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Higgs physics as a window beyond the MSSM

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We interpret the current experimental limit on the lightest Higgs boson mass to suggest that, if nature is supersymmetric, there are additional interactions beyond those of the minimal supersymmetric standard model (MSSM) coming from new degrees of freedom around the TeV scale. Within an effective field theory analysis, the leading order corrections to the MSSM are described in terms of only two operators. This provides a highly constrained description of beyond MSSM physics. The scalar Higgs spectrum as well as the chargino and neutralino spectra and couplings are modified in a distinctive way. These operators can be generated by a variety of microscopic mechanisms.

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I. INTRODUCTION

The CERN LEPII collider placed the problem of Higgs physics at the forefront of supersymmetry phenomenology. While earlier one might have viewed the Higgs fields as just one of the many features of low energy supersymmetric models, the constraints on the Higgs mass are now problematic. Within the minimal supersymmetric standard model (MSSM)[1] the Higgs sector occupies a special place. Unlike the spectrum of squarks, sleptons, and gauginos, which are determined by many parameters, the Higgs spectrum is quite constrained. There are only two, currently unknown, tree-level parameters, and plausible assumptions about fine-tuning restrict these significantly. At the classical (and renormalizable) level, the mass of the lightest Higgs is lighter than the mass of the Z boson. There are significant corrections to the mass arising from loops of top quarks and squarks. But in order that these effects, by themselves, account for the Higgs mass, the top squarks must be quite massive—so massive that the theory appears finely tuned—or the top squarks must be highly mixed. This suggests that *if* low energy supersymmetry is important to the solution of the hierarchy problem, there are likely to be additional degrees of freedom in the theory beyond those of the MSSM.

There are a number of well-studied candidates for such additional physics (beyond the MSSM, or BMSSM physics) [1–5]. The addition of a chiral singlet field gives the next to minimal supersymmetric standard model (NMSSM). One can contemplate structures with more chiral fields, gauge interactions, and strongly interacting gauge theories. This additional physics cannot be too far away; it could well be at scales near the masses of the Higgs particles. However, this dynamics might lie at somewhat higher scales (hierarchies of supersymmetry breaking scales are familiar, for example, within gauge-mediated models). If this new dynamics lies at an energy scale, M, above the typical masses of the MSSM fields, one can organize the analysis by studying an effective Lagrangian from which physics at scale M has been integrated out. The effective action analysis turns out to be *extremely* simple, and will allow us to delineate both phenomenologically interesting regions of parameter space, as well as robust consequences of such extra dynamics for the Higgs spectrum. An effective action analysis of the MSSM Higgs sector has also been considered in earlier work [6].

The power of the effective action follows from the organization of operators in increasing powers of $1/M$. For a given observable, only a small number of operators, with the smallest power of $1/M$, contribute. Normally, the power of $1/M$ follows from the dimension of the operator. However, in theories with approximate symmetries, the notion that the power of $1/M$ can be larger is familiar; chirality violating dipole moment operators are perhaps the most common example. More generally, for any operator $\int d^4x(a/M^n)\mathcal{O}$ (for some constant a), the *effective scaling dimension* with respect to the scale M is $D_{\rm eff}[\mathcal{O}] = 4 + n$. The same phenomenon arises in approximately supersymmetric theories. Because supersymmetry restricts the structure of the operators in the effective theory, the effective dimension can be larger than the naive dimension. In particular, we will see below various dimension four operators, which are suppressed by $1/M$ (or $1/M^2$)—hence their effective dimension is five (or six).

This can happen starting with a dimension five operator which is suppressed by $1/M$, and integrating out an auxiliary field proportional to a scale, μ of order the electroweak scale (such as the familiar μ parameter of the MSSM). In this case the dimensionless coefficient of the dimension four operator is

$$|\mu|/M \ll 1. \tag{1}$$

Supersymmetry-breaking operators also generate such corrections. The MSSM is a renormalizable theory which

[1]The phrase MSSM is used here to refer to the particle content of the model only, not to any features of the Lagrangian. Similar statements apply later to what is referred to as the NMSSM.

includes soft supersymmetry-breaking terms. We can add to this theory hard supersymmetry-breaking terms provided they are suppressed by appropriate powers of $1/M$. Again, such hard supersymmetry-breaking operators have effective dimension larger than their naive dimension. Their coefficients are suppressed by

$$m_{\rm SUSY}/M \ll 1 \tag{2}$$

where $m_{\rm SUSY}$ is a characteristic scale of the supersymmetry-breaking terms in the new physics sector. We will see several examples of this phenomenon in what follows.

We will take $m_{\rm SUSY} \sim \mu$ of order a few hundred GeV and will contemplate M of order a few TeV. Then we can organize our computations in a power series in

$$\epsilon \sim \frac{|\mu|}{M} \sim \frac{m_{\rm SUSY}}{M} \ll 1. \tag{3}$$

At leading order in ϵ, we will see that there are only two operators [7]. In terms of the coefficients of these operators, one can write very simple formulas for the full spectrum. The operators respect different symmetries, with important consequences for the qualitative, as well as detailed quantitative, features of the spectrum.

It is necessary, in the analysis, to distinguish different regions of

$$\eta \equiv \cot\beta. \tag{4}$$

For moderate η, the operators at order ϵ describe the dominant contribution of BMSSM physics to the Higgs spectrum. Indeed, the MSSM particle content with these two operators may be defined as the simplest BMSSM. For sufficiently small η, we have a double expansion in η and in ϵ. Depending on the relative size of ϵ and η, a different subset of terms of order η^2, $\eta\epsilon$, and ϵ^2 are the dominant ones.

These operators can have important effects on the phenomenology of the Higgs sector. Interestingly, the lowest order operators are not at present bounded by, for example, precision electroweak measurements. For quite plausible values of couplings and scales, they can raise the mass of the lightest Higgs particle appreciably, and alter relations among the Higgs masses. Features of the Higgs sector can thus serve as probes for surprisingly high energy phenomena.

In the effective Lagrangian analysis, there are two types of questions one can naturally ask about the Higgs sector: what measurements would establish that additional degrees of freedom beyond those of the MSSM are present, and how can one characterize deviations from MSSM predictions. As we will explain in the next section, it is possible that the MSSM cannot explain the value of the lightest Higgs mass. Measurement of the stop masses could in fact rule out the MSSM; unless the stops are sufficiently heavy or very highly mixed, the Higgs mass computed within the minimal model could simply be too small. Similarly, as we will discuss, the MSSM leads to certain relations among the various Higgs masses. If these relations are not experimentally satisfied, the MSSM will have to be extended to the BMSSM. In this case, precision measurements of the Higgs spectrum could thus determine the coefficients of the new operators.

In the next section we review some well-known facts about the Higgs sector in the MSSM. In Sec. III we introduce the general operator analysis, exhibit the small set of operators at low dimension which can contribute to the Higgs potential, and discuss their properties. We consider their effects on the various Higgs masses and how measurements of the spectrum can establish whether or not additional interactions beyond those of the MSSM are required. In Sec. IV we turn to more microscopic models, discussing how these operators arise in the NMSSM, models with triplets of chiral fields, and theories with extended gauge interactions, establishing plausible values of the coefficients. In these models, we discuss ranges of parameters for which the operator analysis is applicable. In the conclusions, we discuss the prospects for connections with experiment, mentioning the little hierarchy problem and possible scales of new physics. Finally, in the Appendix the (B)MSSM is compared with the most general two Higgs doublet model, and various symmetries which are present for certain values of the parameters are discussed.

