Kantowski-Sachs homogeneous model whereby the line element is given by

$$ds^2 = -N(t)^2 dt^2 + g_{xx}(t) dx^2 + g_{\Omega\Omega}(t) d\Omega^2 \quad (1)$$

with $N(t)$ being the freely specifiable lapse function and $d\Omega^2$ representing the unit two-sphere metric given in polar coordinates,

$$d\Omega^2 = d\theta^2 + \sin^2\theta d\phi^2. \quad (2)$$

The topology of the spatial slices is $\mathbb{R} \times S^2$, hence the coordinates range from $x \in \mathbb{R}$, $\theta \in [0, \pi]$, $\phi \in [0, 2\pi]$. In the standard Schwarzschild solution, the metric components are given by

$$N(t)^2 = \left(\frac{2m}{t} - 1\right)^{-1}, \quad g_{xx}(t) = \left(\frac{2m}{t} - 1\right), \quad (3)$$

$$g_{\Omega\Omega}(t) = t^2$$

for t in the range $t \in [0, 2m]$, where m is the mass of the black hole.

Loop quantum gravity is based on a connection-triad Hamiltonian formulation of general relativity. Considering connections and triads that are symmetric under the Kantowski-Sachs symmetry group $\mathbb{R} \times \text{SO}(3)$, the connection A_a^i and triad E_i^a after gauge fixing of the Gauss constraint are of the form [8]

$$A_a^i \tau_i dx^a = \tilde{c} \tau_3 dx + \tilde{b} \tau_2 d\theta - \tilde{b} \tau_1 \sin\theta d\phi + \tau_3 \cos\theta d\phi, \quad (4)$$

$$E_i^a \tau_i \partial_a = \tilde{p}_c \tau_3 \sin\theta \partial_x + \tilde{p}_b \tau_2 \sin\theta \partial_\theta - \tilde{p}_b \tau_1 \partial_\phi, \quad (5)$$

where the dynamical phase-space variables \tilde{b} , \tilde{c} , \tilde{p}_b , \tilde{p}_c are all only functions of time, and τ_i are $SU(2)$ generators satisfying $[\tau_i, \tau_j] = \epsilon_{ij}^k \tau_k$. The relationship between the triad variables \tilde{p}_b , \tilde{p}_c and the metric variables g_{xx} , $g_{\Omega\Omega}$ is given through the general relation

$$E_i^a E_j^b = \det(q) q^{ab}, \quad (6)$$

which explicitly implies

$$g_{xx} = \frac{\tilde{p}_b^2}{|\tilde{p}_c|}, \quad (7)$$

$$g_{\Omega\Omega} = |\tilde{p}_c|. \quad (8)$$

We thus see that the triad component \tilde{p}_c directly determines the physical radius of the two-sphere which is proportional to $\sqrt{g_{\Omega\Omega}}$. We will use this fact when interpreting the quantum dynamics.

The classical Hamiltonian consists of a single constraint known as the Hamiltonian constraint. The explicit form of the Hamiltonian is given by

$$H = \int \frac{-N}{8\pi G \gamma^2} \left[2\tilde{b} \tilde{c} \sqrt{\tilde{p}_c} + (\tilde{b}^2 + \gamma^2) \frac{\tilde{p}_b}{\sqrt{\tilde{p}_c}} \right] dx d\Omega. \quad (9)$$

Performing the spatial integrations we run into a problem as the integral over x diverges. Following [8], if we restrict the integration over x to a finite interval L_0 , the Hamiltonian becomes

$$H = \frac{-N}{2G\gamma^2} \left[2bc\sqrt{p_c} + (b^2 + \gamma^2) \frac{p_b}{\sqrt{p_c}} \right] \quad (10)$$

written in terms of rescaled variables

$$b = \tilde{b}, \quad c = L_0 \tilde{c}, \quad p_b = L_0 \tilde{p}_b, \quad p_c = \tilde{p}_c. \quad (11)$$

The Poisson structure of these phase-space variables is given by the only nonvanishing Poisson brackets

$$\{b, p_b\} = G\gamma, \quad (12)$$

$$\{c, p_c\} = 2G\gamma. \quad (13)$$

Written in terms of these variables, the metric (1) is given by

$$ds^2 = -N^2 dt^2 + \frac{p_b^2}{|p_c|} \frac{dx^2}{L_0^2} + |p_c| d\Omega^2. \quad (14)$$

The equations of motion are given as in an ordinary Hamiltonian dynamical system based on the Hamiltonian (10). The dynamical behavior of the phase-space variables is governed by Hamilton's equations which give for instance

$$\dot{p}_b = \{p_b, H\} = -G\gamma \frac{\partial H}{\partial b} \quad (15)$$

and so forth. Additionally, the lapse N appears in the Hamiltonian as a Lagrange multiplier, and this implies that the Hamiltonian must vanish leading to an additional constraint on the dynamics

$$H = 0. \quad (16)$$

We will next consider two classes of solutions that will be relevant to the discussion. The first consists of the usual Schwarzschild interior we are most interested in. The second corresponds to the Nariai solution which will be of relevance when we discuss the modified quantum dynamics.

B. Schwarzschild solution

To arrive at the Schwarzschild interior solution let us first choose for simplicity the lapse (for $b \neq 0$) to be given by

$$N = \frac{\gamma\sqrt{p_c}}{b}, \quad (17)$$

whence the Hamiltonian becomes

$$H = \frac{-1}{2G\gamma}[2cp_c + (b + \gamma^2/b)p_b]. \quad (18)$$

The equations of motion are given by

$$\dot{b} = -\frac{1}{2}(b + \gamma^2/b), \quad (19)$$

$$\dot{c} = -2c, \quad (20)$$

$$\dot{p}_b = \frac{p_b}{2}(1 - \gamma^2/b^2), \quad (21)$$

$$\dot{p}_c = 2p_c, \quad (22)$$

$$H = 0 \Rightarrow 2c = -\frac{p_b}{p_c}(b + \gamma^2/b). \quad (23)$$

The solution to these equations of motion is given by

$$p_b(T) = p_b^{(0)} e^T \sqrt{2me^{-T} - 1}, \quad (24)$$

$$p_c(T) = e^{2T}, \quad (25)$$

$$b(T) = \pm \gamma \sqrt{2me^{-T} - 1}, \quad (26)$$

$$c(T) = -\gamma m p_b^{(0)} e^{-2T}, \quad (27)$$

with $p_b^{(0)}$ a constant of motion. Transforming to a new time coordinate $T = \log t$, we have $NdT = \gamma \frac{\sqrt{p_c}}{b} \times \frac{dt}{t} = 1/\sqrt{2m/t - 1} dt$ and the metric (14) is given by

$$ds^2 = -\frac{1}{\frac{2m}{t} - 1} dt^2 + \left(\frac{p_b^{(0)}}{L_0^2}\right)^2 \left(\frac{2m}{t} - 1\right) dx^2 + t^2 d\Omega^2. \quad (28)$$

If we then identify $p_b^{(0)} = L_0$, we arrive at the Schwarzschild interior solution (3).

