

Production of a sterile species via active-sterile mixing: An exactly solvable model

D. Boyanovsky*

Department of Physics and Astronomy, University of Pittsburgh, Pittsburgh, Pennsylvania 15260, USA
(Received 21 June 2007; revised manuscript received 20 September 2007; published 7 November 2007)

The production of a sterile species via active-sterile mixing in a thermal medium is studied in an exactly solvable model. The exact time evolution of the sterile distribution function is determined by the dispersion relations and damping rates $\Gamma_{1,2}$ for the quasiparticle modes. These depend on $\tilde{\gamma} = \Gamma_{aa}/2\Delta E$, with Γ_{aa} the interaction rate of the active species in absence of mixing and ΔE the oscillation frequency in the medium without damping. $\tilde{\gamma} \ll 1$, $\tilde{\gamma} \gg 1$ describe the weak and strong damping limits, respectively. For $\tilde{\gamma} \ll 1$, $\Gamma_1 = \Gamma_{aa} \cos^2 \theta_m$; $\Gamma_2 = \Gamma_{aa} \sin^2 \theta_m$ where θ_m is the mixing angle in the medium and the sterile distribution function does not obey a simple rate equation. For $\tilde{\gamma} \gg 1$, $\Gamma_1 = \Gamma_{aa}$ and $\Gamma_2 = \Gamma_{aa} \sin^2 2\theta_m / 4\tilde{\gamma}^2$, is the sterile production rate. In this regime sterile production is suppressed and the oscillation frequency vanishes at an Mikheyev-Smirnov-Wolfenstein (MSW) resonance, with a breakdown of adiabaticity. These are consequences of quantum Zeno suppression. For active neutrinos with standard model interactions the strong damping limit is only available near an MSW resonance if $\sin 2\theta \ll \alpha_w$ with θ the vacuum mixing angle. The full set of quantum kinetic equations for sterile production for arbitrary $\tilde{\gamma}$ are obtained from the quantum master equation. Cosmological resonant sterile neutrino production is quantum Zeno suppressed relieving potential uncertainties associated with the QCD phase transition.

DOI: [10.1103/PhysRevD.76.103514](https://doi.org/10.1103/PhysRevD.76.103514)

PACS numbers: 98.80.Cq, 11.10.Wx, 14.60.Pq

I. INTRODUCTION

Sterile neutrinos, namely, weak interaction singlets, are compelling candidates to explain a host of cosmological and astrophysical phenomena. They could be a suitable warm dark matter component [1–14], may also be relevant in the latest stages of stellar collapse [15,16], primordial nucleosynthesis [17,18], and provide a potential explanation for the anomalous velocity distributions of pulsars [19–21]. Although sterile neutrinos are ubiquitous in extensions of the standard model [22–25], the MiniBooNE Collaboration [26] has recently reported results in contradiction with those from the Liquid Scintillator Neutrino Detector (LSND) [27,28] that suggested a sterile neutrino with $\Delta m^2 \sim 1 \text{ eV}^2$ scale. Although the MiniBooNE results hint at an excess of events below 475 MeV the analysis distinctly excludes two neutrino appearance-only from $\nu_\mu \rightarrow \nu_e$ oscillations with a mass scale $\Delta m^2 \sim 1 \text{ eV}^2$, perhaps ruling out a light sterile neutrino. However, a recent analysis [29] suggests that while $(3+1)$ schemes are strongly disfavored, $(3+2)$ neutrino schemes provide a good fit to both the LSND and MiniBooNE data, including the low energy events, because of the possibility of CP violation in these schemes, although significant tension remains. These issues notwithstanding the MiniBooNE result does not constrain a heavier variety of sterile neutrinos such as those that could be suitable warm dark matter candidates with masses in the keV range [1–7,9,11,12]. Their radiative decay would contribute to the x-ray background [5,9,30–32] from which constraints on their masses and mixing angles may be extracted [9,31–33]. It

has also been suggested that precision laboratory experiments may be sensitive to $\sim \text{keV}$ neutrinos [34]. Being weak interaction singlets, sterile neutrinos can only be produced via their mixing with an active species, hence any assessment of the possibility of sterile neutrinos as dark matter candidates or their role in supernovae must begin with understanding their production mechanism. To be a suitable dark matter candidate, two important constraints must be satisfied: the correct abundance and a velocity dispersion that restricts the free streaming length to be consistent with the constraints from structure formation. Both ingredients depend directly on the distribution function of the sterile neutrinos, which in turn depend on the dynamics of production and evolution until freeze-out.

Pioneering work on the nonequilibrium dynamics of neutrinos in a medium was cast in terms of kinetic equations for a flavor “matrix of densities” [35] or in terms of 2×2 Bloch-type equations for flavor quantum mechanical states [36,37]. A general field theoretical approach to neutrino mixing and kinetics was presented in [38,39] (see also [25]), however sterile neutrino production in the early Universe is mostly studied in terms of simple phenomenological rate equations [1,4,40–43], and numerical studies [4,43] rely on an approximate semiphenomenological approach [41,42]. A field theoretical study of the hadronic contribution to the sterile production rate near an MSW resonance has been reported in Ref. [44].

Understanding the dynamics of oscillations, decoherence and damping is of fundamental and phenomenological importance not only in neutrino cosmology but also in the dynamics of neutral meson mixing and CP violation [45–47] and axion-photon mixing in the presence of a magnetic field [25], a phenomenon whose interest has

*boyan@pitt.edu

been rekindled by the recent results from the PVLAS Collaboration [48] (see the discussion in Ref. [49]). As argued in [50] the spinorial nature of neutrinos is inessential to describe the dynamics of mixing and decoherence in a medium.

Recently we reported on a study [51] of mixing and decoherence in a theory of mesons that provides an accurate description of similar phenomena for mixed neutrinos. This effective theory incorporates interactions that model the medium effects associated with charge and neutral currents for neutrinos and yields a picture of the dynamics which is remarkably general. The fermion nature of the distributions and Pauli blocking effects can be simply accounted for in the final result [51]. This study implemented quantum field theory methods to obtain the nonequilibrium effective action for the “neutrino” degrees of freedom. More recently this approach was extended to study the production of sterile neutrinos both from the effective action as well as from the correct quantum kinetic equations obtained directly from the quantum master equation [52]. The results obtained in Ref. [52] clarify a host of important aspects, such as the approach to equilibrium and a detailed analysis of quantum Zeno suppression when the decoherence time scale is shorter than the oscillation time scale, thereby confirming previous results obtained for neutrinos with standard model interactions in Refs. [53,54]. The study in Refs. [51,52] relied on integrating out the bath degrees of freedom, assumed to remain in equilibrium, up to second order in a perturbative expansion akin to an expansion in G_F in the standard model. This perturbative treatment restricted the analysis to the weak damping regime in which the decoherence time scale is larger than the oscillation time scale. In Refs. [52,53] it was pointed out that a strong damping regime featuring the opposite relation between these time scales could emerge near an MSW resonance for small vacuum mixing angle consistent with constraints from the x-ray background [9,31–33].

A. Motivation and goals

A sound assessment of sterile neutrinos as warm dark matter candidates requires a reliable description of the kinetics of production and evolution towards freeze-out. Strong departure from equilibrium in the distribution function at freeze-out could lead to significant changes in the abundance or skewed velocity distributions that could affect the free streaming lengths and structure formation [55]. In this article we complement and extend a previous study [52] on the nonequilibrium production of a sterile species via active-sterile mixing. While the previous study [51–53] focused on the weak damping limit consistently with a perturbative expansion in standard model interactions, this article studies an exactly solvable model that allows us to explore systematically the strong damping case and to draw general conclusions on the production dynamics of a sterile species.

The model incorporates all the relevant ingredients: active-sterile mixing via a mass matrix which is off-diagonal in the flavor basis, and the coupling of the active species to a continuum of degrees of freedom which are taken as a thermal bath in equilibrium and includes an index of refraction contribution which modifies the mixing angles and dispersion relations in the same manner as for neutrinos propagating in a medium.

B. Summary of results

The exact solution of the Heisenberg equations of motion allows a complete investigation of the nonequilibrium dynamics of production of the sterile species in the weak and strong damping regimes and to analyze in detail quantum Zeno suppression. We obtain the quantum master equation and from it the complete set of kinetic equations that describe the production and evolution of the active and sterile distribution functions and coherences and reproduce the exact results. Our main results are:

- (i) The exact solution of the Heisenberg (-Langevin) equations of motion for one active and one sterile species yields two different modes of propagation in the medium corresponding to quasiparticles whose dispersion relations and damping rates (widths) depend on the dimensionless ratio $\tilde{\gamma} = \Gamma_{aa}/2\Delta E$ with Γ_{aa} the active species interaction rate in absence of mixing, and ΔE the oscillation frequency in the absence of damping but including the index of refraction in the medium. The weak and strong damping cases correspond to $\tilde{\gamma} \ll 1$ and $\tilde{\gamma} \gg 1$ respectively. The exact distribution functions for the active and sterile species are obtained, their time evolution is completely determined by the widths of these quasiparticles and the oscillation frequency including corrections from the index of refraction and damping.
- (ii) The results in the weak damping regime $\tilde{\gamma} \ll 1$ coincide with those obtained previously in Refs. [51–53]: the dispersion relations are akin to those of neutrinos in a medium with an index of refraction and the damping rates are $\Gamma_1 = \Gamma_{aa}\cos^2\theta_m$; $\Gamma_2 = \Gamma_{aa}\sin^2\theta_m$ where θ_m is the mixing angle in the medium. The generalized active-sterile transition probability obtained from expectation values of Heisenberg operators in the full quantum density matrix is $\frac{\sin^2 2\theta_m}{4}[e^{-\Gamma_1 t} + e^{-\Gamma_2 t} - 2e^{-(1/2)(\Gamma_1 + \Gamma_2)t} \cos[\Delta E t]]$. The production of the sterile species cannot be described by a simple rate equation, since the distribution function depends on the time scales $1/\Gamma_1$, $1/\Gamma_2$, $1/\Delta E$.
- (iii) In the strong damping regime $\tilde{\gamma} \gg 1$ the oscillation frequency vanishes at an MSW resonance signaling a breakdown of adiabaticity, and the widths of the quasiparticles become $\Gamma_1 \sim \Gamma_{aa}$, $\Gamma_2 \sim \Gamma_{aa}\sin^2 2\theta_m/4\tilde{\gamma}^2$. To leading order in $1/\tilde{\gamma}$, the time

evolution of the sterile distribution function simplifies into a rate equation, with the production rate given by $\Gamma_2 \sim \sin^2 2\theta_m (\Delta E)^2 / \Gamma_{aa}$ [see Eq. (3.17)]. The active-sterile transition probability is strongly suppressed $\sim 1/\tilde{\gamma}^2$. The vanishing of the oscillation frequency, the suppression of the transition probability and the production of the sterile species are all manifestations of the quantum Zeno effect emerging in the strong damping limit.

- (iv) For active neutrinos with standard model interactions it is shown that the strong damping limit is only available near an MSW resonance for small vacuum mixing angle θ satisfying the condition $\sin 2\theta \lesssim \alpha_w$ where α_w . This condition is likely satisfied by the constraints on the vacuum mixing angle from the x-ray background [5,30] and entails that sterile neutrino production is strongly suppressed by the quantum Zeno effect near an MSW resonance. This suppression may relieve uncertainties from the QCD phase transition for keV sterile neutrinos.
- (v) The quantum master equation for the reduced density matrix is obtained under standard approximations. From it the generalized transition probability and the complete set of kinetic equations are obtained valid in all regimes of damping. These reproduce the results obtained from the exact treatment. Under simple approximations the full set of kinetic equations is presented in the form of quantum kinetic equations for a ‘‘polarization vector.’’ The complete set of kinetic equations (4.22), (4.23), and (4.24) along with the relations (4.25) and (4.26) provide a complete description of the nonequilibrium evolution of the active and sterile distributions and coherences.

II. THE MODEL

The main ingredients in the dynamics of the production of a sterile species via active-sterile mixing are: (i) a mass matrix off diagonal in the flavor basis which mixes the sterile and active species, (ii) the coupling of the active species to a bath in equilibrium. In the standard model the bath degrees of freedom are quarks, leptons or hadrons, these equilibrate via strong or electromagnetic interactions, and hence can be taken to be in thermal equilibrium.