II. REVIEW OF THE MSSM HIGGS SECTOR

A. Tree level

The tree-level MSSM Higgs potential with fields H_u and H_d receives contributions from the Higgs soft masses, the superpotential Higgs mass μ-term $\int d^2\theta \mu H_u H_d$, and $SU(2)_L \times U(1)_Y$ D-term quartic interactions,

$$\begin{aligned} V = {} & \tilde{m}^2_{H_u} H_u^\dagger H_u + \tilde{m}^2_{H_d} H_d^\dagger H_d - (m^2_{ud} H_u H_d + \text{H.c.}) \\ & + \frac{g^2}{8}[(H_u^\dagger H_u + H_d^\dagger H_d)^2 - 4(H_u H_d)^\dagger (H_u H_d)] \\ & + \frac{g'^2}{8}(H_u^\dagger H_u - H_d^\dagger H_d)^2 \end{aligned} \tag{5}$$

where

$$\tilde{m}^2_{H_u} = |\mu|^2 + m^2_{H_u} \qquad \tilde{m}^2_{H_d} = |\mu|^2 + m^2_{H_d} \tag{6}$$

and $m^2_{H_u}$ and $m^2_{H_d}$ are the Higgs soft masses. Without loss of generality we can take the soft parameter m^2_{ud} to be real. It follows that the tree-level MSSM Higgs potential is CP conserving (even though the full MSSM Lagrangian violates CP). The massive Higgs particles are eigenstates of this approximate CP. The light Higgs h and the heavier Higgs H are CP even, and the Higgs A is CP odd. In addition there is a massive charged Higgs $H^\pm$.

The tree-level potential depends on the known $SU(2)_L \times U(1)_Y$ gauge couplings g and g' and three unknown real mass squared parameters, $\tilde{m}^2_{H_u}$, $\tilde{m}^2_{H_d}$, and m^2_{ud}. It is convenient to parametrize the observables, not in terms of these three real parameters, but in terms of three other quantities. Two of them are

$$v_u = |\langle H_u \rangle| = v \sin\beta, \qquad v_d = |\langle H_d \rangle| = v \cos\beta, \quad (7)$$

and the third is the physical mass of the CP odd Higgs, m_A. Since the expectation value v is known, this leaves m_A and $\tan\beta$ as the two unknown parameters describing the MSSM Higgs sector. Also, instead of using the gauge couplings g and g', we will write expressions in terms of the gauge boson masses, $m_Z^2 = \frac{1}{2}(g^2 + g'^2)v^2$, $m_W^2 = \frac{1}{2}g^2 v^2$.

Then, a straightforward computation leads to the masses

$$m^2_{h,H} = \frac{1}{2}\left[m_Z^2 + m_A^2 \mp \sqrt{(m_A^2 - m_Z^2)^2 + 4m_A^2 m_Z^2 \sin^2 2\beta}\right],$$
$$m^2_{H^\pm} = m_A^2 + m_W^2. \quad (8)$$

We note that for $g = 0$ the expression (8) becomes $m_{H^\pm} = m_A$. In this case the Higgs potential has a custodial $SU(2)_C$ symmetry under which $H^\pm$ and A form a triplet, as discussed in the Appendix [8]. While of little importance in the MSSM, the further breaking of this symmetry in the presence of new physics is distinctly more interesting.

It is worth noticing that the last expression in (8), as well as

$$m_h^2 + m_H^2 = m_Z^2 + m_A^2, \quad (9)$$

is independent of $\tan\beta$ and therefore provides interesting one parameter tests of the MSSM (with of course proper quantum corrections taken into account).

It follows from (8) that $m_h \leq m_Z$, which is incompatible with the LEPII bound. Although radiative corrections and various operator corrections to the Higgs potential discussed below can resolve this contradiction, this problem is less severe when this tree-level inequality is nearly saturated. This is the case for large $\tan\beta$. Therefore, we will expand (8) in powers of $\eta \equiv \cot\beta$.

In treating η as small, we should decide what to hold fixed. In the standard decoupling limit, $m_A \to \infty$. We prefer, instead, to take large $\tan\beta$ with fixed m_A. This limit is more physical because the spectrum remains finite; indeed, if we wish to avoid large, highly tuned hierarchies, we do not expect m_A to be vastly different than the W and Z boson masses. Also, in this limit the parameters in (5) remain finite while m^2_{ud} is taken to zero. In this limit the potential (5) acquires a $U(1)_{PQ}$ symmetry, so the limit $\eta \to 0$ becomes technically natural. A straightforward calculation leads to

$$m_h^2 \simeq m_Z^2 - \frac{4m_Z^2 m_A^2}{m_A^2 - m_Z^2}\eta^2 + \mathcal{O}(\eta^4),$$
$$m_H^2 \simeq m_A^2 + \frac{4m_Z^2 m_A^2}{m_A^2 - m_Z^2}\eta^2 + \mathcal{O}(\eta^4), \quad (10)$$
$$m^2_{H^\pm} = m_A^2 + m_W^2.$$

Note that H and A become degenerate in the large $\tan\beta$ limit. They form a complex boson which carries $U(1)_{PQ}$ charge. Also, the expression for $m_{H^\pm}$, which is independent of η, is not corrected.

B. Radiative corrections

Radiative corrections are known to significantly change the spectrum [9,10]. They are most important for the light Higgs h, for two reasons. First, the tree-level mass is small (because it is proportional to the small gauge couplings). Second, the loop corrections are proportional to four powers of the top Yukawa coupling. These radiative corrections can avoid a contradiction with the LEPII bound. The most important effect is that of virtual loops of tops and stops. The leading one-loop top and stop corrections to m_h^2 in either the large $\tan\beta$ or Higgs decoupling limits including stop mixing effects with arbitrary phases are [11,12]

$$\delta_{1-\mathrm{loop}} m_h^2 \simeq \frac{12}{16\pi^2}\frac{m_t^4}{v^2}\left[\ln\left(\frac{m_{\tilde{t}_1} m_{\tilde{t}_2}}{m_t^2}\right) + \frac{|X_t|^2}{m^2_{\tilde{t}_1} - m^2_{\tilde{t}_2}}\ln\left(\frac{m^2_{\tilde{t}_1}}{m^2_{\tilde{t}_2}}\right) + \frac{1}{2}\left(\frac{|X_t|^2}{m^2_{\tilde{t}_1} - m^2_{\tilde{t}_2}}\right)^2\left(2 - \frac{m^2_{\tilde{t}_1} + m^2_{\tilde{t}_2}}{m^2_{\tilde{t}_1} - m^2_{\tilde{t}_2}}\ln\left(\frac{m^2_{\tilde{t}_1}}{m^2_{\tilde{t}_2}}\right)\right)\right] \quad (11)$$

where $v = (2^{3/2}G_F)^{-1/2} \simeq 174$ GeV, $X_t = A_t - \mu^*\eta$ is the stop mixing parameter, and $m_{\tilde{t}_{1,2}}$ are the stop physical masses (in the limit of large $\tan\beta = 1/\eta$ with m_A held fixed, X_t may be replaced by A_t).

The current bound from LEPII on the mass of a standard-model–like Higgs boson of $m_{h_{SM}} \gtrsim 114$ GeV [13] can be accommodated within the MSSM in certain regions of parameter space. First, the tree-level contribution is maximized at moderate to large $\tan\beta$. For $\tan\beta \gtrsim 10$ the tree-level contribution is already within 2 GeV of its maximum value of m_Z. Moreover, the top and stop loop corrections can be substantial. If stop mixing is small, $|X_t/m_{\tilde{t}_{1,2}}|^2 \ll 1$, the correction depends only on the logarithm of the stop masses, and the stops must be rather heavy in this small mixing limit. Including the full set of two-loop corrections with typical parameters for the remaining MSSM parameters, the LEPII Higgs mass bound requires that $m_{\tilde{t}_{1,2}} \gtrsim 1000$ GeV in the small stop mixing limit [14,15]. However, from (11), one sees that if stop mixing is large, the stop loop correction can be sizable, allowing much lighter stops [14,16,17]. At one loop the stop mixing contribution is maximized for $|X_t/m_{\tilde{t}}|^2 \simeq 6$ where $m_{\tilde{t}} = \frac{1}{2}(m_{\tilde{t}_1} + m_{\tilde{t}_2})$. Including the full set of two-loop corrections with the maximum correction from stop mixing, the LEPII Higgs mass bound is consistent with $m_{\tilde{t}_1} \gtrsim 100$ GeV [14,15], as required by the LEPII direct stop search. While such large mixing is not inconceivable, it is typically hard to obtain in specific mediation schemes, and it generally arises at low energies under renormalization group evolution only from rather special points in parameter space [15].

III. THE HIGGS SECTOR WINDOW TO NEW PHYSICS

A. Operator analysis

The effects of new physics at a sufficiently large mass scale, M, can be encapsulated in new operators. The magnitude of the interactions arising from these operators, in turn, should be organized in inverse powers of the heavy mass scale M. As we explained in the Introduction, the power of $1/M$ is related to the *effective dimension* of the operator.

