Primordial non-Gaussianity from two curvaton decays

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We study a model where two scalar fields, that are subdominant during inflation, decay into radiation some time after inflation has ended but before primordial nucleosynthesis. Perturbations of these two curvaton fields can be responsible for the primordial curvature perturbation. We write down the full nonlinear equations that relate the primordial perturbation to the curvaton perturbations on large scales, calculate the power spectrum of the primordial perturbation, and finally go to second order to find the nonlinearity parameter, f_{NL}. We find large positive values of f_{NL} if the energy densities of the curvatons are subdominant when they decay, as in the single curvaton case. But we also find a large f_{NL} even if the curvatons dominate the total energy density in the case when the inhomogeneous radiation produced by the first curvaton decay is diluted by the decay of a second nearly homogeneous curvaton. The minimum value $\min(f_{NL}) = -5/4$ which we find is the same as in the single-curvaton case.

I. INTRODUCTION

Theories beyond the standard model often contain a large number of scalar fields in addition to the standard-model fields. In the very early Universe it is natural to expect the initial values of these fields to be displaced from the minimum of their potential. If they are displaced by more than the Planck scale then they can drive a period of inflation. But if they are displaced from their minimum by less than the Planck scale they will oscillate about the minimum of their potential once the Hubble rate, H, drops below their effective mass. An oscillating massive field has the equation of state (averaged over several oscillations) of a pressureless fluid. Thus the energy density of a weakly interacting massive field tends to grow relative to radiation in the early Universe. Such fields must therefore decay before the primordial nucleosynthesis era to avoid spoiling the standard, successful hot big bang model. And if the energy density of a late-decaying scalar is non-negligible when it decays then any inhomogeneity in its energy density will be transferred to the primordial radiation [1,2]. This is the curvaton scenario for the origin of structure [3–5].

A curvaton field, χ, is supposed to have a negligible energy density during inflation, but once the Hubble rate drops below the curvaton mass after inflation, the curvaton energy density grows relative to radiation, reaching its maximum value, $\Omega_{\chi,\text{decay}}$, just before the curvaton decays. If all the species are in thermal equilibrium and the baryon asymmetry is generated after the curvaton decays, then the curvaton mechanism generates adiabatic density perturbations. If not then the curvaton can leave a residual isocurvature perturbation [6,7] correlated with the curvature perturbation [8]. The amplitude of isocurvature modes are severely constrained by current data [9–12]. Thus if, for example, the baryon asymmetry is produced by the out-of-equilibrium curvaton decay, then we require $\Omega_{\chi,\text{decay}} \sim 1$.

If we take seriously the multiplicity of scalar fields in the early Universe then we should consider models where more than one field can contribute to the primordial density perturbation on large scales [13]. Several authors have considered the combination of perturbations from a curvaton field and the inflaton field driving inflation [14–16]. More recently Choi and Gong [17] considered the primordial perturbations that may result from multiple curvaton fields, showing that the presence of more than one curvaton field affects the amplitude of residual isocurvature perturbations and their correlation with the curvature perturbation. In this paper we extend their analysis to study the nonlinear curvature perturbation and how the multiple late-decaying scalar fields may affect the non-Gaussianity of the primordial curvature perturbation. For simplicity we restrict our analysis to the case of two curvatons, but it should be straightforward to extend our analysis to three or more curvatons.

Deviations from an exactly Gaussian distribution of the primordial density perturbation is conventionally given in terms of a nonlinearity parameter, f_{NL} [18]. The current upper bound from the WMAP three-year data [19] is $|f_{NL}| < 114$ while Planck is expected to bring this down to $|f_{NL}| < 5$ [18]. Galaxy cluster surveys can offer complementary constraints [20]. Measurement of f_{NL} would give a valuable test of inflation. If primordial perturbations originate from fluctuations in a canonical inflaton field, driving slow-roll inflation, then f_{NL} is less than unity [21,22]. However, if primordial perturbations originate from fluctuations in a single curvaton field, then $f_{NL} \sim 1/\Omega_{\chi,\text{decay}}$ [8,23–27]. Therefore f_{NL} could thus be large if the curvaton does not dominate the energy density of the Universe when it decays. In this paper we shall show that when we consider the decay of two curvaton fields it is also possible that f_{NL} is large even when the energy densities of both curvaton fields are dominant when they decay.

This paper is organized as follows. In Sec. II we introduce a nonlinear definition of curvature perturbations,

define the perturbation power spectrum and bispectrum, and define the nonlinearity parameter $f_{\rm NL}$ which describes the non-Gaussianity of perturbations at leading order. In Sec. III we write down the full nonlinear equations that relate the curvaton perturbations to the total and radiation perturbations at the first curvaton decay and at the second curvaton decay, and finally to the primordial curvature perturbation. Then we solve these equations up to the second order and find the resulting $f_{\rm NL}$ of the primordial perturbation before nucleosynthesis. The general expression for $f_{\rm NL}$ is a nontrivial function of four parameters. To get an insight into its behavior we analyze some special cases in Sec. IV. Finally, in Sec. V we present concluding remarks. As the notation becomes rather heavy in the two-curvaton scenario, we list most of the symbols used in this paper in Appendix A.

II. PRELIMINARIES

A. Primordial curvature perturbation

The primordial density perturbation can be described in terms of the nonlinearly perturbed expansion on uniform-density hypersurfaces [28] (see also [29,30])

$$\zeta(t, \mathbf{x}) = \delta N(t, \mathbf{x}) + \frac{1}{3} \int_{\bar{\rho}(t)}^{\rho(t,\mathbf{x})} \frac{d\tilde{\rho}}{\tilde{\rho} + \tilde{p}}, \qquad (1)$$

where $N = \int H dt$ is the integrated local expansion, $\tilde{\rho}$ the local density and $\tilde{p}$ the local pressure, and $\bar{\rho}$ is the homogeneous density in the background model.

We will expand the curvature perturbation at each order (n) as

$$\zeta(t, \mathbf{x}) = \sum_{n=1}^{\infty} \frac{1}{n!} \zeta_{(n)}(t, \mathbf{x}), \qquad (2)$$

where we assume that the first-order perturbation, $\zeta_{(1)}$, is Gaussian as it is proportional to the initial Gaussian field perturbations. Higher-order terms describe the non-Gaussianity of the full nonlinear ζ.

Working in terms of the Fourier transform of ζ, we define the primordial power spectrum as

$$\langle \zeta(\mathbf{k_1}) \zeta(\mathbf{k_2}) \rangle = (2\pi)^3 P_\zeta(k_1) \delta^3(\mathbf{k_1} + \mathbf{k_2}). \qquad (3)$$

The average power per logarithmic interval in Fourier space is given by

$$\mathcal{P}_\zeta(k) = \frac{4\pi k^3}{(2\pi)^3} P_\zeta(k), \qquad (4)$$

and is roughly independent of wavenumber k.

The primordial bispectrum is given by

$$\langle \zeta(\mathbf{k_1}) \zeta(\mathbf{k_2}) \zeta(\mathbf{k_3}) \rangle = (2\pi)^3 B(k_1, k_2, k_3) \delta^3(\mathbf{k_1} + \mathbf{k_2} + \mathbf{k_3}). \qquad (5)$$

The bispectrum vanishes for a purely Gaussian distribution, and hence is nonzero only at fourth and higher order.

The amplitude of the bispectrum relative to the power spectrum is commonly parametrized in terms of the non-linearity parameter, $f_{\rm NL}$, defined such that [18]

$$B(k_1, k_2, k_3) = (6/5) f_{\rm NL} [P(k_1) P(k_2) + 2 \text{ perms}]. \qquad (6)$$

Higher-order statistics, like trispectrum (see, e.g., [27,31,32]) or full nonlinear probability density function of the primordial ζ [27], can give also valuable information on non-Gaussianity in the curvaton model, but in this paper we consider the bispectrum only.

B. Curvaton perturbations

We will consider the primordial curvature perturbation produced by the decay of two scalar fields a and b. Without loss of generality, we assume that the curvaton a decays first when $H = \Gamma_a$ followed by the decay of the curvaton b when $H = \Gamma_b$, where $\Gamma_b < \Gamma_a$.

Small-scale (sub-Hubble) vacuum fluctuations of any light scalar field are stretched by the expansion to large (super-Hubble) scales during a period of inflation in the early Universe. If the curvaton fields are weakly coupled, massive scalar fields whose masses are less than the Hubble scale, $H_* \gg m$, during inflation, then they acquire an almost scale-invariant spectrum of perturbations on super-Hubble scales,

$$\mathcal{P}_{\delta a_*} = \mathcal{P}_{\delta b_*} = \left(\frac{H_*}{2\pi} \right)^2, \qquad (7)$$

where H_* is the Hubble rate at Hubble exit. In Refs. [33–37] it has been shown that in general, if slow-roll conditions are satisfied, the non-Gaussianity of field perturbations at Hubble exit is small (at least for the three-point and four-point correlators). Consequently, in what follows, we assume that the field perturbations at Hubble exit have Gaussian and independent distributions, consistent with weakly coupled isocurvature fluctuations of light scalar fields [38,39].

