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Exotic eikosiheptaplet in the chiral quark soliton model

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We use the chiral quark soliton model to estimate masses and widths of the two eikosiheptaplets (27-plets of SU(3) flavor) of spin $3/2$ and $1/2$ that emerge in the rigid rotator quantization. We use as input: hyperon decays, Θ^+ mass, and width. While $27_{3/2}$ has small widths (although much larger than the values allowed by the partial wave analysis), $27_{1/2}$ has large decay widths to antidecuplet. However, exactly for this decay channel the widths are suppressed in the large N_c limit.

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I. INTRODUCTION

One of the most puzzling results of the chiral quark-soliton model (χQSM) for exotic baryons consists in a very small hadronic decay width, governed by the decay constant $G_{\overline{10}}$. While the small mass of exotic states is rather generic for all chiral models [1–3] the smallness of the decay width appears as a subtle cancellation of three different terms [3] that contribute to $G_{\overline{10}}$. We are therefore trapped between two extremes. On one hand Δ decay width which is suppressed in large N_c limit is numerically rather large, above 100 MeV, on the other hand Θ^+ decay width which scales like N_c^0, is numerically tiny, below 1 MeV. If narrow pentaquarks exist, the large N_c argument is not enough to claim consistency of the model, and some degree of cancellation in the decay coupling is needed. In this paper we investigate this problem for the next exotic SU(3) representation, namely, 27-plet, called eikosiheptaplet.

Following the prescription of Adkins, Nappi, and Witten [4] (criticized recently in Ref. [5]) the decay width in solitonic models is calculated in terms of a matrix element $\mathcal{M}$ of the collective axial current operator corresponding to the emission of a pseudoscalar meson φ:

$$\begin{aligned}\hat{O}_{\varphi}^{(8)} &= 3 \times \mathrm{const} \times p_{\varphi}^{i} \\ &\quad \times \sum_{i=1}^{3}\left(a_1 D_{\varphi i}^{(8)} + a_2 d_{ibc} D_{\varphi b}^{(8)} \hat{S}_c + \frac{a_3}{\sqrt{3}} D_{\varphi 8}^{(8)} \hat{S}_i\right) \\ &= 3 \times p_{\varphi}^{i} \\ &\quad \times \sum_{i=1}^{3}\left(G_0 D_{\varphi i}^{(8)} - G_1 d_{ibc} D_{\varphi b}^{(8)} \hat{S}_c - \frac{G_2}{\sqrt{3}} D_{\varphi 8}^{(8)} \hat{S}_i\right)\end{aligned} \tag{1}$$

where in the last line of Eq. (1) we have displayed the operator in the form often used in the literature. Here $D_{\varphi i}^{(8)}$ are SU(3) Wigner matrices, $\hat{S}_i$ collective spin operator, p_{φ}^{i} meson momentum (for more details on the collective quantization and baryon wave functions see e.g. Ref. [6]). Constants $a_{1,2,3}$ are constructed from the so called *moments of inertia* that are calculable in χQSM [7,8]. A multiplicative constant has to be fixed from the generalized Goldberger-Treiman relation [3,9]. Alternatively, following the *model-independent approach* of Adkins and Nappi [10], one can treat $a_{1,2,3}$ as *free* constants and try to extract their phenomenological values from the hyperon decays [7,9].

The predictive power of the model-independent approach for exotic baryons is, however, hampered by the fact that only one linear combination constructed from two free parameters $a_{1,2}$, namely

$$a_1 - \tfrac{1}{2}a_2,$$

enters the hyperon decay widths, whereas for the decay widths of exotic states both a_1 and a_2 are needed separately. The same problem occurs for baryonic masses [2,3] where no information on the exotica can be retrieved from the regular baryon spectra alone (and similarly for magnetic moments [11]).

One is therefore forced to introduce some additional assumptions to fix the remaining coefficient. In the original work of Ref. [3] masses were fixed by a requirement that nucleon resonance N*(1710) was a member of antidecuplet. Decay widths were estimated with the help of hyperon semileptonic decays and $g_{\pi NN}$ used as an input with some other simplifying assumptions. A complete phenomenological analysis in this spirit can be found in Ref. [12].

Another possibility to constrain the undetermined parameter is to go beyond the SU(3) symmetry limit and include higher order symmetry breaking terms [9]. Why may going off the symmetry limit be at all helpful? The answer is very simple: the baryonic wave functions belong no longer to pure SU(3) multiplets, but contain m_s dependent admixtures of higher representations. For example, a nucleon contains an admixture of antidecuplet cryptoex-

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otic nucleonic state. As a result, the matrix element of any operator [e.g. the decay operator (1)] contains—apart from the leading term—exotic transitions from antidecuplet to octet as a nonleading correction. By fitting the decay rates with m_s corrections one is therefore able to constrain the otherwise undetermined parameter.

The first estimate of the Θ^+ mass in the Skyrme model has been done in this way already long ago [2]. More recently magnetic moments [11] and Θ^+ decay width [9] have been evaluated by applying the above-mentioned procedure. There all higher representations are treated as stable hadronic states, rather than as wide resonances. In particular admixture of eikosiheptaplet (27-plet) here is of importance (see Fig. 1 of Ref. [13]). Indeed the contamination of baryonic wave functions by eikosiheptaplet reaches 20%–30%.

It is therefore of importance to check whether the eikosiheptaplet may be indeed considered as a (semi) stable exotic representation. Not only can it mix with ordinary baryons, but it contains a number of exotic states that may be of interest by themselves, the isotriplet of Θ states being the most prominent example. Since transitions to exotic representations enter through representation mixing which itself is of the order of m_s, (semi) stability of eikosiheptaplet has to be valid in the leading order of perturbartive expansion in the strange quark mass. Therefore in our analysis of the decay widths we work in the chiral limit.

In chiral models all baryon representations have positive parity and spin corresponding to the isospin of states with $Y = 1$. For eikosiheptaplet that means that we have two distinct representations, one of spin $3/2$ and the second one of spin $1/2$ (i.e. $27_{3/2}$ and $27_{1/2}$ respectively), the latter being heavier. In this work we shall concentrate on the lightest states, namely, on the isospin triplet of Θ_{27}, on Δ_{27} of isospin $3/2$ and on N_{27} states of isospin $1/2$. These states are light (for $27_{3/2}$) and have been looked for in various experiments. Apart from still unconfirmed reports by STAR [14], recent partial wave (PW) analysis of meson-nucleon scattering data puts stringent limits on the possible existence of Θ_{27} and Δ_{27} states [15]. These states may be incorporated into the PW analysis provided that their widths are of the order of tens keV. As we shall see, χQSM predicts that their widths are an order of magnitude larger. Although still small on a hadronic scale, they are much too large to be accommodated by PW analysis.

Throughout this paper we shall assume that Θ^+ exists with mass 1535 MeV and width smaller than 1 MeV. This input allows us to constrain all model parameters except $\Sigma_{\pi\mathrm{N}}$. If additionally we assume that $\Xi_{3/2}$ has mass $\sim$1860 MeV, also pion-nucleon sigma term is fixed $\Sigma_{\pi\mathrm{N}} = 73$ MeV.