In the supersymmetric limit the leading interactions arise from a single operator presented below, which is suppressed by a single power of $1/M$. In terms of component fields, this operator generates both renormalizable and nonrenormalizable interactions. The leading component operator is of dimension four but its coefficient is suppressed by the dimensionless combination

$$\epsilon \sim \frac{|\mu|}{M} \tag{12}$$

where μ is the coefficient of the term $H_u H_d$ in the superpotential. Therefore, this operator has effective dimension five.

For operators which include supersymmetry breaking, an important consideration is the scale of supersymmetry breaking, $m_{\rm SUSY}$, within the new physics sector. We will assume

$$m_{\rm SUSY} \sim |\mu| \tag{13}$$

in this section. Since $m_{\rm SUSY} \ll M$, the effects of supersymmetry breaking may be described through supersymmetric operators which contain spurions with supersymmetry-breaking auxiliary component expectation values. The leading interactions which include supersymmetry breaking arise from a single operator presented below, suppressed by one power of

$$\epsilon \sim \frac{m_{\rm SUSY}}{M}. \tag{14}$$

In terms of component fields, this operator generates a renormalizable interaction, but is suppressed by one power of $1/M$, so it is of effective dimension five. It is important to stress that the explicit supersymmetry breaking due to these operators is not soft. These operators are of dimension four and the breaking is hard in the low energy effective theory. Yet, this is perfectly consistent because their coefficients are suppressed by powers of M.

Let us be more explicit, starting with the supersymmetric operators. Consider first superpotential interactions. The most general superpotential for the MSSM Higgs sector up to dimension five is

$$\int d^2\theta\left(\mu H_u H_d + \frac{\lambda}{M}(H_u H_d)^2\right). \tag{15}$$

After eliminating Higgs auxiliary fields with the leading order canonical Kahler potential, to leading order in μ/M this gives a correction to the renormalizable MSSM Higgs potential,

$$\delta_1 V = 2\epsilon_1 H_u H_d (H_u^\dagger H_u + H_d^\dagger H_d) + \text{H.c.} \tag{16}$$

where

$$\epsilon_1 \equiv \frac{\mu^* \lambda}{M}. \tag{17}$$

The superpotential (15) also gives a dimension five correction to the Higgs-Higgsino Lagrangian interaction:

$$\frac{\epsilon_1}{\mu^*}[2(H_u H_d)(\tilde{H}_u \tilde{H}_d) + 2(\tilde{H}_u H_d)(H_u \tilde{H}_d) + (H_u \tilde{H}_d)(H_u \tilde{H}_d) + (\tilde{H}_u H_d)(\tilde{H}_u H_d)] + \text{H.c.}, \tag{18}$$

where here round parentheses indicate the $SU(2)_L$ singlet. Substituting the scalar Higgs expectation values in this expression, the neutralino and chargino masses are corrected at order ϵ_1.

We should also consider Kahler potential interactions. Gauge invariance requires that all such operators which involve only Higgs fields are functions of an even number of fields. The lowest dimension nontrivial Kahler potential operators beyond the leading kinetic terms are therefore dimension six with four Higgs fields and are suppressed by $1/M^2$. There are several such operators and their effects are typically smaller than those of (16) and (18), which are suppressed by one power of $1/M$. We will discuss some of them below.

The second class of operators involves supersymmetry breaking. These effects may be included by considering superpotential and Kahler potential operators with additional powers of spurion superfields with auxiliary expectation values. For definiteness we consider here the phenomenologically interesting case of an F-term auxiliary expectation value. This may be represented by a dimensionless chiral superfield spurion

$$\mathcal{Z} = \theta^2 m_{\rm SUSY}, \tag{19}$$

where $m_{\rm SUSY}$ is the supersymmetry-breaking scale. Superpotential operators give nonvanishing interactions only with a single power of the spurion $\mathcal{Z}$. The dimension five operator similar to that of (15), but with a spurion field

$$\int d^2\theta \mathcal{Z} \frac{\lambda}{M}(H_u H_d)^2, \tag{20}$$

gives a holomorphic correction to the renormalizable MSSM Higgs potential:

$$\delta_2 V = \epsilon_2 (H_u H_d)^2 + \text{H.c.} \tag{21}$$

where

$$\epsilon_2 = -\frac{m_{\rm SUSY}\lambda}{M}. \tag{22}$$

This correction is also of dimension four, but effective dimension five.

Kahler potential operators give nonvanishing interactions with either one or two powers of the spurion $\mathcal{Z}$. To zeroth order in $1/M$ (as in the MSSM) only the Kahler potential kinetic terms can contribute with one or two powers of the spurion field. As is well known, the terms with one spurion like[2]

$$\int d^4\theta(\mathcal{Z} + \mathcal{Z}^\dagger)X^\dagger e^V X, \tag{23}$$

where X is any of the MSSM fields, can be removed by making holomorphic field redefinitions

$$X \to (1 - \mathcal{Z})X. \tag{24}$$

This restores the canonical kinetic terms for X and generates various operators which are already included in the MSSM Lagrangian and the operator (20). Therefore, the operator (23) is not an independent operator. Terms which are second order in the spurion

$$\int d^4\theta \mathcal{Z}\mathcal{Z}^\dagger X X^\dagger \tag{25}$$

lead to standard soft masses.

Proceeding to first order in $1/M$, i.e. effective dimension five, we easily see that there are no terms which involve only the Higgs fields. Allowing for couplings to other fields, there are operators such as[3]

$$\int d^4\theta \frac{1}{M}(1 + \mathcal{Z} + \mathcal{Z}^\dagger + \mathcal{Z}\mathcal{Z}^\dagger)(H_d^\dagger Q u^c + H_u^\dagger Q d^c + H_u^\dagger L e^c). \tag{26}$$

Here we have suppressed order one coefficients, so even with a single generation, there are 12 independent operators; allowing for generation indices on the quark and lepton doublets Q and L, the anti-up quarks u^c, the anti-down quarks d^c, and the antileptons e^c, there are many independent operators. Such operators are potentially dangerous because they might lead to flavor-changing neutral-current interactions (FCNC). We will assume that the short distance physics is such that, if these operators are generated, their coefficients are sufficiently small. It should be stressed, however, that this assumption is not that strong, because a similar assumption is standard already in the zeroth order terms (25).

In addition there are various dimension five superpotential operators which violate baryon and lepton number [18]. We do expect such operators to be present and to lead to neutrino masses and proton decay. But we expect their scale to be much larger than the scale M we contemplate here, which we assume to be in the TeV range.

The Higgs interactions (16) and (21) and Higgs-Higgsino interactions (18) exhaust all the MSSM Higgs sector effective dimension five operators that arise either in the supersymmetric limit or from chiral superfield spurions. None of these component interactions arise at tree level for the renormalizable MSSM Higgs sector with general soft supersymmetry breaking. All these interactions violate the continuous $U(1)_{PQ}$ symmetry under which both the H_u and H_d superfields have unit charge. In the MSSM this symmetry is, however, already broken by the superpotential Higgs mass μ-term. So the requisite breaking of $U(1)_{PQ}$ by the new physics sector need not necessarily lead to any additional suppression of the effective dimension five interactions. This can be true even in the case that m_{ud}^2 in the tree-level potential (5) is small since this term breaks different discrete symmetries than the μ-term or effective dimension five operators.

There are a large number of interactions with MSSM particle content at effective dimension six. A subset of these have the property that their component expansion involves terms which depend only on H_u. These play an important role in the discussion in the next subsection of the leading effects which can modify the light Higgs boson mass for sufficiently small η, since their effects are independent of η in this limit. Effective dimension six component interactions which involve only the scalars in H_u come entirely from the dimension six Kahler potential operators

$$\int d^4\theta \frac{1}{M^2}[\xi_1(H_uH_d)^\dagger(H_uH_d) + \xi_2(H_u e^V H_u^\dagger)(H_d e^V H_d^\dagger) + \xi_3(H_u^\dagger e^V H_u)^2 + (\xi_4 \mathcal{Z}(H_uH_d)^\dagger(H_u^\dagger e^V H_u) + \text{H.c.}) + \xi_5 \mathcal{Z}^\dagger \mathcal{Z}(H_u^\dagger e^V H_u)^2]. \tag{27}$$

Using $F_{H_d} \simeq -(\mu H_u)^\dagger$ these operators modify the scalar potential for H_u by operators of dimension four and six, and also modify its kinetic term.

