Searching for inflation in simple string theory models: An astrophysical perspective

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Attempts to connect string theory with astrophysical observation are hampered by a jargon barrier, where an intimidating profusion of orientifolds, Kähler potentials, etc. dissuades cosmologists from attempting to work out the astrophysical observables of specific string theory solutions from the recent literature. We attempt to help bridge this gap by giving a pedagogical exposition with detailed examples, aimed at astrophysicists and high energy theorists alike, of how to compute predictions for familiar cosmological parameters when starting with a 10-dimensional string theory action. This is done by investigating inflation in string theory, since inflation is the dominant paradigm for how early universe physics determines cosmological parameters. We analyze three explicit string models from the recent literature, each containing an infinite number of vacuum solutions. Our numerical investigation of some natural candidate inflatons, the so-called ''moduli fields,'' fails to find inflation. We also find in the simplest models that, after suitable field redefinitions, vast numbers of these vacua differ only in an overall constant multiplying the effective inflaton potential, a difference which affects neither the potential's shape nor its ability to support slow-roll inflation. This illustrates that even having an infinite number of vacua does not guarantee having inflating ones. This may be an artifact of the simplicity of the models that we study. Instead, more complicated string theory models appear to be required, suggesting that identifying the inflating subset of the string landscape will be challenging.

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I. INTRODUCTION

String theory is currently the most popular candidate for a consistent theory of quantum gravity, but the goal of confronting it with observation remains elusive. It is sometimes said that testing string theory requires prohibitively high energy accelerators, in order to probe the Planck scale predictions of the theory. There are nontrivial tests of string theory at low energies, however, such as the requirement to have a solution with the standard model of particle physics. Furthermore, it is plausible that we can test string theory by turning to cosmology. The earliest moments of our universe involved extreme energies and the fingerprints of its birth are revealed today by precision measurements of the cosmic microwave background [1] and the large-scale structure of the universe [2,3]. A highly nontrivial test of string theory then is whether it can reproduce our cosmology. With inflation emerging as the paradigm of early universe phenomena, string theory or any competing theory of quantum gravity must be able to realize this. Moreover, merely producing many e-foldings of inflation is not good enough: the details of inflation must give correct predictions for as many as eight cosmological parameters which have been measured or constrained [4].

Although there have been substantial efforts in the string theory literature aimed at identifying and counting long-lived potential energy minima (so-called vacua) [5–10], the key cosmological observables depend also on the history of how our space-time region evolved to this minimum by slow-roll inflation and/or tunneling. This article is aimed at discussing one of the simplest realizations of slow-roll inflation from string theory, where the inflaton fields are the so-called moduli which, loosely speaking, correspond to the size and shape of curled up extra dimensions (see Table I) [11,12]. An alternative scenario involves dynamical branes [13–19], with the most explicit models to date appearing in [20,21]. Some other possibilities include [22,23]. For recent reviews of inflation in string theory, see [24–27].

This paper is aimed at anyone who is intrigued by the possibility of connecting string theory and cosmology. We hope that it is accessible to nonstring theorists, and have therefore tried hard to minimize string theory specific terminology and notation, referring the interested reader to more technical references for further detail. Table I provides a ''Stringlish to English'' reference dictionary for the most central string theory terms. Bridging the gap between string theory and observational astrophysics is important for both fields: not only does it offer potential tests of string theory as mentioned above, but it also offers an opportunity for cosmologists to move beyond the tradition of putting in inflaton potentials by hand.

A. Can string theory describe inflation?

Answering the question of whether inflation can be embedded in string theory is very difficult. First of all, there is no known complete formulation of string theory, so the theory is not fully understood. In particular there does

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TABLE I. Dictionary of some basic string theory terminology.

Symbol	Name	Approximate meaning
α'	Regge parameter	Inverse string tension
l_s	String length	$= 2\pi\sqrt{\alpha'}$ (in our convention)
κ_{10}	10d gravitational strength	$= \sqrt{8\pi G_{10}} = l_s^4/\sqrt{4\pi}$, gravitational strength in 10 dims
$\bar{m}_{\mathrm{P}}$	(Reduced) Planck mass	$= 1/\sqrt{8\pi G}$, mass scale of quantum gravity in 4 dims
ϕ	Dilaton	Scalar field that rescales the strength of gravity
a_i	Axions	Pseudoscalars that appear in the 4d theory
b_i	Geometric moduli	Scalar fields describing ϕ and the size and shape of the compact space
	—Dilaton modulus[a]	$\sim e^{-\phi}$ (explicit form is model dependent)
	—Kähler moduli	Scalar fields that specify the *size* of the compact space
	—Complex structure moduli	Scalar fields that specify the *shape* of the compact space
ψ_i	Complex moduli	$= a_i + ib_i$
ψ	Complex inflaton vector	$= (\psi_1, \ldots, \psi_n)$, the complex moduli vector that can evolve during inflation
ϕ	Real inflaton vector	$= (a_1, b_1, \ldots, a_n, b_n)$, the real moduli vector that can evolve during inflation
g_s	String coupling	$= e^{\phi}$, the string loop expansion parameter
F_p	p-form field strength	Generalized electromagnetic field strength carrying p indices
f_p	Flux	$\propto \int F_p$ (normally integer valued), equivalent to a generalized electric or magnetic charge, but can arise purely due to nontrivial topology
g_{10}/R_{10}	10d string metric/Ricci scalar	Metric/Ricci scalar in the fundamental 10d action in string frame
g_4/R_4	4d string metric/Ricci scalar	Metric/Ricci scalar in the effective 4d action in string frame
g_E/R_E	4d Einstein metric/Ricci scalar	Metric/Ricci scalar in the effective 4d action after a conformal transformation to Einstein frame
g_6	Metric on compact space	2nd block of $g_{10} = \mathrm{diag}(g_4, g_6)$, describing the geometry of compact space
Vol	6d volume of compact space	$= \int_{\mathrm{cs}} d^6x\sqrt{g_6}$ ($cs \equiv$ compact space)
T^6	6d torus	A 6d manifold that is Riemann flat, defined by periodic identifications
K	Kähler potential	Scalar function whose Hessian matrix is the metric on moduli space
W	Superpotential	Scalar function that describes the interactions between moduli set up by fluxes, etc. in a supersymmetric theory in 4 dims
V	Supergravity potential energy	Potential energy function governing the fields a_i, b_i in 4 dims as set up by the supergravity formula in Eq. (11)
$\bar{V}$	Potential energy	$= \bar{m}_{\mathrm{P}}^4 V/4\pi$, potential energy function in 4 dims in conventional units
ϵ	First slow-roll parameter	See Eq. (18), quantifies the magnitude of the 1st derivatives of V
η	Second slow-roll parameter	See Eq. (19), quantifies the minimum of the 2nd derivatives of V
—	D(irichlet)p brane	A $(p+1)$-d object that contributes positive energy and can source F_{p+2}
—	O(rientifold)p plane	A $(p+1)$-d plane that can contribute negative energy; it arises at fixed points in so-called orientifold models

[a]When partnered with its axion, the dilaton modulus is known as the ‘‘axio-dilaton.’’

not seem to be a dynamical mechanism which selects the way in which the theory, which lives in 10 dimensions, should be compactified to 4 dimensions. Instead, there is apparently a ‘‘landscape’’ of possible 4-dimensional effective physics theories. Each so-called vacuum in the landscape corresponds to a stable or very long-lived configuration, containing, amongst other things, gravity, scalar fields (the above-mentioned moduli), and various potential energies. We would like to know if some of these vacua reproduce our observed large and rather uniform patch of $3+1$-dimensional space-time. We return below to the process by which this 4-dimensional picture emerges from the 10-dimensional picture. What is exciting is that these ingredients in 4 dimensions are precisely those used in inflationary model building.

There are several difficulties that must be overcome to build reasonable 4-dimensional models. First, it is difficult to stabilize the moduli. One reason this is problematic is that one of the moduli corresponds to the size of the compact space. If it were not stabilized then the field may roll to very large values and the ‘‘compact space’’ would decompactify. Furthermore, stabilizing all moduli is important to reproduce our universe which exhibits (approximate) Poincaré invariance, and a notable absence of long-range fifth forces. Authors have discussed various ingredients that may be included for such stabilization, e.g., nonperturbative effects that go by names such as gluino condensation and instantons. A second difficulty arises due to supersymmetry; most well-understood vacua have negative cosmological constants, i.e., correspond to anti-de Sitter spaces (we will discuss this issue in some detail later on). One resolution of these problems was provided by KKLT [28], who included nonperturbative phenomena for stabilization and broke supersymmetry to achieve a 4-dimensional solution with positive cosmological constant. (Earlier constructions of de Sitter vacua in noncritical string theory appeared in [29]).

Within this framework (where nonperturbative corrections play an important role, and a supersymmetry breaking sector is incorporated to generate positive vacuum energy) various plausible models of inflation using moduli fields have been suggested in the literature: "N-flation" [30,31], "Kähler moduli inflation" [32–34], "Inflating in a Better Race track" [35], and using brane moduli "KKLMMT" and related scenarios [19–21,36]. These models, however, share a common property: they are not entirely explicit constructions, though steady progress in that direction has been made. This leads us to the obvious and important question: *Can we realize inflation explicitly and reliably in string theory?*

B. Explicit string theory inflation

One of the difficulties with making fully explicit models of inflation has been that most of the methods of moduli stabilization involve an interplay of classical effects in the potential (which are easily computable), and quantum effects whose existence is well established, but for which precise computations are often difficult.

However, recently, models which stabilize all moduli using classical effects alone have been constructed. These manage to stabilize all moduli in a regime where all approximations are parametrically under control [37–41]. These examples are all explicit stable compactifications. They primarily achieve this stability by using potential energy contributions from generalized electric and magnetic fields (so-called fluxes; see Table I) whose combined energy are minimized when the moduli fields take some particular values. To borrow the language of quantum field theory, the potential functions in these models are generated at "tree level," and quantum corrections are shown to be small. This stabilization of all moduli at tree level is what distinguishes these models from their earlier counterparts.[1]

In this paper, we take three such recently found models and analyze each from the point of view of inflation. Specifically, we consider the models of DeWolfe, Giryavets, Kachru, and Taylor (DGKT) [39], Villadoro and Zwirner (VZ) [40], and Ihl and Wrase (IW) [41]. All of these arise in the string theory known as type IIA. Each of these models possesses an infinite number of vacua, distinguished by fluxes.

We wish to examine whether the tree-level potential for moduli fields in these models can support inflation. A well-known challenge for generating inflation within string theory is that generic potentials will not be sufficiently flat, a point we will expand on below. However, one might hope that when vast or infinite numbers of vacua are available, some of them would by chance have sufficiently flat directions to support inflationary slow roll even if generic ones do not.[2] One of our key findings below is that in the case of the (simplest moduli in the simplest examples of) IIA flux vacua that we study here, the distributions of the quantities relevant for inflation are narrow enough that the large number of vacua does not help. Instead, the candidate inflaton potentials have the same shape in many (sometimes all) of the vacua, differing only in overall normalization. So, somewhat surprisingly, our search below does not turn up a single vacuum supporting inflation.

We hasten to emphasize that since these models are in many ways the simplest possible models in their class (involving the simplest compactification geometry, the six-torus, in a crucial way), and since we focus on a simple subset of the moduli (the "untwisted moduli") even in these models, our results should only be viewed as a first pass through this class of models. It is possible (but by no means certain) that more generic vacua in this class (based on compactification manifolds which have more complicated geometry, or based on studies of other moduli) would yield different results. More generally, flux potentials in other classes of vacua may well have broader distributions of the relevant physical quantities for inflation, allowing one to tune fluxes to achieve inflation.[3]

The rest of this paper is organized as follows. Section II is a basic review of string compactification aimed at the nonspecialist. Here we review the process of moving from the 10-dimensional theory to the 4-dimensional theory in fairly simple terms, showing how the 4-dimensional picture has the ingredients of inflation (gravity as well as kinetic and potential energies for scalar fields). We then show how the familiar slow-roll conditions for inflation become slightly generalized due to the nonstandard kinetic

[1]It should be noted that the parametric control (which arises at very large values of fluxes) comes with various features which are undesirable for phenomenology: the extra dimensions become large at large flux values, the moduli masses become small, and the coupling constants become extremely weak. So for real world model building, one would place a cutoff on the flux values, and lose parametric control. However, as simple and explicit examples of stable compactifications, these examples provide a useful setting to address theoretical questions, like our question about explicit computable models of moduli inflation.

[2]This of course depends on the extent to which the inflationary slow-roll parameters ϵ and η vary as the fluxes are changed: if they densely sample a wide range including $\epsilon < 1$, $|\eta| < 1$, then flux tuning can allow inflation, otherwise even large numbers of vacua may not help.

[3]Very concrete reasons to expect that the distributions are broader in IIB flux vacua are described in Sec. 6.2 of [39], for instance.

terms from string theory. In Sec. III, we present and analyze the three explicit models analytically and numerically. We summarize our conclusions in Sec. IV.

II. STRING THEORY AND DIMENSIONAL REDUCTION

In this section we give a gentle introduction and review of the study of compactification in string theory, with a focus on the ingredients that are relevant for the specific models we will investigate in Sec. III. Much more complete and technical reviews are given in [5–10], while a qualitative introductory review appears in [42]. We begin by mentioning the basic ingredients of type IIA string theory, with a focus on fluxes.

A. Supergravity

String theory is believed to be a consistent theory of quantum gravity. One curious feature of the theory is that this *consistency* suggests a special role for 10 dimensions, where consistent string theories are in correspondence with the so-called "maximal supergravities."[4] Furthermore, a remarkable feature of the theory is that its dynamics in 10 dimensions can be *derived*, rather than *guessed*, by demanding consistency. For comparison, consider the familiar case of a charged point particle moving in a background curved space-time $g_{\mu\nu}$ with a background electromagnetic field strength $F_{\mu\nu}$. There is no reasonable way to uniquely derive the dynamical equations governing the time evolution of $g_{\mu\nu}$ and $F_{\mu\nu}$ from any consistency arguments about the behavior of the point particle. However, in the case of the string, this is precisely what happens.

In this paper, we will focus on what is known as the type IIA string theory. It can be derived that part of the 10-dimensional action governing gravity and the field strength of the string in this theory is [43]

$$S = \frac{1}{2\kappa_{10}^2}\int d^{10}x\sqrt{-g_{10}}e^{-2\phi} \times \left(R_{10} + 4(\partial_\mu\phi)^2 - \frac{1}{2}F_{\mu\nu\rho}F^{\mu\nu\rho}\right). \quad (1)$$

Here R_{10} is the 10-dimensional Ricci scalar, ϕ is a scalar field known as the "dilaton," and $F_{\mu\nu\rho}$ is a generalized electromagnetic field strength; it carries 3 indices (making it a so-called 3-form) since it is sourced by the $(1+1)$-dimensional string, just like the familiar electromagnetic field strength $F_{\mu\nu}$ carries 2 indices (a 2-form) since it is sourced by a $(0+1)$-dimensional point particle. The overall prefactor sets the gravitational strength in 10 dimensions $\kappa_{10}^2 = 8\pi G_{10}$. It is related to the so-called Regge parameter α' (units of length-squared) by $2\kappa_{10}^2 = (2\pi)^7\alpha'^4$. The inverse of the Regge parameter is the string tension ($=1/2\pi\alpha'$); the tension of a string is an absolute constant and is analogous to the mass of a particle. Furthermore, α' is related to the string length by (in our convention) $l_s = 2\pi\sqrt{\alpha'}$. Later we will see that it is convenient to measure a number of dimensionful quantities in units of l_s. Table I provides a hopefully useful dictionary of key string theory notation and the symbols used in this paper, including a summary of the above.

