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# Inflationary cosmology connecting dark energy and dark matter

Daniel J. H. Chung* and Lisa L. Everett†

*Department of Physics, University of Wisconsin, Madison, Wisconsin 53706, USA*

Konstantin T. Matchev‡

*Institute for Fundamental Theory, Physics Department, University of Florida, Gainesville, Florida 32611, USA*

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Kination dominated quintessence models of dark energy have the intriguing feature that the relic abundance of thermal cold dark matter can be significantly enhanced compared to the predictions from standard cosmology. Previous treatments of such models do not include a realistic embedding of inflationary initial conditions. We remedy this situation by constructing a viable inflationary model in which the inflaton and quintessence field are the same scalar degree of freedom. Kination domination is achieved after inflation through a strong push or ''kick'' of the inflaton, and sufficient reheating can be achieved depending on model parameters. This allows us to explore both model-dependent and model-independent cosmological predictions of this scenario. We find that measurements of the B-mode cosmic microwave background polarization can rule out this class of scenarios almost model independently. We also discuss other experimentally accessible signatures for this class of models.

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## I. INTRODUCTION

The discovery that the universe is dominated by dark energy strongly suggests that the predictions of standard cosmology should be reevaluated. An intriguing possible explanation of the nature of dark energy arises within the quintessence paradigm, in which the dark energy takes the form of a slowly evolving scalar field (see, e.g., [1–5]). This scenario is extremely difficult (if not impossible) to test directly in collider experiments, since quintessence models generically require the quintessence field to have gravitationally suppressed interactions with the fields of the standard model (SM).

However, if the dark energy is in the form of quintessence, the presence of the quintessence field can modify the cosmological evolution and lead to significant departures from standard cosmology. A striking example is the possible interconnection of dark matter (DM) and dark energy within quintessence scenarios [6–10]. As first pointed out by Salati [6], the freeze-out of thermal relics can be strongly enhanced in scenarios in which the energy density is dominated by the kinetic energy of the quintessence field (kination domination) during the time of freeze-out, but dilutes away by the time of big bang nucleosynthesis (BBN). (Related scenarios were also suggested before by [11,12].) Such kination dominated freeze-out scenarios are then consistent with standard cosmology and predict that the standard relic abundance computed from the parameters extracted from collider measurements will be mismatched from the relic abundance deduced by observational cosmology. This has implications for TeV physics models with thermal dark matter candidates [e.g., models with low energy supersymmetry such as the minimal supersymmetric extension of the SM (MSSM), technicolor models, models with large/warped extra dimensions, or certain classes of little Higgs models], which will be probed at the LHC and other experiments in the foreseeable future.

In most of the previous discussions of this class of scenarios [6–9], the initial condition that the quintessence field kinetic energy density is the dominant component was put in as an ansatz, without a complete picture of the inflationary dynamics. In this paper, we address this issue by constructing an inflationary scenario that dynamically leads to a kination dominated quintessence period. This yields robust predictions which can be used to experimentally support or rule out this class of scenarios.

To construct viable inflationary models which lead to kination domination, the following constraints must be satisfied:

(1) The energy density in a coherent quintessence field must dominate over radiation after the end of inflation.
(2) The quintessence field must be kinetic energy dominated.
(3) The inflaton potential must satisfy the usual requirements of a sufficient number of e-foldings, the right amplitude of density perturbations, and a nearly scale invariant spectral index (with a slight preference for a red spectrum [13]).
(4) There must not be too much reheating in the phase transition at the end of inflation (when the quasi-

*danielchung@wisc.edu
†leverett@wisc.edu
‡matchev@phys.ufl.edu

 

de Sitter phase ends), such that the ratio of the kination energy density to the radiation energy density can be large.

To satisfy these constraints, we build a model in which the quintessence field is the inflaton field $\Phi$, which has sufficiently weak couplings to the SM fields at the end of inflation such that most of the energy density responsible for inflation gets converted to $\Phi$ coherent kinetic energy in a runaway potential. Radiation domination is achieved because the coherent kinetic energy dilutes as $1/a^6$, while the suppressed radiation produced at the end of inflation gets relatively amplified since it dilutes as $1/a^4$. The choice of the inflaton as the quintessence field in kination domination is natural in this framework, since the inflaton possesses the required qualities for the kination domination construction: energy dominance and coherence. Other quintessence models have been constructed in which the inflaton is the same field as the quintessence field (see, e.g., [14–18]), but we treat the reheating more precisely and derive new predictions in the context of kination dominated quintessence models relevant for dark matter. The work [10] considers the connection between dark energy and dark matter, but it differs from the present treatment in that their scenario is model specific and has multiple inflationary phases.

In addition to the prediction that the relic abundance inferred from collider measurements can measurably disagree with the cosmologically inferred dark matter relic abundance (in the context of a thermal freeze-out scenario), many experimentally accessible cosmological predictions can be made by embedding the kination domination scenario in inflationary models. The most *model-independent* nontrivial prediction is the absence of a measurable tensor perturbation induced B-mode cosmic microwave background (CMB) polarization. Hence, the next generation of CMB experiments can falsify this class of models.

Other predictions include a shift in the peak of the gravity wave signal from the electroweak phase transition, shifts in the verifiable leptogenesis/baryogenesis scenarios whose out of equilibrium ingredient is furnished by $H$, and shifts in any residual annihilation effects. In general, the indirect dark matter detection signals are enhanced in this scenario [19]. In particular, it would be interesting to see whether one can explain the positron excess as observed by HEAT and other experiments near 7 GeV within this scenario [20], since most attempts to explain this excess in terms of dark matter annihilations require sufficiently large cross sections that the relic abundance would be negligible with standard cosmological assumptions.

The order of presentation will be as follows. We begin by analyzing both analytically and numerically the constraints on quintessence potentials to achieve a period of kination domination, and look for equation of state signatures. In Sec. III, we construct a class of inflationary models embedding the kination domination scenario and discuss a robust prediction that can observationally falsify this class of models. In Sec. IV, we discuss other observationally accessible cosmological predictions which may corroborate this class of scenarios. We then summarize and conclude. In the Appendix, we provide the details of the particle production computation for the unusual reheating scenario associated with this class of inflationary scenarios.

Throughout this paper, we use the convention of $M_p$ to denote the reduced Planck mass of approximately $2.4 \times 10^{18}$ GeV.

## II. MAPPING KINETIC BEHAVIOR TO QUINTESSENCE POTENTIALS

The standard procedure in constructing scenarios of quintessence dynamics is to focus on potentials, which is a sensible approach since a negative equation of state requires potential energy domination. However, when considering quintessence dynamics with a period of kination domination, it is more natural to focus on the behavior of the field velocity, as this is the quantity which characterizes the energy density of the quintessence field. Here we develop a formalism to map the desired behavior of the kinetic energy to a class of scalar potentials. Our results show that the allowed quintessence potentials are not severely restricted by the requirement of a period of kination domination, since the time at which the equation of state is close to $-1$ is necessarily much later than the time of dark matter freeze-out due to the strong constraints from BBN. In what follows, we will not restrict ourselves to "tracker" models, in order to separate the difficulties of constructing good trackers from the constraints imposed by kination domination. It is worth noting that kination dominated initial conditions can be obtained in tracking potentials, as discussed in [8,21], although clearly there will be constraints depending on the exact form of the potential.

