Width of the 511 keV line from the bulge of the galaxy

Ariel Zhitnitsky

Department of Physics and Astronomy, University of British Columbia, Vancouver, BC, V6T 1Z1, Canada

(Received 31 July 2006; revised manuscript received 1 June 2007; published 12 November 2007)

In this paper I present the detail estimations for the width of the 511 keV line produced by a mechanism when dark matter is represented by macroscopically large dense nuggets. I argue that the width of 511 keV emission in this mechanism is very narrow (in a few keV range) in agreement with all observations. The dominant mechanism of the annihilation in this case is the positronium formation $e^+e^- \to {}^1S_0 \to 2\gamma$ rather than a direct $e^+e^- \to 2\gamma$ annihilation. I also discuss some generic features of the γ rays spectrum (in few MeV range) resulting from this mechanism.

DOI: 10.1103/PhysRevD.76.103518 PACS numbers: 98.80.Cq, 12.38.−t, 95.30.Cq, 95.35.+d

I. INTRODUCTION

Recent observations of the galactic center have presented a number of puzzles for our current understanding of galactic structure and astrophysical processes. In particular a series of independent observations have detected an excess flux of photons across a broad range of energies. Specifically, SPI/INTEGRAL observations of the galactic center have detected an excess of 511 keV gamma rays resulting from low momentum electron-positron annihilations. The observed intensity is a mystery. After accounting for known positron sources, only a small fraction of the emission may be explained [1–6]. Motivated by this observation, it has been suggested recently [7] that the observed flux can be explained by the idea that dark matter (DM) particles are strongly interacting composite macroscopically large objects which are made of well-known light quarks or even antiquarks [8,9], similar to Witten's strangelets [10].

The width of the 511 keV line has been measured by SPI/INTEGRAL on the level of few keV [1,2], but has not been calculated in the original paper [7]. The goal of the present work is to fill this gap. More precisely, in this paper I will estimate the probability for the positronium formation and argue that the positronium formation (rather than direct annihilation) plays a dominant role in e^+e^- annihilation when an electron from visible matter hits the antimatter nugget. In this case the estimated width of the 511 keV line is determined by the velocity distribution of the positroniums which move with typical velocities $v \sim \alpha$; see Sec. III. Consequently, this motion determines the width of the 511 keV line to be $\Gamma \sim m_e\alpha \sim$ few keV in agreement with measurements. The direct annihilation $e^+e^- \to 2\gamma$ which lead to the continuum spectrum with typical photons in MeV range is a subleading process as we argue below. This direct annihilation $e^+e^- \to 2\gamma$ might be interesting on its own as it may explain a well-known mystery on the excess of gamma-ray photons detected by COMPTEL in ~1–20 MeV energy range. However, the corresponding analysis is the subject of a different paper [11] and shall not be discussed here.

II. COMPACT COMPOSITE OBJECTS (CCOS)

Unlike conventional dark matter candidates, dark matter/antimatter nuggets are strongly interacting, macroscopically large objects [12]. Such a "counterintuitive" proposal does not contradict any of the many known observational constraints on dark matter or antimatter in our universe due to three main reasons: (1) the nuggets carry a huge (anti)baryon charge $|B| \approx 10^{20}$–10^{33}, so they have a macroscopic size and a tiny number density. (2) They have nuclear densities in the bulk, so their interaction cross section per unit mass is small $\sigma/M \approx 10^{-13}$–$10^{-9}\ \mathrm{cm}^2/\mathrm{g}$. This small factor effectively replaces a condition on weakness of interaction of conventional dark matter candidates such as WIMPs. (3) They have a large binding energy (gap $\Delta \approx 100$ MeV) such that baryons in the nuggets are not available to participate in big bang nucleosynthesis (BBN) at $T \approx 1$ MeV. Therefore, compact composite objects (CCOs) do not contribute to Ω_B, but rather, they do contribute to the "nonbaryonic" cold dark matter Ω_{DM} of the universe. On large scales, the CCOs are sufficiently dilute that they behave as standard collisionless cold dark matter. As we mentioned above, CCOs can be made from matter as well as from antimatter. Precisely these nuggets made of antimatter represent an unlimited source of positrons which can annihilate with visible electrons and produce observed photons.