Let us summarize: In Eq. (1) we see that the 10-dimensional universe of string theory contains gravity and a field strength $F_{\mu\nu\rho}$, and that they appear in the same way as gravity and electromagnetism does in 4 dimensions. We further note that there exists the dilaton ϕ which is nonminimally coupled to gravity. Because the coefficient of $F_{\mu\nu\rho}F^{\mu\nu\rho}$ is proportional to $e^{-2\phi}$, one identifies $g_s \equiv e^\phi$ as the string coupling; it is the string loop expansion parameter analogous to e in electromagnetism.

Now we must mention some other features of string theory in 10 dimensions that we did not include in Eq. (1). First of all, it turns out that the 3-form $F_{\mu\nu\rho}$ (which we will later denote simply F_3, and which is often denoted H_3 in the string literature) is *not* the only field strength that appears in string theory. Rather, there are also various other fields; various so-called p forms with p indices, where p takes various integer values, and whose interactions are also uniquely determined by consistency. In addition, there are extended objects of various dimensionality in the theory known as Dirichlet branes and orientifold planes, which are charged under these p forms [44]. Also, there are fermions which give rise to a collection of terms to be added to Eq. (1), since we are describing a supersymmetric theory, but we have set their values to zero here. We focus on cosmologies that have maximal space-time symmetry (Minkowski, anti-de Sitter, de Sitter) which means that the vacuum expectation values of the fermion fields must vanish. Finally, what appears in Eq. (1) is only the first term in a perturbative expansion in powers of α' and g_s. For length scales large compared to the string length l_s and for small g_s we can ignore such corrections; this is known as the supergravity approximation.

B. Compactification and fluxes

1. Calabi-Yau manifolds

Of most interest to us is what this theory predicts in 4 dimensions. Currently there is no background independent formulation of string theory, so the *compactification* of the 10-dimensional geometry to 4 large dimensions is specified by hand and is not unique. The most commonly studied compact spaces are Calabi-Yau manifolds (e.g., see [7,43] for a technical definition). They are useful for at least two reasons: First, Calabi-Yau manifolds preserve some unbroken supersymmetry which allows for better computational control. Second, and most importantly for us, Calabi-Yau manifolds are spaces that possess a metric

[4] Here we are referring to the $\mathcal{N}=1$ and $\mathcal{N}=2$ theories in 10 dimensions, where $\mathcal{N}$ is the number of supersymmetries.

that is Ricci flat (has $R_{\mu\nu} = 0$ like any vacuum metric). This is very convenient in finding solutions to the 10-dimensional equations of motion. The simplest example is that of the torus T^6, which is not only Ricci flat, but also flat (with vanishing Riemann tensor). In this paper we will focus on this space (T^6), since this has been studied the most intensely in the literature. It is also a very useful pedagogical device, and we will make some comments below on the connection of our results to more general compactifications.

2. Orbifolds and orientifolds

Although understanding them in detail is not central to following our examples below, let us briefly mention orbifolding [45] and orientifolding [46–48], two technical operations that string theorists perform on the compact space, since they occur in all the models we will investigate. It often proves to be important to reduce the number of points in a manifold by declaring some of them identical. In technical jargon, one forms the quotient space with some finite symmetry group of the manifold, for example $T^6/\mathbb{Z}_p$, where $\mathbb{Z}_p$ is the group of integers modulo p. The specific $\mathbb{Z}_p$ symmetry is model dependent. This defines a so-called orbifold. Certain toroidal orbifolds are of interest since they are a special singular limit of some (nontoroidal) Calabi-Yau.[5] Also, by performing additional discrete operations one can form what is known as an orientifold. It is related to forming unoriented strings out of oriented strings. It will be important for us in what follows that at fixed points of the group action on the internal manifold, in orientifold models, one gets so-called *orientifold planes* (or O-planes). These O-planes provide a negative contribution to the vacuum energy (see ahead to Eq. (13)) and this is important for stabilization.

Both of these operations, orbifolding and orientifolding, serve the purpose of allowing for chiral fermions and reducing the amount of supersymmetry in 4 dimensions. For suitable choices of symmetry groups, there can however be a residual amount of supersymmetry in 4 dimensions.[6]

3. Moduli

In general, there are scalar fields characterizing the size and shape of any compact manifold: Kähler moduli (roughly specifying size) and complex structure moduli (roughly specifying shape). Table I summarizes all fields whose dynamics we will keep track of in 4 dimensions: besides gravity $g_{\mu\nu}$, we have "geometric" moduli: a dilaton ϕ, Kähler moduli (also known as "radions"), and complex structure moduli. In addition, each of these geometric moduli are accompanied by a field that is generically referred to as an "axion." The reason these are called axions is not important here, but suffice it to say that they are all pseudoscalars and some are coupled to a generalized $\mathbf{E}\cdot\mathbf{B}$ term in the action, reminiscent of the axion of quantum chromodynamics (see, e.g., [49,50]). If we denote the various geometric moduli by b_i ($i = 1, 2, \ldots$) and the axions by a_i, then these 2 degrees of freedom can be put into a complex pair $\psi_i = a_i + ib_i$.[7] We will see that this construction of forming a complex scalar is quite useful.[8] We will group all these complex fields into a single vector ψ, which will act as our complex inflaton vector. When separated into its real components, we denote this ϕ; our real inflaton vector.

4. Fluxes and potential energies

Strictly speaking, moduli are defined as those scalars that have vanishing potential. Without including any extra ingredients (such as field strengths), the above-mentioned fields would indeed be massless and free. This is very problematic. For example, if the radions are freely propagating fields then the size of the compact space could take on any value, including unacceptably large values. Indeed, there are constraints from 5th force experiments showing that these fields must be stabilized with a large effective mass (moderately large compared to the inverse millimeter scale to which gravity has been tested, and huge compared to today's Hubble scale), i.e., that there must be contributions to the potential energy density of the form $m_i^2 b_i^2$ (where the coefficients m_i are large). Furthermore, we are interested in whether any of these scalars could be the inflaton. Since a free field by definition is one that does not feel a potential, it cannot possibly drive inflation.

However, an important feature of string theory is the existence of various field strengths, and these induce interactions for the moduli. We have already introduced the field strength $F_{\mu\nu\rho}$, hereafter abbreviated F_3. We will focus on what is known as type IIA string theory in this paper, in which there are also forms with even numbers of indices, such as F_2 and F_4, but more general forms F_p occur in other models. In our $3 + 1$ large dimensions, Lorentz invariance prevents any cosmological field strengths, however such restrictions do not apply to the components of F_p in the compact space.[9] Assuming $p \leq$

[5]E.g., the orbifold $T^4/\mathbb{Z}_2$ is a limit of the Calabi-Yau K3.

[6]For example, in commonly studied orientifolds of Calabi-Yau manifolds, the $\mathcal{N} = 2$ theory in 10 dimensions becomes an $\mathcal{N} = 1$ theory in 4 dimensions in type II string theory.

[7]In the string literature, there are many symbols used for the different moduli, such as T, U, v, etc., but we will just use the common notation $\psi_i = a_i + ib_i$ for all moduli.

[8]In fact this construction is integral to $\mathcal{N} = 1$ supersymmetric models, where such pairs are unified in a chiral multiplet, a representation of the supersymmetry algebra.

[9]In the presence of field strengths in the compact space, the Ricci tensor on the compact space is in general nonzero, and so the space is strictly no longer Calabi-Yau. However, when the moduli masses are light compared to the inverse size of the compact space (the "Kaluza-Klein scale") then this *backreaction* is small, and we can continue to treat the compact space as Calabi-Yau. This is a property of the models we study.

6, then the fields satisfy

$$\frac{1}{l_s^{p-1}}\int F_p = f_p, \tag{2}$$

where the integral is over some p-dimensional internal manifold of the compact space. Such integrals appear when we compactify the theory. Here f_p is an integer, corresponding to a generalized Dirac charge quantization condition. These quantized integrals of the field strengths are known as "fluxes." They correspond to wrapped field lines in the compact space. Such fields can be thought of as being sourced by generalized electric and magnetic charges provided by the various branes of the theory [7]. Note, however, that this is just an incomplete analogy, since the fluxes we are referring to here thread topologically nontrivial internal submanifolds of the compact space; therefore, Gauss' law does not require charges to source the flux. (There are, however, other space-filling branes in the theory, which will appear in the models). Since there is an energy cost associated with deformations of the compact space in the presence of field strengths, these fluxes induce a potential energy $V = V(\boldsymbol{\psi})$ for the moduli. This potential is necessary for stabilization, and we will investigate if it can also drive inflation.

To get an idea of the form that these energies will take, consider a generalized electric field $\mathbf{E}_p$ set up by a stationary source; a point source for $\mathbf{E}_2$, a string for $\mathbf{E}_3$, a membrane for $\mathbf{E}_4$, etc. If there are f_p units of charge contained in a compact space of size r, then the energy density is given roughly by (ignoring factors of l_s)

$$|\mathbf{E}_p|^2 \sim \frac{f_p^2}{r^{2p}}, \tag{3}$$

which reduces to the familiar result f_2^2/r^4 for a point source. The total energy density will involve a sum of such terms. Equation (12) below illustrates the form this takes when expressed in a 4-dimensional action.

C. The 4-dimensional action and slow roll

Here our aim is to move from the 10-dimensional theory to the 4-dimensional theory.

1. Integrating out the compact space

To understand the 4-dimensional action, let us begin by focusing on the gravity sector. For simplicity, we will assume that the 10-dimensional metric is in block diagonal form: $g_{AB}^{10} = \mathrm{diag}(g_{\mu\nu}^4, g_{ab}^6)$, i.e., that it separates into a piece governing the 4 large space-time dimensions and a piece governing the 6 compact dimensions. Furthermore, let us assume that the Ricci scalar is independent of the compact coordinates (the usual assumption in Kaluza-Klein models), so that we can integrate over d^6x. This assumption is valid when one considers an (approximately) "unwarped compactification," as we do in this paper. The relevant piece in the action is the first term in Eq. (1), involving the 10-dimensional Ricci scalar R_{10} and the dilaton (scalar field). This gives

$$\int d^{10}x\sqrt{-g_{10}}e^{-2\phi}R_{10} = \int d^4x\sqrt{-g_4}\mathrm{Vol}e^{-2\phi}R_{10}, \tag{4}$$

where "Vol" is the 6-dimensional volume of the compact space. The particular form of Vol is model dependent, and the relationship between the 10-dimensional Ricci scalar R_{10} and the 4-dimensional Ricci scalar R_4 is also model dependent through the form of the metric on the compact space. However, for this class of models it is true that

$$R_{10} = R_4 + \mathrm{func}(\text{compact space fields}). \tag{5}$$

Since both the volume Vol and the dilaton ϕ are allowed to be dynamical, the action in Eq. (4) is evidently not in canonical form. In order to bring the action into canonical form, we introduce the so-called Einstein metric $g_{\mu\nu}^E$, which is defined via a conformal transformation as

$$g_{\mu\nu}^E \equiv \frac{\mathrm{Vol}\, e^{-2\phi}}{\bar{m}_\mathrm{P}^2\kappa_{10}^2} g_{\mu\nu}^4, \tag{6}$$

where $g_{\mu\nu}^4$ is the 4-dimensional string metric—the metric in the "string frame" of Eq. (1). In this transformation we have introduced the (reduced) Planck mass $\bar{m}_\mathrm{P} = 1/\sqrt{8\pi G} \approx 2\times 10^{18}$ GeV, where G is the 4-dimensional Newton constant. The gravity sector, written in terms of the corresponding Einstein Ricci scalar R_E, then appears in canonical form

$$\frac{1}{2\kappa_{10}^2}\int d^{10}x\sqrt{-g_{10}}e^{-2\phi}R_{10} = \int d^4x\sqrt{-g_E}\left(\frac{1}{16\pi G}R_E + \cdots\right) \tag{7}$$

and this is referred to as the "Einstein frame."

Let us clarify a feature of the conformal transformation. Recall that during inflation, both Vol and ϕ may be dynamical, since they in fact depend on the inflaton vector ψ. Well after inflation, we expect these fields to be stabilized at some fixed values: $\langle\mathrm{Vol}\rangle$ and $\langle\phi\rangle$. At such values we require that the conformal transformation in Eq. (6) be the identity transformation. This implies a relationship between κ_{10}, $\langle\mathrm{Vol}\rangle$, $\langle\phi\rangle$, and $\bar{m}_\mathrm{P}$:

$$\bar{m}_\mathrm{P}^2 = \frac{\langle\mathrm{Vol}\rangle e^{-2\langle\phi\rangle}}{\kappa_{10}^2}. \tag{8}$$

Note that the Planck mass is defined in terms of the fields at their *stabilized values*, so it is a constant. Since it is natural for $\langle\mathrm{Vol}\rangle$ to be given in units of l_s^6 and since $\kappa_{10}^2 = l_s^8/4\pi$, one can in fact use this equation to determine the string length l_s in terms of the Planck length for any particular model.

2. Kinetic energy

In general, the kinetic energy of the moduli is not in canonical form. Recall that the moduli are a combination of not only the dilaton, but also the Kähler and complex structure moduli which describe the size and shape of the particular Calabi-Yau compact space. An important property of every Calabi-Yau (related to the underlying supersymmetry it preserves) is that there exists a scalar function K of the moduli, known as the Kähler potential, whose Hessian matrix is the metric on moduli space:

$$K_{i\bar{j}} \equiv K_{,i\bar{j}} = \frac{\partial^2 K}{\partial \psi^i \partial \psi^{\bar{j}}}, \tag{9}$$

where ψ^i is a generic name for a complex modulus (axion partnered with geometric moduli), and barred variables denote the complex conjugate. Contracting space-time derivates of the moduli with this metric gives the kinetic energy in the Einstein frame:

$$T = -\bar{m}_{\rm P}^2 K_{i\bar{j}} \partial_\mu \psi^i \partial^\mu \psi^{\bar{j}}. \tag{10}$$

There is an implicit sum over all i, j. Since we shall deal with dimensionless Kähler potentials, the factor of $\bar{m}_{\rm P}^2$ is necessary on dimensional grounds. In the limit in which we shall work, K does *not* depend on fluxes, which means that there is 1 Kähler potential for each of the three models that we will investigate, applicable to any vacuum in their respective landscapes. In other words, fluxes affect only the potential energy, not the kinetic energy.

3. Potential energy

In addition to the Kähler potential K, there exists another object which contains information about the particular compactification. For supersymmetric compactifications, of the type we will focus on, this object is the so-called superpotential W. W is a complex analytic function of the complex moduli ψ^i and also depends on the fluxes. If we turn on fluxes, then there are in general energies induced associated with distortions of the compact space and displacements of the dilaton and axions. These interactions are all contained in W. The potential energy term in the 4-dimensional Lagrangian density is given by

$$V = e^K (D_i W K^{i\bar{j}} \overline{D_j W} - 3|W|^2), \tag{11}$$

which is sometimes referred to as the supergravity formula. Here $D_i W \equiv \partial_i W + W \partial_i K$ and $K^{i\bar{j}}$ is the matrix inverse of $K_{i\bar{j}}$. Again there is an implicit sum over all i, j. A reader familiar with supersymmetry in 4 dimensions will recognize that these are the so-called "F terms" and that there are no so-called "D terms."

