PHYSICAL REVIEW D **76,** 101501(R) (2007)

Geometry for the accelerating universe

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(Received 15 December 2006; published 9 November 2007)

The Lorentzian spacetime metric is refined to an area metric which naturally emerges as a generalized geometry in quantum string and gauge theory. Employing the area metric curvature scalar, the Einstein-Hilbert gravitational action is reinterpreted as dynamics for an area metric. The area metric cosmology of the radiation-dominated early universe does not depart from general relativity, enabling successful nucleosynthesis. But intriguingly, without the need for dark energy or fine-tuning, area metric cosmology explains the observed small acceleration of the late universe.

DOI: 10.1103/PhysRevD.76.101501 PACS numbers: 04.20.Cv, 04.50.+h, 98.80.−k

Assuming Einstein's equations for the gravitational field, explanations for the observed accelerating expansion of our Universe [1,2] require some form of dark energy [3]. In its simplest incarnation, dark energy amounts to a positive cosmological constant, but its conceivable physical origins predict a value not even close to the required one [4]. This has prompted attempts to deform the Einstein-Hilbert action for the spacetime metric [5], so to avoid the introduction of dark energy. While partly successful, this typically just shifts the problem to the introduction of an unusually small deformation scale, and a largely *ad hoc* choice of the deformed action. The most serious stroke, however, is delivered by Lovelock's theorem [6] which asserts that, in four dimensions, any action for a spacetime metric different from the Einstein-Hilbert action results in field equations of higher than second derivative order, with all the associated problems [7].

In this paper, we circumvent both Lovelock's theorem and the introduction of a deformation scale by reading the Einstein-Hilbert action not as an action for a spacetime metric, but as an action for an area metric. An area metric is a fourth-rank tensor field, which measures two-dimensional tangent areas in close analogy to the way a metric measures tangent vectors [8]. This yields a consistent classical gravity theory with second order field equations, which is a structural, and hence rigid, refinement of general relativity. We show that the predictions of area metric gravity coincide with those of general relativity for radiation-dominated phases of the early universe, but they stunningly differ at late times. The observed small accelerating expansion of our Universe appears as a new exact solution of the field equations, without any additional assumptions such as dark energy or any form of fine-tuning, see Fig. 1.

Far from being an exotic structure, area metrics naturally emerge in the quantization of fundamental theories whose classical formulation is based on a metric spacetime structure. This applies to gauge theories, gravity, and string theory: backreacting photons in quantum electrodynamics effectively propagate in an area metric background [9]; canonical quantization of gravity *à la* Ashtekar [10] naturally leads to an area operator [11]; the massless states of quantum string theory give rise to the Neveu-Schwarz two-form and dilaton besides the graviton, producing a generalized geometry which may be neatly absorbed into an area metric; the low energy action for D-branes [12,13] is a true area metric volume integral [8]. But even for classical electrodynamics, the historical birthplace of metric spacetime, it has been observed early on that not a spacetime metric, but an area metric, presents the natural and most general background structure [14]; the deflection of light in gravitational fields then is as rich as that in known optical media. In the mathematical literature, the idea to base geometry on an area measure goes back to Cartan [15], who demonstrated that metric and area metric geometry are equivalent in three dimensions. This reconciles the postulate of an area metric spacetime with the fact that we can measure lengths and angles in three-dimensional spatial sections.

In four dimensions, area metric geometry is a generalization of metric geometry: while every metric induces an area metric, not every area metric comes from an underlying metric. The additional degrees of freedom may be viewed as arising from string theory, where generalized geometries play an increasingly important role: different ideas by Hitchin [16] and Hull [17] have been applied to understand flux compactifications acted on with T-dualities [18] or mirror symmetry [19], and compactifications with duality twists [20]. The intimate relation of an area metric spacetime structure to string theory is further revealed by the fact that the minimal classical mechanical object one may discuss in an area metric background is a string. In the present note this plays a role in our discussion of fluids in area metric cosmology; these cannot consist of particles, but must be based on an integrable distribution of string world sheets, which leads us to develop the notion of a string fluid.