For our purposes we adopt a simple model for a compact composite object when all quarks form one of the color superconducting (CS) phases with densities few times the nuclear density [13], while the electrons in CCOs can be treated as noninteracting Fermi gas with density $n_e \simeq (\mu^2 - m_e^2)^{3/2}/3\pi^2$, with μ being the electron chemical potential. A numerical estimation of μ strongly depends on the specific details of CS phase under consideration, and varies from few MeV to tens (or even hundred) MeV [13–16]. It is also assumed that the nuggets have very thin electrosphere with a "transition region" of a microscopical scale separating the bulk of the dense matter (with large μ) from vacuum (with $\mu = 0$). The existence of this "transition region" is a very generic feature of the system and is the direct consequence of Maxwell's equations and the chemical equilibrium requirement [16].

Our goal here is to argue that the photon spectrum resulting from a CCO-based mechanism of e^+e^- annihilation has the following main features: The dominant fraction of incoming electrons will form positroniums. As is known, once the positroniums are formed, one-quarter of them (in $^1S_0$ state) will eventually decay to two 511 keV photons, while three-quarters of them (in $^3S_1$ states) will produce a continuum with the typical energies in 100 keV range. A small fraction of electrons will experience the direct annihilation $e^+e^- \to 2\gamma$. The typical photons produced in direct annihilation will have energies of order $\mu \sim$ few MeV. Such photons from the direct annihilation must always accompany the 511 keV line as they are produced by the same mechanism within our framework. We shall not discuss the spectrum and intensity of the ~1–20 MeV gamma rays in the present paper referring to [11]. However, we would like to remark here that the excess of photons measured by COMPTEL precisely in this band, ~1–20 MeV [17], can be naturally explained by this mechanism if one assumes that the fraction of incoming electrons (which avoid the positronium formation and can reach the nugget's surface with large μ) is on the level of ~10% while the dominant fraction of incoming electrons ~90% will form positroniums [11].

It is important to remark here that μ is always in MeV region, much larger than the typical atomic energy scale which is in eV range. In this case the results which follow are not very sensitive to the specific properties of CS phase in the bulk. Therefore, our simplified treatment of the leptons as noninteracting Fermi gas is a sufficiently good approximation for this problem: any changes (due to the interactions in the bulk of nuggets) are happening at the $\mu \sim$ MeV scale. These changes do not affect physics on eV scale which is the subject of this paper.

III. POSITRONIUM FORMATION

We now consider the probability for the positronium formation when electrons hit the CCO made of antimatter. What is the fate of these nonrelativistic electrons? We shall argue below that the most likely outcome of these events is a formation of the bound states (positroniums with arbitrary quantum numbers $|n, l, m\rangle$) which eventually decay to two photons with $\hbar\omega \simeq 511$ keV and with width $\Gamma \sim m_e\alpha \sim$ few keV or three photons with well-known continuum spectrum $0 < \hbar\omega < m_e$.

Indeed, consider a system of an incoming electron and a positron from a nugget with momenta $\vec{q}_1$ and $\vec{q}_2$ correspondingly. Assuming that both particles are nonrelativistic we can calculate the probability of positronium formation with quantum numbers $|n, l, m\rangle$ by expanding the original wave functions (plane waves with momenta $\vec{q}_1$ and $\vec{q}_2$) in terms of the new basis of positronium's bound states (plus continuum),

$$\Psi_{q_1,q_2}(r_1, r_2) = e^{iQR}\sum_{nlm} c_{nlm}(q)\psi_{nlm}(r) + \text{cont.}, \qquad (1)$$

where $r \equiv (r_1 - r_2)$, $q \equiv 1/2(q_1 - q_2)$ correspond to the relative coordinate and momenta, while $R \equiv 1/2(r_1 + r_2)$, $Q \equiv (q_1 + q_2)$ describe the center of mass of the e^+e^- system. By definition, $|c_{nlm}(q)|^2$ gives the probability to find the e^+e^- system in the positronium state with quantum numbers $|n, l, m\rangle$ if initial e^+e^- states had momenta $\vec{q}_1$ and $\vec{q}_2$ with proper normalization. In particular, for the ground state,

$$|c_{100}(q)|^2 \sim \left| \int e^{-r/a} e^{i\vec{q}\,\vec{r}} d^3r \right|^2 \sim (a^2q^2 + 1)^{-4}, \qquad (2)$$

where $a \equiv 1/(m\alpha) \simeq 10^{-8}$ cm is the Bohr radius.

