Thermodynamical model for nonextremal black p-brane

Ken-ichi Ohshima*

Institute of Physics, University of Tokyo, Komaba, Meguro-ku 153 Tokyo

(Received 31 December 2006; revised manuscript received 27 August 2007; published 8 November 2007)

We show that the correct entropy, temperature (and absorption probability) of nonextremal black p-brane can be reproduced by a certain thermodynamical model when maximizing its entropy. We show that the form of the model is related to the geometrical similarity of nonextremal and near extremal black p-brane at near horizon region, and argue about the appropriateness of the model.

DOI: 10.1103/PhysRevD.76.104011 PACS numbers: 04.70.Dy, 04.60.−m, 04.70.−s

I. INTRODUCTION

The microscopic origin of black hole entropy has been considered to be an important subject for years, since the mechanism of black hole thermodynamics is regarded to be explained by quantum gravity theory. String theory, the leading candidate of quantum gravity theory, actually explained a black hole entropy by a microscopic model [1]. A certain D-brane model exactly reproduces the black hole entropy including its coefficient. It is based on supersymmetric nature of the black hole, which ensures vanishing quantum corrections so that the string—D-brane model defined at weak coupling region reproduces entropy in strong coupling region. Because of similar supersymmetric nature, AdS/CFT correspondence [2] has been providing various important results.

On the other hand, when supersymmetry is broken, the relation between D-brane model in weak coupling region and supergravity (of strong coupling region) is unclear, since the quantum correction is unknown. It is difficult to relate string theoretical models to black hole thermodynamics in general. However, it is important to look for thermodynamical models which can describe black hole thermodynamics, since one might find a clue to understand nonperturbative property of quantum gravity theory.

For example, [3] showed that D3-brane and open string gas model provides the microscopic description of near extremal black 3-brane thermodynamics, up to numerical factor. The reason of this agreement without supersymmetry is explained in [4–6], and the reason of the discrepancy of the coefficient is studied in [7,8]. In addition, [9] showed that the entropy of near extremal black p-brane is described as $S \propto q^a T^b$ where T is the temperature and q is the charge.

Surprisingly, [10] showed that D3-brane—anti–D3-brane model also provides the microscopic description of nonextremal (and, in particular, Schwarzschild) thermodynamics, including its absorption probability [11], up to numerical factor. References [12,13] extended it to general neutral p-brane and showed that the similar model reproduces the correct entropy up to numerical factor. It is also applied to nonextremal black p-branes, multicharged branes, rotating branes [14–18]. Recently, [19] showed a relation between the thermodynamics of chargeless black 3-brane and D3-brane—anti–D3-brane system, in the supergravity framework.

All those thermodynamical models for black 3-brane are T^4 model, i.e., the thermodynamics of black 3-brane is described like $\mathbb{R}^{3+1}$ massless field in finite temperature. This might indicate a correspondence between gravity and other massless theory. In addition, all those models are "extended near extremal model," i.e., when the thermal energy of the near extremal p-brane is described by T^α, that of the nonextremal p-brane (far from extremality) is also described by T^α.

In this paper, we propose an expression of a "phenomenological" thermodynamical model which provides the correct entropy and temperature and absorption probability including coefficients. This expression does not yield additional discrepancy in the coefficient, and exactly reproduces the supergravity results. Next we show a geometrical similarity between the near extremal black p-brane and nonextremal black p-brane. This geometrical similarity results in the "extended near extremal model" which we mentioned above. We also show that our thermodynamical model is in fact appropriate for blackbody radiation of nonextremal black 3-brane, by considering the action of dilaton field in the low energy region. And we show that the correct absorption probability and graybody factor for nonextremal black 3-brane are also obtained. We show that this exact agreement is due to the equality of the scalar field equation in black 3-brane background and parameter changed near extremal 3-brane background.

The organization of this paper is as follows. In Sec. II, we briefly review black p-brane and the thermodynamic quantities of black p-brane and the thermodynamical models proposed in the past. In Sec. III, we explain our thermodynamical model ansatz. After introducing the ansatz, we show a geometrical similarity between nonextremal black p-brane and near extremal black p-brane. We also show that the agreement of the temperature, the entropy, and the absorption probability is related to the geometrical similarity. We also argue about the appropriateness of our ansatz in terms of the geometrical similarity and the action

*ohshimak@hep1.c.u-tokyo.ac.jp

of massless scalar field. Section IV is the conclusion and discussion.

II. BLACK *p*-BRANE THERMODYNAMICS AND THERMODYNAMICAL MODELS

A. Black *p*-brane and its thermodynamical quantities

The black p-brane solution in 10 dimensional supergravity of 1/2 BPS in the extremal limit is described as extrema of the following supergravity action [9,20–26]:

$$S = -\frac{1}{2\kappa^2}\int d^{10}x\sqrt{-g}\left(R - \frac{1}{2}(\partial\phi)^2 - \frac{1}{2(8-p)!}e^{a\phi}{F_{8-p}}^2\right), \tag{2.1}$$

the metric of black p-brane solution is

$$ds^2 = H^{(p+1)/8}(r)(H^{-1}(r)[-f(r)dt^2 + {dy_1}^2 + \ldots + {dy_p}^2] + f^{-1}(r)dr^2 + r^2 d\Omega_{d+1}^2). \tag{2.2}$$

where

$$H(r) = 1 + \frac{R^d}{r^d}, \qquad f(r) = 1 - \frac{\mu^d}{r^d}, \qquad R^d = \mu^d \sinh^2\gamma, \tag{2.3}$$

$$d = 7 - p, \tag{2.4}$$

$$\mu\text{: the horizon radius.} \tag{2.5}$$

The extremal limit corresponds to

$$\mu \to 0, \tag{2.6}$$

$$\gamma \to \infty, \tag{2.7}$$

$$\mu^d \sinh 2\gamma\text{: fixed.} \tag{2.8}$$

The total energy E, the charge per unit volume q of the black p-brane are [9,21,26,27],

$$E = \frac{\omega_{d+1}}{2\kappa^2}\mu^d V(d + 1 + d\sinh^2\gamma), \tag{2.9}$$

$$q = \frac{\omega_{d+1}}{2\sqrt{2}\kappa}d\mu^d \sinh 2\gamma, \tag{2.10}$$

where ω_{d+1} is volume of a $d+1$-dimensional unit sphere, V is volume of the torus which the brane wrapped.

