# **Probing minimal flavor violation at the CERN LHC**

<span id="page-0-3"></span>Yuval Grossman,<sup>1[,\\*](#page-0-0)</sup> Yosef Nir,<sup>2[,†](#page-0-1)</sup> Jesse Thaler,<sup>3,4[,‡](#page-0-2)</sup> Tomer Volansky,<sup>2,§</sup> and Jure Zupan<sup>5,6,||</sup>

<sup>1</sup>Department of Physics, Technion-Israel Institute of Technology, Technion City, Haifa 32000, Israel<br><sup>2</sup>Department of Particle Physics, Weirmann Institute of Science, Peheyot 76100, Israel

*Department of Particle Physics, Weizmann Institute of Science, Rehovot 76100, Israel*

<sup>3</sup> Department of Physics, University of California, Berkeley, California 94720, USA<br><sup>4</sup>Theoretical Physics Group, Lawrence Barkeley National Laboratory, Berkeley, California 94

*Theoretical Physics Group, Lawrence Berkeley National Laboratory, Berkeley, California 94720, USA* <sup>5</sup>

*Department of Physics, University of Ljubljana, Jadranska 19, 1000 Ljubljana, Slovenia* <sup>6</sup>

*J. Stefan Institute, Jamova 39, P.O. Box 3000, 1001 Ljubljana, Slovenia*

(Received 4 July 2007; published 28 November 2007)

If the LHC experiments discover new particles that couple to the standard model fermions, then measurements by ATLAS and CMS can contribute to our understanding of the flavor puzzles. We demonstrate this statement by investigating a scenario where extra SU(2)-singlet down-type quarks are within the LHC reach. By measuring masses, production cross sections, and relative decay rates, minimal flavor violation (MFV) can in principle be excluded. Conversely, these measurements can probe the way in which MFV applies to the new degrees of freedom. Many of our conclusions are valid in a much more general context than this specific extension of the standard model.

DOI: [10.1103/PhysRevD.76.096006](http://dx.doi.org/10.1103/PhysRevD.76.096006) PACS numbers: 11.30.Hv, 12.15.Ff, 12.20.Fv

## **I. INTRODUCTION**

Significant progress in our knowledge of flavor physics has been achieved in recent years. The main contributions have come from the two B factories, Belle and *BABAR*, and from the Tevatron detectors, CDF and D0  $[1,2]$  $[1,2]$  $[1,2]$ . The absence of evidence in the data for new physics poses a puzzle: there are good reasons to think that there is new physics at or below the TeV scale, yet any effective nonrenormalizable flavor-changing operators suppressed by the TeV scale must mimic the standard model (SM) flavor structure with excellent accuracy, reaching in some cases a level of one part in  $10<sup>6</sup>$ . The question of how and why that happens is often referred to as the ''new physics flavor puzzle.''

We will soon enter a new era in high energy physics the LHC era. The LHC experiments should first answer the crucial question of whether there is indeed new physics at the TeV scale, as suggested by the hierarchy problem and weakly interacting dark matter proposals. If the answer is in the affirmative, then the LHC also offers new opportunities in exploring the new physics flavor puzzle. If new particles that couple to SM fermions are discovered, then measurements of their spectrum and of their couplings will help elucidate the basic mechanism that has so far screened the flavor effects of new physics. The main goal of this work is to demonstrate how high- $p<sub>T</sub>$  processes, measured by ATLAS and CMS, can shed light on flavor issues.

Of course, the implications of new physics on flavor are highly model dependent. At energies much below the electroweak scale, the flavor effects of new physics can be entirely captured by a series of higher-dimension operators, but at LHC energies, flavor-changing processes can occur via the production and decay of new on-shell particles. In models like supersymmetry with numerous new partners and the potential for long cascade decays, flavor questions can in principle be addressed [\[3](#page-7-2)], but in the quark sector this is only possible after disentangling many model-dependent effects like gaugino-Higgsino mixing angles and the mass ordering of left- vs right-handed squarks. For purposes of studying how flavor might be probed at the LHC, it is therefore desirable to analyze models (which might be one sector of a more complete theory) for which flavor has an unambiguous effect on LHC signatures.

A simple and rather generic principle that can guarantee that low energy flavor-changing processes would show no deviations from SM predictions is that of *minimal flavor violation* (MFV) [\[4](#page-7-3)–[6\]](#page-7-4). The basic idea can be described as follows (a more rigorous definition is given in the next section). The gauge interactions of the SM are universal in flavor space. The only breaking of this flavor universality comes from the three Yukawa matrices,  $Y_U$ ,  $Y_D$ , and  $Y_E$ . If this remains true in the presence of new physics—namely  $Y_U$ ,  $Y_D$ , and  $Y_E$  are the only flavor nonuniversal parameters—then the model belongs to the MFV class. We use the concrete question of whether ATLAS and CMS can *test* the principle of MFV in order to explore the flavor physics potential of these experiments.

To do so, we further choose a specific example of new physics. We augment the SM with down-type, vectorlike heavy fermions,  $B_L$  and  $B_R$ , that transform as  $(3, 1)_{-1/3}$ under the SM gauge group (for a review see, for example, [\[7\]](#page-7-5)). To be relevant to our study, at least some of the new quarks must be within the reach of the LHC, and they must couple to the SM quarks. We assume that MFV applies to this extension of the SM, and we ask the following ques-

<span id="page-0-0"></span>[<sup>\\*</sup>y](#page-0-3)uvalg@physics.technion.ac.il

<span id="page-0-1"></span>[<sup>†</sup>](#page-0-3) yosef.nir@weizmann.ac.il

<span id="page-0-2"></span>[<sup>‡</sup>](#page-0-3) jthaler@jthaler.net

s<br>
stomer.volansky@weizmann.ac.il

llet in the sense of the sense o

tions:

- (i) What are the possible spectra of the new quarks?
- (ii) What are the possible flavor structures of their couplings to the SM quarks?
- (iii) Can the LHC exclude MFV by measurements related to these quarks?
- (iv) In case that MFV is not excluded, can the LHC be used to support MFV?

While in this study we concentrate only on a specific representation of the extra quarks, many of the lessons that we draw have a much more general applicability beyond our specific example.

In Sec. II we introduce the notion of minimal flavor violation and its consequences for a SM extension with extra vectorlike down-type quarks. The resulting spectrum and decay patterns are discussed in Sec. III. In Sec. IV we examine how experiments at LHC can refute or give support to the MFV hypothesis, and then summarize our conclusions in Sec. V.

