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Width difference of ρ vector mesons

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We compute the difference in decay widths between charged and neutral $\rho(770)$ vector mesons. The isospin breaking arising from mass differences of neutral and charged π and ρ mesons, radiative corrections to $\rho \to \pi\pi$, and the $\rho \to \pi\pi\gamma$ decays are taken into account. It is found that the width difference $\Delta\Gamma_\rho$ is very sensitive to the isospin breaking in the ρ meson mass, Δm_ρ. This result can be useful to test the correlations observed between the values of these parameters extracted from experimental data.

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I. INTRODUCTION

Lowest-lying vector mesons undergo predominantly strong interaction decays. The masses and decay widths of members of the same isomultiplet will therefore look very similar [1], with small differences induced by the breaking of isospin symmetry. The isospin breaking effects in the ρ meson parameters have raised an interest recently, due to both experimental and theoretical reasons [2–6]. According to the PDG [1], the weighted averages of available measurements are

$$\Delta m_\rho \equiv m_{\rho^0} - m_{\rho^\pm} = (-0.7 \pm 0.8)\ \text{MeV}, \tag{1}$$

$$\Delta\Gamma_\rho \equiv \Gamma_{\rho^0} - \Gamma_{\rho^\pm} = (0.3 \pm 1.3)\ \text{MeV}. \tag{2}$$

These results are consistent with the absence of isospin breaking in the $\rho^0 - \rho^\pm$ system. Note however that the scale factors associated with the above averages are, respectively, 1.5 and 1.4 [1] which reflects an important spread in the yields from different experiments.

Some recent theoretical calculations of Δm_ρ seem to confirm the above result. Using a vector-meson dominance model to parametrize the $\gamma\rho\rho$ vertex, the authors of Ref. [4] have obtained $\Delta m_\rho = (-0.02 \pm 0.02)$ MeV. Also, using $1/N_c$ expansion techniques, the authors of Ref. [3] have obtained $-0.4\ \text{MeV} \leq \Delta m_\rho \leq 0.7\ \text{MeV}$. On another hand, it has been found that the width difference of ρ mesons is of great importance to understand the current discrepancy between the hadronic vacuum polarization contributions to the muon anomalous magnetic moment obtained from τ decay and e^+e^- annihilation data [2,5,6].

In this paper we provide an estimate of $\Delta\Gamma_\rho$ by considering the isospin breaking corrections in the exclusive modes that contribute to the decay widths of $\rho^{0,\pm}$ vector mesons. A previous estimate of this effect was done in Ref. [2] taking into account several sources of isospin breaking, as mass differences and other subleading ρ meson decays. Their result $\Delta\Gamma_\rho \approx (-0.42 \pm 0.59)$ MeV [2] is consistent with the world average given in Eq. (2). Additional contributions to isospin breaking in $\Delta\Gamma_\rho$, including the radiative corrections to the dominant $\rho \to \pi\pi$ decays, are considered in this paper.

II. SOURCES OF ISOSPIN BREAKING

At a fundamental level, isospin symmetry is broken by the different masses of u and d quarks and by the effects of electromagnetic interactions. At the hadronic level all manifestations of isospin breaking can be traced back to such fundamental sources. In the absence of isospin breaking, the $\rho^{0,\pm}$ mesons must have equal masses and decay widths, thus $\Delta m_\rho = \Delta\Gamma_\rho = 0$.

The dominant decay modes of ρ mesons that are common to charged and neutral ρ's are the $\pi\pi$ decay and its radiative mode. The branching fraction of other modes contributing only to the ρ^0 meson adds up to [1]

$$B^0_{\text{rest}} = B(\pi^0\pi^0\gamma) + B(\eta\gamma) + B(\mu^+\mu^-) + B(e^+e^-) + B(\pi^+\pi^-\pi^0) \approx 5.3 \times 10^{-4}. \tag{3}$$

There is also a dipole transition $\rho \to \pi\gamma$ which is common to $\rho^{\pm,0}$ vector mesons with branching fractions of a few times 10^{-4} [1]. Since the ρ meson widths are of order 150 MeV, all these subleading decay modes will contribute to the width difference at the tiny level of

$$\Delta\Gamma^{\text{sub}}_\rho \approx 0.08\ \text{MeV}. \tag{4}$$

Thus, any sizable difference in the decay widths can only originate from the dominant decay modes. To be more precise, we will define explicitly the contributions to the width difference as follows:

$$\begin{aligned}\Delta\Gamma_\rho = {}& \Gamma(\rho^0 \to \pi^+\pi^-(\gamma), \omega \le \omega_0) \\ & - \Gamma(\rho^+ \to \pi^+\pi^0(\gamma), \omega \le \omega_0) \\ & + \Gamma(\rho^0 \to \pi^+\pi^-\gamma, \omega \ge \omega_0) \\ & - \Gamma(\rho^+ \to \pi^+\pi^0\gamma, \omega \ge \omega_0) + \Delta\Gamma_\rho^{\rm sub}. \end{aligned} \quad (5)$$

The first two terms in Eq. (5) denote the $\pi\pi$ decay rates that include virtual and soft-photon corrections and the next two terms are related to hard photons in $\pi\pi\gamma$ decays. In the above equation, ω_0 is an arbitrary value of the photon energy that separates the decay rates of soft- and hard-photon bremsstrahlung. It is expected that the ω_0 dependence will cancel in the sum of the first (second) and third (fourth) terms of Eq. (5).