The Bekenstein-Hawking entropy S and the temperature of the black p-brane are

$$S = \frac{2\pi\omega_{d+1}}{\kappa^2}\mu^{d+1}VH^{1/2}(\mu) = \frac{2\pi\omega_{d+1}}{\kappa^2}\mu^{d+1}V\cosh\gamma, \tag{2.11}$$

$$T = \frac{d}{4\pi\mu\cosh\gamma}. \tag{2.12}$$

The absorption probability of black 3-brane for lth partial wave of dilaton to the lowest order in the radiation frequency is [11]

$$P^{(l)} = \frac{2^{-3l-3}\pi^2\Gamma(1 + l/4)}{(l+2)!^2\Gamma(1/2 + l/4)^2}(\omega\mu)^{2l+5}\cosh\gamma, \tag{2.13}$$

the absorption cross-section (the graybody factor) of the black 3-brane is written [28]

$$\sigma^{(l)} = \frac{8\pi^2}{3\omega^5}(l+1)(l+2)^2(l+3)P^{(l)}. \tag{2.14}$$

B. Thermodynamical models for black *p*-brane

In this subsection, we review some of thermodynamical models for nonextremal black p-brane proposed in the past [3,9,10]. In addition, we argue about relation between those models and this paper's model.

1. Near extremal *p*-brane

At the beginning, we review thermodynamical model of near extremal black 3-brane [3]. "Near extremal" corresponds to the region of $\frac{\mu^4}{R^4} \ll 1$ in (2.2). In this region, the near horizon geometry of the near extremal black 3-brane is

$$ds^2 = -\frac{r^2}{R^2}\left(1 - \frac{\mu^4}{r^4}\right)dt^2 + \frac{r^2}{R^2}\sum_{i=1}^{3}dx_i^2 + \frac{R^2}{r^2}\frac{1}{(1 - \frac{\mu^4}{r^4})}dr^2 + R^2 d\Omega_5^2. \tag{2.15}$$

The entropy is determined by the area of the horizon and the temperature is surface gravity on the horizon. Thus the near horizon geometry determines S and T. The entropy and the temperature are written

$$S = \frac{2\pi^4}{\kappa^2}VR^2\mu^3, \tag{2.16}$$

$$T = \frac{\mu}{\pi R^2}. \tag{2.17}$$

The total energy E can be written as [3]

$$E = M_0 + \delta M \tag{2.18}$$

where M_0 is the mass of extremal 3-brane and δM is small mass added to M_0. δM is written as

$$\delta M = \frac{3\pi^3}{2\kappa^2}V\mu^4 \tag{2.19}$$

S and δM can be expressed by R and T,

$$S = \frac{2\pi^7}{\kappa^2}R^8VT^3, \tag{2.20}$$

$$\delta M = \frac{3\pi^7}{2\kappa^2}R^8VT^4. \tag{2.21}$$

Since $\frac{\delta M}{M_0} \ll 1$, R in (2.20) and (2.21) can be written

$$R^4 = \frac{\sqrt{2}\kappa}{4\pi^3} q, \tag{2.22}$$

where q is the charge of the near extremal 3-brane. Reference [3] proposed a D3-brane model for near extremal 3-brane. The number of the D3-brane N is [9]

$$N = \frac{1}{\sqrt{2\pi}} q, \tag{2.23}$$

therefore R^4 is expressed by N as

$$R^4 = \frac{\kappa}{2\pi^{5/2}} N. \tag{2.24}$$

Thus, S and δM are written

$$S = \frac{2\pi^7}{\kappa^2} R^8 V T^3 = \frac{\pi^2}{2} N^2 V T^3, \tag{2.25}$$

$$\delta M = \frac{3\pi^7}{2\kappa^2} R^8 V T^4 = \frac{3\pi^2}{8} N^2 V T^4, \tag{2.26}$$

which corresponds to massless open string gas on the D3-branes (however, the numerical factor differs by $\frac{3}{4}$ from free case).

Note that the parameter R^4 in the metric (2.15) corresponds to N, in the above argument. When R in the near horizon geometry (2.15) changes to R',

$$ds^2 = -\frac{r^2}{R'^2}\left(1 - \frac{\mu^4}{r^4}\right) dt^2 + \frac{r^2}{R'^2} \sum_{i=1}^{3} dx_i^2 + \frac{R'^2}{r^2} \frac{1}{(1 - \frac{\mu^4}{r^4})} dr^2 + R'^2 d\Omega_5^2, \tag{2.27}$$

the entropy is written

$$S = \frac{2\pi^7}{\kappa^2} R'^8 V T^3 = \frac{\pi^2}{2} N'^2 V T^3, \tag{2.28}$$

which means the number of degrees of freedom of the open string N^2 is replaced by N'^2. We will show in the next section that the near horizon geometry of the black 3-brane far from extremality is written in similar metric, and the entropy and the energy can be obtained by replacing the number of degree of freedom.

