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Do $c\bar{c}n\bar{n}$ bound states exist?

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The four-quark system $c\bar{c}n\bar{n}$ is studied in the framework of the constituent quark model. Using different types of quark-quark potentials, we solve the four-body Schrödinger equation by means of the hyperspherical harmonic formalism. Exploring the low laying J^{PC} states for different isospin configurations no four-quark bound states have been found. Of particular interest is the possible four-quark structure of the $X(3872)$. We rule out the possibility that this particle is a compact tetraquark system, unless additional correlations, either in the form of diquarks or at the level of the interacting potential, not considered in simple quark models do contribute.

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I. INTRODUCTION

In the last few years the discovery of several new heavy hadrons containing charm quarks has renewed the interest in heavy quark spectroscopy. In 2003 a new resonance named $X(3872)$ was reported by the Belle Collaboration in the invariant mass distribution of $J/\psi\pi^+\pi^-$ mesons produced in $B^\pm \to K^\pm X(3872) \to K^\pm J/\psi\pi^+\pi^-$ decays. It appeared as a narrow peak with a mass 3871.2 ± 0.5 MeV and a width $\Gamma < 2.3$ MeV, consistent with the detector resolution [1]. This state was confirmed by *BABAR* [2], CDF [3], and D0 Collaborations [4]. In 2004 Belle reported the observation of a new charmonium state in the $\omega J/\psi$ invariant mass distribution for exclusive $B \to K\omega J/\psi$ decays [5]. This state, named $Y(3940)$, has a mass and width of $3943 \pm 11 \pm 13$ MeV and $87 \pm 22 \pm 26$ MeV, respectively, and it has not been seen in the decay modes $Y(3940) \to D\bar{D}$ and $D\bar{D}^*$. In July 2005 Belle claimed the observation of a second charmonium resonance named $X(3940)$ with a mass of $3943 \pm 6 \pm 6$ MeV and total width of less than 52 MeV [6]. This state has been measured in the $e^+e^- \to J/\psi X(3940)$ reaction and it has been seen to decay to $D\bar{D}^*$ and not to $\omega J/\psi$ or $D\bar{D}$. A third state, $Z(3940)$, was reported almost simultaneously by Belle in $\gamma\gamma \to D\bar{D}$ with a mass of $3931 \pm 4 \pm 2$ MeV and a width of $20 \pm 8 \pm 3$ MeV [7]. Being its helicity distribution consistent with $J = 2$, an identification with the excited χ_{c2} seems to be natural. Up to now, the last experimental neighbor of the charmonium tribe has been reported by *BABAR*, the $Y(4260)$ [8]. This state, with quantum numbers $J^{PC} = 1^{--}$, is a broad resonance in the invariant mass spectrum of $\pi^+\pi^-J/\psi$ with mass $4259 \pm 8 \pm 4$ MeV and width $83 \pm 23 \pm 5$ MeV.

While some members of this new hadronic zoo may fit in the simple quark model description as quark-antiquark pairs [$X(3940)$, $Y(3940)$, and $Z(3940)$ may fit into the χ_{c0}, χ_{c1}, and χ_{c2} quark model structure] others appear to be more elusive [$X(3872)$ and $Y(4260)$]. With the advent of all these new resonances, theoretical speculations about the existence of four-quark states mixed with $c\bar{c}$ quark-antiquark bound states have been reinforced, the $X(3872)$ being primarily responsible for that. Before its discovery, only a few attempts were made to look for $c\bar{c}n\bar{n}$ states. In the early 80's, Gelmini [9] studied the S-wave $c\bar{c}n\bar{n}$ states using the one-gluon exchange potential and virtual annihilation of color pairs, obtaining some candidates that could lie below any of the dissociation channels. Chao [10] explored the decay, hadronic production, production in e^+e^- annihilation, and photoproduction of various types of $c\bar{c}n\bar{n}$ states using the quark-gluon model proposed by Chan and Hogaasen [11]. Using a potential derived from the MIT bag model in the Born-Oppenheimer approximation Chao also concluded that $c\bar{c}n\bar{n}$ states lie in the 3.2–3.7 GeV energy range, and therefore below the lowest two-meson thresholds [12]. Silvestre-Brac and Semay [13] analyzed $L = 0$ four-quark systems by means of the Bhaduri potential through a variational method in a harmonic oscillator basis, suggesting the existence of several $(Q_1\bar{Q}_2)(n\bar{n})$ bound states, among them the $J^P = 0^+$ and $J^P = 1^+$ $c\bar{c}n\bar{n}$. After the discovery of the $X(3872)$ the question about the possible existence of $c\bar{c}n\bar{n}$ bound states was posed again. Maiani *et al.* [14] constructed a model of the $X(3872)$ in terms of diquark-antidiquark degrees of freedom. Using the $X(3872)$ as input they predict other $c\bar{c}n\bar{n}$ states with quantum numbers 0^{++}, 1^{+-}, and 2^{++}. Ebert *et al.* [15] addressed heavy tetraquarks with hidden charm and beauty in a diquark-antidiquark relativistic quark model, concluding that the $X(3872)$ could be identified with the 1^{++} neutral charm tetraquark state. The existence of the $X(3872)$ would imply another three four-quark states close in energy. Hogaasen *et al.* [16] explained the mass and coupling properties of the $X(3872)$ resonance as a 1^{++} four-quark state using a chromomagnetic interaction once all spin-color configurations compatible with these quantum numbers were included. Matheus *et al.* [17] used QCD spectral sum rules to test the nature of the $X(3872)$ within a diquark-antidiquark scheme, identified as a possible 1^{++} $c\bar{c}n\bar{n}$ candidate.

Analogous alternatives have been scrutinized to interpret the experimental data of the open-charm meson sector. Several new states with intriguing properties hard to accommodate in a standard quark-antiquark scheme were reported in recent years. In 2003 *BABAR* reported a charm-strange state, the $D_{sJ}^*(2317)$, with a mass of $2316.8 \pm 0.4 \pm 3$ MeV and a width of less than 4.6 MeV [18]. It was confirmed by CLEO Collaboration [19] and also by Belle [20]. Besides, *BABAR* had also pointed out the existence of another charm-strange meson, the $D_{sJ}(2460)$ [18]. This resonance was measured by CLEO [19] and confirmed by Belle [20] with a mass of $2457.2 \pm 1.6 \pm 1.3$ MeV and a width less than 5.5 MeV. Belle results are consistent with the spin-parity assignments of $J^P = 0^+$ for the $D_{sJ}^*(2317)$ and $J^P = 1^+$ for the $D_{sJ}(2460)$. In the nonstrange sector Belle reported the observation of a nonstrange broad scalar resonance, named D_0^*, with a mass of $2308 \pm 17 \pm 15 \pm 28$ MeV and a width $276 \pm 21 \pm 18 \pm 60$ MeV [21]. A state with similar properties has been suggested by FOCUS Collaboration [22] during the measurement of excited charm mesons D_2^*. Although there are several theoretical interpretations for these states (see Ref. [23]), the difficulties to identify some of them with conventional mesons (rather similar to those appearing in the light-scalar meson sector) were interpreted as signals indicating that other configurations, for example, four-quark contributions, could be playing a significant role [24,25]. The idea behind this interpretation is rather simple. Physical mesons are easily identified with $q\bar{q}$ states when virtual quark loops are not important. This is the case of the pseudoscalar and vector mesons, mainly due to the P-wave nature of the hadronic dressing. However, in the positive parity sector the $q\bar{q}$ pair is the one that appears in a P-wave state, whereas quark loops may be in a S-wave. In this case the intermediate hadronic states that are created may play a crucial role in the composition of the resonance, in other words unquenching may be important. The vicinity of these components to the lightest $q\bar{q}$ state implies that they have to be considered either as mixed states or compact structures [26]. This has been shown as a possible interpretation of the low-lying light-scalar mesons [27].

As a consequence, the solution of the four-body problem to analyze the contribution of four-quark components to the meson spectra has become recently a basic tool. Most of the approaches found in the literature are variational calculations with different types of trial wave functions. The rather important interest of this problem requires numerical methods able to provide with solutions free of numerical uncertainties. Recently, a new approach based on the hyperspherical formalism was proposed to solve exactly the four-quark problem [28]. The idea is to perform an expansion of the trial wave function in terms of hyperspherical harmonic (HH) functions. This allows us to generalize the simplicity of the spherical harmonic expansion for the angular functions of a single particle motion to a system of particles by introducing a global length ρ, called the hyperradius, and a set of angles, Ω. For the HH expansion to be practical, the evaluation of the potential energy matrix elements must be feasible. The main difficulty of this method is to construct HH functions of proper symmetry for a system of identical particles. This is a difficult problem that may be overcome by means of the HH formalism based on the symmetrization of the N-body wave function with respect to the symmetric group using the Barnea and Novoselsky algorithm [29]. This method, widely used in nuclear physics, was applied in Ref. [28] to the analysis of four-charm quark systems.