We propose a simple exactly solvable model that includes all these ingredients, it is a generalization of a model for quantum Brownian motion [56,57] which has long served as a paradigm for the study of quantum dissipative systems in condensed matter [58] and quantum optics [59]. It consists of a set of coordinates \vec{q} that describe the ‘‘system’’ coupled to a continuum of harmonic oscillators Q_p that describe a thermal bath in equilibrium. This simple model is generalized so that the coordinates $q_{a,s}$ stand for the active and sterile ‘‘neutrinos’’; these are mixed by off-diagonal elements in a frequency matrix but

only the ‘‘active’’ coordinate couples to the bath degrees of freedom. The motivation for studying this model stems from the realization that the spinorial degrees of freedom are not relevant to describe the nonequilibrium dynamics [50], a statement confirmed by previous studies of mixing, oscillations and decoherence in a theory mesons [51,52] which yields a remarkably robust picture of the dynamics of neutrinos.

The Lagrangian for this model is

$$L = \frac{1}{2} [\vec{\dot{q}}^T \cdot \vec{\dot{q}} - \vec{q}^T (k^2 \mathbb{1} + \mathbb{M}^2 + \mathbb{V}) \vec{q}] + \frac{1}{2} \sum_p [\dot{Q}_p^2 - W_p^2 Q_p^2] + q_a \sum_p C_p Q_p, \quad (2.1)$$

where the flavor vector is given by

$$\vec{q} = \begin{pmatrix} q_a \\ q_s \end{pmatrix} \quad (2.2)$$

and k is a momentum label, which is assumed but not included as an argument of $q_{a,s}$ for compact notation, $\mathbb{1}$ is the 2×2 identity matrix and

$$\mathbb{M}^2 = \begin{pmatrix} M_{aa}^2 & M_{as}^2 \\ M_{as}^2 & M_{ss}^2 \end{pmatrix}; \quad \mathbb{V} = \begin{pmatrix} V_{aa}(k) & 0 \\ 0 & 0 \end{pmatrix}. \quad (2.3)$$

The off-diagonal elements of the mass matrix \mathbb{M} lead to active-sterile mixing and the matrix \mathbb{V} models a ‘‘momentum dependent matter potential’’ for the active species.

A sum over k makes explicit the field theoretical nature of the model, however just as in the case of neutrinos, we are interested on the dynamics of a given k mode in interaction with the ‘‘bath’’ degrees of freedom.

The correspondence with neutrinos is manifest by assuming that the matter potential is obtained from one-loop charged and neutral current contributions of $\mathcal{O}(G_F)$ from a background of leptons, quarks or hadrons (or neutrinos in equilibrium) and features a CP -odd term proportional to the lepton and baryon asymmetries and a CP -even term that only depends on energy and temperature [60,61]. The linear coupling of the active species to the bath degrees of freedom with $C_p \propto G_F$ models the charged current interaction, for example, the coupling between the electron neutrino and protons, neutrons and electrons in a medium, $G_F \bar{\psi}_p (C_V - C_A \gamma^5) \gamma^\mu \psi_N \psi_e \gamma_\mu (1 - \gamma^5) \nu_e$ (see a similar description in [25,39]). The label p will be taken to describe a continuum when the density of states is introduced below. Obviously the model (2.1) affords an exact solution and yields a remarkably general description of the dynamics. The main ingredient is the coupling of a degree of freedom to a continuum of bath or environmental degrees of freedom. Such coupling to a continuum is also at the heart of particle-antiparticle oscillations in neutral meson systems ($K^0 - \bar{K}^0$; $B^0 - \bar{B}^0$) as described in Refs. [46,47]. Other versions of this model, without mixing have been

studied with focus on the dynamics of equilibration [62,63].

For vanishing matter potential \mathbb{V} the flavor $q_{a,s}$ and the mass coordinates $q_{1,2}$ are related by an orthogonal transformation

$$\begin{pmatrix} q_a \\ q_s \end{pmatrix} = U(\theta) \begin{pmatrix} q_1 \\ q_2 \end{pmatrix}; \quad U(\theta) = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \quad (2.4)$$

where the orthogonal matrix $U(\theta)$ diagonalizes the mass matrix \mathbb{M}^2 , namely

$$U^{-1}(\theta)\mathbb{M}^2U(\theta) = \begin{pmatrix} M_1^2 & 0 \\ 0 & M_2^2 \end{pmatrix} \quad (2.5)$$

and θ is the ‘‘vacuum’’ mixing angle in absence of the ‘‘matter potential’’ \mathbb{V} .

In the flavor basis the mass matrix \mathbb{M} can be written in terms of the vacuum mixing angle θ and the eigenvalues of the mass matrix as

$$\mathbb{M}^2 = \bar{M}^2 \mathbb{1} + \frac{\delta M^2}{2} \begin{pmatrix} -\cos 2\theta & \sin 2\theta \\ \sin 2\theta & \cos 2\theta \end{pmatrix} \quad (2.6)$$

where we introduced

$$\bar{M}^2 = \frac{1}{2}(M_1^2 + M_2^2); \quad \delta M^2 = M_2^2 - M_1^2. \quad (2.7)$$

The frequencies of the flavor modes are determined by the diagonal entries of the matrix \mathbb{M}^2 in the flavor basis, introducing

$$\bar{\omega}(k) = \sqrt{k^2 + \bar{M}^2}. \quad (2.8)$$

These are given by

$$\begin{aligned} \omega_a(k) &= \bar{\omega}(k) \left[1 - \frac{\delta M^2}{2\bar{\omega}(k)^2} \cos 2\theta \right]^{1/2}; \\ \omega_s(k) &= \bar{\omega}(k) \left[1 + \frac{\delta M^2}{2\bar{\omega}(k)^2} \cos 2\theta \right]^{1/2}. \end{aligned} \quad (2.9)$$

Focusing on the relevant case of ultrarelativistic neutrinos, we anticipate that the only approximation to be invoked is the one in which $\bar{\omega}(k)$ is larger than any other energy scale. It is convenient to introduce

$$\begin{aligned} \mathbb{K} &\equiv k^2 \mathbb{1} + \mathbb{M} + \mathbb{V} \\ &= \left(\bar{\omega}(k)^2 + \frac{V_{aa}}{2} \right) \mathbb{1} + \frac{\delta M^2}{2} \\ &\quad \times \begin{bmatrix} -(\cos 2\theta - \frac{V_{aa}}{\delta M^2}) & \sin 2\theta \\ \sin 2\theta & (\cos 2\theta - \frac{V_{aa}}{\delta M^2}) \end{bmatrix}. \end{aligned} \quad (2.10)$$

The exact solution will be presented in the Heisenberg picture, in which the density matrix is time independent and determined by its initial value, which is assumed to be uncorrelated and of the form

$$\hat{\rho}(0) = \hat{\rho}_q \otimes \hat{\rho}_Q. \quad (2.11)$$

The bath is taken to be in thermal equilibrium with density matrix $\hat{\rho}_Q = \text{Tr} e^{-H_Q/T}$ where H_Q is the Hamiltonian for the sum of free harmonic oscillators of frequencies W_p .

The Heisenberg equations of motion for the coordinates $q_{a,s}$, Q_p are the following

$$\ddot{q}_\alpha + \mathbb{K}_{\alpha\beta} q_\beta = \eta_\alpha; \quad \alpha, \beta = a, s \quad (2.12)$$

$$\ddot{Q}_p + W_p^2 Q_p = q_a C_p, \quad (2.13)$$

where we have introduced the flavor vector

$$\vec{\eta} = \sum_p C_p Q_p \begin{pmatrix} 1 \\ 0 \end{pmatrix}. \quad (2.14)$$

The solution of Eq. (2.13) is

$$Q_p(t) = Q_p^{(0)}(t) + \frac{C_p}{W_p} \int_0^t \sin[W_p(t-t')] q_a(t') dt', \quad (2.15)$$

where

$$Q_p^{(0)}(t) = \frac{1}{\sqrt{2W_p}} [A_p e^{-iW_p t} + A_p^\dagger e^{iW_p t}], \quad (2.16)$$

is a solution of the homogeneous equation and A_p , A_p^\dagger are free field annihilation and creation operators with the usual canonical commutation relations. The distribution function for the bath degrees of freedom is

$$\text{Tr} \hat{\rho}_Q A_p^\dagger A_p = \frac{1}{e^{W_p/T} - 1} = n(W_p). \quad (2.17)$$

Introducing the solution (2.15) into (2.12) we find the Heisenberg-Langevin equations [59]

$$\ddot{q}_\alpha(t) + \mathbb{K}_{\alpha\beta} q_\beta(t) + \int_0^t \Sigma_{\alpha\beta}(t-t') q_\beta(t') = \xi_\alpha(t) \quad (2.18)$$

where the self-energy is diagonal in the flavor basis and given by

$$\Sigma_{\alpha\beta}(t-t') = -\sum_p \frac{C_p^2}{W_p} \sin[W_p(t-t')] \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}. \quad (2.19)$$

The stochastic quantum noise is

$$\vec{\xi}(t) = \sum_p C_p Q_p^{(0)}(t) \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad (2.20)$$

and we note that

$$\text{Tr} \hat{\rho} \vec{\xi}(t) = 0. \quad (2.21)$$

The self-energy Σ is written in dispersive form by passing to a continuum description of the bath degrees of freedom, writing

$$- \sum_p \frac{C_p^2}{W_p} \sin[W_p(t - t')] = i \int_{-\infty}^{\infty} \frac{d\omega}{\pi} \text{Im}\Sigma_{aa}(\omega) e^{i\omega(t-t')} \quad (2.22)$$

where the density of states

$$\text{Im}\Sigma_{aa}(\omega) = \sum_p \frac{\pi C_p^2}{2W_p} [\delta(\omega - W_p) - \delta(\omega + W_p)] \quad (2.23)$$

has the properties

$$\begin{aligned} \text{Im}\Sigma_{aa}(-\omega) &= -\text{Im}\Sigma_{aa}(\omega); \\ \text{Im}\Sigma_{aa}(\omega) &> 0 \quad \text{for } \omega > 0. \end{aligned} \quad (2.24)$$

The density of states $\text{Im}\Sigma_{aa}$ contains all of the relevant information of the bath. The Heisenberg-Langevin equation (2.18) is solved by Laplace transform, introduce

$$\tilde{q}_\alpha(s) = \int_0^\infty e^{-st} q_\alpha(t) dt; \text{ etc.}, \quad (2.25)$$

in terms of which the equation of motion (2.18) becomes an algebraic equation

$$\begin{aligned} [s^2 \delta_{\alpha\beta} + \mathbb{K}_{\alpha\beta} + \tilde{\Sigma}_{\alpha\beta}(s)] \tilde{q}_\beta(s) \\ = \dot{q}_\alpha(0) + s q_\alpha(0) + \tilde{\xi}_\alpha(s), \end{aligned} \quad (2.26)$$

where in the flavor basis

$$\tilde{\Sigma}(s) = -\frac{1}{\pi} \int_{-\infty}^{\infty} \frac{\text{Im}\Sigma_{aa}(\omega')}{\omega' + is} d\omega' \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}. \quad (2.27)$$

In what follows we need the analytic continuation of the self-energy to real frequencies $s \rightarrow i\omega + 0^+$

$$\tilde{\Sigma}_{aa}(s = i\omega + 0^+) = \text{Re}\Sigma_{aa}(\omega) + i \text{Im}\Sigma_{aa}(\omega) \quad (2.28)$$

with the dispersive relation

$$\text{Re}\Sigma_{aa}(\omega) = -\frac{1}{\pi} \mathcal{P} \int_{-\infty}^{\infty} \frac{\text{Im}\Sigma_{aa}(\omega')}{\omega' - \omega} d\omega', \quad (2.29)$$

and \mathcal{P} stands for the principal part.