To carry out the engineering of quintessence potentials which match the desired kinetic term histories, we begin with the familiar equations of motion for the quintessence field $\Phi$ and the Friedmann-Robertson-Walker scale factor $H \equiv \dot{a}/a$:

$$\ddot{\Phi} + 3H\dot{\Phi} + V'(\Phi) = 0 \tag{1}$$

$$H = \sqrt{\frac{1}{3M_p^2}\left(\frac{1}{2}\dot{\Phi}^2 + V(\Phi) + \rho_{RM}\right)}, \tag{2}$$

in which $\rho_{RM} \equiv \rho_R + \rho_M$ corresponds to the energy densities of radiation and matter. Using the following definitions,

$$q(\Phi) \equiv \ln a^3(\Phi) \tag{3}$$

$$f(\Phi) \equiv \dot{\Phi}(t) \tag{4}$$

$$\gamma \equiv \frac{1}{3M_p^2}, \tag{5}$$

we rewrite Eqs. (1) and (2) as follows:

$$q'(\Phi) = -\left[\frac{V'(\Phi)}{f^2(\Phi)} + \frac{f'(\Phi)}{f(\Phi)}\right] \tag{6}$$

$$q'(\Phi) = \frac{3\sqrt{\gamma}}{f(\Phi)}\sqrt{\frac{1}{2}f^2(\Phi) + V(\Phi) + \rho_{RM}(q(\Phi))}. \tag{7}$$

In the above, we have assumed that $\dot{\Phi}$ is a single valued function of $\Phi$ (which would exclude, e.g., the situation of oscillations). For the scenarios considered here this is typically not a good assumption throughout the entire evolution, since the kinetic energy is usually large enough that $\Phi$ can overshoot the minimum and eventually hit the "other side" of the potential before the quintessence equation of state reaches $-1$. However, the condition on the quintessence potential during kination domination does not change even if this behavior is properly accounted for since the "bounce" occurs long after the kination period is over.

We can use Eqs. (6) and (7) to solve for $q(\Phi)$ and $f(\Phi)$ if $V(\Phi)$ is given; alternatively, we can solve for $V(\Phi)$ and $q(\Phi)$ if the field velocity function $f(\Phi)$ (i.e., the kinetic energy) is specified. For generic potentials, there is no obvious obstruction to achieving a period of kination domination which leads to a period of potential energy domination, as $f(\Phi)$ can be chosen to vanish as $\Phi$ approaches a particular asymptotic value, and the Hubble friction naturally allows for $f(\Phi)$ to vanish without $V$ also vanishing.

In this paper, we restrict our attention to the specific case of the quintessence field velocity function with the initial behavior (during the time relevant for dark matter freeze-out) of

$$f \approx f_i e^{q_i - q} \propto \frac{1}{a^3}, \tag{8}$$

in which $f_i$ and $q_i$ denote initial values of the functions $f(\Phi)$ and $q(\Phi)$. This corresponds to the kination regime in which the universe is driven by quintessence kinetic energy, with $V'/f^2$ playing a subdominant role compared to $f'/f$ in the equation of motion for $\Phi$ (i.e., the force term is subdominant to the Hubble friction term). The reason of course for this restriction is that kination domination gives rise to an energy density which dilutes as $1/a^6$, which allows the quintessence energy to be important during DM freeze-out but disappear by the time of BBN as required by phenomenology.

Let us now study what a potential that leads to the kination behavior of Eq. (8) looks like by constructing it using Eqs. (6) and (7). To obtain a closed form solution, we take the ansatz of neglecting $V'/f^2$, in which case we obtain

$$q = q_i - \ln\frac{f(\Phi)}{f_i} \tag{9}$$

$$V(\Phi) = \frac{-1}{2}f^2(\Phi) - \rho_{RM}(q(\Phi)) + \frac{1}{9\gamma}[f'(\Phi)]^2. \tag{10}$$

One possible procedure is then as follows: choose any desired behavior of $f$ and obtain using Eqs. (9) and (10) a corresponding expansion history (specified by $q$) and the potential $V$, then check that $V'/f^2$ can be neglected in the equation of motion. The validity of neglecting $V'/f^2$ can be rewritten as

$$\left|-1 + \frac{1}{f^2}\frac{d\rho_{RM}}{dq} + \frac{2}{9\gamma}\frac{f''}{f}\right| \ll 1. \tag{11}$$

One trivial way to make $V'/f^2$ negligible is to have $V$ much smaller than any other energy component and to have $V$ be a smooth function. Another way to satisfy Eq. (11) is to set its left-hand side equal to an arbitrary small function $h(\Phi) \ll 1$ and solve for $f(\Phi)$. For example, if the universe is radiation dominated, we can write

$$\rho_{RM}(q) = \rho_i\left(\frac{a_i}{a}\right)^4 = \rho_i e^{(4/3)(q_i - q)} \tag{12}$$

to obtain the following equation:

$$f''(\Phi) - 6\gamma\left(\frac{f(\Phi)}{f_i}\right)^{1/3}\kappa f_i - \frac{9\gamma}{2}(1 + h(\Phi))f(\Phi) = 0, \tag{13}$$

in which

$$\kappa = \rho_i/f_i^2 \approx \frac{5 \times 10^{-7}}{\eta_\Phi}, \tag{14}$$

with $\eta_\Phi \equiv \rho_\Phi/\rho_\gamma|_{T=1\,\mathrm{MeV}}$, and $f_i \equiv f(\Phi_i) \approx 6 \times 10^3\sqrt{\eta_\Phi}\ \mathrm{GeV}^2$ (the initial field velocity) as defined at a temperature of approximately 1 GeV. Note that since $h(\Phi) \ll 1$ is arbitrary, nearly any smoothly varying $f(\Phi)$ can be obtained, which in turn means any smoothly varying shape for the potential can be obtained even with the constraint of Eq. (8). (This corresponds of course to the case in which the $V$ energy density is negligible during kination domination.) To obtain intuition, if we set $h(\Phi) = h_0 = \mathrm{const} \ll 1$ and define $y \equiv f(\Phi)/f_i$ and $x = \Phi\sqrt{\gamma}$, we can rewrite Eq. (13) in terms of a first order equation:

$$\frac{dy}{dx} = -\sqrt{9\kappa y^{4/3} + \frac{9}{2}(1 + h_0)y^2 + C}, \tag{15}$$

in which $C$ is an integration constant. Note that we have chosen the negative square root to coincide with the convention in this paper that $\Phi$ is moving in the positive direction during kination domination (recall $y$ is decreasing by construction).

Although Eq. (15) can be used to write the solution as a single integral, the phase space of solutions can also be visualized by examining this equation. As long as $C \geq 0$, $y$ moves towards $y = 0$ ($y \sim 2 \times 10^{-10}$ at the time of BBN). The existence of a large class of solutions given by $C \geq 0$ is not surprising given that the Hubble expansion generically provides friction for the field velocity $f$. The time it takes for the field to achieve a final velocity $f_f$ can be written as an integral:

$$\Delta t = \frac{1}{f_i\sqrt{\gamma}} \int_1^{f_f/f_i} \frac{dy}{y\frac{dy}{dx}}. \tag{16}$$

As a consistency check, it can be seen from Eq. (15) that generically it takes an infinitely long time to achieve $f_f = 0$ (an infinite time is necessary since we are engineering $f \propto a^{-3}$). Although a larger $C$ seems to correspond to a shorter time to achieve the desired $f_f$, it also implies that the potential is larger, which places severe constraints on $C$. More explicitly, it can be shown that the potential of Eq. (10) implied by Eq. (15) is of the form

$$V(\Phi) = \left(\frac{2C}{9} + h_0 y^2(\sqrt{\gamma}\Phi)\right)\frac{f_i^2}{2}, \tag{17}$$

where $C$ is then explicitly seen to control the cosmological constant. (If $C < 0$, the cosmological constant will be negative, which will cause the universe to eventually contract. This is consistent with the $C \geq 0$ condition described above.) Note that the potential is independent of $\kappa$ except through $y(x)$. It can be seen from this expression that if $h(\Phi)$ were not a constant $h_0$, a richer shape of potentials can easily be attained (i.e., it is easy to show that the $y$ dependent term generalizes to $f_i^2 \int dyyh$ when $h$ is not a constant). Furthermore, we see the condition that $V(\Phi)$ be a subdominant energy component to $f^2/2$ implies that $C \ll 1$. Specifically, if $Cf_i^2/9$ is identified as the cosmological constant that persists to today, one would impose the constraint $C \lesssim 9\Omega_\Lambda \rho_C/f_i^2 \approx 6 \times 10^{-54}/\eta_\Phi$. If we insist on this form of the potential until BBN, we have a bound of

$$C \ll \frac{10^{-18}}{\eta_\Phi} \tag{18}$$

even if this piece of the potential were piecewise glued to other functional forms for the potential once the evolution has persisted past the kination dominated period. Since $C$ is generally required to be small during kination domination, we can now solve Eq. (15) exactly neglecting $C$, which leads to the expression

$$y(x) = \frac{1}{64(1+h_0)^{3/2}} \exp(-3\sqrt{1+h_0}(x - C_2)/\sqrt{2}) \times (1 - 8\kappa \exp[\sqrt{2(1+h_0)}(x - C_2)])^3, \tag{19}$$

in which $C_2$ is another integration constant that is specified by $y(x_i) = 1$:

$$C_2 = x_i + \frac{\sqrt{2}}{\sqrt{1+h_0}} \ln[2(\sqrt{1+h_0} + \sqrt{1+h_0+2\kappa})]. \tag{20}$$