In order to develop some intuition, we would like to get an idea of the typical form of V. In Eq. (3) we estimated the energy density established by a p-form field strength, but must now take into account the integration over the compact space and the conformal transformation. For a compact space of size r, the contribution to V is given roughly by (ignoring factors of l_s)

$$\Delta V \sim f_3^2 \frac{e^{2\phi}}{r^{12}} \quad \text{for } F_3, \qquad \Delta V \sim f_p^2 \frac{e^{4\phi}}{r^{6+2p}} \quad \text{for } F_{p\neq 3}. \tag{12}$$

We also mention an estimate for the contribution to V from N_1 D6-branes and N_2 O6-planes

$$\begin{aligned} \Delta V &\sim N_1 \frac{e^{3\phi}}{r^9} \quad \text{for } D6\text{-branes}, \\ \Delta V &\sim -N_2 \frac{e^{3\phi}}{r^9} \quad \text{for } O6\text{-planes}. \end{aligned} \tag{13}$$

Note that the O6-plane makes a negative contribution.

In the models studied, when calculating the potential from the supergravity formula V, we will for simplicity work in units where the string length $l_s = 1$. This must be rescaled in order to obtain the potential in Einstein frame $\bar{V}$ in conventional units:

$$\bar{V} = \frac{\bar{m}_{\rm P}^4}{4\pi} V. \tag{14}$$

(This comes from: $\bar{m}_{\rm P}^4 \kappa_{10}^2 = \bar{m}_{\rm P}^4 l_s^8/4\pi = \bar{m}_{\rm P}^4/4\pi$.) In Secs. III B and III E we will just refer to V, rather than $\bar{V}$.

4. Putting it all together

Altogether, the effective 4-dimensional action in the Einstein frame is a familiar sum of a gravity term, kinetic energy, and potential energy, i.e.,

$$S = \int d^4x \sqrt{-g_E} \times \left[\frac{1}{16\pi G} R_E - \bar{m}_{\rm P}^2 K_{i\bar{j}} \partial_\mu \psi^i \partial^\mu \psi^{\bar{j}} - \bar{V}(\psi) \right]. \tag{15}$$

Keeping the kinetic term general, the Euler-Lagrange equations of motion for a flat Friedmann-Robertson-Walker (FRW) universe $g_{\mu\nu}^E = \text{diag}(-1, a(t)^2, a(t)^2, a(t)^2)$ are in terms of K and its derivatives:

$$\ddot{\psi}^i + 3H\dot{\psi}^i + \Gamma^i_{jk} \dot{\psi}^j \dot{\psi}^k + K^{i\bar{j}} \bar{V}_{,\bar{j}} / \bar{m}_{\rm P}^2 = 0, \tag{16}$$

$$H^2 = \frac{8\pi G}{3} [\bar{m}_{\rm P}^2 K_{i\bar{j}} \dot{\psi}^i \dot{\psi}^{\bar{j}} + \bar{V}(\psi)], \tag{17}$$

where $H = \dot{a}/a$ is the Hubble parameter and $\Gamma^i_{jk} = K^{i\bar{n}} K_{jk,\bar{n}}$ is the Christoffel symbol on moduli space.

To investigate inflation, we must compute the slow-roll parameters. The first slow-roll condition is that the kinetic energy in (17) should be small compared to the potential energy when the acceleration term in (16) is negligible. The second condition is that the acceleration is and remains small along the slow-roll direction (quantified by differentiating and demanding self-consistency). These two conditions define the two slow-roll parameters ϵ and η and correspond to the requirements that $\epsilon < 1$ and $|\eta| < 1$. We find that

$$\epsilon = \frac{K^{i\bar{j}} V_{,i} V_{,\bar{j}}}{V^2} \left(= \frac{g^{ab} V_{,a} V_{,b}}{2V^2} \right), \tag{18}$$

$$\eta = \min \text{eigenvalue} \left\{ \frac{g^{ab}(V_{,bc} - \Gamma^d_{bc} V_{,d})}{V} \right\}. \tag{19}$$

Here η (and ϵ) is written in terms of the metric g_{ab} governing real scalar fields ϕ^a: $K_{i\bar{j}} \partial_\mu \psi^i \partial^\mu \psi^{\bar{j}} = \frac{1}{2} g_{ab} \partial_\mu \phi^a \partial^\mu \phi^b$, with $\phi^{2i-1} = a^i$ and $\phi^{2i} = b^i$. Note that we can use either V or $\bar{V}$ in the slow-roll conditions, as $V \propto \bar{V}$ (see Eq. (14)). In regions where inflation occurs, these three functions (V, ϵ, η) can be used to predict several cosmological parameters, as detailed in Appendix D. A comparison between the theoretical predictions and observational data provides a precision test of the model.

D. de Sitter vacua

There are two good reasons to want de Sitter "vacua," namely, that there are at least two eras of our universe that are approximately de Sitter; during inflation, which exhibits slow roll, and at late times, which appears consistent with a positive cosmological constant. This article is focussed on the former epoch. If we have a region in moduli space that is de Sitter, namely, a region in which the gradient of the 4-dimensional potential V (from Eq. (15)) is zero with $\bar{V} > 0$, we are a significant step closer to realizing inflation. In such a region (which may be a single point) we at least know that the first slow-roll parameter $\epsilon = 0$, although we may still face the so-called η problem [51] (for a recent discussion in the string theory context, see [52]).

Let us make some comments about supersymmetric vacua and anti-de Sitter (AdS) space. The condition for supersymmetry (SUSY) is that the covariant derivative of the superpotential vanishes, i.e.,

$$D_i W = \partial_i W + W \partial_i K = 0 \tag{20}$$

for all i. It is simple to show from Eq. (11) that at any such SUSY point the supergravity potential V is stationary. This constitutes some "vacuum" (stable or unstable) of the theory. At such a point the potential is

$$V_{\text{SUSY}} = -3e^K |W|^2, \tag{21}$$

so we see it is necessarily nonpositive. It may happen that $W = 0$, corresponding then to Minkowski space. But what is *much more common* is for $W \neq 0$, corresponding then to anti-de Sitter space.

In fact it is known that any vacuum that is supersymmetric (in supergravity, superstring theory, M theory, etc.) is necessarily non-de Sitter. But it may be the case that in such a model, a non-SUSY minimum in the space of scalar field expectation values is de Sitter (spontaneously broken SUSY). Achieving this is not easy, as described by various "no-go theorems." In particular, under mild assumptions on the nature of the compact space (namely that it is nonsingular, etc.), one can show that inclusion of fluxes alone does not allow one to find any de Sitter vacua [53,54].

There are, however, other structures besides fluxes in string theory, e.g., D-branes and O-planes. In [55] it is shown that the argument of [54] may be extended to include most forms of D-branes, but it cannot be extended to include O-planes. The energies associated with such structures were described in Eq. (13). Recent work has shown that the realization of de Sitter vacua is possible with such ingredients, as in the constructions of [29]. However, we do not find any de Sitter vacua in the simple IIA models we study.

Given that we are considering models with AdS vacua, what is the implication for inflation? Suppose that the inflaton eventually settles down to some such AdS vacuum. There the potential has a (negative) value that we will call the "cosmological constant." This may well compromise any chance to obtain many e-folds of inflation, which requires $\bar{V} > 0$. However, we can imagine *a priori* a scenario where this is *not* catastrophic for inflation: We will see later that we have fluxes that can be used to dial the cosmological constant toward zero. Hence the depth of AdS space can be tuned very small. Furthermore, well away from the SUSY vacuum there are regions in moduli space were the potential is large and *positive*. Then, as long as $\bar{V}$ during inflation is much greater than the depth of the AdS minimum, it is plausible that inflation could be realized.

At the end of inflation one should in principle enter the radiation era. Normally this occurs through the decay of the inflaton to various fields including the standard model particles. However, the standard model is not contained in the models that we investigate, so this is an issue that we do not tackle. Furthermore, we do not address the late-time problem of the smallness of the (positive) cosmological constant. (A popular explanation of the smallness of the cosmological constant appeals to the existence of exponentially many vacua realizing different vacuum energies, see [56]).

III. TYPE IIA MODELS

Here we investigate the cosmology of three explicit models. Many choices of compactification are possible. However, the torus is flat and is perhaps the most well-studied compact manifold in the literature, so we will focus on (orbifolds and orientifolds of) this. We will investigate the resulting inflation picture for three explicit models: DGKT [39], VZ [40], and IW [41]. In this section we use units $l_s = 1$.

A. Diagonal torus models

For clarity, let us describe the properties of the Kähler potential and the slow-roll conditions in more detail for a particular class of examples. The first two torus models to be discussed have the property that the Kähler potential is the logarithm of a product of moduli. Writing

$$\psi_i = a_i + ib_i \tag{22}$$

for all moduli,[10] we have

$$K = -\ln\left(\prod_i b_i^{n_i}\right) + \text{const}, \tag{23}$$

where n_i are $\mathcal{O}(1)$ integers (e.g., in the VZ model described below, there are 7 moduli ($i = 1, \ldots, 7$) with $n_i = 1$). The kinetic energy for such models is then

$$T = -\bar{m}_{\rm P}^2 \sum_i \frac{n_i}{4} \frac{(\partial_\mu a_i)^2 + (\partial_\mu b_i)^2}{b_i^2}. \tag{24}$$

In this case, the equations of motion for the moduli are

$$0 = \ddot{b}_i + 3H\dot{b}_i + \frac{\dot{a}_i^2 - \dot{b}_i^2}{b_i} + \frac{1}{\bar{m}_{\rm P}^2}\frac{2b_i^2}{n_i}\frac{\partial \bar{V}}{\partial b_i}, \tag{25}$$

$$0 = \ddot{a}_i + 3H\dot{a}_i - 2\frac{\dot{a}_i\dot{b}_i}{b_i} + \frac{1}{\bar{m}_{\rm P}^2}\frac{2b_i^2}{n_i}\frac{\partial \bar{V}}{\partial a_i}. \tag{26}$$

The first slow-roll parameter for inflation then takes the form

$$\epsilon = \frac{1}{V^2}\sum_i \frac{b_i^2}{n_i}\left[\left(\frac{\partial V}{\partial a_i}\right)^2 + \left(\frac{\partial V}{\partial b_i}\right)^2\right]. \tag{27}$$

It is important to take note of the form of the Kähler potential; it is independent of all axions (general fact). Hence if we shift an axion by a constant, it has no effect on the kinetic energy. Also, if we rescale ψ_i by a real number, the kinetic energy is also unchanged. In short,

$$a_i \to d_i a_i + c_i, \; b_i \to d_i b_i \tag{28}$$

leaves the kinetic energy unchanged for any constants $c_i, d_i \in \mathbb{R}$. In turn, the form of the slow-roll parameters ϵ and η are unaffected. These shift and scaling symmetries allow one to eliminate some flux parameters that appear in the superpotential. In the first model, we will see that these symmetries allow all fluxes to be absorbed into an overall multiplicative factor, while in the second and third models we will have one additional nontrivial flux parameter to dial. There will in general be ambiguities associated with positive/negative values of the fluxes that one should keep careful track of.

[10]More precisely, we study all moduli that arise from metric deformations of the torus, the dilaton, and their superpartners. We neglect so-called "twisted moduli" or "blow-up modes" originating from singularities of the orbifold group action, though we briefly discuss them in Sec. III E.

In order to emphasize that the Lagrangian in 4 dimensions is reminiscent of that in standard inflation, let us perform a field redefinition for the simple case where we ignore the axions and focus on $\bar{V}(b_i)$. By defining

$$\phi_i \equiv \sqrt{\frac{n_i}{2}}\bar{m}_{\rm P}\log b_i, \tag{29}$$

the kinetic energy is put in canonical form and the action in Eq. (15) becomes

$$S = \int d^4x\sqrt{-g_E} \times \left[\frac{1}{16\pi G}R_E - \sum_i \frac{1}{2}(\partial_\mu \phi_i)^2 - \bar{V}(e^{\sqrt{2}\phi_i/\sqrt{n_i}\bar{m}_{\rm P}})\right]. \tag{30}$$

Note that the argument of $\bar{V}$ is now an exponential. The first slow-roll parameter then takes the canonical form for multifield inflation:

$$\epsilon = \frac{\bar{m}_{\rm P}^2}{2}\frac{|\nabla_\phi V|^2}{V^2}. \tag{31}$$

We point out that this is only true when ignoring the axions and relies upon the assumed simple form of the Kähler potential.

B. The model of DeWolfe, Giryavets, Kachru, and Taylor (DGKT)

In May 2005, DeWolfe *et al* [39] (DGKT) found an explicit *infinite* class of stable vacua in type IIA string theory. In their model they found that all moduli are stabilized by including the 3-form, 4-form (and less importantly the 2-form) fluxes of type IIA, and also including a 0-form flux. The 0-form flux plays the role of a mass term in the theory. Its presence induces several extra pieces into the action, which can only be derived from the so-called M theory. This framework is known as "massive type IIA supergravity."

Starting from the torus, they built the orbifold $T^6/\mathbb{Z}_3$ and projected it to the orientifold $T^6/\mathbb{Z}_3^2$. In addition, they introduced a static (6 + 1)-dimensional plane that carries charge (an O6-plane), for the purpose of satisfying a constraint known as a tadpole condition. We will focus here on reporting the salient features of the geometry after these technical operations have been performed; the reader is referred to the original paper [39] for details.

The torus of DGKT takes on essentially the simplest possible form (we will see more complexity in the later models of VZ and IW). The orbifolding and orientifolding act to reduce the number of degrees of freedom of the metric on the compact space to just 3. Here we neglect the moduli which arise at the orbifold fixed points (the so-called "blow-up modes" or "twisted sector" moduli).[11]

[11]We will briefly discuss the proper inclusion of the blow-up modes in III E. They alter the discussion in various important ways, but do not at first sight seem to change our conclusions.

These 3 are the (untwisted) Kähler moduli of the theory. There are no complex structure moduli left. The metric on the compact space and the volume are given by

$$(ds^2)_6 = \sum_{i=1}^{3} \gamma_i[(dx^i)^2 + (dy^i)^2], \tag{32}$$

$$\mathrm{Vol} = \int_{T^6/\mathbb{Z}_3^2} d^6x\sqrt{g_6} = \frac{1}{8\sqrt{3}}\gamma_1\gamma_2\gamma_3 = b_1b_2b_3. \tag{33}$$

Here the elements of the metric are called γ_i. The volume is proportional to the determinant of the square root of the metric ($\gamma_1\gamma_2\gamma_3$), the factor of $1/8\sqrt{3}$ comes from performing the peculiar integration over $T^6/\mathbb{Z}_3^2$, but is not important for us. What is important is the identification of the good Kähler coordinates b_1, b_2, b_3 whose product is the volume[12] (up to a prefactor, they are just the components of the metric).

