Constraints on the very early universe from thermal WIMP dark matter

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We investigate the relic density n_χ of nonrelativistic long-lived or stable particles χ in nonstandard cosmological scenarios. We calculate the relic abundance starting from arbitrary initial temperatures of the radiation-dominated epoch, and derive the lower bound on the initial temperature $T_0 \gtrsim m_\chi/23$, assuming that thermally produced χ particles account for the dark matter energy density in the Universe; this bound holds for all χ annihilation cross sections. We also investigate cosmological scenarios with modified expansion rate. Even in this case an approximate formula similar to the standard one is capable of predicting the final relic abundance correctly. Choosing the χ annihilation cross section such that the observed cold dark matter abundance is reproduced in standard cosmology, we constrain possible modifications of the expansion rate at $T \sim m_\chi/20$, well before big bang nucleosynthesis.

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I. INTRODUCTION

One of the most notable recent developments in cosmology is the precise determination of cosmological parameters from observations of the large-scale structure of the Universe, most notably by the Wilkinson Microwave Anisotropy Probe (WMAP). In particular, the accurate determination of the nonbaryonic cold dark matter (DM) density [1],

$$0.08 < \Omega_{\rm CDM} h^2 < 0.12 \qquad (95\%\ \text{C.L.}), \qquad (1)$$

has great influence on particle physics models which possess dark matter candidates [2,3]. The requirement that the predicted DM density falls in the range (1) is a powerful tool for discriminating between various models and for constraining the parameter space of surviving models.

Many dark matter candidate particles have been proposed. In particular long-lived or stable weakly interacting massive particles (WIMPs) with weak-scale masses are excellent candidates. In standard cosmology WIMPs decoupled from the thermal background during the radiation-dominated epoch after inflation. In this framework convenient and accurate analytic approximate solutions for the relic abundance have been derived [4,5]. One of the best motivated candidates for WIMPs is the lightest neutralino in supersymmetric (SUSY) models. Assuming that the neutralino is the lightest supersymmetric particle (LSP) stabilized due to R-parity, its relic abundance has been extensively discussed [3]. Similar analyses have also been performed for other WIMPs whose existence is postulated in other extensions of the standard model (SM) of particle physics. In many cases the cosmologically favored parameter space of WIMP models can be directly tested at the CERN Large Hadron Collider (LHC) in a few years [6]. The same parameter space often also leads to rates of WIMP interactions with matter within the sensitivity of near-future direct DM detection experiments.

This discussion shows that we are now entering an interesting time where the standard cosmological scenario can be examined by experiments at high-energy colliders as well as DM searches [7]. In this respect we should emphasize that the relic abundance of thermally produced WIMPs depends not only on their annihilation cross section, which can be determined by particle physics experiments, but in general is also very sensitive to cosmological parameters during the era of WIMP production and annihilation. Of particular importance are the initial temperature T_0 at which WIMPs began to be thermally produced, and the expansion rate of the Universe H.

In the standard cosmological scenario, the expansion rate is uniquely determined through the Friedmann equation of general relativity. In this scenario the density of WIMPs with mass m_χ followed its equilibrium value until the freeze-out temperature $T_F \simeq m_\chi/20$. Below T_F, interactions of WIMPs are decoupled, and thus the present density is independent of T_0 as long as $T_0 > T_F$.[1]

It should be noted that in nonstandard scenarios the relic density can be larger or smaller than the value in the standard scenario. One example is the case where T_0 is smaller than or comparable to T_F, which can be realized in inflationary models with low reheat temperature. Since in many models the inflationary energy scale must be much

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[1] Note that T_F can be formally defined in the standard way even if $T_0 < T_F$. In this case WIMPs never were in full equilibrium, and correspondingly never "froze out."

higher than m_χ in order to correctly predict the density perturbations [8], the standard assumption $T_0 > T_F$ is not unreasonable. On the other hand, the constraint on the reheat temperature from big bang nucleosynthesis (BBN) is as low as $T_0 \gtrsim$ MeV [9,10]. From the purely phenomenological viewpoint, it is therefore also interesting to investigate the production of WIMPs in low reheat temperature scenarios [9,11–14].

The standard scenario also assumes that entropy per comoving volume is conserved for all temperatures $T \leq T_F$. Late entropy production can dilute the predicted relic density [15,16]. The reason is that the usual calculation actually predicts the *ratio* of the WIMP number density to the entropy density. On the other hand, if late decays of a heavier particle nonthermally produce WIMPs in addition to the usual thermal production mechanism, the resulting increase of the WIMP density competes with the dilution caused by the decay of this particle into radiation, which increases the entropy density [13,17–21].

Another example of a nonstandard cosmology changing the WIMP relic density is a modified expansion rate of the Universe. This might be induced by an anisotropic expansion [16], by a modification of general relativity [16,22], by additional contributions to the total energy density from quintessence [23], by branes in a warped geometry [24], or by a superstring dilaton [25].

These examples show that, once the WIMP annihilation cross section is fixed, with the help of precise measurements of the cold dark matter density we can probe the very early stage of the Universe at temperatures of $\mathcal{O}(m_\chi/20) \sim 10$ GeV. This is reminiscent of constraining the early evolution of the Universe at $T = \mathcal{O}(100)$ keV using the primordial abundances of the light elements produced by BBN.

The goal of this paper is to investigate to what extent the constraint (1) on the WIMP relic abundance might allow us to derive quantitative constraints on modifications of standard cosmology. So far the history of the Universe has been established by cosmological observations as far back as the BBN era. In this paper we try to derive bounds on cosmological parameters relevant to the era before BBN. Rather than studying specific extensions of the standard cosmological scenario, we simply parametrize deviations from the standard scenario, and attempt to derive constraints on these new parameters. Since we only have the single constraint (1), for the most part we only allow a single quantity to differ from its standard value. We expect that varying two quantities simultaneously will allow us to get the right relic density for almost any WIMP annihilation cross section. This has been shown explicitly in [13] for the case that both late entropy production and nonthermal WIMP production are considered, even if both originate from the late decay of a single scalar field.

We first analyze the dependence of the WIMP abundance on the initial temperature T_0 of the conventional radiation-dominated epoch. We showed in [14] that for fixed T_0 the predicted WIMP relic density reaches a maximum as the annihilation cross section is varied from very small to very large values. A small annihilation cross section corresponds to a large $T_F > T_0$; in this case the relic density increases with the annihilation cross section, since WIMP production from the thermal plasma is more important than WIMP annihilation. On the other hand, increasing this cross section reduces T_F; once $T_F < T_0$ a further increase of the cross section leads to smaller relic densities since in this case WIMPs continue to annihilate even after the temperature is too low for WIMP production. Here we turn this argument around, and derive the lower bound on $T_0 \geq m_\chi/23$ under the assumption that all WIMPs are produced thermally. Note that we do not need to know the WIMP annihilation cross section to derive this bound.

We then examine the dependence of the predicted WIMP relic density on the expansion rate in the epoch prior to BBN, where we allow the Hubble parameter to depart from the standard value. The standard method of calculating the thermal relic density [2,4] is found to be still applicable in this case. Our working hypothesis here is that the standard prediction for the Hubble expansion rate is essentially correct, i.e. that the true expansion rate differs by at most a factor of a few from the standard prediction. We then simply employ a generic Taylor expansion for the temperature dependence of this modification factor; note that the success of standard BBN indicates that this factor cannot deviate by more than ~20% from unity at low temperatures, $T \lesssim 1$ MeV. Similarly, we assume that the WIMP annihilation cross section has been determined (from experiments at particle colliders) to have the value required in standard cosmology. Our approach is thus quite different from that taken in [7], where present upper bounds on the fluxes of WIMP annihilation products are used to place upper bounds on the Hubble expansion rate during WIMP decoupling. The advantage of their approach is that no prior assumption on the WIMP annihilation cross section needs to be made, whereas we assume a cross section that reproduces the correct relic density in the standard scenario. On the other hand, the bounds derived in Refs. [7] are still quite weak, allowing the Hubble parameter to exceed its standard prediction by a factor $\gtrsim 30$; moreover, no lower bound on H can be derived in this fashion.

The remainder of this paper is organized as follows: In Sec. II we will briefly review the calculation of the WIMP relic abundance assuming a conventional radiation-dominated universe, and derive the lower bound on the initial temperature T_0. In Sec. III we discuss the case where the pre-BBN expansion rate is allowed to depart from the standard one. Using approximate analytic formulas for the predicted WIMP relic density for this scenario, we derive constraints on the early expansion parameter. Finally, Sec. IV is devoted to summary and conclusions.