As expected, initially the radiation energy density encoded in $\kappa$ is unimportant, but once the radiation catches up with the quintessence kinetic energy, the potential must compensate to (artificially) maintain the $1/a^6$ behavior for the quintessence energy density while keeping the left-hand side of Eq. (11), $h(\Phi)$, a constant. Once again, it is important to note that the shape of the potential dictated by Eq. (19) is not fundamental because this solution is only valid for a constant $h(\Phi)$.

The main lesson from the discussion thus far is that potentials can be chosen to not interfere with $1/a^6$ behavior of the quintessence energy density even past the point in which the radiation starts to dominate. [As discussed earlier, this follows from the fact that the solution to the $\Phi$ equation of motion with $V'(\Phi) = 0$ is $\dot{\Phi} \propto a^{-3}$.] To obtain a potential that is manifestly independent of $\kappa$, we can choose implicitly a nonconstant $h(\Phi)$. For example, inspired by Eq. (19), we can choose the potential to be

$$V = \Omega_\Lambda \rho_c[1 + b\cosh^2(\lambda\Phi)], \tag{21}$$

in which

$$\lambda \equiv \sqrt{\frac{9\gamma}{2}(1+b)}, \tag{22}$$

with $b \ll 1$. This form of the potential results in $V'(\Phi)$ being unimportant during the $\Phi$ evolution until the temperature reaches

$$T \sim 10^{-10} b^{2/9}(1+b)^{1/9} \exp\left(-10\sqrt{1+b} + \frac{4n}{9}\right) \times \eta_\Phi^{-1/9-(2/3)\sqrt{1+b}} \text{ GeV}, \tag{23}$$

where the initial value of $\Phi$ was parametrized as $-n/\lambda$. The values $n = 30$, $b = 10^{-6}$, and $\eta_\Phi = 1$ result in a temperature during matter domination but close to the matter-radiation equality (i.e., long after the end of BBN). Therefore, this potential provides the desired kination behavior. We will be using this form of the potential for the rest of the paper.

To check the stability of the background scalar field solutions, we set $\Phi = \phi_B(t) + a^{-3/2}\delta\tilde{\phi}(t, \vec{x})$, in which $\phi_B(t)$ is the background solution, and note that upon neglecting the metric fluctuations to a leading order approximation, the equations of motion for the field fluctuations take the form

$$\delta\tilde{\phi} - \frac{1}{a^2}(\partial_i \delta\tilde{\phi})^2 + \left[V''(\phi_B) + \frac{9}{4}\gamma P\right]\delta\tilde{\phi} = 0, \tag{24}$$

in which $P$ is the pressure of an ideal fluid (our stress tensor approximation). The condition for stability on all length scales is

$$V''(\phi_B) + \frac{9}{4}\gamma P > 0. \tag{25}$$

Therefore, if $V''(\phi_B) > 0$ [which is true, for example, in the case of constant $h(\Phi)$ in Eq. (17) if $h_0 > 0$ or in the case of Eq. (21) if $b > 0$], the solution will always be stable for the case of $P > 0$ relevant for kination domination scenarios.

The viability of the quintessence picture also requires that the background solution tends toward a potential energy dominated regime with $V(\Phi) \sim \Omega_\Lambda \rho_c$, in which $\rho_c$ is the critical energy density today. Since the equation of state can be written as

$$w = \frac{1 - 2V/f^2 + \frac{2}{3}\rho_R/f^2}{1 + 2V/f^2 + 2(\rho_M + \rho_R)/f^2}, \tag{26}$$

it is straightforward to see that as long as $V$ asymptotically dominates the energy density as the universe expands, the kination scenario naturally leads to the desired late time quintessence behavior of $w \to -1$. The phenomenological requirement is that $w \approx -1$ by redshift of about $z \approx 1$.

A final issue to address is the underlying dynamics which leads to the required initial conditions to achieve the desired $f_i$. We will return to this question later and will discuss interesting observational consequences, but first analyze an explicit example numerically to illustrate the possible resulting equation of state.

In solving the quintessential cosmology specified by Eq. (21) numerically, we choose the following representative parameters: $\Omega_\Lambda = 0.72$, $\Omega_M = 0.28$ (including baryons), $\Omega_R = 4.6 \times 10^{-5}$, and $H_0 = 73\ \mathrm{km\,s^{-1}\,Mpc^{-1}}$. We further set $b = 10^{-6}$, $\lambda\Phi(t_{\mathrm{initial}}) = \{-30, -20, -10\}$, $\eta_\Phi \equiv \rho_\Phi/\rho_\gamma|_{T=1\ \mathrm{MeV}} = 0.5$, and $\lambda\dot{\Phi}(t_{\mathrm{initial}}) > 0$ (the magnitude is fixed by $\eta_\Phi$). We also take the effective degrees of freedom to evolve approximately as

$$g_*(T) \approx g_{*S}(T) \approx \begin{cases} 90 & T > 1\ \mathrm{GeV} \\ 60 & 1 > T > 0.1\ \mathrm{GeV} \\ 10.75 & 0.1 > T > 10^{-4}\ \mathrm{GeV} \\ 3.36 & 10^{-4}\ \mathrm{GeV} > T. \end{cases} \tag{27}$$

Although the relation $g_*(T) \approx g_{*S}(T)$ breaks down at late times, these subtleties do not affect the main results for kination domination. In evolving the coupled differential equations for $\{a(t), \Phi(t)\}$ as given in Eq. (1) and (2), we take

$$\rho_R(t) = \rho_R(\mathrm{today})\left(\frac{g_*(T_{\mathrm{today}})}{g_*(T_{\mathrm{approx}}(t))}\right)^{1/3}\left(\frac{a(\mathrm{today})}{a(t)}\right)^4 \tag{28}$$

and

$$\rho_M(t) = \rho_M(\mathrm{today})\left(\frac{a(\mathrm{today})}{a(t)}\right)^3. \tag{29}$$

This neglects the fact that $\rho_M$ has a nontrivial time behavior due to annihilations; however, the annihilation corrections are negligible by the time $\rho_M$ becomes a significant component of the energy density. We are also using an approximate temperature,

$$T_{\mathrm{approx}}(t) \equiv T_{\mathrm{today}}\frac{a_{\mathrm{today}}}{a(t)} \tag{30}$$

in $g_*$ in Eq. (28). This should be sufficiently accurate at the current level of approximation, since $g_*$ is a very flat function except in critical regions of Eq. (27) where the transition temperatures are only accurate to an order of magnitude.