We now turn to a precise geometric formulation of the above ideas in four dimensions. Where detailed proofs are omitted, we refer the reader to our companion paper [21] for full detail. Our guiding principle in the construction of area metric gravity is downward compatibility to metric spacetime, paying tribute to the phenomenal success of standard general relativity as a theory of gravity. So first consider a familiar metric manifold (M, g) which naturally induces the area metric

$$C_{g\,abcd} = g_{ac}g_{bd} - g_{ad}g_{bc}, \tag{1}$$

since $C_{g\,abcd}X^aY^bX^cY^d$ measures the squared area of a parallelogram spanned by vectors X and Y. The basic idea of area metric geometry is to promote area metrics to a structure in their own right, by keeping salient algebraic properties of the metric-induced case: we define an area metric spacetime (M, G) as a smooth four-dimensional manifold M equipped with a fourth-rank covariant tensor field G with the symmetries

$$G_{abcd} = G_{cdab}, \qquad G_{abcd} = -G_{bacd}, \tag{2}$$

and the property that G is invertible. Here invertibility is understood as follows: due to its symmetries the indices of G may be combined to antisymmetric Petrov pairs $[ab]$ so that G can be represented by a symmetric 6×6 matrix which is required to be nondegenerate. The determinant $\mathrm{Det}\,G$ over this 6×6 matrix gives rise to a volume form ω_G with components $\omega_{G\,abcd} = |\mathrm{Det}\,G|^{1/6}\epsilon_{abcd}$, where ϵ is the Levi-Civita tensor density normalized such that $\epsilon_{0123} = 1$. The inverse area metric G decomposes uniquely into a cyclic area metric $C^{a[bcd]} = 0$ and a totally antisymmetric four-tensor that is dual to a scalar,

$$G^{abcd} = C^{abcd} + \phi\omega_C^{abcd}. \tag{3}$$

We now construct area metric curvature tensors, which are downward compatible to their metric counterparts, in two steps. First, note that (M, G) gives rise to an effective metric g_G fully defined by area metric data. Using the above decomposition,

$$g_G^{ab} = \frac{1}{2}\frac{\partial^2}{\partial p_a \partial p_b}\bigg|_{p=d\phi}(\mathcal{G}^{ijkl}p_ip_jp_kp_l)^{1/2} \tag{4}$$

for the Fresnel tensor

$$\mathcal{G}^{ijkl} = -\frac{1}{24}\omega_{C\,abcd}\omega_{C\,pqrs}C^{abp(i}C^{j|cq|k}C^{l)drs}. \tag{5}$$

This construction can be motivated in detail by considering wave propagation on area metric manifolds in the geometric optics limit [21], where wave vectors k obey the quartic Fresnel equation $\mathcal{G}^{abcd}k_ak_bk_ck_d = 0$, see [22,23]. The effective metric generally does not contain all information of the area metric, which follows from a simple counting of the independent components. In particular, the reinduced area measure C_{g_G} generally does not agree with G in dimensions greater than three. But if we consider almost metric area metrics for which $C = C_g$ for some metric g in the decomposition (3), so that

$$G^{abcd} = g^{ac}g^{bd} - g^{ad}g^{bc} + \phi\omega_g^{abcd}, \tag{6}$$

then the effective metric g_G recovers g up to a sign, showing downward compatibility at the level of the effective metric. An important example for almost metric spacetimes is area metric cosmology: imposing isotropy and homogeneity on a four-dimensional area metric, by requiring that $\mathcal{L}_K G = 0$ for the symmetry-generating vector fields K, leaves us with an area metric of the form (6), where g takes the standard Robertson-Walker form, and the additional scalar ϕ depends only on time.