In the following section we will consider in more detail the isospin breaking corrections to $\rho \to \pi\pi$ and their radiative decays. Later, we will evaluate the contributions of such corrections to the right-hand side (rhs) of Eq. (5).

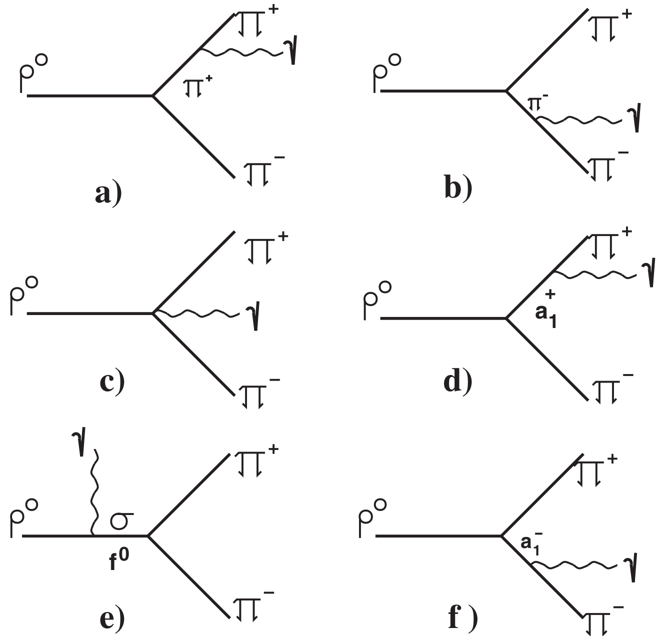

FIG. 1. Feynman graphs for $\rho^0 \to \pi^+\pi^-\gamma$ decays.

III. RADIATIVE CORRECTIONS TO $\rho \to \pi\pi$ DECAYS

At the lowest order (indicated with superscript 0), the rates of $\rho \to \pi\pi$ decays are given by

$$\Gamma_0^0 \equiv \Gamma^0(\rho^0 \to \pi^+\pi^-) = \frac{g_{+-}^2}{48\pi} m_{\rho^0} v_0^3, \quad (6)$$

$$\Gamma_+^0 \equiv \Gamma^0(\rho^+ \to \pi^+\pi^0) = \frac{g_{+0}^2}{48\pi} m_{\rho^+} v_+^3, \quad (7)$$

where

$$v_0 \equiv \sqrt{1 - \frac{4m_{\pi^+}^2}{m_{\rho^0}^2}}, \quad \text{and}$$

$$v_+ \equiv \sqrt{\left(1 - \frac{(m_{\pi^+} - m_{\pi^0})^2}{m_{\rho^+}^2}\right)\left(1 - \frac{(m_{\pi^+} + m_{\pi^0})^2}{m_{\rho^+}^2}\right)} \quad (8)$$

are the pion velocities in the rest frame of the ρ^i meson and g_{ij} are the $\rho\pi^i\pi^j$ coupling constants such that $g_{+-} = g_{+0}$ owing to the isospin symmetry of strong interactions. In the limit of isospin symmetry, the masses of neutral and charged pions (ρ's) are the same, $v_0 = v_+$, and consequently the three-level decay rates are equal ($\Gamma_0^0 = \Gamma_+^0$). In the following subsections we will consider the different ingredients to get the $O(\alpha)$ radiative corrections to the decay rates given above.

A. Real soft-photon corrections

In order to get an infrared safe result, the radiative corrections of order α to Eqs. (6) and (7) must include the sum of virtual corrections and soft-photon bremsstrahlung. In order to define the soft-photon contribution, let us consider the radiative $\rho \to \pi\pi\gamma$ decays shown in Figs. 1 and 2. In each of Figs. 1 and 2, the first three diagrams correspond to the model-independent contributions and the other graphs denote the model-dependent terms.

The decay amplitude of the process $\rho^0(d, \eta) \to \pi^+(p)\pi^-(p')\gamma(k, \epsilon^*)$ (four-momenta and polarization four-vectors are indicated within parenthesis), see Fig. 1, is given by [7,8]