The resulting evolution of $\Phi$ is shown in Fig. 1 as a function of temperature. For $\lambda\Phi(t_{\mathrm{initial}}) = -30$, which we denote as model C, the field does not reach the origin (the minimum of the potential) by today. However, when the initial value of $\lambda\Phi$ is less negative, the field overshoots the minimum and climbs up the potential for $\Phi > 0$, and eventually falls back towards the origin. An inspection of Fig. 2, which shows the dark energy equation of state as a function of redshift, demonstrates that acceptable phenomenology can be obtained for the "overshooting" case of $\lambda\Phi(t_{\mathrm{initial}}) = -20$ (model B), but not for $\lambda\Phi(t_{\mathrm{initial}}) = -10$ (model A), where there is still too much kinetic energy by redshifts of less than one. In fact, the equation of state $w$ for model A actually reaches $-1$ during the bounce and then a time period in which $w = 0$ is sustained until the kinetic energy is finally dissipated. This nontrivial evolution of the equation of state for larger redshifts than those shown in Fig. 2 can be seen in Fig. 3.

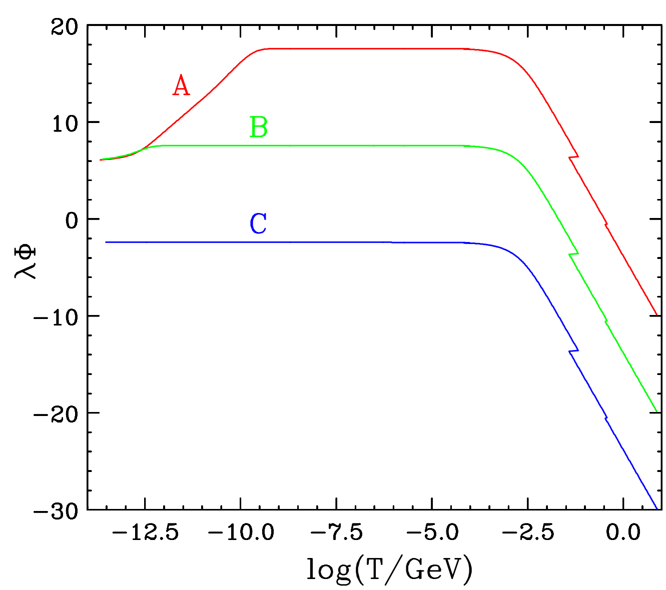


FIG. 1 (color online). $\Phi$ as a function of $T$. The evolution for $\lambda\Phi(t_{\mathrm{initial}}) = \{-10, -20, -30\}$ is depicted by the curves denoted by models A, B, and C, respectively. The little jump in temperature at $T \approx 10^{-1}$ GeV corresponds to the change in $g_*$ at that temperature; a much smaller jump can be seen at $T \approx 1$ GeV.

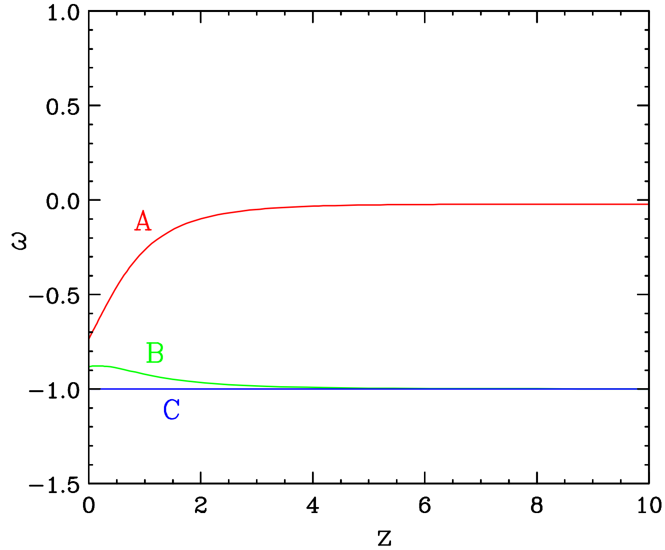


FIG. 2 (color online). The dark energy equation of state $w$ as a function of redshift $z$ for the same cases shown in Fig. 1.

Furthermore, it can also be shown that the scalar field energy density always dominates in model A, which makes such scenarios incompatible with the successes of cold dark matter large scale structure phenomenology. Clearly, even with exactly the same potential function $V(\Phi)$ and initial value of quintessence kinetic energy, the dark energy phenomenology crucially depends on the initial value of $\Phi$.

In all of these cases, the initial condition of $|\lambda\Phi(t_{\mathrm{initial}})| > 10$ corresponds to situations in which the initial displacement is trans-Planckian. This is not attractive from an effective field theory point of view since the

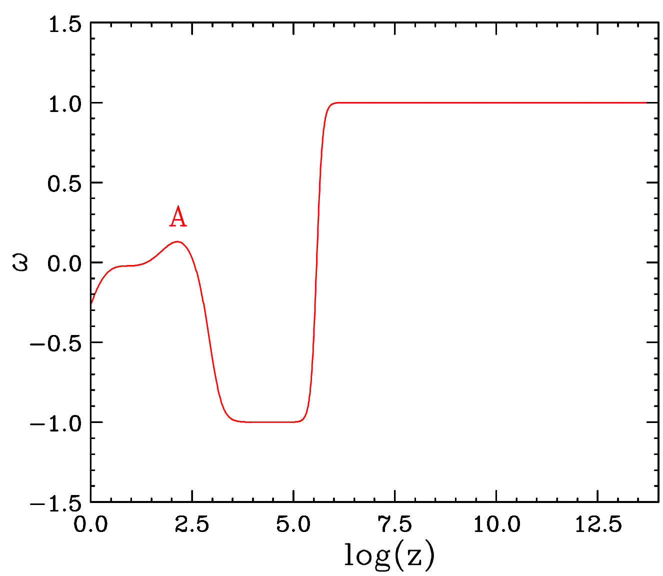


FIG. 3 (color online). The equation of state $w$ of the $\Phi$ component as a function of redshift $z$ for large redshifts for model A $[\lambda\Phi(t_{\mathrm{initial}}) = -10]$.

theory then is sensitive to all powers of the field operator; however, such initial field values are typical in quintessence scenarios [22]. Indeed, although quintessence is generically not attractive as an effective field theory candidate, its classical dynamics may still be useful to parametrize vacuum dynamics, since effective field theory has grossly failed as far as understanding the vacuum structure is concerned, i.e., in giving a plausible explanation for the cosmological constant problem. In fact, we have implicitly been assuming throughout this paper that the cosmological constant problem has been solved by some unknown mechanism which leaves the quintessence dynamics responsible only for the nontrivial vacuum energy dynamics.

The main lesson to be learned from this exercise is that in viable kination scenarios, the dark energy equation of state can exhibit a wide range of behavior both because of the shape of the potential and because of the unknown initial conditions. However, a typical behavior more likely in quintessence scenarios with a period of kination domination than in generic quintessence models is the bounce or turnaround behavior, in which $\Phi$ bounces off the "other" end of the potential barrier and slowly rolls toward the minimum. With one bounce, the behavior of the model A case in Fig. 2 in which $dw/dz \lesssim 0$ today is typical in viable models. The complement, $dw/dz > 0$, is less generic for large initial kinetic energies, but may be possible in models in which the kinetic energy during the kination period is sufficiently small and tuned appropriately. Multiple bounce scenarios may also be possible in kination scenarios, but they require a special conspiracy between the shape of the potential and initial conditions, as exemplified by the unsuccessful case shown in Fig. 2.

In the present model, we can compute the minimum initial displacement $|\lambda\Phi(t_{\mathrm{initial}})|$ to avoid the turnaround behavior. The total displacement of $\Phi$ for the case in which $V'(\Phi)$ plays a negligible role in the equation of motion is

$$\Delta\Phi \approx \sqrt{6}M_p \ln\left[\frac{a(t_{\mathrm{BBN}})}{a(t_i)}\right] + 3\sqrt{\eta_\Phi}M_p, \qquad (31)$$

in which $a(t_i)$ is the scale factor at the time of the initial position of $\Phi$, and the second term on the right-hand side of Eq. (31) arises from the movement of $\Phi$ after BBN. For typical situations in which $a(t_{\mathrm{BBN}})/a(t_i) = a(t_{\mathrm{BBN}})/a(t_F) \gtrsim 10^3$, where $a(t_F)$ is the scale factor at the time of freeze-out, we find

$$\Delta\Phi \gtrsim [17 + 3\sqrt{\eta_\Phi}]M_p. \qquad (32)$$

This result implies that most of the displacement of $\Phi$ occurs during the kination period and not after BBN, and that the numerical result of Fig. 1 is reasonable. In addition, the field displacement is generically trans-Planckian (as discussed previously).