The effective metric g_G with its torsion-free Levi-Civita connection ∇^{LC} are used to define the following covariant derivative on antisymmetric tensors Ω:

$$(\nabla_f\Omega)^{ab} = (\nabla_f^{\mathrm{LC}}\Omega)^{ab} + \tfrac{1}{2}X^{ab}{}_{cdf}\Omega^{cd}, \tag{7}$$

where $X^{ab}{}_{cdf} = \frac{1}{4}G^{abij}\nabla_f^{\mathrm{LC}}G_{ijcd}$. Up to a symmetry condition, ∇ is the unique covariant derivative with $\nabla G = 0$, so that, for instance, areas are preserved under parallel transport. This construction provides a true area metric connection, without using data additional to G. This completes the second step in our construction of the area metric curvature tensor, whose definition is now standard: $\mathcal{R}_G(X, Y)\Omega = [\nabla_X, \nabla_Y]\Omega - \nabla_{[X,Y]}\Omega$, so that

$$\mathcal{R}_{G\,cdij}^{ab} = 4\delta^{[a}_{[c}R^{b]}{}_{d]ij} + (\nabla_i^{\mathrm{LC}}X^{ab}{}_{cdj} + \tfrac{1}{2}X^{ab}{}_{sti}X^{st}{}_{cdj} - (i \leftrightarrow j)). \tag{8}$$

Natural contraction yields the area metric Ricci tensor, and the unique area metric curvature scalar of linear order in $\mathcal{R}_G^{a_1a_2}$, the area metric Ricci scalar:

$$\mathcal{R}_{G\,mn} = \mathcal{R}^{pq}_{G\,pmqn}, \qquad \mathcal{R}_G = g_G^{mn}\mathcal{R}_{G\,mn}. \tag{9}$$

All of the above curvature tensors reduce to their metric counterparts if the area metric is almost metric (6). Thus area metric geometry allows us to devise a gravity theory different from Einstein's without modifying the form of the Einstein-Hilbert action; replacing all metric quantities by their area metric counterparts, the latter is reinterpreted as an action on an area metric manifold:

$$S_{\mathrm{grav}} + S_m = \frac{1}{2\kappa}\int_M \omega_G\mathcal{R}_G + \int_M \mathcal{L}_m. \tag{10}$$

A gravitational constant κ appears, and $\mathcal{L}_m$ represents the matter Lagrangian density. Importantly, the equations of motion are now derived by variation with respect to the inverse area metric G, which leads to a fourth-rank tensor equation

$$K_{abcd} = \kappa T_{abcd}, \tag{11}$$

where we have, schematically, the gravitational variation $K = |\mathrm{Det}\, G|^{-1/6}\delta(\kappa S_{\mathrm{grav}})/\delta G^{-1}$ and the energy-momentum tensor $T = -|\mathrm{Det}\, G|^{-1/6}\delta S_m/\delta G^{-1}$. Diffeomorphism invariance of the action implies an area metric Bianchi identity for the gravitational part of the action, and a conservation law for the area metric energy-momentum tensor; see [21] for the full computation, and [24] for the relation to conventional energy momentum.

For applications to cosmology, where the area metric takes the almost metric form (6), the full equations of motion simplify considerably. Lengthy algebra reveals that in the almost metric case the gravitational variation K is induced from a symmetric two-tensor and a scalar. If the tensor T_{abcd} happens to be likewise induced from a symmetric second rank tensor T_{ab} and a scalar T^{ϕ}, which is the case if and only if the irreducible Weyl component of T_{abcd} vanishes [24], then the full field equations (11), for $\tilde{\phi} = (1+\phi^2)^{-1/2}$, reduce to

$$\begin{aligned} \kappa T_{ab} &= R_{ab} - \tfrac{1}{2}R g_{ab} - \tilde{\phi}^{-1}(\nabla_a \partial_b \tilde{\phi} - g_{ab}\Box\tilde{\phi}), \\ \kappa T^{\phi} &= -\tilde{\phi}(1-\tilde{\phi}^2)^{1/2} R. \end{aligned} \tag{12}$$

In particular, these equations are valid *in vacuo*; suitable field redefinitions reveal conformal equivalence to Einstein gravity minimally coupled to a massless scalar field. So the original vacuum theory is causal. We also conclude that any vacuum solution (M, g) of Einstein gravity is a vacuum solution of area metric gravity (setting $\tilde{\phi} = 1$ or $\tilde{\phi} \to 0$ with appropriate conditions on the derivatives). Including matter, however, it is no longer true that all solutions of Einstein gravity lift to solutions of area metric gravity. The energy-momentum tensor for Maxwell electrodynamics on area metric spacetime, for instance, is not induced from a symmetric two-tensor and a scalar. In other words, not only is an area metric spacetime a possible background for electrodynamics, but the backreaction via the area metric Einstein-Hilbert action requires a truly area metric background.