### **II. THE THEORETICAL FRAMEWORK**

The SM with vanishing Yukawa couplings has a large global symmetry,  $U(3)^5$ . In this work we concentrate only on the quarks. The non-Abelian part of the flavor symmetry for the quarks can be decomposed as follows:

$$
G_{\text{flavor}} = SU(3)_Q \otimes SU(3)_D \otimes SU(3)_U. \tag{1}
$$

<span id="page-1-2"></span>The Yukawa interactions ( $H_c = i\tau_2 H^*$ ),

$$
\mathcal{L}_Y = \bar{Q}_L Y_D D_R H + \bar{Q}_L Y_U U_R H_c, \tag{2}
$$

break the  $G<sub>flavor</sub>$  symmetry. The Yukawa couplings can thus be thought of as spurions with the following transformation properties under  $G<sub>flavor</sub>$ :

$$
Y_D \sim (3, \bar{3}, 1), \qquad Y_U \sim (3, 1, \bar{3}). \tag{3}
$$

We extend the SM by adding vectorlike quarks  $B_L$  and  $B_R$  of electric charge  $-1/3$ . In general, extending the SM with the  $B_L$  and  $B_R$  fields gives three new types of Yukawa and mass terms:

<span id="page-1-1"></span>
$$
\mathcal{L}_B = \frac{m_2}{v} \bar{Q}_L Y_B B_R H + M_1 \bar{B}_L X_{BD} D_R + M_2 \bar{B}_L X_{BB} B_R.
$$
\n(4)

Our assumption is that the mass parameters  $M_1$  and  $M_2$  are much larger than the weak scale, while  $m_2$  is of order the weak scale. If the three new matrices  $Y_B$ ,  $X_{BD}$ , and  $X_{BB}$  had a generic flavor structure, unrelated to that of  $Y_D$  and  $Y_U$ , the deviations from the SM predictions for flavor-changing processes would exclude the model, unless the mass scale for the new quarks is very high, well beyond the LHC reach [\[8](#page-7-6)–[10](#page-7-7)]. We thus impose the criterion of minimal flavor violation (MFV): all the Lagrangian terms constructed from the SM fields, the  $B_L$  and  $B_R$  fields, and  $Y_{D,U}$ , must be (formally) invariant under the flavor group  $G<sub>flavor</sub>$ .

We are interested in the case that the new quarks couple to the SM ones at the renormalizable level. Then, we are led to models where the  $B_L$  and  $B_R$  fields cannot be singlets of  $G_{\text{flavor}}$ . (In fact, the same result follows from the demand that the new fields have allowed decays into SM fields.) This is a general result: MFV (and the requirement of coupling to SM fields) implies that the number of extra vectorlike quarks is at least three. Since there are many options for *G*<sub>flavor</sub> charge assignments, for concreteness we further narrow our scope to the cases where  $B_L$  and  $B_R$  are singlets of  $SU(3)_U$  and transform as (3, 1) or (1, 3) under  $SU(3)_Q \otimes SU(3)_D$ . There are four possible combinations of flavor-charge assignments to the  $B_{L,R}$  fields. These assignments are given in Table [I.](#page-1-0)

Once the G<sub>flavor</sub> representations of the new fields are defined, the flavor structure of their couplings in Eq. ([4\)](#page-1-1) is determined. The flavor structures are also given in Table [I](#page-1-0). For the examples we are considering, there are only two relevant spurions,  $Y_D$  and  $Y_U Y_U^{\dagger}$ . Without loss of generality, we work in a basis where  $Y_U$  is diagonal. To a good approximation we can neglect the Yukawa couplings of the up and charm quarks, and take  $Y_U Y_U^{\dagger} \sim \text{diag}(0, 0, 1)$ . The effect of  $Y_U Y_U^{\dagger}$  can be captured by the combination

$$
D_3 \equiv 1 + d_3 Y_U Y_U^{\dagger} \sim \text{diag}(1, 1, 1 + d_3), \tag{5}
$$

where 1 is the  $3 \times 3$  unit matrix and  $d_3 = O(1)$ . In models where more than a single  $D_3$ -spurion appear, we distinguish between the different  $D_3$ 's with an upper index, to emphasize the fact that  $d_3$  is different.

<span id="page-1-0"></span>TABLE I. The possible flavor assignments for vectorlike quarks that transform as  $(3, 1)_{-1/3}$  under the SM gauge group. Here, we assume that  $B_L$  and  $B_R$  transform either as (1, 3) or (3, 1) under  $SU(3)_Q \times SU(3)_D$ . The model names are determined in a self-evident way from the flavor assignments. The last three columns give the flavor structure for the new Lagrangian terms in Eq. ([4](#page-1-1)), assuming MFV. The matrices  $D_3 \sim \text{diag}(1, 1, 1 + d_3)$ parametrize the breaking of  $SU(3)_Q$  by the top Yukawa. In models QD and DD,  $X_{BD}$  can be taken to be zero by a  $D_R$  –  $B_R$  rotation. The "(0)" in model **DQ** indicates a value that must be fine-tuned to get the right SM quark spectrum.

	Model Quark field $SU(3)_0$ $SU(3)_D$ $Y_B$				$X_{BB}$	$X_{BD}$
	$\varrho_{\scriptscriptstyle L}$	3				
	$D_R$		3			
	$Y_D$	3	$\overline{3}$			
	$Y_U Y_U^{\dagger}$	$1 + 8$				
QD	$B_L$	3				
	$B_R$		3		$D_3^m Y_D$ $D_3^M Y_D$	
DD	$B_L$		3			
	$B_R$	1	3	$D_3Y_D$	1	
DQ	$B_L$		3			
	$B_R$	3		$D_3^m$	$Y_D^{\dagger}D_3^M$	(0)
QQ	$B_L$	3				
	$B_R$	3		$D_3^m$	$D_3^M$	$D_3^Y Y_D$

In terms of symmetries, the significance of  $D_3$  is that it implies a possible  $\mathcal{O}(1)$  breaking of  $SU(3)_Q \to SU(2)_Q \times$  $U(1)_Q$  by the top Yukawa. The remaining symmetries are broken only by small parameters and therefore constitute approximate symmetries in MFV models. This is an important point that is valid in all single-Higgs MFV models.<sup>1</sup> We return to this point in the Conclusions.