In these coordinates, the singularity is located at $t = 0$, while the horizon is at $t = 2m$. We can identify the singularity and horizon in a coordinate invariant fashion by noting that at the singularity both triad components p_c and p_b vanish. The horizon then occurs where $p_b = 0$ and $p_c = 4m^2$.

Before moving to the second class of solutions let us discuss certain gauge freedoms of the Schwarzschild solution. Classically there are two gauge freedoms corresponding to coordinate changes in either t or x variables. A change in the time coordinate t leads to a change in the lapse leaving all physics unchanged. On the other hand if we consider a coordinate rescaling of x , L_0 is correspondingly rescaled such that the ratio dx/L_0 is left invariant.

This implies that a rescaling of x does not rescale the value of p_b . Note that this point was mistaken in [8], where it was mentioned that a rescaling of $p_b^{(0)}$ can be accommodated by a rescaling in x .

In addition to the classical coordinate gauge freedoms, there is an additional gauge in the loop setup pertaining to the choice in restriction of the spatial integration leading to the L_0 parameter. From the definitions of p_b and c in (11), it is easy to see that if we choose a different size of L_0 then this will rescale the value of p_b and c and hence in terms of the Schwarzschild solution, $p_b^{(0)}$ will rescale accordingly.

The loop quantization is based on the phase-space variables p_b , p_c , b , c , all of which are invariant under a coordinate rescaling of x . The quantization is thus manifestly invariant under the first gauge transformation. However the quantum effects in principle can depend on the second gauge freedom which does not appear classically. This will be manifested in the quantum dynamics by a dependence of the physical results on the parameter $p_b^{(0)}$. Any dependence of the quantum physics on this gauge freedom implies the need for additional input into the theory (namely the specification of the value of $p_b^{(0)}$) which must be done by hand. We will show that the improved quantization we focus on in Sec. III B will be independent of $p_b^{(0)}$ and hence will not require additional input of its value.

C. Nariai solution

An additional classical solution that will be of relevance to the discussion of the effective dynamics is the Nariai solution [12,13]. It differs from the Schwarzschild solution in that the two-sphere radius determined from p_c is assumed to be constant and the model is sourced by a cosmological constant with the possible inclusion of charge. The assumption that p_c is a constant requires that $b = 0$ and hence the lapse (17) used in the Schwarzschild case becomes degenerate. We must therefore choose another form of the lapse to generate the Nariai solution.

Choosing the lapse to be equal to one and adding in a cosmological constant Λ leads to the Hamiltonian

$$H = \frac{-1}{2G\gamma^2} \left[2bc\sqrt{p_c} + (b^2 + \gamma^2) \frac{p_b}{\sqrt{p_c}} \right] + \frac{\Lambda}{2G} p_b \sqrt{p_c}. \quad (29)$$

We are interested in a solution where the radius of the two-sphere is constant implying $p_c = \text{const}$, such that $\dot{p}_c = 0$. The resulting equations of motion imply $b = 0$. Putting $b = 0$ into the equation for \dot{b} yields the relation between the constant radius of the sphere and the cosmological constant $p_c = 1/\Lambda$. The remaining two equations of motion simplify to

$$\dot{c} = \gamma\Lambda\sqrt{\Lambda}p_b, \quad (30)$$

$$\dot{p}_b = \frac{1}{\gamma} \frac{1}{\sqrt{\Lambda}} c, \quad (31)$$

which can be written as a single second order differential equation for p_b given by

$$\ddot{p}_b = \Lambda p_b. \quad (32)$$

The general solution is given by

$$p_b = c_1 \cosh(\sqrt{\Lambda}t) + c_2 \sinh(\sqrt{\Lambda}t) \quad (33)$$

where c_1 and c_2 are two constants of integration. To obtain the global cosmological metric of the Nariai universe we set $c_1 = 1/(\Lambda)$ and $c_2 = 0$ to get

$$p_b = \frac{1}{\Lambda} \cosh(\sqrt{\Lambda}t). \quad (34)$$

Hence, we obtained the following solution of the classical field equations:

$$ds^2 = -dt^2 + \frac{1}{\Lambda} \cosh^2(\sqrt{\Lambda}t) \frac{dx^2}{L_0^2} + \frac{1}{\Lambda} d\Omega^2; \quad (35)$$

compare for example [14,15], which corresponds to the uncharged Nariai universe.

Geometrically Nariai type spacetimes can be understood as four dimensional submanifolds of a flat six dimensional Lorentzian manifold being the product of two (three dimensional) spaces of constant curvature. If the respective length scales are denoted by a and b say, the generic Nariai type metric takes the form

$$ds^2 = a^2(-d\tau^2 + \cosh^2\tau d\chi^2) + b^2 d\Omega^2. \quad (36)$$

For $a^2 = b^2 = 1/\Lambda$, this corresponds to the uncharged Nariai universe, given by (35) upon a rescaling of the time $\tau = t/a$. If $a \neq b$, the spacetime corresponds to the charged Nariai universe. It should be noted that the knowledge of the metric alone does not suffice to distinguish between the two types of possible charges, electric or magnetic.