A few remarks are now in order.

(a) The probability for the positronium formation is large when q is sufficiently small, $q \sim 1/a \sim m\alpha$. This justifies our treatment of positrons as nonrelativistic particles. In other words, a nonrelativistic incoming electron will pick up a positron from Fermi gas with a small (rather than large) momenta $q \sim m\alpha$ to form a positronium. The probability of formation of the positroniums with large $q \gg a^{-1}$ is exceedingly small.

(b) The expression for the probability of the positronium formation does not contain a small factor α^2 which is an inherent feature of the direct annihilation process; see below Eq. (4).

(c) Once positroniums are formed, they will eventually decay much later (within or outside CCO) to two or three photons producing the low energy spectrum discussed above: 511 keV line + well known continuum spectrum $0 < \hbar\omega < m_e$.

(d) One may wonder why a small coupling constant α^2 does not enter the expression for the process which eventually leads to the photon's emission. The answer of course is related to the resonance nature of the phenomena. A similar situation occurs, e.g., in charge exchange processes such as capture of an electron from a hydrogen atom by a slow moving proton.

(e) The fact that the positronium formation plays a crucial role in the theory of positron annihilation in solids has been known for 50 years [18]; see also recent review on the subject [19]. Positronium formation always takes place whenever it is energetically allowed and velocities are small (when the so-called "Ore gap" is not destroyed by a complex condensed matter system).

(f) The magnitude of width of the 511 keV line in our framework is determined by the velocity distribution of the positroniums. Indeed, the positroniums in our framework are formed not at rest, but instead they carry a nonzero momenta $Q \equiv (q_1 + q_2)$ as Eq. (1) suggests. As was argued above, parametrically $Q \sim a^{-1} \sim m_e \alpha$. It implies that once the positroniums are formed, they do move with typical velocities $v \sim Q/m_e \sim \alpha$. Consequently, this motion leads to the width of the 511 keV line to be $\Gamma \sim m_e \alpha \sim$ few keV due to the Doppler effect.

(g) The probability for the positronium formation is order of 1 for small q as follows from Eqs. (1) and (2). However, these equations do not say what is the time scale saturating this large probability.

Therefore, the crucial question is, What is the time scale τ_{Ps} for the positronium formation in our specific circumstances? If this time scale is sufficiently short, then an incident electron has a great chance to form the positronium (which eventually leads to the 511 keV line) before it reaches the nugget's surface where the typical positron energies are large $\sim \mu$. If, on the other hand, this time scale is very large, then an incident electron very likely will reach the surface of the nugget and will experience the direct $e^+e^- \rightarrow 2\gamma$ annihilation with emission of ~MeV photons.

The cross section for the resonance positronium formation in atomic units is order of 1. In conventional units it is $\sigma(e^+e^- \rightarrow Ps) \sim a^2$ [20]. In order to estimate τ_{Ps}^{-1} we have to multiply $\sigma(e^+e^- \rightarrow Ps)$ by the density of positrons which effectively participate in the positronium formation and atomic velocity which is order of $v \sim \alpha$.

The density of positrons surrounding the antimatter nugget can be easily estimated in the transition region by using the Thomas-Fermi (mean field)-like approximation [16]. In the relativistic regime the density behaves like $n(z) \sim 1/z^3$ where z is the distance from the nugget's surface [16]. One can show that this behavior slowly changes to $n(z) \sim a^3/z^6$ in nonrelativistic regime where $a \sim (m\alpha)^{-1}$ is the Bohr radius. We do not need to know an exact numerical coefficient in this formula. The important thing for our discussions in what follows is the existence of a transition region ("electrosphere" [16]) where chemical potential μ interpolates between a large value on the surface of the nugget and zero value far away from the nugget. This interpolation always includes a region with a typical atomic density $n \sim a^{-3}$ at distance $z \sim a \sim 10^{-8}$ cm from the nugget's surface.