Once the Hubble rate drops below the mass of each curvaton field the field begins to oscillate. The local value of each curvaton can evolve between Hubble exit during inflation and the beginning of the field oscillations. We parametrize this evolution by functions g_a and g_b, but we assume the two fields remain decoupled so that their perturbations remain uncorrelated. Thus at the beginning of curvaton oscillations, the curvaton fields have values

$$a_{\rm osc} = g_a(a_*), \qquad (8)$$

$$b_{\rm osc} = g_b(b_*). \qquad (9)$$

We can define nonlinear perturbation for each curvaton analogous to the total perturbation (1)

$$\zeta_a = \delta N + \frac{1}{3} \ln\left(\frac{\rho_a}{\bar{\rho}_a} \right) = \delta N + \frac{2}{3} \ln\left(\frac{a_{\rm osc}}{\bar{a}_{\rm osc}} \right), \qquad (10)$$

where we take the energy density to be proportional to the

square of the field value, $\rho_a \propto a_{\text{osc}}^2$, when the field begins to oscillate, and similarly for ζ_b. On uniform-curvaton-density hypersurfaces [40] ζ_a and ζ_b are curvature perturbations. The curvature perturbation ζ_a (and ζ_b) becomes constant on scales larger than the Hubble scale once each curvaton starts oscillating, and while we can neglect energy transfer to the radiation, $\Gamma_a < H$. Thus we require

$$\Gamma_a \ll m_a \quad \text{and} \quad \Gamma_b \ll m_b, \tag{11}$$

consistent with the assumption that we are dealing with weakly coupled fields. For simplicity we will further assume that $\Gamma_a \ll m_b$ and that the energy density of both curvatons is negligible when they begin oscillating, which ensures that we can neglect any gravitational coupling between fields in the overdamped regime before they begin oscillating.

At first order the curvature perturbation (10) for each curvaton can thus be given in terms of the field perturbations on spatially flat hypersurfaces ($\delta N = 0$)

$$\zeta_{a(1)} = \frac{2}{3} \frac{\delta a}{\bar{a}}\bigg|_{\text{osc}} = \frac{2}{3} \frac{g_a'}{g_a} \delta a_*, \tag{12}$$

$$\zeta_{b(1)} = \frac{2}{3} \frac{\delta b}{\bar{b}}\bigg|_{\text{osc}} = \frac{2}{3} \frac{g_b'}{g_b} \delta b_*, \tag{13}$$

where we have Taylor expanded the functions g_a and g_b, defining $g_a' \equiv \partial g_a / \partial a_*$, and $g_b' \equiv \partial g_b / \partial b_*$.

The power spectra P_{ζ_a} and $P_{\zeta_{a(1)}}$ are the same at leading order. The higher-order corrections are generally so small that we use P_{ζ_a} and $P_{\zeta_{a(1)}}$ interchangeably in the following. Therefore the power spectra of ζ_a and ζ_b when the curvatons start to oscillate are related by

$$P_{\zeta_b} = \beta^2 P_{\zeta_a}, \tag{14}$$

where

$$\beta = \frac{g_b'/g_b}{g_a'/g_a}. \tag{15}$$

If we assume linear evolution between Hubble exit and the beginning of curvaton oscillations, this factor reduces to the ratio of the background curvaton field values at Hubble exit,

$$\beta = \frac{a_*}{b_*}. \tag{16}$$

At second order Eq. (10) gives on spatially flat hypersurfaces ($\delta N = 0$)

$$\zeta_{a(2)} = -\frac{3}{2}\left(1 - \frac{g_a g_a''}{g_a'^2}\right)\zeta_{a(1)}^2, \tag{17}$$

$$\zeta_{b(2)} = -\frac{3}{2}\left(1 - \frac{g_b g_b''}{g_b'^2}\right)\zeta_{b(1)}^2, \tag{18}$$

where $\zeta_{a(1)}$ and $\zeta_{b(1)}$ are given in (12) and (13). Analogous

to (6) we can define the nonlinearity parameters of each curvaton f_{NL}^a and f_{NL}^b. Then comparing to (17) and (18) we find

$$f_{\text{NL}}^a = -\frac{5}{4}\left(1 - \frac{g_a g_a''}{g_a'^2}\right), \tag{19}$$

$$f_{\text{NL}}^b = -\frac{5}{4}\left(1 - \frac{g_b g_b''}{g_b'^2}\right). \tag{20}$$

If the evolution of field values is linear between Hubble exit and the beginning of curvaton oscillations, we have $f_{\text{NL}}^a = f_{\text{NL}}^b = -5/4$. Although it is possible to construct potentials which lead to a nonlinear evolution [41], we assume henceforth in this paper that $g_a'' = g_b'' = 0$, consistent with weakly interacting fields.

C. Calculating f_{NL}

Before calculating how the two curvaton perturbations contribute to the primordial curvature perturbation in detail, we can first consider the general form of the primordial power spectrum and bispectrum that will result from a generic model where the local values of two Gaussian fields determine the primordial perturbation.

The primordial curvature perturbation after both curvatons decay can be written up to second order in terms of the first-order curvaton perturbations as

$$\zeta \equiv \zeta_2 = \zeta_{2(1)} + \frac{1}{2}\zeta_{2(2)}$$

$$= A\zeta_{a(1)} + B\zeta_{b(1)} + \frac{1}{2}C\zeta_{a(1)}^2 + \frac{1}{2}D\zeta_{b(1)}^2 + \frac{1}{2}E\zeta_{a(1)}\zeta_{b(1)}, \tag{21}$$

where ζ_2 is the total curvature perturbation after the second decay. The first-order part $\zeta_{2(1)}$ is Gaussian, since it is a linear function of the field perturbations at Hubble exit (and for weakly coupled light scalar fields these field perturbations are Gaussian). The second-order part $\zeta_{2(2)}$ is non-Gaussian, since it contains squares of Gaussian variables. A, B, C, D, and E are coefficients that will depend on background quantities at the time of the first and second decay. In Sec. III we derive the expressions for them in the two-curvaton decay scenario.

The primordial power spectrum at leading order is

$$P_\zeta = A^2 P_{\zeta_a} + B^2 P_{\zeta_b}. \tag{22}$$

Employing the relation (14) we can write

$$P_{\zeta_a} = \frac{1}{A^2 + \beta^2 B^2} P_\zeta, \tag{23}$$

$$P_{\zeta_b} = \frac{\beta^2}{A^2 + \beta^2 B^2} P_\zeta. \tag{24}$$

Since the two curvatons are uncorrelated with one another, i.e.,

$$\langle \zeta_{a(1)}(\mathbf{k_1})\zeta_{b(1)}(\mathbf{k_2})\rangle = 0 \tag{25}$$

for any wave vectors $\mathbf{k_1}$ and $\mathbf{k_2}$, we find that at leading order the three-point correlator of the primordial perturbation is

$$\langle \zeta(\mathbf{k_1})\zeta(\mathbf{k_2})\zeta(\mathbf{k_3})\rangle$$
$$= \left[\frac{1}{2}A^2 C\langle \zeta_{a(1)}(\mathbf{k_1})\zeta_{a(1)}(\mathbf{k_2})(\zeta_{a(1)}*\zeta_{a(1)})(\mathbf{k_3})\rangle \right.$$
$$+ \frac{1}{2}B^2 D\langle \zeta_{b(1)}(\mathbf{k_1})\zeta_{b(1)}(\mathbf{k_2})(\zeta_{b(1)}*\zeta_{b(1)})(\mathbf{k_3})\rangle$$
$$\left. + \frac{1}{2}ABE\langle \zeta_{a(1)}(\mathbf{k_1})\zeta_{b(1)}(\mathbf{k_2})(\zeta_{a(1)}*\zeta_{b(1)})(\mathbf{k_3})\rangle \right]$$
$$+ 2 \text{ permutations of } \{\mathbf{k_1}, \mathbf{k_2}, \mathbf{k_3}\}, \tag{26}$$

where $*$ denotes a convolution. For example,

$$(\zeta_{a(1)}*\zeta_{a(1)})(\mathbf{k_3}) = \frac{1}{(2\pi)^3}\int d^3\mathbf{q}\,\zeta_{a(1)}(\mathbf{q})\zeta_{a(1)}(\mathbf{k_3}-\mathbf{q}). \tag{27}$$

Note that there are two ways to form two pairs of correlators in the first two terms on the right-hand-side of (26). Therefore, the three-point correlator simplifies to

$$\langle \zeta(\mathbf{k_1})\zeta(\mathbf{k_2})\zeta(\mathbf{k_3})\rangle = \left\{ \left[A^2 CP_{\zeta_{a(1)}}(k_1)P_{\zeta_{a(1)}}(k_2) \right. \right.$$
$$+ B^2 DP_{\zeta_{b(1)}}(k_1)P_{\zeta_{b(1)}}(k_2)$$
$$\left. + \frac{1}{2}ABEP_{\zeta_{a(1)}}(k_1)P_{\zeta_{b(1)}}(k_2) \right]$$
$$+ 2 \text{ perms} \Big\}$$
$$\times (2\pi)^3\delta^{(3)}(\mathbf{k_1}+\mathbf{k_2}+\mathbf{k_3}). \tag{28}$$

Substituting (23) and (24) into (28) leads to

$$\langle \zeta(\mathbf{k_1})\zeta(\mathbf{k_2})\zeta(\mathbf{k_3})\rangle = \frac{A^2 C + \beta^4 B^2 D + \frac{1}{2}\beta^2 ABE}{(A^2 + \beta^2 B^2)^2}$$
$$\times \{P_\zeta(k_1)P_\zeta(k_2) + 2 \text{ perms}\}$$
$$\times (2\pi)^3\delta^{(3)}(\mathbf{k_1}+\mathbf{k_2}+\mathbf{k_3}). \tag{29}$$

Hence the bispectrum, defined in Eq. (5), is in the form given by Eq. (6), where the nonlinearity parameter is

$$f_{\text{NL}} = \frac{5}{6}\frac{CA^2 + \frac{1}{2}\beta^2 EAB + \beta^4 DB^2}{(A^2 + \beta^2 B^2)^2}. \tag{30}$$

III. FULL NONLINEAR EQUATIONS

We will estimate the primordial density perturbation produced by the decay of two curvaton fields some time after inflation has ended using the sudden-decay approximation [8], generalizing the nonlinear analysis of Ref. [27] to the case of two curvatons. In this approximation the curvaton fields and the radiation are treated as noninteract-

ing fluids except at the instant of decay when all the energy density of the curvaton is transferred to radiation. This should be a good approximation for scales much larger than the decay time, Γ^{-1}, and has been shown to give a good estimate of the primordial non-Gaussianity when compared against numerical simulations [25–27].