D. Radial geodesics

In order to interpret the quantum dynamics, we will also consider the trajectory of a radially in-falling test particle. The geodesics can be derived from the line element

$$ds^2 = -N(t)^2 dt^2 + \frac{p_b^2(t)}{|p_c(t)|} \frac{dx^2}{L_0^2} + |p_c(t)| d\Omega^2. \quad (37)$$

Since we are mainly interested in radial geodesics we henceforth assume $d\Omega^2 = \text{constant}$. The geodesic equations are most easily read off from the Lagrangian

$$-2\mathcal{L} = -N^2 \left(\frac{dt}{d\tau} \right)^2 + \frac{p_b^2}{p_c L_0^2} \left(\frac{dx}{d\tau} \right)^2, \quad (38)$$

where $\mathcal{L} = 0$ for a massless particle and $\mathcal{L} = 1/2$ for a massive particle, and τ refers to the proper time for a

massive particle. Since the Lagrangian (38) is independent of the variable x , its conjugate momentum π_x is conserved:

$$\pi_x = \frac{\partial \mathcal{L}}{\partial \dot{x}} = -\frac{dx}{d\tau} \frac{p_b^2}{p_c L_0^2} = \text{const} = -\mathcal{E} \quad (39)$$

where \mathcal{E} represents the total energy including the gravitational potential energy for a static observer at infinity. Since $dx/d\tau = \mathcal{E} p_c L_0^2 / p_b^2$ the Lagrangian can now be written as

$$-2\mathcal{L} = -N^2 \left(\frac{dt}{d\tau} \right)^2 + \frac{p_c L_0^2}{p_b^2} \mathcal{E}^2. \quad (40)$$

Hence, we obtain

$$\left(\frac{dt}{d\tau} \right)^2 = \left(\frac{p_c L_0^2}{p_b^2} \mathcal{E}^2 + 2\mathcal{L} \right) \frac{1}{N^2}. \quad (41)$$

In the classical Schwarzschild case, we have $p_c L_0^2 / (N^2 p_b^2) = 1$ and hence the geodesic equation can be understood as a point particle with energy \mathcal{E} moving in an effective potential given by $2\mathcal{L}/N^2$. Moreover, the geodesic equation is simple to interpret because of the simple relation between the coordinate parameter t and the two-sphere radius determined from p_c since we have $p_c = t^2$. However, when we come to the quantum dynamics, the relationship between the coordinate parameter t and p_c will become complicated and we will no longer be interested in the behavior of the coordinate parameter t as a function of proper time. We will therefore be most interested in $p_c(\tau)$ which can be derived from

$$\frac{dp_c}{d\tau} = \frac{dp_c}{dt} \frac{dt}{d\tau}, \quad (42)$$

where dp_c/dt is to be determined from the equations of motion derived from the Hamiltonian, and $dt/d\tau$ is given by (41).

Classically, for any given energy \mathcal{E} , the in-falling particle reaches the singularity at $p_c = 0$ in finite proper time. We will be interested in whether the quantum dynamics implies that the singularity is never reached by the in-falling particle. We will see that the loop quantum dynamics imply a minimum value of the two-sphere radius p_c and hence the solutions are nonsingular with an in-falling particle never reaching the classical singularity.

III. EFFECTIVE LOOP QUANTUM DYNAMICS

In attempting to ascertain the quantum corrections to the classical behavior implied by the loop quantization, we will work in an effective semiclassical description that leads to modifications of the classical Hamiltonian constraint. The effective description is motivated from the construction of the quantum Hamiltonian operator, although a more rigorous understanding of the quantum dynamics would require more sophisticated machinery. Our aim here is to describe the expected dominant correc-

tions arising from the quantum theory and as such, there is room for additional corrections that could modify some of the results presented here. Evidence that the effective theory we describe provides an accurate description of the quantum dynamics is given by homogeneous and isotropic loop quantum cosmology with a massless scalar field with or without a cosmological constant [5–7].

The main quantum correction arises from the fact that in the loop quantization no direct operators exist corresponding to the connection components b and c . Instead, the quantization proceeds as in loop quantum gravity by considering holonomies of the connection. The holonomies roughly consist of exponential terms like $\exp(ib\delta_b)$ with δ_b corresponding to the *edge length* of the holonomy. The end result in the effective Hamiltonian is to replace the classical b terms in the Hamiltonian (10) with $\sin(b\delta_b)/\delta_b$ terms for instance. Thus the δ_b, δ_c factors play a crucial role in determining the effects of the quantum corrections. The classical limit is recovered by taking the $\delta_b, \delta_c \rightarrow 0$ limits. The δ_b, δ_c parameters, arising from the holonomy edge lengths, are a measure of the quantum discreteness arising from the loop quantization.

In the original loop quantization of the Schwarzschild interior [8], the δ_b, δ_c terms were assumed to be a constant value labeled by δ analogous to the original quantization of loop quantum cosmology where similar terms appear in the Hamiltonian operator. In [5], it was shown that because of the assumption that δ is a constant (in loop quantum cosmology the parameter is labeled μ_0), loop quantum cosmology does not have the correct semiclassical limit. The solution to this issue presented in that article is to relax the assumption that the δ factors are constant and instead consider them to be functions of the triad variables. The precise prescription given leads to the nonconstant factor labeled $\bar{\mu}$ which varies inversely with the scale factor of the universe, and with this behavior the correct semiclassical limit is achieved [5].

Thus our goal will be to consider the effective dynamics with a quantization scheme arising from the improved prescription for loop quantum cosmology given in [5]. In particular, the δ_b, δ_c factors will depend explicitly on the triad components p_b and p_c . We will start however, with the case of constant δ since the effective dynamics are exactly solvable and allow us to contrast the modifications from those arising from the new nonconstant δ quantization. The results for the constant δ case have been previously derived in [11]. We merely repeat the results that are relevant to our discussion.

A. Constant δ quantum Hamiltonian

With the considerations of the previous discussion the quantum modifications can be understood as leading to an effective Hamiltonian with the holonomy parameter δ playing a crucial role in determining the magnitude of the corrections. The effective Hamiltonian constraint reads

[11]

$$H_\delta = -\frac{N}{2G\gamma^2} \left[2 \frac{\sin b \delta}{\delta} \frac{\sin c \delta}{\delta} \sqrt{p_c} + \left(\frac{\sin^2 b \delta}{\delta^2} + \gamma^2 \right) \frac{p_b}{\sqrt{p_c}} \right]. \quad (43)$$

It is easy to see that in the limit $\delta \rightarrow 0$, the classical Hamiltonian (10) is recovered. In analogy to the classical case, we choose $N = \gamma \sqrt{p_c} \delta / (\sin b \delta)$ and the equations for the pairs (b, p_b) and (c, p_c) decouple to two independent pairs of differential equations:

$$\dot{c} = 2\gamma G \frac{\partial H_\delta}{\partial p_c} = -2 \frac{\sin c \delta}{\delta}, \quad (44)$$

$$\dot{p}_c = -2\gamma G \frac{\partial H_\delta}{\partial c} = 2p_c \cos c \delta, \quad (45)$$

$$\dot{b} = \gamma \frac{\partial H_\delta}{\partial p_b} = -\frac{1}{2} \left(\frac{\sin b \delta}{\delta} + \frac{\gamma^2 \delta}{\sin b \delta} \right), \quad (46)$$

$$\dot{p}_b = -\gamma \frac{\partial H_\delta}{\partial b} = \frac{1}{2} \cos b \delta \left(1 - \frac{\gamma^2 \delta^2}{\sin^2 b \delta} \right) p_b. \quad (47)$$