Two comments are in order:

- (1) In models **QD** and **DD**, the  $B_R$  and  $D_R$  fields transform in precisely the same way under both the gauge group and the global flavor group. We thus have freedom in choosing our basis in the  $D_R - B_R$ space. We use this freedom to set  $X_{BD} = 0$ .
- (2) Without fine-tuning, model **DQ** predicts nonhierarchical masses for the SM down quarks. Two viable but fine-tuned solutions are to set  $M_1 = 0$  or  $m_2 =$ 0. We choose to work with the first,  $M_1 = 0$ . In Table [I](#page-1-0) we denote a fine-tuned value by parentheses.

### **III. SPECTRUM AND COUPLINGS**

To understand the phenomenological aspects that are relevant to the LHC, we have to find the spectrum and the couplings of the heavy quarks. Our starting point is the Lagrangian terms of Eqs. [\(2](#page-1-2)) and [\(4\)](#page-1-1). We construct the down-sector mass matrices, diagonalize them, and obtain the spectrum of the heavy and the light (i.e. SM) quarks and the couplings of the heavy mass eigenstates to the SM fields (a more detailed account of this procedure will be given in subsequent work  $[11]$  $[11]$  $[11]$ ). We use *B'* and *D'* to denote the heavy and the light down-quark mass eigenstates, respectively. We write the relevant couplings schematically as follows:

<span id="page-2-1"></span>
$$
\mathcal{L}_{B'} = \bar{B}'_L M_{B'} B'_R + \bar{D}'_L Y^L_{B'} B'_R h + \bar{D}'_L \gamma_\mu Y^T_{B'} B'_L Z^\mu
$$
  
+  $\bar{U}'_L \gamma_\mu V_{CKM} Y^T_{B'} B'_L W^\mu,$  (6)

where *h* is the physical Higgs field.  $M_{B}$ <sup>0</sup> is the diagonal mass matrix of the heavy states. In the  $M_{B'} \gg v$  limit, the  $B' \rightarrow ZD'$  and  $B' \rightarrow WU'$  decays are dominated by longitudinally polarized *Z* and *W* final states. According to the Goldstone equivalence theorem, the sizes of the corresponding decay rates are then given by  $Y_{B'}^L$  and  $V_{CKM}Y_{B'}^L$ , respectively,<sup>2</sup> with corrections of order  $M_W^2/M_{B'}^2$ . The  $Y_{B'}^T$ matrix, on the other hand, parametrizes the couplings of the transverse *W* and *Z* bosons.

If the  $Y_U Y_U^{\dagger}$  spurions could be neglected, then the flavor structures would only depend on the Cabibbo-Kobayashi-Maskawa (CKM) matrix  $V_{CKM}$  and the diagonal down <span id="page-2-3"></span>Yukawa coupling matrix  $\hat{\lambda}$ . Expressed in approximate powers of the Wolfenstein parameter  $\lambda \sim 0.2$ , we have

$$
V_{CKM} \sim \begin{pmatrix} 1 & \lambda & \lambda^3 \\ \lambda & 1 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix},
$$
  

$$
\hat{\lambda} = \begin{pmatrix} y_d & & \\ & y_s & \\ & & y_b \end{pmatrix} \sim y_b \begin{pmatrix} \lambda^4 & & \\ & \lambda^2 & \\ & & 1 \end{pmatrix}.
$$
 (7)

<span id="page-2-2"></span>When the  $Y_U Y_U^{\dagger}$  effects are significant, the results are modified in a simple way: the modification of the spectrum may involve matrices of the form  $D_3$ , while the couplings may involve a matrix  $\hat{1}$ :

$$
\tilde{1} = V_{\text{CKM}}^{\dagger} D_3 V_{\text{CKM}} \sim \begin{pmatrix} 1 & 0 & \lambda^3 \\ 0 & 1 & \lambda^2 \\ \lambda^3 & \lambda^2 & d_3 \end{pmatrix}, \qquad (8)
$$

or matrices that scale in the same way with  $\lambda$ , for which we use the same symbol  $\tilde{1}$ .

The masses and couplings for the various models are given in Table  $II$  with additional details of the derivation given in the Appendix. We define a small parameter

$$
\epsilon \equiv \frac{\nu}{M},\tag{9}
$$

where *v* is the electroweak breaking scale, and  $M \sim$  $max(M_1, M_2)$  is the heavy mass scale that fixes the masses of the heavy quarks. Since the spectrum of the heavy quarks can be hierarchical (models **QD** and **DQ**) or (pseudo)degenerate (models **DD** and **QQ**), the heavy mass scale *M* differs significantly in the two cases. From the requirement that the lightest  $B'$  state has a mass in the TeV range, one finds  $\epsilon \sim 10^{-1}$  in models **DD** and **QQ**, and  $\epsilon \sim 10^{-5}$ in models **QD** and **DQ**.

We learn the following points regarding the spectrum:

(1) If the vectorlike quarks are  $SU(3)_Q$ -singlets (model **DD**), the spectrum is fully degenerate. This degeneracy is lifted by effects of order  $m_b^2/M^2$  that can be safely neglected.

<span id="page-2-0"></span>TABLE II. The spectrum and couplings of the heavy quarks from Eq. ([6](#page-2-1)), given the flavor charges from Table [I.](#page-1-0)  $\hat{\lambda}$  is the diagonalized down Yukawa matrix,  $\epsilon$  is the ratio of the electroweak scale to the heavy quark mass scale, and  $\tilde{1} \equiv$  $V_{\text{CKM}}^{\dagger}D_3V_{\text{CKM}}$  parametrizes the effect of  $SU(3)_Q$  breaking from the top Yukawa on the  $B'$  couplings.

$M_{B'}/M$	$Y^L_{B'}$	R'
	Ĩλ	$\epsilon 1$
	$\lambda$	$\epsilon \tilde{1} \hat{\lambda}$
		$\epsilon \tilde{1} \hat{\lambda}$
$D_3$		E١
	$D_3\hat{\lambda}$ $D_3 \hat{\lambda}$	

<sup>&</sup>lt;sup>1</sup>In multi-Higgs models at large tan $\beta$ , the bottom Yukawa could provide an  $\mathcal{O}(1)$  breaking of  $SU(3)_D \to SU(2)_D \times U(1)_D$ .<br><sup>2</sup>This is best seen in the Feynman t'. Hooft gauge where the

<sup>&</sup>lt;sup>2</sup>This is best seen in the Feynman-t' Hooft gauge where the decays are predominantly into unphysical Higgs states, with the relevant terms in the Lagrangian  $\overline{D}_L' Y_{B'}^L B_R' h + \overline{D}_L' Y_{B'}^L B_R' h^3 +$ relevant terms in the Lagrangian  $D'_L Y^L_B B'_R h$ <br> $\bar{U}'_L (\sqrt{2} V_{CKM} Y^L_B) B'_R h^+$ . See, for example, [\[12\]](#page-7-9).