Having discussed the consequences of having large quintessence kinetic energy for late time cosmology, we now consider how such initial conditions can be established within the context of inflationary cosmology.

## III. INFLATIONARY COSMOLOGY

In this section, we address the issue of how kination domination might be achieved within theories of cosmological initial conditions such as inflation. To demonstrate an existence proof of a viable scenario, we construct an inflationary model in which a single scalar field plays the role of both the inflaton and the quintessence field. We will not only show that kination domination can be achieved after inflation, but that sufficient reheating can be achieved as well. The main benefits of embedding quintessence within an inflationary scenario are the resulting cosmological predictions which can be experimentally verified or falsified. We will see that a robust prediction of the kination scenario relevant for dark matter abundance is an absence of a measurable B-mode CMB polarization signal. Other predictions will include those connected to the fact that the Hubble expansion rate during kination domination is different from that within standard cosmological scenarios even when the radiation is in equilibrium.

The basic idea we implement to construct this inflationary model is the assumption that the inflaton receives a kick (or a strong push) at the end of inflation to achieve kination domination. Indeed, since slow roll inflationary models already require a coherent homogeneous scalar field to dominate the energy density in the universe, the inflaton is an ideal candidate field to be converted into a quintessence driving kination-dominated universe. By comparing a multiscalar field system in which the inflaton energy density converts efficiently into a coherent scalar field kinetic energy of another field direction, it is straightforward to see that the only significant simplification being made by considering one scalar field is in the neglect of the acceleration of the field velocity vector direction in field space.

To construct this model for kination cosmology in the context of inflation, we will assume the following degrees of freedom:

(1) *Inflaton and quintessence*. A real scalar field degree of freedom $\Phi$ plays the role of both the inflaton and quintessence. Of course, a complete supersymmetric embedding of this scenario would require a complex scalar degree of freedom. We neglect this detail here for simplicity, as we are first concerned with generic dynamical settings, and note that there is no straightforward insurmountable obstacle to extending this scenario into a fully supersymmetric framework.

(2) *MSSM fields*. We assume the presence of electroweak to TeV-scale MSSM field degrees of freedom; the lightest neutralinos are stable lightest supersymmetric particles (LSPs). We will denote the neutralino LSP as $\chi$ and other generic fields as $\psi$.

(3) *Couplings*. We assume for simplicity that $\Phi$ is coupled to MSSM fields only through the minimal gravitational coupling. Scenarios with additional couplings are of course plausible, but we do not consider them in detail here because the resulting constraints are highly model dependent. Furthermore, since viable phenomenology requires the quintessence field to be very weakly coupled to the (MS)SM fields, the minimal gravitational coupling scenario is ''natural'' within the quintessence paradigm (of course, quintessence itself has a doubtful status from the point of view of effective field theory).

For simplicity, we will also functionally tune the potential (i.e., assume a specific form of the nonrenormalizable operators). Such simplifications are reasonable for this first attempt at model building of this kind, given the current incomplete understanding of possible UV completions of the MSSM as well as the typical difficulties of embedding inflation and quintessence in the context of effective field theories. Future model-building attempts will need to address this issue.

Our scenario is predicated on the physical picture that $\Phi$ receives a kick at the end of the inflationary period. To this end, we consider a scalar field potential with a step-function-like behavior, such that $\Phi$ is potential energy dominated at the top of the step and then becomes kinetic energy dominated when $\Phi$ drops off the cliff of the step. As an example, consider the ansatz

$$V(\Phi) \approx \Omega_\Lambda \rho_c[1 + b\cosh^2(\lambda\Phi)] + \left[V_0 + \beta \ln\left(\frac{(\Phi - \Phi_c)^2}{\mu^2} + \delta^2\right)\right]S(\Phi), \quad (33)$$

in which $\beta$ is a constant energy density scale controlling the slow roll properties of the period of inflation, $\delta$ is a small constant inserted purely to regularize the logarithm when $\Phi - \Phi_c$ vanishes, and $S(\Phi)$ is a steplike function. An example would be $S(\Phi) = (1 - \tanh[\alpha(\Phi - \Phi_c)])/2$, which is unity for $\alpha(\Phi - \Phi_c) < 1$ and smoothly goes to zero after $\alpha(\Phi - \Phi_c) \gg 1$ for sufficiently large values of $\alpha$. This potential is identical to Eq. (21) except with the addition of the $[V_0 + \beta \ln((\Phi - \Phi_c)^2/\mu^2)]S(\Phi)$ term responsible for the inflationary period, which in turn shuts off at $\Phi = \Phi_c$. Note that the constant $\delta$ plays no important role but to make the logarithmic function regular, and as we will comment more explicitly later, we can choose a sufficiently small $\delta$ as not to change any of the inflationary analysis. Hence, it can be dropped from the analysis of the inflationary period.

Let us consider the constraints on the scales $V_0$, $\alpha$, and $\beta$ in this scenario (we will set $\mu = \Phi_c$ without any loss of generality). Inflation occurs for $(\Phi - \Phi_c) < 0$, and ends when the inflaton receives a hard kick at $\Phi \approx \Phi_c$, leading to kination domination. During the kick at the time $t_e$ of the end of inflation, the nonadiabatic time variation of the background gravitational field generates particles with energy density $\rho_\psi(t_e)$ (a similar reheating scenario was considered in [23]). The mechanics of this is explained in the Appendix. If the particles that are produced have masses much smaller than the expansion rate $H_e \sim \sqrt{V_0/3}/M_p$ at the end of inflation, then

$$\rho_\psi(t_e) \sim \frac{\pi^2}{30} g_*(T_e \approx H_e/2\pi)\left(\frac{H_e}{2\pi}\right)^4, \tag{34}$$

in which $g_*$ counts the number of light degrees of freedom. This can be seen as the situation where the all of the species which couple to the large $\Phi$ vacuum expectation value (VEV) have decoupled and the others are lighter than $H_e$. We have made a special simplifying assumption that there is a large number of light species despite the large $\Phi$ VEV. (Indeed, the lack of such light particles can lead to a moduli problem as the VEV can induce large masses to the fields to which it couples. Note that even in regions where the $\Phi$ VEV is zero, large finite density masses can be induced for what would otherwise be light fields [24,25].) Because of the kick and the sudden drop of the potential, the kinetic energy of $\Phi$ just after inflation ends will be of order $V_0$ and will dilute as $1/a^6$. Although the relativistic species $\psi$ initially is out of equilibrium, the energy density can be characterized by an approximate temperature $T_e \sim (\rho_\psi/g_*(T_e))^{1/4}$. As we will discuss below, $\psi$ will eventually equilibrate and the relativistic energy density will dilute as $1/a^4$.

In this class of scenarios, the initial values of $\rho_\psi$ and $\rho_\Phi$ are interconnected, such that for any given reheating energy density $\rho_\psi$, there is a predicted value of $\eta_\Phi \equiv \rho_\Phi/\rho_\gamma|_{T=1\,\mathrm{MeV}}$, which is phenomenologically required to be less than about 1 (at $2\sigma$) by the time the photon temperature is of order 1 MeV. Since $\rho_\Phi/\rho_\gamma \approx a^{-2} \propto T^2$ during the kination period, $\eta_\Phi$ is given by

$$\eta_\Phi \sim 33 \frac{(1\ \mathrm{MeV})^2 V_0}{\rho_\psi^{3/2}(T_e)\sqrt{g_*(T_e)}}. \tag{35}$$

Combining Eqs. (34) and (35), we find that in order to obtain a desired $\eta_\Phi$, the inflationary energy density must be of the form

$$V_0 \sim (3.9 \times 10^{13}\ \mathrm{GeV})^4 \eta_\Phi^{-1/2}\left(\frac{g_*(T_e)}{100}\right)^{-1}. \tag{36}$$

A larger $\eta_\Phi$ requires a smaller $V_0$, because the radiation energy density at the end of inflation is proportional to $V_0^2$, and an increased radiation energy density corresponds to an increased scale factor growth before the temperature reaches 1 MeV.