II. RELIC ABUNDANCE IN THE RADIATION-DOMINATED UNIVERSE

We start the discussion of the relic density n_χ of stable or long-lived particles χ by reviewing the structure of the Boltzmann equation which describes their creation and annihilation. The goal of this section is to find the lowest possible initial temperature of the radiation-dominated universe, assuming that the present relic abundance of cold dark matter is entirely due to thermally produced χ particles.

As usual, we will assume that χ is self-conjugate,[2] $\chi = \bar{\chi}$, and that some symmetry, for example, R parity, forbids decays of χ into SM particles; the same symmetry then also forbids single production of χ from the thermal background. However, the creation and annihilation of χ pairs remains allowed. The time evolution of the number density n_χ of particles χ in the expanding universe is then described by the Boltzmann equation [2],

$$\frac{\mathrm{d}n_\chi}{\mathrm{d}t} + 3Hn_\chi = -\langle\sigma v\rangle(n_\chi^2 - n_{\chi,\mathrm{eq}}^2), \tag{2}$$

where $n_{\chi,\mathrm{eq}}$ is the equilibrium number density of χ, and $\langle\sigma v\rangle$ is the thermally averaged annihilation cross section multiplied with the relative velocity of the two annihilating χ particles. Finally, the Hubble parameter $H = \dot{R}/R$ is the expansion rate of the Universe, R being the scale factor in the Friedmann-Robertson-Walker metric. The first (second) term on the right-hand side of Eq. (2) describes the decrease (increase) of the number density due to annihilation into (production from) lighter particles. Equation (2) assumes that χ is in kinetic equilibrium with standard model particles.

It is useful to rewrite Eq. (2) in terms of the scaled inverse temperature $x = m_\chi/T$ as well as the dimensionless quantities $Y_\chi = n_\chi/s$ and $Y_{\chi,\mathrm{eq}} = n_{\chi,\mathrm{eq}}/s$. The entropy density is given by $s = (2\pi^2/45)g_{*s}T^3$, where

$$g_{*s} = \sum_{i=\mathrm{bosons}} g_i\left(\frac{T_i}{T}\right)^3 + \frac{7}{8}\sum_{i=\mathrm{fermions}} g_i\left(\frac{T_i}{T}\right)^3. \tag{3}$$

Here g_i denotes the number of intrinsic degrees of freedom for particle species i (e.g. due to spin and color), and T_i is the temperature of species i. Assuming that the Universe expands adiabatically, the entropy per comoving volume, sR^3, remains constant, which implies

$$\frac{\mathrm{d}s}{\mathrm{d}t} + 3Hs = 0. \tag{4}$$

The time dependence of the temperature is then given by

$$\frac{\mathrm{d}x}{\mathrm{d}t} = \frac{Hx}{1 - \frac{x}{3g_{*s}}\frac{\mathrm{d}g_{*s}}{\mathrm{d}x}}. \tag{5}$$

[2] The case $\chi \neq \bar{\chi}$ differs in a nontrivial way only in the presence of a $\chi - \bar{\chi}$ asymmetry, i.e. if $n_\chi \neq n_{\bar{\chi}}$.

Therefore the Boltzmann equation (2) can be written as

$$\frac{\mathrm{d}Y_\chi}{\mathrm{d}x} = -\frac{\langle\sigma v\rangle s}{Hx}\left(1 - \frac{x}{3g_{*s}}\frac{\mathrm{d}g_{*s}}{\mathrm{d}x}\right)(Y_\chi^2 - Y_{\chi,\mathrm{eq}}^2). \tag{6}$$

Thermal production of WIMPs takes place during the radiation-dominated epoch, when the expansion rate is given by

$$H = \frac{\pi T^2}{M_{\mathrm{Pl}}}\sqrt{\frac{g_*}{90}}, \tag{7}$$

with $M_{\mathrm{Pl}} = 2.4 \times 10^{18}$ GeV being the reduced Planck mass and

$$g_* = \sum_{i=\mathrm{bosons}} g_i\left(\frac{T_i}{T}\right)^4 + \frac{7}{8}\sum_{i=\mathrm{fermions}} g_i\left(\frac{T_i}{T}\right)^4. \tag{8}$$

In the following we use H_{st} to denote the standard expansion rate (7). If the postinflationary reheat temperature was sufficiently high, WIMPs reached full thermal equilibrium. This remains true for temperatures well below m_χ. We can therefore use the nonrelativistic expression for the χ equilibrium number density,

$$n_{\chi,\mathrm{eq}} = g_\chi\left(\frac{m_\chi T}{2\pi}\right)^{3/2}\mathrm{e}^{-m_\chi/T}. \tag{9}$$

In the absence of nonthermal production mechanisms, $n_\chi \leq n_{\chi,\mathrm{eq}}$ at early times. The annihilation rate $\Gamma = n_\chi\langle\sigma v\rangle$ then depends exponentially on T, and thus drops more rapidly with decreasing temperature than the expansion rate H_{st} of Eq. (7) does. When the annihilation rate falls below the expansion rate, the number density of WIMPs ceases to follow its equilibrium value and is frozen out.

For $T \ll m_\chi$ the annihilation cross section can often (but not always [5]) be approximated by a nonrelativistic expansion in powers of v^2. Its thermal average is then given by

$$\langle\sigma v\rangle = a + b\langle v^2\rangle + \mathcal{O}(\langle v^4\rangle) = a + \frac{6b}{x} + \mathcal{O}\left(\frac{1}{x^2}\right). \tag{10}$$

In this standard scenario, the following approximate formula has been shown [2,4,5] to accurately reproduce the exact (numerically calculated) relic density:

$$Y_{\chi,\infty} \equiv Y_\chi(x \to \infty) \simeq \frac{1}{1.3m_\chi M_{\mathrm{Pl}}\sqrt{g_*(x_F)}(a/x_F + 3b/x_F^2)}, \tag{11}$$

with $x_F = m_\chi/T_F$, T_F being the decoupling temperature. For WIMPs, $x_F \simeq 22$. Here we assume $g_* \simeq g_{*s}$ and $\mathrm{d}g_*/\mathrm{d}x \simeq 0$. It is useful to express the χ mass density as $\Omega_\chi = \rho_\chi/\rho_c$, $\rho_c = 3H_0^2M_{\mathrm{Pl}}^2$ being the critical density of the Universe. The present relic mass density is then given by $\rho_\chi = m_\chi n_{\chi,\infty} = m_\chi s_0 Y_{\chi,\infty}$; here $s_0 \simeq 2900\ \mathrm{cm}^{-3}$ is the present entropy density. Equation (11) then leads to

$$\Omega_\chi h^2 = 2.7 \times 10^{10} Y_{\chi,\infty}\left(\frac{m_\chi}{100\ \mathrm{GeV}}\right) \simeq \frac{8.5 \times 10^{-11} x_F\ \mathrm{GeV}^{-2}}{\sqrt{g_*(x_F)}(a + 3b/x_F)}, \quad (12)$$

where $h \simeq 0.7$ is the scaled Hubble constant in units of $100\ \mathrm{km\,sec^{-1}\,Mpc^{-1}}$. We defer further discussions of this expression to Sec. III, where scenarios with modified expansion rate are analyzed. Note that in the standard scenario leading to Eq. (12), the present χ relic density is inversely proportional to its annihilation cross section and has no dependence on the reheat temperature. Recall that this result depends on the assumption that the highest temperature in the postinflationary radiation-dominated epoch, which we denote by T_0, exceeded T_F significantly.

On the other hand, if T_0 was too low to fully thermalize WIMPs, the final result for Ω_χ will depend on T_0. In particular, if WIMPs were thermally produced in a completely out-of-equilibrium manner starting from vanishing initial abundance during the radiation-dominated era, such that WIMP annihilation remains negligible, the present relic abundance is given by [14]

$$Y_0(x \to \infty) \simeq 0.014 g_\chi^2 g_*^{-3/2} m_\chi M_{\mathrm{Pl}} e^{-2x_0} x_0\left(a + \frac{6b}{x_0}\right). \quad (13)$$

Note that the final abundance depends exponentially on T_0, and *increases* with increasing cross section.