The equation of motion for c can be integrated to give

$$c(T) = \frac{1}{\delta} \arctan \left(\mp \gamma m p_b^{(0)} \frac{\delta}{2} e^{-2T} \right), \quad (48)$$

where the constant of integration has been fixed so that the limit of the effective $c(T)$ as $\delta \rightarrow 0$ coincides with the classical expression (27). Next, the equations for p_c can be solved:

$$p_c(T) = e^{2T} + \gamma^2 m^2 (p_b^{(0)})^2 \frac{\delta^2}{4} e^{-2T}. \quad (49)$$

Also the equation for b can be integrated and yields

$$b(T) = \frac{1}{\delta} \arccos \left[\sqrt{1 + \gamma^2 \delta^2} \times \tanh \left\{ \sqrt{1 + \gamma^2 \delta^2} \left(\frac{T - \log 2m}{2} - \log \frac{\gamma \delta}{2} \right) \right\} \right]. \quad (50)$$

Finally, $p_b(T)$ can be obtained from the vanishing of the Hamiltonian (43), which gives

$$p_b = -\frac{\sin b \delta}{\delta} \frac{\sin c \delta}{\delta} \frac{2p_c}{\sin^2 b \delta / \delta^2 + \gamma^2}. \quad (51)$$

From the form of $b(T)$, it is evident that the evolution ends at a minimum value T_{\min} where the absolute value of the argument of the arccos function equals 1. Moreover, from the form of $p_c(T)$ it is evident that the group orbits also have a minimal value at $T = T_{\min} = \log(\gamma m p_b^{(0)} \delta / 2) / 2$, with p_c reaching its maximum value at the classical horizon and at T_{\min} . This minimal two-sphere radius is given from $p_c(T_{\min}) = \gamma m p_b^{(0)} \delta$, which depends on the mass of the black hole and also on $p_b^{(0)}$.

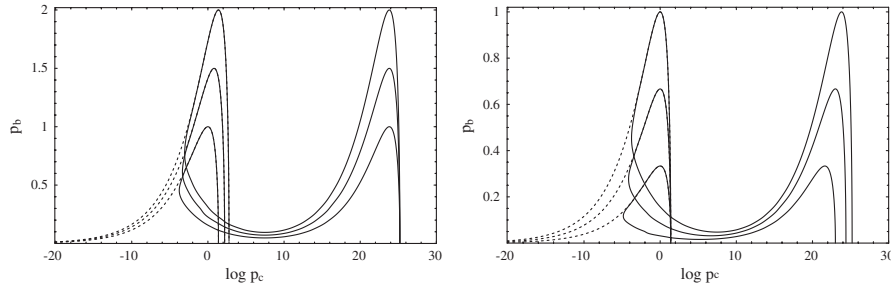


FIG. 1. These figures show the effective plots of $p_b = p_b(p_c)$ for the case with $\delta = \text{const}$. The dashed lines represent the classical solution while the solid lines correspond to the solution of the effective quantum theory with $\delta = 0.1$. Clearly, the initial singularity is removed and the spacetime contains a second black hole. The three plots on the left show the solution for varying mass parameter, $m = 1, 1.5, 2$, respectively. On the right $p_b^{(0)}$ is varied in the three plots, $p_b^{(0)} = 1, 2/3, 1/3$, respectively. One clearly notes (left) that the mass parameter m controls the mass of the first (classical) black only, while (right) the parameter $p_b^{(0)}$ controls the mass of the second black hole.

Figure 1 shows the plots of $p_b = p_b(p_c)$ for various values of m and $p_b^{(0)}$. The singularity is thus avoided through a bounce in the two-sphere radius and the quantum dynamics matches two black holes together though in a generically nonsymmetric value. A symmetric solution can be constructed by fine-tuning the value of $p_b^{(0)}$.

As we have seen in the figures, the mass parameter m controls the mass of the first black hole whereas $p_b^{(0)}$ controls the mass of the second black hole. This suggests that in the effective quantum theory, $p_b^{(0)}$ would not be regarded as a meaningless constant of integration or a certain choice of gauge. For the constant δ quantization of the Schwarzschild interior spacetime this parameter acquires an important physical meaning, and two different choices correspond to different effective quantum theories. Any physical results derived from the constant δ quantization must include a prescription for specifying the value of $p_b^{(0)}$ which has to be input by hand.

B. Improved quantum Hamiltonian

We now wish to relax the assumption that the δ parameters are constant. In accordance with the results from loop quantum cosmology [5], we will assume that δ_b, δ_c depend on the triad variables p_c and p_b . The method to constrain the exact dependence relies on the fact that the holonomies used to construct the quantum Hamiltonian operator form closed loops. The behavior of δ_b and δ_c can then be constrained if we demand that the physical area of the closed loop be equal to the minimum area gap predicted by loop quantum gravity,

$$A_{\min} = \Delta = 2\sqrt{3}\pi\gamma l_{\text{Pl}}^2. \quad (52)$$

This procedure can be executed in different possible ways in the Kantowski-Sachs model considered here, whereas homogeneous isotropic models lead to a unique choice. We discuss in this section what we feel to be the most geometrically natural scheme. We will show in this

section that this scheme has the advantage that the resulting dynamics is independent of the $p_b^{(0)}$ parameter. We will discuss an alternative scheme in the appendix.

The procedure considered in this section amounts to evaluating the classical area of the holonomy loops and constraining the area based on this. For instance if we consider the holonomy loop in the x, θ plane, the classical physical area is given by

$$A_{x\theta} = \delta_b \delta_c p_b, \quad (53)$$

which is evident from the form of the metric (14). For the holonomies on the two-sphere, the loop does not close as discussed in [8], but we can assign an effective area given by

$$A_{\theta\phi} = \delta_b^2 p_c. \quad (54)$$

If we constrain these areas to be equal to A_{\min} we have the following behavior:

$$\delta_b = \frac{\sqrt{\Delta}}{\sqrt{p_c}}, \quad (55)$$

$$\delta_c = \sqrt{\Delta} \frac{\sqrt{p_c}}{p_b}. \quad (56)$$

With this, the effective Hamiltonian constraint reads

$$H_\delta = -\frac{N}{2G\gamma^2} \left[2 \frac{\sin b \delta_b}{\delta_b} \frac{\sin \delta_c c}{\delta_c} \sqrt{p_c} + \left(\frac{\sin^2 b \delta_b}{\delta_b^2} + \gamma^2 \right) \frac{p_b}{\sqrt{p_c}} \right]. \quad (57)$$

Once again, in the limit $\Delta \rightarrow 0$ the effective Hamiltonian (57) reduces to the classical Hamiltonian (10).