Collecting all these factors together we arrive at the following estimation for the probability P that an incident electron entering the "electrosphere" of the nugget will form positronium

$$\tau_{Ps}^{-1} \equiv \frac{dP}{dt} \sim v \cdot \sigma(e^+e^- \rightarrow Ps) \cdot n(z \sim a) \sim v/a, \quad (3)$$

where we use $n \sim a^{-3}$ for $z \sim a$. This expression clearly shows that the total probability for the positronium formation (which consequently decay producing 511 keV line) becomes of order of 1 at atomic distances $z \sim a$ from the nugget's surface, i.e., long before the incident electron reaches the region of large positron densities close to the nugget's surface.

This result is in clear contrast to estimations presented in [21] where a MeV broad spectrum is predicted resulting from the same, CCO-based mechanism. The crucial ingredient in our estimates is, of course, the resonance behavior for the cross section $\sigma(e^+e^- \rightarrow Ps) \sim a^2 \sim (m\alpha)^{-2}$ in contrast with nonresonance formula for the direct $e^+e^- \rightarrow 2\gamma$ annihilation when small parameter α^2 enters the numerator in the corresponding formula, see, e.g., [22],

$$\sigma(e^+e^- \rightarrow 2\gamma) \simeq \frac{2\pi\alpha^2}{s} \ln\left(\frac{s}{4m_e^2}\right), \qquad s \gg 4m_e^2. \quad (4)$$

To conclude this section, the dominant portion of all electrons falling into CCO (made of antimatter) will form the positroniums which eventually decay with low energy spectrum described above. The typical width of the outgoing flux of the 511 keV photons is of order $\Gamma \sim \alpha m \sim$ few keV. These features are very universal and do not depend on specific details of the nugget's internal structure (such as a large variation of possible CS phases in the bulk). Some incident electrons entering the nugget's surface will experience the direct annihilation with emission of gamma rays in MeV band. These photons may even have been already observed [11]. However, the direct annihilation plays a subleading role as argued above.

IV. CONCLUSION

We present a generic picture of the γ spectrum which results from the CCO-based mechanism. As we argued above, the vast majority of e^+e^- annihilations go through the positronium formation with the width of the 511 keV line to be $\Gamma \sim m_e\alpha \sim$ few keV. This is precisely what has been observed [1,23,24]. Also, this line is always accompanied by the well-known continuum with energies $\hbar\omega <$ 511 keV from the $^3S_1$ positronium decays (with the ratio 3:1). Amazingly, this is precisely the spectrum obtained in the recent analysis with a fraction of the observed positroniums estimated to be $(96.7 \pm 2.2)\%$ [23], $(92 \pm 9)\%$ [24]. Undoubtedly, these observations are consistent with almost $\simeq 100\%$ positronium fraction predicted by CCO-based mechanism due to the strong suppression of the direct annihilation in the region $\hbar\omega \leq 511$ keV.

Our mechanism also suggests that the 511 keV line must be accompanied by very broad (1–20 MeV) spectrum with the spectral density $\frac{d\Phi}{d\omega}$ at $\hbar\omega \simeq 511$ keV few orders of magnitude smaller than from the positronium decays. However, the total integrated flux over the large region $\int_0^\mu \frac{d\Phi}{d\omega} d\omega$ could be sufficiently large. Amazingly, there is indeed observational evidence for an excess of photons in the (1–20) MeV region; see [17,25] and references on the original works therein. It has been argued that the soft gamma-ray spectrum in the (1–20) MeV region cannot fully be attributed to either active galactic nuclei or type Ia supernovae or a combination of the two [25]. Therefore, the (1–20) MeV observed excess may find its natural explanation as a result of the direct annihilation of visible electrons with CCO's positrons. Such an explanation can be confirmed (or ruled out) if the correlation between (1–20) MeV photons and the 511 keV line is established. We shall not discuss this problem in the present paper by referring the reader to [11]. However, we would like to remark here that the excess of photons measured by COMPTEL can be naturally explained by this mechanism [11] if one assumes that a small fraction (on the level of ~10%) of incoming electrons can avoid the positronium formation and can reach the nugget's surface where μ is large.