In in-between cases where WIMPs are not completely thermalized but WIMP annihilation can no longer be neglected, we have shown [14] that resumming the first correction term δ enables us to reproduce the full temperature dependence of the density of WIMPs:

$$Y_\chi \simeq \frac{Y_0}{1 - \delta/Y_0} \equiv Y_{1,r}. \quad (14)$$

Here $\delta < 0$ describes the annihilation of WIMPs produced according to Eq. (13):

$$\delta(x \to \infty) \simeq -2.5 \times 10^{-4} g_\chi^4 g_*^{-5/2} m_\chi^3 M_{\mathrm{Pl}}^3 e^{-4x_0} x_0\left(a + \frac{3b}{x_0}\right) \times \left(a + \frac{6b}{x_0}\right)^2. \quad (15)$$

Since δ is proportional to the third power of the cross section, the resummed expression $Y_{1,r}$ is inversely proportional to the cross section for large cross section. In Ref. [14] we have shown that this feature allows the approximation (14) to be smoothly matched to the standard result (12). Not surprisingly, as long as we only consider thermal χ production, decreasing T_0 can only reduce the final χ relic density.

With the help of these results, we can explore the dependence of the χ relic density on T_0 as well as on the annihilation cross section. Some results are shown in Fig. 1, where we take (a) $a \neq 0$, $b = 0$, and (b) $a = 0$, $b \neq 0$. We choose $Y_\chi(x_0) = 0$, $m_\chi = 100$ GeV, $g_\chi = 2$ and $g_* = 90$.

The results depicted in this figure can be understood as follows. For small T_0, i.e. large x_0, Eq. (13) is valid, leading to a very strong dependence of $\Omega_\chi h^2$ on x_0. Recall that in this case the relic density is proportional to the cross section. In this regime one can reproduce the relic density (1) with quite small annihilation cross section, $a + 6b/x_0 \lesssim 10^{-9}\ \mathrm{GeV}^{-2}$, for some narrow range of initial temperature, $x_0 \lesssim 22.5$. Note that this allows much smaller

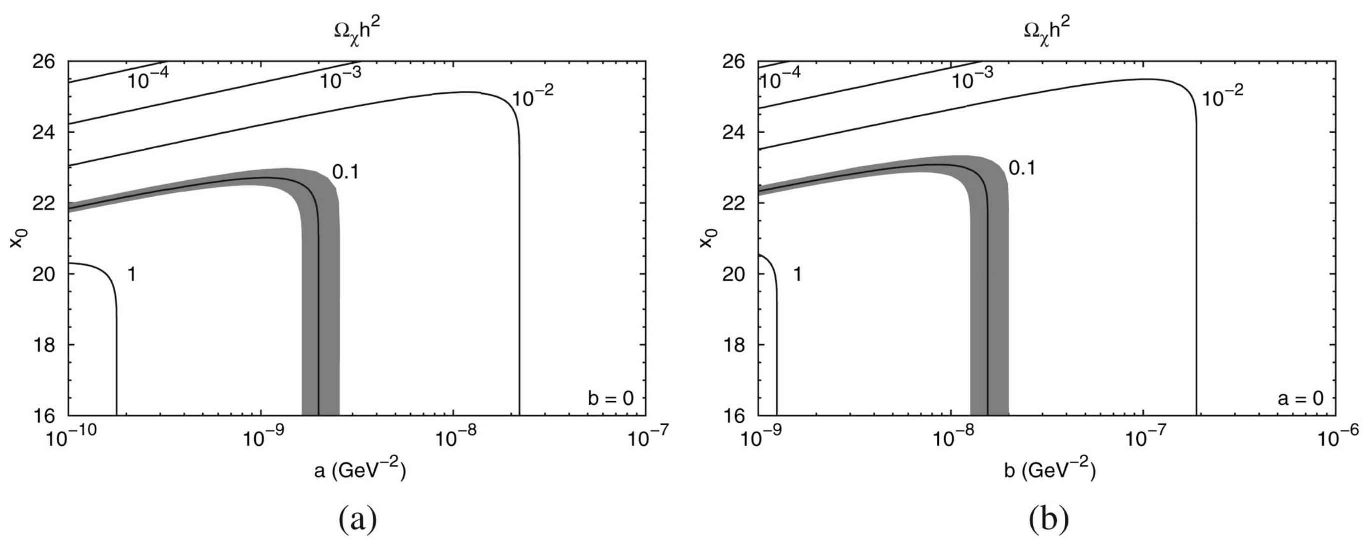

FIG. 1. Contour plots of the present relic abundance $\Omega_\chi h^2$. Here we take (a) $a \neq 0$, $b = 0$, and (b) $a = 0$, $b \neq 0$. We choose $Y_\chi(x_0) = 0$, $m_\chi = 100$ GeV, $g_\chi = 2$, $g_* = 90$. The shaded region corresponds to the WMAP bound on the cold dark matter abundance, $0.08 < \Omega_{\mathrm{CDM}} h^2 < 0.12$ (95% C.L.).

annihilation cross sections than the standard result, at the cost of a very strong dependence of the final result on the initial temperature T_0.

In this section we set out to derive a lower bound on T_0. In this regard the region of parameter space described by Eq. (13) is not optimal. Increasing the χ annihilation cross section at first allows us to obtain the correct relic density for larger x_0, i.e. smaller T_0. However, the correction δ then quickly increases in size; as noted earlier, once $|\delta| > Y_0$ a further increase of the cross section will lead to a decrease of the final relic density. The lower bound on T_0 is therefore saturated if $\Omega_\chi h^2$ as a function of the cross section reaches a maximum. From Fig. 1 we read off

$$T_0 \geq m_\chi/23, \tag{16}$$

if we require $\Omega_\chi h^2$ to fall in the range (1).

We just saw that in the regime where this bound is saturated, the final relic density is (to first order) independent of the annihilation cross section, $\partial(\Omega_\chi h^2)/\partial\langle\sigma v\rangle = 0$. If T_0 is slightly above the absolute lower bound (16), the correct relic density can therefore be obtained for a rather wide range of cross sections. For example, if $x_0 = 22.5$, the entire range $3 \times 10^{-10}\ \mathrm{GeV}^{-2} \lesssim a \lesssim 2 \times 10^{-9}\ \mathrm{GeV}^{-2}$ is allowed. Of course, the correct relic density can also be obtained in the standard scenario of (arbitrarily) high T_0, if $a + 3b/22$ falls within $\sim$20% of $2 \times 10^{-9}\ \mathrm{GeV}^{-2}$.

III. RELIC ABUNDANCE FOR MODIFIED EXPANSION RATE

In this section we discuss the calculation of the WIMP relic density n_χ in modified cosmological scenarios where the expansion parameter of the pre-BBN universe differed from the standard value $H_{\rm st}$ of Eq. (7). For the most part we will assume that WIMPs have been in full thermal equilibrium. Various cosmological models predict a nonstandard early expansion history [22–25]. Here we analyze to what extent the relic density of WIMP dark matter can be used to constrain the Hubble parameter during the epoch of WIMP decoupling. As long as we assume large T_0 we can use a modification of the standard treatment [2,4] to estimate the relic density for given annihilation cross section and expansion rate. We will show that the resulting approximate solutions again accurately reproduce the numerically evaluated relic abundance.

Let us introduce the modification parameter $A(x)$, which parametrizes the ratio of the standard value $H_{\rm st}(x)$ to the assumed $H(x)$:

$$A(x) = \frac{H_{\rm st}(x)}{H(x)}. \tag{17}$$

Note that $A > 1$ means that the expansion rate is smaller than in standard cosmology. Allowing for this modified expansion rate, the Boltzmann equation (6) is altered to

$$\frac{\mathrm{d}Y_\chi}{\mathrm{d}x} = \frac{4\pi}{\sqrt{90}} G(x) m_\chi M_{\rm Pl} \frac{\langle\sigma v\rangle A(x)}{x^2}(Y_\chi^2 - Y_{\chi,\rm eq}^2), \tag{18}$$

where

$$G(x) = \frac{g_{*s}}{\sqrt{g_*}}\left(1 - \frac{x}{3g_{*s}}\frac{\mathrm{d}g_{*s}}{\mathrm{d}x}\right). \tag{19}$$

Following Refs. [2,4], we can obtain an approximate solution of this equation by considering the differential equation for $\Delta = Y_\chi - Y_{\chi,\rm eq}$. For temperatures higher than the decoupling temperature, Y_χ tracks $Y_{\chi,\rm eq}$ very closely and the Δ^2 term can be ignored:

$$\frac{\mathrm{d}\Delta}{\mathrm{d}x} \simeq -\frac{\mathrm{d}Y_{\rm eq}}{\mathrm{d}x} - \frac{4\pi}{\sqrt{90}} m_\chi M_{\rm Pl} \frac{G(x)\langle\sigma v\rangle A(x)}{x^2}(2Y_{\chi,\rm eq}\Delta). \tag{20}$$