It is important to stress again that the effective operator analysis given above for the MSSM Higgs sector differs significantly from that of a nonsupersymmetric theory with the same field content. Here the analysis is best organized in terms of the effective dimension of component interactions. In contrast, the analysis for a generic nonsupersymmetric theory is most naturally organized in terms of the operator dimension of the interactions.

The effective operator analysis presented above, including the effects of supersymmetry breaking by F-term auxiliary expectation values through the spurion $\mathcal{Z}$, may be extended to include D-terms. In most microscopic models of supersymmetry breaking such auxiliary fields

[2]Here and in the rest of this note e^V represents the exponential of the gauged superfield in the appropriate representation.

[3]We thank L. Randall for a useful discussion about this point.

are suppressed and unimportant, and therefore they are somewhat less familiar. However, for completeness it is interesting to classify the effects of such fields in the effective operator analysis.

A D-term expectation value resides in a gauge vector superfield. If MSSM fields carry the corresponding charge, the relevant spurion is a standard vector field of dimension zero and its contribution to the scalar masses generally requires a suppression of the D-term vacuum expectation values (vevs). If instead all the MSSM fields are uncharged under the associated gauge symmetry, we may use a spurion vector superfield of dimension -1, and the effects of D-term breaking are represented by a spinor chiral superfield strength spurion of scaling dimension $1/2$,

$$\mathcal{W}_\alpha = \xi_{\rm SUSY}\theta_\alpha \quad \text{with} \quad \xi_{\rm SUSY} \sim m_{\rm SUSY}. \tag{28}$$

The leading coupling of this spurion to the MSSM is the dimension three operator

$$M \int d^2\theta \mathcal{W}^\alpha W_\alpha^{(1)} = M\xi_{\rm SUSY} D^{(1)} \tag{29}$$

where $W_\alpha^{(1)}$ is the $U(1)_Y$ hypercharge spinor superfield strength and $D^{(1)}$ the hypercharge D-term. This is the standard supersymmetric Fayet-Iliopoulos term for $U(1)_Y$ hypercharge. Since for $\xi_{\rm SUSY} \sim m_{\rm SUSY}$ this term is too large, its coefficient must be suppressed. This is easy to arrange in specific microscopic models.

Up to redundancies due to the lowest order equations of motion and holomorphic field redefinitions, there are no dimension four couplings of the spurion (28) to MSSM fields. At dimension five there are two couplings which involve Higgs sector fields (again, we limit ourselves to baryon and lepton number conserving operators),

$$\begin{aligned}\frac{1}{M}\int d^2\theta \mathcal{W}^\alpha H_u H_d W_\alpha^{(1)} &= \frac{\xi_{\rm SUSY}}{M}\bigg[H_u H_d D^{(1)} + \frac{i}{\sqrt{2}} \\ &\quad \times (H_u\psi_{H_d} + \psi_{H_u}H_d)\lambda^{(1)}\bigg], \\ \frac{1}{M}\int d^2\theta \mathcal{W}^\alpha H_u H_d W_\alpha^{(2)} &= \frac{\xi_{\rm SUSY}}{M}\bigg[H_u H_d D^{(2)} + \frac{i}{\sqrt{2}} \\ &\quad \times (H_u\psi_{H_d} + \psi_{H_u}H_d)\lambda^{(2)}\bigg],\end{aligned} \tag{30}$$

where $W_\alpha^{(2)}$ is the $SU(2)_L$ spinor superfield strength. Using the lowest order values of the D-terms, the bosonic terms give corrections to the Higgs potential of the form $H_uH_d(H_u^\dagger H_u)$ and $H_uH_d(H_d^\dagger H_d)$. These are similar to the effective dimension five terms (16). However, unlike those, in (30) the relative coefficients of the two component interactions are not related. In addition, the Higgs potential correction (16) is related to the Higgs-Higgsino interaction (18), whereas in (30) there are no such component interactions. Instead, there are new dimension four but effective dimension five Higgs-Higgsino-gaugino interactions. Substituting the Higgs expectation values, these interactions correct the chargino and neutralino masses at order $1/M^2$.

B. The Higgs spectrum

We now consider the effect of the operators above on the Higgs masses. We add to the MSSM potential (5) the two terms $\delta_1 V$ (16) and $\delta_2 V$ (21). Again, we express the coefficients of the quadratic terms in terms of v, $\tan\beta$, and m_A and expand the answers to leading order in ϵ:

$$\delta_\epsilon m_h^2 = 2v^2\bigg(\epsilon_{2r} + 2\epsilon_{1r}\sin(2\beta) + \frac{2\epsilon_{1r}(m_A^2+m_Z^2)\sin(2\beta) - \epsilon_{2r}(m_A^2-m_Z^2)\cos^2(2\beta)}{\sqrt{(m_A^2-m_Z^2)^2 + 4m_A^2m_Z^2\sin^2(2\beta)}}\bigg) \simeq \frac{16m_A^2}{m_A^2-m_Z^2}v^2\eta\epsilon_{1r} + \mathcal{O}(\eta^2\epsilon), \tag{31}$$

$$\begin{aligned}\delta_\epsilon m_H^2 &= 2v^2\bigg(\epsilon_{2r} + 2\epsilon_{1r}\sin(2\beta) - \frac{2\epsilon_{1r}(m_A^2+m_Z^2)\sin(2\beta) - \epsilon_{2r}(m_A^2-m_Z^2)\cos^2(2\beta)}{\sqrt{(m_A^2-m_Z^2)^2 + 4m_A^2m_Z^2\sin^2(2\beta)}}\bigg) \\ &\simeq 4v^2\epsilon_{2r} - \frac{16m_Z^2}{m_A^2-m_Z^2}v^2\eta\epsilon_{1r} + \mathcal{O}(\eta^2\epsilon),\end{aligned} \tag{32}$$

$$\delta_\epsilon m_{H^\pm}^2 = 2\epsilon_{2r}v^2, \tag{33}$$

where ϵ_{1r} and ϵ_{2r} are the real parts of ϵ_1 and ϵ_2.

We would like to make some simple comments about these expressions:

(1) These expressions, at first order in ϵ, depend only on the real parts of $\epsilon_{1,2}$. This can be understood as follows. At the zeroth order in ϵ the problem is CP invariant. At first order in ϵ, the effect of $\text{Im}\,\epsilon_{1,2}$ is to mix CP even and odd states. But the spectrum is only affected in second order. Interactions (decays, dipole moments, and the like) will be affected at order ϵ.

(2) The expressions (31) and (32) also receive corrections of higher order in $\epsilon_{1,2}$ (from the potential

evaluated to first order in ϵ). For example,

$$\delta_{\epsilon^2} m_h^2 = -\frac{16\epsilon_{1r}^2 + 48\epsilon_{1i}^2 - 4\frac{m_Z^2}{m_A^2}\epsilon_{2i}^2}{m_A^2 - m_Z^2} v^4 + \mathcal{O}(\eta\epsilon^2), \quad (34)$$

where we see that, as we commented above, the order ϵ^2 corrections depend on $\mathrm{Im}\,\epsilon_1 = \epsilon_{1i}$ and $\mathrm{Im}\,\epsilon_2 = \epsilon_{2i}$. We should point out that with CP violation the limits $\epsilon \to 0$ and $\eta \to 0$ are singular and more care is needed in the expansions. Note that the operator (15) also leads to a sixth order term in the potential which is suppressed by ϵ_1^2. This term does not contribute to (34) because it includes at least two factors of H_d.

(3) Although the expressions for both (31) and (32) receive higher order corrections in $\epsilon_{1,2}$ [as can be seen in (34)], for $\epsilon_{1i} = \epsilon_{2i} = 0$ the sum $m_h^2 + m_H^2$ (9) and $m_{H^\pm}$ (33) are given exactly by the order ϵ correction above. This comment, however, is only of academic interest, because corrections to the potential of higher effective dimension will modify both $m_h^2 + m_H^2$ and $m_{H^\pm}$.

(4) The expression for $\delta_\epsilon m_{H^\pm}^2$ (33) is very simple and is independent of $\tan\beta$ and ϵ_1. This can be understood as a consequence of an approximate global $SU(2)_C$ custodial symmetry which is discussed in the Appendix.