- (2) If the vectorlike quarks are  $SU(3)_Q$ -triplets (model **QQ**), the spectrum could have an interesting structure of  $2 + 1$ : two degenerate quarks and one with a mass of the same order of magnitude but not degenerate. This is a manifestation of the  $O(1)$  breaking of  $SU(3)_Q \rightarrow SU(2)_Q \times U(1)_Q$  due to  $y_t$ . The two degenerate states are split by effects of order  $m_c^2/v^2 \sim$  $10^{-4}$  that we neglect.
- (3) If the vectorlike quarks are chiral (triplet  $+$  singlet) under  $SU(3)_Q$  (model **QD** and **DQ**), the spectrum is hierarchical, with the hierarchy  $y_d$ : $y_s$ : $\mathcal{O}(y_b)$ . In that case, only one heavy quark is at the TeV scale.

As for the decay rates, we learn the following:

- (1) The decays to the transverse *W* and *Z* are always negligible, that is,  $Y_{B'}^T \ll Y_{B'}^L$ .
- (2) The couplings to longitudinal  $W/Z$  and to *h* are the same to a very good approximation. This implies that up to phase space effects, the heavy quarks decay rates to *W*, *Z*, and *h* are in ratios 2:1:1 [\[12\]](#page-7-9).
- (3) The flavor diagonal couplings dominate, that is  $Y_{B'}^{T,L}$ is close to a unit matrix. The most significant flavorchanging *Z* coupling is  $(Y_{B}^L)_{23} \sim 0.04(Y_{B}^L)_{33}$  and the most significant flavor-changing *W* coupling is  $(V_{CKM}Y_{B}^L)_{12} \sim 0.23(V_{CKM}Y_{B}^L)_{22}.$

Finally, adding vectorlike quarks to the SM affects, in general, the low energy phenomenology of both flavor and electroweak precision measurements. As concerns flavor, the CKM matrix is not unitary and the *Z*-boson acquires flavor-changing couplings to the down sector. In the framework of MFV, the flavor-changing *Z* couplings are suppressed by  $\epsilon^2$ , by small mixing angles, and, in some models, by down-sector Yukawa couplings. Consequently, these contributions are safely within bounds. The effects of the extra quarks on electroweak precision measurements are also suppressed by  $\epsilon^2$  [[13](#page-7-10)]. In some of the models, MFV leads to further suppression of these effects [\[11\]](#page-7-8). For  $M \geq TeV$ , the deviations of the *S* and *T* parameters from their SM values are of  $\mathcal{O}(0.01)$  in model **QQ**, and considerably smaller in all other models. Thus, the models we study are generically allowed by present data.

### **IV. LHC PHENOMENOLOGY**

We are now ready to discuss the phenomenology of the model. Our main task is to check if the idea of MFV can be tested by direct measurements at the LHC. Clearly, we need to establish the fact that new downlike quarks exist to start any probe of their flavor structure. An ATLAS study of vectorlike down-type quarks using only  $2Z \rightarrow 4\ell$  final states found a  $B'$  mass reach of 920 GeV with 300 fb<sup>-1</sup> of data  $[14]$ , but the inclusion of other  $B'$  decay modes is likely to improve the reach, given the small leptonic branching fraction of the *Z*. For various models with



<span id="page-3-0"></span>FIG. 1. Leading order cross section for  $B<sup>1</sup>$  pair production at the LHC calculated at leading order using PYTHIA 6.4.10 [\[21\]](#page-7-15) with CTEQ5L parton distribution functions [[20](#page-7-17)]. From bottom to top, the total cross section for 1, 2, and 3 generations of  $B<sup>1</sup>$ quarks. See [[9\]](#page-7-18) for the variation of the cross section from different choices of factorization scale.

vectorlike up-type quarks, the mass reach was found to range from 1 to 2.5 TeV for 100–300 fb<sup>-1</sup> of data  $[15-17]$  $[15-17]$  $[15-17]$ .

The high end of the above discovery range is due to large mixing angles with SM quarks, when the heavy quarks can be singly produced using quark-*W* fusion [\[17](#page-7-13)[–19\]](#page-7-14). In our case, such channels are particularly interesting for models **DQ** and **QQ**, where the couplings to longitudinal gauge bosons are unsuppressed for the first generation, allowing the possibility for  $uW$  fusion to create a heavy  $B'_1$ . Depending on the interplay between parton distribution functions and flavor structures, the single  $B'$  channel may offer an interesting probe of minimal flavor violation [[11\]](#page-7-8).

We focus on the QCD pair production channel  $pp \rightarrow$  $B'\bar{B}'$  which is flavor diagonal by  $SU(3)_C$  gauge invariance. In Fig. [1,](#page-3-0) we show the estimated cross section for  $B<sup>1</sup>$  pair production, calculated at leading order using PYTHIA 6.4.10 [\[21\]](#page-7-15). After production, each  $B<sup>1</sup>$  quark decays to a SM quark and either a Higgs-, *Z*-, or *W*-boson, leading to final states with multiple gauge bosons and hard jets.

An important simplification of the analysis arises due to the absence of missing energy involved with the new flavor physics. Indeed by assumption, the only new states are the heavy quarks and, except for neutrinos from gauge boson decays, all final states can be observed. Putting aside the question of backgrounds and signal efficiencies, this would allow a determination of the  $B<sup>1</sup>$  production cross sections and the relative decay rates into *Wq*, *Zq*, and *hq* (here *q* stand for any  $SM$  quark).<sup>3</sup> With large enough statistics, the *W* and *Z* helicities could be determined as well [\[22\]](#page-7-16).

<sup>&</sup>lt;sup>3</sup>Depending on the Higgs mass and decay modes, this might be an interesting discovery channel for the Higgs. See, for example, [\[9\]](#page-7-18).