Let us summarize the moduli of this model. As mentioned, all complex structure moduli are projected out by the orientifolding, leaving 4 moduli: 3 Kähler moduli $\psi_i = a_i + ib_i, i = 1, 2, 3$ and an axio-dilaton $\psi_4 = a_4 + ib_4(b_4 = e^{-\phi}\sqrt{\mathrm{Vol}}/\sqrt{2})$.[13] We note that the axio-dilaton appears in the compactification, not through an explicit appearance in the compact metric, but through its direct appearance in the action, as discussed in Sec. II.

The Kähler potential takes on the form promised in Sec. III A, namely, the logarithm of the product of geometric moduli. All that is left is to specify the values of n_i and the constant. One finds that

$$K = -\ln(32b_1b_2b_3b_4^4). \tag{34}$$

The superpotential is set by the interactions: DGKT turned on fluxes coming from F_3, F_2, F_4, and a zero form F_0, a so-called mass term, as well as an F_6. By studying the work of Grimm and Louis [37] they find

$$W = \frac{f_6}{\sqrt{2}} + \sum_{i=1}^{3}\frac{f_{4,i}}{\sqrt{2}}\psi_i - \frac{f_0}{\sqrt{2}}\psi_1\psi_2\psi_3 - 2f_3\psi_4, \tag{35}$$

the flux integers $f_6, f_{4,i}, f_0, f_3$ arising from F_6, F_4, F_0, F_3, respectively. We have turned F_2 off, as all results are qualitatively similar, although it is simple to include.

As mentioned, DGKT are able to satisfy the "tadpole condition" by including an O6-plane. In order to so, the following relationship between two of the flux integers must hold:

$$f_0f_3 = -2. \tag{36}$$

At this point there are several flux integers in the problem. However, we can simplify the problem greatly by exploiting the shift and scaling symmetries that we discussed in Sec. III A. Let us perform the following transformations of our fields:

$$\begin{aligned}\psi_i &\to \frac{1}{|f_{4,i}|}\sqrt{\frac{|f_{4,1}f_{4,2}f_{4,3}|}{|f_0|}}\psi_i \quad (i = 1, 2, 3),\\ \psi_4 &\to \frac{1}{|f_3|}\sqrt{\frac{|f_{4,1}f_{4,2}f_{4,3}|}{|f_0|}}\psi_4 + \frac{1}{2\sqrt{2}}\frac{f_6}{f_3}\end{aligned} \tag{37}$$

which leaves the form of the kinetic terms invariant. In terms of these new variables, the superpotential becomes

$$W = \sqrt{\frac{|f_{4,1}f_{4,2}f_{4,3}|}{|f_0|}} \times \left(\sum_{i=1}^{3}\frac{\hat{f}_{4,i}}{\sqrt{2}}\psi_i - \frac{\hat{f}_0}{\sqrt{2}}\psi_1\psi_2\psi_3 - 2\hat{f}_3\psi_4\right), \tag{38}$$

where the "hat" fluxes are just the signs of the fluxes, e.g., $\hat{f}_0 \equiv f_0/|f_0|$. This has a very interesting form: apart from an overall multiplicative factor, the superpotential is *independent of the magnitude of the fluxes* (although their sign will be important).

We now have all the tools we need; the Kähler potential and the superpotential. Using these, we can compute the 4-dimensional potential V using the supergravity formula (11). Focusing on the symmetric case, i.e., $\psi_1 = \psi_2 = \psi_3$, we find

$$\begin{aligned}V = V_{\mathrm{flux}}[&2(3a_1 + 2\sqrt{2}a_4)^2 - 4\delta a_1^3(3a_1 + 2\sqrt{2}a_4)\\ &+ 2a_1^6 + 6b_1^2 + 4b_4^2 - 12\delta a_1^2b_1^2 + 6a_1^4b_1^2 + 6a_1^2b_1^4\\ &+ 2b_1^6 - 8\sqrt{2}b_1^3b_4]/(32b_1^3b_4^4),\end{aligned} \tag{39}$$

where

$$V_{\mathrm{flux}} \equiv \frac{|f_0|^{5/2}|f_3|^4}{|f_{4,1}f_{4,2}f_{4,3}|^{3/2}} \tag{40}$$

is an overall multiplicative scale that depends on the fluxes. Note that since f_0 and f_3 are tightly constrained by the tadpole condition (36), V_{flux} is bounded from above and approaches 0 as $f_{4,1}f_{4,2}f_{4,3} \to \infty$. Also, $\delta \equiv \hat{f}_0\hat{f}_{4,1}\hat{f}_{4,2}\hat{f}_{4,3} = \pm 1$, delineates two independent families of V. The more general result, without simplifying to the symmetrical case, is given in Appendix A Eq. (A1).

Here we make a parenthetical comment: One can perform direct dimensional reduction from the 10-dimensional action without using the Kähler potential or superpotential. In the DGKT paper this is done explicitly with the axions (a_i, a_4) set to their SUSY values. Having set the axions to their SUSY values, the natural (and perhaps most intuitive) coordinates are then the original fields: dilaton ϕ and radions b_i. They find

[12] In the DGKT paper: $\mathrm{Vol} = \kappa b_1b_2b_3$, with $\kappa = 81$. By rescaling $b_i \to \kappa^{-1/3}b_i$, $i = 1, 2, 3$ we obtain Eq. (33) and κ is eliminated.

[13] The canonical model-independent axion is $\xi = 2a_4$.

$$V = \frac{f_3^2}{4}\frac{e^{2\phi}}{\mathrm{Vol}^2} + \frac{1}{4}\left(\sum_{i=1}^{3} f_{4,i}^2 b_i^2\right)\frac{e^{4\phi}}{\mathrm{Vol}^3} + \frac{f_0^2}{4}\frac{e^{4\phi}}{\mathrm{Vol}} - 2\frac{e^{3\phi}}{\mathrm{Vol}^{3/2}}, \tag{41}$$

where the first 3 terms come from fluxes: 3-form, 4-form, and 0-form, respectively, and the final term comes from the O6-plane. The first 3 terms take on the form we indicated in Eq. (12) for p forms. The final term carries a minus sign, since O6-planes carry *negative* tension, as we indicated in Eq. (13). This term is crucial to achieve stability. In this form it is not clear that the fluxes scale out, however. By rewriting this in terms of the variables $b_i, b_4 = e^{-\phi}\sqrt{\mathrm{Vol}}/\sqrt{2}$, and then scaling according to Eq. (37), it is simple to show that one recovers a simpler version of (39), one with a_i and a_4 set to zero.

We plot V in Fig. 1. In order to discuss the properties of this potential, let us begin by discussing its supersymmetric properties. The SUSY vacuum lies at

$$a_1 = 0, \qquad b_1 \approx 1.29, \qquad a_4 = 0, \qquad b_4 \approx 1.22. \tag{42}$$

For $\delta = -1$, this has a corresponding positive definite mass matrix and is clearly stable. However, for $\delta = +1$ the mass matrix has negative eigenvalues and so is tachyonic, as reported by DGKT. Nevertheless, it is stable as it satisfies the Breitenlohner-Freedman bound [57] which states that tachyonic vacua can be stable if the cosmological constant is large and negative. In Fig. 1 (top), we set the axions to zero and plot V as a function of b_1 and b_4 in the vicinity of the SUSY vacuum. We see that with respect to these two coordinates, the potential has a regular stable minimum. Note also, that with $a_1 = a_4 = 0$ then the values of the potential in (39) for $\delta = \pm 1$ coincide. In Fig. 1 (bottom), we plot V with b_1 and b_4 fixed at their SUSY values, and allow a_1 and a_4 to vary. We have plotted the case $\delta = +1$ as its behavior is the most interesting.

Now, let us investigate the potential further away from the SUSY point. We find that in the $\delta = +1$ case (tachyonic), there is a second stationary point of the potential. It is nonsupersymmetric, and lies at

$$\begin{aligned} a_1 &\approx \pm 0.577, & b_1 &\approx 1.15, \\ a_4 &\approx \mp 0.544, & b_4 &\approx 1.09. \end{aligned} \tag{43}$$

Given these two stationary points of V (one SUSY, one non-SUSY) we choose to plot V as a function of λ, where λ is a parameter that linearly interpolates between these two points. With the SUSY point denoted by a vector of moduli $\psi_{i,\mathrm{susy}}$ and the other (non-SUSY) stationary point denoted by a vector $\psi_{i,\mathrm{stat}}$, we form the interpolating vector:

$$\psi_i(\lambda) \equiv (1-\lambda)\psi_{i,\mathrm{susy}} + \lambda\psi_{i,\mathrm{stat}} \tag{44}$$

so that $\lambda = 0$ is the SUSY vacuum and $\lambda = 1$ is the second stationary point. We plot this in Fig. 2 (top). Also, in Fig. 2 (bottom), we plot V as a function of b_4 with all other moduli at their SUSY values. As already stated, $b_4 \approx 1.217$ is the SUSY point (a minimum with respect to b_4) and this exists on the left-hand side of the figure with the potential much lower than shown. However, the interesting feature is that for $b_4 \approx 7.912$ there exists a local maximum (with $V > 0$) with respect to this modulus (but not stationary with respect to the other moduli) and then V approaches zero from above as $b_4 \to \infty$. There is quite similar behavior when one plots V versus the radial modulus, with the dilaton fixed.

Let us recapitulate the salient features of this class of vacua. We have come to an important realization: the potential V is of the form $V = V_{\mathrm{flux}}(f_i)\mathrm{func}(\psi^i)$, where f_i are flux integers and $\mathrm{func}(\psi^i)$ is some function of the (rescaled) moduli, *independent of fluxes*. Hence, apart from the overall multiplicative scale (which is proportional to the cosmological constant) all vacua look the same. This

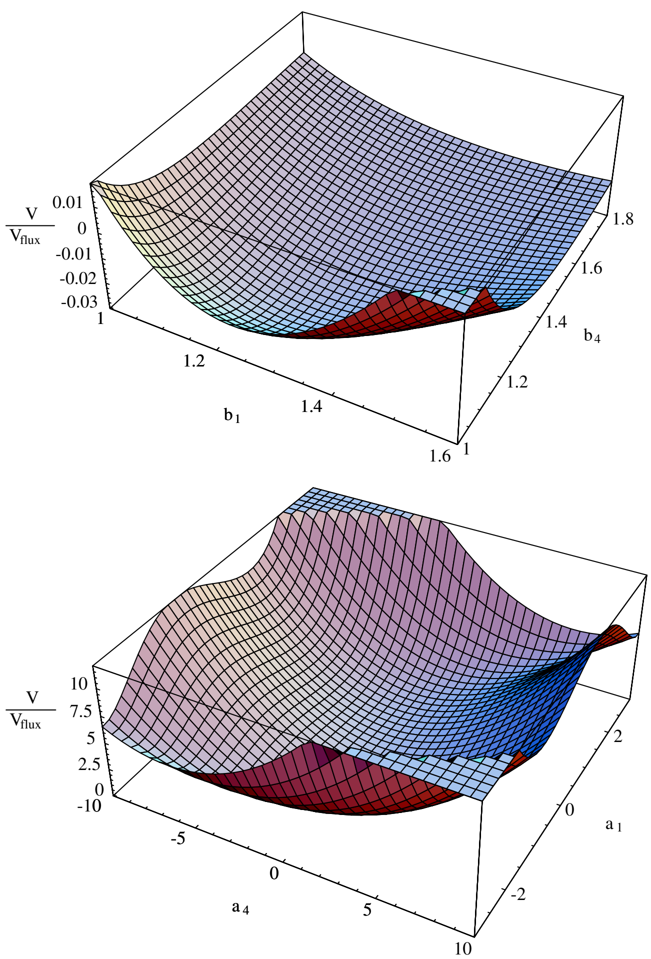

FIG. 1 (color online). Top: The potential $V = V(b_1, b_4)$ with axions at their SUSY values: $a_1 = a_4 = 0$. Bottom: The potential $V = V(a_1, a_4)$ with $\delta = +1$ and geometric moduli at their SUSY values: $b_1 \approx 1.291$, $b_4 \approx 1.217$.

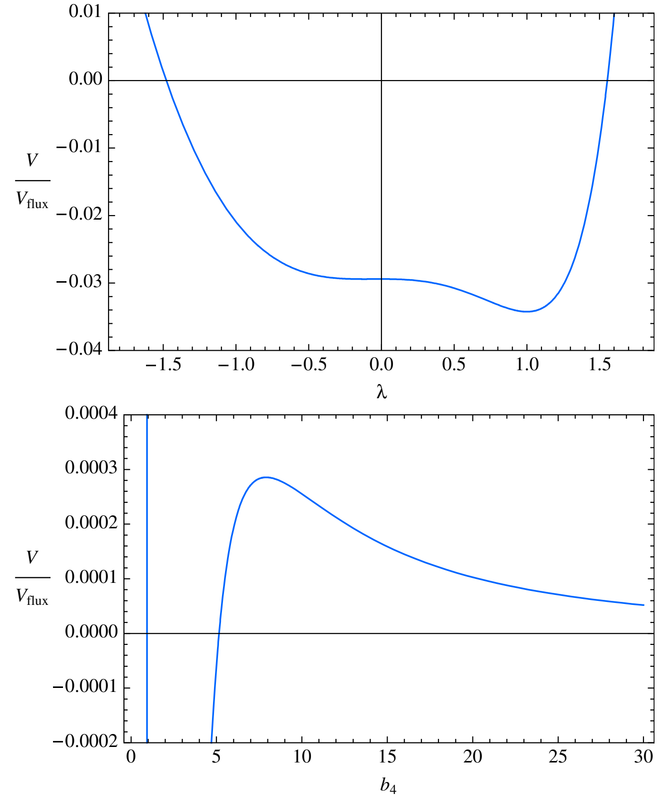

FIG. 2 (color online). Top: We plot $V = V(\lambda)$ by interpolating between the two stationary points of the potential, which exists for the "lower case." $\lambda = 0$ corresponds to the (tachyonic) SUSY vacuum and $\lambda = 1$ corresponds to a local (non-SUSY) minimum. Bottom: We plot $V = V(b_4)$, focusing on large b_4, with all other moduli fixed at their SUSY values.

means that the slow-roll parameters ϵ and η are independent of the fluxes. So for this model inflation is realized by all or none of the flux vacua.

Of course we wish to know if the potential is sufficiently flat in some region to exhibit slow roll. Here we see a general barrier to this. Note that the potential is a polynomial in $\{a_i, b_i, 1/b_i\}$. Naively, this may look as if it allows for inflation due to some form of power law potential, e.g., the potential is quadratic in a_4, so this may look like Linde's $\sim\phi^2$ "chaotic inflation" [58,59]. However, by inspecting the form of Eq. (27), we see that this is not at all the case. The factor of b_i^2 in the summand changes the picture significantly. It means that the typical contribution to ϵ is not $\mathcal{O}(\phi^{-2})$ but $\mathcal{O}(1)$, and cannot be tuned small by taking ϕ large, as in chaotic inflation. In fact, an extensive numerical search of moduli space (detailed below) suggests that $\epsilon > 1$ whenever $V > 0$ (of course $\epsilon \rightarrow 0$ at the stationary point(s) of the potential, but $V < 0$ there). With the axions set to zero, it is simple to analytically prove the nonexistence of inflation. With axions nonzero, we produced vast tables of ϵ supporting this result. We will give a representative plot of ϵ in the upcoming VZ model (see Fig. 4).