Equation (36) is a remarkable result as it sets an approximate upper bound on $V_0$ if a non-negligible $\eta_\Phi$ is to be achieved. Note that this did not depend on the details of the inflationary model, but only on the fact that a period of kination domination occurs just following inflation together with reheating. Although such upper bounds have not been imposed in previous studies [6,8], this result appears to be quite generic within a large class of inflationary models.

An interesting ramification of this bound on $V_0$ results from the fact that the detection of inflationary tensor perturbations in the foreseeable future requires $V_0 \gtrsim (3 \times 10^{15}\ \mathrm{GeV})^4$ (corresponding to a tensor to scalar ratio of about $10^{-4}$) [26–29]. Hence, if tensor perturbations are detected, $\eta_\Phi$ must satisfy the following approximate bound:

$$\eta_\Phi \lesssim 10^{-13}\left(\frac{\theta}{10}\right)^2\left(\frac{r_{\min}}{10^{-4}}\right)^{-2}\left(\frac{g_*(T_e)}{100}\right)^{-2}. \tag{37}$$

In the above, $\theta \equiv V(\Phi_N)/V(\Phi_e)$ is the ratio of the potential between the time when the largest observable scales left the horizon and the time when inflation ends, and $r_{\min}$ is the minimum detectable tensor to scalar ratio defined as $16\epsilon$ evaluated at the 0.002 $\mathrm{Mpc}^{-1}$ Hubble crossing scale, where $\epsilon \equiv (M_p^2/2)(V'(\phi)/V(\phi))^2$ is the usual potential expansion slow roll parameter. Therefore, larger values of $\eta_\Phi$ would be ruled out in this scenario if tensor perturbations are measured. Note that a nonzero $\eta_\Phi \ll 10^{-6}$ causes an $\mathcal{O}(10^6\eta_\Phi)$ change in the prediction of the relic abundance compared to the standard scenario [6,8,30]. In the event of a positive tensor perturbation measurement, $\eta_\Phi$ would be bounded to be so small that kination dominated scenarios cannot be effective in changing the thermal relic abundance from the values obtained using standard cosmology (i.e., the connection between collider measurements and dark energy would be lost).

The constraint for obtaining the required number of $e$-folds is given by

$$N \gtrsim \ln\left(\sqrt{\theta}\frac{H_e}{H_0}\right) + \frac{1}{3}\ln\left(\frac{g_{*S}(T_{RH})}{g_{*S}(T_0)}\right) + \ln\frac{T_0}{T_{RH}}, \tag{38}$$

in which $H_0$ is the expansion rate today, and in our scenario the reheating temperature $T_{RH} \sim H_e/2\pi$, indicating that the explicit $H_e$ dependence drops out of Eq. (38). The requisite number of e-foldings is slightly larger than the usual inflationary scenario with reheating during coherent oscillations since there is no corresponding scale stretching that would have occurred during the coherent oscillations phase. As there is no explicit $H_e$ dependence, for $\theta \sim \mathcal{O}(10)$ the required number of $e$-folds of inflation is

$$N \gtrsim 71, \tag{39}$$

nearly independently of the inflationary potential. Note that in the absence of an unusual initial state of the vacuum, the number of $e$-folds required after the onset of inflation to achieve the usual inflationary predictions is very small [31].

Since $V_0$ is fixed by Eq. (36) and $\epsilon$ is fixed by the density perturbation constraint

$$P_{\mathcal{R}}(0.002\ \mathrm{Mpc}^{-1}) = \frac{\theta V_0}{24\pi^2 \epsilon M_p^4} \approx 2 \times 10^{-9}, \qquad (40)$$

$\beta$ is approximately given by

$$\beta \approx (8.3 \times 10^{10}\ \mathrm{GeV})^4 \left(\frac{g_{*S}(T_e)}{100}\right)^{-2} \left(\frac{\theta}{\eta_\Phi}\right). \qquad (41)$$

For this class of models $V_0 \gg \beta$, which implies that $\theta \approx 1$ during the approximate 70 e-foldings of interest since the potential depends only logarithmically on $\Phi$ during inflation. Nonetheless, it is still useful to keep the $\theta$ dependence explicitly to keep track of the model dependence, since expressions like Eq. (37) are not strongly dependent on the shape of the inflationary potential.

To determine the very model-dependent field value during which $k \approx 0.002\ \mathrm{Mpc}^{-1}$ leaves the horizon (corresponding to about 68 e-foldings before the end of inflation), we use the usual field integral over $1/\sqrt{2\epsilon(\Phi)}$, which yields

$$\Phi - \Phi_c \approx -8 \times 10^{-5} (\eta_\Phi)^{-1/4} \left(\frac{g_{*S}(T_e)}{100}\right)^{-1/2} M_p \ll \Phi_c. \qquad (42)$$

Equation (42) implies that unless there is fine-tuning of the initial conditions, the number of e-foldings will be much larger than the requisite number of Eq. (39). Finally, the end of inflation, which is determined by $|M_p^2 V''(\Phi)/V(\Phi)| \approx 1$ (since afterwards $\Phi$ rolls quickly to make $\epsilon = 1$), is given by

$$\Phi_e - \Phi_c \approx -6 \times 10^{-6} (\eta_\Phi)^{-1/4} \left(\frac{g_{*S}(T_e)}{100}\right)^{-1/2} M_p, \qquad (43)$$

which means that inflation ends very close to $\Phi_c$ (by construction) and far away from scales of interest of Eq. (42). This places a constraint on the steplike function $S$, since the slow roll behavior should not be disrupted by the slope of this steplike function before the required number of e-foldings of inflation. For example, if $S(\Phi) = (1 - \tanh[\alpha(\Phi - \Phi_c)])/2$, $\alpha$ is bounded by

$$\alpha \gtrsim \frac{10^6}{M_p} \eta_\Phi^{1/4} \left(\frac{g_{*S}(T_e)}{100}\right)^{1/6}. \qquad (44)$$

It is also straightforward to show that in order for $\Phi$ kination domination to occur at the end of inflation, the potential that kicks $\Phi$ must satisfy the following condition:

$$-\sqrt{2} M_p \frac{V'(\Phi)}{V(\Phi)} \gtrsim 6, \qquad (45)$$

a much less stringent constraint than Eq. (44). Finally, as we have commented near Eq. (33), we now see explicitly that if we make $\delta \lesssim 10^{-7}$ for $|\Phi_c|/M_p \gtrsim 10$, $\delta$ does not change the inflationary analysis.

We have checked the details of this inflationary scenario with explicit numerical computations. The analytic discussion above is in good agreement with the numerical results.