The most prominent example of matter which *is* compatible with almost metric backgrounds arises in area metric cosmology. Recall that in general relativity perfect fluids are the most general matter consistently coupling to cosmology, due to the way symmetries restrict the energy-momentum tensor. This is also the case in area metric geometry, which however does not admit fluids based on point particles in the first place; we have to resort to string fluids based on continuous distributions of world sheets. A three-component string fluid with local tangent surfaces $\Omega_I = \partial_t \wedge v_I$ for three g-spacelike vectors v_I is required to isotropically fill the spatial sections. Then the general source tensor T_{abcd} equals

$$\frac{\tilde{\rho}+\tilde{p}}{4}\sum_I G_{abij}\Omega_I^{ij} G_{cdkl}\Omega_I^{kl} + \tilde{p} G_{abcd} + (\tilde{\rho}+\tilde{q})G_{[abcd]} \tag{13}$$

in terms of three time-dependent functions $\tilde{\rho}$, $\tilde{p}$, and $\tilde{q}$. This string fluid tensor is induced by a two-tensor and by a scalar, as is needed for the equations to take the simple form (12). Using a time/space split,

$$\begin{aligned} T_{00} &= 12\frac{\tilde{\rho}-\tilde{q}\phi^2}{1+\phi^2}, \qquad T^{\phi} = \frac{24\phi\tilde{q}}{1+\phi^2}, \\ T_{\alpha\beta} &= 4\frac{-\tilde{\rho}+2\tilde{p}+(2\tilde{\rho}+2\tilde{p}+3\tilde{q})\phi^2}{1+\phi^2} g_{\alpha\beta}. \end{aligned} \tag{14}$$

Careful evaluation of the Eqs. (12) now reveals a remarkable correspondence between, on the one hand, area metric cosmology (determined by g and ϕ) plus string fluid matter ($\tilde{\rho}$, $\tilde{p}$, and $\tilde{q}$), and, on the other hand, Einstein cosmology (g) plus perfect fluid matter (ρ and p). The effective energy density and pressure of the perfect fluid emerge as

$$\rho = 3(x-y), \qquad p = x+y \tag{15}$$

for $x = -H\dot{\tilde{\phi}}\tilde{\phi}^{-1} + 4\kappa(\tilde{\rho}+\tilde{q})\tilde{\phi}^2$ and $y = 4\kappa\tilde{q}$, where H is the Hubble function. Recall $\tilde{\phi} = (1+\phi^2)^{-1/2}$ which obeys $\Box\tilde{\phi} = \partial V(\tilde{\phi})/\partial\tilde{\phi}$ with potential

$$V(\tilde{\phi}) = 4\kappa(\tilde{\rho}+\tilde{p}+\tilde{q})\tilde{\phi}^2 - 4\kappa(\tilde{\rho}+\tilde{q})\tilde{\phi}^4. \tag{16}$$

We now discuss area metric cosmology for specific epochs in the evolution of the universe.

The early universe is dominated by radiation consisting of ultrarelativistic fermions and gauge bosons. While the latter directly couple to the area metric G [21], the spin structure needed for fermions derives from the effective metric g_G determined by G. Radiation fields are, as usual, identified as those with null physical momentum with respect to the Fresnel tensor. In cosmology this leads to the condition $\omega_G^{abcd}T_{abcd} = 0$ for the energy-momentum four-tensors of both massless Dirac spinors and gauge fields, see [24]. Imposing this condition on the string fluid energy-momentum tensor (13) leads to $\tilde{q} = 0$. Hence, as can be seen from (15), area metric cosmology with a radiation string fluid is equivalent to Einstein cosmology with a standard perfect fluid with fixed constitutive relation

$$p = \tfrac{1}{3}\rho. \tag{17}$$

It follows that all results of Einstein cosmology in radiation-dominated epochs directly carry over to area metric cosmology without modification. In particular, nucleosynthesis is unaffected.