C. The model of Villadoro and Zwirner (VZ)

In March 2005, Villadoro and Zwirner (VZ) [40] constructed a class of orientifold compactifications based on toroidal orbifolds, where the dilaton and all the moduli associated with the torus are stabilized through the inclusion of p-form field strength fluxes (as discussed earlier) and other sorts of fluxes, simply referred to as *general fluxes*. Their model is strongly motivated by the work of Derendinger *et al.* [38]. In particular, VZ include what are known as Scherk-Schwarz geometrical fluxes[14] which provides a large class of vacua. With many fluxes in the model, there are a number of Bianchi identities and tadpole constraints that the fluxes must satisfy. This is achieved by including D6-branes and O6-planes. The interested reader is referred to the original paper [40] for details.

As originally studied by Derendinger *et al.* [38] the orbifold is $T^6/\mathbb{Z}_2$. A further $\mathbb{Z}_2$ projection is performed to obtain an O6 orientifold. This particular orientifold permits 6 degrees of freedom in the metric on the compact space. The torus takes the form $T_6 = T_2 \times T_2 \times T_2$ and possesses a diagonal metric. Then, without loss of generality, the 6-dimensional metric can be parametrized by 6 variables γ_i and β_i ($i = 1, 2, 3$) as follows:

$$(ds^2)_6 = \sum_{i=1}^{3}((\gamma_i/\beta_i)(dx^i)^2 + (\gamma_i\beta_i)(dy^i)^2), \tag{45}$$

$$\mathrm{Vol} = \int_{T^6/\mathbb{Z}_2^2} d^6x\sqrt{g_6} = \gamma_1\gamma_2\gamma_2 = b_1b_2b_3. \tag{46}$$

The form of the volume explains the choice in decomposing the metric as above, namely, that the product of the γ_i is proportional to the volume, as it was for DGKT. In turn, we again identify 3 good Kähler coordinates b_1, b_2, b_3. The β_i, on the other hand, are related to the complex structure and axio-dilaton moduli.

Let us now provide the full list of moduli (again ignoring "blow-up modes"). In this case there are seven complex moduli that survive the orientifold projection: 3 Kähler moduli $\psi_i = a_i + ib_i$, $i = 1, 2, 3$, an axio-dilaton $\psi_4 = a_4 + ib_4(b_4 = e^{-\phi}\sqrt{\mathrm{Vol}}/\sqrt{\beta_1\beta_2\beta_3})$, and 3 complex structure moduli $\psi_i = a_i + ib_i$, $i = 5, 6, 7(b_5 = e^{-\phi}\sqrt{\mathrm{Vol}}\sqrt{\beta_2\beta_3/\beta_1}$, etc.).

The Kähler potential takes on an extremely simple form: in the notation of Sec. III A it has all 7 $n_i = 1$. Explicitly, it is[15]

$$K = -\ln(b_1b_2b_3b_4b_5b_6b_7). \tag{47}$$

[14]Geometric flux here refers to a particular kind of topologically nontrivial alteration of the metric on the compact space which yields a contribution to the scalar potential analogous to the contributions from the p-form fluxes.

[15]We follow the convention of Ref. [40] where an overall factor of 2^7 was removed from the argument of the logarithm, since it can be simply reabsorbed into W.

By way of comparison to the DGKT model, it is as though $b_4^4 \to b_4 b_5 b_6 b_7$, in order to accommodate the 3 complex structure moduli that appear here.

The superpotential incorporates geometric flux, in addition to the familiar p-form flux, and satisfies the "tadpole condition" with D-branes. The superpotential, as derived in [38], is given by

$$\begin{aligned} W = {}& f_{111} - f_{112}(\psi_1 + \psi_2 + \psi_3) + f_{222}\psi_1\psi_2\psi_3 \\ & + f_{122}(\psi_1\psi_2 + \psi_1\psi_3 + \psi_2\psi_3) - f'_{111}\psi_4 \\ & + f'_{112}\psi_4(\psi_1 + \psi_2 + \psi_3) - f_{114}(\psi_5 + \psi_6 + \psi_7) \\ & + f_{124}(\psi_1(\psi_6 + \psi_7) + \psi_2(\psi_5 + \psi_7) + \psi_3(\psi_5 + \psi_6)) \\ & - f_{113}(\psi_1\psi_5 + \psi_2\psi_6 + \psi_3\psi_7). \end{aligned} \tag{48}$$

Here we have designated the fluxes by f_{ijk}, which correspond to different choices of p form and geometric flux wrapped on various cycles of the torus. There are constraints that the fluxes f_{ijk} must satisfy, namely

$$f_{124}(f_{124} - f_{113}) = 0, \qquad f'_{112}(f_{124} - f_{113}) = 0. \tag{49}$$

VZ find a family of SUSY vacua by choosing the following parametrization of fluxes:

$$\begin{aligned} & f_{111} = -15p_1, \qquad f_{112} = \frac{3p_2}{q_1}, \qquad f_{122} = \frac{p_1}{q_1^2}, \\ & f_{222} = -\frac{3p_2}{q_1^3}, \qquad f'_{111} = -\frac{2p_2}{q_2}, \qquad f'_{112} = -\frac{2p_1}{q_1 q_2}, \\ & f_{114} = -\frac{2p_2}{q_3}, \qquad f_{113} = f_{124} = -\frac{6p_1}{q_1 q_3}. \end{aligned} \tag{50}$$

We note that the f_{ijk} are actually noninteger. Here we do not record the conditions that p_1, p_2, q_1, q_2, q_3 must satisfy, but refer the reader to [40]. We do note that $\{q_1, q_2, q_3\} \in \mathbb{R}^+$. What is important is that this designates an infinite family of vacua with fluxes parametrized by the set of five parameters p_1, p_2, q_1, q_2, q_3. So we have started with a superpotential with 9 fluxes: $f_{111}, \ldots, f_{124}$, one has been eliminated by the conditions (49) ($f_{113} = f_{124}$), three have been eliminated by demanding that the SUSY condition (20) be satisfied for each ψ_i, leaving five independent parameters.

For this family of vacua it is rather straightforward to show that we can scale out the fluxes q_1, q_2, q_3 by making the following rescaling of our fields:

$$\begin{aligned} & \psi_i \to q_1\psi_i \quad (i = 1, 2, 3), \qquad \psi_4 \to q_2\psi_4, \\ & \psi_i \to q_3\psi_i \quad (i = 5, 6, 7). \end{aligned} \tag{51}$$

This leaves only p_1 and p_2 of which we can scale out one of them, leaving only their ratio as a tunable parameter

$$s \equiv \frac{p_1}{p_2} \tag{52}$$

($p_2 = 0$ can be handled separately).

Now let us focus on the symmetric case, in which $\psi_1 = \psi_2 = \psi_3$ and $\psi_5 = \psi_6 = \psi_7$, and keep track of the fields ψ_1, ψ_4, ψ_5. We find that W is simplified to

$$\begin{aligned} W = {}& -15p_1 - 9p_2\psi_1 + 3p_1\psi_1^2 - 3p_2\psi_1^3 \\ & + 2p_2(\psi_4 + 3\psi_5) - 6p_1\psi_1(\psi_4 + 3\psi_5). \end{aligned} \tag{53}$$

We note that since W only depends on a linear combination of ψ_4 and ψ_5, namely $\psi_4 + 3\psi_5$, the potential V only depends on the same linear combination of the corresponding axions, namely $\hat{a}_4 \equiv a_4 + 3a_5$.

By using (11), it is a straightforward matter to obtain the potential. The result is a rather long expression that we report in Appendix B Eq. (B1). The leading prefactor

$$V_{\text{flux}} \equiv \frac{p_2^2}{q_1^3 q_2 q_3^3} \tag{54}$$

is an overall multiplicative scale that depends on the fluxes. At fixed s there exists a family of solutions for p_1, p_2, q_1, q_2, q_3 for which $V_{\text{flux}} \to 0$ parametrically. However, one should note the explicit appearance of $s = p_1/p_2$ in the potential, which controls its *shape*. We mention that without loss of generality we can focus on s non-negative, since $s \to -s$ and $a_i \to -a_i$ leaves V unchanged.

By solving the equations $D_i W = 0$, one can show that the SUSY vacuum lies at

$$a_1 = a_4 + 3a_5 = 0, \qquad b_1 = b_4 = b_5 = \sqrt{\tfrac{5}{3}}, \tag{55}$$

for all s. We note that this (AdS) SUSY vacuum is tachyonic but stable, as is satisfies the Breitenlohner-Freedman bound [57]. The potential here takes on the value

$$V_{\text{SUSY}} = -V_{\text{flux}} \tfrac{432\sqrt{15}}{125}(1 + 15s^2). \tag{56}$$

Now, an important special case is when $p_1 = p_2$ ($s = 1$), since as VZ describe, this provides this $\mathcal{N} = 1$ supergravity theory with an interpretation in terms of an $\mathcal{N} = 4$ supergravity theory with extended (gauged) symmetry. This may be of some interest [60]. In this case, the potential in Eq. (B1) may be simplified to

$$\begin{aligned} V = {}& V_{\text{flux}}(36\tilde{a}_1^6 + 108b_1^2\tilde{a}_1^4 + 144\tilde{a}_4\tilde{a}_1^4 + 1280\tilde{a}_1^3/3 \\ & + 108b_1^4\tilde{a}_1^2 + 144\tilde{a}_4^2\tilde{a}_1^2 + 144\tilde{a}_4 b_1^2\tilde{a}_1^2 + 144b_4^2\tilde{a}_1^2 \\ & + 432b_5^2\tilde{a}_1^2 + 2560\tilde{a}_4\tilde{a}_1/3 + 36b_1^6 + 48\tilde{a}_4^2 b_1^2 \\ & + 48b_1^2 b_4^2 - 432b_1^2 b_5^2 - 576b_1^2 b_4 b_5 \\ & + 102\,400/81)/(b_1^3 b_4 b_5^3) \end{aligned} \tag{57}$$

with $\tilde{a}_1 \equiv a_1 - 1/3$ and $\tilde{a}_4 = a_4 + 3a_5 + 4/3$. In addition to the SUSY vacua, we find three additional AdS vacua given by

$$a_1 = a_4 + 3a_5 = \tfrac{1}{3}, \qquad b_1 \approx 1.38, \qquad b_4 = b_5 \approx 1.26$$
$$a_1 = a_4 + 3a_5 = \tfrac{1}{3}, \qquad b_1 \approx 1.38, \qquad b_4 \approx 2.87,$$
$$b_5 \approx 0.958 \qquad a_1 = 1, \qquad a_4 + 3a_5 = -4,$$
$$b_1 = b_4 = b_5 = \tfrac{2}{\sqrt{3}}. \tag{58}$$

In Fig. 3 (top) we plot V as a function of a_1 and a_4 with other parameters fixed at their SUSY values. We see that V is relatively flat along each axis, while V is steep along diagonals.

Now, this potential contains one modulus that is not stabilized. One linear combination of the axions is left exactly massless at the SUSY vacuum. We will return to this later in the discussion. This is a result of the fact that the superpotential in Eq. (53) only depends on the combination $a_4 + 3a_5$. A plot of V as a function of a_4 and a_5, with all other moduli fixed at their SUSY values, is given in Fig. 3 (bottom). We see the flat "valley." The existence of such a flat direction certainly seems useful from the point of view of inflation, however one should recall that this flat direction emanates from an AdS vacuum.

For slow roll it is again evident that this is very difficult, due to the argument presented at the end of Sect. III B, namely, that the characteristic value of ϵ in these types of tree-level toroidal models is $\mathcal{O}(1)$. However, it is important to investigate the effect of our tunable parameter s. To get a flavor of its effect, in Fig. 4 (top) we plot $\epsilon = \epsilon(a_1)$ for $2 < s < 20$. We see that $\epsilon > 1$ in this region. Indeed our numerical studies indicate (detailed below) that there is no inflating region anywhere in moduli space. Again this is based on the results of vast tables of ϵ over moduli space. For a more conventional representation, in Fig. 4 (bottom) we plot ϵ as a function of a pair of moduli, namely a_1 and b_1, with $s = 1$. Here ϵ is large in all regions in which $V > 0$. ($\epsilon \to \infty$ as $V \to 0$ and $\epsilon \to 0$ at the AdS minimum). The plot displays a dip in ϵ as $a_1 \to -1$, $b_1 \to 0$. At this point we find $\epsilon \to 4$.

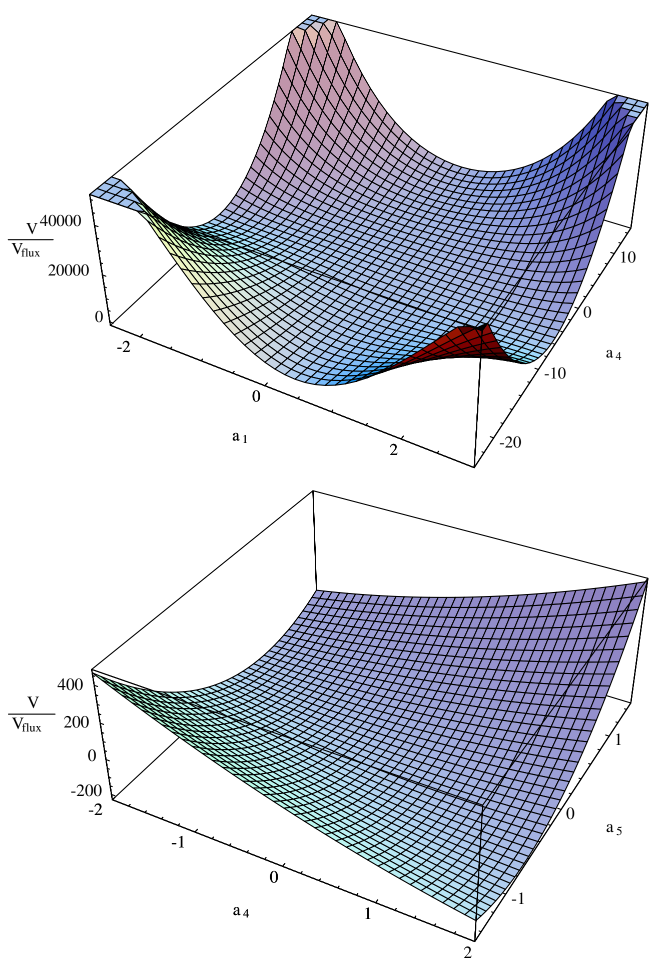

FIG. 3 (color online). Top: The potential $V = V(a_1, a_4)$ with $a_5 = 0$ and other moduli taking on their SUSY values. Bottom: The potential $V = V(a_4, a_5)$ with other moduli taking on their SUSY values.