Before either curvaton has decayed, the radiation (from inflaton decay products) has a definite equation of state, $p_\gamma = (1/3)\rho_\gamma$, and so do the oscillating curvaton fields, $p_a = p_b = 0$. Thus we have three noninteracting fluids with barotropic equations of state and hence three curvature perturbations (1) which are constant on large scales [40]:

$$\zeta_\gamma = \delta N + \frac{1}{4}\ln\left(\frac{\rho_\gamma}{\bar{\rho}_\gamma}\right), \tag{31}$$

$$\zeta_a = \delta N + \frac{1}{3}\ln\left(\frac{\rho_a}{\bar{\rho}_a}\right), \tag{32}$$

$$\zeta_b = \delta N + \frac{1}{3}\ln\left(\frac{\rho_b}{\bar{\rho}_b}\right). \tag{33}$$

On the spatial hypersurface where $H = \Gamma_a$, there is an abrupt jump in the overall equation of state due to the sudden decay of the first curvaton into radiation, but the total energy density is continuous,

$$\rho_{\gamma_1 1} + \rho_{b1} = \rho_{\gamma_0 1} + \rho_{a1} + \rho_{b1}. \tag{34}$$

Here $\rho_{\gamma_1 1}$ is the radiation energy density immediately after the first curvaton decay, ρ_{b1} is density of the second curvaton at the time of the first curvaton decay, $\rho_{\gamma_0 1}$ is the density of preexisting radiation just before the first curvaton decay, and ρ_{a1} is the density of the first curvaton just before the first decay when it is converted to radiation. Equation (34) simplifies to

$$\rho_{\gamma_1 1} = \rho_{\gamma_0 1} + \rho_{a1}. \tag{35}$$

Note that the first-decay hypersurface is a uniform-density hypersurface and thus, from Eq. (1) the perturbed expansion on this hypersurface is $\delta N = \zeta_1$, where ζ_1 denotes the total curvature perturbation at the first-decay hypersurface. Hence on the first-decay hypersurface we have

$$\rho_{\gamma_0 1} = \bar{\rho}_{\gamma_0 1}e^{4(\zeta_{\gamma_0}-\zeta_1)}, \tag{36}$$

$$\rho_{\gamma_1 1} = \bar{\rho}_{\gamma_1 1}e^{4(\zeta_{\gamma_1}-\zeta_1)}, \tag{37}$$

$$\rho_{a1} = \bar{\rho}_{a1}e^{3(\zeta_a-\zeta_1)}, \tag{38}$$

$$\rho_{b1} = \bar{\rho}_{b1}e^{3(\zeta_b-\zeta_1)}, \tag{39}$$

where we employed Eqs. (31)–(33). In the curvaton scenario we usually assume the initial radiation is homogeneous before the curvaton decays, so that $\zeta_{\gamma_0} = 0$, but in

this section we keep ζ_{γ_0} in our calculations for generality. On a uniform-density hypersurface the total energy density is homogeneous. Therefore an infinitesimal time before the first decay we have

$$\rho_{\gamma_0 1} + \rho_{a1} + \rho_{b1} = \bar{\rho}_1. \qquad (40)$$

Substituting here Eqs. (36), (38), and (39), and dividing by the total energy density $\bar{\rho}_1$ we end up with

$$\Omega_{\gamma_0 1}e^{4(\zeta_{\gamma_0} - \zeta_1)} + \Omega_{a1}e^{3(\zeta_a - \zeta_1)} + \Omega_{b1}e^{3(\zeta_b - \zeta_1)} = 1, \quad (41)$$

where $\Omega_{\gamma_0 1} = \rho_{\gamma_0 1}/\bar{\rho}_1$, $\Omega_{a1} = \rho_{a1}/\bar{\rho}_1$, and $\Omega_{b1} = \rho_{b1}/\bar{\rho}_1$ are the energy density parameters of the preexisting radiation, the first curvaton, and the second curvaton at the first curvaton decay, respectively. On the other hand, an infinitesimal time after the first curvaton has decayed into radiation the total energy density can be written as

$$\rho_{\gamma_1 1} + \rho_{b1} = \bar{\rho}_1, \qquad (42)$$

which gives an equation

$$\Omega_{\gamma_1 1}e^{4(\zeta_{\gamma_1} - \zeta_1)} + \Omega_{b1}e^{3(\zeta_b - \zeta_1)} = 1. \qquad (43)$$

The total curvature perturbation ζ is the perturbation on the decay hypersurface and thus is continuous between the two phases. However the energy density of radiation changes abruptly at the decay time and the radiation curvature perturbation is discontinuous, $\zeta_{\gamma_0} \neq \zeta_{\gamma_1}$.

Similarly, an infinitesimal time before the second decay we have

$$\Omega_{\gamma_1 2}e^{4(\zeta_{\gamma_1} - \zeta_2)} + \Omega_{b2}e^{3(\zeta_b - \zeta_2)} = 1, \qquad (44)$$

while an infinitesimal time after the second decay we have

$$\Omega_{\gamma_2 2}e^{4(\zeta_{\gamma_2} - \zeta_2)} = 1. \qquad (45)$$

Above, $\Omega_{\gamma_1 2}$ and Ω_{b2} are the radiation and the second curvaton energy density parameters just before the second curvaton decay, $\Omega_{\gamma_2 2}$ is the radiation density parameter just after the second curvaton decay, and ζ_2 is the total curvature perturbation at the second curvaton decay hypersurface. As the radiation is the only constituent left after the second decay, the radiation curvature perturbation ζ_{γ_2} equals the total curvature perturbation ζ_2, and Eq. (45) reduces to $\Omega_{\gamma_2 2} = 1$ as we will see in the next subsection. After both curvatons have decayed we have a single radiation fluid with equation of state $p = \rho/3$ and hence the curvature perturbation remains constant. Thus we identify the primordial curvature perturbation,

$$\zeta = \zeta_{\gamma_2}. \qquad (46)$$

We can Taylor expand the exponential functions, $e^x = 1 + x + x^2/2 + \ldots$, in Eqs. (41)–(45) to express the final curvature perturbation, ζ, in terms of the initial curvaton perturbations, ζ_a and ζ_b, at any given order.

A. Zeroth order

Taylor expanding Eq. (41) at zeroth order gives

$$\Omega_{\gamma_0 1} + \Omega_{a1} + \Omega_{b1} = 1. \qquad (47)$$

This is identically true, since we are studying spatially flat models with $\Omega_{\text{tot}} = 1$. Similarly Eq. (43) gives

$$\Omega_{\gamma_1 1} + \Omega_{b1} = 1. \qquad (48)$$

Therefore $\Omega_{\gamma_1 1}$ is a redundant variable, and in what follows we can replace it by $1 - \Omega_{b1}$. To zeroth order, Eq. (44) is

$$\Omega_{\gamma_1 2} + \Omega_{b2} = 1. \qquad (49)$$

Therefore $\Omega_{\gamma_1 2}$ is a redundant variable, and in what follows we can replace it by $1 - \Omega_{b2}$. Finally Eq. (45) is identically true as it states $\Omega_{\gamma_2 2} = 1$. At all higher orders Eq. (45) states $\zeta_{\gamma_2} = \zeta_2$. After the second decay only radiation is left.

B. First order

1. First decay

At first order, Eq. (41) gives

$$4\Omega_{\gamma_0 1}[\zeta_{\gamma_0(1)} - \zeta_{1(1)}] + 3\Omega_{a1}[\zeta_{a(1)} - \zeta_{1(1)}]$$
$$+ 3\Omega_{b1}[\zeta_{b(1)} - \zeta_{1(1)}] = 0. \qquad (50)$$

From this we solve for the total curvature perturbation, $\zeta_{1(1)}$, at the first-decay surface. It is

$$\zeta_{1(1)} = f_{\gamma_0 1}\zeta_{\gamma_0(1)} + f_{a1}\zeta_{a(1)} + f_{b1}\zeta_{b(1)}, \qquad (51)$$

where

$$f_{\gamma_0 1} = \frac{4\Omega_{\gamma_0 1}}{4\Omega_{\gamma_0 1} + 3\Omega_{a1} + 3\Omega_{b1}}, \qquad (52)$$

$$f_{a1} = \frac{3\Omega_{a1}}{4\Omega_{\gamma_0 1} + 3\Omega_{a1} + 3\Omega_{b1}}, \qquad (53)$$

$$f_{b1} = \frac{3\Omega_{b1}}{4\Omega_{\gamma_0 1} + 3\Omega_{a1} + 3\Omega_{b1}}. \qquad (54)$$

These first-order curvature transfer efficiency parameters at the first decay obey the relation $f_{\gamma_0 1} + f_{a1} + f_{b1} = 1$. In what follows we use this to eliminate $f_{\gamma_0 1}$.

After the first curvaton decays, but before the second curvaton decays, the curvature perturbation in the radiation, ζ_{γ_1}, will remain constant on large scales. Equation (43) relates the radiation curvature perturbation and the total curvature perturbation immediately after the first decay. At first order it reads

$$4\Omega_{\gamma_1 1}[\zeta_{\gamma_1(1)} - \zeta_{1(1)}] + 3\Omega_{b1}[\zeta_{b(1)} - \zeta_{1(1)}] = 0, \quad (55)$$

which gives

$$\zeta_{\gamma_1(1)} = R_1\zeta_{1(1)} - (R_1 - 1)\zeta_{b(1)}, \tag{56}$$

where

$$R_1 = \frac{4 - \Omega_{b1}}{4 - 4\Omega_{b1}} \tag{57}$$

$$= \frac{3 + f_{a1}}{3(1 - f_{b1}) + f_{a1}}. \tag{58}$$

Note that if the density of the second curvaton is negligible when the first curvaton decays then we have $R_1 = 1$ and $\zeta_{\gamma_1(1)} = \zeta_{1(1)}$.