For the late universe, matter has spread out so much that interactions are no longer important, so that we may specialize to noninteracting string dust, whose parameters $\tilde{p}$, $\tilde{\rho}$, and $\tilde{q}$ can be identified from the condition that energy conservation should be implied by the minimal surface equation and by a generalized continuity equation [21], which yields

$$\tilde{p} = 0, \qquad \tilde{q} = -\tilde{\rho}. \tag{18}$$

We now provide exact solutions for the string dust case with $\tilde{\rho} \neq 0$. Using $R_{00} = -3\dot{H} - 3H^2$ and $R_{\alpha\beta} = (2ka^{-2} + \dot{H} + 3H^2)g_{\alpha\beta}$, one collects the full set of equations from (12), (14), and (16). The equations of area metric cosmology filled with string dust then become

$$\tilde{\phi} = \lambda \dot{a}, \qquad \tilde{\rho} = \zeta a^{-2}, \qquad 0 = \ddot{a} + \frac{\dot{a}^2}{a} + \frac{k - 4\kappa\zeta}{a} \tag{19}$$

for $\xi = k - 4\kappa\zeta$, and integration constants λ and ζ. These equations are exactly solved by the scale factor

$$a(t) = \begin{cases} \sqrt{c(t - t_0)} & \text{for } \xi = 0, \\ \sqrt{c\xi^{-1} - \xi(t - t_0)^2} & \text{for } \xi \neq 0, \end{cases} \tag{20}$$

with integration constants c and t_0. The different types of solutions are presented in Fig. 1, and discussed in detail in [21]. Remarkably we find that area metric cosmology allows accelerating late-time evolution of a dust-filled universe for $\xi < 0$, $c < 0$, and any value of the cosmological curvature k.

So we have shown that area metric cosmology provides an appropriate description for the early and late-time evolution of our universe: first, by explaining the radiation-dominated early epochs in precisely the same manner as does Einstein cosmology, enabling standard nucleosynthesis; second, and more strikingly, by explaining the present acceleration of the Universe. This explanation neither invokes concepts such as a cosmological constant or dark energy nor fine-tuning since the acceleration automatically tends to small values at late time. While these results are pure consequences of area metric geometry, the discussion of the very early universe in its inflationary epoch would require further assumptions, just as in the standard model of cosmology, and so would not present a genuine test of the area metric spacetime structure. Hence, observations of the cosmic microwave background, whose understanding is based on an inflationary phase, cannot confidently distinguish area metric from metric spacetime.

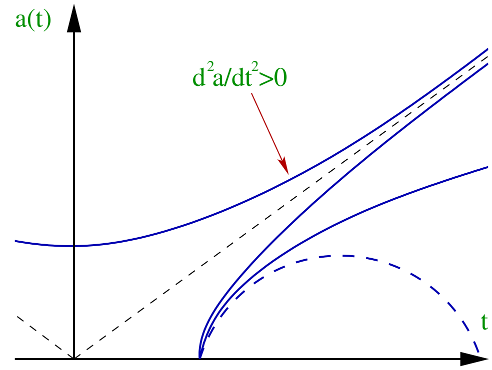

FIG. 1 (color online). The solid curves sketch the string dust-filled area metric cosmologies: from bottom to top: $\xi = 0$, $\xi < 0$ with $c > 0$, and $\xi < 0$ with $c < 0$. The dashed curve depicts the case $\xi > 0$. The top curve is an important new result from area geometry (and cannot be obtained as a dust-filled Einstein cosmology): late-time acceleration tending to zero.

It is remarkable that the single fundamental hypothesis of area metric geometry turns Einstein gravity into a consistent alternative gravity theory which reproduces the successes of general relativity in the early universe, but surprisingly, without further assumptions, explains the present acceleration of the universe, one of the most intriguing cosmological observations of recent times. The fact that these results crucially depend on the observed standard model fields being naturally coupled to area metric backgrounds shows the consistency, and relevance, of our findings far beyond pure gravity.

We have the pleasure to acknowledge useful comments by Gary Gibbons, Julius Wess, Alexander Turbiner, Albert Schwarz, Friedrich Hehl, and Marcus Werner. This work has been supported by the European Union (R. P.), DGAPA-UNAM Grant No. IN121306 (F. P. S.), and the DFG Emmy Noether Grant No. WO 1447/1-1 (M. N. R. W.).

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