The above argument can easily be extended to general p-brane. The near horizon geometry of the near extremal black p-brane is

$$ds^2 = -\frac{r^{d(1-\alpha)}}{R^{d(1-\alpha)}}\left(1 - \frac{\mu^d}{r^d}\right) dt^2 + \frac{r^{d(1-\alpha)}}{R^{d(1-\alpha)}} \sum_{i=1}^{p} dx_i^2 + \frac{R^{\alpha d}}{r^{\alpha d}(1 - \frac{\mu^d}{r^d})} dr^2 + R^{\alpha d} r^{2-\alpha d} d\Omega_{d+1}^2, \tag{2.29}$$

where $\alpha = \frac{p+1}{8}$, and the entropy and the temperature are

$$S = \frac{2\pi\omega_{d+1}}{\kappa^2} R^{d/2} \mu^{(d/2)+1} V = \frac{2\pi\omega_{d+1}}{\kappa^2} \left(\frac{4\pi}{d}\right)^{\lambda/(1-\lambda)} R^{d/2(1-\lambda)} V T^{\lambda/(1-\lambda)}, \tag{2.30}$$

$$\delta M = \frac{\omega_{d+1}}{2\kappa^2} V \left(\frac{d}{2} + 1\right) \mu^d = \frac{\omega_{d+1}}{2\kappa^2} \left(\frac{d}{2} + 1\right) \left(\frac{4\pi}{d}\right)^{1/(1-\lambda)} R^{d/2(1-\lambda)} V T^{1/(1-\lambda)}, \tag{2.31}$$

where $\lambda = \frac{8-p}{7-p} - \frac{1}{2}$. Those are expressed by the number of Dp-branes N

$$N = \frac{1}{\sqrt{2\pi}} \frac{1}{(2\pi l_s)^{3-p}} q, \tag{2.32}$$

as

$$S = \frac{1}{\lambda} C_p N^{1/2(1-\lambda)} T^{\lambda/(1-\lambda)}, \tag{2.33}$$

$$\delta M = C_p N^{1/2(1-\lambda)} T^{1/(1-\lambda)}, \tag{2.34}$$

as shown in [9], where C_p is a constant. R^d is proportional to N as in 3-brane case. When R in (2.29) changes, the N in (2.33) and (2.34) changes.

2. Far from extremality

For the black 3-brane far from extremality, [10] proposed the thermodynamical model

$$E = (N + \bar{N})\tau_3 V + \frac{3\pi^2}{8} N^2 V T^4 + \frac{3\pi^2}{8} \bar{N}^2 V \bar{T}^4, \tag{2.35}$$

where $\bar{N}$ is the number of anti–D3-branes and $\bar{T}$ is the temperature of the anti–D3-branes. Note that the coefficient $\frac{3\pi^2}{8}$ already includes the $\frac{3}{4}$ numerical factor of the near extremal case (2.26). The entropy is

$$S = \frac{\pi^2}{2} N^2 V T^4 + \frac{\pi^2}{2} \bar{N}^2 V \bar{T}^4. \tag{2.36}$$

The correct entropy up to numerical factor ($2^{3/4}$) is reproduced when maximizing S by N and $\bar{N}$. Note that the above argument contains two kinds of additional numerical factor. The one is the $\frac{3}{4}$ factor built in the D3-brane model (2.35), another is the discrepancy that emerged after the maximization of the entropy, $2^{3/4}$ [10].

A similar argument can be applied for general p-brane [12,13]. Those models also yield a discrepancy even though they already include the numerical factor of the near extremal region.

3. Relation to this paper's model

In the above model for the black p-brane far from extremality [10,12,13], the total energy after maximizing the entropy is

$$E = (N + \bar{N})\tau_p V + 4\lambda\sqrt{N\bar{N}}\tau_p V. \tag{2.37}$$

where N, $\bar{N}$ are written by the supergravity parameters as

$$N - \bar{N} = \frac{\omega_{d+1} d}{4\kappa^2 \tau_p}\mu^d \sinh 2\gamma = \frac{\omega_{d+1} d}{2\kappa^2 \tau_p} R^{d/2}(R^d + \mu^d)^{1/2}, \tag{2.38}$$

$$\sqrt{N\bar{N}} = \frac{\omega_{d+1} d}{8\kappa^2 \tau_p}\mu^d. \tag{2.39}$$

The second term of (2.37) corresponds to the energy of gas on the branes and the antibranes. If we rewrite (2.37) by $Q = N - \bar{N}$,

$$E = \sqrt{Q^2 + (2\sqrt{N\bar{N}})^2}\tau_p V + 4\lambda\sqrt{N\bar{N}}\tau_p V. \tag{2.40}$$

E and Q are fixed when maximizing the entropy. We have now one parameter $\sqrt{N\bar{N}}$ instead of N and $\bar{N}$. The $\sqrt{N\bar{N}}$ determines the degree of nonextremality. If we denote

$$n = 2\sqrt{N\bar{N}}, \tag{2.41}$$

the energy is

$$E = \sqrt{Q^2 + n^2}\tau_p V + 2\lambda n \tau_p V \tag{2.42}$$

$$= \sqrt{Q^2 + n^2}\tau_p V + C_p(\sqrt{Q^2 + n^2} + n)^{1/2(1-\lambda)} V T^{1/(1-\lambda)}. \tag{2.43}$$

where C_p is a constant. The above (2.42) and (2.43) is the energy *after maximizing the entropy.*

Our model is as follows. The total energy *before maximizing the entropy* is

$$E = \sqrt{Q^2 + n^2}\tau_p V + C_p(\sqrt{Q^2 + n^2} + n)^{1/2(1-\lambda)} V T^{1/(1-\lambda)}, \tag{2.44}$$

where n is a free parameter, and the correct entropy and temperature are obtained when maximizing the entropy by n. This model smoothly reduces to the near extremal model [3,9] when $\frac{n}{N} \ll 1$. The constant C_p is determined by comparing with the near extremal models [3,9] in the near extremal region. Once we include the numerical factor in the near extremal model as C_p, our model does not yield any other discrepancies, and correctly reproduces entropy and temperature through all the nonextremal region.

III. THERMODYNAMICS OF NONEXTREMAL BLACK p-BRANE

A. An ansatz of thermodynamical model

First, we introduce an ansatz of microcanonical thermodynamical model for nonextremal black p-brane as follows. In the next subsection, we argue about the appropriateness of this ansatz.