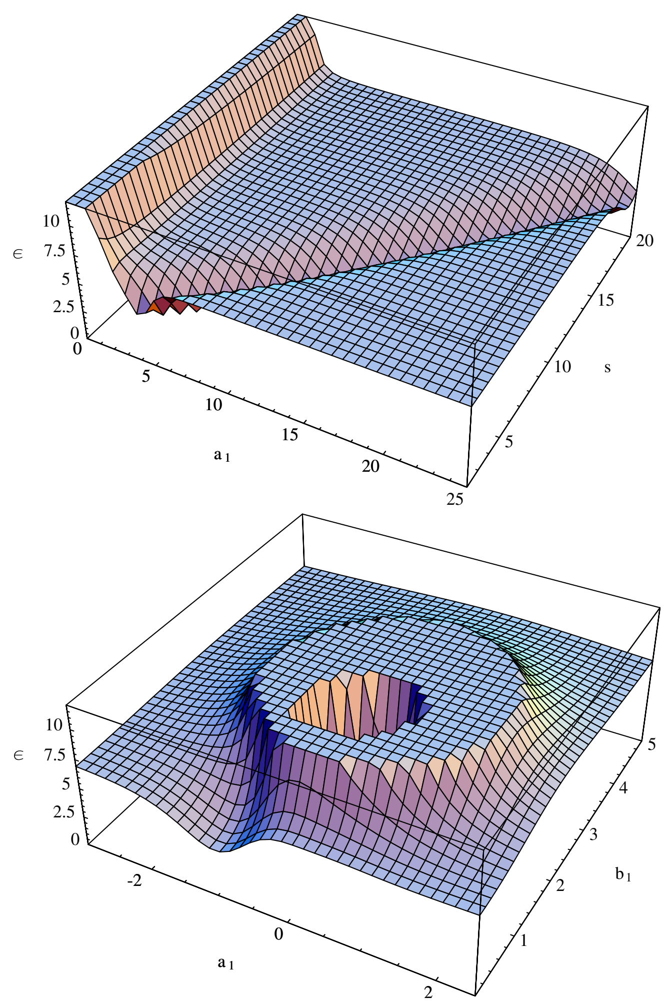

FIG. 4 (color online). Top: The slow-roll parameter $\epsilon(a_1, s)$ with other moduli taking on their values from the third set of Eq. (58). Bottom: The slow-roll parameter $\epsilon(a_1, b_1)$ with $s = 1$ and other moduli taking on their SUSY values.

D. The model of Ihl and Wrase (IW)

In the previous two models, the Kähler potential took on the form of Eq. (23), which we referred to as diagonal torus models. It followed from this that the first slow-roll parameter took on the form as given in Eq. (27). For potentials V that were rational in the moduli, this meant that $\epsilon = \mathcal{O}(1)$ was quite natural. We would like then to investigate more complicated models in which this does not occur. In April 2006 Ihl and Wrase [41] obtained an explicit example of this nature. Their work is strongly motivated by the work of DGKT. Indeed they also consider massive type IIA supergravity. However, unlike the DGKT model, we find that one tunable parameter remains in the potential, as we found in the VZ model.

The orientifold is $T^6/\mathbb{Z}_4$. Unlike the torii of DGKT and VZ, this orientifold *does not* permit the decomposition of T^6 to $T_2 \times T_2 \times T_2$ with identical T_2s. Instead the T_2s must have *different* metrics. The interested reader is referred to the original paper [41] for details.

The metric on the compact space is somewhat more complicated than our previous models. When expressed in terms of the most useful coordinates (those that are readily related to Kähler and complex structure moduli) the metric on the compact space is nondiagonal. Here there are 4 independent degrees of freedom that appear explicitly in the metric. Denoting them as usual by γ_i, the metric is given by

$$(ds^2)_6 = \sum_{i=1}^{3} \gamma_i[(dx^i)^2 + (dy^i)^2] + 2\gamma_4\left(dx^1dx^2 + dy^1dy^2 - \sum_{i,j=1}^{2} \epsilon_{ij}dx^i dy^j\right), \quad (59)$$

$$\begin{aligned}\mathrm{Vol} &= \int_{T^6/\mathbb{Z}_4} d^6x\sqrt{g_6} = U_2\gamma_3(\gamma_1\gamma_2 - 2\gamma_4^2)/4 \\ &= b_3(b_1b_2 - b_4^2/2),\end{aligned} \quad (60)$$

where ϵ_{ij} is the Levi-Civita symbol defined by $\epsilon_{12} = -\epsilon_{21} = 1$, $\epsilon_{11} = \epsilon_{22} = 0$. The form of the volume requires a little explanation: The square root of the determinant of the metric is easily shown to be $\gamma_3(\gamma_1\gamma_2 - 2\gamma_4^2)$, and indeed Vol is proportional to this. There is also a factor of U_2, which is related to the canonical complex structure moduli. It is known as a "pure type" contribution that is in some sense hidden in the metric. The interested reader is referred to footnote 11 of the IW paper for clarification. Again we have written the volume in terms of some b_i that are good Kähler coordinates.[16] Note that the volume is *not* simply a product of the Kähler coordinates.

[16]In the IW paper: $\mathrm{Vol} = \kappa b_3(b_1b_2 - b_4^2/2)$. By rescaling $b_i \to \kappa^{-1/3}b_i$, $i = 1, 2, 3, 4$ we obtain Eq. (60) and κ is eliminated.

In summary, after the orientifolding there remains *one* complex structure modulus U_2. In total we have 4 Kähler moduli $\psi_i = a_i + ib_i$, $i = 1, \ldots, 4$ and 2 other moduli which mix the axio-dilaton and the complex structure modulus: $\psi_5 = a_5 + ib_5(b_5 = e^{-\phi}\sqrt{\mathrm{Vol}}/\sqrt{U_2})$ and $\psi_6 = a_6 + ib_6(b_6 = 2\sqrt{U_2}e^{-\phi}\sqrt{\mathrm{Vol}})$.[17] (We again ignore the "blow-up modes").

Here the Kähler potential *does not* take the form of the previous 2 models, i.e., it is not of the form of a logarithm of a product of geometric moduli and so is not quite of the form discussed in Sec. III A. There is a modification due to the nontrivial form of the volume, namely $\mathrm{Vol} = b_3(b_1b_2 - b_4^2/2)$. The Kähler potential is

$$K = -\ln(2b_3(b_1b_2 - b_4^2/2)b_5^2b_6^2) \quad (61)$$

—see Eq. (23) for comparison.

The ingredients for the superpotential are just the same as the DGKT model. A 0-form, 3-form, 4-form (and an unimportant 2-form) are included. The superpotential is a simple modification of the DGKT model, namely,

$$\begin{aligned}W = \frac{f_6}{\sqrt{2}} + \sum_{i=1}^{4}\frac{f_{4,i}}{\sqrt{2}}\psi_i - \frac{f_0}{\sqrt{2}}\psi_3(\psi_1\psi_2 - \psi_4^2/2) \\ - 2f_3(\psi_5 + \psi_6),\end{aligned} \quad (62)$$

which is to be compared to Eq. (35). We note that there is one additional flux component: $f_{4,4}$, which is due to the presence of a 4th Kähler modulus ψ_4.

Furthermore, just as in the DGKT model, an 06-plane is introduced in order to satisfy the tadpole condition. This occurs in precisely the same way as before (see Eq. (36)), i.e., $f_0f_3 = -2$.

Again let us exploit all existing shift and scaling symmetries. We perform the following transformations on our fields:

$$\begin{aligned}\psi_i &\to \frac{1}{|f_{4,i}|}\sqrt{\frac{|f_{4,1}f_{4,2}f_{4,3}|}{|f_0|}}\psi_i \quad (i = 1, 2, 3), \\ \psi_4 &\to \sqrt{\frac{|f_{4,3}|}{|f_0|}}\psi_4, \\ \psi_5 &\to \frac{1}{|f_3|}\sqrt{\frac{|f_{4,1}f_{4,2}f_{4,3}|}{|f_0|}}\psi_5 + \frac{1}{2\sqrt{2}}\frac{f_6}{f_3}, \\ \psi_6 &\to \frac{1}{|f_3|}\sqrt{\frac{|f_{4,1}f_{4,2}f_{4,3}|}{|f_0|}}\psi_6,\end{aligned} \quad (63)$$

which was chosen in such a way as to leave the kinetic terms invariant. This allows one to rewrite the superpotential as

[17]The canonical axions are $\xi_5 = 2a_5$ and $\xi_6 = 2a_6$.

$$W = \sqrt{\frac{|f_{4,1}f_{4,2}f_{4,3}|}{|f_0|}}\bigg(\sum_{i=1}^{3}\frac{\hat{f}_{4,i}}{\sqrt{2}}\psi_i + \frac{\hat{f}_{4,4}}{\sqrt{2}}t\psi_4 - \frac{\hat{f}_0}{\sqrt{2}}\psi_3(\psi_1\psi_2 - \psi_4^2/2) - 2\hat{f}_3(\psi_5 + \psi_6)\bigg). \quad (64)$$

Here there exists one combination of the fluxes that does *not* scale out:

$$t \equiv \frac{|f_{4,4}|}{\sqrt{|f_{4,1}f_{4,2}|}}. \quad (65)$$

This is a result of the nontrivial ("intersection") form for the volume, which puts ψ_4 on a different footing from the other Kähler moduli.

We turn now to the 4-dimensional potential V. In the presence of all axions, the result is somewhat complicated: see Appendix C Eq. (C1). Here we note that a consistent solution is found with all (shifted) axions vanishing, and so we will focus on this case: $a_1 = \ldots = a_6 = 0$. Also note that ψ_5 and ψ_6 are treated on equal footing. Inspired by this fact, we will concentrate on the case $\psi_5 = \psi_6$. This sets $U_2 = 1/2$, as reported in the IW paper. We also make a final set of multiplicative transformations by ± 1, namely: $b_1 \to \hat{f}_{4,1}b_1$, $b_2 \to \hat{f}_{4,1}b_2$, $b_4 \to \hat{f}_{4,4}b_4$, which *still* preserves the form of the kinetic energy. We find

$$\begin{aligned} V = V_{\rm flux}[&2(b_1^2 + b_2^2 + b_3^2) + 2\delta_{12}b_4^2 + 16b_5^2 \\ &- 16\sqrt{2}b_1b_2b_3b_5 + 8\sqrt{2}b_3b_4^2b_5 + 2b_1^2b_2^2b_3^2 \\ &- 2b_1b_2b_3^2b_4^2 + b_3^2b_4^4/2 + 4(b_1 + \delta_{12}b_2)b_4t \\ &+ (b_4^2 + 2b_1b_2)t^2]/(2b_3(b_1b_2 - b_4^2/2)b_5^4), \end{aligned} \quad (66)$$

where

$$V_{\rm flux} \equiv \frac{|f_0|^{5/2}|f_3|^4}{|f_{4,1}f_{4,2}f_{4,3}|^{3/2}} \quad (67)$$

as we defined it for the DGKT model. Note, however, that this is not the only piece that depends on the magnitude of the fluxes, since the flux parameter t also appears in (66). So there is one combination of the fluxes that describes the *shape* of the potential. We have defined $\delta_{12} \equiv \hat{f}_{4,1}\hat{f}_{4,2} = \pm 1$ which delineates two families of potentials. We should also keep track of the physical constraints that the area of the third torus and the compact volume Vol are both positive, so $b_3 > 0$ and $b_1b_2 - b_4^2/2 > 0$.

All stationary points are AdS (even the non-SUSY ones). The interested reader is referred to the IW paper [41] for a detailed description of the locations of these stationary points. Here we begin by noting that when $\delta_{12} = +1$ and $t = 0$, there is a SUSY AdS minimum, which coincides with that of the DGKT model:

$$b_1 = b_2 = b_3 \approx 1.29, \qquad b_4 = 0, \qquad b_5 \approx 0.609 \quad (68)$$

(compare to Eq. (42) with renaming of variables; b_5 of IW replaced by $b_4/2$ of DGKT).

Since the Kähler potential is not simply the logarithm of a product of moduli, ϵ is *not* given by Eq. (27). Instead we revert to Eq. (18). We emphasize that the particular transformations we have performed on the ψ_i have left the form of $K^{i\bar{j}}V_iV_{\bar{j}}$ unchanged. With this in mind, we find the following:

$$\epsilon = \frac{1}{V^2}\bigg\{\sum_{i,j=1}^{4} M_{ij}\bigg(\frac{\partial V}{\partial a_i}\frac{\partial V}{\partial a_j} + \frac{\partial V}{\partial b_i}\frac{\partial V}{\partial b_j}\bigg) + \sum_{i=5}^{6}\frac{b_i^2}{2}\bigg[\bigg(\frac{\partial V}{\partial a_i}\bigg)^2 + \bigg(\frac{\partial V}{\partial b_i}\bigg)^2\bigg]\bigg\}, \quad (69)$$

where

$$M \equiv \begin{pmatrix} b_1^2 & b_4^2/2 & 0 & b_1b_4 \\ b_4^2/2 & b_2^2 & 0 & b_2b_4 \\ 0 & 0 & b_3^2 & 0 \\ b_1b_4 & b_2b_4 & 0 & b_1b_2 + b_4^2/2 \end{pmatrix}, \quad (70)$$

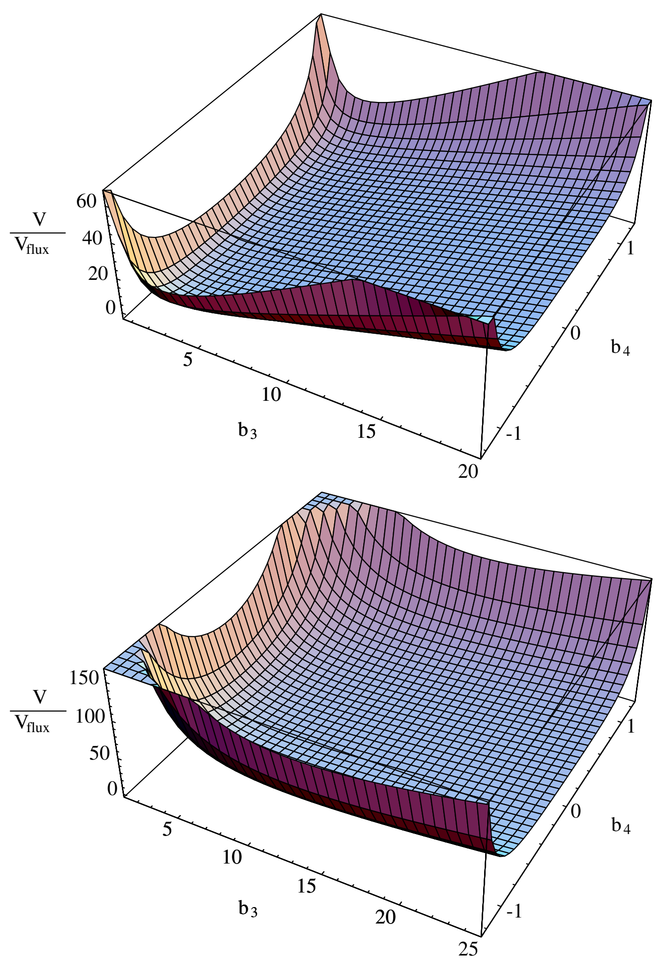

FIG. 5 (color online). The potential $V = V(b_3, b_4)$ with $b_1 = b_2 = b_5 = 1$ and $\delta_{12} = -1$. Top: $t = 1$. Bottom: $t = 10$.

and V given in terms of our rescaled variables, i.e., by Eq. (66) for the simple vanishing axion case, and by Eq. (C1) for the general nonvanishing axion case. In Fig. 5 we give a representative plot of a piece of moduli space. We see significant variation as we change the flux parameter from $t = 1$ in (top) to $t = 10$ in (bottom). In this plot we have ensured that b_4 has remained in the physical region given by $b_1 b_2 - b_4^2/2 > 0$. We note that the potential becomes singular at this boundary; this follows again from the nontrivial form of Vol. Our numerical investigations into the slow-roll parameter ϵ have again yielded $\epsilon > 1$ whenever $V > 0$, although we have not investigated the full moduli space—we did not include all axions in our search.