The solution of Eq. (2.18) in real time is given by

$$\begin{aligned} q_\alpha(t) &= \dot{G}_{\alpha\beta}(t) q_\beta(0) + G_{\alpha\beta}(t) \dot{q}_\beta(0) \\ &+ \int_0^t G_{\alpha\beta}(t') \xi_\beta(t - t') dt' \end{aligned} \quad (2.30)$$

with

$$G_{\alpha\beta}(t) = \int_C \frac{ds}{2\pi i} \tilde{G}_{\alpha\beta}(s) e^{st}. \quad (2.31)$$

The Laplace transform of the propagator is given by

$$\tilde{G}(s) = [s^2 \mathbb{1} + \mathbb{K} + \tilde{\Sigma}(s)]^{-1} \quad (2.32)$$

and C is the Bromwich contour that runs parallel to the imaginary axis and to the right of all the singularities of \tilde{G}

in the complex s plane. It follows from Eqs. (2.26) and (2.32) that the propagator matrix $G_{\alpha\beta}(t)$ is a homogeneous solution of the equation of motion (2.18) with initial conditions

$$G_{\alpha\beta}(0) = 0; \quad \dot{G}_{\alpha\beta}(0) = 1. \quad (2.33)$$

It is convenient to introduce the following combinations

$$\tilde{\Delta}(s) = \frac{1}{\delta M^2} [\tilde{\Sigma}_{aa}(s) + V_{aa}] \quad (2.34)$$

$$\tilde{\rho}(s) = [(\cos 2\theta - \tilde{\Delta}(s))^2 + \sin^2 2\theta] \quad (2.35)$$

and the matrix

$$\mathbb{A}(s) = \frac{1}{\tilde{\rho}(s)} \begin{bmatrix} \cos 2\theta - \tilde{\Delta}(s) & -\sin 2\theta \\ -\sin 2\theta & -\cos 2\theta + \tilde{\Delta}(s) \end{bmatrix}, \quad (2.36)$$

in terms of which we find

$$\begin{aligned} \tilde{G}(s) &= \frac{1}{2} \frac{\mathbb{1} + \mathbb{A}(s)}{s^2 + \bar{\omega}^2(k) + \frac{\delta M^2}{2} (\tilde{\Delta}(s) - \tilde{\rho}(s))} \\ &+ \frac{1}{2} \frac{\mathbb{1} - \mathbb{A}(s)}{s^2 + \bar{\omega}^2(k) + \frac{\delta M^2}{2} (\tilde{\Delta}(s) + \tilde{\rho}(s))}. \end{aligned} \quad (2.37)$$

Each term in this expression features poles in the complex s plane near $s \approx \pm i\bar{\omega}(k)$ which are found by first performing the analytic continuation $s \rightarrow i\omega + 0^+$ upon which the denominators in $\tilde{G}(s)$ become

$$\begin{aligned} s^2 + \bar{\omega}^2(k) + \frac{\delta M^2}{2} (\tilde{\Delta}(s) \mp \tilde{\rho}(s)) &\rightarrow -\omega^2 + \bar{\omega}^2(k) \\ &+ \frac{1}{2} [\text{Re}\Sigma_{aa}(\omega) \\ &+ i \text{Im}\Sigma_{aa}(\omega) + V_{aa}] \\ &\mp \frac{\delta M^2}{2} \rho(\omega) \end{aligned} \quad (2.38)$$

where the analytic continuations are given by

$$\rho(\omega) = [(\cos 2\theta - \Delta_R(\omega) - i\Delta_I(\omega))^2 + (\sin 2\theta)^2]^{1/2} \quad (2.39)$$

$$\Delta_R(\omega) = \frac{[\text{Re}\Sigma_{aa}(\omega) + V_{aa}]}{\delta M^2}; \quad \Delta_I(\omega) = \frac{\text{Im}\Sigma_{aa}(\omega)}{\delta M^2}. \quad (2.40)$$

The complex poles describe quasiparticles, the real part determines their dispersion relation and the imaginary part their damping rate in the medium. At this stage it is convenient to introduce the following variables

$$\bar{\Delta}_R \equiv \Delta_R(\bar{\omega}(k)) = \frac{[V_{aa} + \text{Re}\Sigma_{aa}(\bar{\omega}(k))]}{\delta M^2} \quad (2.41)$$

$$\tilde{\gamma} \equiv \frac{\Delta_I(\bar{\omega}(k))}{\rho_0} = \frac{\text{Im}\Sigma_{aa}(\bar{\omega}(k))}{\delta M^2 \rho_0} \quad (2.42)$$

and write

$$\rho(\bar{\omega}(k)) = \rho_0 r e^{-i\alpha} \quad (2.43)$$

where

$$\rho_0 = [(\cos 2\theta - \bar{\Delta}_R)^2 + (\sin 2\theta)^2]^{1/2} \quad (2.44)$$

$$r = [(1 - \tilde{\gamma}^2)^2 + (2\tilde{\gamma} \cos 2\theta_m)^2]^{1/4}, \quad (2.45)$$

$$\alpha = \frac{1}{2} \text{arctg} \left[\frac{2\tilde{\gamma} \cos 2\theta_m}{1 - \tilde{\gamma}^2} \right] \quad (2.46)$$

and the branch is chosen such that $0 \leq \text{arctg}[\dots] \leq \pi$. The mixing angle in the medium, θ_m , is defined by the relations

$$\cos 2\theta_m = \frac{\cos 2\theta - \bar{\Delta}_R}{\rho_0}; \quad \sin 2\theta_m = \frac{\sin 2\theta}{\rho_0}, \quad (2.47)$$

an MSW resonance in the medium occurs whenever [22–24]

$$\cos 2\theta = \bar{\Delta}_R. \quad (2.48)$$

The only approximations to be used are the following

$$\frac{\delta M^2}{\bar{\omega}(k)} \ll 1; \quad \frac{\text{Re}\Sigma_{aa}(\omega)}{\bar{\omega}(k)} \ll 1; \quad \frac{\text{Im}\Sigma_{aa}(\omega)}{\bar{\omega}(k)} \ll 1 \quad (2.49)$$

these are all consistent with the ultrarelativistic limit, small radiative corrections and the narrow width limit, all approximations used in the case of neutrinos. Using these approximations we find the following complex poles:

(i) The first term in (2.37) features complex poles at

$$\omega = \pm \Omega_1 + i \frac{\Gamma_1}{2} \quad (2.50)$$

with

$$\Omega_1 = \bar{\omega}(k) + \frac{1}{4\bar{\omega}(k)} [\text{Re}\Sigma_{aa}(\bar{\omega}(k)) + V_{aa} - \delta M^2 \rho_0 r \cos \alpha] \quad (2.51)$$

$$\Gamma_1 = \frac{\Gamma_{aa}}{2} \left[1 + \frac{r \sin \alpha}{\tilde{\gamma}} \right]. \quad (2.52)$$

(ii) The second term in (2.37) features complex poles at

$$\omega = \pm \Omega_2 + i \frac{\Gamma_2}{2} \quad (2.53)$$

with

$$\Omega_2 = \bar{\omega}(k) + \frac{1}{4\bar{\omega}(k)} [\text{Re}\Sigma_{aa}(\bar{\omega}(k)) + V_{aa} + \delta M^2 \rho_0 r \cos \alpha] \quad (2.54)$$

$$\Gamma_2 = \frac{\Gamma_{aa}}{2} \left[1 - \frac{r \sin \alpha}{\tilde{\gamma}} \right] \quad (2.55)$$

where

$$\Gamma_{aa} = \frac{\text{Im}\Sigma_{aa}(\bar{\omega}(k))}{\bar{\omega}(k)} \quad (2.56)$$

is the interaction rate for the active species in the absence of mixing in the limit $\bar{\omega}(k) \gg \delta M^2$, which is of relevance for ultrarelativistic or nearly degenerate neutrinos. In what follows we suppress the argument $\bar{\omega}(k)$ in the quantities $\Delta_{R,I}$, etc., to simplify notation.

Near the complex poles the analytic continuation $\tilde{G}(s = i\omega + 0^+)$ features a Breit-Wigner form, and the inverse Laplace transform can be performed by approximating the analytic continuation by the Breit-Wigner Lorentzian. We find

$$G(t) = \frac{e^{i\Omega_1 t} e^{-(\Gamma_1/2)t}}{2i\Omega_1} \frac{1}{2} [\mathbb{I} + \mathbb{T}] - \frac{e^{-i\Omega_1 t} e^{-(\Gamma_1/2)t}}{2i\Omega_1} \times \frac{1}{2} [\mathbb{I} + \mathbb{T}^*] + \frac{e^{i\Omega_2 t} e^{-(\Gamma_2/2)t}}{2i\Omega_2} \frac{1}{2} [\mathbb{I} - \mathbb{T}] - \frac{e^{-i\Omega_2 t} e^{-(\Gamma_2/2)t}}{2i\Omega_2} \frac{1}{2} [\mathbb{I} - \mathbb{T}^*] \quad (2.57)$$

where we have neglected wave function renormalization (residues at the poles) and introduced the complex matrix

$$\mathbb{T} = \frac{e^{i\alpha}}{r} \begin{bmatrix} \cos 2\theta_m - i\tilde{\gamma} & -\sin 2\theta_m \\ -\sin 2\theta_m & -\cos 2\theta_m + i\tilde{\gamma} \end{bmatrix} \quad (2.58)$$

where all quantities are evaluated at $\omega = \bar{\omega}(k)$ and used the approximations (2.49). Inserting the result (2.57) into the solution (2.30) we obtain the complete solution for the time evolution of the Heisenberg operators. The Breit-Wigner approximation leading to exponential damping in (2.57) is a Markovian approximation [59]. The full solution requires the initial conditions on the Heisenberg operators $q(0)$, $\dot{q}(0)$ it is convenient to expand these in a basis of creation and annihilation operators of flavor states,

$$q_\beta(0) = \frac{1}{\sqrt{2\omega_\beta}} [a_\beta(0) + a_\beta^\dagger(0)];$$

$$\dot{q}_\beta(0) = -i \frac{\omega_\beta}{\sqrt{2\omega_\beta}} [a_\beta(0) - a_\beta^\dagger(0)]; \quad \beta = a, s \quad (2.59)$$

where $\omega_{a,s}$ are the frequencies associated with flavor eigenstates given by Eq. (2.9). Under the validity of the approximations (2.49), we can approximate

$$\omega_a \sim \omega_s \sim \Omega_1 \sim \Omega_2 \sim \bar{\omega}(k) \quad (2.60)$$

leading to a simplified form

$$\begin{aligned} q_\alpha(t) \approx & \frac{1}{\sqrt{2\bar{\omega}(k)}} \left\{ e^{-i\Omega_1 t} e^{-(\Gamma_1/2)t} \frac{1}{2} [\mathbb{I} + \mathbb{T}^*] \right. \\ & \left. + e^{-i\Omega_2 t} e^{-(\Gamma_2/2)t} \frac{1}{2} [\mathbb{I} - \mathbb{T}^*] \right\}_{\alpha\beta} a_\beta(0) + \text{H.c.} \\ & + \int_0^t G_{\alpha\beta}(t') \xi_\beta(t-t') dt'. \end{aligned} \quad (2.61)$$

Under the same approximations, we find the Heisenberg annihilation operators at an arbitrary time from

$$a_\alpha(t) = \sqrt{\frac{\omega_\alpha}{2}} \left[q_\alpha(t) - \frac{p_\alpha(t)}{i\omega_\alpha} \right]; \quad p_\alpha(t) = \dot{q}_\alpha(t), \quad (2.62)$$

these are given by

$$\begin{aligned} a_\alpha(t) \approx & \left\{ e^{-i\Omega_1 t} e^{-(\Gamma_1/2)t} \frac{1}{2} [\mathbb{I} + \mathbb{T}^*] + e^{-i\Omega_2 t} e^{-(\Gamma_2/2)t} \right. \\ & \left. \times \frac{1}{2} [\mathbb{I} - \mathbb{T}^*] \right\}_{\alpha\beta} a_\beta(0) \\ & + \sqrt{\frac{\bar{\omega}(k)}{2}} \int_0^t \left[G(t') + \frac{i\dot{G}(t')}{\bar{\omega}(k)} \right]_{\alpha\beta} \xi_\beta(t-t') \end{aligned} \quad (2.63)$$

where we have used the initial condition $G_{\alpha\beta}(0) = 0$ [see Eq. (2.33) and (2.49)]. Under these same approximations we find

$$\begin{aligned} G(t') + \frac{i\dot{G}(t')}{\bar{\omega}(k)} \approx & \frac{i}{\bar{\omega}(k)} \left\{ e^{-i\Omega_1 t'} e^{-(\Gamma_1/2)t'} \frac{1}{2} [\mathbb{I} + \mathbb{T}^*] \right. \\ & \left. + e^{-i\Omega_2 t'} e^{-(\Gamma_2/2)t'} \frac{1}{2} [\mathbb{I} - \mathbb{T}^*] \right\}. \end{aligned} \quad (2.64)$$

A. Transition probability

The result (2.63) allows us to obtain the generalized transition probability from expectation values of these operators in the initial density matrix. Denoting $\langle a(t) \rangle = \text{Tr} \hat{\rho} a(t)$ and using the result (2.21) we find

$$\begin{aligned} \langle a_\alpha(t) \rangle = & \left\{ e^{-i\Omega_1 t} e^{-(\Gamma_1/2)t} \frac{1}{2} [\mathbb{I} + \mathbb{T}^*] + e^{-i\Omega_2 t} e^{-(\Gamma_2/2)t} \right. \\ & \left. \times \frac{1}{2} [\mathbb{I} - \mathbb{T}^*] \right\}_{\alpha\beta} \langle a_\beta(0) \rangle. \end{aligned} \quad (2.65)$$

Consider an initial density matrix that yields an initial nonvanishing expectation value for the annihilation operator of the active component, but a vanishing expectation value for the sterile one, namely

$$\langle a_a(0) \rangle \neq 0; \quad \langle a_s(0) \rangle = 0. \quad (2.66)$$

From the form of the matrix \mathbb{T} given by Eq. (2.58) we find the generalized active-sterile transition probability

$$\begin{aligned} \mathcal{P}_{a \rightarrow s}(t) = & \left| \frac{\langle a_s(t) \rangle}{\langle a_a(0) \rangle} \right|^2 \\ = & \frac{\sin^2 2\theta_m}{4r^2} [e^{-\Gamma_1 t} + e^{-\Gamma_2 t} \\ & - 2e^{-(1/2)(\Gamma_1 + \Gamma_2)t} \cos[(\Omega_2 - \Omega_1)t]] \end{aligned} \quad (2.67)$$

where

$$\begin{aligned} \Omega_2 - \Omega_1 = & \frac{\delta M^2 \rho_0 r}{2\bar{\omega}(k)} \cos \alpha; \\ \Gamma_1 + \Gamma_2 = & \frac{\text{Im} \Sigma_{aa}(\bar{\omega}(k))}{\bar{\omega}(k)} = \Gamma_{aa}. \end{aligned} \quad (2.68)$$

and $\Gamma_{1,2}$ are given by Eqs. (2.52) and (2.55). The expression (2.67) is similar to the transition probability for particle-antiparticle mixing of neutral mesons [45,46].