There have been already a few calculations in the literature of the eikosiheptaplet masses and widths in chiral soliton models [16–23]. In this paper we use the mass formula of Ref. [18]. Generically the mass of the lowest $I = 1$ multiplet of Θ_{27} states in $27_{3/2}$ is almost degenerate with Θ^+ of $\overline{10}$. On the contrary, $27_{1/2}$ is substantially heavier.

As far as widths are concerned our calculations differ in three aspects from those of Refs. [16–19]. First, in Ref. [16] one considers only the leading G_0-term, whereas in Refs. [17–19] the constant G_2 has been neglected. Indeed, G_2 (or more precisely a_3) is small as it is directly related to the singlet axial current. Even though it is really small, it can be safely neglected *only* if there is no cancellation between G_0 and G_1, so that the pertinent linear combination of G_0 and G_1 is much larger than G_2 itself. In the decays of antidecuplet, $27_{3/2} \rightarrow 8 + \text{meson}$ and $27_{1/2} \rightarrow 10 + \text{meson}$ strong cancellations are indeed present and G_2 cannot be neglected. In this paper we use a_3 extracted from the chiral limit fits to the semileptonic hyperon decays that is definitely not consistent with zero. Second, we use the Goldberger-Treiman relation to fix the constant entering Eq. (1), so that $G_{0,1,2}$ depend on the decay in question, whereas in Refs. [16–19] they were considered as universal. Third, instead of calculating the decay widths and masses for a fixed choice of model parameters, we explore the residual freedom within the model and calculate the *range* of values, rather than only one number. Finally, some calculations [19] took partially into account the effects of the symmetry breaking, which is neglected in our paper.

We show that $27_{3/2}$ is in a sense "well behaving" having small widths to octet with most other channels kinematically suppressed. On the contrary, $27_{1/2}$ has large decay widths to antidecuplet, with small decay widths to other channels. However, precisely in the case of $27 \rightarrow \overline{10}$ transition the phase space is formally suppressed in the large N_c limit. The situation reminds the decay of Δ and Θ^+, the first one being numerically large, but formally damped in the large N_c limit with the second one being numerically small but $\mathcal{O}(1)$ as far as N_c counting is concerned.

The paper is organized as follows. In Sec. II we give an overview of the nonrelativistic formalism to calculate the decay widths using the generalized Goldberger-Treiman relation. We fix two out of three axial constants and define model parameters. Finally, we calculate the masses of antidecuplet and eikosiheptaplet. In Sec. III we express antidecuplet and decuplet amplitudes entering the decay widths through couplings G_{10} and $G_{\overline{10}}$ and the SU(3) isoscalar factors. By fixing Θ^+ decay width to be below 1 MeV we constrain the axial coupling parameter space and give results for the decay widths of other members of antidecuplet. In Sec. IV we repeat the calculations from the preceding section for eikosiheptaplets of spin $3/2$ and $1/2$. We perform phenomenological analysis of the pertinent decay couplings—the analogs of G_{10} and $G_{\overline{10}}$—and calculate the decay widths. We summarize our findings in Sec. V. Some useful group-theoretical formulas are collected in the Appendix.

II. GENERAL FORMALISM

Throughout this paper we shall use the nonrelativistic formula for the decay width [3,24]

$$\Gamma_{B\to B'+\varphi} = \frac{1}{8\pi}\frac{p_\varphi}{MM'}\bar{\mathcal{M}}^2 = \frac{1}{8\pi}\frac{p_\varphi^3}{MM'}\bar{\mathcal{A}}^2. \tag{2}$$

The "bar" over the amplitude squared denotes averaging over initial and summing over final spin and over isospin. Anticipating linear momentum dependence of the decay amplitude $\mathcal{M}$

$$\mathcal{M}_{B\to B'+\varphi} = \langle \mathcal{R}'_{S'}, B'|\hat{O}^{(8)}_\varphi|\mathcal{R}_S, B\rangle \tag{3}$$

we have introduced reduced amplitude $\mathcal{A}$ where the momentum of the outgoing meson

$$p_\varphi = \frac{\sqrt{(M^2-(M'+m_\varphi)^2)(M^2-(M'-m_\varphi)^2)}}{2M} \tag{4}$$

has been factored out. Here $\mathcal{R}$ stands for the SU(3) representation and S for spin. In Ref. [3] following the approach of Ref. [25] MM' in Eq. (2) was replaced by $(M+M')^2/4$, and, furthermore, the additional factor M/M' was inserted to sum up certain kinematical effects. We will not make such alterations in the following. Instead, we will apply the generalized Goldberger-Treiman relation that allows to relate the axial constants $a_{1,2,3}$ to the constants $G_{0,1,2}$ by means of the following relation [9]:

$$G_0 = -\frac{M+M'}{3f_\varphi}a_1, \qquad G_{1,2} = \frac{M+M'}{3f_\varphi}a_{2,3} \tag{5}$$

where M and M' stand for the baryonic masses involved in the decay $B \to B' + \varphi$ and f_φ denotes pseudoscalar meson decay constant in the normalization where $f_\pi = 93$ MeV, $f_K = 115$ MeV, and $f_\eta = 1.2 f_\pi$ [26] (we neglect $\eta - \eta'$ mixing). The use of Eq. (5) makes constants $G_{0,1,2}$ *decay dependent* in contrast to previous analysis where they were considered to be universal, with possible modification of the formula for the width (2).

In contrast to the early exploratory works we now *know for sure* that if Θ^+ exists it is light and its width is small. Therefore we use these two pieces of information to constrain the mass and the decay width of Θ^+ for which we take $M_\Theta = 1535$ MeV and $\Gamma_{\Theta\to \mathrm{N}+\mathrm{K}} \sim 1$ MeV. With these parameters fixed we calculate the decay widths of decuplet, antidecuplet, and eikosiheptaplet and discuss uncertainties of our results coming from the m_s corrections. In this respect we differ from Ref. [9] where m_s corrections were used—as explained in the Introduction—to constrain input parameters.

In order to fix the input parameters $a_{1,2,3}$ we use a fit from Refs. [7,27] where one uses two linear combinations of known hyperon decays, that in χQSM are free of the linear m_s corrections

$$a_1 - \tfrac{1}{2}a_2 = -2.675, \qquad a_3 = 0.678. \tag{6}$$

With these parameters one obtains:

$$g_A^{(3)} = 1.27, \qquad g_A^{(8)} = 0.43, \qquad g_A^{(0)} = 0.68. \tag{7}$$

These values overshoot present experimental results, especially for $g_A^{(0)}$ (that ranges between 0.15–0.35 [28]). It should, however, be remembered that $g_A^{(0)}$ is sensitive to the corrections of higher order in m_s that pull it down with respect to the chiral limit estimate (see Fig. 2 in [27]).