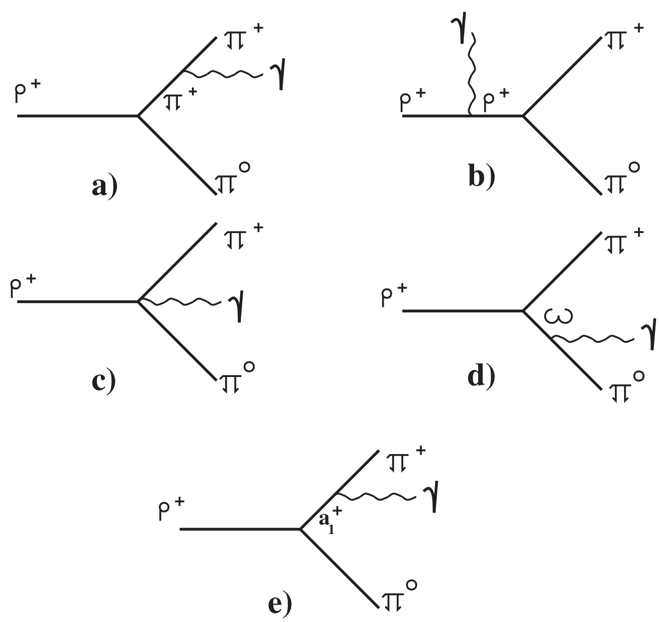

FIG. 2. Feynman graphs for $\rho^+ \to \pi^+\pi^0\gamma$ decays.

$$\mathcal{M}(\rho^0 \to \pi^+\pi^-\gamma) = ieg_{+-}\left\{(p-p')\cdot\eta\left(\frac{p'\cdot\epsilon^*}{p'\cdot k}-\frac{p\cdot\epsilon^*}{p\cdot k}\right) - k\cdot\eta\left(\frac{p'\cdot\epsilon^*}{p'\cdot k}+\frac{p\cdot\epsilon^*}{p\cdot k}\right) + 2\epsilon^*\cdot\eta\right\} + \mathcal{M}_{\rm d,e,f} \tag{9}$$

where the model-independent pieces of the amplitude are given explicitly, and the remaining terms $\mathcal{M}_{\rm d,e,f}$ correspond to the model-dependent amplitudes arising from Figs. 1(d)–1(f). Similarly, the decay amplitude for the decay $\rho^+(d,\eta)\to\pi^+(p)\pi^0(p')\gamma(k,\epsilon^*)$), see Fig. 2, is given by the following expression [7,9–11]:

$$\begin{aligned}\mathcal{M}(\rho^+\to\pi^+\pi^0\gamma) = ieg_{+0}&\left\{\left(\frac{p\cdot\epsilon^*}{p\cdot k}-\frac{d\cdot\epsilon^*}{d\cdot k}\right)(p-p')\cdot\eta + \left(\frac{p\cdot\epsilon^*}{p\cdot k}-\frac{d\cdot\epsilon^*}{d\cdot k}\right)k\cdot\eta\right.\\ &\left.+\left[2+\frac{\Delta\kappa}{2}\left(1+\frac{\Delta_\pi^2}{m_{\rho^+}^2}\right)\right]\left(\frac{d\cdot\epsilon^*}{d\cdot k}k\cdot\eta-\epsilon^*\cdot\eta\right) - (2+\Delta\kappa)\left(\frac{p\cdot\epsilon^*}{p\cdot k}k\cdot\eta-\epsilon^*\cdot\eta\right)\frac{p\cdot k}{d\cdot k}\right\} + \mathcal{M}_{\rm d,e},\end{aligned} \tag{10}$$

where $\Delta\kappa$ denotes the anomalous magnetic dipole moment of the ρ^+ vector meson in units of $e/2m_{\rho^+}$ (in our numerical evaluations we will set $\Delta\kappa = 0$), and we have defined $\Delta_\pi^2 \equiv m_{\pi^+}^2 - m_{\pi^0}^2$. Once again, only the model-independent amplitudes for Figs. 2(a)–2(c) have been written explicitly.

The soft-photon bremsstrahlung is the divergent piece of the amplitudes (9) and (10) in the infrared region ($k\to 0$). The diagrams contributing to the soft-photon amplitude are shown in Figs. 1(a) and 1(b) for $\rho^0\to\pi^+\pi^-\gamma$ and in Figs. 2(a) and 2(b) for $\rho^+\to\pi^+\pi^0\gamma$ decays. As usual, only photons of energy smaller than ω_0 must be considered in the real-photon emission rate that is included in radiative corrections. Thus, the radiative corrected rates of $\rho^i\to\pi\pi$ decays ($i = 0, +$) can be written in the following general form:

$$\Gamma(\rho^i\to\pi\pi(\gamma),\,\omega\le\omega_0) = \Gamma_i^0 + \Gamma_i^{1\rm v} + \Gamma_i^{\rm soft}(\omega_0) = \Gamma_i^0\left[1+\frac{\Gamma_i^{1\rm v}}{\Gamma_i^0}+\frac{\Gamma_i^{\rm soft}(\omega_0)}{\Gamma_i^0}\right] = \Gamma_i^0[1+\delta_{\rho^i}]. \tag{11}$$

where $\Gamma_i^{1\rm v}$ denotes the virtual corrected rate at order α for $\rho^i\to\pi\pi$ decay and $\Gamma_i^{\rm soft}(\omega_0)$ is the soft-photon rate of $\rho^i\to\pi\pi\gamma$ obtained from the first term in Eqs. (9) and (10).