Here $\mathrm{d}Y_{\chi,\rm eq}/\mathrm{d}x \simeq -Y_{\chi,\rm eq}$ for $x \gg 1$. In order to keep $|\Delta|$ small, the derivative $\mathrm{d}\Delta/\mathrm{d}x$ must also be small, which implies

$$\Delta \simeq \frac{x^2}{(8\pi/\sqrt{90}) m_\chi M_{\rm Pl} G(x)\langle\sigma v\rangle A(x)}. \tag{21}$$

This solution is used down to the freeze-out temperature T_F, defined via

$$\Delta(x_F) = \xi Y_{\chi,\rm eq}(x_F), \tag{22}$$

where ξ is a constant of order of unity. This leads to the following expression:

$$x_F = \ln\left[\sqrt{\frac{45}{\pi^5}}\xi m_\chi M_{\rm Pl} g_\chi \frac{\langle\sigma v\rangle A(x)}{\sqrt{x g_*(x)}}\left(1 - \frac{x}{3g_{*s}}\frac{\mathrm{d}g_{*s}}{\mathrm{d}x}\right)\right]\Bigg|_{x=x_F}, \tag{23}$$

which can e.g., be solved iteratively. In our numerical calculations we will choose $\xi = \sqrt{2} - 1$ [2,4].

On the other hand, for low temperatures ($T < T_F$), the production term $\propto Y_{\chi,\rm eq}^2$ in Eq. (18) can be ignored. In this limit, $Y_\chi \simeq \Delta$, and the solution of Eq. (18) is given by

$$\frac{1}{\Delta(x_F)} - \frac{1}{\Delta(x \to \infty)} = -\frac{4\pi}{\sqrt{90}} m_\chi M_{\rm Pl} I(x_F), \tag{24}$$

where the annihilation integral is defined as

$$I(x_F) = \int_{x_F}^{\infty} \mathrm{d}x \frac{G(x)\langle\sigma v\rangle A(x)}{x^2}. \tag{25}$$

Assuming $\Delta(x \to \infty) \ll \Delta(x_F)$, the final relic abundance is

$$Y_{\chi,\infty} \equiv Y_\chi(x \to \infty) = \frac{1}{(4\pi/\sqrt{90}) m_\chi M_{\rm Pl} I(x_F)}. \tag{26}$$

Plugging in numerical values for the Planck mass and for today's entropy density, the present relic density can thus be written as

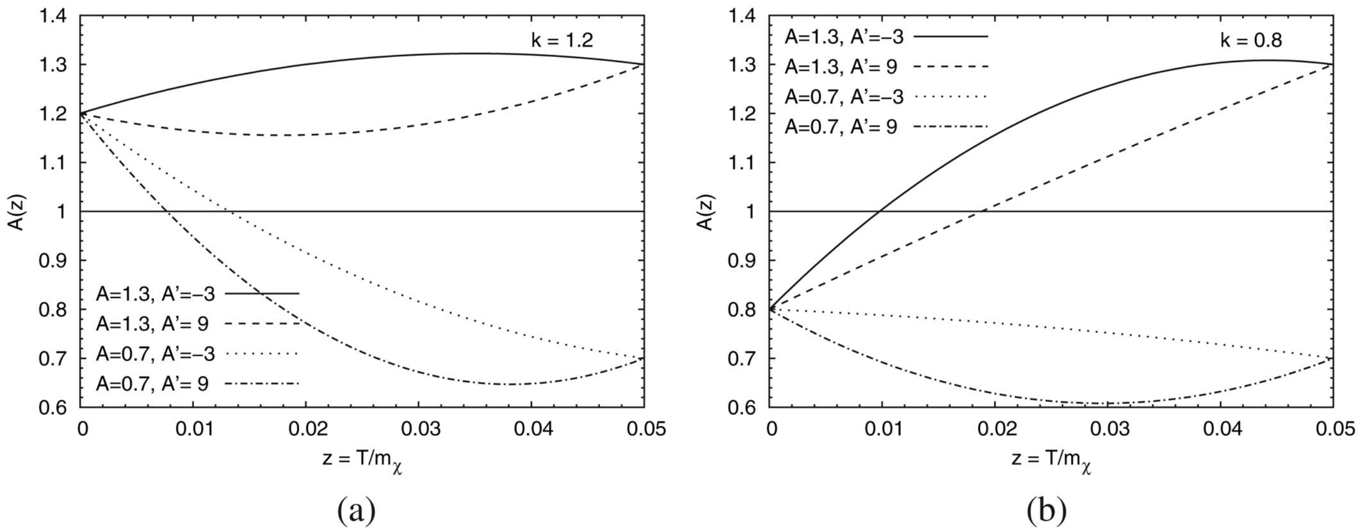

FIG. 2. Examples of possible evolutions of the modification parameter $A(z)$ as a function of z for $z_F = 0.05$. Here we take $k = 1.2$ (left frame) and $k = 0.8$ (right). In each frame we choose $A(z_F) = 1.3$, $A'(z_F) = -3$ (thick line), $A(z_F) = 1.3$, $A'(z_F) = 9$ (dashed), $A(z_F) = 0.7$, $A'(z_F) = -3$ (dotted), $A(z_F) = 0.7$, $A'(z_F) = 9$ (dot-dashed).

$$\Omega_\chi h^2 = \frac{8.5 \times 10^{-11}}{I(x_F)\ \mathrm{GeV}^2}. \tag{27}$$

The constraint (1) therefore corresponds to the allowed range for the annihilation integral

$$7.1 \times 10^{-10}\ \mathrm{GeV}^{-2} < I(x_F) < 1.1 \times 10^{-9}\ \mathrm{GeV}^{-2}. \tag{28}$$

The standard formula (12) for the final relic density is recovered if $A(x)$ is set to unity and $G(x)$ is replaced by the constant $\sqrt{g_*(x_F)}$.

The further discussion is simplified if we use the normalized temperature $z = T/m_\chi \equiv 1/x$, rather than x. Phenomenologically $A(z)$ can be any function subject to the condition that $A(z)$ approaches unity at late times in order not to contradict the successful predictions of BBN. We need to know $A(z)$ only for the interval from around the freeze-out to BBN: $z_{\mathrm{BBN}} \sim 10^{-5}$–$10^{-4} \lesssim z \lesssim z_F \sim 1/20$. This suggests a parametrization of $A(z)$ in terms of a power series in $(z - z_{F,\mathrm{st}})$:

$$A(z) = A(z_{F,\mathrm{st}}) + (z - z_{F,\mathrm{st}})A'(z_{F,\mathrm{st}}) + \frac{1}{2}(z - z_{F,\mathrm{st}})^2 A''(z_{F,\mathrm{st}}), \tag{29}$$

where $z_{F,\mathrm{st}}$ is the normalized freeze-out temperature in the standard scenario and a prime denotes a derivative with respect to z. The ansatz (29) should be of quite general validity, so long as the modification of the expansion rate is relatively modest. This suits our purpose, since we wish to find out what constraints can be derived on the expansion history if standard cosmology leads to the correct WIMP relic density.

We further introduce the variable

$$k = A(z \to 0) = A(z_{F,\mathrm{st}}) - z_{F,\mathrm{st}}A'(z_{F,\mathrm{st}}) + \frac{1}{2} z_{F,\mathrm{st}}^2 A''(z_{F,\mathrm{st}}), \tag{30}$$

which describes the modification parameter at late times. Since z_{BBN} is almost zero, we treat k as the modification parameter at the era of BBN in this paper.[3] Deviations from $k = 1$ are conveniently discussed in terms of the equivalent number of light neutrino degrees of freedom N_ν. BBN permits that the number of neutrinos differs from the standard model value $N_\nu = 3$ by $\delta N_\nu = 1.5$ or so [26]. We therefore take the uncertainty of k to be 20%. In the following we treat $A(z_{F,\mathrm{st}})$, $A'(z_{F,\mathrm{st}})$ and k as free parameters; $A''(z_{F,\mathrm{st}})$ is then a derived quantity.