Finally, we substitute (51) into (56):

$$\zeta_{\gamma_1(1)} = R_1(1 - f_{a1} - f_{b1})\zeta_{\gamma_0(1)} + R_1 f_{a1}\zeta_{a(1)}$$
$$+ [1 - R_1(1 - f_{b1})]\zeta_{b(1)}. \tag{59}$$

This remains constant between the first and second decay and hence, at the second decay we can use this as the incoming radiation perturbation.

2. Second decay

Following the same procedure as above, we find from Eq. (44) at first order [or equivalently using (51) but dropping curvaton b, then relabeling $a \to b$, $1 \to 2$, and $0 \to 1$]

$$\zeta_{2(1)} = f_{\gamma_1 2}\zeta_{\gamma_1(1)} + f_{b2}\zeta_{b(1)}, \tag{60}$$

where

$$f_{\gamma_1 2} = \frac{4\Omega_{\gamma_1 2}}{4\Omega_{\gamma_1 2} + 3\Omega_{b2}}, \tag{61}$$

$$f_{b2} = \frac{3\Omega_{b2}}{4\Omega_{\gamma_1 2} + 3\Omega_{b2}}. \tag{62}$$

These first-order curvature transfer efficiency parameters at the second decay obey the relation $f_{\gamma_1 2} + f_{b2} = 1$, and hence Eq. (60) reads

$$\zeta_{2(1)} = (1 - f_{b2})\zeta_{\gamma_1(1)} + f_{b2}\zeta_{b(1)}. \tag{63}$$

Finally, using Eq. (59) for $\zeta_{\gamma_1(1)}$ we obtain

$$\zeta_{2(1)} = R_1(1 - f_{a1} - f_{b1})(1 - f_{b2})\zeta_{\gamma_0(1)} + r_a\zeta_{a(1)}$$
$$+ r_b\zeta_{b(1)}, \tag{64}$$

where

$$r_a = R_1 f_{a1}(1 - f_{b2}) \tag{65}$$

$$= \frac{(1 - f_{b2})(3 + f_{a1})f_{a1}}{3(1 - f_{b1}) + f_{a1}}, \tag{66}$$

$$r_b = 1 - R_1(1 - f_{b1})(1 - f_{b2}) \tag{67}$$

$$= \frac{(1 - f_{b1})f_{b2}(3 + f_{a1}) + f_{b1}f_{a1}}{3(1 - f_{b1}) + f_{a1}}. \tag{68}$$

From Eqs. (45) and (46) we have $\zeta_{2(1)} = \zeta_{\gamma_2(1)} = \zeta_{(1)}$. Therefore, if the preexisting radiation perturbation vanishes, $\zeta_{\gamma_0(1)} = 0$, the primordial first-order curvature perturbation after the second decay, is

$$\zeta_{(1)} = r_a\zeta_{a(1)} + r_b\zeta_{b(1)}. \tag{69}$$

Then in Eq. (21) at first order we can identify

$$A = r_a, \quad \text{and} \quad B = r_b, \tag{70}$$

and the power spectrum for the primordial curvature perturbation is given by Eq. (22),

$$\frac{P_{\zeta_{(1)}}}{P_{\zeta_{a(1)}}} = r_a^2 + \beta^2 r_b^2, \tag{71}$$

where β^2 defined in Eq. (14) gives the ratio between the initial power in the curvaton a and that in the curvaton b.

Note that our result differs from that presented recently by Choi and Gong [17] due to the presence of the extra factor R_1 which arises due to the difference between the uniform total density hypersurface and the uniform radiation density hypersurface when curvaton a decays, if the density of the curvaton b is not negligible. When $\Omega_{b1} = 0$ we find from Eq. (58) that $R_1 = 1$ and we recover the simpler result [17]

$$\zeta_{\gamma_2(1)} = (1 - f_{b2})f_{a1}\zeta_{a(1)} + f_{b2}\zeta_{b(1)}. \tag{72}$$

The transfer coefficients r_a and r_b in this case are shown by the thick solid lines in Figs. 1 and 2.
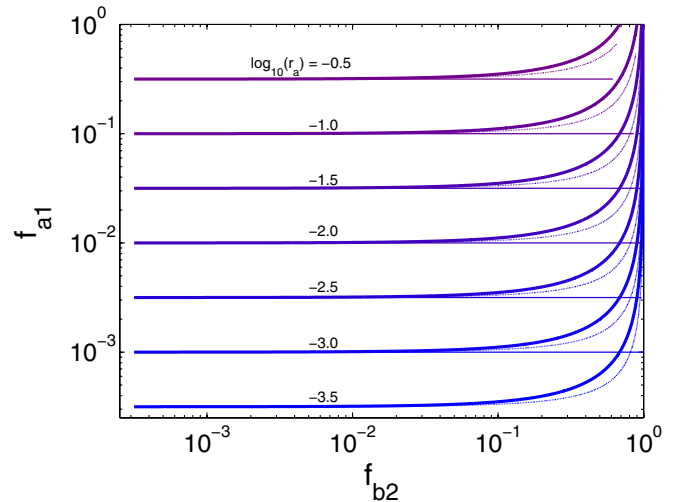

FIG. 1 (color online). $\log_{10} r_a(f_{b2}, f_{a1})$ for $f_{b1} = 0$ (thick solid lines), for $f_{b1} = f_{b2}/2$ (dotted lines), and for $f_{b1} = (1 + f_{a1}/3)f_{b2}$ (thin solid lines). In the latter two cases the constraint (73) excludes a small region in the top right corner. The contours of equal r_a are shown, from bottom to top, for $r_a = 10^{-3.5}, 10^{-3}, 10^{-2.5}, 10^{-2}, 10^{-1.5}, 10^{-1},$ and $10^{-0.5}$.
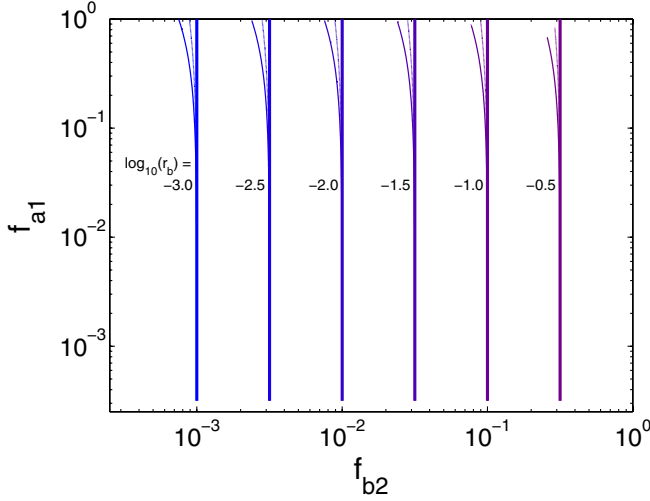
FIG. 2 (color online).   Same as Fig. 1 but now for $\log_{10} r_b$. The contours of equal r_b are shown, from left to right, for $r_b = 10^{-3}$, $10^{-2.5}$, 10^{-2}, $10^{-1.5}$, 10^{-1}, and $10^{-0.5}$.

However in general when $\Omega_{b1} \neq 0$ and $R_1 \neq 1$ we find a novel effect where the inhomogeneous density of the curvaton b may lead to a perturbation in the radiation density after the curvaton a decays, even if the curvaton a is homogeneous; see Eq. (59). A local overdensity of the curvaton b delays the decay of curvaton a due to a local gravitational time dilation, resulting in a local overdensity in the radiation after decay. In practice $f_{b2} \geq (3/4) f_{b1}$ so this is usually a small correction. The dotted lines in Figs. 1 and 2 show this effect for the case $f_{b1} = f_{b2}/2$.

When considering r_a and r_b it should be noted that the range of f_{a1}, f_{b1}, and f_{b2} is $[0, 1]$, but the allowed range of f_{b1} is constrained for given f_{a1} and f_{b2}. Since $f_{a1} + f_{b1} = 1 - f_{\gamma 01}$, and $f_{\gamma 01} \in [0, 1]$, we always have a trivial constraint

$$f_{b1} \leq 1 - f_{a1}. \tag{73}$$

Furthermore, we can rewrite f_{b1} in a form

$$f_{b1} = \frac{4 - \Omega_{b1}}{4 - \Omega_{a1} - \Omega_{b1}} \frac{3\Omega_{b1}}{4 - \Omega_{b1}}. \tag{74}$$

Since $\frac{3\Omega_b}{4 - \Omega_b}$ is an increasing function of time, and $f_{b2} = \frac{3\Omega_{b2}}{4 - \Omega_{b2}}$, we find

$$f_{b1} \leq \frac{4 - \Omega_{b1}}{4 - \Omega_{a1} - \Omega_{b1}} f_{b2}. \tag{75}$$

The multiplier of f_{b2} simplifies to $1 + f_{a1}/3$. So we have a constraint

$$f_{b1} \leq (1 + f_{a1}/3) f_{b2}. \tag{76}$$

For example, when $f_{b1} = f_{b2}/2$ the constraint (73) forbids a small region in the upper right corner of Figs. 1 and 2 where $f_{b2} \sim 1$ and $f_{a1} \sim 1$.

Equality in Eq. (76) corresponds to the case where $\Omega_{b1} = \Omega_{b2}$, i.e., the second curvaton decays at the same moment as the first curvaton. Then the transfer efficiencies simplify to $r_a = f_{a1}$, and $r_b = (1 + f_{a1}/3) f_{b2} = f_{b1}$, indicated by the thin solid lines in Figs. 1 and 2. (Note therefore that the case where $f_{b1} = f_{b2}$ describes a situation where the curvaton b decays some time *after* the curvaton a.)