We can now derive the equations of motion as in the previous case. Choosing the lapse function to be $N = \gamma \sqrt{p_c} \delta_b / (\sin b \delta_b)$, the resulting equations of motion are given by

$$\begin{aligned} \dot{c} = & -\frac{\text{sinc}\delta_c}{\delta_c} - \frac{1}{2} \frac{\text{sin}\delta_b b}{\delta_c} - c \text{csc}\delta_c \\ & + \frac{1}{2} \frac{\delta_b}{\delta_c} b \text{cos}b\delta_b \left(1 - \frac{\gamma^2 \delta_b^2}{\text{sin}^2 b \delta_b}\right) + \frac{\gamma^2}{2} \frac{\delta_b^2}{\delta_c} \frac{1}{\text{sin}b\delta_b}, \end{aligned} \quad (58)$$

$$\dot{p}_c = 2p_c \text{csc}\delta_c, \quad (59)$$

$$\dot{b} = -\frac{1}{2} \left(\frac{\text{sin}b\delta_b}{\delta_b} + \frac{\gamma^2 \delta_b}{\text{sin}b\delta_b} \right) - \frac{\text{sinc}\delta_c}{\delta_b} + \frac{\delta_c}{\delta_b} c \text{csc}\delta_c, \quad (60)$$

$$\dot{p}_b = \frac{1}{2} \text{cos}b\delta_b \left(1 - \frac{\gamma^2 \delta_b}{\text{sin}^2 b \delta_b}\right) p_b. \quad (61)$$

Before considering numerical solutions to these equations, we can get an idea on where the quantum modifications should be appreciable. It is evident from the effective Hamiltonian that the modifications arise when either $c\delta_c$ or $b\delta_b$ are of the order of 1. Near the classical singularity we have $p_b \rightarrow 0$, $p_c \rightarrow 0$, $b \rightarrow \pm\infty$, and $c \rightarrow -\infty$ in such a way that $b\delta_b$ and $c\delta_c$ blow up, hence we expect the quantum corrections to become important near the singularity. If we also consider the horizon, we have $p_b \rightarrow 0$, $p_c \rightarrow (2m)^2$, $b \rightarrow \pm 0$, and $c \rightarrow -\gamma m p_b^{(0)}/(2m)^2$ such that $b\delta_b$ is small, but $c\delta_c$ diverges. Thus we can expect quantum corrections also as the classical horizon is approached. Therefore, the picture that arises is a region in the interior where classical behavior is recovered with quantum effects near the classical singularity and also near the classical horizon. This behavior is borne out in the numerical simulations which we now discuss. We focus in this article on the effects near the singularity and will comment later on the horizon effects. Accordingly, in the numerical simulations we will specify initial conditions starting in the interior region where the quantum effects are negligible and evolve towards the singularity.

To solve the equations of motion numerically, we start at some initial point near the horizon with initial conditions $p_c(T_0)$, $p_b(T_0)$, $b(T_0)$ that match the classical solutions (24)–(26) for some initial T_0 , and the constraint $H_\delta = 0$ is then solved to get $c(T_0)$. The coupled differential equations of motion are then solved for the approach towards the classical singularity ($T = -\infty$). The results of the simulations for p_b as a function of p_c appear in Fig. 2 for various initial conditions corresponding to different values of the classical black-hole mass m as well as different values of $p_b^{(0)}$. The results all indicate that the classical singularity is resolved with the two-sphere radius p_c being bounded from below. The second figure in Fig. 2 indicates that choosing different values of $p_b^{(0)}$ only rescale $p_b(T)$ as in the classical case, and thus the effective dynamics predicted is insensitive to this classical gauge choice as opposed to the constant δ case.

Unlike the constant δ case of Sec. III A, the solutions do not match up to a separate black-hole solution. Instead, the solutions asymptote to a Nariai type solution for $T \rightarrow -\infty$. The solution is characterized by

$$b = \bar{b}, \quad pb = \bar{p}_b e^{-\alpha T}, \quad c = \bar{c} e^{-\alpha T}, \quad p_c = \bar{p}_c, \quad (62)$$

where the barred quantities and α are constants. Figure 3 shows this behavior with plots of p_c , b and the ratio of c/p_b as functions of time. It is clear that p_c undergoes damped oscillations settling in to a fixed finite value as T goes to minus infinity. Similarly the ratio of c/p_b settles into a finite value given by \bar{c}/\bar{p}_b as expected from a Nariai type solution.

From the equations of motion we can determine the asymptotic values of the constants \bar{p}_c , \bar{b} , α , and \bar{c}/\bar{p}_b . Assuming $p_c = \bar{p}_c$ is a constant, Eq. (59) implies

$$\text{csc}\delta_c = \cos\left(\sqrt{\Delta} \frac{\sqrt{\bar{p}_c} \bar{c}}{\bar{p}_b}\right) = 0, \quad (63)$$

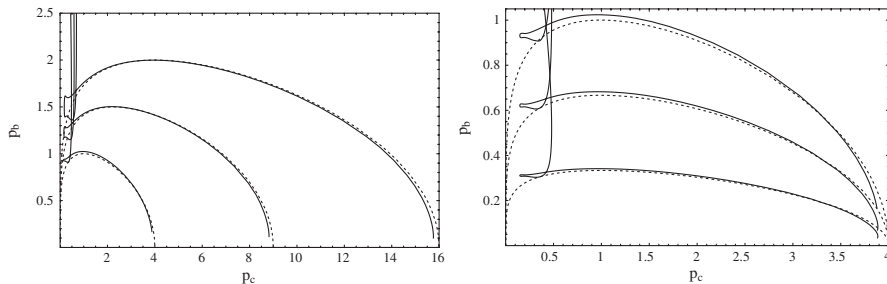


FIG. 2. These figures show the effective plots of $p_b = p_b(p_c)$ for the case where δ_b , δ_c are not constants. The dashed lines represent the classical solution while the solid lines correspond to the solution of the effective quantum theory. The initial singularity is removed, however, not with a second black hole. The three plots on the left show the solution for varying mass parameter, $m = 1, 1.5, 2$, respectively. On the right $p_b^{(0)}$ is varied in the three plots, $p_b^{(0)} = 1, 2/3, 1/3$, respectively. One verifies that this quantization scheme does not depend on the choice of $p_b^{(0)}$ and thus the effective theory is independent of this gauge freedom.