Note that we allow $A(z)$ to cross unity, i.e. to switch from an expansion that is faster than in standard cosmology to a slower expansion or vice versa. This is illustrated in Fig. 2, which shows examples of possible evolutions of $A(z)$ as a function of z for $z_F = 0.05$. Here we take $k = 1.2$ (left frame) and $k = 0.8$ (right). In each case we consider scenarios with $A(z_F) = 1.3$ (slower expansion at T_F than in standard cosmology) as well as $A(z_F) = 0.7$ (faster expansion); moreover, we allow the change of A at $z = z_F$ to be either positive or negative. However, we insist that H remains positive at all times, i.e. $A(z)$ must not cross zero. This excludes scenarios with very large positive $A'(z_{F,\mathrm{st}})$, which would lead to $A < 0$ at some $z < z_F$.

[3] Presumably the Hubble expansion rate has to approach the standard rate even more closely for $T < T_{\mathrm{BBN}}$. However, since all WIMP annihilation effectively ceased well before the onset of BBN, this epoch plays no role in our analysis.

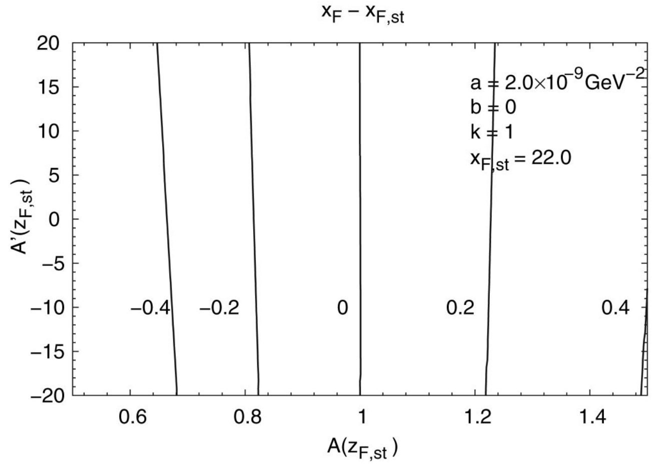

FIG. 3. Contour plot of $x_F - x_{F,\mathrm{st}}$ in the $(A(z_{F,\mathrm{st}}), A'(z_{F,\mathrm{st}}))$ plane. Here we take $a = 2.0 \times 10^{-9}\ \mathrm{GeV}^{-2}$, $b = 0$, $m_\chi = 100$ GeV, $g_\chi = 2$, $g_* = 90$ (constant) and $k = 1$. This parameter set produces $x_{F,\mathrm{st}} = 22.0$ and $\Omega_\chi h^2 = 0.099$ for the standard approximation.

Similarly, demanding that our ansatz (29) remains valid for some range of temperatures above T_F excludes scenarios with very large negative $A'(z_{F,\mathrm{st}})$. We will come back to this point shortly.

Equation (23) shows that $z_F \neq z_{F,\mathrm{st}}$ $(x_F \neq x_{F,\mathrm{st}})$ if $A(z_F) \neq 1$. This is illustrated by Fig. 3, which shows the difference between x_F and $x_{F,\mathrm{st}}$ in the $(A(z_{F,\mathrm{st}}), A'(z_{F,\mathrm{st}}))$ plane. Here we take parameters such that $\Omega_\chi h^2 = 0.099$ in the standard cosmology, which is recovered for $A(z_{F,\mathrm{st}}) = 1$, $A'(z_{F,\mathrm{st}}) = 0$. Because of the logarithmic dependence on A, x_F (or z_F) differs by at most a few percent from its standard value if $A(z_{F,\mathrm{st}})$ is $\mathcal{O}(1)$. Since T_F only depends on the expansion rate at T_F, it is essentially insensitive to the derivative $A'(z_{F,\mathrm{st}})$.

In our treatment the modification of the expansion parameter affects the WIMP relic density mostly via the annihilation integral (25). In terms of the normalized temperature z, the latter can be rewritten as

$$I(z_F) = \int_0^{z_F} \mathrm{d}z G(z) \langle \sigma v \rangle A(z). \tag{31}$$

One advantage of the expansion (29) is that this integral can be evaluated analytically:

$$I(z_F) \simeq G(z_F)\Big[k(az_F + 3bz_F^2) + (A'(z_{F,\mathrm{st}}) - z_{F,\mathrm{st}} A''(z_{F,\mathrm{st}})) \times \Big(\frac{a}{2} z_F^2 + 2bz_F^3\Big) + \frac{A''(z_{F,\mathrm{st}})}{2}\Big(\frac{a}{3} z_F^3 + \frac{3b}{2} z_F^4\Big)\Big]. \tag{32}$$

Here we have assumed that $G(z)$ varies only slowly.

Before proceeding, we first have to convince ourselves that the analytic treatment developed in this section still works for $A \neq 1$. This is demonstrated by Fig. 4, which shows the ratio of the analytic solution obtained from Eqs. (27) and (32) to the exact one, obtained by numerically integrating the Boltzmann equation (18), assuming constant g_*. We see that our analytical treatment is accurate to better than 1%, and can thus safely be employed in the subsequent analysis.

We are now ready to analyze the impact of the modified expansion rate on the WIMP relic density. In Fig. 5, we show contour plots of $\Omega_\chi h^2$ in the $(A(z_{F,\mathrm{st}}), A'(z_{F,\mathrm{st}}))$ plane. Recall that large (small) values of A correspond to a small (large) expansion rate. Since a smaller expansion rate allows the WIMPs more time to annihilate, $A > 1$ leads

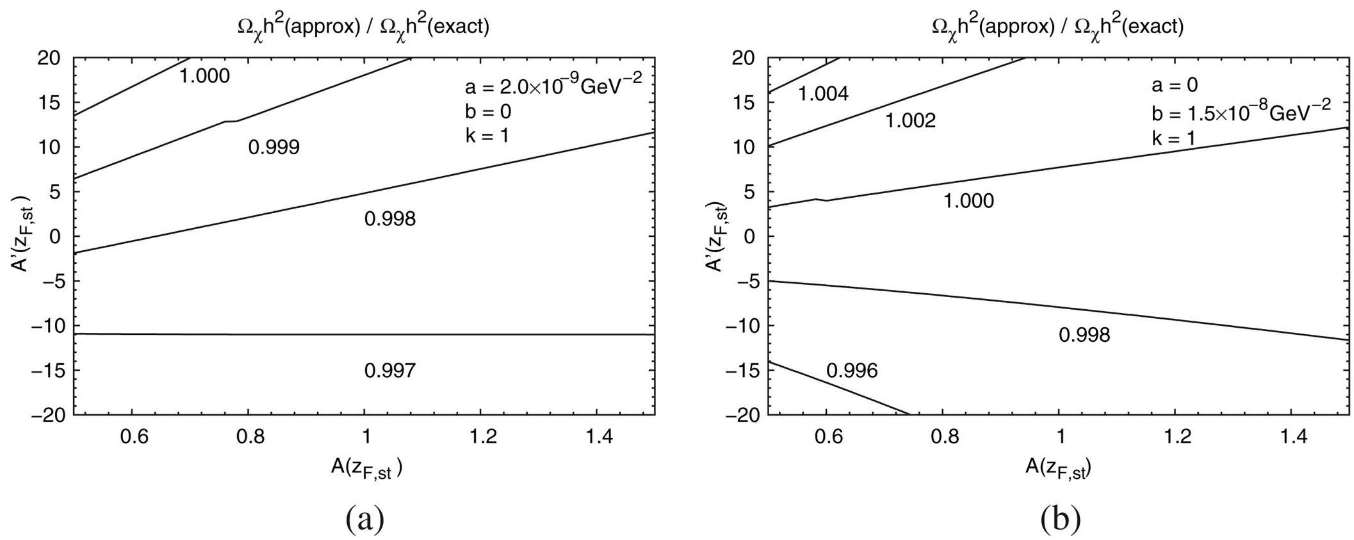

FIG. 4. Ratio of the analytic result of the relic density to the exact value in the $(A(z_{F,st}), A'(z_{F,st}))$ plane for $a = 2.0 \times 10^{-9}\ \mathrm{GeV}^{-2}$, $b = 0$ (left frame) and for $a = 0$, $b = 1.5 \times 10^{-8}\ \mathrm{GeV}^{-2}$ (right). The other parameters are as in Fig. 3.