In this work we present a study of the $c\bar{c}n\bar{n}$ ground states using the HH technique. For this purpose we have generalized the HH formalism of Ref. [28] to describe four-quark states of different flavors. The manuscript is organized as follows. In Sec. II the procedure necessary to generalize the hyperspherical formalism for studying quark systems of different flavors is described. In Sec. III we review two different quark models we will make use of to test our method and compare with existing results and experiments. In Sec. IV we present the results and the analysis of the $c\bar{c}n\bar{n}$ spectroscopy. Finally, we summarize in Sec. V our conclusions.

II. TECHNICAL DETAILS

A. Basis functions

Within the HH expansion, the four-quark wave function can be written as a sum of outer products of color, isospin, spin, and configuration terms

$$|\phi_{\rm CISR}\rangle = |{\rm Color}\rangle|{\rm Isospin}\rangle[|{\rm Spin}\rangle \otimes |{\rm R}\rangle]^{\rm JM}, \qquad (1)$$

such that the four-quark state is a color singlet with well-defined parity, isospin, and total angular momentum. In the following we shall present the construction of the basis functions for the $QQ\bar{n}\,\bar{n}$ and $Q\bar{Q}n\bar{n}$ tetraquark systems. We shall assume that particles 1 and 2 are the Q-quarks and particles 3 and 4 are the n-quarks. In the $QQ\bar{n}\,\bar{n}$ case particles 1 and 2 are identical, and so are 3 and 4. Consequently, the Pauli principle leads to the following conditions,

$$\hat{P}_{12}|\phi_{\rm CISR}\rangle = \hat{P}_{34}|\phi_{\rm CISR}\rangle = -|\phi_{\rm CISR}\rangle, \qquad (2)$$

$\hat{P}_{ij}$ being the permutation operator of particles i and j.

Coupling the color states of two quarks (antiquarks) can yield two possible representations, the symmetric 6-dimensional, 6 ($\bar{6}$), and the antisymmetric 3-dimensional, $\bar{3}$ (3). Coupling the color states of the quark pair with that of the antiquark pair must yield a color singlet. Thus, there are only two possible color states for a $QQ\bar{q}\,\bar{q}$ system [30],

$$|{\rm Color}\rangle = |C_{12}C_{34}\rangle = \{|\bar{3}_{12}3_{34}\rangle, |6_{12}\bar{6}_{34}\rangle\}. \qquad (3)$$

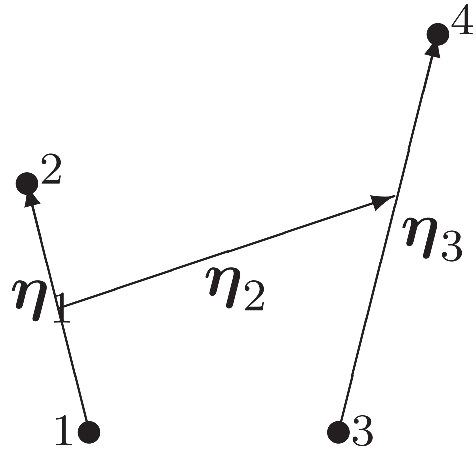

FIG. 1. *H*-type Jacobi vectors.

These states have well-defined symmetry under permutations, Eq. (2). Spin states with such symmetry can be obtained coupling the particle spins in the following way,

$$|\mathrm{Spin}\rangle = |((s_1, s_2)S_{12}, (s_3, s_4)S_{34})S\rangle = |(S_{12}S_{34})S\rangle. \quad (4)$$

The same holds for the isospin, $|\mathrm{Isospin}\rangle = |(i_3, i_4)I_{34}\rangle$, which applies only to the n-quarks, thus $I = I_{34}$.

As said, we use the HH expansion to describe the spatial part of the wave function. We choose for convenience the H-type Jacobi coordinates (see Fig. 1),

$$\begin{aligned} \boldsymbol{\eta}_1 &= \mu_{1,2}(\boldsymbol{r}_2 - \boldsymbol{r}_1) \\ \boldsymbol{\eta}_2 &= \mu_{12,34}\left(\frac{m_3\boldsymbol{r}_3 + m_4\boldsymbol{r}_4}{m_{34}} - \frac{m_1\boldsymbol{r}_1 + m_2\boldsymbol{r}_2}{m_{12}}\right) \\ \boldsymbol{\eta}_3 &= \mu_{3,4}(\boldsymbol{r}_4 - \boldsymbol{r}_3), \end{aligned} \quad (5)$$

where $m_{ij} = m_i + m_j$, $\mu_{i,j} = \sqrt{m_i m_j / m_{ij}}$, and $m_{1234} = m_1 + m_2 + m_3 + m_4$. Using these vectors, it is easy to obtain basis functions that have well-defined symmetry under permutations of the pairs (12) and (34). The hyperspherical coordinates (ρ, Ω) are defined through the relation

$$\boldsymbol{\eta}_1 = \rho\cos\alpha_2\cos\alpha_1\hat{\eta}_1 \qquad \boldsymbol{\eta}_2 = \rho\cos\alpha_2\sin\alpha_1\hat{\eta}_2 \\ \boldsymbol{\eta}_3 = \rho\sin\alpha_2\hat{\eta}_3, \quad (6)$$

where

$$\rho = \sqrt{\eta_1^2 + \eta_2^2 + \eta_3^2}, \quad (7)$$

is the hyperradius, $\hat{\eta}_j \equiv (\theta_j, \phi_j)$ is the unit vector of $\boldsymbol{\eta}_j$, and $\Omega \equiv (\alpha_1, \alpha_2, \hat{\eta}_1, \hat{\eta}_2, \hat{\eta}_3)$ is a hyperangle that represents the location on the 8-dimensional sphere.

By using hyperspherical coordinates one can write the Laplace operator as a sum of two terms

$$\Delta = \frac{1}{\rho^8}\frac{\partial}{\partial\rho}\rho^8\frac{\partial}{\partial\rho} - \frac{1}{\rho^2}\hat{K}^2, \quad (8)$$

where the hyperspherical, or grand angular momentum, operator $\hat{K}^2$ is the 9 dimensional analogous of the angular momentum operator associated with the 3-dimensional Laplacian.

The hyperspherical harmonic functions $\mathcal{Y}_{[K]}$ are the eigenfunctions of this hyperangular momentum operator, labeled by the quantum numbers $[K] \equiv \{K_3K_2L_3M_3L_2\ell_3\ell_2\ell_1\}$. The quantum number K_3 is the grand hyperangular momentum associated with the 3 Jacobi vectors, L_3M_3 are the usual orbital angular momentum quantum numbers of the system, and ℓ_i is the angular momentum associated with the Jacobi vector η_i. The quantum numbers K_2, L_2 correspond to intermediate coupling of η_1 and η_2. The explicit expression for the HH functions is given by [31]

$$\begin{aligned} \mathcal{Y}_{[K]}(\Omega) = \Bigg[&\sum_{m_1,m_2,m_3} \langle \ell_1 m_1 \ell_2 m_2 | L_2 M_2\rangle \\ &\times \langle L_2 M_2 \ell_3 m_3 | L_3 M_3\rangle \prod_{j=1}^{3} Y_{\ell_j,m_j}(\hat{\eta}_j)\Bigg] \\ &\times \Bigg[\prod_{j=2}^{3} \mathcal{N}_{n_j}^{a_j,b_j}(\sin\alpha_j)^{\ell_j} \\ &\times (\cos\alpha_j)^{K_{j-1}} P_{n_j}^{(a_j,b_j)}(\cos(2\alpha_j))\Bigg], \end{aligned} \quad (9)$$

where $Y_{\ell,m}$ are the spherical harmonic functions, $P_n^{(a,b)}$ are the Jacobi polynomials, $a_j = \ell_j + \frac{1}{2}$, $b_j = K_{j-1} + \frac{3j-5}{2}$ and $\mathcal{N}_{n_j}^{a_j,b_j}$ are normalization constants given by [32]:

$$\mathcal{N}_{n_j}^{a_j b_j} = \left[\frac{2(2n_j + a_j + b_j + 1)n_j!\Gamma(n_j + a_j + b_j + 1)}{\Gamma(n_j + a_j + 1)\Gamma(n_j + b_j + 1)}\right]^{1/2}. \quad (10)$$

The quantum numbers K_j are given by

$$K_j = 2n_j + K_{j-1} + \ell_j; \qquad n_1 \equiv 0, \quad (11)$$

where the n_j's are non-negative integers. In the following we shall use the notations $K \equiv K_3$ and $L \equiv L_3$ for the total hyperangular and angular quantum numbers. By construction, $\rho^K \mathcal{Y}_{[K]}$ is a harmonic polynomial of degree K, therefore the HH function $\mathcal{Y}_{[K]}$ is an eigenfunction of $\hat{K}^2$ with eigenvalues $K(K + 7)$.