Let us stress that parametrization (6) is theoretically very appealing, because one does not need to refit leading order parameters $a_{1,2,3}$ when m_s corrections are included. Nevertheless the overall quality of the fit is of course better when full formula with m_s corrections is used [9]. To check sensitivity of our results to the fitting procedure, we have also used different set of parameters (that will be called fit 2 in the following) which better fits $g_A^{(0)}$ in the leading order:

$$a_1 - \tfrac{1}{2}a_2 = -5.4, \qquad a_3 = 0.3 \tag{8}$$

which gives:

$$g_A^{(3)} = 1.27, \qquad g_A^{(8)} = 0.36, \qquad g_A^{(0)} = 0.3. \tag{9}$$

Contenting ourselves with input parameters (6) and (8) we can check our formalism against the hadronic data. First, let us compute the pion-nucleon coupling constant $g_{\pi\mathrm{NN}}$ that for both fits reads

$$g_{\pi\mathrm{NN}} = \tfrac{7}{10}(G_0 + \tfrac{1}{2}G_1 + \tfrac{1}{14}G_2) = 12.8 \tag{10}$$

vs. experimental value of 13.1–13.3 [12]. Here the numerical result has been obtained by putting $M = M' = M_\mathrm{N}$ in Eq. (5). Secondly, anticipating results of the next section, we can also quote our prediction for the decay width of Δ obtained by means of Eq. (2)

$$\Gamma_\Delta = 104\ (106)\ \mathrm{MeV} \tag{11}$$

in fair agreement with experiment [the number in parenthesis refers to the parameters of Eq. (8)]. Note that one may improve this result by including a phenomenological factor M_Δ/M_N [3,25] that would scale (11) up to 134 MeV. Also m_s corrections increase the Δ width (in this case by 25%–30% [13]).

For the decays of exotic states we have to know a_1 and a_2 separately. We therefore parametrize

$$a_1 = \rho, \qquad a_2 = 5.352 + 2\rho, \qquad a_3 = 0.68. \tag{12}$$

It follows from the phenomenological analysis of Ref. [9] that the realistic range for ρ lies within -3 to -1.9. In what follows we shall fix ρ to *fit* the "experimental" width for Θ^+. As will be shown in Eq. (25), if we require $\Gamma_\Theta <$ 1 MeV then $\rho_1 = -1.98 < \rho < \rho_2 = -1.814$. For comparison we will also use fit 2

$$a_1 = \rho, \qquad a_2 = 5.4 + 2\rho, \qquad a_3 = 0.3 \tag{13}$$

varying ρ within the limits $\rho_1 = -1.933 < \rho < \rho_2 = -1.767$. All numerical results in the following will be presented for fit 1 (12), modifications due the second choice of input parameters (13) will be discussed in Sec. V.

Finally, in order to use formula (2) we have to specify masses of exotic states. To this end we parametrize all exotic masses in terms of one parameter: $\Sigma_{\pi N}$, i.e. the pion nucleon sigma term that we will vary within the range of 40–70 MeV.

In the chiral quark soliton model baryon masses can be read off from the collective hamiltonian

$$\hat{H} = M_{cl} + \frac{1}{2I_1}S(S+1) + \frac{1}{2I_2}\left(C_2(\mathrm{SU}(3)) - S(S+1) - \frac{N_c^2}{12}\right) + \hat{H}' \quad (14)$$

where the symmetry breaking Hamiltonian takes the following form:

$$\hat{H}' = \alpha D_{88}^{(8)} + \beta Y + \frac{\gamma}{\sqrt{3}} D_{8i}^{(8)} \hat{S}_i. \quad (15)$$

Matrix elements of $\hat{H}'$ can be found e.g. in Refs. [12,18]. For $M_\Theta = 1535$ MeV the model parameters take the following values (in MeV) as functions of $\Sigma_{\pi N}$ [12,13]:

$$\frac{1}{I_2} = 152.4, \qquad \frac{1}{I_2} = 608.7 - 2.9\Sigma_{\pi N} \quad (16)$$

and

$$\alpha = 336.4 - 12.9\Sigma_{\pi N}, \qquad \beta = -336.4 + 4.3\Sigma_{\pi N}, \qquad \gamma = -475.94 + 8.6\Sigma_{\pi N}. \quad (17)$$

Numerical results for antidecuplet obtained with the help of Eqs. (16) and (17) are summarized in Table I.

Our choice for the values of $\Sigma_{\pi N}$ in Table I is not accidental. For $\Sigma_{\pi N} = 42$ MeV the mass of the cryptoexotic nucleon resonance corresponds to the original choice of [3] who associated it with the known resonance N*(1710). Almost for sure this choice is now ruled out, and this implies that the new, narrow (as we will see below) nucleon resonance needs to be still discovered. There are several candidates for such states found both in partial wave analysis [29], η photoproduction on nucleon (see Ref. [30] and references therein), and at STAR [31]. Next, the value of 55 MeV corresponds to $\Sigma_{\pi N}$ calculated within the model [32], and moreover it is the value for which one of the symmetry breaking parameters (15) $\gamma \approx 0$. Let us note that $\gamma = 0$ in the nonrelativistic limit. Finally for $\Sigma_{\pi N} = 73$ MeV the mass of $\Xi_{3/2}$ corresponds to the estimate of NA49 [33]. This is also the value preferred by the recent analysis of πN scattering [34].

TABLE I. Masses of antidecuplets for different values of $\Sigma_{\pi N}$.

$\Sigma_{\pi N}$	42 MeV	55 MeV	73 MeV
Θ	1535	1535	1535
N	1709	1681	1642
Σ	1883	1827	1750
$\Xi_{3/2}$	2057	1974	1857

For eikosiheptaplet the masses (in MeV) are listed in the Table II.

TABLE II. Masses of eikosiheptaplets for different values of $\Sigma_{\pi N}$.

$\Sigma_{\pi N}$	42 MeV		55 MeV		73 MeV	
Spin	3/2	1/2	3/2	1/2	3/2	1/2
Θ	1568	1999	1578	1965	1593	1919
Δ	1721	2213	1688	2165	1642	2098
N	1715	2158	1717	2087	1721	1988
Γ	1875	2439	1798	2365	1691	2264
Σ	1866	2358	1837	2261	1796	2126
Λ	1862	2318	1856	2209	1850	2058
$\Xi_{3/2}$	2018	2558	1956	2435	1872	2264
Ξ	2011	2521	1986	2399	1951	2230
Ω	2160	2677	2115	2504	2052	2265

Table II deserves a few comments. The first two columns corresponding to $\Sigma_{\pi N} = 42$ MeV are in agreement with the numerical values from Ref. [16] where N*(1710) was taken as input. The last two columns corresponding to the antidecuplet masses: $M_{\Theta^+} = 1535$ and $M_{\Xi_{3/2}} = 1860$ are in agreement with Refs. [17–19]. Finally, let us observe that—as can be also seen from Fig. 1—the spin 1/2 eikosiheptaplet is squeezed for smaller values of the hypercharge making the heaviest isospin submultiplets al-