E. Comments on blow-up modes

In the three models that we have investigated, we have ignored a class of moduli known as "twisted moduli" or "blow-up modes." Recall that apart from the dilaton, the geometric moduli describe the size and shape of the compact space, i.e., its *geometry*. These are the Kähler and complex structure moduli. For a smooth compact space, this is fully general. However, the models investigated here are not smooth; they are all *orbifolds*, which have fixed points. These fixed points correspond to conical singularities. In the large volume limit, these conical singularities are "blown up" and smoothed out. The effective 4-dimensional description then captures this aspect of the geometry by a modulus for each fixed point; the so-called blow-up modes.

These blow-up modes can be included in the analysis in a straightforward fashion through the use of the Kähler potential K and superpotential W. Let us give an explicit example; the DGKT model. Here there are 9 fixed points, and so there are 9 blow-up modes. We call these: $\psi_i = a_i + ib_i$ for $i = 5, \ldots, 13$. The expression for the volume in Eq. (33) is modified to

$$\mathrm{Vol} = b_1 b_2 b_3 - \frac{1}{54}\sum_{i=5}^{13} b_i^3. \tag{71}$$

The Kähler potential (34) and the superpotential (35) are modified to

$$K = -\log\left[32\left(b_1 b_2 b_3 - \frac{1}{54}\sum_{i=5}^{13} b_i^3\right) b_4^4\right], \tag{72}$$

$$W = \frac{f_6}{\sqrt{2}} + \sum_{i=1}^{3}\frac{f_{4,i}}{\sqrt{2}}\psi_i + \sum_{i=5}^{13}\frac{f_{4,i}}{\sqrt{2}}\psi_i - \frac{f_0}{\sqrt{2}}\left(\psi_1\psi_2\psi_3 - \frac{1}{54}\sum_{i=5}^{13}\psi_i^3\right) - 2f_3\psi_4. \tag{73}$$

In principle we could now explore this larger moduli space for inflation. However, it is numerically difficult; we have moved from 4 complex moduli (axio-dilation plus 3 Kähler moduli) to 13 complex moduli through the addition of 9 complex blow-up modes. Instead of a full investigation into the effects of dynamical blow-up modes, we shall freeze the blow-up modes at some vacuum expectation value (vev). Such a vev is explicitly found in the DGKT paper. We then explore the effect of a nonzero vev for the blow-up modes on the original moduli. Suppose the b_i are frozen in at some value, which we characterize as: $B \equiv \frac{1}{54}\sum_{i=5}^{13} b_i^3$. With B taken as a constant, our Kähler potential becomes

$$K = -\log[32(b_1 b_2 b_3 - B) b_4^4]. \tag{74}$$

Also, since we are treating the blow-up modes as constants, the superpotential is, for all intents and purposes, unchanged from its value in Eq. (35). This is because we can always shift the real part (axion) of ψ_4 to eliminate any constants. We perform the same scalings as before in Eqs. (37), with an extra shift on ψ_4 to eliminate any constants, giving Eq. (38). Under such field redefinitions we introduce $\bar{B}$, defined such that: $(b_1 b_2 b_3 - B) \to (b_1 b_2 b_3 - \bar{B})$.

Let us focus on the case in which the axions are vanishing and $b_1 = b_2 = b_3$, leaving 2 moduli: b_1 and b_4. We find the potential

$$\begin{aligned} V = V_{\mathrm{flux}}[&(6b_1^2 + 4b_4^2 + 2b_1^6 - 8\sqrt{2}b_1^3 b_4) \\ &+ \bar{B}(12\sqrt{2}b_4 - 6b_1^3 + (4\delta_a - 6)/b_1 + 4\sqrt{2}\delta_b b_4/b_1^2) \\ &+ \bar{B}^2(6 + 4\delta_b/b_1^2 + 6/b_1^4)]/[32(b_1^3 - \bar{B})b_4^4], \end{aligned} \tag{75}$$

where $\delta_a \equiv \hat{f}_1\hat{f}_2 + \hat{f}_2\hat{f}_3 + \hat{f}_3\hat{f}_1$, $\delta_b \equiv \hat{f}_0(\hat{f}_1 + \hat{f}_2 + \hat{f}_3)$, and V_{flux} is given in Eq. (40). There are physical constraints: $0 < \bar{B} < b_1^3$. Note that the presence of the $\bar{B}$ parameter breaks the scaling of the model that occurs in the absence of blow-up modes, i.e., scaling only occurs in the $\bar{B} \to 0$ limit. This is because the nonvanishing blow-up modes introduce a nontrivial intersection form, as we encountered previously in the IW model, which prevents the flux parameters from being scaled out completely. However, several flux parameters can still be eliminated for finite $\bar{B}$. Note that in the limit $\bar{B} \to 0$ this potential gives precisely the potential in Eq. (39) with $a_1 = a_4 = 0$.

Our numerical investigation into V of Eq. (75) has again yielded no inflating region, despite the presence of the tunable parameter $\bar{B}$.

IV. DISCUSSION AND CONCLUSIONS

We have presented an explicit investigation into three explicit string models. Although this represents only a rather small part of the landscape, this acts as a starting point for further investigation into moduli driven inflation.

The nonstring theorist should note that despite the inherent complexity of string theory, M theory, etc., it is possible to strip down the physics in 4 dimensions to familiar territory. Equation (15) gives a familiar 4-dimensional action for coupled scalar fields minimally coupled to gravity. We note, however, that the kinetic energy is in general noncanonical since $K_{i\bar{j}}$ is typically not equal to $\delta_{i\bar{j}}$ and furthermore the geometric moduli and axions appear in the action differently.[18] We have proceeded in the usual fashion to check for inflation by examining the slow-roll conditions (18) and (19).

We have not found inflation in any of the specific models presented. In the absence of blow-up modes, the DGKT, VZ, and IW models involved 8, 14, and 12 real-valued inflaton fields, respectively, making a full numerical exploration of the inflaton potential $V(\phi)$ (which can also be flux dependent) computationally challenging. We have therefore performed as comprehensive a search as feasible given our available resources:

(i) For the DGKT model we derived an analytic expression for the 8-dimensional potential $V(\phi)$, finding $V(\phi)$ to be flux independent (up to an overall scale), and searched the 8-dimensional moduli space for vacua, finding one AdS vacuum in addition to the known SUSY vacuum from [39].

(ii) For the VZ model, we derived an analytic expression for the 14-dimensional potential $V(\phi)$, finding that $V(\phi)$ depended on the fluxes via a single parameter s (up to an overall scale). We found the potential to be invariant under permutation of two triplets of complex moduli, and searched the full 6-dimensional subspace corresponding to $\psi_1 = \psi_2 = \psi_3$, $\psi_5 = \psi_6 = \psi_7$ for vacua for the cases $s \in \{0, 1/2, 1, 2, 5, \infty\}$, finding three new AdS vacua in addition to the known SUSY vacuum from [40].

(iii) For the IW model, we derived an analytic expression for the 12-dimensional potential $V(\phi)$, finding that $V(\phi)$ depended on the fluxes via a single parameter t (up to an overall scale). We searched the 6-dimensional subspace corresponding to vanishing axions, finding no new vacua in addition to the five AdS vacua reported by [41] for various flux sign combinations.

We performed this search for vacua using the numerical packages Mathematica[19] and Singular[20] to algebraically (using Gröbner bases) solve the set of high order coupled polynomial equations that follow from setting $\nabla V = 0$.

We then performed a numerical investigation of the slow-roll conditions, evaluating slow-roll parameters for the three models on a multidimensional grid (of dimension 8, 6, and 6,[21] respectively), involving of order 10^9 grid points each, and found $\epsilon > 1$ for all grid points where $V > 0$. Although we cannot claim to have a *proof* of the nonexistence of inflation in these models, as the moduli space is rather large and the potentials V are rather complicated, we do suspect this to be true. We also performed a partial investigation into the consequences of (frozen) blow-up modes, as described in Sec. III E, again finding no inflation.

In the type of models presented we have identified at least three obstacles to realizing inflation: the vacua are AdS, there is a logarithmic Kähler potential K, and suitable redefinitions allow one to scale many of the fluxes out of the potential. None of these features forbid a realization of inflation. Each is probably a reflection of the simple starting point we took, studying models closely based on toroidal compactification and focusing on the moduli of the underlying torus. It is certainly known that each of these three points may be avoided in other regions of the landscape. Nevertheless, our result does underscore that slow-roll inflation may be a rare and delicate phenomenon in the landscape. We will now discuss each of the three obstacles in turn.

A. The potential energy challenge

As we discussed earlier in Sec. II D it is somewhat difficult to realize de Sitter vacua with $V > 0$ in string theory. If we break supersymmetry, then existing analyses suggest that such vacua are rare, but plentiful in absolute number. Let us truncate our discussion here to supersymmetric vacua, which we know must not be de Sitter. A good starting point would be Minkowski vacua which are allowed. Again focusing on toroidal orientifolds in type IIA string theory, a detailed investigation is given in [61], in which a host of fluxes are included. In addition to the geometric fluxes that we have described, they turn on so-called nongeometric fluxes [62], and additionally turn on fluxes associated with S-duality (strong-weak coupling duality). In this framework, although they are nongeneric, Minkowski vacua are explicitly found (see also [63]). This may be a good starting point for considering inflation models where the inflaton eventually settles down to zero energy density. However, the Minkowski vacua given in [61] are not under good perturbative control. In other words, it is expected that there are large α' and g_s (loop) corrections to the potential. This is in contrast to the models we have investigated in this article. In each case we could dial the fluxes in a particular fashion so that all quantum corrections were small. This justifies the supergravity treatment and makes the results of our investigation particularly informative.

[18] If we ignore the axions and focus on a certain class of simple models, we can perform field redefinitions to obtain a canonical action, as given in Eq. (30).
[19] http://www.wolfram.com
[20] http://www.singular.unikl.de
[21] In fact we did a little more than this: In the IW model we did not *fully* include the axions, which would be a 12-dimensional space, but did so *partially*.

B. The kinetic energy challenge

Let us turn to the form of the kinetic energy, which is governed by the Kähler potential K. As we have pointed out several times, in supergravity models this is typically logarithmic. The Hessian matrix of K determines the form of the kinetic terms. The tree-level form of this for the diagonal torus model is given in Eq. (24). At the level of supergravity (i.e., ignoring quantum corrections) this form is rather generic for nontorus models also [64]. So for instance, this kind of metric on moduli space will generically occur for the volume modulus b. Let us write this form as

$$T \sim -\partial_\mu(\log b)\partial^\mu(\log b) \qquad (76)$$

(suppressing factors of $\bar{m}_{\rm P}$). Although we shall not go through the explicit details here, this is fairly simple to show from the fact that in performing the dimensional reduction from 10 to 4 dimensions, we pick up factors of the volume modulus. In order to move to the Einstein frame, we must then compute the transformation of the Ricci scalar, which is a contraction of the Riemann tensor. Since the mixed derivative terms of the Riemann tensor are proportional to the Christoffel symbols, and since the Christoffel symbols essentially perform *logarithmic* derivatives of the metric, the result in (76) follows.

For models involving fluxes, etc., it is rather generic that the potential V be given (at large volume) by some rational function of b (or more generally of the full set of Kähler moduli). We have given several explicit examples of this in this paper. Let us drastically simplify the form by writing

$$V \sim \sum_i c_i b^{-k_i}, \qquad (77)$$

where the k_i are positive integers and the c_i are some coefficients.[22] We note that by defining $\phi \equiv \log b$, the kinetic energy is in canonical form and $V \sim \sum_i c_i e^{-k_i\phi}$. (This property was alluded to earlier in Eq. (30)). Of course exponentials have slow-roll parameters $\epsilon \sim \eta \sim k^2$, which are not generically small for positive integers k. A recent discussion of how one can perhaps construct working models by fine-tuning similar potentials with several terms appears in [66]. We point out that in [66], only the volume modulus is considered and other moduli are treated as fixed; we have seen explicit examples in our models where although one partial derivative of the potential may be small, another one will often be large, hence making ϵ large and spoiling inflation.

[22]Locality in the extra dimensions allows one to prove that any contribution to the Einstein-frame potential should fall at least as quickly as $1/r^6$ at large radius r for the extra dimensions (as described on e.g. page 12 of [65]). This puts a bound on the $|k_i|$, and explains why we have disallowed contributions which grow at large radius.

1. α' corrections

Let us comment now on the effect that α' corrections have. According to [67], in type IIB string theory there exists an α' correction to the Kähler potential from an $\mathcal{O}(R^4)$ term in the 10-dimensional action. In units where $2\pi\alpha' = 1$, the piece coming from the volume modulus is found to be

$$K = -2\ln[(2b)^{3/2} + \hat{\xi}], \qquad (78)$$

where $\hat{\xi} = -\zeta(3)\chi e^{-3\phi/2}/4$ with χ the Euler characteristic of the Calabi-Yau. The dilaton ϕ, and hence $\hat{\xi}$, is assumed to be fixed. Although we are considering IIA orientifolds (which are not directly related to IIB models, because of the flux), let us imagine the effects of a similar correction in our context. Our models have χ which is of $\mathcal{O}(1-100)$—although the torus has vanishing χ, the fixed points of the orbifold group action introduce blow-up modes that generate nontrivial χ. However, in the regime where our classical analysis is trustworthy (and by choosing sufficiently large fluxes we can make it arbitrarily reliable [39]), we can neglect this effect. One could imagine that for more general Calabi-Yau's χ, and hence the $\hat{\xi}$ correction, is sometimes large.

It is known that such a term can indeed be important for inflation, see e.g. [68]. For $b^{3/2} \gg \hat{\xi}$, this correction is irrelevant and we return to the previous analysis. For $b^{3/2} \ll \hat{\xi}$, however, this changes the situation considerably. In this case, one finds that the kinetic term for b is modified to, roughly

$$T \sim -\frac{1}{\hat{\xi}\sqrt{b}}(\partial_\mu b)^2. \qquad (79)$$

So the inverse of the metric on moduli space is $K^{i\bar{j}} \sim \hat{\xi}\sqrt{b}$. The inclusion of α' corrections into the Kähler potential also induces corrections to the potential V through the supergravity formula. However, let us again assume the form for V as given in Eq. (77) for the purposes of illustration. We find that generically $\epsilon \sim \eta \sim k^2\hat{\xi}/b^{3/2}$, which is large in the assumed regime $b^{3/2} \ll \hat{\xi}$. Reference [68] shows that even in the setups containing nonperturbative corrections, e.g., race track, etc., this α' correction usually makes achieving inflation harder or impossible. Of course this entire discussion should be viewed with caution: in the regime where the $\hat{\xi}$ correction has a significant effect, one would have to carefully justify any analysis which neglects additional α' corrections.