Ansatz.—Partition function Z for nonextremal black p-brane thermodynamics:

$$\ln Z = -\frac{1}{T}(\sqrt{N^2 + n^2} - N)\tau_p V + \frac{1-\lambda}{\lambda} f(N, n) V T^{\lambda/(1-\lambda)}, \tag{3.1}$$

where

$$f(N, n) = C_p(\sqrt{N^2 + n^2} + n)^{1/2(1-\lambda)}, \tag{3.2}$$

$$C_p = 2^{2(15-2p)/(5-p)} d^{-[2(8-p)/(5-p)]}\lambda\pi^{2(7-p)/5-p} \times \omega_{d+1}^{-[2/(5-p)]}\kappa^{4/(5-p)}\tau_p^{d/(5-p)}, \tag{3.3}$$

$$\lambda = \frac{8-p}{7-p} - \frac{1}{2}, \tag{3.4}$$

N: charge, V: The volume of the brane,

and n is determined so that the entropy is maximized under fixed energy and charge. The resulted n is $n = \frac{\omega_{d+1} d}{4\kappa^2 \tau_p}\mu^d$.

Total energy of the black p-brane E_{total} is

$$E_{\text{total}} = E_{\text{thermal}} + N\tau_p V, \tag{3.5}$$

where $N\tau_p V$ is the mass of the black p-brane in the extremal limit (i.e., zero temperature brane).

From the ansatz (3.1) we get (thermal) energy and free energy and entropy:

(i) Energy:

$$E_{\text{thermal}} = T^2 \frac{\partial}{\partial T}\ln Z = (\sqrt{N^2 + n^2} - N)\tau_p V + f(N, n) V T^{1/(1-\lambda)} \tag{3.6}$$

(ii) Free energy:

$$F = -T\ln Z = (\sqrt{N^2 + n^2} - N)\tau_p V - \frac{1-\lambda}{\lambda} f(N, n) V T^{1/(1-\lambda)} \tag{3.7}$$

(iii) Entropy:

$$S = \frac{E_{\text{thermal}} - F}{T} = \frac{1}{\lambda} f(N, n) V T^{\lambda/(1-\lambda)} \tag{3.8}$$

The n and temperature T are determined by the condition of maximum entropy, as follows.

Rewrite the entropy as

$$S = \frac{1}{\lambda} f V T^{\lambda/(1-\lambda)} = \frac{1}{\lambda} f^{1-\lambda} V^{1-\lambda} (E_{\rm thermal} - \sqrt{N^2+n^2}\tau_p V + N\tau_p V)^{\lambda}. \quad (3.9)$$

The condition

$$\frac{\partial S}{\partial n} = 0, \quad (3.10)$$

at fixed $E_{\rm thermal}$ (or $E_{\rm total}$) and fixed charge N yields the following equation:

$$(E_{\rm thermal} - \sqrt{N^2+n^2}\tau_p V + N\tau_p V) = 2\lambda n \tau_p V. \quad (3.11)$$

Then the total energy $E_{\rm total}$ is written as

$$E_{\rm total} = \sqrt{N^2+n^2}\tau_p V + 2\lambda n \tau_p V. \quad (3.12)$$

From above (3.12) and the given total energy and charge, we get n. Also, we get the temperature as

$$T = \left(\frac{2\lambda n \tau_p}{f(N,n)}\right)^{1-\lambda}. \quad (3.13)$$

The second order derivative of the entropy is

$$\frac{\partial^2 S}{\partial n^2} = -\lambda\sqrt{N^2+n^2} - 2\lambda^2(N^2+n^2) < 0, \quad (3.14)$$

and the first order derivative vanishes at the one point, thus the above solution is the solution at the maximum entropy.

The model has the following characteristics:

(i) For a given total energy and charge, the ansatz yields the correct temperature and entropy, which agree with the supergravity result.

(ii) The thermal energy $E_{\rm thermal}$ vanishes at extremal limit.

(iii) The model reduces to near extremal black p-brane thermodynamical model [3,9] at $\frac{n}{N} \to 0$ limit.

(iv) The second term of (3.6) can be obtained from the thermodynamical model of *near extremal* black p-brane by replacing the "freedom of the gas" by $f(N,n)$.

(v) When we substitute the "freedom of the gas" $f(N,n)$ and the temperature to the absorption probability of *near extremal* black 3-brane, we get the correct absorption probability and graybody factor of the *nonextremal* black 3-brane.

We explain 4 and 5 in the following.

Characteristic 4.—The thermodynamical model of the near extremal black 3-brane is [3]

$$E_{\rm thermal} = C_3 N^2 V T^4, \quad (3.15)$$

where C_3 is constant $(3\pi^2/8)$. The N is the number of coincident D3-branes, and N^2 is the number of degrees of freedom of the open string. Replacing the C_3N^2 by $f(N,n)$

$$C_3 N^2 \to C_3(\sqrt{N^2+n^2}+n)^2 \quad (3.16)$$

yields the second term of the (3.6). For $p \neq 3$, the replacement

$$C_p N^{1/[2(1-\lambda)]} \to C_p(\sqrt{N^2+n^2}+n)^{1/[2(1-\lambda)]} \quad (3.17)$$

in the near extremal p-brane model [9] yields the second term of (3.6)

Characteristic 5.—In the following, we show that the absorption probability (and the graybody factor) of *non-extremal* black 3-brane can be obtained by replacing "the number of the degrees of freedom" and temperature of the *near extremal* result.