(5) Focusing on the leading order in ϵ and assuming that η is very small, the corrections to the heavy Higgs masses m_H and $m_{H^\pm}$ are nonzero and depend only on ϵ_{2r}. Since the corrections to these two masses depend on only one real number, experimental values of these masses will over-determine it (or bound it) and will serve as a nontrivial test of the existence of this operator. In particular, the ϵ_2 independent relation

$$2m_{H^\pm}^2 = m_H^2 + m_A^2 + 2m_W^2 + \mathcal{O}(\eta^2, \eta\epsilon, \epsilon^2) \quad (35)$$

serves as a test of the BMSSM effective dimension five parametrization (with, of course, proper quantum corrections taken into account).

(6) The corrections to the light Higgs mass m_h^2 are of order $\epsilon\eta$ and hence they are negligible for very small η. If η is such that this contribution is measurable, it will determine the value of ϵ_{1r}.

One of the motivations for our analysis is to classify effects which may lift the light Higgs mass. We now see that for η parametrically very small the effect of terms at first order in ϵ is negligible. More precisely, if $\eta \gg \epsilon$ then the result (31) is useful. However, if $\eta \lesssim \epsilon$, then the order ϵ^2 corrections are comparable, and perhaps even larger than the order $\epsilon\eta$, and they are parametrically the leading correction to m_h^2.

First, let us ignore the various order ϵ^2 corrections. Then, the total shift of the light Higgs mass depends on both the radiative corrections (11) and the corrections (31) from the operators discussed above. As a conservative numerical example of the magnitude of the radiative corrections, stop squark masses of $m_{\tilde{t}_{1,2}} \simeq 300$ GeV with small mixing, $X_t \simeq 0$, yield a Higgs mass at moderate to large $\tan\beta$ of $m_h \simeq 100$ GeV. Even in this small mixing case, and with $m_A \gg m_Z$, the additional correction (31) can accommodate the LEPII Higgs mass bound of $m_h \gtrsim 114$ GeV for $\eta\epsilon_{1r} \gtrsim 6 \times 10^{-3}$. This could be achieved, for example, with $\eta \sim 0.1$ and $\epsilon_{1r} \sim 0.06$, which for $\mu \sim 300$ GeV would correspond to a scale in the operator (15) of $M/\lambda \sim 5$ TeV. The leading correction (31) grows with decreasing m_A, and so is slightly larger away from the Higgs decoupling limit.

Next, consider the limit $\eta \ll 1$, where ϵ^2 corrections to the light Higgs mass are important. In this case, h arises predominantly from H_u. Therefore, we should examine the corrections to the potential and the kinetic term of H_u. The relevant operators are enumerated in (27). At order ϵ^2 we need to consider the leading order effect of these operators as well as the order ϵ^2 corrections to the masses computed with the order ϵ correction to the potential as in (34). Since we are interested only in the zeroth order in η we can absorb all the unknowns in one number $\epsilon_3 \sim \epsilon$ and write (for real $\epsilon_{1,2}$)

$$\delta_{\epsilon^2} m_h^2 = -\frac{16v^4}{m_A^2 - m_Z^2}\epsilon_{1r}^2 + \epsilon_3^2 v^2. \quad (36)$$

For the small mixing example given above, the LEPII Higgs mass bound can be accommodated with the second order corrections (36) for $\epsilon_3 \gtrsim 0.3$. With $m_{\mathrm{SUSY}} \sim 300$ GeV and $\epsilon_3 \sim m_{\mathrm{SUSY}}/M$ this corresponds to $M \sim 1$ TeV.

In addition to modifications of the scalar Higgs masses, the operator (15) also modifies the Higgsino masses through the interactions (18). The first and second terms in (18) with scalar Higgs expectation values correct the charged and neutral Higgsino Dirac masses. The third and fourth terms in (18) with scalar Higgs expectation values give rise to neutral Higgsino Majorana masses which are absent in the tree-level neutralino mass matrix. Precision fits to both masses and couplings of neutralinos and charginos would be sensitive to the dimension five Higgs-Higgsino interactions. It is important to note that the interactions (18) are all proportional to a single coupling, ϵ_1, which is the same as the coupling affecting the Higgs mass. They are not the most general component couplings at this operator dimension which would arise among the b-ino, W-ino, Higgsinos, and Higgs scalars in a general nonsupersymmetric theory. So a precision fit to the BMSSM with the two effective dimension five operators is still highly over-constrained.

IV. MICROSCOPIC MODELS

Various types of dynamics might give rise to the various operators we have enumerated above. The most widely explored are the NMSSM [1], the addition of a singlet to the MSSM, and the possibility of additional gauge interactions at low scales [2,3]. Here we analyze both types of models in the framework we have described above. Although we will not discuss it here, it is clear that the dynamics leading to these operators can involve new strongly coupled sectors in the TeV range.

A. Adding a singlet: The NMSSM

The most studied extension of the MSSM is a model with an additional singlet, S, the NMSSM [1]. This model is usually motivated as an explanation of the μ-term, or, more generally, to avoid the appearance of any parameter with dimensions of mass in the low energy Lagrangian. As a result, one usually writes

$$\int d^2\theta\left(\lambda_S S H_u H_d + \frac{\lambda'}{3} S^3\right). \tag{37}$$

This structure can be enforced, for example, by a discrete R symmetry. Supersymmetry breaking (e.g. through the judicious choice of soft mass terms for S and H) then leads to an expectation value for S, which in turn generates an effective μ-term.

In a model of this sort, the S field is unlikely to be significantly more massive than the Higgs field, and it does not make sense to integrate it out to obtain a local action. The problem is that the couplings λ_S and λ' cannot be too large, at least if the model is to remain perturbative up to high energies; typically one requires $\lambda_S < 0.7$. At the same time, one cannot make λ_S very small, if one is to obtain a substantial Higgs mass. Given that $\langle S\rangle = \mu/\lambda_S$, any *supersymmetric* mass term for S is given by $M_S = 2\lambda'\mu/\lambda_S$, and this cannot be much larger than μ.

It has been appreciated for a long time that dimensionful parameters such as μ and a supersymmetric mass term for S can arise through couplings to some nonperturbative dynamics. More specifically, following [19–23], if one has some new, pure gauge theory, with a characteristic scale Λ, then couplings such as

$$\frac{W_\alpha^2}{M_0^2}\left(a_1 H_u H_d + \frac{a_2}{2} S^2\right) + \lambda_S S H_u H_d, \tag{38}$$

where a_1 and a_2 are numerical constants and M_0 is some heavy scale, give rise to μ and M_S of order Λ^3/M_0^2. As in the case of the conventional NMSSM (37), this structure can be enforced by discrete symmetries (in which case, necessarily, a "bare" mass for Higgs or S is forbidden).

We work, then, with a model with superpotential:

$$\int d^2\theta\left(\mu H_u H_d + \frac{1}{2} M_S S^2 + \lambda_S S H_u H_d\right). \tag{39}$$

If $M_S \gg \mu$ [which arises in (38) if $a_2 \gg a_1$], the heavy singlet may be integrated out through its holomorphic equation of motion:

$$S = -\frac{\lambda_S}{M_S} H_u H_d. \tag{40}$$

Here we neglected corrections involving covariant derivatives because they lead to higher order corrections to the effective action. In the supersymmetric limit this gives a tree-level Kahler potential and superpotential,

$$\int d^4\theta\left(H_u^\dagger e^V H_u + H_d^\dagger e^V H_d + \left|\frac{\lambda_S}{M_S}\right|^2 (H_u H_d)^\dagger (H_u H_d)\right), \tag{41}$$

$$\int d^2\theta\left(\mu H_u H_d - \frac{\lambda_S^2}{2M_S}(H_u H_d)^2\right). \tag{42}$$

From (42) we can read off the coefficient of the operator of (15):

$$\frac{\lambda}{M} = -\frac{\lambda_S^2}{2M_S}; \qquad \epsilon_1 = -\frac{\mu^* \lambda_S^2}{2M_S}. \tag{43}$$