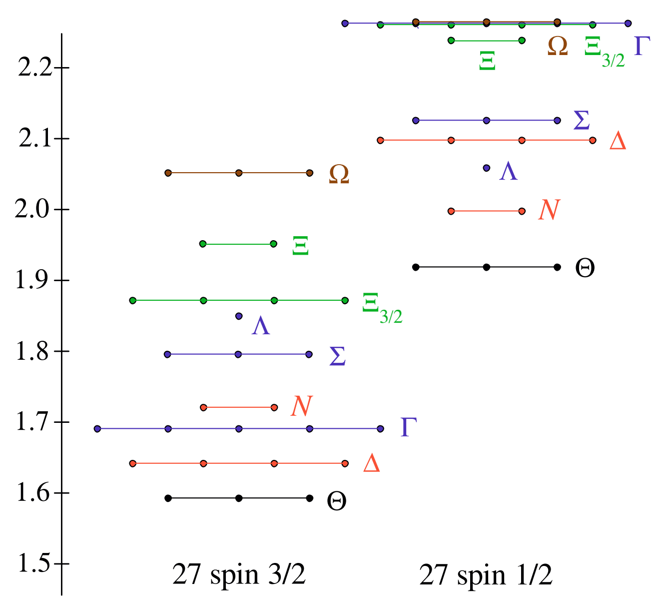

FIG. 1 (color online). Spectrum of eikosiheptaplet (in GeV) of spin 3/2 (left) and spin 1/2 (right) for $\Sigma_{\pi N} = 73$ MeV. Note large splittings of equal hypercharge multiplets.

most degenerate. On the other hand, Θ_{27} in $27_{3/2}$ is only a few tens of GeV above the Θ^+ of antidecuplet.

III. DECAY CONSTANTS FOR DECUPLET AND ANTIDECUPLET

The matrix elements for decuplet and antidecuplet with $S_3 = S_3' = 1/2$ read:

$$\mathcal{A}\,(B_{10_{3/2}} \to B_8' + \varphi) = 3\left(\begin{matrix} 8 & 8 \\ \varphi & B' \end{matrix}\middle|\begin{matrix} 10 \\ B \end{matrix}\right)\frac{2}{\sqrt{15}} \times G_{10}, \tag{18}$$

$$\mathcal{A}\,(B_{\overline{10}_{1/2}} \to B_8' + \varphi) = -3\left(\begin{matrix} 8 & 8 \\ \varphi & B' \end{matrix}\middle|\begin{matrix} \overline{10} \\ B \end{matrix}\right)\frac{1}{\sqrt{15}} \times G_{\overline{10}}, \tag{19}$$

where

$$G_{10} = G_0 + \tfrac{1}{2}G_1, \qquad G_{\overline{10}} = G_0 - G_1 - \tfrac{1}{2}G_2. \tag{20}$$

In order to have an estimate of the width (2) the authors of Ref. [3] calculated $G_{\overline{10}}$ in the nonrelativistic limit of χQSM [35] and got $G_{\overline{10}} \equiv 0$. It has been shown [36] that this cancellation between terms that scale differently with N_c ($G_0 \sim N_c^{3/2}$, $G_{1,2} \sim N_c^{1/2}$) is actually consistent with large N_c counting, since in fact

$$G_{\overline{10}} = G_0 - \frac{N_c + 1}{4}G_1 - \frac{1}{2}G_2 \tag{21}$$

where the explicit N_c dependence comes from the SU(3) Clebsch-Gordan coefficients calculated for large N_c (note that for arbitrary N_c baryons are built from N_c quarks rather than from 3). In the nonrelativistic limit (NRL) [36]:

$$G_0 = -(N_c + 2)G, \qquad G_1 = -4G, \qquad G_2 = -2G, \qquad G \sim N_c^{1/2}. \tag{22}$$

Similar cancellations occur also for the decays of the eikosiheptaplet [37]. From now on we will keep $N_c = 3$.

Following steps described in the appendix we obtain the averaged matrix elements

$$\bar{\mathcal{A}}^2(B_{10_{3/2}} \to B_8' + \varphi) = \frac{6}{5}\left[\begin{matrix} 8 & 8 \\ \varphi & B' \end{matrix}\middle|\begin{matrix} 10 \\ B \end{matrix}\right]^2 \times G_{10}^2, \tag{23}$$

$$\bar{\mathcal{A}}^2(B_{\overline{10}_{1/2}} \to B_8' + \varphi) = \frac{3}{5}\left[\begin{matrix} 8 & 8 \\ \varphi & B' \end{matrix}\middle|\begin{matrix} \overline{10} \\ B \end{matrix}\right]^2 \times G_{\overline{10}}^2 \tag{24}$$

where the squares of the isoscalar factors [the quantities in the square brackets in Eqs. (23) and (24)] are listed in Table III.

In Fig. 2 we plot *scaled* coupling constants G_{10} and $G_{\overline{10}}$ [i.e. without Goldberger-Treiman factors $(M + M')/3f_\varphi$ (5)] as functions of parameter ρ, where ρ is given by Eq. (12). As already explained in the Introduction, G_{10} is constant, as the ρ dependence cancels out, while $G_{\overline{10}}$ steeply decreases reaching zero for $\rho_0 = -1.897$. This is a reflection of the nonrelativistic cancellation (22) observed for the first time in Ref. [3]. It is obvious that by an appropriate choice of ρ in the vicinity of ρ_0 we can make $G_{\overline{10}}$ arbitrarily small. By plugging in parameters (6) and (12) into (5) and (20) we get that

$$\Gamma_\Theta < 1\ \text{MeV} \to \rho_1 = -1.98 < \rho < \rho_2 = -1.814. \tag{25}$$

In Table IV we list the decay widths for the remaining members of antidecuplet for $\rho = -1.98$ (or equivalently -1.814) for various choices of the masses from Table I parametrized by the pion-nucleon sigma term $\Sigma_{\pi N}$:

We see from Table IV that the widths of cryptoexotic nucleon and Σ resonances exceed 1 MeV, the width of $\Xi_{3/2}$ is even larger, however within the limits set by NA49. It is

TABLE III. Isoscalar factors squared for the decays of decuplet and antidecuplet.