The soft-photon rates can be computed in analytical form by integrating over photon energies smaller than ω_0, with the following results for ρ^0 and ρ^+ radiative decays, respectively:

$$\begin{aligned}\frac{\Gamma_0^{\rm soft}(\omega_0)}{\Gamma_0^0} = \frac{\alpha}{\pi}&\left\{2\ln\left(\frac{\lambda}{2\omega_0}\right)\left(1+\frac{1+v_0^2}{2v_0}\ln\left[\frac{1-v_0}{1+v_0}\right]\right) - \frac{1}{v_0}\ln\left(\frac{1-v_0}{1+v_0}\right)\right.\\ &\left.+\frac{1+v_0^2}{2v_0}\left[\mathrm{Li}_2\left(\frac{1-v_0}{1+v_0}\right) - \mathrm{Li}_2\left(\frac{1+v_0}{1-v_0}\right) + \imath\pi\ln\left(\frac{1-v_0}{1+v_0}\right) + \ln\left(\frac{1-v_0}{1+v_0}\right)\ln\left(\frac{4v_0^2}{1-v_0^2}\right)\right]\right\},\end{aligned} \tag{12}$$

and

$$\begin{aligned}\frac{\Gamma_+^{\rm soft}(\omega_0)}{\Gamma_+^0} = \frac{\alpha}{\pi}&\left\{1-2\ln 2+2\ln\left[\frac{\lambda}{\omega_0}\right]\left(1+\frac{1}{2v'_+}\ln\left[\frac{1-v'_+}{1+v'_+}\right]\right) + \frac{1}{2v'_+}\ln\left(\frac{1+v'_+}{1-v'_+}\right)\right.\\ &\left.-\frac{1}{2v'_+}\left[\mathrm{Li}_2\left(\frac{1+v'_+}{1-v'_+}\right) - \mathrm{Li}_2\left(\frac{1-v'_+}{1+v'_+}\right) + \ln\left(\frac{v'^2_+}{1-v'^2_+}\right)\ln\left(\frac{1+v'_+}{1-v'_+}\right) + \imath\pi\ln\left(\frac{1+v'_+}{1-v'_+}\right)\right]\right\},\end{aligned} \tag{13}$$

where we have defined $v'_+ = v_+/\xi$, with $\xi = 1 + \Delta_\pi^2/m_{\rho^+}^2$, and $\mathrm{Li}(x)$ denotes the Spence function.

Observe that the soft-photon rates depend logarithmically upon the infrared cutoff λ (the photon mass regulator) and the photon energy parameter ω_0. We have checked that Eq. (12) coincides numerically with the result reported long ago by Cremmer and Gourdin [12] in the case of $\phi\to K^+K^-\gamma$ decays, although our corresponding expressions are written in a different form. Finally, let us point out that a very small contribution arising from the regular part (finite when the photon energy goes to zero) of the radiative decay rate must be added to the rhs of Eq. (11).

B. Virtual corrections

The Feynman diagrams of virtual corrections of $O(\alpha)$ to the $\rho\to\pi\pi$ amplitudes are shown in Figs. 3 and 4. As in the real-photon case, we will use scalar QED for the electromagnetic couplings of charged pions and assume that the $\rho^\pm$ electromagnetic vertex (see for example [9,10]) is similar to the $WW\gamma$ gauge boson vertex.

The sum of virtual corrections to the $\rho^0\to\pi^+\pi^-$ amplitude is finite in the ultraviolet region owing to a Ward identity satisfied by the vertex and self-energy corrections [12]. In the case of the $\rho^+\to\pi^+\pi^0$ decay, the sum of virtual corrections diverges. One way of getting rid of such

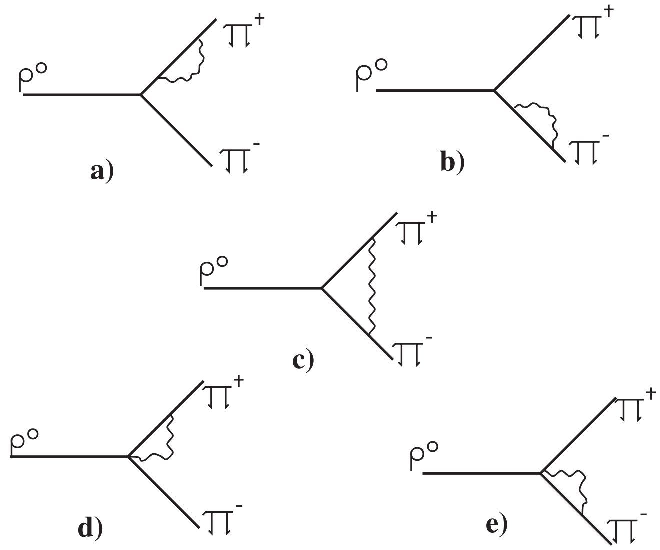

FIG. 3. Feynman graphs for the virtual photonic corrections to $\rho^0 \to \pi^+\pi^-$ decays.