Supersymmetry-breaking can be described to leading order by a spurion coupling

$$\int d^2\theta Z \frac{1}{2} M_S S^2. \tag{44}$$

After integrating out the singlet with this soft breaking, the operator (20) is generated with

$$\epsilon_2 = \frac{m_{\rm SUSY}\lambda_S^2}{M_S}. \tag{45}$$

The first order corrections in ϵ can be substantial. For example, taking $\tan\beta = 4$, the stop soft masses 300 GeV, and some sample values of X_t, the mass of the lightest Higgs in the MSSM (i.e. the mass including radiative corrections but excluding the contributions of the higher dimension operators), $m_{h\,\rm MSSM}$, is given in the second column of the table below. Adding the singlet S, with $M = 5\mu$, $\lambda_s = 0.7$ and different values of ϵ_2, the mass of the light Higgs, m_h, is readily pushed above the LEPII bound:

$X_t/m_{\tilde t}$	$m_{h\,\rm MSSM}$ (GeV)	ϵ_2	m_h (GeV)
0	91	0	120
0	91	ϵ_1	124
1	98	0	126
1	98	ϵ_1	129

In this model, we can assess the validity of the expansion in ϵ_i and η. There are a number of terms at second order in ϵ which can increase or decrease the mass of h. One set of contributions is indicated in (34). The final term in (41) is one of the Kahler potential operators (27) which can modify the H_u potential at effective dimension six. With $F_{H_d} \simeq -(\mu H_u)^\dagger$, it gives an η-independent correction to the light Higgs mass at order ϵ^2. In the normalization (36) this correction is

$$\epsilon_3^2 = -4\left|\frac{\mu\lambda_S}{M_S}\right|^2. \tag{46}$$

This contribution is negative definite, so it tends to decrease the Higgs mass. In the supersymmetric limit, it dominates for small η. Even for moderate $\tan\beta = 1/\eta$ (of order 3–4), the contributions we have enumerated are substantial. Including supersymmetry-breaking effects, there are additional contributions, however, to ϵ_3^2, which can have either sign.

B. Triplets

For the mass of the lightest Higgs, the effects of singlets are suppressed for small η, because they only couple to the Higgs combination H_uH_d, so it is interesting to consider triplets [4,5]. We will introduce two such fields, T, $\bar{T}$, with hypercharge $+2$ and -2, respectively. As in the case of the singlets, we will take the triplets to be heavy, with Kahler potential and superpotential

$$\int d^4\theta(H_u^\dagger e^V H_u + H_d^\dagger e^V H_d + T^\dagger e^V T + \bar{T}^\dagger e^V \bar{T}), \tag{47}$$

$$\int d^2\theta(\mu H_u H_d + M_T T\bar{T} + \lambda_T T H_u H_u + \lambda_{\bar{T}}\bar{T} H_d H_d). \tag{48}$$

Note the couplings to H_uH_u and H_dH_d. To leading order the heavy triplets may be integrated out through the chiral superfield holomorphic equation of motion, as in the NMSSM;

$$\bar{T} = -\frac{\lambda_{\bar{T}}}{M_T}H_u^2; \qquad T = -\frac{\lambda_T}{M_T}H_d^2. \tag{49}$$

In the supersymmetric limit this gives a tree-level Kahler and superpotential

$$\int d^4\theta\Big(H_u^\dagger e^V H_u + H_d^\dagger e^V H_d + \frac{|\lambda_T|^2}{M_T^2}(H_u^\dagger e^V H_u)^2 + \frac{|\lambda_{\bar{T}}|^2}{M_T^2}(H_d^\dagger e^V H_d)^2\Big), \tag{50}$$

$$\int d^2\theta\Big(\mu H_u H_d - \frac{\lambda_T\lambda_{\bar{T}}}{M_T}(H_u H_d)^2\Big). \tag{51}$$

From (51) we can read off the coefficient of the operator of (15):

$$\frac{\lambda}{M} = -\frac{\lambda_T\lambda_{\bar{T}}}{M_T}; \qquad \epsilon_1 = -\frac{\mu^*\lambda_T\lambda_{\bar{T}}}{M_T}. \tag{52}$$

Supersymmetry-breaking can be described to leading order by a spurion coupling

$$\int d^2\theta Z\frac{1}{2}M_T T\bar{T}. \tag{53}$$

After integrating out the triplets with this soft breaking, the operator (20) is generated with

$$\epsilon_2 = \frac{m_{\rm SUSY}\lambda_T\lambda_{\bar{T}}}{M_T}. \tag{54}$$

As in the singlet model, there are calculable effects of order ϵ^2. At zeroth order in η, the leading contributions come from the term (34), the third term in the Kahler potential (50), and from the additional spurion couplings

$$\int d^4\theta ZZ^\dagger(T^\dagger e^V T). \tag{55}$$

Unlike the singlet case, the sign of the η-independent order ϵ_3^2 term depends on the precise value of the soft breaking in this final operator.

C. Additional gauge interactions

Another approach to generating quartic couplings has been widely studied [2,3]. Gauge interactions beyond those of the standard model, broken at a scale M_V, can generate new contributions to the quartic couplings. In the supersymmetric limit, properly integrating out the massive gauge fields at M_V eliminates the quartic coupling associated with these gauge interactions, but creates higher order terms in the Kahler potential, as in (27). These include supersymmetric terms and supersymmetry violating terms. The former are of order μ^2/Λ^2 (Λ is the vev which breaks the gauge symmetry) and the latter are of order $m_{\rm SUSY}^2/\Lambda^2$.

We illustrate the basic phenomenon in $U(1)'$ theories. We add to the MSSM two charged fields, $\phi^\pm$, and a neutral field, ϕ^0, and assume that some of the MSSM fields, e.g. H_u, are charged under $U(1)'$. The superpotential is

$$\int d^2\theta(\phi_0(\phi_+\phi_- - \Lambda^2) + W_{\rm MSSM}), \tag{56}$$

where $W_{\rm MSSM}$ is the MSSM superpotential. The Kahler potential is

$$\int d^4\theta(\phi_+^\dagger e^{V'}\phi_+ + \phi_-^\dagger e^{-V'}\phi_- + K_{\rm MSSM}), \tag{57}$$

where V' is the $U(1)'$ gauge superfield, and $K_{\rm MSSM}$ is the MSSM Kahler potential with appropriate insertions of $e^{\pm V'}$.

Consider, first, this model without supersymmetry breaking. Ignoring covariant derivative terms, the ϕ_0 and $\phi_\pm$ equations of motion lead to

$$\phi_+\phi_- = \Lambda^2, \qquad \phi_0 = 0, \tag{58}$$

and therefore the gauge symmetry is Higgsed.

We want to integrate out the massive fields $\phi_\pm$, ϕ_0, and V'. We use the unitary gauge $\phi_+ = \Lambda$, and then the equations of motion (58) allow us to set

$$\phi_+ = \phi_- = \Lambda. \tag{59}$$

[Note that in the equations of motion leading to (58) we neglected the kinetic terms. Including them affects only higher derivative terms in the effective action.] Using (59) in the Kahler potential we find an effective Kahler potential for V',

$$\int d^4\theta(|\Lambda|^2(e^{V'} + e^{-V'}) + K_{\mathrm{MSSM}}) = \int d^4\theta(|\Lambda|^2(2 + V'^2 + \mathcal{O}(V'^4)) + K_{\mathrm{MSSM}}) \tag{60}$$

and hence the gauge boson mass is

$$M_{V'}^2 = 4g_{V'}^2|\Lambda|^2. \tag{61}$$

It is now straightforward to use (60) and integrate out V'. Its equation of motion is

$$|\Lambda|^2(e^{V'} - e^{-V'}) + D_{\mathrm{MSSM}} = 2|\Lambda|^2V'(1 + \mathcal{O}(V'^2)) + D_{\mathrm{MSSM}} = 0, \quad D_{\mathrm{MSSM}} = \frac{\delta K_{\mathrm{MSSM}}}{\delta V'}. \tag{62}$$

We recognize the $\theta = \bar{\theta} = 0$ component of this equation as the standard D-term equation. As above, here we neglect the kinetic term for V' because we are interested only in terms without covariant derivatives in the low energy theory. We can now solve for V',

$$V' = -\frac{D_{\mathrm{MSSM}}(V' = 0)}{2\Lambda^2} + \mathcal{O}(1/|\Lambda|^4). \tag{63}$$

Substituting this in (60) with the expansion

$$K_{\mathrm{MSSM}} = K_{\mathrm{MSSM}}(V' = 0) + V'\frac{\delta K_{\mathrm{MSSM}}}{\delta V'}(V' = 0) + \mathcal{O}(V'^2), \tag{64}$$

we find the effective Kahler potential

$$\int d^4\theta K_{\mathrm{eff}} = \int d^4\theta\Big(K_{\mathrm{MSSM}}(V' = 0) - \frac{D_{\mathrm{MSSM}}^2(V' = 0)}{4|\Lambda|^2} + \mathcal{O}(1/|\Lambda|^4)\Big). \tag{65}$$

We note that, unlike the examples of integrating out chiral superfields where the quartic correction to the Kahler potential was positive, here it is negative.