The oscillatory term is a result of the coherent interference between the quasiparticle states in the medium and its exponential suppression in (2.67) identifies the decoherence time scale $\tau_{\text{dec}} = 2/(\Gamma_1 + \Gamma_2) = 2/\Gamma_{aa}$.

B. Weak and strong damping: Quantum Zeno suppression

The above expressions for the propagation frequencies and damping rates of the quasiparticle excitations in the medium lead to two different cases:

$$|\tilde{\gamma}| \ll 1 \Rightarrow \text{weak damping} \quad (2.69)$$

$$|\tilde{\gamma}| \gtrsim 1 \Rightarrow \text{strong damping}. \quad (2.70)$$

These conditions can be written in a more illuminating manner, from the definitions (2.42) and (2.56) it follows that

$$\tilde{\gamma} = \frac{\Gamma_{aa}}{2\Delta E} \quad (2.71)$$

where

$$\Delta E = \frac{\delta M^2 \rho_0}{2\bar{\omega}(k)} \quad (2.72)$$

is the oscillation frequency in the medium in the absence of damping, namely ΔE is given by $|\Omega_2 - \Omega_1|$ setting $\Delta_I = 0$, i.e, the difference in the propagation frequencies only arising from the index of refraction in the medium. The dimensionless quantity $\tilde{\gamma}$ is the ratio between the oscillation time scale $1/\Delta E$ and the decoherence time scale $2/\Gamma_{aa}$. When $\tilde{\gamma} \gg 1$ the environment induced decoherence occurs on time scales much shorter than the oscillation scale and active-sterile oscillations are strongly suppressed. In the opposite limit $\tilde{\gamma} \ll 1$ there are many oscillations before the environment induces decoherence.

The strong damping condition (2.70) is then recognized with the condition for quantum Zeno suppression by scattering in a medium [4,36]. It corresponds to the limit in

which the active mean free path is shorter than the oscillation length and decoherence by the medium suppresses active-sterile oscillations.

1. Weak damping case: $|\tilde{\gamma}| \ll 1$

For weak damping it follows that

$$r \approx 1; \quad \sin\alpha \approx \tilde{\gamma} \cos 2\theta_m \quad (2.73)$$

and the widths $\Gamma_{1,2}$ given by (2.52) and (2.55) become

$$\Gamma_1 = \Gamma_{aa} \cos^2 \theta_m; \quad \Gamma_2 = \Gamma_{aa} \sin^2 \theta_m. \quad (2.74)$$

For the oscillation frequency we obtain

$$\Omega_2 - \Omega_1 = \Delta E = \frac{\delta M^2 \rho_0}{2\bar{\omega}(k)} \quad (2.75)$$

and

$$\begin{aligned} \mathbb{T} &\simeq \begin{pmatrix} \cos 2\theta_m & -\sin 2\theta_m \\ -\sin 2\theta_m & -\cos 2\theta_m \end{pmatrix} \\ &= U^{-1}(\theta_m) \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} U(\theta_m) \end{aligned} \quad (2.76)$$

where $U(\theta)$ is the unitary matrix given by Eq. (2.4). Introducing the Heisenberg annihilation and creation operators in the medium as

$$\begin{pmatrix} a_1(t) \\ a_2(t) \end{pmatrix} = U^{-1}(\theta_m) \begin{pmatrix} a_a(t) \\ a_s(t) \end{pmatrix} \quad (2.77)$$

and similarly with the creation operators, the time evolution (2.65) in the weakly damped case yields

$$\begin{aligned} \begin{pmatrix} \langle a_1(t) \rangle \\ \langle a_2(t) \rangle \end{pmatrix} &= \begin{pmatrix} e^{-i\Omega_1 t} e^{-(\Gamma_1/2)t} & 0 \\ 0 & e^{-i\Omega_2 t} e^{-(\Gamma_2/2)t} \end{pmatrix} \\ &\times \begin{pmatrix} \langle a_a(0) \rangle \\ \langle a_s(0) \rangle \end{pmatrix}. \end{aligned} \quad (2.78)$$

Therefore, in the weak damping regime, the Heisenberg operators $a_{1,2}^\dagger$, $a_{1,2}$ create and annihilate the in-medium states that propagate with frequencies $\Omega_{1,2}$ and their ensemble averages damp out with the widths $\Gamma_{1,2}$. The active-sterile transition probability in this limit, is obtained from Eq. (2.67), and is given by

$$\begin{aligned} \mathcal{P}_{a \rightarrow s}(t) &= \left| \frac{\langle a_s \rangle(t)}{\langle a_a \rangle(0)} \right|^2 \\ &= \frac{\sin^2 2\theta_m}{4} [e^{-\Gamma_1 t} + e^{-\Gamma_2 t} \\ &\quad - 2e^{-(1/2)(\Gamma_1 + \Gamma_2)t} \cos[\Delta E t]]. \end{aligned} \quad (2.79)$$

In the weakly damped case the decoherence time scale $\tau_{\text{dec}} = 2/\Gamma_{aa}$ is much larger than the oscillation time scale $1/\Delta E$, hence many oscillations take place before the interaction with the environment leads to decoherence.

These results reproduce those of Refs. [51–53] and confirm their generality and applicability to the case of

neutrinos with standard model interactions studied in Ref. [53].

2. Strong damping case: $|\tilde{\gamma}| \gg 1$

The case of (very) strong damping yields the following simplifications:

$$r^2 \sim \tilde{\gamma}^2 - 1 + 2\cos^2 2\theta_m \quad (2.80)$$

$$r \sin\alpha \sim \tilde{\gamma} \left[1 - \frac{\sin^2 2\theta_m}{2\tilde{\gamma}^2} \right], \quad (2.81)$$

leading to the damping rates

$$\Gamma_1 \simeq \Gamma_{aa} \left[1 - \frac{\sin^2 2\theta_m}{4\tilde{\gamma}^2} \right] \approx \Gamma_{aa} \quad (2.82)$$

$$\Gamma_2 \simeq \Gamma_{aa} \frac{\sin^2 2\theta_m}{4\tilde{\gamma}^2}. \quad (2.83)$$

This is a remarkable result, the quasiparticle width Γ_2 becomes vanishingly small in the strong damping regime, with important consequences for production of the sterile species as seen below. Furthermore, the oscillation frequency is found to be

$$\Omega_2 - \Omega_1 = \frac{\delta M^2 \rho_0}{2\bar{\omega}(k)} \cos 2\theta_m = \Delta E \cos 2\theta_m, \quad (2.84)$$

this is another remarkable result in the strong damping regime: the oscillation frequency vanishes at the MSW resonance. It follows from Eqs. (2.46), (2.51), and (2.54) that the vanishing of the oscillation frequency at an MSW resonance is an exact result for any $\tilde{\gamma}^2 > 1$. This result implies that there is a degeneracy right at the resonance, and unlike the quantum mechanical case in which there is no level crossing, in the presence of strong environmental damping the two propagating states in the medium become degenerate at the resonance leading to a breakdown of adiabaticity. Furthermore, in this regime the transition probability (2.67) is strongly suppressed by the factor $1/\tilde{\gamma}^2 \ll 1$, it is given by

$$\begin{aligned} \mathcal{P}_{a \rightarrow s}(t) &= \left| \frac{\langle a_s \rangle(t)}{\langle a_a \rangle(0)} \right|^2 \\ &= \frac{\sin^2 2\theta_m}{4\tilde{\gamma}^2} [e^{-\Gamma_1 t} + e^{-\Gamma_2 t} \\ &\quad - 2e^{-(1/2)(\Gamma_1 + \Gamma_2)t} \cos[(\Omega_2 - \Omega_1)t]]. \end{aligned} \quad (2.85)$$

In the strong damping limit $\Delta\Omega = |\Omega_2 - \Omega_1| \leq \Delta E$ hence it follows that $\tau_{\text{dec}} \ll 1/\Delta\Omega$ and the interference term is strongly damped out before one oscillation takes place. This is the quantum Zeno effect in which the rapid scattering in the medium prevents the buildup of coherence [36].

The vanishing of the oscillation frequency, the suppression of the transition probability and Γ_2 in the strong

damping case are all manifestations of the quantum Zeno effect. Of particular importance is the vanishing of the oscillation frequency at the MSW resonance because this entails a breakdown of adiabaticity.

III. PRODUCTION OF THE STERILE SPECIES

The number of sterile particles is given

$$N_s(t) = \langle a_s^\dagger(t) a_s(t) \rangle \quad (3.1)$$

where the Heisenberg operators are given by Eqs. (2.63) and (2.64) and the expectation value is in the density matrix (2.11). Let us consider the case in which the initial density matrix $\hat{\rho}_q$ is diagonal in the flavor basis with initial populations

$$N_a(0) = \langle a_a^\dagger(0) a_a(0) \rangle; \quad N_s(0) = \langle a_s^\dagger(0) a_s(0) \rangle. \quad (3.2)$$

Using the results (2.63) and (2.64) and the stochastic noise given by Eq. (2.16) and (2.20) with the averages (2.17) and (2.21) we find

$$N_s(t) = \mathcal{P}_{a \rightarrow s}(t) N_a(0) + \mathcal{P}_{s \rightarrow s}(t) N_s(0) + N_s^\xi(t) \quad (3.3)$$

where $\mathcal{P}_{a \rightarrow s}(t)$ is the active-sterile transition probability given by Eq. (2.67), and

$$\mathcal{P}_{s \rightarrow s}(t) = |e^{-i\Omega_1 t} e^{-(\Gamma_1/2)t} f_- + e^{-i\Omega_2 t} e^{-(\Gamma_2/2)t} f_+|^2 \quad (3.4)$$

$$f_\pm = \frac{1}{2} \left(1 \pm \frac{e^{-i\alpha}}{r} (\cos 2\theta_m + i\tilde{\gamma}) \right). \quad (3.5)$$

The contribution $N_s^\xi(t)$ is completely determined by the correlation function of the noise in the initial density matrix, it is given by

$$N_s^\xi(t) = \frac{\sin^2 2\theta_m}{4r^2} \int \frac{d\omega}{\pi} \frac{\text{Im} \Sigma_{aa}(\omega)}{2\bar{\omega}(k)} n(\omega) |F_1(\omega; t) - F_2(\omega; t)|^2 \quad (3.6)$$

where $n(\omega) = [e^{\omega/T} - 1]^{-1}$ and

$$F_i(\omega; t) = \frac{e^{-i(\Omega_i - \omega)t} e^{-(\Gamma_i/2)t} - 1}{\omega_i - \omega - \frac{\Gamma_i}{2}}; \quad i = 1, 2. \quad (3.7)$$

The frequency integral is carried out by approximating the functions $F_i(\omega; t)$ as Breit-Wigner Lorentzians near their complex poles, the result is found to be

$$N_s^\xi(t) = \frac{\sin^2 \theta_m \cos^2 \theta_m}{r^2} \left\{ \frac{\Gamma_{aa}}{\Gamma_1} n(\Omega_1) (1 - e^{-\Gamma_1 t}) + \frac{\Gamma_{aa}}{\Gamma_2} n(\Omega_2) (1 - e^{-\Gamma_2 t}) - e^{-(1/2)(\Gamma_1 + \Gamma_2)t} \frac{\Gamma_{aa} [n(\Omega_1) + n(\Omega_2)]}{(\frac{\Gamma_{aa}}{2})^2 + (\Omega_2 - \Omega_1)^2} \times \left[\frac{\Gamma_{aa}}{2} (1 - \cos(\Omega_2 - \Omega_1)t) + (\Omega_2 - \Omega_1) \sin(\Omega_2 - \Omega_1)t \right] \right\} \quad (3.8)$$

The set of Eqs. (2.67), (3.3), (3.4), and (3.8) completely determines the time evolution of the sterile distribution function $N_s(t)$.