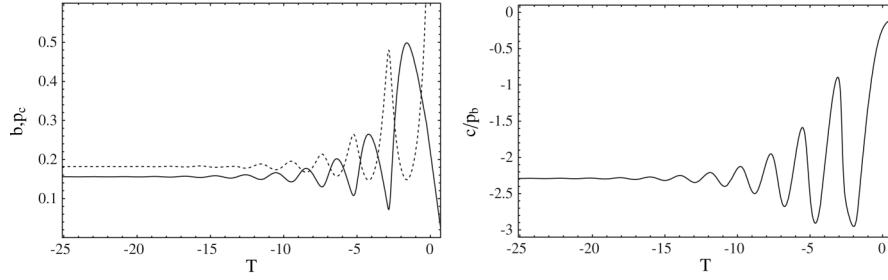


FIG. 3. In the left figure $b(T)$ (solid) and $p_c(T)$ (dashed) are plotted. Both tends towards constants as $T \rightarrow -\infty$. The right figure shows the ratio $c(T)/p_b(T)$ also tends to a constant asymptotically. This asymptotic behavior is indicative of a Nariai type metric of the form given in Eq. (36).

$$\cos\left(\sqrt{\Delta} \frac{\sqrt{\bar{p}_c}}{\bar{p}_b} \bar{c}\right) = 0, \quad \frac{\bar{c}}{\bar{p}_b} = -\frac{\pi}{2\sqrt{\Delta\bar{p}_c}}, \quad (64)$$

which also yield $\sin\bar{\delta}_c\bar{c} = -1$. Assuming furthermore that $b = \bar{b}$ is constant, the resulting equation of motion for \dot{b} , namely, Eq. (60), yields

$$2 \sin\left(\sqrt{\Delta} \frac{\bar{b}}{\sqrt{\bar{p}_c}}\right) - \sin\left(\sqrt{\Delta} \frac{\bar{b}}{\sqrt{\bar{p}_c}}\right)^2 = \frac{\Delta\gamma^2}{\bar{p}_c}. \quad (65)$$

The same equation is obtained by evaluating the vanishing of the Hamiltonian constraint (57). Next, Eq. (61) implies that $p_b(T) = \bar{p}_b e^{-\alpha T}$ and fixes the constant α to be

$$\alpha = -\cos\left(\sqrt{\Delta} \frac{\bar{b}}{\sqrt{\bar{p}_c}}\right) + \cot\left(\sqrt{\Delta} \frac{\bar{b}}{\sqrt{\bar{p}_c}}\right). \quad (66)$$

Finally, the above Eqs. (64)–(66) can be inserted in the equation of motion for c leading to

$$\begin{aligned} & -2 + \sin\left(\sqrt{\Delta} \frac{\bar{b}}{\sqrt{\bar{p}_c}}\right) - \left(\frac{\pi}{2} + \sqrt{\Delta} \frac{\bar{b}}{\sqrt{\bar{p}_c}}\right) \\ & \times \left(\cos\left(\sqrt{\Delta} \frac{\bar{b}}{\sqrt{\bar{p}_c}}\right) - \cot\left(\sqrt{\Delta} \frac{\bar{b}}{\sqrt{\bar{p}_c}}\right)\right) = 0. \end{aligned} \quad (67)$$

For a given value of Δ this equation can be solved for the

value of the ratio $\bar{b}/\sqrt{\bar{p}_c}$. A find root algorithm is necessary since an analytical solution cannot be obtained. The ratio $\bar{b}/\sqrt{\bar{p}_c}$ then specifies the constant in the exponent α by Eq. (66). Moreover, with γ , Δ , and $\bar{b}/\sqrt{\bar{p}_c}$ given, the relation (65) can be used to extract the value of \bar{p}_c which in turn yields the value of \bar{b} . Finally, Eq. (64) fixes the ratio \bar{c}/\bar{p}_b . However, there is a remaining scaling freedom in \bar{c} and \bar{p}_b since only their ratio is specified. Note that this implies that rescalings in \bar{p}_b have to be compensated by simultaneous rescalings in \bar{c} .

From the previous equations it is clear that the asymptotic value of \bar{p}_c depends only on the constants Δ and γ as

$$\bar{p}_c = \gamma^2 g(\Delta) \quad (68)$$

where $g(\Delta)$ is some function determined from the solutions of Eqs. (65) and (67). For the natural choice $\Delta = 2\sqrt{3}\pi\gamma l_{\text{Pl}}^2$ the following asymptotic values are obtained:

$$\begin{aligned} \bar{b} &\approx 0.156, & \bar{p}_c &\approx 0.182l_{\text{Pl}}^2, & \alpha &\approx 0.670 \\ \bar{c}/\bar{p}_b &\approx -2.290m_{\text{Pl}}^2, & \bar{N} &\approx 0.689, \end{aligned} \quad (69)$$

where \bar{N} is the asymptotic value of the lapse which also behaves as a constant. These values agree with the asymp-

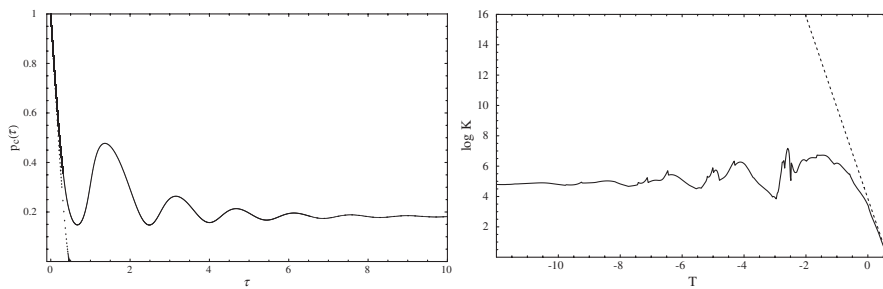


FIG. 4. Left: p_c as a function of proper time of radially in-falling geodesic. The black dashed line represents the classical behavior indicating that the particle reaches the singularity in finite proper time. The solid line is the geodesic inside the quantum black hole. The particle approaches the singularity but undergoes damped oscillations at a fixed two-sphere radius eventually settling in at a fixed value \bar{p}_c . For the natural values of the parameters γ , Δ , the two-sphere radius is on the order of the Planck length. Right: The logarithm of the Kretschmann invariant as a function of the time coordinate T is plotted. The dashed line is the classical value which behaves as $\exp(-6T)$ and diverges at the singularity $T = -\infty$. The solid line represents the quantum value which asymptotes to a fixed value which again indicates a resolution of the singularity. The numerical asymptotic values obtained for p_c and the Kretschmann invariant K are $p_c \approx 0.182l_{\text{Pl}}^2$ and $K \approx 124.35m_{\text{Pl}}^4$ which are both independent of the black-hole mass.