$10 \to 8 + 8$	$\overline{10} \to 8 + 8$	C^2
$\Omega \to \bar{K} + \Xi$	$\Theta \to K + N$	1
$\Xi^* \to \pi + \Xi$	$N \to \pi + N$	1/4
$\Xi^* \to \eta + \Xi$	$N \to \eta + N$	1/4
$\Xi^* \to \bar{K} + \Lambda$	$N \to K + \Lambda$	1/4
$\Xi^* \to \bar{K} + \Sigma$	$N \to K + \Sigma$	1/4
$\Sigma^* \to \bar{K} + N$	$\Sigma \to \bar{K} + N$	1/6
$\Sigma^* \to \pi + \Lambda$	$\Sigma \to \pi + \Lambda$	1/4
$\Sigma^* \to \pi + \Sigma$	$\Sigma \to \pi + \Sigma$	1/6
$\Sigma^* \to \eta + \Sigma$	$\Sigma \to \eta + \Sigma$	1/4
$\Sigma^* \to K + \Xi$	$\Sigma \to K + \Xi$	1/6
$\Delta \to \pi + N$	$\Xi_{3/2} \to \pi + \Xi$	1/2
$\Delta \to K + \Sigma$	$\Xi_{3/2} \to \bar{K} + \Sigma$	1/2

TABLE IV. Decay widths in MeV for the decays of antidecuplet.

$B_{\overline{10}} \to \varphi + B_8'$	$\Gamma_{B\to\varphi+B'}$ [MeV]		
$\Sigma_{\pi N}$ [MeV]	42	55	73
$\Theta \to K + N$	0.95	0.95	0.95
$N \to \pi + N$	4.18	3.77	3.25
$N \to \eta + N$	0.99	0.80	0.56
$N \to K + \Lambda$	0.24	0.14	0.04
$N \to K + \Sigma$	0.02	—	—
$\Sigma \to \bar{K} + N$	1.95	1.53	1.04
$\Sigma \to \pi + \Lambda$	4.40	3.57	2.59
$\Sigma \to \pi + \Sigma$	2.24	1.77	1.22
$\Sigma \to \eta + \Sigma$	0.54	0.25	0.01
$\Sigma \to K + \Xi$	0.10	0.01	—
$\Xi_{3/2} \to \pi + \Xi$	8.41	6.01	3.44
$\Xi_{3/2} \to \bar{K} + \Sigma$	4.52	2.89	1.20

important to observe that the estimate from Ref. [18] is almost 4 times bigger; it is difficult to comment why because the authors of Ref. [18] give no details of their width calculation. One has to remember that the entries in Table IV constitute in fact the *upper* limits, since the widths scale as $(\rho - \rho_0)^2$ (with $\rho_0 = -1.897$), and can be arbitrarily decreased with an appropriate choice of ρ. In the situation when the leading contributions are small, m_s corrections become important, that issue has been studied in Ref. [13].

IV. DECAY CONSTANTS FOR EIKOSIHEPTAPLET

In this section we shall consider decays of eikosiheptaplet (27) that can have either spin 1/2 or 3/2, the latter being lighter. Matrix elements for the decays of eikosiheptaplet of $S = 3/2$ (and with $S_3 = 1/2$) read:

$$\mathcal{A}(B_{27_{3/2}} \to B'_8 + \varphi) = 3\begin{pmatrix} 8 & 8 & \vert & 27 \\ \varphi & B' & \vert & B \end{pmatrix}\frac{2\sqrt{2}}{9} \times G_{27},$$
$$\mathcal{A}(B_{27_{3/2}} \to B'_{10} + \varphi) = -3\begin{pmatrix} 8 & 10 & \vert & 27 \\ \varphi & B' & \vert & B \end{pmatrix}\frac{\sqrt{10}}{36} \times F_{27},$$
$$\mathcal{A}(B_{27_{3/2}} \to B'_{\overline{10}} + \varphi) = 3\begin{pmatrix} 8 & \overline{10} & \vert & 27 \\ \varphi & B' & \vert & B \end{pmatrix}\frac{\sqrt{30}}{9} \times E_{27}, \tag{26}$$

where

$$G_{27} = G_0 - \tfrac{1}{2}G_1, \qquad F_{27} = G_0 - \tfrac{1}{2}G_1 - \tfrac{3}{2}G_2, \qquad E_{27} = G_0 + G_1. \tag{27}$$

For $S = 1/2$ and $S_3 = 1/2$ we have:

$$\mathcal{A}(B_{27_{1/2}} \to B'_8 + \varphi) = -3\begin{pmatrix} 8 & 8 & \vert & 27 \\ \varphi & B' & \vert & B \end{pmatrix}\frac{\sqrt{10}}{45} \times H_{27},$$
$$\mathcal{A}(B_{27_{1/2}} \to B'_{10} + \varphi) = -3\begin{pmatrix} 8 & 10 & \vert & 27 \\ \varphi & B' & \vert & B \end{pmatrix}\frac{\sqrt{2}}{9} \times G'_{27},$$
$$\mathcal{A}(B_{27_{1/2}} \to B'_{\overline{10}} + \varphi) = 3\begin{pmatrix} 8 & \overline{10} & \vert & 27 \\ \varphi & B' & \vert & B \end{pmatrix}\frac{7\sqrt{2}}{36} \times H'_{27}, \tag{28}$$

where

$$H_{27} = G_0 - 2G_1 + \tfrac{3}{2}G_2, \qquad G'_{27} = G_0 - 2G_1, \qquad H'_{27} = G_0 + \tfrac{11}{14}G_1 + \tfrac{3}{14}G_2. \tag{29}$$

In Fig. 2 we plot *scaled* coupling constants [i.e. without Goldberger-Treiman factors $(M + M')/3f_\varphi$ (5)] for decays of $27_{3/2}$ and $27_{1/2}$ together with G_{10} and $G_{\overline{10}}$ (solid lines) as functions of parameter ρ, where ρ is given by Eq. (12). Together with the aforementioned suppression of $G_{\overline{10}}$ we see strong suppression of F_{27} (corresponding to $27_{3/2} \to 10_{3/2} + \varphi$) and H_{27} (corresponding to $27_{1/2} \to 8_{1/2} + \varphi$) for the same range of ρ. Interestingly, both F_{27} and H_{27} vanish [37] in the nonrelativistic limit (22) exactly as $G_{\overline{10}}$. In our parametrization they cross zero for the parameter ρ in the range (25). Somewhat smaller suppression is seen for spin changing transitions G_{27} (corresponding to $27_{3/2} \to 8_{1/2} + \varphi$) and G'_{27} (corresponding to $27_{1/2} \to 10_{3/2} + \varphi$). Interestingly, in the nonrelativistic limit there is a partial cancellation in these couplings, namely, the leading N_c coefficients cancel out [37]. Finally, the remaining couplings E_{27} (corresponding to $27_{3/2} \to \overline{10}_{1/2} + \varphi$) and H'_{27} (corresponding to $27_{1/2} \to \overline{10}_{1/2} + \varphi$) are not suppressed (they are neither suppressed in the nonrelativistic limit). However, decays to antidecuplet have much smaller phase space, and they are totally switched off for $27_{3/2}$. It is remarkable that our simple phenomenological parametrization (6) and (12) respects—for the ρ values of interest (25)—the large N_c suppression in the nonrelativistic limit.