A. Weak and strong damping limits

1. Weak damping: $|\tilde{\gamma}| \ll 1$

In the weak damping limit the results above yield

$$r \sim 1; \quad \sin \alpha \sim \mathcal{O}(\tilde{\gamma}); \quad \cos \alpha \sim 1$$

$$\Gamma_1 \sim \Gamma_{aa} \cos^2 \theta_m; \quad \Gamma_2 \sim \Gamma_{aa} \sin^2 \theta_m \quad (3.9)$$

$$\Omega_2 - \Omega_1 \sim \frac{\delta M^2 \rho_0}{2\bar{\omega}(k)} = \Delta E$$

which lead to the following expression for the number density of the sterile species, valid for an initial density matrix diagonal in the flavor basis and with $N_s(0) = 0$, $N_a(0) \neq 0$,

$$N_s(t) = N_a(0) \frac{\sin^2 2\theta_m}{4} \times [e^{-\Gamma_1 t} + e^{-\Gamma_2 t} - 2e^{-(\Gamma_{aa}/2)t} \cos[\Delta E t]] + \sin^2 \theta_m n(\Omega_1) (1 - e^{-\Gamma_1 t}) + \cos^2 \theta_m n(\Omega_2) (1 - e^{-\Gamma_2 t}) + \mathcal{O}(\tilde{\gamma}) \quad (3.10)$$

this result reproduces those in Ref. [52] for $N_s(0) = 0$. We note that the production of the sterile species cannot be described in terms of a simple rate equation in the weak damping case because it depends on several different time scales.

2. Strong damping limit: $|\tilde{\gamma}| \gg 1$

While the strong damping limit $|\tilde{\gamma}| \geq 1$ must be studied numerically, progress can be made in the very strong damping regime $|\tilde{\gamma}| \gg 1$. It will be seen below that this regime is relevant for sterile neutrinos near an MSW resonance. In this regime the above results yield

$$r^2 \sim \tilde{\gamma}^2 \quad \Gamma_1 \sim \Gamma_{aa} \quad \Gamma_2 \sim \Gamma_{aa} \frac{\sin^2 2\theta_m}{4\tilde{\gamma}^2} \quad (3.11)$$

$$\Omega_2 - \Omega_1 \sim \frac{\delta M^2 \rho_0}{2\bar{\omega}(k)} \cos 2\theta_m = \Delta E \cos 2\theta_m.$$

The coefficients

$$\begin{aligned} \frac{1}{2r^2} \frac{\Gamma_{aa}^2}{(\frac{\Gamma_{aa}}{2})^2 + (\Omega_2 - \Omega_1)^2} &= \frac{2}{\tilde{\gamma}^2 + \cos^2 2\theta_m} \sim \frac{2}{\tilde{\gamma}^2} \ll 1 \\ \frac{1}{r^2} \frac{\Gamma_{aa}(\Omega_2 - \Omega_1)}{(\frac{\Gamma_{aa}}{2})^2 + (\Omega_2 - \Omega_1)^2} &= \frac{2 \cos 2\theta_m}{\tilde{\gamma}(\tilde{\gamma}^2 + \cos^2 2\theta_m)} \sim \frac{1}{\tilde{\gamma}^3} \ll 1 \end{aligned} \quad (3.12)$$

therefore the second line in the noise contribution (3.8) becomes subleading. Furthermore the ratios

$$\frac{\Gamma_{aa}}{r^2 \Gamma_1} \sim \frac{1}{\tilde{\gamma}^2} \ll 1 \quad \frac{\Gamma_{aa}}{r^2 \Gamma_2} \sim \frac{4}{\sin^2 2\theta_m} \quad (3.13)$$

therefore, only the term with Γ_{aa}/Γ_2 survives in the first line in (3.8). Since the transition probability in the first term in Eq. (3.3) $\mathcal{P}_{a \rightarrow s} \propto 1/\tilde{\gamma}^2$ [see Eq. (2.67)] this term is also strongly suppressed, therefore in the strong damping limit

$$N_s(t) \sim N_s^{\xi}(t) \sim n(\Omega_2)(1 - e^{-\Gamma_2 t}). \quad (3.14)$$

Hence in this limit the sterile population obeys a simple rate equation

$$\frac{dN_s(t)}{dt} = \Gamma_2[n(\Omega_2) - N_s(t)], \quad (3.15)$$

however the sterile production rate is

$$\Gamma_2 = \Gamma_{aa} \frac{\sin^2 2\theta_m}{4\tilde{\gamma}^2} \ll \Gamma_{aa} \quad (3.16)$$

becoming vanishingly small in the strong damping case. We conclude that sterile species production is strongly suppressed in the strong damping case as a consequence of the quantum Zeno effect. The nonperturbative nature of this result is manifest by writing

$$\Gamma_2 = \sin^2 2\theta_m \frac{(\Delta E)^2}{\Gamma_{aa}}. \quad (3.17)$$

We note that with $\tilde{\gamma} = \Gamma_{aa}/2\Delta E$ [see Eq. (2.71)] this result coincides with the effective rate in the quantum Zeno limit $2\Delta E/\Gamma_{aa} \ll 1$ obtained in Ref. [42] and implemented in the numerical study in Refs. [4,43]. However, we argue below that in the case of sterile neutrinos, the strong damping limit is only available near an MSW resonance, and far away from this resonance the nonequilibrium dynamics corresponds to weak damping and the time evolution of $N_s(t)$ cannot be described by a simple rate equation.

IV. QUANTUM MASTER AND KINETIC EQUATIONS

Although we have obtained the time evolution of the distribution function from the exact solution of the Heisenberg-Langevin equations [under the approximation (2.60)], within the cosmological setting it is more convenient to obtain a set of quantum kinetic equations for the

distribution functions. This is achieved by obtaining first the quantum master equation for the time evolution of the reduced density matrix. In the case of neutrinos, the index of refraction term V_{aa} is of first order in G_F (Fermi's effective weak coupling) while the self-energy $\Sigma = \Sigma_R + i\Sigma_I$ is of second order. Furthermore the study in the previous sections clearly shows that the contribution of the real part of the self-energy yields a second order renormalization of the index of refraction which can be simply absorbed into a redefinition of V_{aa} . The most important aspect of the second order self-energy correction arises from its imaginary part, which yields the damping rates of the collective quasiparticle excitations. The production of the sterile species is associated with this imaginary part, and not the real part of the self-energy, which only renormalizes the index of refraction in the medium. Therefore it is convenient to include the index of refraction in the “non-interacting” part of the Hamiltonian by first diagonalizing the Hamiltonian for the system's degrees of freedom \vec{q} corresponding to the first term in the Lagrangian (2.1). This is achieved by introducing the mass eigenstates in the medium with the index of refraction as follows. The matrix \mathbb{K} in Eq. (2.10) can be written as

$$\begin{aligned} \mathbb{K} &= \left(k^2 + \bar{M}^2 + \frac{V_{aa}}{2} \right) \mathbb{1} + \frac{\delta M^2 \rho_0}{2} \\ &\times \begin{bmatrix} -\cos 2\theta_m & \sin 2\theta_m \\ \sin 2\theta_m & \cos 2\theta_m \end{bmatrix}, \end{aligned} \quad (4.1)$$

where the expressions for ρ_0 and the mixing angle in the medium are the same as (2.44) and (2.47) but neglecting the second order correction $\text{Re}\Sigma_{aa}$ to the index of refraction. The diagonalization of the Hamiltonian is achieved via the unitary transformation (2.4) but in terms of the mixing angle in the medium θ_m that includes the correction from the index of refraction, namely

$$\begin{pmatrix} q_a \\ q_s \end{pmatrix} = U(\theta_m) \begin{pmatrix} q_1 \\ q_2 \end{pmatrix}; \quad U(\theta) = \begin{pmatrix} \cos \theta_m & \sin \theta_m \\ -\sin \theta_m & \cos \theta_m \end{pmatrix}. \quad (4.2)$$

Again to avoid proliferation of indices we refer to the coordinates that diagonalize the Hamiltonian with the index of refraction with the labels 1, 2, which now should not be identified with those labeling the complex poles in Sec. (II).

Expanding $q_{1,2}$ and their canonical momenta $p_{1,2}$ in terms of Heisenberg annihilation and creation operators

$$q_i = \frac{1}{\sqrt{2\omega_i}} [a_i + a_i^\dagger]; \quad p_i = -i \frac{\omega_i}{\sqrt{2\omega_i}} [a_i - a_i^\dagger] \quad (4.3)$$

where the frequencies in the medium are

$$\begin{aligned}\omega_1 &\sim \bar{\omega}(k) + \frac{V_{aa}}{4\bar{\omega}(k)} - \frac{\delta M^2 \rho_0}{4\bar{\omega}(k)} \\ \omega_2 &\sim \bar{\omega}(k) + \frac{V_{aa}}{4\bar{\omega}(k)} + \frac{\delta M^2 \rho_0}{4\bar{\omega}(k)}.\end{aligned}\quad (4.4)$$

Under the approximation (2.49) the active and sterile annihilation (and creation) operators $a_{a,s}$ are related to $a_{1,2}$ as

$$a_a = \cos\theta_m a_1 + \sin\theta_m a_2; \quad a_s = \cos\theta_m a_2 - \sin\theta_m a_1. \quad (4.5)$$

The total system-bath Hamiltonian becomes $H = H_0 + H_I$ where

$$H_0 = \sum_{i=1,2} a_i^\dagger a_i \omega_i + \sum_p \frac{1}{2} [P_p^2 + W_p^2 Q_p^2] \quad (4.6)$$

$$H_I = (q_1 \cos\theta_m + q_2 \sin\theta_m) \sum_p C_p Q_p. \quad (4.7)$$

The density matrix in the interaction picture of H_0 is

$$\hat{\rho}_i(t) = e^{iH_0 t} e^{-iH_I t} \hat{\rho}(0) e^{iH_I t} e^{-iH_0 t} \quad (4.8)$$

where $\hat{\rho}(0)$ is given by Eq. (2.11). The equation of motion of the density matrix in the interaction picture is

$$\frac{d\hat{\rho}_i(t)}{dt} = -i[H_I(t), \hat{\rho}_i(t)] \quad (4.9)$$

with $H_I(t) = e^{iH_0 t} H_I e^{-iH_0 t}$ is the interaction Hamiltonian in the interaction picture of H_0 . Iteration of this equation up to second order in the interaction yields [59]

$$\begin{aligned}\frac{d\hat{\rho}_i(t)}{dt} &= -i[H_I(t), \hat{\rho}_i(0)] - \int_0^t dt' [H_I(t), [H_I(t'), \hat{\rho}_i(t')]] \\ &+ \dots\end{aligned}\quad (4.10)$$

The reduced density matrix for the system's variables q is obtained from the total density matrix by tracing over the bath degrees of freedom Q_p , which are assumed to remain in equilibrium [59]. The following standard approximations are invoked [59]: (a) *factorization*: the total density matrix is assumed to factorize

$$\hat{\rho}_i(t) = \rho_{q,i}(t) \otimes \rho_Q(0) \quad (4.11)$$

where it is assumed that the bath remains in equilibrium. (b) *Markovian approximation*: the memory of the evolution is neglected and in the double commutator in (4.10) $\hat{\rho}_i(t')$ is replaced by $\hat{\rho}_i(t)$ and taken out of the integral [59]. Taking the trace over the bath degrees of freedom yields the quantum master equation for the reduced density matrix,

$$\frac{d\rho_R(t)}{dt} = - \int_0^t dt' \text{Tr} \rho_Q \{ [H_I(t), [H_I(t'), \hat{\rho}_i(t)]] \} + \dots \quad (4.12)$$

where the first term has vanished because

$\text{Tr} \rho_Q(0) Q_p^{(0)}(t) = 0$ since $Q_p^{(0)}(t)$ is a free harmonic oscillator in the interaction picture of H_0 [see Eq. (2.16)]. The trace over Q in the double commutator requires the following ingredients

$$\begin{aligned}\sum_{p,p'} \frac{C_p C_{p'}}{\sqrt{4W_p W_{p'}}} \text{Tr} \rho_Q(0) Q_p^{(0)}(t) Q_{p'}^{(0)}(t') \\ = \sum_p \frac{C_p^2}{2W_p} [(1 + n(W_p)) e^{-iW_p(t-t')} + n(W_p) e^{iW_p(t-t')}] \\ = \int \frac{d\omega}{\pi} \text{Im} \Sigma_{aa}(\omega) (1 + n(\omega)) e^{-i\omega(t-t')}\end{aligned}\quad (4.13)$$

$$\begin{aligned}\sum_{p,p'} \frac{C_p C_{p'}}{\sqrt{4W_p W_{p'}}} \text{Tr} \rho_Q(0) Q_{p'}^{(0)}(t') Q_p^{(0)}(t) \\ = \sum_p \frac{C_p^2}{2W_p} [(1 + n(W_p)) e^{-iW_p(t'-t)} + n(W_p) e^{iW_p(t'-t)}] \\ = \int \frac{d\omega}{\pi} \text{Im} \Sigma_{aa}(\omega) n(\omega) e^{-i\omega(t-t')}\end{aligned}\quad (4.14)$$

where the interaction picture operators $Q^{(0)}(t)$ are given by Eq. (2.16) and we have used Eqs. (2.23) and (2.24).