It is now easy to repeat this calculation with explicit supersymmetry breaking. For simplicity, we add equal masses for $\phi_\pm$; i.e. we replace (57) with

$$\int d^4\theta((1 - m_{\mathrm{SUSY}}^2\theta^4)(\phi_+^\dagger e^{V'}\phi_+ + \phi_-^\dagger e^{-V'}\phi_-) + D_{\mathrm{MSSM}}). \tag{66}$$

Then, (65) becomes

$$\int d^4\theta\Big(K_{\mathrm{MSSM}}(V' = 0) - (1 + m_{\mathrm{SUSY}}^2\theta^4)\frac{D_{\mathrm{MSSM}}^2(V' = 0)}{4|\Lambda|^2} + \mathcal{O}(1/|\Lambda|^4)\Big). \tag{67}$$

We are interested in the contributions which depend on $H_{u,d}$. From (67) we find

$$-\int d^4\theta\Big((1 + m_{\mathrm{SUSY}}^2\theta^4)\frac{1}{4|\Lambda|^2}(q_{H_u}(H_u^\dagger e^V H_u) + q_{H_d}(H_d^\dagger e^V H_d))^2\Big) \tag{68}$$

where $q_{H_{u,d}}$ are the $U(1)'$ charges of $H_{u,d}$.

We note that in this model there are no superpotential corrections and therefore $\epsilon_1 = \epsilon_2 = 0$. We recognize in the Kahler potential correction (68) some of the operators in (27) with specific relations between their coefficients ξ_i. In particular, setting $M = 2|\Lambda|$, we have $\xi_2 = -2q_{H_u}q_{H_d}$, $\xi_3 = -q_{H_u}^2$, and $\xi_5 = -q_{H_u}^2$, and all other ξ_i vanish. Using these ξ_i it is straightforward to compute ϵ_3 and the shift in m_h^2. In general, there are additional supersymmetry-breaking parameters, which will contribute to the ξ_i's.

There are a number of phenomenological constraints which must be satisfied. Typically it is necessary that $M_V > 1$ TeV; moreover, the gauge coupling constants might be expected to be small [the usual $U(1)$ coupling in the standard model is about 1/3]. In such circumstances, the scale m_{SUSY} must be of order 500 GeV or larger if the corrections to the quartic coupling are to account for the light Higgs mass. If $\mu \sim 200$ GeV, then the effects of ξ_2 and ξ_3 are negligible, and $\delta_{\epsilon^2}m_h^2 \simeq q_{H_u}^2 m_{\mathrm{SUSY}}^2 v^2/|\Lambda|^2$.

While M_V must be rather large, and as a result, the supersymmetry-breaking scale in this sector cannot be too small, there are naturalness upper bounds on these scales as well. It is important that loop corrections to scalar masses of MSSM fields which carry the additional gauge quantum numbers not be too large; this, combined with the requirement that the quartic correction be substantial, constrains the scale M_V. It is possible that m_{SUSY} is comparable to M_V, and that an operator analysis is not appropriate. Simple models of this type can be obtained by taking no superpotential for $\phi^\pm$ and different soft breaking masses for each field.

An interesting model of this kind can be motivated from familiar compactifications of the heterotic string theory. In E_6, there are two $U(1)$'s beyond those of the standard model. It is possible to break one of these, say $B - L$, at very high energies. If one assumes the remaining $U(1)$ is broken at a scale of, say, a few TeV, and that the scale of supersymmetry breaking in the vector multiplet is comparable, a conventional unification calculation leads to a small value of the $U(1)$ coupling, but can readily give a light Higgs mass of order 125 GeV or so. Finally, theories with non-Abelian symmetries can give much larger Higgs mass [3].

V. CONCLUSIONS

The current limit on the Higgs mass may suggest that, if nature is supersymmetric, the underlying model contains degrees of freedom beyond those of the MSSM. If this physics lies at scales a bit above that of the MSSM degrees of freedom, its effects are naturally organized within an effective Lagrangian. In the MSSM Higgs sector there are only two operators at effective dimension five. These operators can have important effects on the scalar Higgs masses. For the light Higgs mass at very large $\tan\beta$, it is necessary to consider operators of effective dimension six; there are potentially five such operators, though many are negligible in simple modes.

It is conceivable that the puzzle of the Higgs mass is resolved by very massive stops, or by large stop mixing. It seems more plausible that, if the hypothesis of low energy supersymmetry is correct, stop squarks with more modest masses and mixings will be discovered. Once the stop masses and mixing are known, we can calculate the radiatively corrected Higgs masses and compare them with the measured values. If a discrepancy with the simple MSSM predictions is found, then the BMSSM operators can correct the masses. At leading order in ϵ, the three masses m_h, m_H, and $m_{H^\pm}$ are corrected only by two real numbers ϵ_{1r} and ϵ_{2r}. Therefore, the measured values over-constrain these two unknowns. In particular, for small η the masses of H and $H^\pm$ are corrected only by one real number ϵ_{2r}, and therefore the BMSSM relation $2m_{H^\pm}^2 = m_H^2 + m_A^2 + 2m_W^2 + \mathcal{O}(\eta^2, \eta\epsilon, \epsilon^2)$ (up to radiative corrections) can be tested.

The new operators of the BMSSM can have interesting effects not only on the Higgs mass spectrum but on decays as well. For example, the Higgs-Higgsino interactions (18) contribute to decay of heavy Higgses to charginos and neutralinos H, $A \to \chi_i^+\chi_j^-$, $\chi_i^0\chi_j^0$ and $H^\pm \to \chi_i^\pm\chi_j^0$ for both $i = j$ and $i \neq j$. These interactions also contribute to neutralino and chargino decays which involve Higgs bosons, $\chi_i^0 \to h\chi_j^0$, $H\chi_j^0$, $A\chi_j^0$, $H^\pm\chi_j^\mp$ and $\chi_i^\pm \to H^\pm\chi_j^0$, $h\chi_j^\pm$, $H\chi_j^\pm$, $A\chi_j^\pm$. In regions of parameter space where the initial and final state charginos or neutralinos are Higgsino-like, the operator (18) can significantly modify these branching ratios compared with the tree-level MSSM. The interactions (18) can also lead to three body decays such as H, $A \to h\chi_i^+\chi_j^-$, $h\chi_i^0\chi_j^0$ and $\chi_i^0 \to hh\chi_j^0$. CP violation in the effective dimension five operators also opens interesting decay modes which are absent in the tree-level MSSM, such as $A \to hh$ and $A \to WW$, ZZ through mixing.

Future precision fits to chargino and neutralino masses, mixings, and couplings in both production and decay would also be affected. However, it is important to stress that at effective dimension five all these observables depend only on ϵ_1, and thus represent a highly overconstrained parameter space for the precision fits. So restricting to the BMSSM effective dimension five parameter space could still allow for nontrivial tests of both supersymmetry and the extension of the MSSM.

The BMSSM operators can also have interesting implications for cosmological supersymmetry signatures. Details of electroweak baryogenesis could be affected in a number of ways.[4] First, the phase transition could be somewhat more strongly first order than in the MSSM as currently constrained, since both of the stop squarks could be fairly light if the BMSSM operators contribute significantly to the zero temperature Higgs mass. Second, the quartic Higgs couplings can provide an interesting source of CP violation within the bubble wall. Finally, the Higgs-Higgsino interactions with CP violating phases could contribute to an axial current of Higgsinos scattered from the bubble wall (which is ultimately processed into a baryon asymmetry by sphalerons).