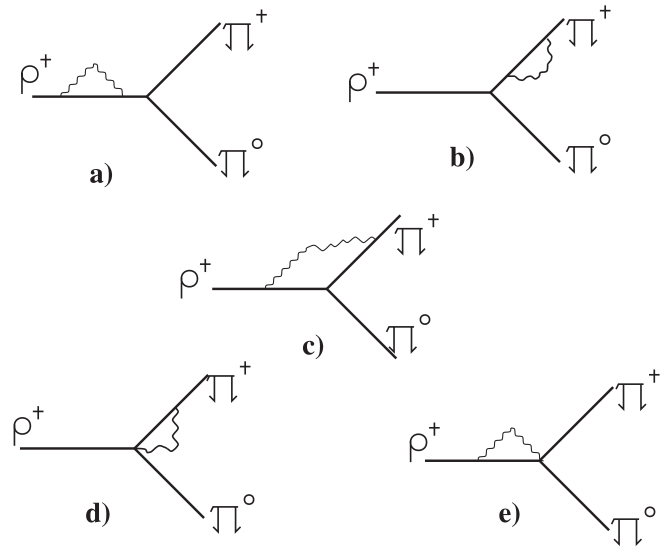

FIG. 4. Feynman graphs for the virtual photonic corrections to $\rho^+ \to \pi^+\pi^0$ decays.

divergences is by absorbing them into a redefinition of the strong coupling constant g_{+0}. Instead of dealing with divergent amplitudes, in the case of $\rho^+ \to \pi^+\pi^0$ decays we will follow a procedure introduced long ago by Yennie [13]. According to this method we pick only the convection terms from virtual corrections in Figs. 4(a)–4(c). The sum of such terms gives rise to a one-loop amplitude which is finite in the ultraviolet region and is also gauge invariant and model independent [13]. Furthermore, such an amplitude contains all the infrared divergent terms of virtual corrections [13].

Just to illustrate the cancellation of infrared divergencies in the sum of virtual and real-photon corrections, we reproduce here the expression for the virtual corrections in the case of $\rho^0 \to \pi^+\pi^-$ decays [12]:

$$\frac{\Gamma_0^{1v}}{\Gamma_0^0} = \frac{\alpha}{\pi}\left[\pi^2\left(\frac{1+v_0^2}{2v_0}\right) - 2\left(1+\ln\left[\frac{\lambda}{m_{\pi^+}}\right]\right)\left(1+\frac{1+v_0^2}{2v_0}\ln\left[\frac{1-v_0}{1+v_0}\right]\right) - \left(\frac{1+v_0^2}{v_0}\right)[\mathrm{Li}_2(v_0) - \mathrm{Li}_2(-v_0)] - \frac{1+v_0^2}{2v_0}\left(\mathrm{Li}_2\left[\frac{2}{1+v_0}\right] - \mathrm{Li}_2\left[\frac{2}{1-v_0}\right]\right)\right]. \tag{14}$$

The corresponding result in the case of $\rho^+ \to \pi^+\pi^0$ decays can be computed using the procedure described above [13]. We obtain

$$\begin{aligned}\frac{\Gamma_+^{1v}}{\Gamma_+^0} = \frac{\alpha}{\pi}\Bigg\{&-1 - 2\ln\left(\frac{\lambda}{m_{\rho^+}}\right)\left[1+\frac{1}{2v'_+}\ln\left(\frac{1-v'_+}{1+v'_+}\right)\right] + \frac{3}{4}\ln\left(\frac{m_{\pi^+}^2}{m_{\rho^+}^2}\right)\\ &+ \frac{m_{\rho^+}^2}{4m_{\pi^0}^2}\left[\ln\left(\frac{m_{\rho^+}^2}{m_{\pi^+}^2}\right)\left(1-\frac{\Delta_\pi^2}{m_{\rho^+}^2}\right) - v_+\left[\ln\left(\frac{1-v_+-\frac{\Delta_\pi^2}{m_{\rho^+}^2}}{1+v_+-\frac{\Delta_\pi^2}{m_{\rho^+}^2}}\right) + \ln\left(\frac{1+v_+-\frac{\Sigma_\pi^2}{m_{\rho^+}^2}}{1-v_+-\frac{\Sigma_\pi^2}{m_{\rho^+}^2}}\right)\right]\right]\\ &+ \frac{1}{2v'_+}\left[\ln\left(\frac{1-v'_+}{1+v'_+}\right)\left[-\frac{1}{4}\ln\left(\frac{1-v'_+}{1+v'_+}\right) + 2\ln\left(\frac{2v'_+}{1+v'_+}\right) - \ln\left(\frac{m_{\rho^+}}{m_{\pi^+}}\right)\right] - \frac{\pi^2}{3} + 2\,\mathrm{Li}_2\left(\frac{1-v'_+}{1+v'_+}\right) + \ln^2\left(\frac{m_{\pi^+}}{m_{\rho^+}}\right)\right.\\ &\left.+ 2\,\mathrm{Li}_2\left(\frac{1+v'_+-2/\xi}{1+v'_+}\right) + 2\,\mathrm{Li}_2\left(\frac{-1+v'_++2/\xi}{2/\xi}\right)\right]\Bigg\},\end{aligned} \tag{15}$$

where we have defined $\Sigma_\pi^2 \equiv m_{\pi^+}^2 + m_{\pi^0}^2$. As pointed out before, this result is free from ultraviolet divergencies, and it depends logarithmically on the infrared mass regulator λ.