Several standard approximations are invoked: terms that feature rapidly varying phases of the form $a_i^\dagger a_j^\dagger e^{i(\omega_i + \omega_j)t}$ and $a_i a_j e^{-i(\omega_i + \omega_j)t}$ are averaged out in time leading to their cancellation, in the quantum optics literature this is known as the ‘‘rotating wave approximation’’ [59], similar terms are discarded in the kinetic approach in Refs. [25,39]. The time integrals are evaluated in the Weisskopf-Wigner approximation [52,59]. Finally we also invoke the ultrarelativistic approximation $\omega_1 \sim \omega_2 \sim \bar{\omega}(k)$. Neglecting the second order energy shift [see Eq. (2.29)], the final result for the quantum master equation is given by

$$\begin{aligned}\frac{d\rho_R(t)}{dt} &= - \frac{\Gamma_{aa}}{2} \left\{ \cos^2\theta_m \mathcal{L}_{11}[\rho_R] + \sin^2\theta_m \mathcal{L}_{22}[\rho_R] \right. \\ &\quad \left. + \frac{1}{2} \sin 2\theta_m (\mathcal{L}_{12}[\rho_R] + \mathcal{L}_{21}[\rho_R]) \right\}\end{aligned}\quad (4.15)$$

where $\mathcal{L}_{ij}[\rho_R]$ are the Lindblad operators [59]

$$\begin{aligned}\mathcal{L}_{ij}[\rho_R] &= (1 + n(\omega_i)) [\rho_R a_i^\dagger a_j + a_j^\dagger a_i \rho_R - a_i \rho_R a_j^\dagger \\ &\quad - a_j \rho_R a_i^\dagger] + n(\omega_i) [\rho_R a_i a_j^\dagger + a_j a_i^\dagger \rho_R \\ &\quad - a_i^\dagger \rho_R a_j - a_j^\dagger \rho_R a_i].\end{aligned}\quad (4.16)$$

In these expressions, the annihilation and creation operators carry the time dependence in the interaction picture, namely

$$a_i^\dagger(t) = a_i^\dagger(0) e^{i\omega_i t}; \quad a_i(t) = a_i(0) e^{-i\omega_i t}. \quad (4.17)$$

The trace of the reduced density matrix is automatically conserved in time as a consequence of unitary time evolution of the full density matrix. Denoting the expectation

value of any interaction picture operator $A(t)$ in the reduced density matrix by

$$\langle A \rangle(t) = \text{Tr} \rho_R(t) A(t), \quad (4.18)$$

we obtain the following equations for the expectation values of the annihilation operators

$$\begin{aligned} \frac{d}{dt} \begin{pmatrix} \langle a_1 \rangle(t) \\ \langle a_2 \rangle(t) \end{pmatrix} &= \begin{pmatrix} -i\omega_1 - \frac{\Gamma_{aa}}{2} \cos^2 \theta_m & -\frac{\Gamma_{aa}}{4} \sin 2\theta_m \\ -\frac{\Gamma_{aa}}{4} \sin 2\theta_m & -i\omega_2 - \frac{\Gamma_{aa}}{2} \sin^2 \theta_m \end{pmatrix} \\ &\times \begin{pmatrix} \langle a_1 \rangle(t) \\ \langle a_2 \rangle(t) \end{pmatrix} \end{aligned} \quad (4.19)$$

The eigenvalues of the matrix in Eq. (4.19) are found to be $-i\tilde{\Omega}_{1,2} - \Gamma_{1,2}/2$ where $\tilde{\Omega}_{1,2}$ are obtained from Eqs. (2.51) and (2.54) by setting the second order contribution to the energy shift $\text{Re}\Sigma_{aa} = 0$, and $\Gamma_{1,2}$ are precisely given by Eqs. (2.52) and (2.55) but again setting $\text{Re}\Sigma_{aa} = 0$ in ρ_0 , which of course is a consequence of having neglected the second order energy shifts (real part of the self-energy) in the quantum master equation. It is a straightforward exercise to obtain the (complex) eigenvectors of the matrix (4.19) and to write $\langle a_{a,s} \rangle$ in terms of these through the relation (4.5). Fixing the initial values of the corresponding eigenvectors to yield the initial values $\langle a_a \rangle(0) \neq 0$; $\langle a_s \rangle(0) = 0$ we find

$$\begin{aligned} \mathcal{P}_{a \rightarrow s}(t) &= \left| \frac{\langle a_s \rangle(t)}{\langle a_a \rangle(0)} \right|^2 \\ &= \frac{\sin^2 2\theta_m}{4r^2} [e^{-\Gamma_1 t} + e^{-\Gamma_2 t} \\ &\quad - 2e^{-(1/2)(\Gamma_1 + \Gamma_2)t} \cos[(\tilde{\Omega}_2 - \tilde{\Omega}_1)t]] \end{aligned} \quad (4.20)$$

which is the same as the transition probability (2.67) but neglecting the second order correction from $\text{Re}\Sigma_{aa}$. These results clearly show that the quantum master Eq. (4.15) correctly describes the nonequilibrium dynamics including the strong damping regime, the only difference with the exact result being that the second order energy shift $\text{Re}\Sigma_{aa}$ is neglected. The quantum master equation (4.15) is exactly the same as the one obtained in Ref. [52].

We now introduce the distribution functions

$$n_{ij} = \text{Tr} \rho_R(t) a_i^\dagger(t) a_j(t), \quad (4.21)$$

the diagonal components describe the population of the in-medium states, and the off-diagonal components the coherences [59]. Accounting for the free field time dependence of the operators a^\dagger, a in the interaction picture, we find the following kinetic equations for the distribution functions

$$\dot{n}_{11} = -\Gamma_{aa} \left\{ \cos^2 \theta_m (n_{11} - n(\omega_1)) + \frac{\sin 2\theta_m}{4} (n_{12} + n_{12}^*) \right\} \quad (4.22)$$

$$\dot{n}_{22} = -\Gamma_{aa} \left\{ \sin^2 \theta_m (n_{22} - n(\omega_2)) + \frac{\sin 2\theta_m}{4} (n_{12} + n_{12}^*) \right\} \quad (4.23)$$

$$\begin{aligned} \dot{n}_{12} &= -i(\omega_2 - \omega_1)n_{12} - \frac{\Gamma_{aa}}{2} \left[n_{12} + \frac{\sin 2\theta_m}{2} \right. \\ &\quad \left. \times (n_{11} + n_{22} - n(\omega_1) - n(\omega_2)) \right] \end{aligned} \quad (4.24)$$

where $n(\omega_i)$ are the equilibrium distribution functions. In terms of the $n_{ij}(t)$ we obtain the time evolution of the active and sterile distribution functions via the relation (4.5), namely

$$\begin{aligned} N_a(t) &= \cos^2 \theta_m n_{11}(t) + \sin^2 \theta_m n_{22}(t) \\ &\quad + \frac{1}{2} \sin 2\theta_m (n_{12}(t) + n_{12}^*(t)) \end{aligned} \quad (4.25)$$

$$\begin{aligned} N_s(t) &= \sin^2 \theta_m n_{11}(t) + \cos^2 \theta_m n_{22}(t) - \frac{1}{2} \sin 2\theta_m (n_{12}(t) \\ &\quad + n_{12}^*(t)). \end{aligned} \quad (4.26)$$

The weak damping limit can be studied in a perturbative expansion in $\tilde{\gamma} \ll 1$ by considering the terms n_{12}, n_{12}^* in Eqs. (4.22) and (4.23) and the terms $n_{ij} - n(\omega_i); i = 1, 2$ in Eq. (4.24) as perturbations. This study was carried out in Ref. [52] and reproduces the result Eq. (3.10) for the sterile population. Therefore the set of quantum kinetic Eqs. (4.22), (4.23), and (4.24) reproduce the exact results both in the weak and strong damping cases.

We can now establish a correspondence with the quantum kinetic equation often quoted in the literature [25,42,43,64,65] by introducing the following ‘‘polarization vector’’ [66]

$$P_0(t) = \langle a_a^\dagger a_a + a_s^\dagger a_s \rangle(t) = N_a(t) + N_s(t) \quad (4.27)$$

$$P_x(t) = \langle a_a^\dagger a_s + a_s^\dagger a_a \rangle(t) \quad (4.28)$$

$$P_y(t) = -i \langle a_a^\dagger a_s - a_s^\dagger a_a \rangle(t) \quad (4.29)$$

$$P_z(t) = \langle a_a^\dagger a_a - a_s^\dagger a_s \rangle(t) = N_a(t) - N_s(t) \quad (4.30)$$

where the creation and annihilation operators for the active and sterile fields are related to those that create and annihilate the propagating modes in the medium 1, 2 by Eq. (4.5), and the angular brackets denote expectation values in the reduced density matrix ρ_R which obeys the quantum master Eq. (4.15). In terms of the population and coherences n_{ij} the elements of the polarization vector are given by

$$P_0 = n_{11} + n_{22} \quad (4.31)$$

$$P_x = -\sin 2\theta_m (n_{11} - n_{22}) + \cos 2\theta_m (n_{12} + n_{12}^*) \quad (4.32)$$

$$P_y = -i(n_{12} - n_{12}^*) \quad (4.33)$$

$$P_z = \cos 2\theta_m(n_{11} - n_{22}) + \sin 2\theta_m(n_{12} + n_{12}^*). \quad (4.34)$$

Using the quantum kinetic Eqs. (4.22), (4.23), and (4.24) we find

$$\begin{aligned} \frac{dP_0}{dt} = & -\frac{\Gamma_{aa}}{2} P_z - \frac{\Gamma_{aa}}{2} [(n_{11} - n(\omega_1)) + (n_{22} - n(\omega_2))] \\ & + \frac{\Gamma_{aa}}{2} \cos 2\theta_m (n(\omega_1) - n(\omega_2)) \end{aligned} \quad (4.35)$$

$$\begin{aligned} \frac{dP_x}{dt} = & -i(\omega_2 - \omega_1) \cos 2\theta_m (n_{12} - n_{12}^*) - \frac{\Gamma_{aa}}{2} P_x \\ & - \frac{\Gamma_{aa}}{2} \sin 2\theta_m (n(\omega_1) - n(\omega_2)) \end{aligned} \quad (4.36)$$

$$\frac{dP_y}{dt} = -(\omega_2 - \omega_1)(n_{12} + n_{12}^*) - \frac{\Gamma_{aa}}{2} P_y \quad (4.37)$$

$$\begin{aligned} \frac{dP_z}{dt} = & -i(\omega_2 - \omega_1) \sin 2\theta_m (n_{12} - n_{12}^*) - \frac{\Gamma_{aa}}{2} P_z \\ & - \frac{\Gamma_{aa}}{2} [(n_{11} - n(\omega_1)) + (n_{22} - n(\omega_2))]. \end{aligned} \quad (4.38)$$

Under the approximation $\omega_1 \sim \omega_2 \sim \bar{\omega}(k)$ we can take

$$(n(\omega_1) - n(\omega_2)) \sim 0, \quad (4.39)$$

and neglect the last terms in Eqs. (4.35) and (4.36). Introducing the vector \vec{V} with components

$$\vec{V} = (\omega_2 - \omega_1)(\sin 2\theta_m, 0, -\cos 2\theta_m) \quad (4.40)$$

we find the following equations of motion for the polarization vector

$$\frac{d\vec{P}}{dt} = \vec{V} \times \vec{P} - \frac{\Gamma_{aa}}{2} (P_x \hat{x} + P_y \hat{y}) + \frac{dP_0}{dt} \hat{z}. \quad (4.41)$$

This equation is exactly of the form

$$\frac{d\vec{P}}{dt} = \vec{V} \times \vec{P} - D\vec{P}_T + \frac{dP_0}{dt} \hat{z} \quad (4.42)$$

often used in the literature [36,42,43,64,65], where

$$D = \frac{\Gamma_{aa}}{2}; \quad \vec{P}_T = (P_x \hat{x} + P_y \hat{y}). \quad (4.43)$$

Therefore the quantum kinetic equation for the polarization vector (4.41) is equivalent to the full set of quantum kinetic equations (4.22), (4.23), and (4.24).