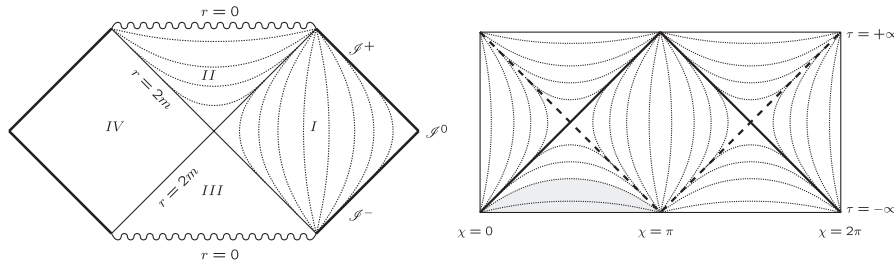


FIG. 5. The left figure shows the Carter-Penrose diagram for the Schwarzschild spacetime. I^+ and I^- denote future and past null infinity, respectively. The usual Schwarzschild metric in curvature coordinates only covers region I . The Kantowski-Sachs form of the line element covers region II . The dotted lines represent the $r = \text{constant}$ lines in the respective regions. The right figure shows the Carter-Penrose diagram of the nonsingular charged Nariai universe. The $\chi = 0$ and $\chi = 2\pi$ lines are identified, and the dotted lines stand for the constant two-sphere radii. The solid and dashed diagonal lines represent the disconnected future and past event horizon, respectively. Every point of this diagram represents a sphere of the same area. For later purpose we have indicated a shaded region.

otic region of the numerical solution that we studied; see, in particular, Fig. 3.

The fact that p_c asymptotes to a fixed value on the order of the Planck length squared implies that the radius of the two-sphere asymptotes to a value on the order of the Planck length. Furthermore this value is independent of the mass of the black hole. The interpretation of the effective dynamics becomes clear if we consider the radially in-falling test particle using Eq. (42). A plot of the numerical integration of $p_c(\tau)$ is shown in Fig. 4. An in-falling particle classically would reach the singularity at $p_c = 0$ in finite proper time as evidenced by the solid black line, however as a result of the quantum effects, the particle oscillates about the fixed value of \bar{p}_c . The particle thus settles in at a fixed Planckian radius. The geodesic reaches this asymptotic value in infinite proper time and hence we can say that the singularity is truly absent in the quantum modified spacetime. The fate of an in-falling test particle is also of interest in the context of the information loss paradox. The results are suggestive that the black hole can store information infinitely long, and strictly speaking no information can ever be lost. However, once the thermal black-hole radiation is taken into account, things are still more involved. In this respect, however, we would like to point out that the improved quantization scheme also leads to quantum modification of the horizon. In the context of

Euclidean quantum gravity, it has been argued in [16] that indeed horizon modification might resolve the paradox.

We have also considered the behavior of the Kretschmann invariant, the square of the Riemann tensor $K = R_{abcd}R^{abcd}$. For the metric (14), written out explicitly, this yields a rather complicated expression. However, plotting this quantity for our numerical solution also shows that the geometry of the spacetime changes significantly, from the classical Schwarzschild-like behavior where $K = 48m^2 \exp(-6T)$ (which diverges at $T = -\infty$) towards a space of constant curvature, as expected for the Nariai type metrics whose form is given by

$$K = 4\left(\frac{\alpha^4}{\bar{N}^2} + \frac{1}{\bar{p}_c^2}\right). \tag{70}$$

Plugging in the theoretical values in (69) gives an expected value

$$K \approx 124.36 m_{\text{pl}}^4 \tag{71}$$

which is in good agreement with the numerical results. Furthermore this asymptotic value is independent of the black-hole mass. We find that the quantum effects become appreciable when the Kretschmann curvature invariant approaches the Planck scale. Figure 4 shows a plot indicating this behavior.

The quantum effects removing the singularity can best be visualized by Carter-Penrose diagrams; see e.g. [17]. The classical diagrams for both the Schwarzschild and Nariai spacetimes are given in Fig. 5. The quantum effects remove the classical Schwarzschild singularity shown as the wavy line at the top of region II in the Carter-Penrose diagram and glue the diagram to the shaded region of the Nariai diagram. The resulting diagram of the interior is shown in Fig. 6.

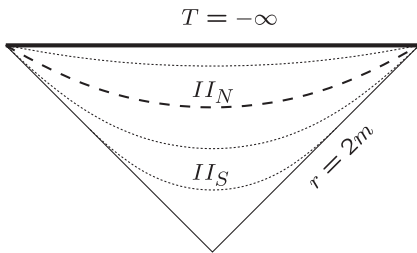


FIG. 6. In region II we have indicated a specific $r = \text{constant}$ line. This line separates the region with decreasing group orbits (II_S) from the region where the 2-spheres have (effectively) constant volume (II_N), which defines the Nariai spacetime.

IV. DISCUSSION

At the level of the effective description we have presented, we have been able to answer the questions posed at

the beginning, namely, what replaces the singularity in the Schwarzschild interior. In all of the quantization schemes we have discussed, singularity avoidance arises naturally thus providing further evidence that loop quantum gravity successfully resolves the classical black-hole singularity. We have seen two possible replacements for the singularity, one in which quantum effects connect two separate (nonidentical) black holes resembling a wormholelike solution. The other solution predicts repulsive gravitational effects leading to an equilibrium two-sphere radius to which an in-falling particle would asymptote.

The results are suggestive but cannot be considered as a proof that the LQG resolves the black-hole singularity. The effective equations considered here are not rigorously derived from the quantum theory and thus can only be considered approximations of the quantum dynamics. Since understanding the quantum dynamics is difficult, we do not have a complete handle on when the approximation might break down. As was mentioned however, the effective equations have approximated the quantum dynamics in the models where both can be developed completely, namely, the homogeneous and isotropic models of LQC sourced with a massless scalar field.