As we can easily check, the sum of the virtual corrections, Eqs. (14) and (15) and the corresponding soft-photon bremsstrahlung, Eqs. (12) and (13), are free from the infrared regulator λ as it must be according to the Bloch-Nordsiek theorem. Note, however, that this sum depends on the photon energy cut ω_0. We can expect that this ω_0 dependence will be largely canceled in the photon-inclusive $\rho \to \pi\pi$ decay rate in such a way that $\Delta\Gamma_\rho$ defined in Eq. (5) does not depend on this parameter. Since the soft- and hard-photon decay rates have a different dependence upon ω_0, we need to choose values of ω_0 which are sufficiently small (typically much smaller than the masses of hadronic particles) [14]. In the following subsection we briefly comment about the decay rates for hard photons.

C. Hard-photon emission

In order to estimate the contributions to $\Delta\Gamma_\rho$ due to photons of energy larger than ω_0, the third and fourth terms in Eq. (5), we require the complete expressions of the radiative decay amplitudes corresponding to the Feynman diagrams in Figs. 1 and 2. These amplitudes include also the model-dependent contributions shown in Figs. 1(d)–1(f), 2(d), and 2(e). The explicit contributions of the model-dependent amplitudes can be found in Refs. [7,8,11].

When we evaluate the decay rates of radiative decays, we note that the contributions of the model-dependent terms (intermediate states with ω, a_1, σ, and f_0 mesons) are typically 2 orders of magnitude below the contributions due to model-independent terms [8,9,11]. Therefore, such model-dependent terms would affect the width difference $\Delta\Gamma_\rho$ at a completely negligible level and will be neglected in our numerical results.

IV. RESULTS AND DISCUSSION

In this section we present the main numerical results of our calculations. The values of the radiative corrections δ_{ρ^i} ($i = +, 0$) defined in Eq. (11) are shown in Tables I and II for $\rho^0 \to \pi^+\pi^-(\gamma)$ and $\rho^+ \to \pi^+\pi^0(\gamma)$ decays, respectively. The values of δ_{ρ^i} are tabulated as a function of the photon energy cut ω_0 for three different values of the ρ^i meson mass which are consistent with a small isospin breaking. As expected, these radiative corrections depend only slightly on the specific value of the ρ meson mass. It is interesting to observe that in $\phi \to K^+K^-$ decays (and other analogous quarkonia decays that occur close to threshold) the radiative corrections are dominated by the Coulombic interaction in the final state [12], while in $\rho^0 \to \pi^+\pi^-$ this is not the case.

Table III shows the radiative decay rates of $\rho^i \to \pi\pi\gamma$ normalized to the corresponding tree-level rates Γ_i^0. The tabulated values are defined as follows ($i = 0, +$):

$$\Delta_{\rho^i} \equiv \frac{\Gamma(\rho^i \to \pi\pi\gamma, \omega \geq \omega_0)}{\Gamma_i^0}. \tag{16}$$

These values were calculated for photon energies larger than ω_0 and for three different values of the ρ meson mass. The values of Δ_{ρ^i} are always positive and exhibit the typical decreasing behavior as the lower energy photon cut ω_0 increases. We observe that these normalized rates are in good agreement with the corresponding results reported long ago in Ref. [7] using slightly different values for the mass and width of the ρ meson.

The predicted branching fraction for the neutral radiative mode,

$$B(\rho^0 \to \pi^+\pi^-\gamma, \omega \geq 50\ \text{MeV}) = 11.5 \times 10^{-3}, \tag{17}$$

compares very well (they are in agreement within 1σ) with

TABLE I. Radiative correction δ_{ρ^0} to the $\rho^0 \to \pi^+\pi^-(\gamma)$ decay rate [see definition in Eq. (5)] as a function of ω_0 and for three different values of m_{ρ^0}.

ω_0 (MeV)	$m_{\rho^0} = 772$ MeV δ_{ρ^0}	$m_{\rho^0} = 775$ MeV δ_{ρ^0}	$m_{\rho^0} = 778$ MeV δ_{ρ^0}
2	−0.036 70	−0.036 92	−0.037 14
4	−0.029 10	−0.029 30	−0.029 49
6	−0.024 65	−0.024 83	−0.025 01
8	−0.021 50	−0.021 67	−0.021 83
10	−0.019 05	−0.019 21	−0.019 37
12	−0.017 05	−0.017 20	−0.017 36
14	−0.015 36	−0.015 50	−0.015 65
16	−0.013 89	−0.014 03	−0.014 77
18	−0.012 60	−0.012 73	−0.012 87
20	−0.011 44	−0.011 57	−0.011 70
30	−0.006 97	−0.007 08	−0.007 20
40	−0.003 78	−0.003 88	−0.003 99
50	−0.001 30	−0.001 39	−0.001 50
60	0.000 74	−0.000 65	0.000 56
70	0.002 49	0.002 40	0.002 32
80	0.004 01	0.003 93	0.003 84
90	0.005 36	0.005 29	0.005 21
100	0.006 59	0.006 51	0.006 43

TABLE II. Radiative corrections δ_{ρ^+} to the $\rho^+ \to \pi^+\pi^0(\gamma)$ decay rate [see definition in Eq. (5)] as a function of ω_0 and for three different values of m_{ρ^i}.