C. Quasi-second-order

In the next section we present the full second-order calculation of ζ, but here we consider a simplified treatment, which will give the correct form of $f_{\rm NL}$ generated by multiple curvaton decays in the limit where the non-Gaussianity is large.

In this quasi-non-linear approximation we will use the linearized form for the full curvature perturbation (32) and (33) in terms of the density perturbations on spatially flat hypersurfaces:

$$\zeta_a = \frac{1}{3} \frac{\delta \rho_a}{\bar{\rho}_a}, \tag{77}$$

$$\zeta_b = \frac{1}{3} \frac{\delta \rho_b}{\bar{\rho}_b}, \tag{78}$$

but we will include nonlinear contributions to the curvaton densities. The energy density of the curvaton a is $\rho_a \propto a^2 = (\bar{a} + \delta a)^2$ and hence we have

$$\frac{\delta \rho_a}{\bar{\rho}_a} = 2\frac{\delta a}{\bar{a}} + \left(\frac{\delta a}{\bar{a}}\right)^2. \tag{79}$$

For simplicity we neglect any nonlinear evolution of the curvaton field [i.e., we take $g_a'' = g_b'' = 0$ in Eq. (8) and (9)] so that the curvaton field perturbations can be taken to be Gaussian and we have

$$\zeta_a = \zeta_{a(1)} + \frac{3}{4} \zeta_{a(1)}^2, \tag{80}$$

where

$$\zeta_{a(1)} = \frac{2}{3} \frac{\delta a_*}{\bar{a}_*}, \tag{81}$$

and similarly for the curvaton b.

Now we use the first-order result for the primordial curvature perturbation produced by the two curvaton decays, but instead of $\zeta_{a(1)}$ and $\zeta_{b(1)}$ we use the quasi-nonlinear ζ_a and ζ_b of Eqs. (77) and (78). Then we have from Eq. (69)

$$\zeta = r_a \zeta_a + r_b \zeta_b$$
$$= r_a \left[\zeta_{a(1)} + \frac{3}{4} \zeta_{a(1)}^2 \right] + r_b \left[\zeta_{b(1)} + \frac{3}{4} \zeta_{b(1)}^2 \right]. \tag{82}$$

Hence, comparing this result with Eq. (21), we have in our quasi-second-order approximation

$$A = r_a, \tag{83}$$

$$B = r_b, \tag{84}$$

$$C = \frac{3}{2} r_a, \tag{85}$$

$$D = \frac{3}{2} r_b, \tag{86}$$

$$E = 0. \tag{87}$$

Substituting these into Eq. (30) the nonlinearity parameter reads

$$f_{\mathrm{NL}}^{\mathrm{quasi}} = \frac{5}{4} \frac{r_a^3 + \beta^4 r_b^3}{(r_a^2 + \beta^2 r_b^2)^2}. \tag{88}$$

We derived this result using only first-order formulas for ζ, but including the full second-order expression for the energy density. The true second-order ζ includes corrections of order ζ^2, but we expect the above result to give a good approximation when $\zeta_{(2)} \gg \zeta_{(1)}^2$, i.e., when the non-Gaussianity is large. In the next section we present the full second-order calculation.

D. Second order
1. First decay

At second order, Eq. (41) reads

$$\Omega_{\gamma_0 1}[4(\zeta_{\gamma_0(1)} - \zeta_{1(1)})]^2 + \Omega_{a1}[3(\zeta_{a(1)} - \zeta_{1(1)})]^2$$
$$+ \Omega_{b1}[3(\zeta_{b(1)} - \zeta_{1(1)})]^2 + 4\Omega_{\gamma_0 1}[\zeta_{\gamma_0(2)} - \zeta_{1(2)}]$$
$$+ 3\Omega_{a1}[\zeta_{a(2)} - \zeta_{1(2)}] + 3\Omega_{b1}[\zeta_{b(2)} - \zeta_{1(2)}] = 0. \tag{89}$$

From this, the solution for the total curvature perturbation at the first decay is

$$\zeta_{1(2)} = 4f_{\gamma_0 1}[\zeta_{\gamma_0(1)} - \zeta_{1(1)}]^2 + 3f_{a1}[\zeta_{a(1)} - \zeta_{1(1)}]^2$$
$$+ 3f_{b1}[\zeta_{b(1)} - \zeta_{1(1)}]^2 + f_{\gamma_0 1}\zeta_{\gamma_0(2)}$$
$$+ f_{a1}\zeta_{a(2)} + f_{b1}\zeta_{b(2)}. \tag{90}$$

At second order, Eq. (43) reads

$$\Omega_{\gamma_1 1}[4(\zeta_{\gamma_1(1)} - \zeta_{1(1)})]^2 + \Omega_{b1}[3(\zeta_{b(1)} - \zeta_{1(1)})]^2$$
$$+ 4\Omega_{\gamma_1 1}(\zeta_{\gamma_1(2)} - \zeta_{1(2)}) + 3\Omega_{b1}(\zeta_{b(2)} - \zeta_{1(2)}) = 0, \tag{91}$$

which gives

$$\zeta_{\gamma_1(2)} = -4[\zeta_{\gamma_1(1)} - \zeta_{1(1)}]^2 + 3(1 - R_1)[\zeta_{b(1)} - \zeta_{1(1)}]^2$$
$$+ \zeta_{1(2)} + (1 - R_1)\zeta_{b(2)} - (1 - R_1)\zeta_{1(2)}. \tag{92}$$

2. Second decay

We find the second-order radiation perturbation after the second decay from Eq. (44) [or from (92) by first dropping curvaton b (which also implies $R_1 = 1$), then relabeling $a \to b$, and $1 \to 2$ and $0 \to 1$]. The result is

$$\zeta_{\gamma_2(2)} = -4[\zeta_{\gamma_2(1)} - \zeta_{2(1)}]^2 + \zeta_{2(2)}. \tag{93}$$

But recalling that $\zeta_{2(1)} = \zeta_{\gamma_2(1)}$ this is just

$$\zeta_{2(2)} = \zeta_{\gamma_2(2)}, \tag{94}$$

exactly as it should be since after the second decay only radiation is left. Now $\zeta_{2(2)}$ is found from (90) by first dropping curvaton b, and then relabeling $a \to b$, $1 \to 2$, and $0 \to 1$. Thus we have

$$\zeta_{2(2)} = 4(1 - f_{b2})[\zeta_{\gamma_1(1)} - \zeta_{2(1)}]^2 + 3f_{b2}[\zeta_{b(1)} - \zeta_{2(1)}]^2$$
$$+ (1 - f_{b2})\zeta_{\gamma_1(2)} + f_{b2}\zeta_{b(2)}. \tag{95}$$

Here we substitute the curvature perturbations from our calculations above and assume $\zeta_{\gamma_0(1)} = \zeta_{\gamma_0(2)} = 0$. Then we end up with the second-order part of the primordial curvature perturbation

$$\zeta_{(2)} \equiv \zeta_{2(2)}$$
$$= \tilde{C}\zeta_{a(1)}^2 + \tilde{D}\zeta_{b(1)}^2 + E\zeta_{a(1)}\zeta_{b(1)} + F\zeta_{a(2)} + G\zeta_{b(2)}, \tag{96}$$

where

$$\tilde{C} = -2R_1^2 f_{a1}^2 f_{b2}^2 - R_1^2 f_{a1}^2 f_{b2}^3 + 7R_1^2 f_{a1}^2 f_{b2} - 4R_1^2 f_{a1}^2$$
$$+ 3R_1 f_{a1}^2 - f_{a1}^2 - R_1 f_{a1}^3 - R_1 f_{b1} f_{a1}^2 + 3R_1 f_{a1}$$
$$- 3f_{b2}R_1 f_{a1}^2 + f_{b2}f_{a1}^2 + f_{b2}R_1 f_{a1}^3$$
$$+ f_{b2}R_1 f_{b1} f_{a1}^2 - 3f_{b2}R_1 f_{a1}, \tag{97}$$

$$\tilde{D} = -1 + f_{b2}R_1 f_{a1}f_{b1}^2 - 7R_1 f_{b1} - f_{b1}^2 + 2f_{b1} + 5R_1$$
$$+ f_{b2} - 4R_1^2 f_{b1}^2 + 8R_1^2 f_{b1} + 7f_{b2}R_1 f_{b1}$$
$$+ 7R_1^2 f_{b1}^2 f_{b2} - 14R_1^2 f_{b1}f_{b2} - 2f_{b2}^2 R_1^2 f_{b1}^2$$
$$+ 4f_{b2}^2 R_1^2 f_{b1} - f_{b2}^3 R_1^2 f_{b1}^2 + 2f_{b2}^3 R_1^2 f_{b1} - 4R_1^2$$
$$- 5f_{b2}R_1 + 7f_{b2}R_1^2 - 2f_{b2}^2 R_1^2 - f_{b2}^3 R_1^2 + 3R_1 f_{b1}^2$$
$$- R_1 f_{b1}^3 + f_{b2}f_{b1}^2 - 2f_{b2}f_{b1} - R_1 f_{a1}f_{b1}^2$$
$$- 3f_{b2}R_1 f_{b1}^2 + f_{b2}R_1 f_{b1}^3, \tag{98}$$

$$E = 2f_{a1} + 2f_{b2}R_1 f_{a1}f_{b1}^2 - 2R_1 f_{a1}f_{b1}^2 - 10R_1 f_{a1}$$
$$- 2R_1 f_{b1}f_{a1}^2 + 10f_{b2}R_1 f_{a1} + 2f_{b2}R_1 f_{b1}f_{a1}^2$$
$$+ 14R_1^2 f_{a1}f_{b2}f_{b1} - 4f_{b2}^2 R_1^2 f_{a1}f_{b1} - 2f_{b2}^3 R_1^2 f_{a1}f_{b1}$$
$$- 6f_{b2}R_1 f_{a1}f_{b1} + 8R_1^2 f_{a1} - 8R_1^2 f_{a1}f_{b1}$$
$$- 14R_1^2 f_{a1}f_{b2} + 4f_{b2}^2 R_1^2 f_{a1} + 2f_{b2}^3 R_1^2 f_{a1} + 6R_1 f_{a1}f_{b1}$$
$$- 2f_{a1}f_{b1} - 2f_{b2}f_{a1} + 2f_{b2}f_{a1}f_{b1}, \tag{99}$$

$$F = r_a = (1 - f_{b2})f_{a1}R_1, \tag{100}$$

$$G = r_b = 1 - R_1 + f_{b1}R_1 + f_{b2}R_1 - f_{b2}f_{b1}R_1. \tag{101}$$

In Appendix B we rewrite these coefficients with R_1 from (58) substituted in and organized in order of raising powers of f_{a1}, f_{b2}, and f_{b1}.