The HH functions are a complete basis set for the hyperangular coordinate Ω. For the hyperradial coordinate our basis functions read

$$L_n(\rho) = \sqrt{\frac{n!}{(n+\nu)!}}\rho_0^{-9/2}\left(\frac{\rho}{\rho_0}\right)^{(\nu-8)/2} L_n^\nu\left(\frac{\rho}{\rho_0}\right) e^{-(1/2)(\rho/\rho_0)}, \quad (12)$$

where $L_n^\nu(x)$ are the associated Laguerre polynomials. The range parameter ρ_0 and the parameter ν are varied to get optimal results.

By inspection it can be verified that the spatial basis states, given by

$$\langle\rho\Omega|R\rangle \equiv \langle\rho\Omega|n[K]\rangle = L_n(\rho)\mathcal{Y}_{[K]}(\Omega), \tag{13}$$

have the following symmetry properties with respect to particle permutations

$$\begin{aligned}\hat{P}_{12}|n[K]\rangle &= (-1)^{\ell_1}|n[K]\rangle;\\ \hat{P}_{34}|n[K]\rangle &= (-1)^{\ell_3}|n[K]\rangle\end{aligned} \tag{14}$$

Application of the Pauli principle, Eq. (2), to the basis function

$$|\phi_{\rm CISR}\rangle = |C_{12}C_{34}\rangle|(S_{12}S_{34})S\rangle|I_{34}\rangle|n[K]\rangle \tag{15}$$

leads to the following restrictions on the allowed combinations of basis states,

(i) $(-1)^{S_{12}+\ell_1} = +1$, $(-1)^{S_{34}+I+\ell_3} = -1$ for the $|6_{12}\bar{6}_{34}\rangle$ color state.

(ii) $(-1)^{S_{12}+\ell_1} = -1$, $(-1)^{S_{34}+I+\ell_3} = +1$ for the $|\bar{3}_{12}3_{34}\rangle$ color state.

In the $Q\bar{Q}n\bar{n}$ case particle 2 is the antiparticle of particle 1, and particle 4 is the antiparticle of particle 3. Assuming that C-parity is a good symmetry of QCD we can regard quarks and antiquarks as identical particles and impose the symmetry condition, Eq. (2), on the $Q\bar{Q}n\bar{n}$ system as well. Coupling the color states of a quark and an antiquark can yield two possible representations: the singlet and the octet. These representations should be combined in the following way $\{|1_{12}1_{34}\rangle, |8_{12}, 8_{34}\rangle\}$ to yield a total color singlet state [30]. However, these states have no definite symmetry under the particle permutations (12) and (34). To construct symmetrized states for the $Q\bar{Q}$ pair we consider the following combinations,

$$|C_{12}^{\Gamma_{12}}\rangle = \frac{1}{\sqrt{2}}(|C_{12}\rangle + \Gamma_{12}|C_{21}\rangle), \tag{16}$$

where $C_{12} = \{1_{12}, 8_{12}\}$, and $\Gamma_{12} = +1$ for a symmetric combination and -1 for antisymmetric. For light quarks the color and isospin states should be combined together to form states with well-defined symmetry. For $I_z = 0$, as an example, these states take the form,

$$\begin{aligned}|(C_{34}I_{34})^{\Gamma_{34}}\rangle = &+\tfrac{1}{2}[|C_{34}\rangle(|u\bar{u}\rangle \pm |d\bar{d}\rangle)\\ &+ \Gamma_{34}|C_{43}\rangle(|\bar{u}u\rangle \pm |\bar{d}d\rangle)],\end{aligned} \tag{17}$$

where the plus sign stands for $I_{34} = 0$ state and the minus sign for the $I_{34} = 1$ state. As before, C_{34} stands for either the singlet or the octet representation. The total color-isospin states, $|C_{12}^{\Gamma_{12}}(C_{34}I_{34})^{\Gamma_{34}}\rangle$ are not only good symmetry states, but also good C-parity states with, $C = \Gamma_{12}\Gamma_{34}$. Imposing the Pauli principle for the $Q\bar{Q}n\bar{n}$ system we get the following restrictions, $\Gamma_{12}(-1)^{S_{12}+\ell_1} = +1$, $\Gamma_{34}(-1)^{S_{34}+\ell_3} = +1$, on the basis states.

B. Calculation of the matrix elements

Because of the recursive nature of the HH functions, evaluation of potential matrix elements become simpler for the pair (34). In this case, the matrix elements of an isoscalar two-body potential

$$V_{ij} = \sum_p V_p(r_{ij})O_p^S O_p^I O_p^C, \tag{18}$$

written as a sum of products of spatial, spin, isospin, and color operators, respectively, are diagonal in the quantum numbers ℓ_1, ℓ_2, L_2, K_2, S_{12}, C_{12}, I_{34}. The matrix elements of central potentials are further diagonal also in the quantum numbers ℓ_3, L, S_{34}, S thus reducing the calculation of the matrix elements

$$\begin{aligned}\langle\phi_{\rm CISR}|V_{34}|\phi'_{\rm CISR}\rangle = &\sum_p \langle nK|V_p|n'K'\rangle_{K_2\ell_3}\langle S_{34}|O_p^S|S_{34}\rangle\\ &\times\langle I_{34}|O_p^I|I_{34}\rangle\langle C_{34}|O_p^C|C_{34}\rangle,\end{aligned} \tag{19}$$

into a product of the internal matrix elements times a two dimensional integral

$$\begin{aligned}\langle nK|V_p|n'K'\rangle_{K_2\ell_3} = &\frac{\mathcal{N}_{n_3}^{a,b}\mathcal{N}_{n_3'}^{a,b}}{2^{a+b+2}}\int\rho^8 d\rho L_n(\rho)L_{n'}(\rho)\\ &\times\int_{-1}^{1}dx(1-x)^a(1+x)^b P_{n_3}^{(a,b)}(x)\\ &\times P_{n_3'}^{(a,b)}(x)V_p(\rho\sqrt{1-x})\end{aligned} \tag{20}$$

where $x = \cos(2\alpha_2)$, $a = \ell_3 + \frac{1}{2}$, and $b = K_2 + 2$. Noncentral potentials lead to similar expressions after rearrangement of the angular momentum couplings.

Direct evaluation of the potential matrix elements for other pairs will lead to multidimensional integrals much more complicated than Eq. (20). These complicated integrals can be avoided through rearrangement of the basis functions. If we denote by $|\phi_{\rm CISR}(1234)\rangle$ the basis function defined in Eq. (15), and by $|\phi_{\rm CISR}(ijkl)\rangle$ basis function associated with the particles arranged in the order $ijkl$, then the interaction matrix element for the pair $(ij) \neq (34)$ can be evaluated in the following way,

$$\begin{aligned}&\langle\phi_{\rm CISR}(1234)|V_{ij}|\phi'_{\rm CISR}(1234)\rangle\\ &\quad= \sum\langle\phi_{\rm CISR}(1234)|\phi''_{\rm CISR}(klij)\rangle\\ &\quad\times\langle\phi'''_{\rm CISR}(klij)|\phi'_{\rm CISR}(1234)\rangle\\ &\quad\times\langle\phi''_{\rm CISR}(klij)|V_{ij}|\phi'''_{\rm CISR}(klij)\rangle.\end{aligned} \tag{21}$$