The absorption probability of near extremal black 3-brane for the lth partial wave calculated by supergravity theory is [11,29]

$$P^{(l)} = \frac{2^{-2l-3}\pi\Gamma(1+l/4)^4}{(l+2)!^2\Gamma(1+l/2)^2}\omega^{2l+5}\mu^{2l+3}R^2. \quad (3.18)$$

In the near extremal region, the parameter R and μ in supergravity can be written by N and T as [11]

$$R^4 = \frac{\kappa N}{2\pi^{5/2}}, \qquad \mu = T\sqrt{\frac{\kappa N}{2\pi^{1/2}}}. \quad (3.19)$$

Rewriting the absorption probability by N and T,

$$P^{(l)} = \frac{2^{-2l-3}\pi\Gamma(1+l/4)^4}{(l+2)!^2\Gamma(1+l/2)^2} \times \omega^{2l+5}N^{l+2}T^{2l+3}\kappa^{l+2}\left(\frac{1}{2}\right)^{l+2}\left(\frac{1}{\pi}\right)^{(l+4)/2}. \quad (3.20)$$

Replacing N by $\sqrt{N^2+n^2}+n$ and substituting the correct T, and rewriting those by supergravity parameters, we get

$$P^{(l)} = \frac{2^{-3l-3}\pi^2\Gamma(1+l/4)}{(l+2)!^2\Gamma(1/2+l/4)^2}(\omega\mu)^{2l+5}\cosh\gamma. \quad (3.21)$$

This result agrees with the result from the supergravity (2.13). The graybody factor is calculated as (2.14), thus we reproduced graybody factor of black 3-brane by this "replacing."

B. Near extremal and nonextremal geometry

In this section, we show that the nonextremal black p-brane has a relation to near extremal black p-brane geometry. In the vicinity of the horizon, the nonextremal p-brane (far from extremality) has similar geometry with near extremal p-brane, and this leads to the temperature and the entropy described by the similar form. We also show that this geometrical similarity has a relation to various properties of our thermodynamical model.

1. The vicinity of the horizon geometry

A nonextremal black p-brane (arbitrary far from extremality) with parameter R_0 and μ_0 is

$$ds^2 = -\frac{(1-\frac{\mu_0^d}{r^d})}{(1+\frac{R_0^d}{r^d})^{1-\alpha}}dt^2 + \frac{1}{(1+\frac{R_0^d}{r^d})^{1-\alpha}}\sum_{i=1}^{p} dx_i^2 + \frac{(1+\frac{R_0^d}{r^d})^{\alpha}}{(1-\frac{\mu_0^d}{r^d})}dr^2 + \left(1+\frac{R_0^d}{r^d}\right)^{\alpha} r^2 d\Omega_{d+1}^2, \quad (3.22)$$

where

$$\alpha = \frac{p+1}{8}. \quad (3.23)$$

On the other hand, the near horizon limit of *near extremal* black p-brane parametrized by R and μ is

$$ds^2 = -\frac{r^{d(1-\alpha)}}{R^{d(1-\alpha)}}\left(1-\frac{\mu^d}{r^d}\right)dt^2 + \frac{r^{d(1-\alpha)}}{R^{d(1-\alpha)}}\sum_{i=1}^{p} dx_i^2 + \frac{R^{\alpha d}}{r^{\alpha d}(1-\frac{\mu^d}{r^d})}dr^2 + R^{\alpha d} r^{2-\alpha d} d\Omega_{d+1}^2. \quad (3.24)$$

When we change the parameters R and μ as

$$R^d = R_0^d + \mu_0^d, \quad (3.25)$$

$$\mu = \mu_0, \quad (3.26)$$

(3.24) is

$$ds^2 = -\frac{r^{d(1-\alpha)}}{(R_0^d+\mu_0{}^d)^{(1-\alpha)}}\left(1-\frac{\mu_0^d}{r^d}\right)dt^2 + \frac{r^{d(1-\alpha)}}{(R_0{}^d+\mu_0{}^d)^{(1-\alpha)}}\sum_{i=1}^{p} dx_i^2 + \frac{(R_0{}^d+\mu_0{}^d)^{\alpha}}{r^{\alpha d}(1-\frac{\mu_0^d}{r^d})}dr^2 + (R_0{}^d+\mu_0{}^d)^{\alpha} r^{2-\alpha d} d\Omega_{d+1}^2. \quad (3.27)$$

First, the nonextremal p-brane (3.22) and the parameter changed near extremal p-brane (3.27) are equal at the horizon ($r = \mu_0$).

Next, we consider the vicinity of the horizon. When we expand the metric for the time direction of the nonextremal p-brane (3.22) [we denote it as $g_{tt}(r)$] in the vicinity of the horizon,

$$g_{tt}(\mu_0+\epsilon) = g_{tt}(\mu_0) - \frac{\mu_0^{d(1-\alpha)-1}d}{(R_0^d+\mu_0^d)^{1-\alpha}}\epsilon. \quad (3.28)$$

On the other hand, when we expand the metric for the time direction of the parameter changed near extremal p-brane (3.27) [we denote it as $h_{tt}(r)$] in the vicinity of the horizon,

$$h_{tt}(\mu_0+\epsilon) = h_{tt}(\mu_0) - \frac{\mu_0^{d(1-\alpha)-1}d}{(R_0^d+\mu_0^d)^{1-\alpha}}\epsilon. \quad (3.29)$$

The both agree. The same agreement holds for the r direction. Both metrics for r direction: g_{rr} and h_{rr} also agree in the vicinity of the horizon.

2. The entropy and the temperature

Consider the following two types of geometry:

(i) Geometry A: Geometry (3.27) in near horizon region, and asymptotically flat.

(ii) Geometry B: Nonextremal black p-brane background (3.22).

In the following, we show that the entropy and the temperature of the both geometry agree.

Entropy.—As we mentioned in the previous subsection, the metric of the geometry A and the geometry B agree at the horizon $r = \mu_0$. Thus their entropies agree.