The relic abundance of neutralino dark matter in the well-know gaugino-Higgsino region of parameter space is also modified in the BMSSM. In this region of the MSSM the observed relic dark matter abundance is obtained for the μ parameter and the b-ino or W-ino mass, m_1 or m_2, very close in value, resulting in sizable b-ino–Higgsino or W-ino–Higgsino mixing. Details of this mixing as well as couplings to final states with or through Higgs bosons are sensitive to the dimension five Higgs-Higgsino interactions. In particular, CP violation in these interactions would open up S-wave annihilation channels through light s-channel Higgs bosons and could quantitatively shift the region of parameter space which results in the proper relic abundance.

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APPENDIX: TWO DOUBLET HIGGS SECTOR SYMMETRIES

The most general $SU(2)_L \times U(1)_Y$ invariant renormalizable scalar potential for two Higgs doublets, H_u and H_d, with hypercharge $Y = \pm 1$, is given in standard notation by [24]

[4] We thank Ann Nelson for pointing out this possibility.

$$\begin{aligned} V = {} & \tilde{m}^2_{H_u} H_u^\dagger H_u + \tilde{m}^2_{H_d} H_d^\dagger H_d - (m^2_{ud} H_u H_d + \text{H.c.}) \\ & + \tfrac{1}{2}\lambda_1 (H_u^\dagger H_u)^2 + \tfrac{1}{2}\lambda_2 (H_d^\dagger H_d)^2 \\ & + \lambda_3 (H_u^\dagger H_u)(H_d^\dagger H_d) + \lambda_4 (H_u^\dagger H_d)(H_d^\dagger H_u) \\ & + [\tfrac{1}{2}\lambda_5 (H_u H_d)^2 + (\lambda_6 (H_u^\dagger H_u) \\ & + \lambda_7 (H_d^\dagger H_d)) H_u H_d + \text{H.c.}]. \end{aligned} \tag{A1}$$

The Higgs potential (5) of the renormalizable MSSM, along with the effective dimension five interactions (16) and (21), corresponds to the potential (A1) with

$$\begin{aligned} &\lambda_1 = \lambda_2 = \tfrac{1}{4}(g'^2 + g^2), \qquad \lambda_3 = \tfrac{1}{4}(g^2 - g'^2), \\ &\lambda_4 = -\tfrac{1}{2}g^2, \qquad \lambda_5 = 2\epsilon_2, \qquad \lambda_6 = \lambda_7 = 2\epsilon_1. \end{aligned} \tag{A2}$$

In order to classify patterns within the physical Higgs mass spectrum and interactions, it is useful to determine how the Higgs sector couplings transform under background global symmetries. The largest possible symmetry of the scalar potential (A1) is an $SO(8)$ under which the eight real components of H_u and H_d transform in the $\mathbf{8}_v$ representation. An interesting subgroup of this maximal symmetry is $SU(2)_{L_u} \times SU(2)_{R_u} \times SU(2)_{L_d} \times SU(2)_{R_d} \subset SO(8)$ under which the Higgs fields transform as bi-doublets,

	$SU(2)_{L_u}$	$SU(2)_{R_u}$	$SU(2)_{L_d}$	$SU(2)_{R_d}$
H_u	**2**	**2**	**1**	**1**
H_d	**1**	**1**	**2**	**2**

The $SU(2)_L \times U(1)_Y$ gauge symmetry is embedded in the diagonal subgroups of this decomposition as

$$\begin{aligned} &SU(2)_L \subset SU(2)_{L_u} \times SU(2)_{L_d}, \\ &U(1)_Y \subset SU(2)_R \subset SU(2)_{R_u} \times SU(2)_{R_d}. \end{aligned} \tag{A3}$$

Expectation values for the Higgs fields spontaneously break each $SU(2)_L \times SU(2)_R$ to a diagonal custodial subgroup,

$$\begin{aligned} &\langle H_u \rangle\colon SU(2)_{L_u} \times SU(2)_{R_u} \to SU(2)_{C_u}, \\ &\langle H_d \rangle\colon SU(2)_{L_d} \times SU(2)_{R_d} \to SU(2)_{C_d}. \end{aligned} \tag{A4}$$

The diagonal subgroup of these custodial symmetries,

$$SU(2)_C \subset SU(2)_{C_u} \times SU(2)_{C_u}, \tag{A5}$$

provides a useful symmetry for understanding features of the Higgs spectrum. It reduces to the usual $SU(2)_C$ custodial symmetry of the one Higgs doublet model in the limit in which the second Higgs doublet is decoupled. Since the most general expectation values (A4) leave $SU(2)_C$ unbroken, any violations arise from the interactions.

If the potential preserves the $SU(2)_C$ symmetry, then the three eaten Goldstone modes are in an $SU(2)_C$ triplet, the two massive Higgses h and H are $SU(2)_C$ singlets, and there is a massive triplet H^+, A, H^-. The couplings of the Higgs potential (A1) transform as the components with vanishing $U(1)_Y \subset SU(2)_C$ representations,

$$\begin{array}{cc} & SU(2)_C \\ \tilde{m}^2_{H_{u,d}},\ \text{Re}(m^2_{ud} e^{i\varphi}),\ \lambda_{1,2,3},\ \lambda_4 + \text{Re}(\lambda_5 e^{2i\varphi}),\ \text{Re}(\lambda_{6,7} e^{i\varphi}) & \mathbf{1} \\ \text{Im}(m^2_{ud} e^{i\varphi}),\ \text{Im}(\lambda_5 e^{2i\varphi}),\ \text{Im}(\lambda_{6,7} e^{i\varphi}) & \mathbf{3} \\ \lambda_4 - \text{Re}(\lambda_5 e^{2i\varphi}) & \mathbf{5} \end{array} \tag{A6}$$

where, in a general basis, φ is the phase of the expectation value,

$$\varphi = \arg\langle H_u H_d \rangle. \tag{A7}$$

If CP symmetry is unbroken, either explicitly or spontaneously, there is a basis in which $\varphi = 0$ and the imaginary components of all potential couplings vanish. In this basis the $SU(2)_C$ transformation properties of the Higgs potential couplings reduce to those under the diagonal $SU(2)_R$ in (A3), since the phase φ vanishes and all couplings are $SU(2)_L$ invariant. In addition, the component field A is a mass eigenstate if CP is unbroken. Since H^+, A, H^- transform as a **3** of $SU(2)_C$, the mass squared terms for these fields transform as the symmetric representations $\mathbf{1} + \mathbf{5} \subset \mathbf{3} \times \mathbf{3}$ of $SU(2)_C$,

$$V = m^2_{\mathbf{1}}(\tfrac{1}{2}A^2 + H^+H^-) + m^2_{\mathbf{5}}(\tfrac{1}{2}A^2 - H^+H^-) + \cdots. \tag{A8}$$

In the CP conserving limit, $\lambda_4 - \lambda_5$ is the only combination of renormalizable Higgs potential couplings which transforms under **5** of $SU(2)_C$ and leads to splittings between $H^\pm$ and A,

$$m^2_{H^\pm} = m^2_A + v^2(\lambda_5 - \lambda_4). \tag{A9}$$

Another useful global symmetry which provides selection rules is the well-known $U(1)_{PQ}$ symmetry under which both H_u and H_d have the same charge. This symmetry is generated by the Cartan subalgebra element orthogonal to the gauged $U(1)_Y$,

$$U(1)_{PQ} \subset SU(2)_{R_u} \times SU(2)_{R_d}. \tag{A10}$$

If the potential preserves this $U(1)_{PQ}$ symmetry, the massive fields $H^\pm$ and $H \pm iA$ are charged under it, and therefore H and A are degenerate.

The terms in the more general potential are classified by the $U(1)_{PQ}$ charge as follows:

$$\begin{array}{cc} & U(1)_{PQ} \\ \tilde{m}^2_{H_{u,d}},\ \lambda_{1,2,3,4} & 0 \\ m^2_{ud},\ \lambda_{6,7} & -2 \\ \lambda_5 & -4 \end{array} \tag{A11}$$

One application of this symmetry is the limit $m^2_{ud}, \lambda_{6,7} \to 0$ with $\tilde{m}^2_{H_u} < 0$ and $\tilde{m}^2_{H_d} > 0$ for which only the up-type Higgs gains an expectation value. This is the large $\tan\beta$ limit of the (B)MSSM with m_A, or equivalently $\tilde{m}^2_{H_u}$ and $\tilde{m}^2_{H_d}$, held fixed.

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