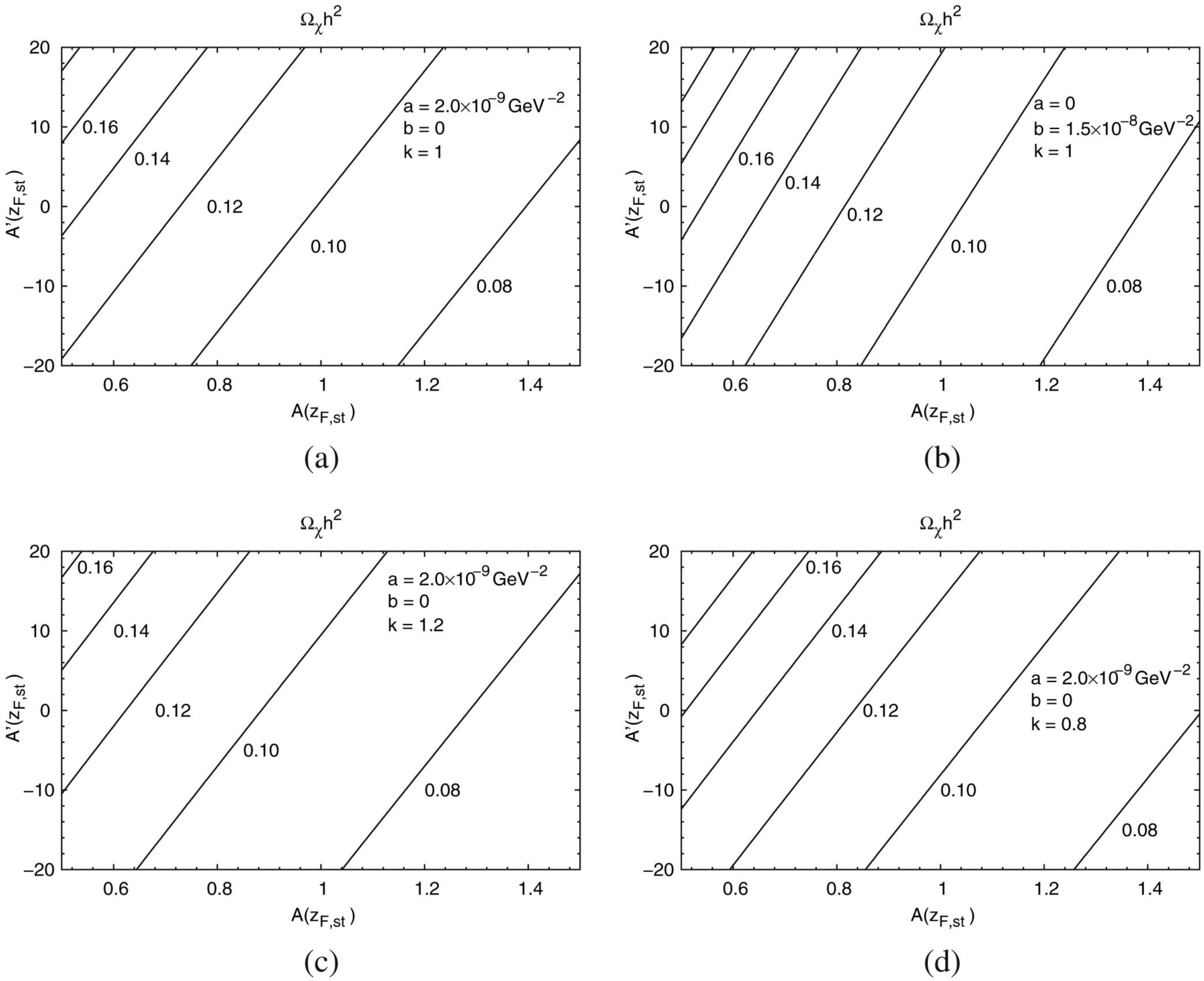

FIG. 5. Contour plots of the relic abundance in the $(A(z_{F,st}), A'(z_{F,st}))$ plane. Here we choose (a) $a = 2.0 \times 10^{-9}$ GeV^{-2}, $b = 0$, $k = 1$; (b) $a = 0$, $b = 1.5 \times 10^{-8}$ GeV^{-2}, $k = 1$; (c) $a = 2.0 \times 10^{-9}$ GeV^{-2}, $b = 0$, $k = 1.2$; (d) $a = 2.0 \times 10^{-9}$ GeV^{-2}, $b = 0$, $k = 0.8$. The other parameters are as in Fig. 3.

to a reduced WIMP relic density, whereas $A < 1$ means larger relic density, if the cross section is kept fixed.

However, unlike the freeze-out temperature, the annihilation integral is sensitive to $A(z)$ for all $z \le z_F$. Note that $A'(z_{F,\mathrm{st}}) > 0$ implies $A(z) < A(z_{F,\mathrm{st}})$ for $z < z_{F,\mathrm{st}} \simeq z_F$. A positive first derivative, $A'(z_{F,\mathrm{st}}) > 0$, can therefore to some extent compensate for $A(z_{F,\mathrm{st}}) > 1$; analogously, a negative first derivative can compensate for $A(z_{F,\mathrm{st}}) < 1$. This explains the slopes of the curves in Fig. 5. Recall also that $A'(z_{F,\mathrm{st}}) = 0$ does not imply a constant modification factor $A(z)$; rather, the term $\propto A''(z_{F,\mathrm{st}})$ in Eq. (29) makes sure that A approaches k as $z \to 0$. This explains why a change of A by some given percentage leads to a *smaller* relative change of $\Omega_\chi h^2$, as can be seen in the figure. This also illustrates the importance of ensuring appropriate (near-standard) expansion rate in the BBN era. Finally, since the expansion rate at late times is given by H_{st}/k, bigger (smaller) values of k imply that the χ relic density is reduced (enhanced).

Figure 5 shows that we need additional physical constraints if we want to derive bounds on $A(z_{F,\mathrm{st}})$ and/or $A'(z_{F,\mathrm{st}})$. Once the annihilation cross section is known, the requirement (1) will single out a region in the space spanned by our three new parameters (including k) which describe the nonstandard evolution of the Universe, but this region is not bounded. Such additional constraints can be derived from the requirement that the Hubble parameter should remain positive throughout the epoch we are considering. As noted earlier, requiring $H > 0$ for all $T < T_{F,\mathrm{st}}$ leads to an upper bound on $A'(z_{F,\mathrm{st}})$; explicitly,

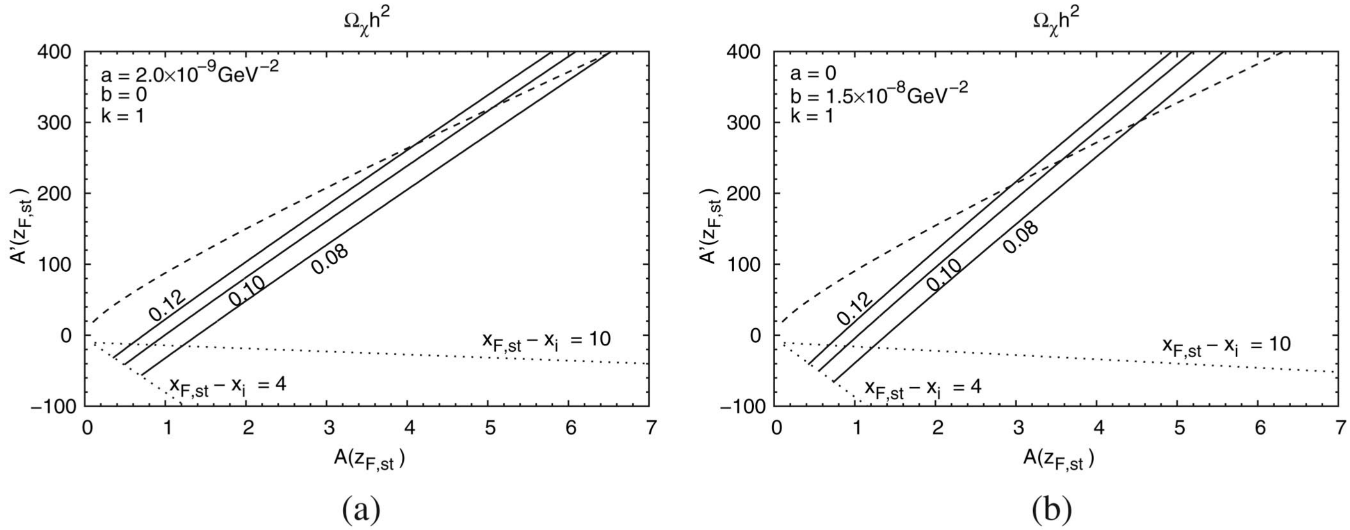

FIG. 6. Contour plots of the relic abundance $\Omega_\chi h^2$ in the $(A(z_{F,\text{st}}), A'(z_{F,\text{st}}))$ plane. The dashed line corresponds to the upper bound on $A'(z_{F,\text{st}})$. The dotted lines correspond to the lower bounds calculated for $x_{F,\text{st}} - x_i = 4$, 10. We take $a = 2.0 \times 10^{-9}$ GeV^{-2}, $b = 0$ (left frame) and $a = 0$, $b = 1.5 \times 10^{-8}$ GeV^{-2} (right frame). The other parameters are as in Fig. 3.