Averaging over spin and isospin, as described in the Appendix gives:

$$\bar{\mathcal{A}}^2(B_{27_{3/2}} \to B'_8 + \varphi) = \frac{4}{9}\begin{bmatrix} 8 & 8 & \vert & 27 \\ \varphi & B' & \vert & B \end{bmatrix}^2 \times G_{27}^2,$$
$$\bar{\mathcal{A}}^2(B_{27_{3/2}} \to B'_{10} + \varphi) = \frac{25}{72}\begin{bmatrix} 8 & 10 & \vert & 27 \\ \varphi & B' & \vert & B \end{bmatrix}^2 \times F_{27}^2,$$
$$\bar{\mathcal{A}}^2(B_{27_{3/2}} \to B'_{\overline{10}} + \varphi) = \frac{5}{3}\begin{bmatrix} 8 & \overline{10} & \vert & 27 \\ \varphi & B' & \vert & B \end{bmatrix}^2 \times E_{27}^2, \tag{30}$$

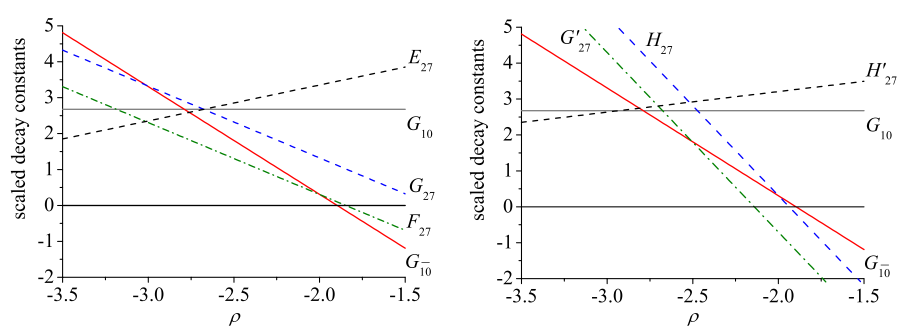

FIG. 2 (color online). Scaled coupling constants G_{10} and $G_{\overline{10}}$ together with couplings of $27_{3/2}$ (first panel) and $27_{1/2}$ (second panel) defined in Eqs. (27) and (29) as functions of parameter ρ, where ρ is given by Eq. (12).

TABLE V. Isoscalar factors squared for the decays of eikosiheptaplet.

$27 \to 8 + 8$	C^2	$27 \to 8 + 10$	C^2	$27 \to 8 + \overline{10}$	C^2
				$\Theta \to \pi + \Theta$	3/4
$\Theta \to K + N$	1	$\Theta \to K + \Delta$	1	$\Theta \to K + N$	1/4
$N \to \eta + N$	9/20			$N \to \eta + N$	9/80
$N \to \pi + N$	1/20	$N \to \pi + \Delta$	1/5	$N \to \pi + N$	49/80
$N \to K + \Sigma$	1/20	$N \to K + \Sigma$	4/5	$N \to K + \Sigma$	1/20
$N \to K + \Lambda$	9/20			$N \to \bar{K} + \Theta$	9/40
		$\Delta \to \eta + \Delta$	9/16		
$\Delta \to \pi + N$	1/2	$\Delta \to \pi + \Delta$	5/16	$\Delta \to \pi + N$	1/2
$\Delta \to K + \Sigma$	1/2	$\Delta \to K + \Sigma$	1/8	$\Delta \to K + \Sigma$	1/2

where the quantities in the square brackets denote SU(3) isoscalar factors. For $27_{1/2}$ we get

$$\bar{\mathcal{A}}^2(B_{27_{1/2}} \to B'_8 + \varphi) = \frac{2}{45}\left[\begin{array}{cc|c} 8 & 8 & 27 \\ \varphi & B' & B \end{array}\right]^2 \times H_{27}^2,$$
$$\bar{\mathcal{A}}^2(B_{27_{1/2}} \to B'_{10} + \varphi) = \frac{2}{9}\left[\begin{array}{cc|c} 8 & 10 & 27 \\ \varphi & B' & B \end{array}\right]^2 \times G_{27}^{\prime 2},$$
$$\bar{\mathcal{A}}^2(B_{27_{1/2}} \to B'_{\overline{10}} + \varphi) = \frac{49}{72}\left[\begin{array}{cc|c} 8 & \overline{10} & 27 \\ \varphi & B' & B \end{array}\right]^2 \times H_{27}^{\prime 2}. \tag{31}$$

The squares of the relevant SU(3) isoscalar factors are listed in Table V.

Now we are ready to calculate the decay widths for $27_{3/2}$. In fact, only decays to the octet baryons have nonvanishing widths, we list them in the Table VI (''~ 0'' denotes the decay width below 1 MeV, whereas "$-$" means that the decay is kinematically forbidden).

Decays of $27_{3/2}$ to decuplet are kinematically forbidden except for the decays $N_{27} \to \pi + \Delta$ and $\Delta_{27} \to \pi + \Delta$ which have widths smaller than 1 MeV. All decays to antidecuplet are kinematically forbidden. We can therefore conclude that eikosiheptaplet of spin 3/2 has widths small enough to justify the rigid rotor quantization. Not only are the widths numerically smaller than the one of Δ, but also in the large N_c limit with the partial nonrelativistic cancellation taking place, $\Gamma_{27_{3/2} \to 8+8} \to 0$ [37].

TABLE VI. Decay widths in MeV for the decays of $27_{3/2}$.

$27_{3/2} \to 8 + 8$		Γ [MeV]			Γ [MeV]	
ρ		ρ_1			ρ_2	
$\Sigma_{\pi N}$	42	55	73	42	55	73
$\Theta_{27} \to K + N$	29	33	39	16	18	21
$N_{27} \to \eta + N$	37	37	38	20	20	21
$N_{27} \to \pi + N$	17	17	17	9	9	9
$N_{27} \to K + \Sigma$	~ 0	~ 0	~ 0	~ 0	~ 0	~ 0
$N_{27} \to K + \Lambda$	10	10	10	5	5	6
$\Delta_{27} \to \pi + N$	172	152	128	94	83	70
$\Delta_{27} \to K + \Sigma$	2	—	—	1	—	—

Our results for Θ_{27} presented in Table VI are smaller than the estimate of Ref. [18]. Although the widths of the order of tens of MeV can be considered small, one has to remember that partial wave analysis requires Δ_{27} and Θ_{27} widths to be of the order of 100 keV [15].