The potential matrix element on the right-hand side of Eq. (21) can be calculated now using Eqs. (19) and (20). The permutation matrix elements are trivial for the isospin and can be calculated using standard $SU(2)$ recoupling techniques for the spin part. The color terms can be obtained for the $QQ\bar{n}\,\bar{n}$ from the relation

$$\begin{aligned} |1_{13}1_{24}\rangle &= \sqrt{\tfrac{1}{3}}|\bar{3}_{12}3_{34}\rangle + \sqrt{\tfrac{2}{3}}|6_{12}\bar{6}_{34}\rangle \\ |8_{13}8_{24}\rangle &= -\sqrt{\tfrac{2}{3}}|\bar{3}_{12}3_{34}\rangle + \sqrt{\tfrac{1}{3}}|6_{12}\bar{6}_{34}\rangle, \end{aligned} \tag{22}$$

and for the $Q\bar{Q}n\bar{n}$ from the relations

$$\begin{aligned} |\bar{3}_{13}3_{24}\rangle &= \sqrt{\tfrac{1}{3}}|1_{12}1_{34}\rangle - \sqrt{\tfrac{2}{3}}|8_{12}8_{34}\rangle \\ |6_{13}\bar{6}_{24}\rangle &= \sqrt{\tfrac{2}{3}}|1_{12}1_{34}\rangle + \sqrt{\tfrac{1}{3}}|8_{12}8_{34}\rangle; \end{aligned} \tag{23}$$

and

$$\begin{aligned} |1_{32}1_{14}\rangle &= \sqrt{\tfrac{1}{9}}|1_{12}1_{34}\rangle + \sqrt{\tfrac{8}{9}}|8_{12}8_{34}\rangle \\ |8_{32}8_{14}\rangle &= \sqrt{\tfrac{8}{9}}|1_{12}1_{34}\rangle - \sqrt{\tfrac{1}{9}}|8_{12}8_{34}\rangle. \end{aligned} \tag{24}$$

In the last case we should also consider the symmetrized states, Eqs. (16) and (17). For these states it can be easily shown that

$$\begin{aligned} &\langle (CI)^{|\Gamma_{12}\Gamma_{34}}(1234)|(C'I')^{|\Gamma'_{12}\Gamma'_{34}}(ijkl)\rangle \\ &\quad = \delta_{\Gamma_{12},\Gamma'_{12}}\delta_{\Gamma_{34},\Gamma'_{34}}\delta_{I,I'}\langle C(1234)|C'(ijkl)\rangle. \end{aligned} \tag{25}$$

Thus reducing the rearrangement matrix elements to Eqs. (23) and (24).

The permutation matrix elements for the HH functions are obtained using a numerical trick due to Efros [33]. Consider the particle positions $\mathbf{r}_i$. Using Eqs. (5) and (6) these positions are translated into a hyperradius ρ and a point on the hypersphere Ω. Under particle permutations $(1234) \to (ijkl)$ the point on the hypersphere will move to a new position, i.e., $\Omega \to \Omega_{ijkl}$, where the hyperradius remains invariant. Using the completeness of the HH basis and the fact the subspace defined by the quantum numbers K, L is invariant under particle permutations, we can express the HH functions of Ω_{ijkl} in the following way

$$\mathcal{Y}_{[K]}(\Omega_{ijkl}) = \sum_{[K']\in KL} \langle [K']|\hat{P}_{ijkl}|[K]\rangle \mathcal{Y}_{[K']}(\Omega), \tag{26}$$

where $\hat{P}_{ijkl}$ is the permutation operator. The sum contains N_{KL} terms, which amounts to the number of HH states with hyperspherical angular momentum K and orbital angular momentum L. Equation (26) is valid for any hyperangular point Ω. Therefore we can choose N_{KL} hyperangular points $\Omega(p)$, $p = 1 \ldots N_{KL}$ and get a set of N_{KL} equations which can be inverted to yield

$$\langle [K']|\hat{P}_{ijkl}|[K]\rangle = \sum_{p=1}^{N_{KL}} \mathcal{Y}_{[K]}(\Omega_{ijkl}(p))(\mathcal{Y}_{[K']}(\Omega(p)))^{-1}. \tag{27}$$

Here $(\mathcal{Y}_{[K']}(\Omega(p)))^{-1}$ stands for the inverse of the $N_{KL} \times N_{KL}$ matrix M whose entries are defined as $M_{[K'],p} = \mathcal{Y}_{[K']}(\Omega(p))$.

III. CONSTITUENT QUARK MODELS

For our study we will use two standard constituent quark models providing a reasonable description of the hadron spectra. A summary of the energies obtained within both models for selected meson states is given in Table I, in comparison with the corresponding experimental energies. In the following we draw the basic properties of the interacting potentials.

A. Bhaduri, Cohler, and Nogami model (BCN)

This model was proposed in the early 1980's by Bhaduri *et al.* in an attempt to obtain a unified description of meson and baryon spectroscopy [35]. It was later on applied to study the baryon spectra [36] and four-quark ($qq\bar{q}\bar{q}$) systems [13]. The model retains the most important terms of the one-gluon exchange interaction proposed by

TABLE I. Meson energies (in MeV) obtained with the quark models described in Sec. III. Experimental data (Exp.) are taken from Ref. [34], except for the state denoted by a dagger that has been taken from Ref. [21]. See text for the meaning of the different columns.

(L, S, J, I)	State	Exp.	CQC_{18}	CQC	BCN
		$n\bar{n}$			
(0,0,0,1)	π	139.0	139	496	136
(0,0,0,0)	$\eta(547)$	547.51 ± 0.18	572	772	136
(0,1,1,1)	$\rho(770)$	775.5 ± 0.4	772	744	777
(0,1,1,0)	$\omega(782)$	782.65 ± 0.12	691	651	777
(1,0,1,1)	$b_1(1235)$	1229.5 ± 3.2	1234	1232	1118
(1,0,1,0)	$h_1(1170)$	1170 ± 20	1257	1253	1118
(1,1,0,1)	$a_0(980)$	984.7 ± 1.2	1079	1269	1254
(1,1,0,0)	$f_0(600)$	400–1200	648	1262	1254
(1,1,1,1)	$a_1(1269)$	1230 ± 40	1221	1269	1254
(1,1,1,0)	$f_1(1285)$	1281.8 ± 0.6	1289	1262	1254
(1,1,2,1)	$a_2(1320)$	1318.3 ± 0.6	1315	1269	1254
(1,1,2,0)	$f_2(1270)$	1275.4 ± 1.1	1298	1262	1254
		$c\bar{n}$			
(0,0,0,0)	D	1864.5 ± 0.4	1883	1936	1886
(0,1,1,0)	$D^*(2007)$	2006.7 ± 0.4	2010	2001	2020
(1,1,0,0)	D_0^*	$2308.0 \pm 17 \pm 12^\dagger$	2465	2498	2491
(1,0,1,0)	$D_1(2420)$	2422.3 ± 1.3	2492	2490	2455
(1,1,1,0)	$D_1^*(2430)$	2427 ± 40	2504	2498	2491
(1,1,2,0)	$D_2^*(2460)$	2461.1 ± 1.6	2496	2498	2491
		$c\bar{c}$			
(0,0,0,0)	$\eta_c(1S)$	2980.4 ± 1.2	2990	3032	3038
(0,1,1,0)	$J/\psi(1S)$	3096.916 ± 0.011	3097	3094	3097
(1,0,1,0)	$h_c(1P)$	3525.93 ± 0.27	3507	3506	3502
(1,1,0,0)	$\chi_{c0}(1P)$	3414.76 ± 0.35	3443	3509	3519
(1,1,1,0)	$\chi_{c1}(1P)$	3510.66 ± 0.07	3496	3509	3519
(1,1,2,0)	$\chi_{c2}(1P)$	3556.20 ± 0.09	3525	3509	3519

de Rújula *et al.* [37], namely, coulomb and spin-spin terms, and a linear confining potential, having the form

$$V(\vec{r}_{ij}) = -\frac{3}{16}(\vec{\lambda}_i^c \cdot \vec{\lambda}_j^c) \times \left(\frac{r_{ij}}{a^2} - \frac{\kappa}{r_{ij}} - D + \frac{\kappa}{m_i m_j}\frac{e^{-r_{ij}/r_0}}{r_{ij} r_0^2}(\vec{\sigma}_i \cdot \vec{\sigma}_j)\right), \tag{28}$$

where $\vec{\sigma}_i$ are the Pauli matrices and $\vec{\lambda}_i^c$ are the $SU(3)$ color matrices. The parameters $\kappa = 102.67$ MeV fm, $D = 913.5$ MeV, $a = 0.0326$ MeV$^{-1/2}$ fm$^{1/2}$, $r_0 = 2.2$ fm, $m_{u,d} = 337$ MeV, and $m_c = 1870$ MeV are taken from Ref. [13].

B. Constituent quark cluster model (CQC)

This model was proposed in the early 1990's in an attempt to obtain a simultaneous description of the nucleon-nucleon interaction and the baryon spectra [38]. It was later on generalized to all flavor sectors giving a reasonable description of the meson [39] and baryon spectra [40]. The possible existence of four-quark states within this model has also been addressed [25,27,28].