$$A'(z_{F,\text{st}}) < \frac{2(A(z_{F,\text{st}}) + \sqrt{kA(z_{F,\text{st}})})}{z_{F,\text{st}}}. \tag{33}$$

On the other hand, a lower bound on $A'(z_{F,\text{st}})$ is obtained from the condition that the modified Hubble parameter is positive between the highest temperature T_i where the ansatz (29) holds and $T_{F,\text{st}}$:

$$A'(z_{F,\text{st}}) > -\left[\frac{1}{z_i - z_{F,\text{st}}}\left(2 - \frac{z_i}{z_{F,\text{st}}}\right)A(z_{F,\text{st}}) + k\left(\frac{1}{z_{F,\text{st}}} - \frac{1}{z_i}\right)\right], \tag{34}$$

for $(1 - z_{F,\text{st}}/z_i)^2 k < A(z_{F,\text{st}})$, and

$$A'(z_{F,\text{st}}) > \frac{2(A(z_{F,\text{st}}) - \sqrt{kA(z_{F,\text{st}})})}{z_{F,\text{st}}}, \tag{35}$$

for $A(z_{F,\text{st}}) < (1 - z_{F,\text{st}}/z_i)^2 k$, where $z_i = T_i/m_\chi$.

Evidently the lower bound on $A'(z_{F,\text{st}})$ depends on z_i, i.e. on the maximal temperature where we assume our ansatz (29) to be valid. In Ref. [14] we have shown that in standard cosmology ($A \equiv 1$) essentially full thermalization is already achieved for $x_i \lesssim x_F - 0.5$, even if $n_\chi(x_i) = 0$. However, it seems reasonable to demand that H should remain positive at least up to $x_i = x_F -$ (a few). In Fig. 6 we therefore show the physical constraints on the modification parameter $A(z)$ for $x_{F,\text{st}} - x_i = 4$, 10 and $k = 1$. The dashed and dotted lines correspond to the upper and lower bounds on $A'(z_{F,\text{st}})$, described by Eqs. (33)–(35), respectively. We see that when $x_{F,\text{st}} - x_i = 4$ the allowed region is $0.4 \lesssim A(z_{F,\text{st}}) \lesssim 6.5$ with $-60 \lesssim A'(z_{F,\text{st}}) \lesssim 400$ for $b = 0$ (left frame), and $0.4 \lesssim A(z_{F,\text{st}}) \lesssim 4.5$ with $-60 \lesssim A'(z_{F,\text{st}}) \lesssim 300$ for $a = 0$ (right frame). When $x_{F,\text{st}} - x_i = 10$, the lower bounds are altered to $0.6 \lesssim A(z_{F,\text{st}})$, $-10 \lesssim A'(z_{F,\text{st}})$ for $b = 0$ (left frame), and $0.6 \lesssim A(z_{F,\text{st}})$, $-20 \lesssim A'(z_{F,\text{st}})$ for $a = 0$ (right frame). Note that the lower bounds on $A(z_{F,\text{st}})$, which depend only weakly on x_i so long as it is not very close to x_F, are almost the same in both cases, which also lead to very similar relic densities in standard cosmology. However, the two upper bounds differ significantly. The reason is that large values of $A(z_{F,\text{st}})$, i.e. a strongly suppressed Hubble expansion, require some degree of fine-tuning: One also has to take large positive $A'(z_{F,\text{st}})$, so that A becomes smaller than 1 for some range of z values below z_F, leading to an annihilation integral of similar size as in standard cosmology. Since the b terms show different z_F dependence in the annihilation integral (32), the required tuning between $A(z_{F,\text{st}})$ and $A'(z_{F,\text{st}})$ is somewhat different than for the a terms, leading to a steeper slope of the allowed region. This allowed region therefore saturates the upper bound (33) on the slope for somewhat smaller $A(z_{F,\text{st}})$.

The effect of this tuning can be seen by analyzing the special case where $A''(z_{F,\text{st}}) = 0$. The modification parameter then reads

$$A(z) = \frac{A(z_{F,\text{st}}) - k}{z_{F,\text{st}}} z + k. \tag{36}$$

Note that A is now a monotonic function of z, making large cancellations in the annihilation integral impossible. Imposing that $A(z)$ remains positive for $z_{F,\text{st}} \leq z \leq z_i$

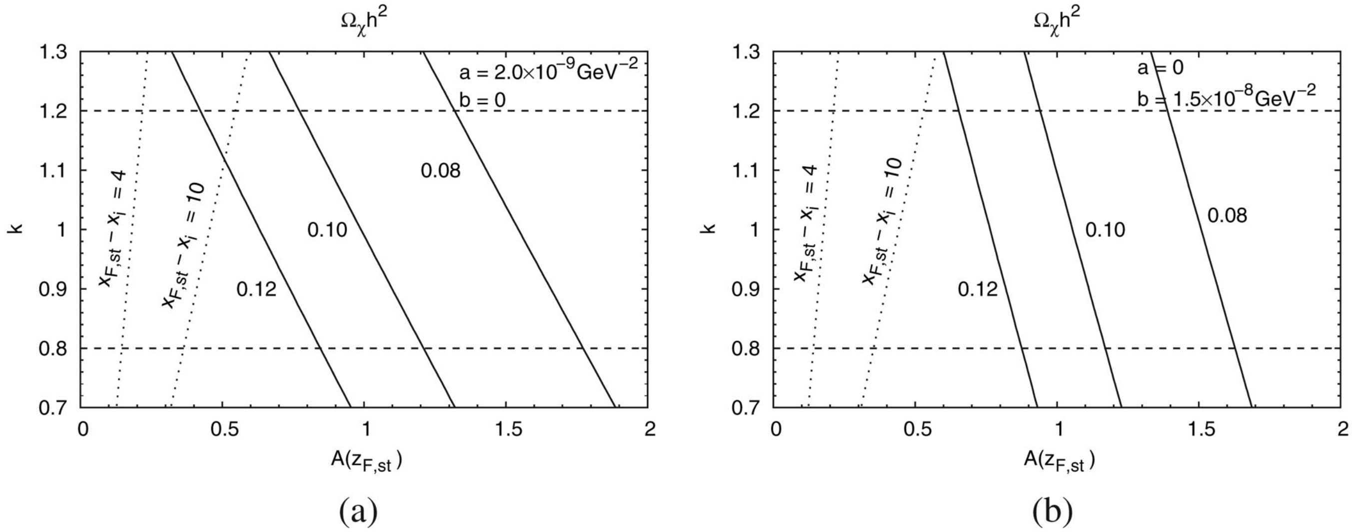

FIG. 7. Contour plots of the relic abundance $\Omega_\chi h^2$ in the $(A(z_{F,\mathrm{st}}), k)$ plane for $A''(z_{F,\mathrm{st}}) = 0$. The dotted lines correspond to the lower bounds of $A(z_{F,\mathrm{st}})$, calculated for $x_{F,\mathrm{st}} - x_i = 4$, 10. We take $a = 2.0 \times 10^{-9}$ GeV^{-2}, $b = 0$ (left frame) and $a = 0$, $b = 1.5 \times 10^{-8}$ GeV^{-2} (right frame). The other parameters are as in Fig. 3.

leads to the lower limit

$$A(z_{F,\mathrm{st}}) > \left(1 - \frac{z_{F,\mathrm{st}}}{z_i}\right)k. \tag{37}$$

There is no upper bound, since $A(z)$ is now automatically positive for all $z \in [0, z_{F,\mathrm{st}}]$ if $A(z_{F,\mathrm{st}})$ and $A(0) \equiv k$ are both positive. Figure 7 shows constraints on the relic abundance in the $(A(z_{F,\mathrm{st}}), k)$ plane for $A''(z_{F,\mathrm{st}}) = 0$. The dotted lines correspond to the lower bounds (37) on $A(z_{F,\mathrm{st}})$ for $x_{F,\mathrm{st}} - x_i = 4$, 10. As noted earlier, k is constrained by the BBN bound. This leads to the bounds $0.5 \lesssim A(z_{F,\mathrm{st}}) \lesssim 1.8$ for $b = 0$ (left frame), and $0.65 \lesssim A(z_{F,\mathrm{st}}) \lesssim 1.6$ for $a = 0$ (right frame), when $x_{F,\mathrm{st}} - x_i = 10$. Evidently the constraints now only depend weakly on whether the a or b term dominates in the annihilation cross section. As the initial temperature is lowered, the impact of the constraint (37) disappears.