Some initial attempts at constructing semiclassical states for the constant δ quantization have been made in [18,19], and it is worthwhile to compare our results with the previous work. In the works cited, numerical instabilities of the difference equation are found which bring into question whether any effective picture is valid near the singularity. In [18] it is argued that no proper semiclassical states exist unless the Immirzi parameter γ is less than the accepted value in the LQG literature raising a further question. In this article, our aim has been to probe the improved, non-constant δ quantization and thus we do not see a direct contradiction. In particular, the bound on the Immirzi parameter depends explicitly on the constant value of δ and so it is not clear if a bound exists in the improved quantization. In addition, both papers evolve Gaussian semiclassical states with the quantum difference equation which seem to imply that semiclassicality breaks down near the singularity. The numerical results are based on using the triad component p_c as an internal clock, however, our results suggest that p_c bounces and hence would not play the role of a good clock. Thus to reconcile our results with those of [18,19], the numerical evolutions would probably require different initial conditions to incorporate both the expanding and contracting parts of the wave-packets at the initial instance in “time.” This issue is solved in the isotropic case by including a scalar field that plays the role of a global internal clock. We thus feel that our results are not in conflict with [18,19], and that all of our works require further development of the quantum dynamics including the specification of a good internal clock, physical inner product, and Dirac observables.

Additionally, the validity of the Kantowski-Sachs description of the interior could be questioned. Boundary

conditions must be provided for the numerical evolution at the horizon which are prescribed using classical solutions. However, this could be modified when considering a true quantization of the full spacetime. In particular, the improved quantization scheme described in Sec. III B leads to modifications at the horizon which may require input from the full quantization. Additionally, by considering the homogeneous interior, we cannot probe a realistic collapse scenario.

Therefore, an interesting next step would be to apply the techniques described here to the inhomogeneous spherically symmetric quantization. The results described in [20] provide the initial steps at an amenable quantization, though the results presented there are only valid in the exterior. If the results can be extended to cover the whole spacetime, then an effective dynamics analysis could be extended. With this model, one could analyze a genuine collapse scenario.

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APPENDIX: ALTERNATIVE QUANTUM HAMILTONIAN

Here we consider an alternate behavior for the δ factors that are more heuristically motivated. In the language of [21] this corresponds to a lattice refinement model whereby the number of vertices is proportional to the transverse area. In addition this scheme has been considered in anisotropic Bianchi I models in [22,23]. A stability analysis indicates that this scheme leads to an unstable difference equation [21], thus it may not represent a good quantization scheme. An advantage of this scheme is that the quantum difference equation is much simpler than in the previous one and more amenable to a proper analysis of the true quantum dynamics, although the instability of the difference equation could spoil this.

In this scheme we have the δ factors behaving as

$$\delta_b = \frac{\sqrt{\Delta}}{\sqrt{p_b}}, \quad (\text{A1})$$

$$\delta_c = \frac{\sqrt{\Delta}}{\sqrt{p_c}}. \quad (\text{A2})$$

Again the effective Hamiltonian is given by

$$H_\delta = -\frac{N}{2G\gamma^2} \left[2 \frac{\sin b \delta_b}{\delta_b} \frac{\sin \delta_c c}{\delta_c} \sqrt{p_c} + \left(\frac{\sin^2 b \delta_b}{\delta_b^2} + \gamma^2 \right) \frac{p_b}{\sqrt{p_c}} \right], \quad (\text{A3})$$

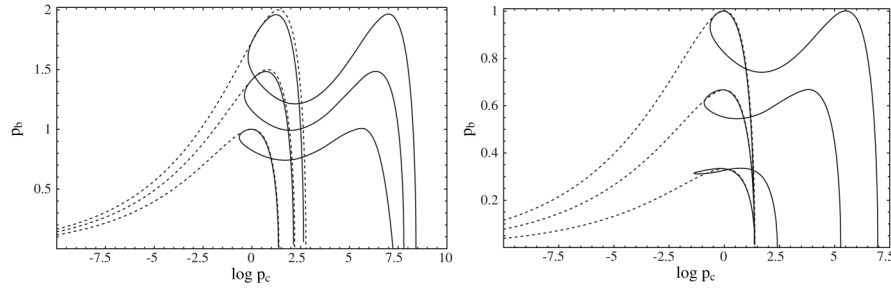


FIG. 7. These figures show the effective plots of $p_b = p_b(p_c)$. The dashed lines represent the classical solution while the solid lines correspond to the solution of the effective quantum theory. The initial singularity is removed and the spacetime is connected to a second black hole. The three plots on the left show the solution for varying mass parameter, $m = 1, 1.5, 2$, respectively. On the right $p_b^{(0)}$ is varied in the three plots, $p_b^{(0)} = 1, 2/3, 1/3$, respectively.

leading to equations of motion with lapse $N = \gamma\sqrt{p_c}\delta_b/(\sin b\delta_b)$,

$$\dot{c} = c \cos c \delta_c - 3 \frac{\sin c \delta_c}{\delta_c}, \quad (\text{A4})$$

$$\dot{p}_c = 2p_c \cos c \delta_c, \quad (\text{A5})$$

$$\dot{b} = -\frac{3 \sin b \delta_b}{4\delta_b} + \frac{b \cos b \delta_b}{4} - \frac{\gamma^2 \delta_b}{4 \sin b \delta_b} - \frac{\gamma^2 b \delta_b^2 \cos b \delta_b}{4 \sin^2 b \delta_b}, \quad (\text{A6})$$

$$\dot{p}_b = \frac{1}{2} p_b \cos b \delta_b - \frac{\gamma^2 p_b \delta_b^2 \cos b \delta_b}{2 \sin^2 b \delta_b}. \quad (\text{A7})$$

Again we can examine where the quantum corrections are appreciable by determining where $b\delta_b$ and $c\delta_c$ are on the order of 1. From the classical behavior, both $b\delta_b$ and $c\delta_c$ blow up at the singularity and thus quantum effects are expected there. At the horizon, $b\delta_b$ vanishes and $c\delta_c$ is small provided Δ is small. Hence with this quantization scheme, the effects near the horizon are minimal. The resulting numerical solutions are qualitatively very similar to those obtained for the constant δ quantization scheme, as shown in Fig. 7. As discussed above, the parameter $p_b^{(0)}$ controls the mass of the second black hole in the effective quantum theory which can be tuned to give a symmetric spacetime. We note that similarly to the other quantization schemes, the singularity is resolved dynamically.

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