Assuming linear evolution between Hubble exit and the beginning of curvaton oscillation, the genuine second-order curvaton perturbations in (96) read [see Eqs. (17) and (18)]

$$\zeta_{a(2)} = -\frac{3}{2}\zeta_{a(1)}^2, \tag{102}$$

$$\zeta_{b(2)} = -\frac{3}{2}\zeta_{b(1)}^2. \tag{103}$$

Comparing now (96) with (21) we conclude

$$C = \tilde{C} - \frac{3}{2}F, \tag{104}$$

$$D = \tilde{D} - \frac{3}{2}G. \tag{105}$$

Recalling that $A = r_a$ and $B = r_b$ from the first-order analysis, we now have found the nonlinearity parameter f_{NL} of Eq. (30):

$$f_{\mathrm{NL}} = \frac{5}{6}\frac{r_a^2(\tilde{C} - \frac{3}{2}r_a) + \frac{1}{2}\beta^2 E r_a r_b + \beta^4 r_b^2(\tilde{D} - \frac{3}{2}r_b)}{(r_a^2 + \beta^2 r_b^2)^2}. \tag{106}$$

IV. f_{NL} IN VARIOUS LIMITS

The general expression for f_{NL} in Eq. (106) written in terms of the coefficients $\tilde{C}$, $\tilde{D}$, E, r_a, and r_b in Eqs. (97)–(101) [or equivalently (B1)–(B5) in Appendix B] would be very complicated as a function of all four parameters f_{a1}, f_{b1}, f_{b2}, and β. Hence in this section we identify some useful limiting cases in which the expression significantly simplifies.

A. Single-curvaton limits

As a consistency check we first show that if either of the curvaton densities is negligible when they decay then the result (106) simplifies to the single-curvaton decay result [23,24,27]

$$f_{\mathrm{NL}}^{\mathrm{single}}(f) = \frac{5}{4f} - \frac{5}{3} - \frac{5f}{6}, \tag{107}$$

where either $f = f_{a1}$ if $f_{b2} = 0$, or $f = f_{b2}$ if $f_{a1} = 0$.

If the density of the second curvaton is always negligible, then we can set $f_{b1} = f_{b2} = 0$ which leads to

$$r_a = f_{a1}, \tag{108}$$

$$\tilde{C} = 3f_{a1} - 2f_{a1}^2 - f_{a1}^3, \tag{109}$$

$\tilde{D} = E = r_b = 0$, and then Eq. (106) yields

$$f_{\mathrm{NL}} = \frac{5}{4r_a} - \frac{5}{3} - \frac{5r_a}{6} = f_{\mathrm{NL}}^{\mathrm{single}}(f_{a1}). \tag{110}$$

On the other hand, if the density of the first curvaton is negligible, we have $f_{a1} = 0$ which leads to

$$r_b = f_{b2}, \tag{111}$$

$$\tilde{D} = 3f_{b2} - 2f_{b2}^2 - f_{b2}^3, \tag{112}$$

$\tilde{C} = E = r_a = 0$, and then

$$f_{\mathrm{NL}} = \frac{5}{4r_b} - \frac{5}{3} - \frac{5r_b}{6} = f_{\mathrm{NL}}^{\mathrm{single}}(f_{b2}). \tag{113}$$

Hence in the limit where only one curvaton has a non-negligible density, our two-curvaton result for f_{NL} reduces to the well-known single-curvaton result (107).

B. Simultaneous decay of curvatons a and b

In the case where both curvatons decay at the same moment, we have $f_{b1} = (1 + f_{a1}/3)f_{b2}$, as discussed after Eq. (76). Then we find

$$r_a = f_{a1}, \tag{114}$$

$$r_b = f_{b1}, \tag{115}$$

$$\tilde{C} = -f_{a1}(f_{a1}^2 + 2f_{a1} + f_{b1}f_{a1} - 3), \tag{116}$$

$$\tilde{D} = -f_{b1}(f_{b1}^2 + 2f_{b1} + f_{b1}f_{a1} - 3), \tag{117}$$

$$E = -2f_{a1}f_{b1}(f_{a1} + f_{b1} + 2). \tag{118}$$

The nonlinearity parameter f_{NL} which follows from these is indicated by the thin solid lines in Figs. 3–5, for the cases $\beta \to \infty$, $\beta = 1$, and $\beta = 0$ respectively.

In the limiting case where the second curvaton is homogeneous ($\beta = 0$) the expression (106) reduces to

$$f_{\mathrm{NL}} = f_{\mathrm{NL}}^{\mathrm{single}}(f_{a1}) - \frac{5}{6}f_{b1}. \tag{119}$$

Conversely, in the limit when the first curvaton is homogeneous ($\beta \to \infty$) we get

$$f_{\mathrm{NL}} = f_{\mathrm{NL}}^{\mathrm{single}}(f_{b1}) - \frac{5}{6}f_{a1}. \tag{120}$$

Note that in either case the example of two curvatons which decay at the same time does not reduce exactly to the case of a single curvaton. However we can show that taking into account the constraint (73) the minimum value for f_{NL} (for any value of β) is still that found for a single curvaton, $\min(f_{\mathrm{NL}}) = -5/4$.
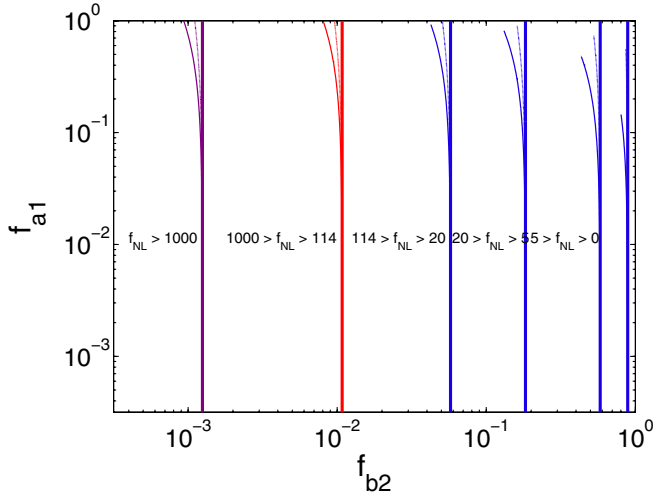
FIG. 3 (color online). $f_{\mathrm{NL}}(f_{b2}, f_{a1})$ when $\beta \to \infty$ and $f_{b1} = 0$ (thick solid lines) or $f_{b1} = f_{b2}/2$ (dotted lines) or $f_{b1} = (1 + f_{a1}/3)f_{b2}$ (thin solid lines). In the latter two cases the constraint (73) excludes a small region in the top right corner. Contours of equal f_{NL} are shown, from left to right, for $f_{\mathrm{NL}} = 1000$, 114, 20, 5, 0, and -1.

C. Both curvatons subdominant at decay

If the energy density of both curvatons is small when they decay then $f_{a1} \ll 1$ and $f_{b2} \ll 1$ [which implies $f_{b1} \ll 1$, from Eq. (76)]. To first order in the f parameters, we find

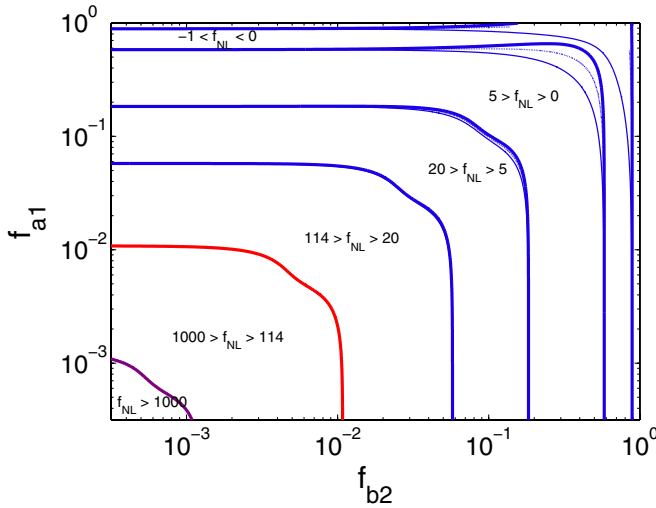$$r_a \simeq f_{a1}, \tag{121}$$

$$r_b \simeq f_{b2}, \tag{122}$$

FIG. 4 (color online). Same as Fig. 3 but now $f_{\mathrm{NL}}(f_{b2}, f_{a1})$ is shown for $\beta = 1$. Contours of equal f_{NL} are shown, from the bottom left corner to the top right corner, for $f_{\mathrm{NL}} = 1000$, 114, 20, 5, 0, and -1.
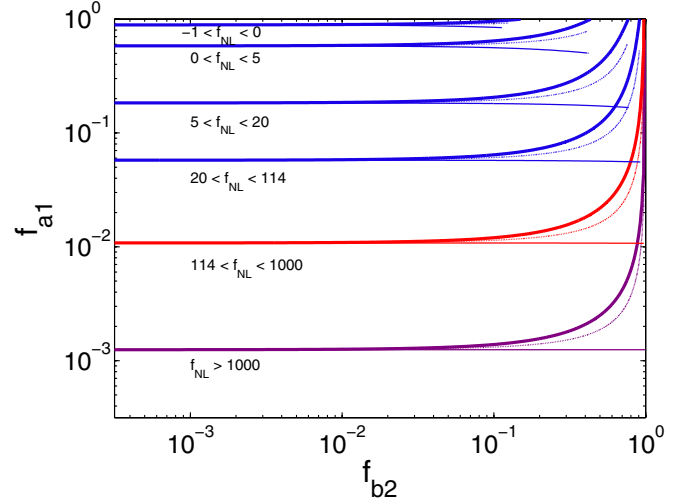