### A. Inflation to kination domination

The steplike feature in the potential presented in Eq. (33) suggests an interesting new classical equation of motion which is exactly solvable that is relevant for this class of scenarios. If gravity is turned off and the ln term is neglected, the equation of motion for the coherent state of $\Phi$ at the end of inflation can be written as

$$\ddot{\Phi} - \frac{V_0 \alpha}{2} \mathrm{sech}^2(\alpha(\Phi - \Phi_c)) = 0, \qquad (46)$$

in which we have omitted the negligible contribution proportional to $\Omega_\Lambda \rho_c$. This class of potentials allows us to now rewrite this equation in the limit $|\alpha \Phi_c| \gg 1$ [the limit of interest according to Eq. (44)] as follows:

$$\ddot{\Phi} - V_0 \delta(\Phi - \Phi_c) \approx 0. \qquad (47)$$

The parameter $\alpha$ has disappeared from the equation of motion; this is not surprising given the step-function-like behavior of the potential in this limit. The beauty of Eq. (47) is that it can be solved exactly. The solution is given by

$$\Phi = \Theta(t_c - t)\{\Phi_i + (t - t_i)\dot{\Phi}_i\} + \Theta(t - t_c)\{\Phi_c + (t - t_c)\gamma\}, \qquad (48)$$

where $\Theta(x)$ is a step function, $\gamma \equiv \sqrt{2V_0 + \dot{\Phi}_i^2}$, $\Phi_i$ and $\dot{\Phi}_i$ are initial values of the field and its velocity at time $t_i$, and $t_c$ is the time at which $\Phi$ reaches $\Phi_c$. Therefore, since the initial time variation of the field $\dot{\Phi}_i$ is small compared to $\sqrt{V_0}$, the field obtains a strong kick at $t = t_c$ to obtain the final state velocity of $\dot{\Phi} \sim \sqrt{2V_0}$.

### B. Does dark matter ever reach equilibrium?

Another difference from more traditional inflationary scenarios is that the dark matter is initially out of thermal equilibrium after reheating (or more accurately, entropy production) in this scenario. To see this, first consider the more traditional inflationary paradigm. The neutralino $\chi$ self-annihilation rate behaves as

$$\Gamma_\chi \sim \langle \sigma v \rangle n_\chi^{eq} \sim \alpha^2 T, \qquad (49)$$

where $\chi$ is relativistic, and $\alpha \equiv g_W^2/(4\pi)$ is the weak coupling expansion parameter. On the other hand, the expansion rate after reheating in standard inflationary scenarios behaves as

$$H \sim \sqrt{g_*} \frac{T^2}{M_p}, \qquad (50)$$

which means that the neutralinos are in equilibrium as long as

$$T \lesssim 2.6 \times 10^{14}\ \mathrm{GeV}\left(\frac{\alpha}{1/30}\right)^2\left(\frac{g_*}{100}\right)^{-1/2}. \qquad (51)$$

In our scenario, however, $H$ is governed by $\dot{\Phi}$ (not $T$) during kination domination, which is also when $\chi$ freezes out:

$$H \approx \sqrt{\frac{V_0}{3M_p^2}}\left(\frac{a_e}{a}\right)^3 \approx \sqrt{\frac{V_0}{3M_p^2}}\left(\frac{2\pi T}{H_e}\right)^3. \qquad (52)$$

In the above, we have used the fact that $\dot{\Phi}^2 \propto 1/a^6$ during kination domination. Hence, the neutralinos reach equilibrium through self-annihilations only for

$$T \lesssim 8 \times 10^5\ \mathrm{GeV}\left(\frac{\alpha}{1/30}\right)\left(\frac{V_0}{[3.9 \times 10^{13}\ \mathrm{GeV}]^4}\right)^{1/2}. \qquad (53)$$

Comparing this temperature with the original relativistic species temperature of

$$\frac{H_e}{2\pi} \approx \left(\frac{V_0}{[3.9 \times 10^{13}\ \mathrm{GeV}]^4}\right)^{1/2} 6 \times 10^7\ \mathrm{GeV}, \qquad (54)$$

we see that there is a long period in which the neutralino self-annihilations are out of equilibrium after the entropy in the universe has been produced. The fact that Eq. (53) exists for $T \gg 1$ GeV is important, since our goal is to embed the modified dark matter freeze-out scenario in which the dark matter was originally in equilibrium.

## IV. OTHER PREDICTIONS FOR COSMOLOGY

One of the main advantages of embedding the kination domination scenario within an inflationary cosmological setting is that other predictions for observables can be made whose experimental confirmation would either support or rule out the scenario. We have already seen that if tensor perturbations are measured in the near future, this scenario is ruled out. There are many other observables correlated with this dark matter scenario. Although an exhaustive analysis of these signatures is beyond the scope of this paper, we briefly discuss several possibilities here.

In the present scenario, an out of equilibrium effective "temperature" scale as high as $10^8 \eta_\Phi^{-1/4}$ GeV is reached at the end of inflation and the equilibration of the (MS)SM particles occurs by temperatures of about $10^6 \eta_\Phi^{-1/4}$ GeV. During the time until the temperature is below ~1 GeV, the Hubble expansion rate differs significantly from the usual radiation dominated universe value. This implies that any physics which depends both on the temperature and the Hubble expansion will be modified.

One testable example is the gravity wave production during the electroweak phase transition (see for example [32–37]), where the peak frequency of the gravity waves are set by the Hubble scale. Since $H$ is about $10^5\sqrt{\eta_\Phi}$ of the usual Hubble value (for the same temperature), the effects can be large even for a very small $\eta_\Phi$. Exact details will require a careful reanalysis of the gravity wave production. The effects on electroweak baryogenesis are expected to be weaker since the out of equilibrium condition is primarily provided by the bubble wall velocity [38,39]. A more careful investigation of this issue is left for future work.

Another prediction of this scenario is that since the temperature at which equilibrium is reached for any heavy lepton number carrying particle (such as a right-handed neutrino) will be relatively low as in Eq. (53), possible leptogenesis mechanisms in this context will necessarily be either nonthermal or nonstandard. It will be interesting to explore what kinds of leptogenesis scenarios are viable for this class of models.

There are also predictions associated with particle astrophysics. For example, it is well known that explaining the HEAT measurement (and other cosmic ray positron measurements) of excess positrons around and above 7 GeV requires an efficient annihilation of neutralinos (or other thermal relics in the context of models of extra dimensions, etc.) within our halo [40–42]. One of the many problems associated with this efficient annihilation scenario is that the relic abundance of neutralinos is generically too low (by a factor of 10 to 100) to explain most of the dark matter energy within the context of standard thermal scenarios. The kination scenario can clearly give the necessary boost to resurrect the neutralino dark matter annihilation explanation of the excess positrons. There are also other cosmic ray signatures which may shed light on nonstandard $H$ behavior [43,44].

Finally, there are other possible signatures such as the change in BBN due to the effects of residual annihilations after freeze-out [45], and the change in cosmic string generated gravity wave signature [46,47] due to the change in $T/H$ involved in the scaling behavior of cosmic strings. Since much of cosmology is about studying the out of equilibrium phenomena generated by the expansion of the universe within a finite temperature setting, further signatures related to a nonstandard relationship between $T$ and $H$ will appear as we learn more about the early history of our universe.

## V. CONCLUSIONS

The possibility of a new scalar is generic in extensions of physics beyond the standard model of particle physics and cosmology (particularly in those containing a dilatonic field degree of freedom). It is natural to expect that the dark energy density is connected with such a new scalar field degree of freedom. However, the elusive nature of dark energy, and its requisite small couplings to observable fields, make such conjectures difficult to verify or disprove.

We have constructed a viable class of inflationary scenarios which exhibit a period of kination domination after inflation where the inflaton plays the role of the quintessence. Such scenarios have the intriguing feature that they can lead to observable consequences for the dark matter freeze-out of thermal relics expected in many TeV-scale extensions of the SM, which can be tested at the LHC and ILC. We have focused here on supersymmetric scenarios as prototype examples, for which the connections between astroparticle and collider physics have been extensively explored [48–65]. However, other WIMP candidates have recently emerged in models with flat [66,67] or warped [68,69] extra dimensions, in little Higgs theories [70–74], or in technicolor models [75], to which the cosmological scenarios described in this paper could also be applied.

The advantage of embedding kination dominated quintessence models within an inflationary context is that it allows for many other correlated cosmological predictions which can corroborate or rule out such kination dominated scenarios. The most robust, nearly model-independent, signature of this class of models is the absence of measurable tensor perturbations, such that any positive detection of tensor perturbations in upcoming experiments can rule out this scenario (at least as far generating an observable shift in the DM abundance is concerned). Other examples are measurements of gravity waves from the electroweak phase transition, shifts in the predictions for baryogenesis/leptogenesis, implications for the cosmic ray flux from dark matter annihilations, and other phenomena which depend on the ratio of the photon temperature to the Hubble expansion rate $T/H$.