However it must be highlighted that the set of equations (4.41) and (4.42) is not closed because it must input the time evolution of P_0 which is obtained from the full set of kinetic equations (4.22), (4.23), and (4.24).

Often the last term in (4.42) (\dot{P}_0) is omitted, however, such omission is not warranted, since it follows from the definition of P_0 , Eqs. (4.25), (4.26), and (4.31), that

$$P_0 = N_a(t) + N_s(t), \quad (4.44)$$

therefore \dot{P}_0 vanishes only when both the active and the sterile species have reached equilibrium. Thus we advocate that the set of kinetic equations (4.22), (4.23), and (4.24) combined with the relations (4.25) and (4.26) provide a complete description of active and sterile production.

V. CONSEQUENCES FOR COSMOLOGICAL PRODUCTION OF STERILE NEUTRINOS

The results obtained above can be straightforwardly adapted to the case of neutrinos by replacing the equilibrium distributions $n(\Omega_{1,2})$ by the Fermi-Dirac distributions in the ultrarelativistic limit and the matter potential from forward scattering in the medium.

While in general $\tilde{\gamma}$, $\Gamma_{1,2}$ and $\Omega_{1,2}$ depend on the details of the interactions, masses and vacuum mixing angles, an assessment of the consequences of the results obtained above on cosmological sterile neutrino production can be obtained for an active neutrino with standard model interactions. In this case the matter potential for temperatures features a CP -odd contribution proportional to the lepton and baryon asymmetries, and a CP -even contribution that depends solely on momentum and temperature. In the ultrarelativistic limit with $\bar{\omega}(k) \sim k$ the matter potential for neutrinos is given by [60,61,67],

$$V_{aa} = \frac{4\sqrt{2}\xi(3)}{\pi^2} G_F k T^3 \left[L - A \frac{Tk}{M_W^2} \right] \quad (5.1)$$

where L is proportional to the lepton and baryon asymmetries and $A \sim 10$ [60,61], for antineutrinos $L \rightarrow -L$. The active neutrino interaction rate (neglecting contributions from the lepton and baryon asymmetries) is given by [25,40,41,60,61]

$$\Gamma_{aa} \sim G_F^2 T^4 k. \quad (5.2)$$

For keV sterile neutrinos an MSW resonance is available only for $L \gg Tk/M_W^2$ when the first term in the bracket in (5.1) dominates [4,6,43,61], while no resonance is available when the second term dominates. We will analyze separately the two different cases

$$L \ll \frac{T^2}{M_W^2} \quad (5.3)$$

$$L \gg \frac{T^2}{M_W^2} \quad (5.4)$$

where we have taken $k \sim T$. In the first case no MSW resonance is possible for keV sterile neutrinos, whereas such resonance is possible in the second case [4,6,43,61].

- (i) *High temperature limit:* At high temperature above the MSW resonance for $V_{aa} \gg \delta M^2$ and neglecting the second order correction to the matter potential ($\text{Re}\Sigma$),

$$\rho_0 \sim \frac{V_{aa}}{\delta M^2}. \quad (5.5)$$

For $L \ll T^2/M_W^2$

$$\frac{\delta M^2 \rho_0}{\bar{\omega}(k)} \sim \frac{G_F T^5}{M_W^2} \quad (5.6)$$

and the ratio

$$\tilde{\gamma} = \left| \frac{\Gamma_{aa}}{\frac{\delta M^2}{\bar{\omega}(k)} \rho_0} \right| \sim G_F M_W^2 \sim \alpha_w \ll 1 \quad (5.7)$$

where α_w is the standard model ‘‘fine structure constant.’’ For $L \gg T^2/M_W^2$ a similar analysis yields

$$\tilde{\gamma} \sim G_F M_W^2 \left(\frac{T^2}{LM_W^2} \right) \sim \alpha_w \left(\frac{T^2}{LM_W^2} \right) \ll 1. \quad (5.8)$$

- (ii) *Low temperature limit:* In the low temperature regime for $V_{aa} \ll \delta M^2$, $\rho_0 \sim 1$ and $\tilde{\gamma}$ becomes

$$\left| \frac{\text{Im} \Sigma_{aa}}{\delta M^2 \rho_0} \right| \sim \left| \frac{\text{Im} \Sigma_{aa}}{\delta M^2} \right| \quad (5.9)$$

however in perturbation theory $V_{aa} \gg \text{Im} \Sigma$ since V_{aa} is of $\mathcal{O}(G_F)$ and $\text{Im} \Sigma_{aa} \sim \mathcal{O}(G_F^2)$. Therefore since in this regime

$$\delta M^2 \gg V_{aa} \gg \text{Im} \Sigma_{aa} \Rightarrow \left| \frac{\text{Im} \Sigma_{aa}}{\delta M^2} \right| \ll 1. \quad (5.10)$$

The conclusion of this analysis is that far away from an MSW resonance, either in the high or low temperature limit damping is weak, namely, at high or low temperature away from the MSW resonance

$$\tilde{\gamma} = \frac{\Gamma_{aa}}{2\Delta E} \ll 1. \quad (5.11)$$

Therefore the strong damping condition may only be fulfilled near an MSW resonance $\theta_m \sim \pi/4$ in which case $\rho_0 \approx |\sin 2\theta|$.

- (iii) *Near an MSW resonance:* As mentioned above a resonance is only possible for keV sterile neutrinos for $L \gg T^2/M_W^2$ [4,6,43,61]. For very small vacuum mixing angle $\sin 2\theta \ll 1$ it proves illuminating to write the resonance condition $\cos 2\theta = V_{aa}/\delta M^2$ as $V_{aa} \sim \delta M^2$ and $\rho_0 \sim |\sin 2\theta|$, with V_{aa} given by Eq. (5.1) for $L \gg T^2/M_W^2$. Therefore $\delta M^2/k \sim G_F T^3 L$, hence using Eq. (5.2) near the MSW resonance, the ratio

$$\left| \frac{\Gamma_{aa}}{\frac{\delta M^2}{\bar{\omega}(k)} \rho_0} \right| \sim \frac{G_F M_W^2}{|\sin 2\theta|} \left(\frac{T^2}{LM_W^2} \right) \sim \frac{\alpha_w}{|\sin 2\theta|} \left(\frac{T^2}{LM_W^2} \right). \quad (5.12)$$

Therefore, the strong damping condition near the resonance is fulfilled provided that $|\sin 2\theta| \ll \alpha_w$. With $\alpha_w \sim 10^{-2}$ the region near an MSW resonance is generally described by the strong damping regime for $|\sin 2\theta| \lesssim 10^{-3}$, which is likely to be the case for sterile neutrinos [4,21] and is consistent with constraints from the x-ray background [5,9,30–32].

In the resonance region the sterile production rate is described by the simple rate equation [see Eq. (3.15)]

$$\dot{N}_s(t) = -\Gamma_2 [N_s(t) - n_{\text{eq}}] \quad (5.13)$$

where the sterile production rate Γ_2 is given by Eq. (3.16) which can be written as

$$\Gamma_2 \sim \sin^2 2\theta \frac{(\delta M^2)^2}{\bar{\omega}(k)^2 \Gamma_{aa}} \quad (5.14)$$

and clearly exhibits the suppression for small vacuum mixing angle and the nonperturbative nature as a function of Γ_{aa} .

This analysis leads to the conclusion that away from an MSW resonance the weak damping condition holds, sterile neutrino production cannot be described by a simple rate equation but involves $\Gamma_{1,2}$ and ΔE . In this regime the quantum kinetic equations (4.22), (4.23), and (4.24) may be simplified [52] by neglecting the terms with n_{12} , n_{12}^* in Eqs. (4.22) and (4.23) and the terms with $n_{11} - n(\omega_1)$; $n_{22} - n(\omega_2)$ in Eq. (4.24). The resulting equations are very simple and their solutions feature the two damping rates $\Gamma_1 = \Gamma_{aa} \cos^2 \theta_m$; $\Gamma_2 = \Gamma_{aa} \sin^2 \theta_m$. This simplification also holds if the lepton asymmetry is of the same order of the baryon asymmetry $L \sim 10^{-9}$ in which case $L \ll T^2/M^2$ for $T \gtrsim 3$ MeV [10,67] and no MSW resonance is available [43,60,61]. Near an MSW resonance for sterile neutrinos with \sim keV mass and $\sin 2\theta \lesssim 10^{-3}$ the strong damping condition holds and $N_s(t)$ obeys a simple rate equation, but the sterile production rate is suppressed by the quantum Zeno effect.

For keV sterile neutrinos with small mixing angle $\sin 2\theta \lesssim 10^{-3}$, the MSW resonance occurs near the scale of the QCD phase transition $T \sim 180$ MeV [4,44] with the inherent uncertainties arising from strong interactions and the rapid change in the effective number of relativistic degrees of freedom in a regime in which hadronization becomes important. However, as argued above, near the MSW resonance the strong damping condition is fulfilled and quantum Zeno suppression hinders the production of sterile neutrinos. As discussed above the sterile distribution function obeys a simple rate equation with a production rate given by Eq. (3.16) or alternatively (3.17) which is strongly suppressed by the factor $1/\tilde{\gamma}^2 \sim \sin^2 2\theta/\alpha_w^2 \ll 1$. This suppression of the sterile production rate makes the production mechanism less efficient near the resonance, thus relieving the uncertainties associated with the strong

interactions, although these remain in the nonresonant scenario [1].

VI. CONCLUSIONS

The production of a sterile species via active-sterile mixing has been studied in a simple, exactly solvable model that includes all the relevant ingredients: active-sterile mixing via an off-diagonal mass matrix and the coupling of the active species to a bath in thermal equilibrium. The exact solution of the Heisenberg-Langevin equations allows us to obtain the exact time evolution of the distribution function for the sterile species and the active-sterile transition probability. Both are determined by the dispersion relations and damping rates (widths) of the two quasiparticle modes in the medium. These depend on

$$\tilde{\gamma} = \frac{\Gamma_{aa}}{2\Delta E} \quad (6.1)$$

where Γ_{aa} is the interaction rate of the active species in the absence of mixing and ΔE is the oscillation frequency with corrections from forward scattering (the index of refraction) but no damping. $\tilde{\gamma} \ll 1$; $\tilde{\gamma} \gg 1$ correspond to the weak and strong damping regimes, respectively. In the weak damping case the damping rates are $\Gamma_1 = \Gamma_{aa}\cos^2\theta_m$; $\Gamma_2 = \Gamma_{aa}\sin^2\theta_m$ the active-sterile transition probability is given by Eq. (2.79), and the time evolution of the sterile distribution function is given by Eq. (3.10) for vanishing initial sterile population, both feature these two scales along with the oscillation time scale. As a result, the time evolution of the sterile distribution function does not obey a simple rate equation. These results confirm those of Refs. [51–53]. The exact solution allows the systematic exploration of the strong damping case for which $\tilde{\gamma} \gg 1$ corresponding to the situation in which the interaction rate in the medium is faster than the oscillation time scale and the quantum Zeno effect is present [36]. In this regime we find that the damping rates of the quasiparticles are $\Gamma_1 = \Gamma_{aa}$; $\Gamma_2 = \Gamma_{aa}\sin^2 2\theta_m/4\tilde{\gamma}^2$ where θ_m is the mixing angle in the medium. The active-sterile (generalized) transition probability is

$$\mathcal{P}_{a \rightarrow s} = \frac{\sin^2 2\theta_m}{4\tilde{\gamma}^2} [e^{-\Gamma_1 t} + e^{-\Gamma_2 t} - 2e^{-(1/2)(\Gamma_1 + \Gamma_2)t} \cos[(\Omega_1 - \Omega_2)t].$$

In the strong damping regime the oscillation frequency $\Omega_1 - \Omega_2 \propto \cos 2\theta_m$ vanishes at an MSW resonance and the two quasiparticle states become degenerate leading to a breakdown of adiabaticity. The sterile distribution function obeys a simple rate equation with a sterile production rate Γ_2 strongly suppressed for $\tilde{\gamma}^2 \gg 1$. The suppression of the active-sterile transition probability and the sterile production rate, and the vanishing of the oscillation frequency in the strong damping limit are all consequences of quantum

Zeno suppression. The quantum master equation for the reduced density matrix is derived and shown to be valid in both limits. From it we obtain the complete set of quantum kinetic equations that yield the nonequilibrium evolution of the active and sterile distribution functions. The complete nonequilibrium time evolution of the active and sterile distribution functions and the coherences are given by the set of equations (4.22), (4.23), and (4.24) along with the identifications (4.25) and (4.26). The set of kinetic equations (4.22), (4.23), and (4.24) is shown to be equivalent to the kinetic equations for the ‘‘polarization vector’’ often quoted in the literature. However, unlike these the set (4.22), (4.23), and (4.24) along with (4.25) and (4.26) yield a complete description of the nonequilibrium dynamics amenable to a straightforward numerical analysis, the extrapolation to fermionic degrees of freedom is a straightforward replacement of the equilibrium distribution functions by the Fermi-Dirac distributions. Furthermore, the analysis based on the exact solution and the quantum master equation yields a wealth of information that cannot be easily gleaned from the set of kinetic equations, for example, the active-sterile transition probability.