Temperature.—Defining u as

$$u = 2\left(\frac{(R_0^d+\mu_0^d)^{\alpha}}{\mu_0^{\alpha d-1}d}\right)^{1/2}(r-\mu_0)^{1/2}, \quad (3.30)$$

both metrics in the vicinity of the horizon can be written in Euclidean form as

$$ds^2 \sim u^2\left(\frac{dt_E}{2\frac{(R_0^d+\mu_0^d)^{1/2}}{\mu_0^{d/2-1}d}}\right)^2 + du^2 + (\text{other terms}). \quad (3.31)$$

By comparing above with the polar coordinates of 2-dimensional plane

$$ds^2 = u^2 d\theta^2 + du^2, \qquad 0 \le \theta \le 2\pi, \quad (3.32)$$

the period of the compactified t_E (which is $\frac{1}{T}$) must be

$$\frac{1}{T} = \frac{4\pi(R_0^d+\mu_0^d)^{1/2}}{\mu_0^{d/2-1}d}, \quad (3.33)$$

so that conical singularity in $t-r$ plane vanishes. As above, the temperature of the two geometries agree because their $tt-$ and $rr-$ components are equal in the vicinity of the horizon.

Near extremal and far from extremality.—As we mentioned in Sec. II B, near extremal region corresponds to the region of $\frac{\mu}{R} \ll 1$. In this region,

$$S \sim \frac{2\pi\omega_{d+1}}{\kappa^2}\mu_0^{(d/2)+1}V\sqrt{R_0^d}, \quad (3.34)$$

$$T \sim \frac{\mu_0^{(d/2)-1}d}{4\pi\sqrt{R_0^d}}. \quad (3.35)$$

On the other hand, $\frac{\mu}{R}$ can not be neglected in the region far from extremality. In this region,

$$S \sim \frac{2\pi\omega_{d+1}}{\kappa^2}\mu_0^{(d/2)+1}V\sqrt{R_0^d+\mu_0^d}, \quad (3.36)$$

$$T \sim \frac{\mu_0^{(d/2)-1} d}{4\pi\sqrt{R_0^d + \mu_0^d}}. \quad (3.37)$$

One can see from (3.34), (3.35), (3.36), and (3.37) that S and T of "far from extremality" region are obtained by replacing $R_0^d \to R_0^d + \mu_0^d$ in the near extremal S and T. The geometric similarity we showed above corresponds to this "replacement." As we have shown in Sec. II B, R_0^d of near extremal p-brane corresponds to the number of Dp-branes N. Thus above replacement corresponds to the replacement of N in the near extremal thermodynamical model. Explicitly, $R_0^d \to R_0^d + \mu_0^d$ corresponds to

$$N \to \sqrt{N^2 + n^2} + n, \quad (3.38)$$

and this leads to the second term of total energy (3.6)

$$E_{\rm thermal} = (\sqrt{N^2 + n^2} - N)\tau_p V + f(N, n) V T^{1/(1-\lambda)} \quad (3.39)$$

of our model, where n is determined by the maximization of entropy in our model as (3.9) to (3.13).

3. Match of absorption probability and graybody factor

We show below that the equation of the motion of the massless scalar field in geometry A 3-brane and geometry B 3-brane is equal in the low energy region. This equality results in the agreement of the absorption probability (and the graybody factor).

The equation of the motion of massless scalar field in nonextremal black 3-brane background (the geometry B) is

$$\partial_r^2\phi + \left(\frac{5}{r} + \frac{4\mu^4}{r^5(1-\frac{\mu^4}{r^4})}\right)\partial_r\phi + \left(\frac{1+\frac{R^4}{r^4}}{(1-\frac{\mu^4}{r^4})^2}\omega^2 - \frac{l(l+4)}{r^2(1-\frac{\mu^4}{r^4})} - \frac{1+\frac{R^4}{r^4}}{1-\frac{\mu^4}{r^4}}k^2\right)\phi = 0, \quad (3.40)$$

where ω: energy (of the scalar field ϕ), l: angular momentum, k: momentum in direction to x_i.

We consider the equation of the motion in the low energy region $\frac{\omega}{T} \ll 1$ (where T is the temperature of the black 3-brane) in the following.

The equation of motion of massless scalar field in the geometry A of near horizon region is

$$\partial_r^2\phi + \left(\frac{5}{r} + \frac{4\mu^4}{r^5(1-\frac{\mu^4}{r^4})}\right)\partial_r\phi + \left(\frac{\frac{R^4+\mu^4}{r^4}}{(1-\frac{\mu^4}{r^4})^2}\omega^2 - \frac{l(l+4)}{r^2(1-\frac{\mu^4}{r^4})} - \frac{\frac{R^4+\mu^4}{r^4}}{1-\frac{\mu^4}{r^4}}k^2\right)\phi = 0. \quad (3.41)$$

At infinity of geometry A, the equation is the same as that in flat spacetime.

The difference of the equation in the geometry A and that in the geometry B is the terms which contain R. When we consider the massless particle propagating perpendicular to the brane ($k = 0$), the difference of the equation in the geometry A and that in the geometry B is the ω term only.

(i) Outer region [$\mu \ll r$ ($\rho_h \ll \rho$)]
With

$$\rho = \omega r, \qquad \rho_h = \omega\mu, \quad (3.42)$$

the equation in the geometry B (black 3-brane) is

$$\partial_\rho^2\phi + \frac{5\rho^4 - \rho_h^4}{\rho(\rho^4 - \rho_h^4)}\partial_\rho\phi - \frac{\rho^2 l(l+4)}{\rho^4 - \rho_h^4}\phi + \frac{\rho^4(\rho^4 + (\omega R)^4)}{(\rho^4 - \rho_h^4)^2}\phi = 0. \quad (3.43)$$

When $\mu < R$, the equation in the outer region of the geometry B (black 3-brane background) is

$$\partial_\rho^2\phi + \frac{5}{\rho}\partial_\rho\phi + \left(1 + \frac{(\omega R)}{\rho^4} - \frac{l(l+4)}{\rho^2}\right)\phi = 0. \quad (3.44)$$

From the low energy condition

$$\frac{(\omega R)^4}{\rho^2} \ll \left(\frac{\omega(R^4 + \mu^4)^{1/2}}{\mu}\right)^2 = \left(\frac{\omega}{\pi T}\right)^2 \ll 1, \quad (3.45)$$

the R term can be ignored. When ignoring the R term, the equation of the motion is the same as that in the flat space. Thus the equation in the geometry B is equal to that in the geometry A.