<span id="page-4-0"></span>TABLE III. Estimated SM backgrounds for three generations of  $B<sup>0</sup>$  pair production with a benchmark  $B'$  mass of 600 GeV. These cross sections were calculated at leading order using ALPGEN 2.11 [\[24\]](#page-7-21) with CTEQ5L parton distribution functions [\[20\]](#page-7-17). Jets have a minimum  $p<sub>T</sub>$  of 100 GeV with a ΔR separation of 1.0. For backgrounds involving *Ws* and *Zs*, the quoted cross section refers to gauge bosons decaying to all three lepton generations (excluding  $Z \rightarrow \nu \nu$ ). To approximate the effect of cuts to isolate the  $B<sup>1</sup>$  pair production channel, the center-of-mass energy of the background events is forced to be above  $2m_{B'}$ . Backgrounds could be further suppressed by insisting on *B*<sup> $\prime$ </sup> mass reconstruction [\[16,](#page-7-19)[23](#page-7-20)]. The *W* + 3*j* and *Z* + 3*j* backgrounds are in parentheses because they are only backgrounds in regions of phase space where the  $W/Z/h$  from a *B*<sup>*I*</sup> decay is boosted enough to form one "fat" jet. The "*B*<sup> $i$ </sup>*B*<sup> $i$ </sup>  $\rightarrow$  *ZX*" cross section assumes that the *W*:*Z*:*h* branching ratios are in a 2:1:1 ratio and the *Z* decays to visible leptons (including taus). The " $B'\overline{B'} \rightarrow \overline{W} \overline{Z} \overline{X}$ " cross section requires an additional leptonic *W*.

								$t\bar{t} + j$ $t\bar{t} + 2j$ $W + 3j$ $W + 4j$ $Z + 3j$ $Z + 4j$ $WZ + 2j$ $WZ + 3j$	
								$\sigma$ 2.9 pb 9.1 pb 3.0 pb (23.3 pb) 4.4 pb (2.0 pb) 0.5 pb 0.020 pb 0.006 pb	
	$R'\bar{R}'$			$R'\bar{R}' \rightarrow ZX$		$B'\overline{B'} \rightarrow WZX$			
$\sigma$	$2.7$ pb				$0.14$ pb		$0.022$ pb		

In order to separate  $B'$  pair production from SM backgrounds, various techniques can be used to reconstruct the  $B'$  masses [\[16,](#page-7-19)[23\]](#page-7-20). Backgrounds for new vectorlike downtype quarks have also been studied in [[9](#page-7-18)]. Because we are interested in studying the flavor structure of  $B<sup>1</sup>$  decays, though, we cannot rely too heavily on *b*-tagging to suppress SM backgrounds. On the other hand, unlike generic fourth generation quarks, the  $B'$  quarks have nonnegligible branching fractions to *Z*s, so requiring leptonic Zs can help suppress the large  $t\bar{t}$  and  $W +$  jets backgrounds without biasing quark flavor determination.

Though a complete background study is beyond the scope of the present paper, example backgrounds calculated using ALPGEN 2.11  $[24]$  $[24]$  $[24]$  for a benchmark *B'* mass of 600 GeV are shown in Table [III](#page-4-0). Even in the most pessimistic case where both a leptonic *Z* and a leptonic *W* are needed to reduce backgrounds to an acceptable level, for three generations of  $600 \text{ GeV } B'$  quarks, there can still be 2000 signal events at 100 fb<sup>-1</sup> with  $O(1)$  signal to background ratio.<sup>4</sup>

To test MFV, one must extract information about the spectrum of the heavy quarks and their partial and total decay widths. Especially important are the tagging capabilities of the LHC. The quark from the  $B'$  decay can be classified either as a light jet, a heavy-flavor (*b* or *c*) jet, or a *t* quark (by identifying *bW* combinations with the right invariant mass). The purities and efficiencies depend on the flavor, the energy of the jet, and the details of the rest of the event.<sup>5</sup> We expect that the ability to distinguish a  $t$  quark from a *c* or *u* quark will be a particularly good handle because of uncertainties in the heavy-flavor tagging efficiencies at high  $p<sub>T</sub>$ . That is, for heavy-flavor and light jets alone, if the flavor violation is not large, it can perhaps be reinterpreted as flavor conservation with modified heavyflavor efficiencies. Top quarks are special because their decays add additional *W*s to events, making it in principle easier to tag the third generation in a robust way. Of course, QCD radiation can easily add 80 GeVof energy to an event, so the ability to ''*t*-tag'' may depend on the ability to simultaneously reconstruct the  $B'$  and top masses. A more detailed study of these issues is needed in order to see how ambitious this flavor program can become [\[11\]](#page-7-8).

In what follows, we investigate what can be learned about MFV if the LHC provides us with (i) the spectrum, (ii) some capability for heavy-flavor tagging, and (iii) some information on the decay widths.

(i) *Spectrum*.—MFV predicts that at the TeV scale, there is either a near degenerate spectrum of  $B<sup>0</sup>$  quarks, or only one kinematically accessible flavor. A discovery of two (or more) nondegenerate states at the TeV scale will disfavor MFV. (MFV will not be excluded because the two nondegenerate states might be the lightest members of two different triplets.) Conversely, by measuring the mass and the production cross section, the LHC might be able to establish that there is a threefold or twofold degeneracy. That will provide evidence that some flavor  $SU(3)$  or  $SU(2)$  is at work.

In principle, the total cross section can tell us the degeneracy of each state by comparing the latter to the  $t\bar{t}$  production rate. The extraction of the degeneracy requires theoretical knowledge of the mass dependence of the production cross section, and experimental measurements of the mass of the heavy quarks, their production cross section, and the  $t\bar{t}$  production rate. A complication in this procedure comes from the different decay modes of the  $B<sup>1</sup>$  compared to the *t*. It would be helpful to measure several decay modes of the *B<sup>1</sup>* to confirm the expected  $W/Z/h$ 

<sup>&</sup>lt;sup>4</sup>These estimates make the unrealistic assumption that taus can be treated on an equal footing with electrons and muons. Given the large next-to-leading-order corrections to both QCD backgrounds and  $B'$  pair production, though, the estimate is still of the right order of magnitude.

Combinatoric background in *t* reconstruction is an obvious challenge for high-multiplicity final states. The large boost factor of the  $B'$  decay products may alleviate some of the combinatoric issues, though.

branching ratios. If it can be determined that  $B'$  decays always involve longitudinally polarized *W*s and *Z*s, then this could be used as a further argument for why the *W*/Z/h branching ratios should be fixed by the Goldstone equivalence theorem.

A threefold degeneracy might get further support by looking at the flavor content of  $B<sup>1</sup>$  pair production events. Since the  $B<sup>1</sup>$  quarks are produced in pairs, MFV predicts that 1/3 of the pairs decay exclusively into third generation quarks, while  $2/3$  into non-third-generation quarks. Such evidence will make the case for degeneracy strong and will provide a rather convincing evidence for MFV.