The model is based on the assumption that the light-quark constituent mass appears because of the spontaneous breaking of the original $SU(3)_L \otimes SU(3)_R$ chiral symmetry at some momentum scale. In this domain of momenta, quarks interact through Goldstone boson exchange potentials. QCD perturbative effects are taken into account through the one-gluon-exchange (OGE) potential as the one used in the BCN model. Finally, it incorporates confinement as dictated by unquenched lattice calculations predicting, for heavy quarks, a screening effect on the linearly dependent interquark potential when increasing the interquark distance [41].

The model parameters have been taken from Ref. [39] with the exception of the OGE regularization parameter. This parameter, taking into account the size of the system, was fitted for four-quark states in the description of the light scalar sector [27], being $\hat{r}_0 = 0.18$ fm for mesons and $\hat{r}_0 = 0.38$ fm for four-quark systems. Let us also notice that the CQC model contains an interaction generating flavor mixing between $n\bar{n}$ and $s\bar{s}$ components. It allows us to exactly reproduce the masses of the η and η' mesons. In the four-quark case this contribution would mix isospin zero $Q\bar{Q}n\bar{n}$ and $Q\bar{Q}s\bar{s}$ components. Such contributions were explicitly evaluated in the variational approach of Ref. [27] for the light isocalar tetraquarks, giving a negligible effect. Therefore, such flavor-mixing components will not be consider in the present calculation. In order to make a proper comparison between thresholds and four-quark states we have recalculated the meson spectra of Ref. [39] with the same $\hat{r}_0$ value and interaction used in the four-quark calculation, neglecting therefore the flavor-mixing terms. The results are summarized in Table I for the original meson parametrization (CQC$_{18}$) and for the one used in this work (CQC) (note that in this case the $\eta(547)$ would corresponds to a pure $n\bar{n}$ state). Explicit expressions of the interacting potentials and a more detailed discussion of the model can be found in Ref. [39].

IV. RESULTS

A. Threshold determination

The existence of the color degree of freedom gives rise to an important difference between four-quark systems and standard baryons or mesons. For baryons and mesons it is not possible to construct a color singlet using a subset of the constituents, thus only $q\bar{q}$ or qqq states are proper solutions of the two- or three-quark interacting Hamiltonian and therefore, all solutions correspond to bound states. However, this is not the case for four-quark systems. The color rearrangement of Eqs. (22) and (24), $(q\bar{q})_1 \otimes (q\bar{q})_1 = (qq\bar{q}\,\bar{q})_1$, makes possible that two isolated mesons are also a solution of the four-quark Hamiltonian. In order to discriminate between four-quark bound states and simple pieces of the meson-meson continuum, one has to analyze the two-meson states that constitute the thresholds for each set of quantum numbers.

TABLE II. Lowest two-meson thresholds requiring J, P, and C quantum number conservation. Energies are in MeV.

	Experiment		CQC		BCN	
J^{PC}	$I=0$	$I=1$	$I=0$	$I=1$	$I=0$	$I=1$
0^{++}	$\eta_c\eta\|_S$ 3528	$\eta_c\pi\|_S$ 3119	$J/\psi\omega\|_{S,D}$ 3745	$\eta_c\pi\|_S$ 3528	$\eta_c\eta\|_S$ 3174	$\eta_c\pi\|_S$ 3174
0^{+-}	$J/\psi f_0\|_P$ 3697	$h_c\pi\|_P$ 3665	$\chi_{cJ}\omega\|_P$ 4160	$h_c\pi\|_P$ 4002	$h_c\eta\|_P$ 3638	$h_c\pi\|_P$ 3638
1^{++}	$\eta_c f_0\|_P$ 3580	$\chi_{c0}\pi\|_P$ 3554	$J/\psi\omega\|_{S,D}$ 3745	$J/\psi\rho\|_{S,D}$ 3838	$\chi_{cJ}\eta\|_P$ 3655	$\chi_{cJ}\pi\|_P$ 3655
1^{+-}	$J/\psi\eta\|_{S,D}$ 3644	$J/\psi\pi\|_{S,D}$ 3236	$\eta_c\omega\|_{S,D}$ 3683	$J/\psi\pi\|_{S,D}$ 3590	$J/\psi\eta\|_{S,D}$ 3233	$J/\psi\pi\|_{S,D}$ 3233
2^{++}	$\eta_c\eta\|_D$ 3528	$\eta_c\pi\|_D$ 3119	$J/\psi\omega\|_{S,D}$ 3745	$\eta_c\pi\|_D$ 3528	$\eta_c\eta\|_D$ 3174	$\eta_c\pi\|_D$ 3174
2^{+-}	$J/\psi\eta\|_D$ 3644	$J/\psi\pi\|_D$ 3236	$\eta_c\omega\|_D$ 3683	$J/\psi\pi\|_D$ 3590	$J/\psi\eta\|_D$ 3233	$J/\psi\pi\|_D$ 3233
0^{-+}	$\eta_c f_0\|_S$ 3580	$\chi_{c0}\pi\|_{S,D}$ 3554	$J/\psi\omega\|_P$ 3745	$J/\psi\rho\|_P$ 3838	$\chi_{cJ}\eta\|_{S,D}$ 3655	$\chi_{cJ}\pi\|_{S,D}$ 3655
0^{--}	$J/\psi\eta\|_P$ 3644	$J/\psi\pi\|_P$ 3236	$\eta_c\omega\|_P$ 3683	$J/\psi\pi\|_P$ 3590	$J/\psi\eta\|_P$ 3233	$J/\psi\pi\|_P$ 3233
1^{-+}	$\eta_c\eta\|_P$ 3528	$\eta_c\pi\|_P$ 3119	$J/\psi\omega\|_P$ 3745	$\eta_c\pi\|_P$ 3528	$\eta_c\eta\|_P$ 3174	$\eta_c\pi\|_P$ 3174
1^{--}	$J/\psi\eta\|_P$ 3644	$J/\psi\pi\|_P$ 3236	$\eta_c\omega\|_P$ 3683	$J/\psi\pi\|_P$ 3590	$J/\psi\eta\|_P$ 3233	$J/\psi\pi\|_P$ 3233
2^{-+}	$\eta_c f_0\|_D$ 3580	$\chi_{c0}\pi\|_D$ 3554	$J/\psi\omega\|_P$ 3745	$J/\psi\rho\|_P$ 3838	$\chi_{cJ}\eta\|_{S,D}$ 3655	$\chi_{cJ}\pi\|_{S,D}$ 3655
2^{--}	$J/\psi\eta\|_P$ 3644	$J/\psi\pi\|_P$ 3236	$\eta_c\omega\|_P$ 3683	$J/\psi\pi\|_P$ 3590	$J/\psi\eta\|_P$ 3233	$J/\psi\pi\|_P$ 3233
3^{-+}	$J/\psi\omega\|_P$ 3880	$\chi_{c1}\pi\|_D$ 3650	$J/\psi\omega\|_P$ 3745	$J/\psi\rho\|_P$ 3838	$\chi_{cJ}\eta\|_D$ 3655	$\chi_{cJ}\pi\|_D$ 3655
3^{--}	$J/\psi f_0\|_D$ 3697	$h_c\pi\|_D$ 3665	$D^*\bar{D}^*\|_P$ 4002	$D^*\bar{D}^*\|_P$ 4002	$h_c\eta\|_D$ 3638	$h_c\pi\|_D$ 3638

TABLE III. Lowest two-meson experimental thresholds imposing L, S, J, P, and C quantum number conservation. Energies are in MeV.