FIG. 5 (color online). Same as Fig. 3 but now $f_{\mathrm{NL}}(f_{b2}, f_{a1})$ is shown for $\beta = 0$. Contours of equal f_{NL} are shown, from bottom to top, for $f_{\mathrm{NL}} = 1000$, 114, 20, 5, 0, and -1.

$$\tilde{C} \simeq 3f_{a1}, \tag{123}$$

$$\tilde{D} \simeq 3f_{b2}, \tag{124}$$

$$E \simeq 0. \tag{125}$$

In this case the nonlinearity parameter Eq. (106) reduces to the quasi-second-order result (88). As expected the quasi-second-order result gives a good approximation when f_{a1} and f_{b2} are small and the non-Gaussianity is thus large.

D. Second curvaton negligible at the first decay

If the energy density of the second curvaton b is negligible when the first curvaton a decays then we can set $f_{b1} = 0$ and the linear result reduces to the simpler result (72) found by Choi and Gong [17]. We then have

$$r_a = (1 - f_{b2})f_{a1}, \tag{126}$$

$$r_b = f_{b2}, \tag{127}$$

$$\tilde{C} = f_{a1}(1 - f_{b2})(3 - 2f_{a1} - f_{a1}^2) + f_{a1}^2 f_{b2}(3 - 2f_{b2} - f_{b2}^2), \tag{128}$$

$$\tilde{D} = f_{b2}(3 - 2f_{b2} - f_{b2}^2), \tag{129}$$

$$E = -2f_{a1}(3f_{b2} - 2f_{b2}^2 - f_{b2}^3), \tag{130}$$

and

$$f_{\rm NL} = \frac{5}{6}\Bigg\{ (1-f_{b2})^3 f_{a1}^3 \left[\frac{3}{2} - 2f_{a1} - f_{a1}^2 + f_{a1}f_{b2}(f_{b2}+3) \right]$$

$$- \beta^2 (1-f_{b2})^2 f_{a1}^2 f_{b2}^2 (f_{b2}+3) + \beta^4 f_{b2}^3 \left(\frac{3}{2} - 2f_{b2} \right.$$

$$\left. \left. - f_{b2}^2 \right) \right\} \Bigg/ \{ (1-f_{b2})^2 f_{a1}^2 + \beta^2 f_{b2}^2 \}^2. \qquad (131)$$

In order to illustrate the general behavior of $f_{\rm NL}$ we discuss three particular cases.

(a) *Case $\beta \to \infty$*: this describes a situation where the first curvaton is effectively homogeneous, $\zeta_a = 0$. This case is shown by the thick solid vertical lines in Fig. 3.

In this case the Universe remains homogeneous until the density of the second curvaton becomes non-negligible. Thus there is no difference from the original single inhomogeneous curvaton scenario, and Eq. (131) gives

$$f_{\rm NL} = f_{\rm NL}^{\rm single}(f_{b2}). \qquad (132)$$

Note that we have assumed here that the density of the second curvaton is negligible when the first decays, $f_{b1} = 0$. If this is not the case then there can be some dependence on f_{a1} (and f_{b1}) illustrated in Fig. 3.

(b) *Case $\beta = 1$*: this illustrates a situation where the fractional density perturbation in each curvaton field is comparable, here $P_{\zeta_a} = P_{\zeta_b}$. The thick solid lines in Fig. 4 show $f_{\rm NL}(f_{b2}, f_{a1})$ corresponding to Eq. (131) with $\beta = 1$. We find that $f_{\rm NL}$ can be large in this case only when $f_{b2} \ll 1$ and $f_{a1} \ll 1$ simultaneously.

(c) *Case $\beta = 0$*: describes a situation where the second curvaton is effectively homogeneous, $(\zeta_b = 0)$. With $\beta = 0$ and $f_{b1} = 0$ we have

$$f_{\rm NL} = \frac{1}{1-f_{b2}} \left[f_{\rm NL}^{\rm single}(f_{a1}) + \frac{5}{6} f_{b2}(f_{b2}+3) \right]. \qquad (133)$$

In this case the inhomogeneous radiation produced by the first curvaton decay is diluted by the decay of the second homogeneous curvaton. The thick solid lines in Fig. 5 show $f_{\rm NL}(f_{b2}, f_{a1})$ of Eq. (133). When the second curvaton's density remains negligible throughout, $f_{b2} \ll 1$, the nonlinearity parameter is a function of a single variable f_{a1}, and we recover the single-curvaton case (110), which gives large $f_{\rm NL}$ when $f_{a1} \ll 1$.

In the opposite limit where $f_{b2} \to 1$, we find a new regime in which the non-Gaussianity becomes large, which only appears in the presence of two (or more) decaying scalar fields. The nonlinearity parameter is given by

$$f_{\rm NL} \to \frac{1}{1-f_{b2}} \left[f_{\rm NL}^{\rm single}(f_{a1}) + \frac{10}{3} \right]. \qquad (134)$$

When $f_{a1} \to 1$, the expression in square brackets approaches $25/12$. Hence we have found an interesting result that $f_{\rm NL}$ can be large even when $f_{a1} \sim 1$ if $f_{b2} \sim 1$. Note that the quasi-second-order approximation (88) correctly predicts $f_{\rm NL} \propto 1/(1-f_{b2})$ in this limit, but fails to reproduce the numerical coefficient $25/12$.

E. General case $f_{b1} \neq 0$

As an example of the general case where the density of the second curvaton is non-negligible when the first curvaton decays ($f_{b1} \neq 0$), we show in Figs. 3–5, $f_{\rm NL}(f_{b2}, f_{a1})$ for the case where $f_{b1} = f_{b2}/2$ and for the simultaneous decay where $f_{b1} = (1 + f_{a1}/3)f_{b2}$. Generally we find only a weak dependence on f_{b1}, although the constraint $f_{b1} \leq 1 - f_{a1}$ excludes a small region in the top right corner where $f_{b2} \sim 1$ and $f_{a1} \sim 1$. For intermediate values of $f_{b1} \in [0, (1 + f_{a1}/3)f_{b2}]$ we find that the values of $f_{\rm NL}$ fall between the cases $f_{b1} = 0$ and $f_{b1} = (1 + f_{a1}/3)f_{b2}$, i.e., the contours of equal values of $f_{\rm NL}$ in Figs. 3–5, lie between the thick and thin solid lines shown.

Finally, we find numerically that the minimum value of the nonlinearity parameter (106) is $\min(f_{\rm NL}) = -5/4$, when the constraints (73) and (76) are taken into account. This is the same as the minimum value found for a single curvaton, $\min(f_{\rm NL}^{\rm single})$, assuming that the evolution between Hubble exit and the beginning of curvaton oscillation is linear, which is the case, for example, for a weakly interacting field with a quadratic potential.

V. CONCLUSION

In this paper we have calculated the nonlinear primordial curvature perturbation, ζ, following the decay of two curvaton fields in the early Universe. The full nonlinear curvature perturbation is given by the series of expressions, (41) and (43)–(45), relating the final perturbation in the radiation after the second curvaton decays, ζ_{γ_2}, to the initial curvature perturbations in each curvaton field, ζ_a and ζ_b, in the sudden-decay approximation. Expanding to first- and then second-order in the perturbations we have derived the general expressions in Eq. (71) for the power spectrum and Eq. (106) for the nonlinearity parameter, $f_{\rm NL}$, which describes the non-Gaussianity of the primordial curvature perturbation at leading order.

Large values of the nonlinearity parameter in the two-curvaton model can qualitatively be understood by a simplified quasi-second-order model

$$\zeta \sim r_a \left[\zeta_{a(1)} + \frac{3}{4}\zeta_{a(1)}^2 \right] + r_b \left[\zeta_{b(1)} + \frac{3}{4}\zeta_{b(1)}^2 \right], \qquad (135)$$

leading to

$$f_{\text{NL}} \sim \frac{5}{4} \frac{r_a^3 + \beta^4 r_b^3}{(r_a^2 + \beta^2 r_b^2)^2}, \qquad (136)$$

where r_a and r_b are given by first-order results (66) and (68).

We easily recover single field results: $f_{\text{NL}} \sim 5/(4r_b)$ if $f_{a1} = 0$, or $f_{\text{NL}} \sim 5/(4r_a)$ if $f_{b1} = 0$ and $f_{b2} = 0$. In either case the nonlinearity becomes large if the transfer of curvaton to curvature perturbation is inefficient ($r_a \ll 1$ and $r_b \ll 1$). More generally in the two-curvaton model we find large f_{NL} if both curvatons are subdominant at their decay time.

We can also obtain large non-Gaussianity in a different way, only possible due to the existence of two curvatons, where the second curvaton is effectively homogeneous: $\beta = 0$. In this case $f_{\text{NL}} \propto 1/r_a$, but the efficiency $r_a \propto f_{a1}(1 - f_{b2})$ becomes small either when f_{a1} is small (as in the one curvaton case) or when $f_{b2} \sim 1$. In this latter case the inhomogeneous radiation produced by the first curvaton decay is diluted by the decay of the second homogeneous curvaton.