When $\mu \sim R$ or $R < \mu$, the equation in the geometry B is the same as the equation in flat spacetime from the condition of the outer region. Thus the equation in the geometry B is equal to that in the geometry A again.

In summary, for every case of R and μ, the equation in the geometry A and that in the geometry B are equal in the outer region.

(ii) Inner region [$\frac{\mu}{r} \sim O(1)$ and $\mu \le r$]
When $\mu < r$, the equation in the geometry B (black 3-brane) is

$$\partial_r^2\phi + \left(\frac{5}{r} + \frac{4\mu^4}{r^5(1-\frac{\mu^4}{r^4})}\right)\partial_r\phi + \left(\frac{1+\frac{R^4}{r^4}}{(1-\frac{\mu^4}{r^4})^2}\omega^2 - \frac{l(l+4)}{r^2(1-\frac{\mu^4}{r^4})}\right)\phi = 0. \quad (3.46)$$

The R term can be ignored since $\frac{\omega}{T} \ll 1$. The equation in the geometry A is

$$\partial_r^2\phi + \left(\frac{5}{r} + \frac{4\mu^4}{r^5(1-\frac{\mu^4}{r^4})}\right)\partial_r\phi + \left(\frac{\frac{R^4+\mu^4}{r^4}}{(1-\frac{\mu^4}{r^4})^2}\omega^2 - \frac{l(l+4)}{r^2(1-\frac{\mu^4}{r^4})}\right)\phi = 0. \quad (3.47)$$

The R term also can be ignored because $\frac{\omega}{T} \ll 1$. Thus the equation in the geometry A and that in the geometry B is the same, in the region of $\mu < r$ in the inner region.

In the region of $\mu \sim r$, the R term cannot be ignored in both geometries, but g^{rr} is the same in the region of $\mu \sim r$. The R term in both equations agree since the R term is actually $(1/g^{rr})^2$. Thus both equations agree in the inner region.

In summary,

(i) $\mu \ll r$: Both equations are the equation in flat space.

(ii) $\mu < r$: Both equations agree by the low energy condition.

(iii) $\mu \sim r$: Both equations agree since the $1/g^{rr}$ is the same in the vicinity of the horizon.

Thus the equation in the geometry A and that in the geometry B are equal in all the regions.

The geometry A is a parameter changed near extremal geometry ($R_0^4 \to R_0^4 + \mu_0^4$) in the near horizon region. Since the scalar field equation in both geometries are the same, the absorption probability of "far from extremality" region is obtained by replacing $R_0^4 \to R_0^4 + \mu_0^4$ in the near extremal absorption probability. As we explained, this replacement corresponds to the replacement $N \to \sqrt{N^2+n^2}+n$, where n is determined by the maximization of entropy in our model. The characteristic 5 of our thermodynamical model ansatz is as follows: When we substitute the "freedom of the gas" $f(N, n)$ and the temperature to the *near extremal* black 3-brane result of absorption probability, we get the correct absorption probability and graybody factor of the *nonextremal* black 3-brane. This is caused by the above equality of the equation in the geometry A and B.

C. Appropriateness of the thermodynamical model

1. Gas term (after maximization of entropy)

As we have shown, nonextremal p-brane geometry is similar to "parameter changed near extremal p-brane geometry" in the vicinity of the horizon. This leads to the agreement of entropy, temperature, and absorption probability in both the geometry A and B. Hence, we can describe thermodynamics of nonextremal p-brane by the near extremal p-brane model with replacement $N \to \sqrt{N^2+n^2}+n$, which results in the second term (gas term) of

$$E_{\text{thermal}} = (\sqrt{N^2+n^2} - N)\tau_p V + f(N, n)VT^{1/(1-\lambda)}, \quad (3.48)$$

where n is determined by the maximization of entropy in our model.

The original explanation of the black hole Hawking radiation [30] is derived by considering the behavior of massless scalar field in black hole background. In order to examine physical appropriateness of our model more strictly, we have to examine the action of massless scalar field (in this case, dilaton field) in black p-brane background. We show below that the action in the geometry A and the geometry B actually agree in low energy region for $p = 3$.

The Lagrangian of massless scalar field in nonextremal black 3-brane background (geometry B) is

$$\mathcal{L} = \sqrt{-g}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi \quad (3.49)$$

$$= r^5\left(-\frac{H}{f}(\partial_t\phi)^2 + f(\partial_r\phi)^2 + \sum_{i=1}^{3} H(\partial_i\phi)^2 + \sum_{j=1}^{5}\frac{1}{r^2\Omega_i(\theta_1,\cdots,\theta_5)}(\partial_{\theta_j}\phi)^2\right), \quad (3.50)$$

where

$$H(r) = 1 + \frac{R^4}{r^4}, \qquad f(r) = 1 - \frac{\mu^4}{r^4}, \quad (3.51)$$

$$r^2\Omega_i\text{: the metric for the angular directions } \theta_i. \quad (3.52)$$

On the other hand, for massless scalar field in the "parameter changed near extremal black 3-brane" background, $H(r) = \frac{R^4+\mu^4}{r^4}$. Note that $\frac{f}{H}$ is the same for both backgrounds at the vicinity of the horizon.

If the x_i direction (3 dimensional directions) are compactified and the size of the compactified manifold is small enough compared to the energy scale of ϕ, the $(\partial_i\phi)^2$ term can be dropped from the Lagrangian. Then the difference is only in the $(\partial_t\phi)^2$ term, and this term is the same in the vicinity of the horizon for both backgrounds.

Now, consider the low energy limit $\omega\mu \to 0$. The Lagrangian of the massless scalar field at the energy scale of $\omega\mu \to 0$ vanishes in almost all the region of the space, except near the horizon, where $f(r)$ diverges. The dominant contribution to the Euclidean action is the action near the horizon, thus in the low energy limit $\omega\mu \to 0$, the Euclidean action in the geometry A and that in the geometry B are approximately the same. The agreement of the Euclidean action means the same quantum phenomena of the same temperature. Hence, applying the thermodynamical model of the geometry A to the thermodynamics of the geometry B is regarded to be valid in this limit.