ω_0 (MeV)	$m_{\rho^+} = 772$ MeV δ_{ρ^+}	$m_{\rho^+} = 775$ MeV δ_{ρ^+}	$m_{\rho^+} = 778$ MeV δ_{ρ^+}
2	−0.019 59	−0.019 68	−0.019 70
4	−0.017 01	−0.017 10	−0.017 18
6	−0.015 51	−0.015 58	−0.015 66
8	−0.014 44	−0.014 51	−0.014 59
10	−0.013 61	−0.013 68	−0.013 75
12	−0.012 93	−0.013 00	−0.013 07
14	−0.012 36	−0.012 42	−0.012 49
16	−0.011 86	−0.011 92	−0.011 99
18	−0.011 42	−0.011 49	−0.011 55
20	−0.011 03	−0.011 09	−0.011 15
30	−0.009 53	−0.009 58	−0.009 63
40	−0.008 44	−0.008 49	−0.008 54
50	−0.007 61	−0.007 65	−0.007 69
60	−0.006 92	−0.006 96	−0.007 00
70	−0.006 33	−0.006 37	−0.006 39
80	−0.005 82	−0.005 84	−0.005 89
90	−0.005 36	−0.005 40	−0.005 44
100	−0.004 95	−0.004 99	−0.005 02

the experimental measurement $(9.9 \pm 1.6) \times 10^{-3}$ reported in [1], which was obtained for the same value of ω_0. For the same value of ω_0, we obtain the isospin breaking in the radiative modes to be (assuming $\Gamma_{\rho+} \approx \Gamma_{\rho^0} = 150$ MeV)

$$\Gamma(\rho^0 \to \pi^+\pi^-\gamma, \omega_0 = 50\text{ MeV}) - \Gamma(\rho^+ \to \pi^+\pi^0\gamma, \omega_0 = 50\text{ MeV}) \approx 1.1\text{ MeV}, \quad (18)$$

which is larger than the estimate (0.45 ± 0.45) MeV considered in Ref. [5] and essentially independent of the ρ meson mass.

In Table IV we display the sum of the radiative corrections δ_{ρ^i} and the radiative branching ratios Δ_{ρ^i} for a common value (775 MeV) of the neutral and charged ρ meson mass. According to the definitions given in the previous sections, we have

TABLE III. Decay rates of $\rho^i \to \pi\pi\gamma$ [normalized to the tree-level rates Γ_i^0, see definition in Eq. (16)] as a function of the low photon energy cut ω_0 and for three different values of m_ρ.

ω_0 (MeV)	$m_{\rho^{+,0}} = 772$ MeV Δ_{ρ^+}	Δ_{ρ^0}	$m_{\rho^{+,0}} = 775$ MeV Δ_{ρ^+}	Δ_{ρ^0}	$m_{\rho^{+,0}} = 778$ MeV Δ_{ρ^+}	Δ_{ρ^0}
2	0.015 44	0.044 75	0.015 53	0.044 97	0.015 61	0.045 18
4	0.012 90	0.037 24	0.012 97	0.037 42	0.013 05	0.037 61
6	0.011 43	0.032 88	0.011 49	0.033 05	0.0115 56	0.033 22
8	0.010 39	0.029 81	0.010 45	0.029 97	0.010 51	0.030 13
10	0.009 59	0.027 45	0.009 65	0.027 60	0.009 70	0.027 75
12	0.008 94	0.025 53	0.009 00	0.025 68	0.009 05	0.025 82
14	0.008 40	0.023 93	0.008 45	0.024 06	0.008 50	0.024 20
16	0.007 93	0.022 55	0.007 98	0.022 68	0.008 03	0.022 81
18	0.007 53	0.021 34	0.007 58	0.021 47	0.007 62	0.021 59
20	0.007 17	0.020 27	0.007 21	0.020 39	0.007 26	0.020 51
30	0.005 81	0.016 24	0.005 85	0.016 35	0.005 89	0.016 45
40	0.004 88	0.013 50	0.004 92	0.013 59	0.004 95	0.013 69
50	0.004 20	0.011 46	0.004 23	0.011 55	0.004 26	0.011 63
60	0.003 66	0.009 87	0.003 69	0.009 94	0.003 72	0.010 02
70	0.003 22	0.008 57	0.003 25	0.008 64	0.003 27	0.008 71
80	0.002 86	0.007 50	0.002 88	0.007 57	0.0029 1	0.007 63
90	0.002 55	0.006 59	0.002 57	0.006 65	0.002 59	0.006 72
100	0.002 28	0.005 82	0.002 30	0.005 88	0.002 32	0.005 93

TABLE IV. Photon-inclusive corrections for $\rho^i \to \pi\pi\gamma$ [see definition in Eq. (19)] as a function of the photon energy ω_0. The masses of charged and neutral ρ mesons were fixed to 775 MeV.