The class of models in this paper is meant to be illustrative and represent early attempts at model building, and as such certain features are not optimal. For example, one might argue that the toy model presented here may not be easily achievable from an effective field theory point of view. However, it is reasonable to believe that this class of models is as potentially viable as any quintessence and most inflationary models that are considered seriously in the current literature. Given that such scenarios may be testable through their potentially dramatic interconnection with dark matter predictions and TeV-scale particle physics, they represent an intriguing and potentially fruitful ground for quintessence model building which warrants further exploration.

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## APPENDIX: PARTICLE PRODUCTION COMPUTATION

In this Appendix, we consider the particle production computation for a real scalar degree of freedom coupled to gravity. We start with the perturbative expression for the Bogoliubov coefficient as an integral over conformal time [76]:

$$\beta_k \approx \int d\eta \frac{w_k'}{2w_k} \exp\left[-2i \int w_k d\eta\right] = \int \frac{d(w_k^2)}{4w_k^2} \exp\left[-i \int \frac{d(w_k^2)}{w_k'}\right], \quad \text{(A1)}$$

where

$$w_k^2 \approx k^2 + (\tfrac{1}{6} + \xi)Ra^2, \quad \text{(A2)}$$

which assumes that the effective mass is dominated by the Ricci scalar $R \gg m_\chi^2$ (note that $\xi = 0$ corresponds to minimal gravitational coupling). Using the approximation

$$H^2 = \begin{cases} V_0/(3M_p^2), & \eta < \eta_c \\ V_0(a_e/a)^6/(3M_p^2), & \eta \geq \eta_c \end{cases} \quad \text{(A3)}$$

we find

$$w_k^2 \approx -6\left(\frac{1}{6} + \xi\right)a^2\left(\frac{2V_0}{3M_p^2}\Theta(\eta_c - \eta) - \frac{V_0}{3M_p^2}\left(\frac{a_e}{a}\right)^6\Theta(\eta - \eta_c)\right) + k^2, \quad \text{(A4)}$$

$$w_k' \approx \frac{-1}{2w_k}\frac{|w_k^2 - k^2|^{3/2}}{\sqrt{\frac{1}{6} + \xi}}\left[\frac{1}{\sqrt{3}}\Theta(\eta_c - \eta) + 2\sqrt{\frac{2}{3}}\Theta(\eta - \eta_c)\right], \quad \text{(A5)}$$

in which $\Theta(z)$ is a unit step function which evaluates to 1 for $z > 0$. For the limits of the integral over $w_k^2$ in Eq. (A1), we have

$$-\left(\frac{1}{6} + \xi\right)a_e^2\frac{4V_0}{M_p^2} + k^2 \leq w_k^2 \leq k^2 \quad \text{(A6)}$$

during inflation, and

$$k^2 \leq w_k^2 \leq \left(\frac{1}{6} + \xi\right)a_e^2\frac{2V_0}{M_p^2} + k^2 \quad \text{(A7)}$$

after the end of inflation. The conformal time $\eta$ is not a single valued function of $w_k^2$ at the transition at the end of inflation. However, since the nonsingle valued time period is arbitrarily short (in the limit that the potential behaves like a step function), this time period can be excised from the computation without loss of numerical accuracy, as long as a UV cutoff is imposed. The reason for the UV cutoff is that the peak strength of the nonadiabaticity responsible for particle production is precisely determined by the detailed gravitational dynamics of the transition time period, which we excise to simplify the computation. Since the particle production is through gravitational curvature, there will generically be an exponential cutoff in momentum of the particles produced at $H_e$, the expansion rate at the end of inflation. With this simplification, Eq. (A1) can be written as

$$\beta_k \approx \frac{1}{4}\int_{k^2}^{x_1}\frac{dx}{x}\exp\biggl[-i2\sqrt{3}\sqrt{\frac{1}{6}+\xi}\times\biggl(\frac{2\sqrt{x}\sqrt{k^2-x}}{x-k^2}-2\arctan\biggl[\frac{\sqrt{k^2-x^2}\sqrt{x}}{x-k^2}\biggr]+C_1(k^2)\biggr)\biggr]+\frac{1}{4}\int_{x_2}^{k^2}\frac{dx}{x}\exp\biggl[-i\sqrt{\frac{3}{2}}\sqrt{\frac{1}{6}+\xi}\times\biggl(\frac{-2\sqrt{x}}{\sqrt{x-k^2}}+2\ln[\sqrt{x-k^2}+\sqrt{x}]+C_2(k^2)\biggr)\biggr], \tag{A8}$$

in which $C_{1,2}$ are constant phase factors which are independent of the integration variable $x$ and depend on $k^2$, while

$$x_1 \equiv k^2 - \frac{4V_0}{M_p^2}\biggl(\frac{1}{6}+\xi\biggr)a_e^2 \tag{A9}$$

$$x_2 \equiv k^2 + \frac{2V_0}{M_p^2}\biggl(\frac{1}{6}+\xi\biggr)a_e^2, \tag{A10}$$

which correspond to $w_k^2$ just before and after the end of inflation (approximately a step function transition).

To obtain an estimate for $|\beta_k|^2$, we will neglect the interference term in $|\beta_k|^2$, which means that we can neglect the integration constants $C_i$. We will also utilize the fact that the contribution to the $dx$ integral in Eq. (A8) is appreciable only when $x$ is near $x_i$ and far away from $k^2$ because of the damping phase oscillations near $x \approx k^2$. Finally, we will only account for contributions with $w_k^2 > 0$, so as to maintain the particle interpretation of the massless modes produced. We then find

$$|\beta_k|^2 \sim \frac{1}{16}\biggl[\biggl(\ln\biggl[1-\frac{4V_0}{M_p^2}\biggl(\frac{1}{6}+\xi\biggr)\frac{a_e^2}{k^2}\biggr]\biggr)^2+\biggl(\ln\biggl[1+\frac{2V_0}{M_p^2}\biggl(\frac{1}{6}+\xi\biggr)\frac{a_e^2}{k^2}\biggr]\biggr)^2\biggr]\times\Theta(k-k_{\rm min})\Theta(aH_e\tilde{\lambda}-k), \tag{A11}$$

where $k_{\rm min} = \sqrt{4V_0/(M_p^2)(1/6+\xi)}$ is an appropriate infrared cutoff imposed to maintain the particle production interpretation of the massless modes, and $aH_e\tilde{\lambda}$ is the UV cutoff, with $\tilde{\lambda} \sim \mathcal{O}(1)$ reflecting the uncertainty in the UV cutoff function.

Taking the approximation that the log factors contribute $\mathcal{O}(1)$, the energy density of the real scalar degree of freedom that is produced is

$$\rho \sim \frac{1}{a^3}\int\frac{d^3k}{(2\pi)^3}\biggl(\frac{k}{a}\biggr)|\beta_k|^2 \sim \frac{\tilde{\lambda}^4}{32\pi^2}H_e^4 = \frac{\tilde{\lambda}^4\pi^2}{2}\biggl(\frac{H_e}{2\pi}\biggr)^4. \tag{A12}$$

Taking the parametrization of the thermal equilibrium abundance of one real scalar degree of freedom as $\rho = (\pi^2/30)T_{\rm eff}^4$, where $T_{\rm eff}$ is the effective temperature and $g_*$ counts the number of degrees of freedom, we find

$$T_{\rm eff} \sim \frac{H_e}{2\pi}(2\tilde{\lambda}). \tag{A13}$$

For an order of magnitude estimate, we will absorb the uncertainty factor $2\tilde{\lambda}$ in the uncertainty in the effective number of degrees of freedom during the "reheating" stage at the end of inflation, and therefore write

$$T_e \sim \frac{H_e}{2\pi} \tag{A14}$$

throughout the paper.


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