The limit $\omega\mu \to 0$ corresponds to blackbody radiation region, as one can see that the graybody factor (2.14) is nonzero only for $l = 0$ where it does not depend on the energy. Thus our thermodynamical model successfully reproduces the blackbody radiation of the nonextremal black 3-brane.

2. Total energy and maximization of entropy

The gas term we explained in the previous subsection requires specific value of n. However, our model does not require n as an input.

The total energy of our model is

$$E_{\mathrm{thermal}} = (\sqrt{N^2 + n^2} - N)\tau_p V + f(N, n)VT^{1/(1-\lambda)}, \tag{3.53}$$

where n is a *free parameter*. When we maximize the entropy by n under fixed E and N, the value of n is automatically determined. The n at the maximum entropy yields correct entropy, temperature, and absorption probability. This property is nontrivial, because free parameters like this generally do not yield correct entropy, as discrepancies found in [10,12,13].

In addition, our model smoothly reaches to near extremal model [3,9] when $\frac{n}{N} \ll 1$. The coefficient in the gas term [the second term of the energy (3.53)] is determined by comparing with the near extremal models. Once we take the near extremal coefficient into our model, our model yields correct entropy and temperature through all the nonextremal region beyond the near extremal region, and does not yield any additional discrepancy.

IV. CONCLUSION AND DISCUSSION

In this paper, we introduced an ansatz of thermodynamical model for nonextremal black p-brane thermodynamics, which yields the correct entropy and temperature and graybody factor when the entropy is maximized. We have shown that the geometrical similarity between nonextremal black p-brane and near extremal black p-brane is related to the various properties of the model. This fact implies that the model is appropriate, and we have actually shown the appropriateness of the model for $p = 3$ by considering the action of massless scalar field.

Comparing with the models proposed in the past, our model can smoothly reach to the near extremal models [3,9] and does not need any additional condition (like equal gas energies on brane and antibrane), and reproduces correct entropy and temperature through all the nonextremal region.

Since the supersymmetry is broken in the nonextremal region, the definite argument including quantum correction is difficult. Our ansatz certainly reproduces the entropy and the temperature of the nonextremal black p-brane, however further study is needed for the explanation beyond the geometrical similarity.

ACKNOWLEDGMENTS

I would like to thank to Tamiaki Yoneya, Mitsuhiro Kato, Koji Hashimoto, Kenji Hotta, and Yuri Aisaka for many helpful discussions, and would like to thank to Makoto Natsuume for helpful discussion and advice.

[1] A. Strominger and C. Vafa, Phys. Lett. B **379**, 99 (1996).
[2] J. Maldacena, Adv. Theor. Math. Phys. **2**, 231 (1998); Int. J. Theor. Phys. **38**, 1113 (1999).
[3] S. S. Gubser, I. R. Klebanov, and A. W. Peet, Phys. Rev. D **54**, 3915 (1996).
[4] J. Maldacena, Phys. Rev. D **55**, 7645 (1997).
[5] S. R. Das, Phys. Rev. D **56**, 3582 (1997).
[6] S. R. Das, Nucl. Phys., Proc. Suppl. **B68**, 119 (1998).
[7] S. S. Gubser, I. R. Klebanov, and A. A. Tseytlin, Nucl. Phys. **B534**, 202 (1998).
[8] A. Fotopoulos and T. R. Taylor, Phys. Rev. D **59**, 061701 (1999).
[9] I. R. Klebanov and A. A. Tseytlin, Nucl. Phys. **B475**, 164 (1996).
[10] U. H. Danielsson, A. Guijosa, and M. Kruczenski, J. High Energy Phys. 09 (2001) 011; Rev. Mex. Fis. **49S2**, 61 (2003).
[11] J. A. Garcia and A. Guijosa, J. High Energy Phys. 09 (2004) 027.
[12] O. Saremi and A. W. Peet, Phys. Rev. D **70**, 026008 (2004).
[13] O. Bergman and G. Lifschytz, J. High Energy Phys. 04 (2004) 060.
[14] S. Kalyana Rama, Phys. Lett. B **593**, 227 (2004).
[15] G. Lifschytz, J. High Energy Phys. 09 (2004) 009.
[16] S. Kalyana Rama and Sanjay Siwach, Phys. Lett. B **596**, 221 (2004).
[17] A. Guijosa, H. H. Hernandez Hernandez, and H. A. Morales Tecotl, J. High Energy Phys. 03 (2004) 069.
[18] Edi Halyo, J. High Energy Phys. 10 (2004) 007.
[19] D. Gluck and Y. Oz, J. High Energy Phys. 03 (2007) 089.
[20] G. T. Horowitz and A. Strominger, Nucl. Phys. **B360**, 197 (1991).
[21] M. J. Duff and J. X. Lu, Nucl. Phys. **B416**, 301 (1994).
[22] G. W. Gibbons, Gary T. Horowitz, and P. K. Townsend, Classical Quantum Gravity **12**, 297 (1995).
[23] H. Lu, C. N. Pope, E. Sezgin, and K. S. Stelle, Nucl. Phys. **B456**, 669 (1995).
[24] H. Lu and C. N. Pope, Nucl. Phys. **B465**, 127 (1996).
[25] M. J. Duff, H. Lu, and C. N. Pope, Phys. Lett. B **382**, 73 (1996).

[26] T. Harmark and N. A. Obers, J. High Energy Phys. 01 (2000) 008.
[27] J. X. Lu, Phys. Lett. B **313**, 29 (1993).
[28] S. S. Gubser, Phys. Rev. D **56**, 4984 (1997).
[29] G. Policastro and A. Starinets, Nucl. Phys. **B610**, 117 (2001).
[30] S. W. Hawking, Commun. Math. Phys. **43**, 199 (1975).