J^{PC}	Experiment					
	$I=0$			$I=1$		
	$(L,S)=(0,0)$	$(L,S)=(0,1)$	$(L,S)=(0,2)$	$(L,S)=(0,0)$	$(L,S)=(0,1)$	$(L,S)=(0,2)$
0^{++}	$\eta_c\eta\|_S$	—	—	$\eta_c\pi\|_S$	—	—
	3528	—	—	3119	—	—
0^{+-}	$J/\psi f_0\|_P$	—	—	$h_c\pi\|_P$	—	—
	3697	—	—	3665	—	—
1^{++}	—	$\eta_c f_0\|_P$	—	—	$\chi_{c0}\pi\|_P$	—
	—	3580	—	—	3554	—
1^{+-}	—	$J/\psi\eta\|_S$	—	—	$J/\psi\pi\|_S$	—
	—	3644	—	—	3236	—
2^{++}	—	—	$J/\psi\omega\|_S$	—	—	$J/\psi\rho\|_S$
	—	—	3880	—	—	3873
2^{+-}	—	—	$J/\psi f_0\|_P$	—	—	$J/\psi a_0\|_P$
	—	—	3697	—	—	4082
	$(L,S)=(1,0)$	$(L,S)=(1,1)$	$(L,S)=(1,2)$	$(L,S)=(1,0)$	$(L,S)=(1,1)$	$(L,S)=(1,2)$
0^{-+}	—	$\eta_c f_0\|_S$	—	—	$\chi_{c0}\pi\|_S$	—
	—	3580	—	—	3554	—
0^{--}	—	$J/\psi\eta\|_P$	—	—	$J/\psi\pi\|_P$	—
	—	3644	—	—	3236	—
1^{-+}	$\eta_c\eta\|_P$	$D\bar{D}^*\|_P$	$J/\psi\omega\|_P$	$\eta_c\pi\|_P$	$\chi_{c1}\pi\|_{S,D}$	$J/\psi\rho\|_P$
	3528	3871	3880	3119	3650	3873
1^{--}	$J/\psi f_0\|_{S,D}$	$J/\psi\eta\|_P$	$J/\psi f_0\|_{S,D}$	$h_c\pi\|_{S,D}$	$J/\psi\pi\|_P$	$D^*\bar{D}^*\|_P$
	3697	3644	3697	3665	3236	4018
2^{-+}	—	$\eta_c f_0\|_D$	$J/\psi\omega\|_P$	—	$\chi_{c0}\pi\|_S$	$J/\psi\rho\|_P$
	—	3580	3880	—	3554	3873
2^{--}	—	$J/\psi\eta\|_P$	$J/\psi f_0\|_D$	—	$J/\psi\pi\|_P$	$D^*\bar{D}^*\|_P$
	—	3644	3697	—	3236	4018
3^{-+}	—	—	$J/\psi\omega\|_P$	—	—	$J/\psi\rho\|_P$
	—	—	3880	—	—	3873
3^{--}	—	—	$J/\psi f_0\|_D$	—	—	$D^*\bar{D}^*\|_P$
	—	—	3697	—	—	4018

These thresholds must be determined assuming quantum number conservation within exactly the same scheme used in the four-quark calculation. Dealing with strongly interacting particles, the two-meson states should have well-defined total angular momentum (J), parity (P), and C-parity (C). When noncentral forces are not considered, orbital angular momentum (L) and total spin (S) are also good quantum numbers. As the systems studied could dissociate either into $(c\bar{c})(n\bar{n})$ or $(c\bar{n})(n\bar{c})$, we indicate the lowest two-meson threshold in both channels, quoting also the final state relative angular momentum. We give in Table II the lowest thresholds requiring J, P, and C conservation, while in Tables III, IV, and V we quote those when L and S are also preserved. We give the experimental thresholds, corresponding to the energies in Ref. [34], the thresholds obtained with the BCN model, and those calculated with the CQC model as described in Sec. III.

A property of $c\bar{c}n\bar{n}$ states, that is crucial for the discussion on the possible existence of bound states, is that two different physical thresholds can always be constructed for any set of quantum numbers, corresponding to the $(c\bar{c})(n\bar{n})$ and $(c\bar{n})(n\bar{c})$ couplings. We show an example in Table VI, where we give the $J^{PC}=1^{++}$ lowest threshold in the two possible couplings. This is not a general property for any four-quark system, note for instance that a $cc\bar{n}\bar{n}$ four-quark state only has one allowed physical threshold, corresponding to the coupling $(c\bar{n})(c\bar{n})$.

B. The four-quark $c\bar{c}n\bar{n}$ spectra

Once we have developed a method to study four-quark systems of different flavor, we will address an important physical question making contact with the actual experimental situation: Does the quark model naturally predict the existence of $c\bar{c}n\bar{n}$ bound states? For this purpose we have performed an exhaustive analysis of the $c\bar{c}n\bar{n}$ spectra by means of the two different quark models, CQC and BCN, described in Sec. III. We have considered all isoscalar states with total orbital angular momentum $L \leq 1$.

TABLE IV. Same as Table III for CQC.

J^{PC}	CQC					
	$I=0$			$I=1$		
	$(L,S)=(0,0)$	$(L,S)=(0,1)$	$(L,S)=(0,2)$	$(L,S)=(0,0)$	$(L,S)=(0,1)$	$(L,S)=(0,2)$
0^{++}	$J/\psi\omega\|_S$	—	—	$\eta_c\pi\|_S$	—	—
	3745	—	—	3528	—	—
0^{+-}	$\chi_{cJ}\omega\|_P$	—	—	$h_c\pi\|_P$	—	—
	4160	—	—	4002	—	—
1^{++}	—	$J/\psi\omega\|_S$	—	—	$J/\psi\rho\|_S$	—
	—	3745	—	—	3838	—
1^{+-}	—	$\eta_c\omega\|_S$	—	—	$J/\psi\pi\|_S$	—
	—	3683	—	—	3590	—
2^{++}	—	—	$J/\psi\omega\|_S$	—	—	$J/\psi\rho\|_S$
	—	—	3745	—	—	3838
2^{+-}	—	—	$\chi_{cJ}\omega\|_P$	—	—	$\chi_{cJ}\rho\|_P$
	—	—	4160	—	—	4253
	$(L,S)=(1,0)$	$(L,S)=(1,1)$	$(L,S)=(1,2)$	$(L,S)=(1,0)$	$(L,S)=(1,1)$	$(L,S)=(1,2)$
0^{-+}	—	$J/\psi\omega\|_P$	—	—	$J/\psi\rho\|_P$	—
	—	3745	—	—	3838	—
0^{--}	—	$\eta_c\omega\|_P$	—	—	$J/\psi\pi\|_P$	—
	—	3683	—	—	3590	—
1^{-+}	$J/\psi\omega\|_P$	$J/\psi\omega\|_P$	$J/\psi\omega\|_P$	$\eta_c\pi\|_P$	$J/\psi\rho\|_P$	$J/\psi\rho\|_P$
	3745	3745	3745	3528	3838	3838
1^{--}	$D\bar{D}\|_P$	$\eta_c\omega\|_P$	$D^*\bar{D}^*\|_P$	$D\bar{D}\|_P$	$J/\psi\pi\|_P$	$D^*\bar{D}^*\|_P$
	3872	3683	4002	3872	3590	4002
2^{-+}	—	$J/\psi\omega\|_P$	$J/\psi\omega\|_P$	—	$J/\psi\rho\|_P$	$J/\psi\rho\|_P$
	—	3745	3745	—	3838	3838
2^{--}	—	$\eta_c\omega\|_P$	$D^*\bar{D}^*\|_P$	—	$J/\psi\pi\|_P$	$D^*\bar{D}^*\|_P$
	—	3683	4002	—	3590	4002
3^{-+}	—	—	$J/\psi\omega\|_P$	—	—	$J/\psi\rho\|_P$
	—	—	3745	—	—	3838
3^{--}	—	—	$D^*\bar{D}^*\|_P$	—	—	$D^*\bar{D}^*\|_P$
	—	—	4002	—	—	4002

For positive parity, the lowest states correspond to $L=0$, while $L=1$ for negative parity ground states. The reason is that the parity of a four-quark state can be written in terms of the relative angular momenta associated with the Jacobi coordinates as $P=(-)^{\ell_1+\ell_2+\ell_3}$. This makes that $P=-1$ states need three units of relative angular momentum to obtain $L=0$ ($\ell_1=\ell_2=\ell_3=1$) while only one is needed for $L=1$. The same reasoning applies for $P=+1$ states. The calculation has been done up to the maximum value of K within our computational capabilities, $K_{\rm max}$.

The absolute energy obtained for each state does not provide much information regarding the stability of the system. The relevant quantity is Δ_E, defined as the energy difference between the mass of the four-quark system and that of the lowest corresponding threshold,

$$\Delta_E = E_{4q} - E(M_1, M_2). \tag{29}$$

Here, E_{4q} stands for the four-quark energy and $E(M_1, M_2)$ for the energy of the corresponding threshold. Using this definition, $\Delta_E < 0$ will indicate that all fall-apart strong decays are forbidden, i.e., one has a proper bound state. On the other hand, $\Delta_E = 0$ will indicate that the four-quark solution corresponds to an unbound threshold (two free mesons) state.

One of the main difficulties in studying four-quark states was discussed in Ref. [28], i.e., the slow convergence to the asymptotic two free mesons unbound states. This makes the identification of threshold states a cumbersome task, demanding large values of K. In Ref. [28] it was proposed that this problem could be overcome by means of an extrapolation of the four-quark energy using the expression

$$E(K) = E(K=\infty) + \frac{a}{K^b}, \tag{30}$$

where $E(K=\infty)$, a and b are fitted parameters.