ω_0 (MeV)	σ_{ρ^0}	σ_{ρ^+}	$\sigma_{\rho^0} - \sigma_{\rho^+}$
2	8.05×10^{-3}	-4.15×10^{-3}	12.20×10^{-3}
4	8.12×10^{-3}	-4.13×10^{-3}	12.25×10^{-3}
6	8.22×10^{-3}	-4.09×10^{-3}	12.31×10^{-3}
8	8.30×10^{-3}	-4.06×10^{-3}	12.36×10^{-3}
10	8.39×10^{-3}	-4.03×10^{-3}	12.42×10^{-3}
12	8.48×10^{-3}	-4.00×10^{-3}	12.48×10^{-3}
14	8.56×10^{-3}	-3.97×10^{-3}	12.53×10^{-3}
16	8.65×10^{-3}	-3.94×10^{-3}	12.59×10^{-3}
18	8.74×10^{-3}	-3.91×10^{-3}	12.65×10^{-3}
20	8.82×10^{-3}	-3.88×10^{-3}	12.70×10^{-3}

$$\frac{\Gamma(\rho^i \to \pi\pi(\gamma), \omega \le \omega_0) + \Gamma(\rho^i \to \pi\pi\gamma, \omega \ge \omega_0)}{\Gamma_i^0} = 1 + \delta_{\rho^i} + \Delta_{\rho^i} \equiv 1 + \sigma_{\rho^i}. \tag{19}$$

As we have pointed out in Sec. II, we expect that the σ_{ρ^i} correction term will be independent of the photon energy cut ω_0 as far as Eq. (19) describes a photon-inclusive rate and ω_0 is an arbitrary reference value used to separate soft- and hard-photon emission. In practice this cancellation is better for values of ω_0 (typically below a few MeV's, see Table IV) because radiative corrections include only the logarithmic dependence in ω_0, while the hard-photon radiative rate contains also linear and other ω_0 dependent terms.

Finally, we can evaluate the difference in decay widths arising from different corrections. Based on previous definitions, we can write the width difference of ρ mesons, Eq. (5), as follows:

$$\Delta\Gamma_\rho = \Gamma_0^0\left[1 + \sigma_{\rho^0} - \left(\frac{m_{\rho^+} v_+^3}{m_{\rho^0} v_0^3}\right)[1 + \sigma_{\rho^+}]\right] + \Delta\Gamma^{\rm sub}. \tag{20}$$

Once the isospin breaking in Δm_ρ is known, the values of $\Delta\Gamma_\rho$ can be easily evaluated from Tables I, II, and III, using the above expression. For illustrative purposes, we provide the values of $\Delta\Gamma_\rho$ for two interesting cases (we have used $\Gamma_0^0 = 150$ MeV and the value of σ_{ρ^i} at $\omega_0 = 10$ MeV):

$$\Delta\Gamma_\rho = \begin{cases} 0.86\ \text{MeV}, & \text{if } \Delta m_\rho = 0 \\ 0.02\ \text{MeV}, & \text{if } \Delta m_\rho = -3\ \text{MeV} \\ 1.70\ \text{MeV}, & \text{if } \Delta m_\rho = +3\ \text{MeV}. \end{cases} \tag{21}$$

Note that the width difference is very sensitive to the size and sign of Δm_ρ. This is very interesting because it can be useful as a test of $(\Delta m_\rho(\text{exp}), \Delta\Gamma_\rho(\text{exp}))$ correlated values extracted from fits to experimental data. In particular, the above results are in good agreement with the values $(-2.4 \pm 0.8, -0.2 \pm 1.0)$ MeV extracted from a combined fit to τ and e^+e^- data [15], but the agreement with the central values $(-3.1 \pm 0.9, -2.3 \pm 1.6)$ MeV reported in [6] is not very good.

V. CONCLUSIONS

The isospin breaking in the $\rho^0 - \rho^\pm$ system is a very important ingredient to understand the current discrepancy in predictions of the hadronic vacuum polarization contributions to the muon magnetic moment based in τ decay and e^+e^- annihilation data. In this paper we have evaluated the difference in decay widths $(\Delta\Gamma_\rho)$ of $\rho(770)$ vector mesons. We have considered in our calculation the isospin breaking in the exclusive decay modes of charged and neutral ρ mesons. In particular, we have carried out a calculation of the radiative corrections to the dominant $\rho \to \pi\pi$ decay modes and have done a careful reevaluation of the differences in their radiative $\rho \to \pi\pi\gamma$ decays.

We found that $\Delta\Gamma_\rho$ is sensitive to the isospin breaking in the $\rho^0 - \rho^\pm$ mass difference. This provides a useful tool to test the (model-dependent) values of these parameters extracted from experimental data. In particular, we have found that positive values of $\Delta\Gamma_\rho$ are favored by $|\Delta m_\rho| \le 3$ MeV.

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