We have concentrated here on the lightest states of eikosiheptaplet that have been looked for in PW analysis [15]. Obviously, we can easily calculate widths for the plethora of the remaining states of eikosiheptaplet. We have checked that for other states widths are smaller than the one of Δ_{27} quoted above. Assuming $\Sigma_{\pi N} = 73$ MeV, we get the following upper bounds for the partial widths of the next isospin multiplets

$$\Gamma_{\Lambda \to \eta + \Lambda} \sim 42\ \text{MeV}, \qquad \Gamma_{\Sigma \to \pi + \Lambda} \sim 75\ \text{MeV},$$
$$\Gamma_{\Xi_{1/2} \to \bar{K} + \Lambda} \sim 68\ \text{MeV}, \qquad \Gamma_{\Xi_{3/2} \to \pi + \Xi} \sim 74\ \text{MeV}. \tag{32}$$

For $27_{1/2}$ we expect larger widths because the available phase space is much larger. Interestingly, this is not the case for the decays to octet. The reason is that H_{27} responsible for these decays is strongly suppressed in the relevant range of ρ. Indeed, H_{27} crosses zero at $\rho = -1.937$ i.e. within the range (25). Moreover, the overall group theoretical factor in Eq. (31) is suppressed by factor of 13 with respect to the decays of antidecuplet (24). These two suppressions overcome the increase of the phase-space volume and the decay widths are comparable to those of $\overline{10}_{1/2}$. A similar effect takes place for the decays to decuplet (although the decay constant G'_{27} does not cross zero in the relevant range of ρ) and the decays are comparable to those of $27_{3/2} \to 8 + 8$. Numerical results are given in Tables VII and VIII.

Unfortunately, as can be seen from Table IX, there is no suppression for the decays of $27_{1/2}$ to antidecuplet. Indeed, the relevant coupling H'_{27} is as large as G_{10} (responsible for Δ decay)—see Fig. 2—and the phase space is also not suppressed: for $\Theta_{27} \to \pi + \Theta_{\overline{10}}$ the pion momentum is of the order of 300–400 MeV depending on $\Sigma_{\pi N}$. Hence the resulting widths are large.

Therefore one would be tempted to conclude that $27_{1/2}$ cannot be considered as a semi-stable multiplet and its description in terms of the rigid rotor fails, at least in the situations where the transitions $27_{1/2} \leftrightarrow \overline{10}$ are of importance. This statement is, however, not supported by the N_c counting [37]. We shall come back to this issue in the next section.

V. SUMMARY

In the present paper we have studied masses and decay widths of exotic baryon eikosiheptaplets (i.e. 27-plets) of spin 3/2 and 1/2 that follow from the chiral quark-soliton model in the rigid rotator quantization approach. We have also reexamined the widely studied by now antidecuplet that we use as an input that constrains model parameters. Rigid rotator quantization predicts a tower of stable exotic representations of different spins and positive parity, antidecuplet, eikosiheptaplet, $\overline{35}$ being most prominent examples. A question arises, where does the rigid rotator approach break? Leaving aside fundamental problems based on claims in the literature that the rigid rotator approach to exotica is not compatible with large N_c expansion for QCD [38], we have taken a more modest phenomenological approach. If the widths of the baryonic states calculated within the model exceed a certain critical value that can be taken to be above the Δ resonance width (one has to remember that Δ can be considered as a well behaved stable state in the large N_c limit), then the model becomes inconsistent. There are two sources that contribute to the increase of the width with the increase of the dimensionality of the SU(3) flavor representation. One is obvious: for higher representations the pertinent states are heavier and the phase space is larger. The second source is the coupling. For the antidecuplet there is only one coupling corresponding to the transition $\overline{10} \to 8$ that is excessively small due to the cancellation found in Ref. [3] and discussed in some detail in Sec. III. For higher representations there are more couplings corresponding to different transitions and some of them are not suppressed. For eikosiheptaplet couplings to the antidecuplet are not suppressed. Obviously if the phase space is large and the coupling is not suppressed then the widths are large. In other cases one has to perform explicit calculations to see what is the interplay between the rising phase space and small coupling.

We have addressed this question by applying the so-called *model-independent approach* [4] in which the general group theoretical structure is taken from the model, while the parameters are fitted to appropriate data. We have used as an input nonexotic masses and the mass of Θ^+, semileptonic decay constants, and the assumption that $\Gamma_{\Theta^+} < 1$ MeV. The residual freedom was parametrized by the value of the pion-nucleon sigma term. We have confined our analysis to eikosiheptaplet (i.e. 27-plet) that is the only exotic representation (apart from antidecuplet) appearing in the direct product of two octets. For this reason eikosiheptaplet might be produced in meson-nucleus scattering and could subsequently decay to meson-nucleon or meson-hyperon final states.

Our findings can be shortly summarized as follows. Based on group theory alone, eikosiheptaplet can decay into octet, decuplet, and exotic antidecuplet. However, for $27_{3/2}$ a regular octet is kinematically the only allowed channel (with two exceptions discussed in Sec. IV). Furthermore, the transition $27 \to 8$ is governed by a small decay coupling, G_{27}. Therefore eikosiheptaplet of spin 3/2 has widths of the order of a few tens MeV with one exception, namely Δ_{27} for which $\Gamma \sim 70$–170 MeV. For spin 1/2 the situation is different. Decays to the octet and decuplet have small transition couplings and the resulting widths are small (see Tables VII and VIII). For the decays to the antidecuplet, the coupling is large. Therefore, whenever the decay is possible the widths are of the order of 500 MeV. This might be interpreted as the signal that the model breaks down and that the assumption that $27_{1/2}$ is stable cannot be justified phenomenologically.

TABLE VII. Decay widths in MeV for the decays of $27_{1/2}$ to octet.

$27_{1/2} \to 8 + 8$	Γ [MeV]			Γ [MeV]		
ρ		ρ_1			ρ_2	
$\Sigma_{\pi N}$	42	55	73	42	55	73
$\Theta_{27} \to K + N$	0.97	0.87	0.73	8.14	7.26	6.12
$N_{27} \to \eta + N$	0.69	0.56	0.40	5.83	4.70	3.36
$N_{27} \to \pi + N$	0.16	0.13	0.11	1.34	1.14	0.89
$N_{27} \to K + \Sigma$	0.04	0.03	0.02	0.32	0.24	0.14
$N_{27} \to K + \Lambda$	0.44	0.34	0.22	3.68	2.85	1.87
$\Delta_{27} \to \pi + N$	1.80	1.62	1.34	15.12	13.60	11.66
$\Delta_{27} \to K + \Sigma$	0.46	0.39	0.30	3.89	3.26	2.48

The situation is, however, more complicated. Since the mass difference

$$\Delta_{27_{1/2}-\overline{10}} = \frac{1}{I_2} \sim \mathcal{O}(1/N_c) \tag{33}$$

as calculated from Eq. (14) is suppressed in the large N_c limit, so is the meson momentum (4). Therefore the widths that depend on the third power of momentum may be suppressed in the large N_c limit despite the fact that they are numerically large. That this is indeed the case was shown in Ref. [37] where e.g.

$$\Gamma_{\Theta_{27}\to\pi+\Theta_{\overline{10}}} \sim \mathcal{O}(1/N_c^2). \tag{34}$$

On the contrary, for the transitions of $27_{1/2}$ to octet which are numerically suppressed [remember that the pertinent coupling H_{27} (29) vanishes in the NR limit (22)] the N_c scaling is different, e.g. [37]:

TABLE VIII. Decay widths in MeV for the decays of $27_{1/2}$ to decuplet.