So far we have assumed in this section that the reheat temperature was high enough for WIMPs to have attained full thermal equilibrium. If this was not the case, the initial temperature as well as the suppression parameter affects the final relic abundance. Here we show that the lower bound on the reheat temperature derived in the previous section survives even in scenarios with altered expansion history as long as WIMPs were only produced thermally.

This can be understood from the observation that the Boltzmann equation with modified expansion rate is obtained by replacing $\langle\sigma v\rangle$ in the radiation-dominated case by $\langle\sigma v\rangle A$. Increasing (decreasing) A therefore has the same effect as an increase (decrease) of the annihilation cross section. Since the lower bound on T_0 was independent of σ (more exactly: we quoted the absolute minimum, for the optimal choice of σ), we expect it to survive even if $A(z) \neq 1$ is introduced.

This is borne out by Fig. 8, which shows the relic abundance $\Omega_\chi h^2$ in the $(A(z_{F,st}), x_0)$ plane for the simplified case $A''(z_{F,\mathrm{st}}) = 0$; similar results can be obtained for the more general ansatz (29). The shaded region corresponds to the bound (1) on the cold dark matter abundance. As expected, this figure looks similar to Fig. 1 if the

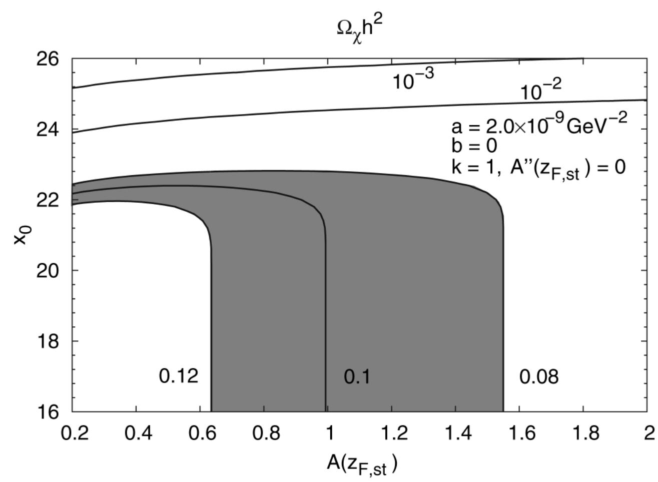

FIG. 8. Contour plot of the relic abundance $\Omega_\chi h^2$ in the $(A(z_{F,st}), x_0)$ plane. Here we choose $a = 2.0 \times 10^{-9}$ GeV^{-2}, $b = 0$, $k = 1$, $A''(z_{F,\mathrm{st}}) = 0$. The other parameters are as in Fig. 3. The shaded region corresponds to the WMAP bound on the cold dark matter abundance, $0.08 < \Omega_{\mathrm{CDM}} h^2 < 0.12$ (95% C.L.).

annihilation cross section in Fig. 1 is replaced by $A(z_{F,\mathrm{st}})$. The maximal value of x_0 consistent with the WMAP data remains around 23 even in these scenarios with modified expansion rate. Figure 8 also shows that $A(z_{F,\mathrm{st}}) \ll 1$ is allowed for some narrow range of initial temperature $T_0 < T_F$. This is analogous to the low cross section branch in Fig. 1.

IV. SUMMARY AND CONCLUSIONS

In this paper we have investigated the relic abundance of WIMPs χ, which are nonrelativistic long-lived or stable particles, in nonstandard cosmological scenarios. One motivation for studying such scenarios is that they allow us to reproduce the observed dark matter density for a large range of WIMP annihilation cross sections. Our motivation was the opposite: we wanted to quantify the constraints that can be obtained on parameters describing the early universe, under the assumption that thermally produced WIMPs form all dark matter. Wherever necessary, we fixed particle physics quantities such that standard cosmology yields the correct relic density.

Specifically, we first considered scenarios with low post-inflationary reheat temperature, while keeping all other features of standard cosmology (known particle content and Hubble expansion parameter during WIMP decoupling; no late entropy production; no nonthermal WIMP production channels). If the temperature was so low that WIMPs could not achieve full thermal equilibrium, the dependence of the abundance on the mass and annihilation cross section of the WIMPs is completely different from that in the standard thermal WIMP scenario. In particular, if the maximal temperature T_0 is much less than the decoupling temperature T_F, n_χ remains exponentially suppressed. By applying the observed cosmological amount of cold dark matter to the predicted WIMP abundance, we therefore found the lower bound of the initial temperature $T_0 \gtrsim m_\chi/23$. One might naively think that this bound could be evaded by choosing a sufficiently large WIMP production (or annihilation) cross section. However, increasing this cross section also reduces T_F. For sufficiently large cross section one therefore has $T_F \leq T_0$ again; in this regime the relic density drops with increasing cross section. Our lower bound is the minimal T_0 required for *any* cross section; once the latter is known, the bound on T_0 might be slightly sharpened. As a by-product, we also noted that the final relic density depends only weakly on the annihilation cross section if T_0 is slightly above this lower bound.

We also investigated the effect of a nonstandard expansion rate of the Universe on the WIMP relic abundance. In general the abundance of thermal relics depends on the ratio of the annihilation cross section to the expansion rate; the latter is determined unambiguously in standard cosmology. We found that even for nonstandard Hubble parameter the relic abundance can be calculated accurately in terms of an annihilation integral, very similar to the case of standard cosmology. We assumed that the WIMP annihilation cross section is such that the standard scenario yields the observed relic density, and parametrized the modification of the Hubble parameter as a quadratic function of the temperature. In this analysis it is crucial to make sure that at low temperatures the Hubble parameter approaches its standard value to within ~20%, as required for the success of big bang nucleosynthesis (BBN).

Keeping the annihilation cross section fixed and allowing a 20% variation in the relic density, roughly corresponding to the present "2σ" band, we found that the expansion of the Universe at $T = T_F$ might have been more than 2 times faster, or more than 6 times slower, than in standard cosmology. These large variations of $H(T_F)$ can only be realized by fine-tuning of the parameters describing $H(T < T_F)$. However, even if we forbid such fine-tuning by choosing a linear parametrization for the modification of the expansion rate, a 20% variation of $\Omega_\chi h^2$ allows a difference between $H(T_F)$ and its standard expectation of more than 50%. This relatively weak sensitivity of the predicted $\Omega_\chi h^2$ on $H(T_F)$ is due to the fact that the relic density depends on *all* $H(T < T_F)$; as stressed above, we have to require that $H(T \ll T_F)$ approaches its standard value to within ~20%. The fact that determining $\Omega_\chi h^2$ will yield relatively poor bounds on $H(T_F)$ remains true even if the annihilation cross section is such that a nonstandard behavior of $H(T)$ is required for obtaining the correct χ relic density. Finally, we showed that the absolute lower bound on the temperature required for thermal χ production is unaltered by allowing $H(T)$ to differ from its standard value.

Of course, in order to draw the conclusions derived in this article, we need to convince ourselves that WIMPs do indeed form (nearly) all dark matter. This requires not only the detection of WIMPs, e.g. in direct search experiments; we also need to show that their density is in accord with the local dark matter density derived from astronomical observations. To that end, the cross sections appearing in the calculation of the detection rate need to be known independently. This can only be done in the framework of a definite theory, using data from collider experiments. For example, in order to determine the cross section for the direct detection of supersymmetric WIMPs, one needs to know the parameters of the supersymmetric neutralino, Higgs and squark sectors [3]. We also saw that inferences about $H(T_F)$ can only be made if the WIMP annihilation cross section is known. This again requires highly nontrivial analyses of collider data [27], as well as a consistent overall theory. We thus see that the interplay of accurate cosmological data with results obtained from dark matter detections and collider experiments can give us insight into the pre-BBN universe, which to date remains unexplored territory.

ACKNOWLEDGMENTS

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