In cases that the  $B<sup>1</sup>$  quarks are too heavy to be pair produced in a statistically significant amount, the single  $B<sup>0</sup>$  production can still be significant. This is particularly true for models **DQ** and **QQ**, where  $(Y_{B}^L)_{11} = \mathcal{O}(1)$ . Here, the single  $B'$  production channel has the peculiar feature that the production rates are determined by parton distribution functions. Furthermore, it can be used to test MFV, because the singly produced  $B_1$  should not decay to third generation quarks.

(ii) *Flavor tagging*.—The hierarchy  $v \ll M$  guarantees that the rates into the three different final bosons are comparable,

$$
\Gamma(B' \to Wq) \approx 2\Gamma(B' \to Zq) \approx 2\Gamma(B' \to hq). \tag{10}
$$

Thus, the LHC can use whichever (or a combination) of these modes that is optimal for flavor tagging. As mentioned above, because of the large  $t\bar{t}$  and  $W + \text{jets back-}$ grounds, events with at least one leptonically decaying *Z* are likely to be the most useful.

The most prominent feature of the MFV models is the suppression of flavor-changing couplings: each mass eigenstate decays to a very good approximation only to SM quarks of the corresponding generation. This property is a direct consequence of MFV. Namely, all flavor violating effects are proportional to the CKM matrix, which is very close to the unit matrix. It is this feature of MFV that can be tested in the decays of the heavy quarks.

Flavor tagging will therefore allow the LHC to put MFV to the test. First, consider events where the heavy quarks are pair produced. MFV predicts that both of them should decay to quarks of the same generation. Since the mixing between the third generation to the light one is of order  $|V_{cb}| \sim 0.04$ , we can test the following prediction:

<span id="page-5-0"></span>
$$
\frac{\Gamma(B'\bar{B}' \to Xq_{1,2}q_3)}{\Gamma(B'\bar{B}' \to Xq_{1,2}q_{1,2}) + \Gamma(B'\bar{B}' \to Xq_3q_3)} \lesssim 10^{-3}. (11)
$$

Here  $q_3$  stands for third generation quarks  $(b, t)$ ,  $q_{1,2}$  stands for first two generation quarks  $(u, d, s, c)$ , and both  $q_3$  and  $q_{1,2}$  stand for both quarks and antiquarks. Note that Eq. ([11\)](#page-5-0) is a nontrivial check of MFV, because constraints from low energy flavor experiments [[10](#page-7-7)] still allow flavorchanging couplings in  $Y_{B}^L$  of Eq. [\(6](#page-2-1)) that are considerably

larger than those predicted by MFV. In fact, this ratio could even be  $\mathcal{O}(1)$ .

Second, in the case that there is no degeneracy at all, MFV predicts that each mass eigenstate decays either to third generation quarks or to light quarks, to an accuracy of  $O(10^{-3})$ . In the case of twofold degeneracy, MFV predicts that the two mass eigenstates decay to light quarks only, up to  $\mathcal{O}(10^{-3})$  effects.

Finally, if charm tagging is also possible, the theory can be tested further. Consider a nondegenerate state that decays into light quarks (for example, model **QD**). MFV implies that this light state must decay predominantly to the first generation with small charm branching ratio, of order  $\lambda^2 \sim 5\%$ . A larger amount of charm will therefore exclude MFV.

(iii) *Decay width*.—In principle, measurements of the total decay widths of degenerate states can provide a smoking gun signal since their ratio is either one to a good accuracy (model **QQ**) or is given by the ratio of light quark masses (model **DD**). Unfortunately, it seems unlikely that the total decay width of the states can be measured. In models **QD** and **DD**, the width is, on one hand, highly suppressed and far below the experimental resolution, and on the other hand, much larger than the width required to generate a secondary vertex.<sup>6</sup> In models **DQ** and **QQ**, the width is roughly of the size of the experimental resolution (3%), which gives hope that we may get some information on the width.

As a final remark, we note that perhaps the most spectacular case will arise if model **QQ** is realized in nature, with all three heavy quarks within reach of the LHC. Establishing a  $2 + 1$  spectrum, with the separated quark decaying exclusively into third generation quarks, and the two degenerate states decaying exclusively into non-thirdgeneration quarks, will provide convincing evidence for MFV. In fact, a twofold degeneracy which involves no third generation quarks will probably be sufficient to support MFV.

### **V. CONCLUSIONS**

We have explored the question of whether high- $p_T$ physics at the LHC can contribute to our understanding of flavor physics. We considered here a specific framework of new physics, that of extra down-type SU(2)-singlet quarks in the simplest representations under the flavor group. Many other possibilities can be considered [\[11\]](#page-7-8): new downlike quarks in other representations of the flavor group, such as triplets of  $SU(3)_U$ ; up-type SU(2)-singlet

<sup>6</sup> There is the amusing possibility in models **QD** and **DD** of fine-tuning the overall magnitude of the  $Y_{B}^L$  coupling to be small while still maintaining MFV, allowing the  $B'_1$  to be long-lived enough to generate a secondary vertex while the  $B_3'$  decays promptly.

quarks; extra weak doublets; or even extra heavy leptons [\[25\]](#page-7-22).

Our scenario spans, however, four representative situations: the spectrum can be degenerate or hierarchical, and the couplings to SM quarks can be universal or hierarchical. Our framework demonstrates that, in spite of this variety of options, there are several features that are common to all MFV models.

In particular, our main result, that extra quarks at the TeV scale will allow the LHC to test MFV, does not depend on the specific implementation of MFV. MFV implies that the new physics is, to a very good approximation, flavor conserving. Thus, by roughly testing the flavor structure of the new quarks, MFV can, in principle, be excluded or, otherwise, supported and probed.