2. Approximate Kähler potentials

One further comment on the form of the kinetic energy comes from ''inflation in supergravity'' treatments, e.g., [69]. There it is often assumed that the Kähler potential takes on the minimal form: $K = \phi^*\phi$ (giving $K_{i\bar{j}} = \delta_{i\bar{j}}$). This form does not literally occur in any string compactifications that we are aware of. It can appear as an approximate Kähler potential in models where one fixes the moduli of the compactification manifold and expands the Kähler potential for brane position moduli (or, sometimes, axions) in a Taylor expansion. So we believe that realizing inflation in these scenarios should be taken with a grain of salt, subject to justifying the appearance of the relevant K, for the relevant range of field space, in a model with fixed moduli.

C. The challenge of fluxes scaling out

Turning to the issue of scaling out fluxes, although this occurs in the DGKT model if one neglects blow-up modes and focuses on untwisted moduli, it did not occur in full in the other VZ and IW models. In the DGKT model, neglecting the blow-up modes, every member of the infinite set of vacua was identical from the point of view of the slow-roll conditions, and there was large, but not complete, degeneracy in the other models. Degeneracy is reduced in the presence of blow-up modes. In general, though, the ability to exploit scaling and shift symmetries reduces the freedom allowed in dialing the shape of a potential in the landscape. Much like the relation whereby unbroken supersymmetry generically implies AdS vacua, or the simple geometric arguments which determine the logarithmic form of the Kähler potential, this all points to the idea that the landscape, although extremely large, has *structure*. On the other hand, the relatively simple form of flux potentials for toroidal moduli, which is behind the existence of some of these scaling symmetries, would not persist in generic Calabi-Yau models. Therefore, it is reasonable to postulate that the degeneracy we found may be an artifact of the particular simple models we have examined.

D. Outlook

Alternatively, it could be that the type of construction discussed in the introduction, where inflation is realized through a combination of ingredients, including nonperturbative corrections to the superpotential, is more promising. For example, let us make a comment on the N-flation idea [30,31], which requires N massless axions at the perturbative level, whose mass is then generated by nonperturbative effects. We have seen one flat direction of the axions in the untwisted modes of the VZ model, and the papers [30,31,39] discuss how one can have $N \gg 1$ for more complicated compact spaces. However, various model-building assumptions made in [30,31] can certainly be questioned, and an explicit realization of this class of scenarios is important to unravel.

In summary, our work should be viewed as a starting point for a much more general study into inflation driven by computable flux potentials. One obvious next step would be to study a similar class of problems in more general Calabi-Yau manifolds, rather than orbifolds of the torus. The more complicated structure of the internal geometry should translate into richer flux potentials, which could solve some of the problems we found in the toroidal models. Another approach could be to develop statistical arguments along the lines of [70] to quantify how generically or nongenerically one expects to find inflation in flux vacua.

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APPENDIX A: DGKT POTENTIAL

In the DGKT setup, there are 4 complex pairs of moduli: $a_1, b_1, \ldots, a_4, b_4$. Using the Kähler potential of Eq. (34) and the superpotential of Eq. (38) we find the following 4-dimensional potential:

$$\begin{aligned} V = V_{\rm flux}[&2(a_1 + a_2 + a_3 + 2\sqrt{2}a_4)^2 \\ &- 4\delta a_1 a_2 a_3 (a_1 + a_2 + a_3 + 2\sqrt{2}a_4) + 2a_1^2 a_2^2 a_3^2 \\ &+ 2(b_1^2 + b_2^2 + b_3^2) + 4b_4^2 - 4\delta(a_2 a_3 b_1^2 + a_1 a_3 b_2^2 \\ &+ a_1 a_2 b_3^2) + 2(a_2^2 a_3^2 b_1^2 + a_1^2 a_3^2 b_2^2 + a_1^2 a_2^2 b_3^2) \\ &+ 2(a_3^2 b_1^2 b_2^2 + a_2^2 b_1^2 b_3^2 + a_1^2 b_2^2 b_3^2) + 2b_1^2 b_2^2 b_3^2 \\ &- 8\sqrt{2} b_1 b_2 b_3 b_4]/(32 b_1 b_2 b_3 b_4^4), \end{aligned} \tag{A1}$$

with $V_{\rm flux}$ given in (40) and $\delta = \hat{f}_0 \hat{f}_{4,1} \hat{f}_{4,2} \hat{f}_{4,3} = \pm 1$. The slow-roll parameter ϵ is given by (27) with $n_1 = n_2 = n_3 = 1$, $n_4 = 4$.

APPENDIX B: VZ POTENTIAL

In the VZ setup, there are 7 complex pairs of moduli: $a_1, b_1, \ldots, a_7, b_7$. Let us focus on the symmetric case $a_1 = a_2 = a_3$, $b_1 = b_2 = b_3$, $a_5 = a_6 = a_7$, $b_5 = b_6 = b_7$. Using the Kähler potential of Eq. (47) and the superpotential of Eq. (53) in the symmetric case, we find the following 4-dimensional potential:

$$V = V_{\rm flux}[36a_1^6 - 72sa_1^5 + 36s^2a_1^4 + 108b_1^2a_1^4 + 144s\hat{a}_4a_1^4 + 216a_1^4 - 144sb_1^2a_1^3 + 144sa_1^3 - 144s^2\hat{a}_4a_1^3 - 48\hat{a}_4a_1^3 + 108b_1^4a_1^2 - 360s^2a_1^2 + 144s^2\hat{a}_4^2a_1^2 + 48s^2b_1^2a_1^2 + 144s\hat{a}_4b_1^2a_1^2 + 216b_1^2a_1^2 + 144s^2b_4^2a_1^2 + 432s^2b_5^2a_1^2 + 480s\hat{a}_4a_1^2 + 324a_1^2 - 72sb_1^4a_1 - 96s\hat{a}_4^2a_1 - 144sb_1^2a_1 - 96s^2\hat{a}_4b_1^2a_1 - 96sb_4^2a_1 - 288sb_5^2a_1 + 1080sa_1 + 720s^2\hat{a}_4a_1 - 144\hat{a}_4a_1 + 36b_1^6 + 12s^2b_1^4 + 900s^2 + 16\hat{a}_4^2 + 48s^2\hat{a}_4^2b_1^2 + 144s\hat{a}_4b_1^2 + 108b_1^2 + 48s^2b_1^2b_4^2 + 16b_4^2 - 432s^2b_1^2b_5^2 + 48b_5^2 - 240s\hat{a}_4 + 48s^2b_1^3b_4 - 48b_1^3b_4 + 144s^2b_1^3b_5 - 144b_1^3b_5 - 576s^2b_1^2b_4b_5]/(b_1^3b_4b_5^3), \tag{B1}$$

with $V_{\rm flux}$ given in (54), $\hat{a}_4 = a_4 + 3a_5$, and $s = p_1/p_2$. The slow-roll parameter ϵ is given by (27) with $n_1 = \ldots = n_7 = 1$.

APPENDIX C: IW POTENTIAL

In the IW setup, there are 6 complex pairs of moduli: $a_1, b_1, \ldots, a_6, b_6$. Using the Kähler potential of Eq. (61) and the superpotential of Eq. (64), we find the following 4-dimensional potential:

$$V = V_{\rm flux}[a_3^2a_4^4/2 + b_3^2a_4^4/2 - 2a_1a_2a_3^2a_4^2 - 2a_1a_2b_3^2a_4^2 + 2b_1b_2b_3^2a_4^2 + a_3^2b_4^2a_4^2 + b_3^2b_4^2a_4^2 + 2\delta a_2a_3a_4^2 + 2a_3^2b_1b_2a_4^2 + 2\delta_{30}a_3^2a_4^2 + 2\delta_{30}b_3^2a_4^2 + 2\delta_{30}a_1a_3a_4^2 + 4\sqrt{2}\delta_{30}a_3a_5a_4^2 + 4\sqrt{2}\delta_{30}a_3a_6a_4^2 - 4a_2b_1b_3^2b_4a_4 - 4a_1b_2b_3^2b_4a_4 - 4a_2a_3^2b_1b_4a_4 - 4a_1a_3^2b_2b_4a_4 + 4\delta a_3b_2b_4a_4 + 4\delta_{30}a_3b_1b_4a_4 + a_3^2b_4^4/2 + b_3^2b_4^4/2 + 2a_1^2 + 2a_2^2 + 2a_1^2a_2^2a_3^2 + 2a_3^2 + 16a_5^2 + 16a_6^2 + 2a_2^2a_3^2b_1^2 + 2b_1^2 + 2a_1^2a_3^2b_2^2 + 2a_3^2b_1^2b_2^2 - 4\delta a_1a_3b_2^2 + 2b_2^2 + 2a_1^2a_2^2b_3^2 + 2a_2^2b_1^2b_3^2 + 2a_1^2b_2^2b_3^2 + 2b_1^2b_2^2b_3^2 + 2b_3^2 + 2a_1a_2a_3^2b_4^2 + 2a_1a_2b_3^2b_4^2 - 2b_1b_2b_3^2b_4^2 - 2\delta a_2a_3b_4^2 - 2a_3^2b_1b_2b_4^2 + 8b_5^2 + 8b_6^2 - 4\delta a_1a_2^2a_3 + 4a_1a_3 + 8\sqrt{2}a_1a_5 + 8\sqrt{2}a_3a_5 + 8\sqrt{2}a_1a_6 + 8\sqrt{2}a_3a_6 + 32a_5a_6 + 4\sqrt{2}b_3b_4^2b_5 - 8\sqrt{2}b_1b_2b_3b_5 + 4\sqrt{2}b_3b_4^2b_6 - 8\sqrt{2}b_1b_2b_3b_6 + 2\delta_{12}b_4^2 + 4\delta_{12}a_1a_2 + 4\delta_{12}a_2a_3 + 8\sqrt{2}a_2a_5\delta_{12} + 8\sqrt{2}\delta_{12}a_2a_6 - 4\delta_{30}a_1a_2a_3^2 - 4\delta_{30}a_2a_3b_1^2 - 4\delta_{30}a_1a_2b_3^2 - 2\delta_{30}a_1a_3b_4^2 - 4\delta_{30}a_1^2a_2a_3 - 8\sqrt{2}\delta_{30}a_1a_2a_3a_5 - 8\sqrt{2}\delta_{30}a_1a_2a_3a_6 + (2\delta_{30}a_3a_4^3 + 4a_1a_4 + 4a_3a_4 + 8\sqrt{2}a_5a_4 + 8\sqrt{2}a_6a_4 + 4\delta_{12}a_2a_4 + 2\delta_{30}a_3b_4^2a_4 - 4\delta_{30}a_1a_2a_3a_4 + 4\delta_{30}a_3b_1b_2a_4 + 4b_1b_4 + 4\delta_{12}b_2b_4 - 4\delta_{30}a_2a_3b_1b_4 - 4\delta_{30}a_1a_3b_2b_4)t + (2a_4^2 + b_4^2 + 2b_1b_2)t^2]/[2(b_1b_2 - b_4^2/2)b_3b_5^2b_6^2], \tag{C1}$$

with $V_{\rm flux}$ given in (67), t given in (65), $\delta = \hat{f}_0\hat{f}_{4,1}\hat{f}_{4,2}\hat{f}_{4,3} = \pm 1$, $\delta_{12} = \hat{f}_{4,1}\hat{f}_{4,2} = \pm 1$, $\delta_{30} = \hat{f}_{4,3}\hat{f}_0 = \pm 1$. The slow-roll parameter ϵ is given by (69).

APPENDIX D: COSMOLOGICAL PARAMETERS FROM SLOW-ROLL INFLATION

The mathematical prescription in this section allows one to compute cosmological parameters corresponding to an arbitrary string potential without understanding the derivation or interpretation of the results.

Suppose from some string model we are given a potential energy function $\bar{V}$ of some complex scalar fields ψ^i in the Einstein frame (see Eq. (15)) and a Kähler potential K. For example, $\bar{V}$ may be given by the supergravity formula in Eq. (11) complemented by the rescaling Eq. (14). We then compute the following slow-roll parameters:

$$\epsilon = \frac{K^{i\bar{j}}\bar{V}_{,i}\bar{V}_{,\bar{j}}}{\bar{V}^2}\left(= \frac{g^{ab}\bar{V}_{,a}\bar{V}_{,b}}{2\bar{V}^2}\right), \tag{D1}$$

$$\eta = \text{min eigenvalue}\left\{\frac{g^{ab}(\bar{V}_{,bc} - \Gamma^d_{bc}\bar{V}_{,d})}{\bar{V}}\right\}, \tag{D2}$$

where η (and ϵ) is written in terms of the metric g_{ab} governing real scalar fields ϕ^a: $K_{i\bar{j}}\partial_\mu\psi^i\partial^\mu\psi^{\bar{j}} = \frac{1}{2}g_{ab}\partial_\mu\phi^a\partial^\mu\phi^b$, with $\phi^{2i-1} = \mathrm{Re}[\psi^i]$ and $\phi^{2i} = \mathrm{Im}[\psi^i]$.

The universe inflates until a time t_e when the slow-roll conditions ($\epsilon < 1$, $|\eta| < 1$) are no longer satisfied. The number of e-foldings from time t to t_e is defined by

$$N = \int_t^{t_e} dt\, H. \tag{D3}$$

All the cosmological parameters defined below are a function of N. A good value to use is 55 (see [71]), with a reasonable range being $50 < N < 60$.

According to inflation, several cosmological parameters flux can be computed as follows:

$$Q_s \equiv \sqrt{\frac{\bar{V}}{150\pi^2\bar{m}_{\rm P}^4\epsilon}}, \tag{D4}$$

$$n_s \equiv 1 - \partial_N \ln Q_s^2 = 1 - 6\epsilon + 2\eta, \tag{D5}$$

$$\alpha_s \equiv -\partial_N^2 \ln Q_s^2, \tag{D6}$$

$$Q_t \equiv \sqrt{\frac{\bar{V}}{75\pi^2 \bar{m}_{\rm P}^4}}, \qquad r \equiv \left(\frac{Q_t}{Q_s}\right)^2 = 16\epsilon, \tag{D7}$$

$$n_t \equiv -\partial_N Q_t^2 = -2\epsilon, \tag{D8}$$

which corresponds to the amplitude, spectral index, and running of spectral index of scalar fluctuations, and the amplitude and spectral index of tensor fluctuations, respectively. The expressions giving Q_t and n_t have general validity. In contrast, the expressions for Q_s, n_s, and α_s are good approximations for the most studied cases of multifield inflation in the literature, where the walls of the multidimensional gorge in which the inflaton slowly rolls are much steeper than the roll direction, but do not hold more generally. The expression for Q_s always provides a lower limit on the correct value.

The predictions for these cosmological parameters can be directly compared with observation. The most recent constraints from combining Wilkinson Microwave Anisotropy Probe (WMAP) microwave background data with Sloan Digital Sky Survey (SDSS) galaxy clustering data are [2]

$$Q_s = 1.945^{+0.051}_{-0.053} \times 10^{-5}, \tag{D9}$$

$$n_s = 0.953^{+0.016}_{-0.016}, \tag{D10}$$

$$\alpha_s = -0.040^{+0.027}_{-0.027}, \tag{D11}$$

$$r < 0.30(95\%), \tag{D12}$$

$$n_t + 1 = 0.9861^{+0.0096}_{-0.0142}. \tag{D13}$$

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