Although the study of the energy is the most powerful way to distinguish between bound and unbound four-quark states, we have taken a step further analyzing in detail the structure of the wave function. In particular, it is possible to

TABLE V. Same as Table III for BCN.

J^{PC}	BCN $I=0$			BCN $I=1$		
	$(L,S)=(0,0)$	$(L,S)=(0,1)$	$(L,S)=(0,2)$	$(L,S)=(0,0)$	$(L,S)=(0,1)$	$(L,S)=(0,2)$
0^{++}	$\eta_c\eta\vert_S$	—	—	$\eta_c\pi\vert_S$	—	—
	3174	—	—	3174	—	—
0^{+-}	$h_c\eta\vert_P$	—	—	$h_c\pi\vert_P$	—	—
	3638	—	—	3638	—	—
1^{++}	—	$\chi_{cJ}\eta\vert_P$	—	—	$\chi_{cJ}\pi\vert_P$	—
	—	3655	—	—	3655	—
1^{+-}	—	$J/\psi\eta\vert_S$	—	—	$J/\psi\pi\vert_S$	—
	—	3233	—	—	3233	—
2^{++}	—	—	$J/\psi\omega\vert_S$	—	—	$J/\psi\rho\vert_S$
	—	—	3874	—	—	3874
2^{+-}	—	—	$\chi_{cJ}\omega\vert_P$	—	—	$\chi_{cJ}\rho\vert_P$
	—	—	4296	—	—	4296
	$(L,S)=(1,0)$	$(L,S)=(1,1)$	$(L,S)=(1,2)$	$(L,S)=(1,0)$	$(L,S)=(1,1)$	$(L,S)=(1,2)$
0^{-+}	—	$\chi_{cJ}\eta\vert_{S,D}$	—	—	$\chi_{cJ}\pi\vert_S$	—
	—	3655	—	—	3655	—
0^{--}	—	$J/\psi\eta\vert_P$	—	—	$J/\psi\pi\vert_P$	—
	—	3233	—	—	3233	—
1^{-+}	$\eta_c\eta\vert_P$	$\chi_{cJ}\eta\vert_{S,D}$	$J/\psi\omega\vert_P$	$\eta_c\pi\vert_P$	$\chi_{cJ}\pi\vert_{S,D}$	$J/\psi\rho\vert_P$
	3174	3655	3874	3174	3655	3874
1^{--}	$h_c\eta\vert_{S,D}$	$J/\psi\eta\vert_P$	$D^*\bar{D}^*\vert_P$	$h_c\pi\vert_{S,D}$	$J/\psi\pi\vert_P$	$D^*\bar{D}^*\vert_P$
	3638	3233	4040	3638	3233	4040
2^{-+}	—	$\chi_{cJ}\eta\vert_{S,D}$	$J/\psi\omega\vert_P$	—	$\chi_{cJ}\pi\vert_{S,D}$	$J/\psi\rho\vert_P$
	—	3655	3874	—	3655	3874
2^{--}	—	$J/\psi\eta\vert_P$	$D^*\bar{D}^*\vert_P$	—	$J/\psi\pi\vert_P$	$D^*\bar{D}^*\vert_P$
	—	3233	4040	—	3233	4040
3^{-+}	—	—	$J/\psi\omega\vert_P$	—	—	$J/\psi\rho\vert_P$
	—	—	3874	—	—	3874
3^{--}	—	—	$D^*\bar{D}^*\vert_P$	—	—	$D^*\bar{D}^*\vert_P$
	—	—	4040	—	—	4040

determine if a four-quark system behaves as a pure meson-meson state or if it has a more involved structure through the analysis of the dominant components of the wave function. Any solution of the four-quark problem that could be identified with a threshold should verify not only that $\Delta_E = 0$ but also that the probability of the threshold within the four-quark wave function should be unity. One can also study the behavior of the root mean square radius (RMS) of the four-quark system as compared to the radii of the two-mesons threshold. The RMS is defined in the usual way for four (two) quark systems

$$\mathrm{RMS}\,|_{4(2)} = \left(\frac{\sum_{i=1}^{4(2)} m_i \langle(\boldsymbol{r}_i - \boldsymbol{R}_{\mathrm{CM}})^2\rangle}{\sum_{i=1}^{4(2)} m_i}\right)^{1/2}. \qquad (31)$$

Combining all this information it could be claimed that any four-quark state with the following characteristics: (i) $\Delta_E \to 0$ with K; (ii) $\mathrm{RMS} \to \infty$ with K, (its value should at least exceed the value corresponding to the threshold system); (iii) The wave-function tends to a single singlet-singlet physical channel; should be considered as an unbound threshold state.

We present in Table VII the results obtained for all possible $L \leq 1$ isoscalar channels with both quark models, CQC and BCN. We indicate the maximum value of K used, $K_{\max}$. We also indicate the probability of the basis vector

TABLE VI. $(L,S)=(0,1)$ $J^{PC}=1^{++}$ lowest two-meson thresholds for S-, P-, and D-wave final state relative angular momentum. Both possible couplings, $(c\bar{n})(n\bar{c})$ and $(c\bar{c})(n\bar{n})$, are considered. Energies are in MeV.

	$(c\bar{n})(n\bar{c})$			$(c\bar{c})(n\bar{n})$		
Experiment	$D\bar{D}^*\vert_S$	$D\bar{D}_0^*\vert_P$	$D_1\bar{D}_0^*\vert_D$	$J/\psi\omega\vert_S$	$\eta_c f_0\vert_P$	$\chi_{c1} f_0\vert_D$
	3871	4176	4731	3880	3580	4111
CQC	$D\bar{D}^*\vert_S$	$D\bar{D}_J^*\vert_P$	$D_1\bar{D}_J^*\vert_D$	$J/\psi\omega\vert_S$	$h_c\omega\vert_P$	$\chi_{cJ} f_J\vert_D$
	3937	4434	4988	3745	4157	4771
BCN	$D\bar{D}^*\vert_S$	$D\bar{D}_J^*\vert_P$	$D_1\bar{D}_J^*\vert_D$	$J/\psi\omega\vert_S$	$\chi_{cJ}\eta\vert_P$	$\chi_{cJ} f_J\vert_D$
	3906	4377	4946	3874	3655	4773

TABLE VII. Energy, $E_{4q} \equiv E_{4q}(K_{\max})$ (in MeV) and probability, P_T, of the basis vector corresponding to the lowest threshold for CQC and BCN models. $T(M_1, M_2)$ indicates the lowest physical threshold, E_T its energy as obtained from Tables III, IV, and V and Δ_E is defined in Eq. (29). We also quote in the last three columns the experimental thresholds.