$27_{1/2} \to 8 + 10$	Γ [MeV]			Γ [MeV]		
ρ		ρ_1			ρ_2	
$\Sigma_{\pi N}$	42	55	73	42	55	73
$\Theta_{27} \to K + \Delta$	21	17	12	86	69	48
$N_{27} \to \pi + \Delta$	24	19	14	97	78	56
$N_{27} \to K + \Sigma^*$	18	11	4	75	45	16
$\Delta_{27} \to \eta + \Delta$	30	24	18	126	103	74
$\Delta_{27} \to \pi + \Delta$	43	37	31	178	155	127
$\Delta_{27} \to K + \Sigma^*$	4	3	2	16	12	8

TABLE IX. Decay widths in MeV for the decays of $27_{1/2}$ to antidecuplet.

$27_{1/2} \to 8 + 10$		Γ [MeV]			Γ [MeV]	
ρ		ρ_1			ρ_2	
$\Sigma_{\pi N}$	42	55	73	42	55	73
$\Theta_{27} \to \pi + \Theta_{\overline{10}}$	658	523	365	697	554	387
$\Theta_{27} \to K + N_{\overline{10}}$	—	—	—	—	—	—
$N_{27} \to \eta + N_{\overline{10}}$	—	—	—	—	—	—
$N_{27} \to \pi + N_{\overline{10}}$	500	364	215	530	385	228
$N_{27} \to K + \Sigma_{\overline{10}}$	—	—	—	—	—	—
$N_{27} \to \bar{K} + \Theta_{\overline{10}}$	72	20	—	76	21	—
$\Delta_{27} \to \pi + N_{\overline{10}}$	579	510	424	614	541	449
$\Delta_{27} \to K + \Sigma_{\overline{10}}$	—	—	—	—	—	—

$$\Gamma_{\Theta_{27} \to K+N} \sim \mathcal{O}(1). \tag{35}$$

Obviously numerical results presented in Sec. IV depend on the choice of input parameters. We have studied this sensitivity by employing another set of parameters (13) that corresponds to more realistic $g_A^{(0)}$. The decay widths of antidecuplet do not change, since we require that $\Gamma_\Theta < 1$ (which is equivalent to a slightly different range of the parameter ρ: $\rho_1 = -1.933 < \rho < \rho_2 = -1.767$) and this condition fixes all remaining decay widths. For eikosiheptaplet some differences appear. For the transitions of $27_{3/2}$ to octet, the decay widths for fit 2 (13) are smaller by a few MeV. More drastic changes appear for $27_{1/2}$. The reason is that the small change in the coupling is magnified by a large phase-space factor. Indeed, for the decays to octet, presented in Table VII, the decay widths for fit 2 are larger by a factor 10–6 (first number refers to $\rho = \rho_1$ whereas the second one to $\rho = \rho_2$ for fit 2). Although this enhancement seems large, the absolute values are still small on a typical hadronic scale. Less drastic enhancement occurs for the decays to decuplet presented in Table VIII, the decay widths for fit 2 are larger by a factor 2–1.4. Finally, large decay widths to antidecuplet remain almost the same as for the fit 1 (12). We see therefore that despite some numerical uncertainties due to the choice of input parameters the general pattern persists and our conclusions still hold.

Summarizing: there is no simple way to judge the quality of the rigid rotator approach to the eikosiheptaplet. On the basis of phenomenology alone one would conclude that $27_{1/2}$ is unstable because of the large numerical values of the decay widths to the antidecuplet. On the other hand precisely these decays are damped in the large N_c limit, similarly to the decays of Δ resonance. Other decays, like the decays to octet scale as $\mathcal{O}(N_c^0)$, have numerical values that are small due to the coupling suppression and additional group theoretical factors. For eikosiheptaplet of spin $3/2$ all kinematically allowed decays have widths small enough to justify the rigid rotor quantization. Not only are the widths numerically smaller than the one of Δ, but also in the large N_c limit partial nonrelativistic cancellation takes place and the pertinent couplings are suppressed.

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APPENDIX A: SUMMING OVER SPINS AND ISOSPINS

We shall use the identity

$$\frac{1}{2S+1}\sum_{S_3',S_3}\left(\begin{array}{cc|c} 1 & S' & S \\ m' & S_3' & S_3 \end{array}\right)\left(\begin{array}{cc|c} 1 & S' & S \\ m' & S_3' & S_3 \end{array}\right) = \frac{1}{3}\delta_{mm'} \tag{A1}$$

to average over the initial spin (and in the same time to sum over the final spin). For spin $3/2$ the amplitude for $10_{3/2} \to 8_{1/2} + \varphi$ and $27_{3/2} \to 8_{1/2}, \overline{10}_{1/2} + \varphi$ is proportional to

$$\left(\begin{array}{cc|c} 1 & 1/2 & 3/2 \\ 0 & -1/2 & -1/2 \end{array}\right) = \sqrt{\frac{2}{3}} \tag{A2}$$

hence

$$\frac{1}{2S+1}\sum_{S_3}|\mathcal{A}(3/2 \to 1/2)|^2 = \frac{1}{2}|\mathcal{A}(3/2 \to 1/2)|^2. \tag{A3}$$

For $27_{3/2} \to 10_{3/2} + \varphi$ the amplitude is proportional to

$$\left(\begin{array}{cc|c} 1 & 3/2 & 3/2 \\ 0 & -1/2 & -1/2 \end{array}\right) = \sqrt{\frac{1}{15}} \tag{A4}$$

and

$$\frac{1}{2S+1}\sum_{S_3}|\mathcal{A}(3/2 \to 3/2)|^2 = 5|\mathcal{A}(3/2 \to 3/2|^2. \tag{A5}$$

Finally, for $\overline{10}_{1/2} \to 8_{1/2} + \varphi$ and $27_{1/2} \to 8_{1/2}, \overline{10}_{1/2} + \varphi$ the amplitude is proportional to

$$\left(\begin{array}{cc|c} 1 & 1/2 & 1/2 \\ 0 & -1/2 & -1/2 \end{array}\right) = \sqrt{\frac{1}{3}} \tag{A6}$$

and for $27_{1/2} \to 10_{3/2} + \varphi$ to

$$\left(\begin{matrix} 1 & 3/2 \\ 0 & -1/2 \end{matrix}\middle|\begin{matrix} 1/2 \\ -1/2 \end{matrix}\right) = -\sqrt{\frac{1}{3}}. \tag{A7}$$

Hence

$$\frac{1}{2S+1}\sum_{S_3}|\mathcal{A}(1/2 \to 1/2, 3/2)|^2 = |\mathcal{A}(1/2 \to 1/2, 3/2)|^2. \tag{A8}$$

Similarly we shall average over initial isospin and sum over the final isospin using the formula

$$\frac{1}{2I+1}\sum_{I_{\varphi 3}, I_3', I_3}\left(\begin{matrix} I_\varphi & I' \\ I_{\varphi 3} & I_3' \end{matrix}\middle|\begin{matrix} I \\ I_3 \end{matrix}\right)^2 = 1. \tag{A9}$$

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