A similar correlation should also exist between the 511 keV line and diffuse x-ray emission as discussed in detail in [26].

ACKNOWLEDGMENTS

I am thankful to I. Khriplovich for discussions during his visit to Vancouver. This work was supported in part by the National Science and Engineering Research Council of Canada.

[1] J. Knödlseder *et al.*, Astron. Astrophys. **411**, L457 (2003).
[2] P. Jean *et al.*, Astron. Astrophys. **407**, L55 (2003).
[3] C. Boehm, D. Hooper, J. Silk, and M. Casse, Phys. Rev. Lett. **92**, 101301 (2004).
[4] J. F. Beacom and H. Yuksel, Phys. Rev. Lett. **97**, 071102 (2006).
[5] L. Zhang, X. L. Chen, Y. A. Lei, and Z. G. Si, Phys. Rev. D **74**, 103519 (2006).
[6] H. Yuksel, arXiv:astro-ph/0609139.
[7] D. H. Oaknin and A. R. Zhitnitsky, Phys. Rev. Lett. **94**, 101301 (2005).
[8] A. R. Zhitnitsky, J. Cosmol. Astropart. Phys. 10 (2003) 010.
[9] D. H. Oaknin and A. Zhitnitsky, Phys. Rev. D **71**, 023519 (2005).
[10] E. Witten, Phys. Rev. D **30**, 272 (1984).
[11] K. Lawson and A. R. Zhitnitsky, arXiv:0704.3064.
[12] A. Zhitnitsky, Phys. Rev. D **74**, 043515 (2006).
[13] K. Rajagopal and F. Wilczek, arXiv:hep-ph/0011333; M. G. Alford, Annu. Rev. Nucl. Part. Sci. **51**, 131 (2001); T. Schafer and E. V. Shuryak, Lect. Notes Phys. **578**, 203 (2001).
[14] A. W. Steiner, S. Reddy, and M. Prakash, Phys. Rev. D **66**, 094007 (2002).
[15] J. Madsen, Phys. Rev. Lett. **87**, 172003 (2001).
[16] C. Alcock, E. Farhi, and A. Olinto, Astrophys. J. **310**, 261 (1986).
[17] A. W. Strong, I. V. Moskalenko, and O. Reimer, Astrophys. J. **537**, 763 (2000); **541**, 1109(E) (2000).
[18] R. Ferrell, Rev. Mod. Phys. **28**, 308 (1956).
[19] M. Puska and R. Nieminen, Rev. Mod. Phys. **66**, 841 (1994).
[20] If relative velocity v_e of colliding e^+e^- particles is much smaller than atomic velocity $\sim\alpha$ there will be an additional enhancement factor α/v_e in the cross section due to the long distance Coulomb interactions [27].
[21] D. T. Cumberbatch, J. Silk, and G. D. Starkman, arXiv:astro-ph/0606429.
[22] M. Peskin and D. Schroeder, *An Introduction to QFT* (Addison-Wesley, Reading, MA, 1996).
[23] P. Jean *et al.*, Astron. Astrophys. **445**, 579 (2006).
[24] G. Weidenspointner *et al.*, arXiv:astro-ph/0601673.
[25] K. Ahn, E. Komatsu, and P. Hoflich, Phys. Rev. D **71**, 121301(R) (2005); K. Ahn and E. Komatsu, Phys. Rev. D **72**, 061301(R) (2005); L. E. Strigari, J. F. Beacom, T. P. Walker, and P. Zhang, J. Cosmol. Astropart. Phys. 04 (2005) 017.
[26] M. M. Forbes and A. R. Zhitnitsky, arXiv:astro-ph/0611506.
[27] L. D. Landau and E. M. Lifshitz, *Quantum Mechanics* (Elsevier Butterworth-Heinemann, Amsterdam, 2005), Vol. III.