In all cases we find $f_{\text{NL}} \geq -5/4$, which seems to be a robust lower bound in single and multifield curvaton models in which the curvaton field perturbations are themselves Gaussian, which should be a good approximation for a weakly interacting field.

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APPENDIX A: NOTATION

The notation becomes quite complicated with the need to specify two different curvaton species and the radiation produced in their decays, which happen at two different moments. Furthermore, we need to keep track of the first- and second-order parts of perturbations. Therefore, to advise the reader, we present in this section a list of some symbols that appear in our calculations and results.

A subscript 1 or 2 after specification of the particle species tells whether the quantity is evaluated at first or second decay. Note that some quantities are constant, and hence do not need this subscript as apparent from the list below.

The energy density parameters ($\Omega_i = \rho_i/\rho_{\text{total}}$):

Ω_{a1} the density of the first curvaton a just before the first decay,

Ω_{b1} the density of the second curvaton b at the first decay,

Ω_{b2} the density of the second curvaton b just before the second decay,

$\Omega_{\gamma_0 1}$ the density of preexisting radiation at the first decay,

$\Omega_{\gamma_1 1}$ the density of all radiation immediately after the first decay,

$\Omega_{\gamma_1 2}$ the density of all radiation just before the second decay,

$\Omega_{\gamma_2 2}$ the density of all radiation immediately after the second decay.

The full nonlinear curvature perturbations:

ζ ($= \zeta_2$) the primordial perturbation after the second curvaton decay, but before the nucleosynthesis,

ζ_1 the total perturbation at the first decay,

ζ_2 the total perturbation at/after the second decay,

ζ_a the perturbation of the first curvaton a,

ζ_b the perturbation of the second curvaton b,

ζ_{γ_0} the preexisting radiation perturbation,

ζ_{γ_1} the radiation perturbation after the first decay,

ζ_{γ_2} ($= \zeta_2 = \zeta$) the radiation perturbation after the second decay.

Subscript in parentheses after all the other subscripts denotes the order considered. For example:

$\zeta_{(1)}$ the first-order part of the primordial perturbation,

$\zeta_{(2)}$ the second-order part of the primordial perturbation,

$\zeta_{1(1)}$ the first-order part of the total perturbation at the first decay,

$\zeta_{1(2)}$ the second-order part of the total perturbation at the first decay,

$\zeta_{2(1)}$ the first-order part of the total perturbation at the second decay,

$\zeta_{2(2)}$ the second-order part of the total perturbation at the second decay,

$\zeta_{a(1)}$ the first-order part of the first-curvaton perturbation,

$\zeta_{a(2)}$ the second-order part of the first-curvaton perturbation,

$\zeta_{b(1)}$ the first-order part of the second-curvaton perturbation,

$\zeta_{b(2)}$ the second-order part of the second-curvaton perturbation,

$\zeta_{\gamma_0(1)}$ the first-order part of the preexisting radiation perturbation,

$\zeta_{\gamma_0(2)}$ the second-order part of the preexisting radiation perturbation,

$\zeta_{\gamma_1(1)}$ the first-order part of the radiation perturbation after the first decay,

$\zeta_{\gamma_1(2)}$ the second-order part of the radiation perturbation after the first decay,

$\zeta_{\gamma_2(1)}$ the first-order part of the radiation perturbation after the second decay,

$\zeta_{\gamma_2(2)}$ the second-order part of the radiation perturbation after the second decay.

The energy density ratios (curvature perturbation transfer efficiencies) at the first decay:

$$f_{a1} = \frac{3\Omega_{a1}}{4\Omega_{\gamma_0 1} + 3\Omega_{a1} + 3\Omega_{b1}},$$
$$f_{b1} = \frac{3\Omega_{b1}}{4\Omega_{\gamma_0 1} + 3\Omega_{a1} + 3\Omega_{b1}},$$

$$f_{\gamma_0 1} = \frac{4\Omega_{\gamma_0 1}}{4\Omega_{\gamma_0 1} + 3\Omega_{a1} + 3\Omega_{b1}},$$

$$R_1 = \frac{4 - \Omega_{b1}}{4 - 4\Omega_{b1}}.$$

The energy density ratios (curvature perturbation transfer efficiencies) at the second decay:

$$f_{b2} = \frac{3\Omega_{b2}}{4\Omega_{\gamma_1 2} + 3\Omega_{b2}},$$

$$f_{\gamma_1 2} = \frac{4\Omega_{\gamma_1 2}}{4\Omega_{\gamma_1 2} + 3\Omega_{b2}}.$$

The total first-order curvaton perturbation transfer efficiencies:

$$r_a = \frac{(1 - f_{b2})(3 + f_{a1})f_{a1}}{3(1 - f_{b1}) + f_{a1}},$$

$$r_b = \frac{(1 - f_{b1})f_{b2}(3 + f_{a1}) + f_{b1}f_{a1}}{3(1 - f_{b1}) + f_{a1}}.$$

The nonlinearity parameters:

f_{NL}^a the nonlinearity parameter of the first-curvaton perturbation ζ_a,

f_{NL}^b the nonlinearity parameter of the second-curvaton perturbation ζ_b,

f_{NL} the nonlinearity parameter of the primordial perturbation $\zeta = \zeta_2 = \zeta_{\gamma_2}$.

APPENDIX B: COEFFICIENTS IN THE SECOND-ORDER PART OF ζ

In this section we express the coefficients $\tilde{C}$, $\tilde{D}$, E, F, and G, Eqs. (97)–(101), with R_1 from Eq. (58) substituted in. We do not write the neatest possible forms, but instead organize the results in order of raising powers of f_{a1}, f_{b2}, and f_{b1}. This is particularly practical for the purposes of Sec. IV, where we express f_{NL} in various limits, for example, in the limit where both curvatons are subdominant at their decay time, i.e., $f_{a1} \ll 1$, $f_{b2} \ll 1$, and $f_{b1} \ll 1$.

The coefficients in the second-order part of ζ in (96) are

$$
\begin{aligned}
\tilde{C} = \frac{1}{(3 - 3f_{b1} + f_{a1})^2} \times \{&[27 - 27f_{b1} + (-27 + 27f_{b1})f_{b2}]f_{a1} + [-27f_{b1} + (27 + 27f_{b1})f_{b2} - 18f_{b2}^2 - 9f_{b2}^3]f_{a1}^2 \\
&+ [-18 + 3f_{b1}^2 + (36 - 3f_{b1}^2)f_{b2} - 12f_{b2}^2 - 6f_{b2}^3]f_{a1}^3 + [-8 + 2f_{b1} + (11 - 2f_{b1})f_{b2} - 2f_{b2}^2 - f_{b2}^3]f_{a1}^4 \\
&+ [-1 + f_{b2}]f_{a1}^5\},
\end{aligned}
\tag{B1}
$$

$$
\begin{aligned}
\tilde{D} = \frac{1}{(3 - 3f_{b1} + f_{a1})^2} \times \{&(27 - 54f_{b1} + 27f_{b1}^2)f_{b2} + (-18 + 36f_{b1} - 18f_{b1}^2)f_{b2}^2 + (-9 + 18f_{b1} - 9f_{b1}^2)f_{b2}^3 \\
&+ [9f_{b1} - 12f_{b1}^2 + 3f_{b1}^4 + (18 - 45f_{b1} + 30f_{b1}^2 - 3f_{b1}^4)f_{b2} + (-12 + 24f_{b1} - 12f_{b1}^2)f_{b2}^2 + (-6 + 12f_{b1} \\
&- 6f_{b1}^2)f_{b2}^3]f_{a1} + [3f_{b1} - 8f_{b1}^2 + 2f_{b1}^3 + (3 - 9f_{b1} + 11f_{b1}^2 - 2f_{b1}^3)f_{b2} + (-2 + 4f_{b1} - 2f_{b1}^2)f_{b2}^2 \\
&+ (-1 + 2f_{b1} - f_{b1}^2)f_{b2}^3]f_{a1}^2 + [-f_{b1}^2 + f_{b2}f_{b1}^2]f_{a1}^3\},
\end{aligned}
\tag{B2}
$$

$$
\begin{aligned}
E = \frac{1}{(3 - 3f_{b1} + f_{a1})^2} \times \{&[18f_{b1} - 18f_{b1}^2 + (-54 + 36f_{b1} + 18f_{b1}^2)f_{b2} + (36 - 36f_{b1})f_{b2}^2 + (18 - 18f_{b1})f_{b2}^3]f_{a1} \\
&+ [-24f_{b1} + 6f_{b1}^3 + (-36 + 60f_{b1} - 6f_{b1}^3)f_{b2} + (24 - 24f_{b1})f_{b2}^2 + (12 - 12f_{b1})f_{b2}^3]f_{a1}^2 + [-16f_{b1} + 4f_{b1}^2 \\
&+ (-6 + 22f_{b1} - 4f_{b1}^2)f_{b2} + (4 - 4f_{b1})f_{b2}^2 + (2 - 2f_{b1})f_{b2}^3]f_{a1}^3 + [-2f_{b1} + 2f_{b1}f_{b2}]f_{a1}^4\},
\end{aligned}
\tag{B3}
$$

$$
F = r_a = \frac{1}{(3 - 3f_{b1} + f_{a1})} \times \{(3 - 3f_{b2})f_{a1} + (1 - f_{b2})f_{a1}^2\},
\tag{B4}
$$

$$
G = r_b = \frac{1}{(3 - 3f_{b1} + f_{a1})} \times \{(3 - 3f_{b1})f_{b2} + [f_{b1} + (1 - f_{b1})f_{b2}]f_{a1}\}.
\tag{B5}
$$

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