The more detailed structure of the MFV principle can be tested in various ways. The full symmetry in the down sector is  $SU(3)_Q \times SU(3)_D$ . In model **DD**, one can achieve evidence for this symmetry from the threefold degeneracy. The only order one breaking of the flavor symmetry in the down sector is due to  $Y_U Y_U^{\dagger}$ . It breaks  $SU(3)_Q \times$  $SU(3)_D \rightarrow SU(2)_Q \times U(1)_Q \times SU(3)_D$ . In model **QQ**, one can see evidence for this breaking by observing a  $2 +$ 1 spectrum. Further evidence for the approximate symmetry can be obtained in all models from the decays of heavy quarks which do not mix third generation with first and second. The down-quark masses  $\hat{\lambda}$  lead to further breaking into  $U(1)_b \times U(1)_s \times U(1)_d$ . Measuring this breaking requires sufficient *c*-tagging (which can perhaps be achieved). The effects of  $U(1)_s \times U(1)_d$  breaking are proportional to  $|V_{us}|^2$ ; measuring them via the small rate of  $B^{\prime} \bar{B}^{\prime} \rightarrow ZdWc$  will be very hard at the LHC without excellent *c*-tagging efficiency. The  $U(1)_b$  breaking effects are proportional to  $|V_{cb}|^2$  and therefore below the observable level. Consequently, they provide the strongest test of MFV.

Going forward, the main experimental issues that must be understood with regard to high- $p<sub>T</sub>$  flavor studies are:

- (i) How well will the heavy-flavor tagging efficiency be known at high- $p_T$ ? Because flavor violation could be masked by adjustments in the *b*-tagging efficiency, it may be desirable to develop less efficient but better calibrated *b*-tagging methods.
- (ii) What are the prospects for ''*t*-tagging'' in highmultiplicity events? The ability to robustly identify when events have extra *W*s from top decays will aid in the identification of  $B'$  decays to the third generation.
- (iii) Assuming the  $B<sup>1</sup>$  mass is measured in a clean channel, to what extent is it possible to separate SM backgrounds from  $B'$  signals using  $B'$  mass reconstruction? Because flavor studies are likely to be statistics limited, it may be desirable to use events with fewer numbers of final state leptons, for which  $t\bar{t}$  and  $W/Z$  + jets backgrounds are substantial.

We conclude that, if the LHC discovers new particles, it can also make a significant contribution to our understanding of flavor physics. The confirmation or invalidation of the MFV hypothesis will illuminate the new physics flavor puzzles, providing insight into the relation between high precision tests at low energy and new discoveries at the energy frontier.

# **ACKNOWLEDGMENTS**

We thank Roni Harnik, Zoltan Ligeti, Michelangelo Mangano, Gilad Perez, and Yael Shadmi for helpful discussions. This project was supported by the Albert Einstein Minerva Center for Theoretical Physics. The work of Y. G. is supported in part by the Israel Science Foundation under Grant No. 378/05. The research of Y. N. is supported by the Israel Science Foundation founded by the Israel Academy of Sciences and Humanities, the United States-Israel Binational Science Foundation (BSF), Jerusalem, Israel, the German-Israeli foundation for scientific research and development (GIF), and the Minerva Foundation. The work of J. T. is supported by the Miller Institute for Basic Research in Science. The work of J. Z. is supported in part by the European Commission RTN network, Contract No. MRTN-CT-2006-035482 (FLAVIAnet), and by the Slovenian Research Agency.

## **APPENDIX: CALCULATION OF SPECTRA AND COUPLINGS**

In this Appendix, we derive the heavy quark spectra that follow from the mass terms in Eqs.  $(2)$  and  $(4)$  $(4)$ . After electroweak symmetry breaking, these terms take the following form:

<span id="page-6-0"></span>
$$
\mathcal{L}_m = \left(\frac{\bar{Q}_L}{\bar{B}_L}\right)^{\text{T}} \underbrace{\begin{pmatrix} \nu Y_D & m_2 Y_B \\ M_1 X_{BD} & M_2 X_{BB} \end{pmatrix}}_{M_d} \underbrace{\begin{pmatrix} D_R \\ B_R \end{pmatrix}}, \tag{A1}
$$

with the transpose acting only on the generation indices. The values of the matrices  $Y_B$ ,  $X_{BD}$ ,  $X_{BB}$  for the four models are compiled in Table [I,](#page-1-0) while the resulting spectra are given in Table [II](#page-2-0). These results are easiest to derive by diagonalizing  $M_d M_d^{\dagger}$ , where  $M_d$  is the mass matrix in Eq. [\(A1](#page-6-0)). Taking  $v \sim m_2 \ll M_{1,2}$  the matrix  $M_d M_d^{\dagger}$  scales as (suppressing the dependence on  $Y_D$  and  $D_3$ )

$$
M_d M_d^{\dagger} \sim \begin{pmatrix} v^2 & vM \\ vM & M^2 \end{pmatrix}.
$$
 (A2)

Up to  $v/M$  suppressed corrections, the mass spectrum of heavy states is thus governed by the lower right  $3 \times 3$ submatrix,

<span id="page-6-1"></span>
$$
(M_d M_d^{\dagger})_{BB} = M_1^2 X_{BD} X_{BD}^{\dagger} + M_2^2 X_{BB} X_{BB}^{\dagger} \approx M_2^2 X_{BB} X_{BB}^{\dagger},
$$
\n(A3)

where the last equality is valid up to Yukawa suppressed

terms. For the **DD** and **QQ** models this is already diagonal, giving spectra as in Table [II](#page-2-0). For the **QD** and **DQ** models it is useful to go to the basis where  $Y_D$  is diagonal. The diagonalization is done by the biunitary transformation,

$$
V_L Y_D V_R^{\dagger} = Y_D^{\text{diag}} \equiv \hat{\lambda}.
$$
 (A4)

In this basis, the spectra in Table  $II$  then follow immediately, if one neglects the remaining small off-diagonal terms and uses the relation  $V_{CKM} = V_L^{\dagger} + O(v^2/M^2)$ . Explicitly, for the **QD** model we have

$$
X_{BB}X_{BB}^{\dagger} \rightarrow V_{L}X_{BB}X_{BB}^{\dagger}V_{L}^{\dagger} = V_{L}D_{3}Y_{D}Y_{D}^{\dagger}D_{3}V_{L}^{\dagger}
$$

$$
= V_{L}D_{3}V_{L}^{\dagger}\hat{\lambda}^{2}V_{L}D_{3}V_{L}^{\dagger}, \tag{A5}
$$

which together with Eq. ([8](#page-2-2)) gives  $M_{B'} \sim MD_3\hat{\lambda}$  as in Table [II](#page-2-0).