For active neutrinos with standard model interactions it is shown that the weak damping limit describes the parameter range away from an MSW resonance and that the strong damping limit only emerges near the resonance for very small vacuum mixing angle, such that $\sin 2\theta \lesssim \alpha_w \sim 10^{-2}$. Such small value is consistent with constraints from the x-ray background. This result bears important consequences for cosmological sterile neutrino production. In the resonant production mechanism of Ref. [1] the production rate peaks at the MSW resonance, however our analysis, which includes consistently the damping corrections, shows that quantum Zeno suppression hinders the sterile production rate near the resonance. For keV sterile neutrinos the MSW resonance occurs in a temperature range too close to the QCD phase transition. Hadronization and strong interactions lead to substantial uncertainties during this temperature regime which translate into uncertainties in the production rate. Quantum Zeno suppression of the production rate in this regime relieves these uncertainties.

In summary, the set of kinetic equations (4.22), (4.23), and (4.24) (with Fermi-Dirac equilibrium distributions) along with the relations (4.25) and (4.26) yield a complete description of the nonequilibrium dynamics of active and sterile neutrino production valid in the weak and strong damping limits. Quantum Zeno suppression is operative near an MSW resonance and suppresses the sterile production rate, thus relieving potential uncertainties associated with the QCD phase transition for keV neutrinos.

ACKNOWLEDGMENTS

The author thanks C.-M. Ho for fruitful discussions and acknowledges support from the U.S. National Science Foundation through Grant No. PHY-0553418.

- [1] S. Dodelson and L. M. Widrow, Phys. Rev. Lett. **72**, 17 (1994).
- [2] T. Asaka, M. Shaposhnikov, and A. Kusenko, Phys. Lett. B **638**, 401 (2006).
- [3] X. Shi and G. M. Fuller, Phys. Rev. Lett. **83**, 3120 (1999).
- [4] K. Abazajian, G. M. Fuller, and M. Patel, Phys. Rev. D **64**, 023501 (2001).
- [5] A. D. Dolgov and S. H. Hansen, Astropart. Phys. **16**, 339 (2002).
- [6] K. Abazajian and G. M. Fuller, Phys. Rev. D **66**, 023526 (2002).
- [7] K. Abazajian, Phys. Rev. D **73**, 063506 (2006); **73**, 063513 (2006).
- [8] P. Biermann and A. Kusenko, Phys. Rev. Lett. **96**, 091301 (2006).
- [9] K. Abazajian and S. M. Koushiappas, Phys. Rev. D **74**, 023527 (2006).
- [10] A. D. Dolgov, Phys. Rep. **370**, 333 (2002); Surv. High Energy Phys. **17**, 91 (2002).
- [11] J. Lesgourgues and S. Pastor, Phys. Rep. **429**, 307 (2006).
- [12] S. Hannestad, Annu. Rev. Nucl. Part. Sci. **56**, 137 (2006).
- [13] P. L. Biermann and F. Munyaneza, arXiv:astro-ph/0702173; arXiv:astro-ph/0702164; F. Munyaneza and P. L. Biermann, arXiv:astro-ph/0609388; J. Stasielak, P. L. Biermann, and A. Kusenko, Astrophys. J. **654**, 290 (2007).
- [14] M. Shaposhnikov, arXiv:astro-ph/0703673.
- [15] G. Raffelt and G. Sigl, Astropart. Phys. **1**, 165 (1993).
- [16] J. Hidaka and G. M. Fuller, Phys. Rev. D **74**, 125015 (2006).
- [17] C. J. Smith, G. M. Fuller, C. T. Kishimoto, and K. Abazajian, Phys. Rev. D **74**, 085008 (2006).
- [18] C. T. Kishimoto, G. M. Fuller, and C. J. Smith, Phys. Rev. Lett. **97**, 141301 (2006).
- [19] A. Kusenko and G. Segre, Phys. Rev. D **59**, 061302 (1999).
- [20] G. M. Fuller, A. Kusenko, I. Mociouiu, and S. Pascoli, Phys. Rev. D **68**, 103002 (2003).
- [21] A. Kusenko, AIP Conf. Proc. **917**, 58 (2007).
- [22] C. W. Kim and A. Pevsner, *Neutrinos in Physics and Astrophysics* (Harwood Academic Publishers, Chur, Switzerland, 1993).
- [23] R. N. Mohapatra and P. B. Pal, *Massive Neutrinos in Physics and Astrophysics* (World Scientific, Singapore, 2004).
- [24] M. Fukugita and T. Yanagida, *Physics of Neutrinos and Applications to Astrophysics* (Springer-Verlag, Berlin, Heidelberg, 2003).
- [25] G. G. Raffelt, *Stars as Laboratories for Fundamental Physics* (The University of Chicago Press, Chicago, 1996); arXiv:astro-ph/0302589; New Astron. Rev. **46**, 699 (2002); arXiv:hep-ph/0208024.
- [26] A. A. Aguilar-Arevalo *et al.* (MiniBooNE Collaboration), Phys. Rev. Lett. **98**, 231801 (2007).
- [27] C. Athanassopoulos *et al.* (LSND Collaboration), Phys. Rev. Lett. **81**, 1774 (1998).
- [28] A. Aguilar *et al.* (LSND Collaboration), Phys. Rev. D **64**, 112007 (2001).
- [29] M. Maltoni and T. Schwetz, arXiv:0705.0107 [Phys. Rev. D (to be published)].
- [30] K. Abazajian, G. M. Fuller, and W. H. Tucker, Astrophys. J. **562**, 593 (2001).
- [31] A. Boyarsky, A. Neronov, O. Ruchayskiy, and M. Shaposhnikov, Mon. Not. R. Astron. Soc. **370**, 213 (2006); JETP Lett. **83**, 133 (2006); Phys. Rev. D **74**, 103506 (2006); A. Boyarsky, A. Neronov, O. Ruchayskiy, M. Shaposhnikov, and I. Tkachev, Phys. Rev. Lett. **97**, 261302 (2006); A. Boyarsky, J. Nevalainen, and O. Ruchayskiy, Astron. Astrophys. **471**, 51 (2007); A. Boyarsky, O. Ruchayskiy, and M. Markevitch, Phys. Rev. D **72**, 085011 (2005).
- [32] S. Riemer-Sorensen, K. Pedersen, S. H. Hansen, and H. Dahle, Phys. Rev. D **76**, 043524 (2007); S. Riemer-Sorensen, S. H. Hansen, and K. Pedersen, Astrophys. J. **644**, L33 (2006).
- [33] K. N. Abazajian, M. Markevitch, S. M. Koushiappas, and R. C. Hickox, Phys. Rev. D **75**, 063511 (2007).
- [34] F. Bezrukov and M. Shaposhnikov, Phys. Rev. D **75**, 053005 (2007).
- [35] A. Dolgov, Sov. J. Nucl. Phys. **33**, 700 (1981); R. Barbieri and A. Dolgov, Nucl. Phys. **B349**, 743 (1991) (see also [10]).
- [36] R. A. Harris and L. Stodolsky, Phys. Lett. B **116**, 464 (1982); L. Stodolsky, Phys. Rev. D **36**, 2273 (1987).
- [37] K. Enqvist, K. Kainulainen, and J. Maalampi, Nucl. Phys. **B349**, 754 (1991); Phys. Lett. B **244**, 186 (1990); K. Enqvist, K. Kainulainen, and M. Thompson, Nucl. Phys. **B373**, 498 (1992).
- [38] G. Raffelt, G. Sigl, and L. Stodolsky, Phys. Rev. Lett. **70**, 2363 (1993); Phys. Rev. D **45**, 1782 (1992).
- [39] G. Sigl and G. Raffelt, Nucl. Phys. **B406**, 423 (1993).
- [40] J. Cline, Phys. Rev. Lett. **68**, 3137 (1992).
- [41] K. Kainulainen, Phys. Lett. B **244**, 191 (1990).
- [42] R. Foot and R. R. Volkas, Phys. Rev. D **55**, 5147 (1997).
- [43] P. Di Bari, P. Lipari, and M. Lusignoli, Int. J. Mod. Phys. A **15**, 2289 (2000).
- [44] T. Asaka, M. Laine, and M. Shaposhnikov, J. High Energy Phys. **06** (2006) 053; **01** (2007) 091.
- [45] R. Fleischer, arXiv:hep-ph/0608010.
- [46] C. Gay, Annu. Rev. Nucl. Part. Sci. **50**, 577 (2000).
- [47] M. Beuthe, Phys. Rep. **375**, 105 (2003); M. Beuthe, G. Lopez Castro, and J. Pestieau, Int. J. Mod. Phys. A **13**, 3587 (1998).
- [48] E. Zavattini *et al.* (PVLAS Collaboration), Phys. Rev. Lett. **96**, 110406 (2006).
- [49] M. Ahlers, H. Gies, J. Jaeckel, and A. Ringwald, Phys. Rev. D **75**, 035011 (2007).
- [50] A. D. Dolgov, O. V. Lychkovskiy, A. A. Mamonov, L. B. Okun, M. V. Rotaev, and M. G. Schepkin, Nucl. Phys. **B729**, 79 (2005).
- [51] D. Boyanovsky and C. M. Ho, Phys. Rev. D **75**, 085004 (2007).
- [52] D. Boyanovsky and C. M. Ho, Phys. Rev. D **76**, 085011 (2007).
- [53] D. Boyanovsky and C. M. Ho, J. High Energy Phys. **07** (2007) 030.
- [54] D. Boyanovsky and C. M. Ho, Astropart. Phys. **27**, 99 (2007).
- [55] X. Shi and G. Fuller, Phys. Rev. Lett. **82**, 2832 (1999).
- [56] R. P. Feynman and F. L. Vernon, Ann. Phys. (N.Y.) **24**, 118 (1963).
- [57] A. O. Caldeira and A. J. Leggett, Physica (Amsterdam)

- 121A**, 587 (1983); H. Grabert, P. Schramm, and G.-L. Ingold, *Phys. Rep.* **168**, 115 (1988); A. Schmid, *J. Low Temp. Phys.* **49**, 609 (1982).
- [58] U. Weiss, *Quantum Dissipative Systems* (World Scientific, Singapore, 1993).
- [59] M.O. Scully and M.S. Zubairy, *Quantum Optics* (Cambridge University Press, Cambridge, U.K., 1997); P. Meystre and M. Sargent, III, *Elements of Quantum Optics* (Springer Verlag, Berlin, 1998), 3rd ed.; C.W. Gardiner and P. Zoller, *Quantum Noise* (Springer, Berlin, Heidelberg, 2004), 3rd ed.
- [60] D. Notzold and G. Raffelt, *Nucl. Phys.* **B307**, 924 (1988).
- [61] N.F. Bell, R.R. Volkas, and Y.Y.Y. Wong, *Phys. Rev. D* **59**, 113001 (1999).
- [62] S.M. Alamoudi, D. Boyanovsky, H.J. de Vega, and R. Holman, *Phys. Rev. D* **59**, 025003 (1998); S.M. Alamoudi, D. Boyanovsky, and H.J. de Vega, *Phys. Rev. E* **60**, 94 (1999).
- [63] D. Boyanovsky, K. Davey, and C.M. Ho, *Phys. Rev. D* **71**, 023523 (2005); D. Boyanovsky and H.J. de Vega, *Nucl. Phys. A* **747**, 564 (2005); *Phys. Rev. D* **68**, 065018 (2003).
- [64] B.H.J. McKellar and M.J. Thompson, *Phys. Rev. D* **49**, 2710 (1994).
- [65] R.R. Volkas and Y.Y.Y. Wong, *Phys. Rev. D* **62**, 093024 (2000); K.S.M. Lee, R.R. Volkas, and Y.Y.Y. Wong, *Phys. Rev. D* **62**, 093025 (2000).
- [66] D. Boyanovsky and C.M. Ho, *Phys. Rev. D* **69**, 125012 (2004); The components of the polarization vector P_x, P_y, P_z are the expectation values of S_y, S_x, S_z of this reference, respectively, in the quantum master equation.
- [67] C.M. Ho, D. Boyanovsky, and H.J. de Vega, *Phys. Rev. D* **72**, 085016 (2005).