In a similar way, the spectrum could be obtained from  $M_d^{\dagger} M_d$ . Note that the 3  $\times$  3 off-diagonal blocks are either  $v^2/M^2$  or  $Y_D$  suppressed (the large right-handed rotations between  $D_R$  and  $B_R$  in **QD** and **DD** models have already been used to set  $X_{BD} = 0$ ). As above, the heavy quark spectra then follow from

$$
(M_d^{\dagger} M_d)_{BB} = m_2^2 Y_B^{\dagger} Y_B + M_2^2 X_{BB} X_{BB}^{\dagger} \sim M_2^2 X_{BB}^{\dagger} X_{BB},
$$
\n(A6)

giving the same result as Eq. ([A3\)](#page-6-1). The diagonalization of  $(M_d^{\dagger} M_d)_{BB}$  also gives the Higgs coupling

$$
\frac{m_2}{v}\bar{Q}_L Y_B B_R H, \tag{A7}
$$

in the  $Q'_{L}$ ,  $B'_{R}$  mass-eigenstate basis. Up to suppressed terms we have

$$
Q_L = U_L^{\dagger} Q_L', \qquad B_R = U_R^{\dagger} B_R', \tag{A8}
$$

where  $U_R$  diagonalizes the  $(M_d^{\dagger} M_d)_{BB}$  matrix and  $U_L \sim$ *V*<sub>CKM</sub>. The similarity sign means that the two sides scale in the same way in terms of the Wolfenstein parameter  $\lambda$ , with the scaling for  $V_L = V_{\text{CKM}}^{\dagger} + O(v^2/M^2)$  given in Eq. [\(7](#page-2-3)). In the mass-eigenstate basis, the Higgs coupling is

$$
\frac{m_2}{v} \bar{Q}'_L U_L Y_B U_R^\dagger B_R H.
$$
 (A9)

From this, the scaling of Higgs couplings with the Wolfenstein parameter  $\lambda$  is obtained as given in Table [II](#page-2-0). In the derivation, one also uses the fact that  $Y_U Y_U^{\dagger}$  insertions preserve an  $SU(2)$  symmetry of the first two generations, so that the Higgs coupling  $U_L Y_B U_R^{\dagger}$  is diagonal in the first two rows and columns.

- <span id="page-7-0"></span>[1] W. M. Yao *et al.* (Particle Data Group), J. Phys. G **33**, 1 (2006).
- <span id="page-7-1"></span>[2] E. Barberio *et al.* (Heavy Flavor Averaging Group), arXiv:0704.3575.
- <span id="page-7-2"></span>[3] I. Hinchliffe and F. E. Paige, Phys. Rev. D **63**, 115006 (2001); D. F. Carvalho, J. R. Ellis, M. E. Gomez, S. Lola, and J. C. Romao, Phys. Lett. B **618**, 162 (2005); K. Agashe and M. Graesser, Phys. Rev. D **61**, 075008 (2000); N. V. Krasnikov, JETP Lett. **65**, 148 (1997); A. Bartl, K. Hidaka, K. Hohenwarter-Sodek, T. Kernreiter, W. Majerotto, and W. Porod, Eur. Phys. J. C **46**, 783 (2006).
- <span id="page-7-3"></span>[4] G. D'Ambrosio, G. F. Giudice, G. Isidori, and A. Strumia, Nucl. Phys. **B645**, 155 (2002).
- [5] A. J. Buras, P. Gambino, M. Gorbahn, S. Jager, and L. Silvestrini, Phys. Lett. B **500**, 161 (2001).
- <span id="page-7-4"></span>[6] A. J. Buras, Acta Phys. Pol. B **34**, 5615 (2003).
- <span id="page-7-5"></span>[7] G. C. Branco, L. Lavoura, and J. P. Silva, *CP Violation* (Clarendon, Oxford, 1999).
- <span id="page-7-6"></span>[8] J. A. Aguilar-Saavedra, Phys. Rev. D **67**, 035003 (2003); **69**, 099901(E) (2004).
- <span id="page-7-18"></span>[9] T. C. Andre and J. L. Rosner, Phys. Rev. D **69**, 035009 (2004).
- <span id="page-7-7"></span>[10] T. Yanir, J. High Energy Phys. 06 (2002) 044; G. Barenboim, F.J. Botella, and O. Vives, Nucl. Phys. **B613**, 285 (2001); T. Morozumi, Z. H. Xiong, and T. Yoshikawa, arXiv:hep-ph/0408297.
- <span id="page-7-8"></span>[11] Y. Grossman, Y. Nir, J. Thaler, T. Volansky, and J. Zupan

(work in progress).

- <span id="page-7-9"></span>[12] M. Perelstein, M. E. Peskin, and A. Pierce, Phys. Rev. D. **69**, 075002 (2004).
- <span id="page-7-10"></span>[13] L. Lavoura and J. P. Silva, Phys. Rev. D **47**, 2046 (1993).
- <span id="page-7-11"></span>[14] R. Mehdiyev, S. Sultansoy, G. Unel, and M. Yilmaz, Eur. Phys. J. C **49**, 613 (2007).
- <span id="page-7-12"></span>[15] J. A. Aguilar-Saavedra, Phys. Lett. B **625**, 234 (2005); **633**, 792(E) (2006).
- <span id="page-7-19"></span>[16] W. Skiba and D. Tucker-Smith, Phys. Rev. D **75**, 115010 (2007).
- <span id="page-7-13"></span>[17] G. Azuelos *et al.*, Eur. Phys. J. C **39S2**, 13 (2005).
- [18] S. S. D. Willenbrock and D. A. Dicus, Phys. Rev. D **34**, 155 (1986).
- <span id="page-7-14"></span>[19] T. Han, H. E. Logan, B. McElrath, and L. T. Wang, Phys. Rev. D **67**, 095004 (2003).
- <span id="page-7-17"></span>[20] H. L. Lai *et al.* (CTEQ Collaboration), Eur. Phys. J. C **12**, 375 (2000).
- <span id="page-7-15"></span>[21] T. Sjostrand, S. Mrenna, and P. Skands, J. High Energy Phys. 05 (2006) 026.
- <span id="page-7-16"></span>[22] F. Hubaut, E. Monnier, P. Pralavorio, K. Smolek, and V. Simak, Eur. Phys. J. C **44S2**, 13 (2005).
- <span id="page-7-20"></span>[23] B. Holdom, J. High Energy Phys. 03 (2007) 063; J. High Energy Phys. 08 (2007) 069.
- <span id="page-7-21"></span>[24] M. L. Mangano, M. Moretti, F. Piccinini, R. Pittau, and A. D. Polosa, J. High Energy Phys. 07 (2003) 001.
- <span id="page-7-22"></span>[25] V. Cirigliano, A. Kurylov, M. J. Ramsey-Musolf, and P. Vogel, Phys. Rev. D **70**, 075007 (2004).