$K_{\max}$	$(L, S)J^{PC}$	E_{4q}	P_T	$T(M_1, M_2)$	E_T	Δ_E	$T(M_1, M_2)$	E_T	Δ_E
		CQC			CQC			Exp.	
24	$(0,0)0^{++}$	3779	0.9954	$J/\psi\omega\|_S$	3745	+34	$\eta_c\eta\|_S$	3528	+251
22	$(0,0)0^{+-}$	4224	0.9995	$\chi_{cJ}\omega\|_P$	4160	+64	$J/\psi f_0\|_P$	3697	+438
20	$(0,1)1^{++}$	3786	0.9968	$J/\psi\omega\|_S$	3745	+41	$\eta_c f_0\|_P$	3580	+206
22	$(0,1)1^{+-}$	3728	0.9983	$\eta_c\omega\|_P$	3683	+45	$J/\psi\eta\|_S$	3644	+84
28	$(0,2)2^{++}$	3774	0.9989	$J/\psi\omega\|_S$	3745	+29	$J/\psi\omega\|_S$	3880	−106
28	$(0,2)2^{+-}$	4214	0.9997	$\chi_{cJ}\omega\|_P$	4160	+54	$J/\psi f_0\|_P$	3697	+517
19	$(1,0)1^{-+}$	3829	0.9998	$J/\psi\omega\|_P$	3745	+84	$\eta_c\eta\|_P$	3528	+301
19	$(1,0)1^{--}$	3969	0.9451	$D\bar{D}\|_P$	3872	+97	$J/\psi f_0\|_{S,D}$	3697	+272
17	$(1,1)(0,1,2)^{-+}$	3839	0.9998	$J/\psi\omega\|_P$	3745	+94	$\eta_c f_0\|_{S,D}$ [a]	3580	+259
							$D\bar{D}^*\|_P$ [b]	3871	−32
17	$(1,1)(0,1,2)^{--}$	3791	0.9997	$\eta_c\omega\|_P$	3683	+108	$J/\psi\eta\|_P$	3644	+147
21	$(1,2)(1,2,3)^{-+}$	3820	0.9999	$J/\psi\omega\|_S$	3745	+75	$J/\psi\omega\|_P$	3880	−60
21	$(1,2)(1,2,3)^{--}$	4054	0.9999	$D^*\bar{D}^*\|_P$	4002	+52	$J/\psi f_0\|_D$	3697	+357
		BCN			BCN			Exp.	
26	$(0,0)0^{++}$	3249	0.9993	$\eta_c\eta\|_S$	3174	+75	$\eta_c\eta\|_S$	3528	−279
24	$(0,0)0^{+-}$	3778	0.9997	$h_c\eta\|_P$	3638	+140	$J/\psi f_0\|_P$	3697	+81
22	$(0,1)1^{++}$	3808	0.9997	$\chi_{cJ}\eta\|_P$	3655	+153	$\eta_c f_0\|_P$	3580	+228
22	$(0,1)1^{+-}$	3319	0.9993	$J/\psi\eta\|_S$	3233	+86	$J/\psi\eta\|_S$	3644	−325
26	$(0,2)2^{++}$	3897	0.9987	$J/\psi\omega\|_S$	3874	+23	$J/\psi\omega\|_S$	3880	+17
28	$(0,2)2^{+-}$	4328	0.9998	$\chi_{cJ}\omega\|_P$	4296	+32	$J/\psi f_0\|_P$	3697	+631
21	$(1,0)1^{-+}$	3331	0.9999	$\eta_c\eta\|_P$	3174	+157	$\eta_c\eta\|_P$	3528	−197
21	$(1,0)1^{--}$	3732	0.9934	$h_c\eta\|_{S,D}$	3638	+94	$J/\psi f_0\|_{S,D}$	3697	+35
19	$(1,1)(0,1,2)^{-+}$	3760	0.9950	$\chi_{cJ}\eta\|_{S,D}$	3655	+105	$\eta_c f_0\|_{S,D^1}$	3580	+180
							$D\bar{D}^*\|_{P^2}$	3871	−111
19	$(1,1)(0,1,2)^{--}$	3405	0.9998	$J/\psi\eta\|_P$	3233	+172	$J/\psi\eta\|_P$	3644	−239
21	$(1,2)(1,2,3)^{-+}$	3929	0.9999	$J/\psi\omega\|_S$	3874	+55	$J/\psi\omega\|_P$	3880	+49
21	$(1,2)(1,2,3)^{--}$	4092	0.9999	$D^*\bar{D}^*\|_P$	4040	+52	$J/\psi f_0\|_D$	3697	+395

[a] S-wave for the $J = 0$ threshold and D-wave for the $J = 2$. [b] $J = 1$ state.

corresponding to the lowest physical threshold. Let us first of all concentrate on the results of the two quark models used, we will comment later on the comparison with the experimental data. There is a first general conclusion immediately derived looking at this table, namely, that no bound state is observed for any set of quantum numbers in any of the models, in all cases $\Delta_E > 0$. Let us analyze the results in detail. The convergence of the results is illustrated in Table VIII, where we show the evolution of the energy, radius and probabilities as a function of K for two different channels with both quark models. We have denoted by $P_{C_{12}C_{34}}(S_{12}, S_{34})$ the probability of the basis vector with color state $C_{12} \otimes C_{34}$ and spin $S = S_{12} \otimes S_{34}$ in the $(c\bar{c})(n\bar{n})$ coupling. For each model, we compare in the bottom part of the table with the lowest energy threshold and its RMS (the sum of the RMS's of the two mesons). One can see how the four-quark energies do converge to the lowest possible threshold at the same time that the radius increases linearly and the probability of the vector that characterizes the threshold tends to unity. When the extrapolation (30) is used, we observe how the four-quark energies reproduce those of the lowest threshold allowed for each set of quantum numbers. This is illustrated in Table IX, where we indicate $E(K = \infty)$ as a function of the initial and final values of K used for the fitting. The thresholds are perfectly reproduced within a difference, due to the extrapolation, of a few MeV. These are general features for all the states in Table VII.

Let us emphasize the importance of comparing the four-quark energies with the proper mathematical threshold, if this is not done it could easily lead to the misidentification of bound states. This is made evident on the last columns of Table VII, where we have quoted the experimental thresholds. Looking at column Δ_E under the epigraph Exp., one can see the abundance of *spurious* bound states predicted by the BCN model (five) if the experimental thresholds are considered instead the correct, theoretical, ones.

Special attention must also be paid to some numerical approximations used for solving the four-quark problem. In the numerical procedure described in Sec. II one can

TABLE VIII. Energy (MeV), radius (fm), and probability of the different components of the four-quark wave function as a function of K for the $J^{PC}=2^{++}$ and $J^{PC}=2^{+-}$ $(L,S)=(0,2)$ channels. The lowest threshold is quoted in the bottom part of the table.

		$J^{PC}=2^{++}$					$J^{PC}=2^{+-}$		
K	E	RMS	$P_{11}(1,1)$	$P_{88}(1,1)$	K	E	RMS	$P_{11}(1,1)$	$P_{88}(1,1)$
				CQC					
0	4140	0.3418	1.0000	0.0000	0	—	—	—	—
2	3986	0.3697	0.9890	0.0110	2	4689	0.4851	0.9998	0.0002
4	3912	0.4010	0.9884	0.0116	4	4508	0.5183	0.9962	0.0038
6	3872	0.4337	0.9913	0.0087	6	4414	0.5567	0.9961	0.0039
8	3846	0.4669	0.9934	0.0066	8	4359	0.5967	0.9974	0.0026
10	3828	0.5001	0.9951	0.0049	10	4321	0.6372	0.9981	0.0019
12	3815	0.5329	0.9962	0.0038	12	4295	0.6775	0.9987	0.0013
14	3805	0.5652	0.9970	0.0030	14	4275	0.7175	0.9990	0.0010
16	3798	0.5970	0.9975	0.0025	16	4260	0.7570	0.9992	0.0008
18	3792	0.6283	0.9980	0.0020	18	4248	0.7959	0.9994	0.0006
20	3787	0.6590	0.9983	0.0017	20	4239	0.8342	0.9995	0.0005
22	3783	0.6889	0.9985	0.0015	22	4231	0.8718	0.9996	0.0004
24	3779	0.7185	0.9987	0.0013	24	4224	0.9087	0.9997	0.0003
26	3776	0.7475	0.9989	0.0011	26	4219	0.9451	0.9997	0.0003
28	3774	—	—	—	28	4214	—	—	—
$J/\psi\omega\vert_S$	3745	0.5745	1	0	$\chi_{cJ}\omega\vert_P$	4160	0.6873	1	0
				BCN					
0	4196	0.3393	1.0000	0.0000					
2	4057	0.3778	0.9862	0.0138	2	4732	0.4628	0.9943	0.0057
4	3999	0.4168	0.9865	0.0135	4	4557	0.5127	0.9929	0.0071
6	3968	0.4568	0.9899	0.0101	6	4478	0.5639	0.9939	0.0061
8	3948	0.4972	0.9923	0.0077	8	4433	0.6163	0.9964	0.0036
10	3935	0.5375	0.9944	0.0056	10	4405	0.6694	0.9977	0.0023
12	3925	0.5775	0.9957	0.0043	12	4385	0.7226	0.9986	0.0014
14	3918	0.6168	0.9966	0.0034	14	4371	0.7758	0.9990	0.0010
16	3913	0.6561	0.9973	0.0027	16	4360	0.8288	0.9993	0.0007
18	3908	0.6946	0.9978	0.0022	18	4352	0.8816	0.9995	0.0005
20	3905	0.7309	0.9981	0.0019	20	4345	0.9341	0.9996	0.0004
22	3902	0.7720	0.9984	0.0016	22	4340	0.9863	0.9997	0.0003
24	3899	0.8091	0.9987	0.0013	24	4335	1.0382	0.9998	0.0002
26	3897	—	—	—	26	4332	1.0899	0.9998	0.0002
28					28	4328	—	—	—
$J/\psi\omega\vert_S$	3874	0.6133	1	0	$\chi_{cJ}\omega\vert_P$	4296	0.7259	1	0

easily restrict the method to perform a calculation considering only a limited set of relative angular momenta between the quarks. We show in Table X the results obtained neglecting large relative orbital angular momenta ($\sum_i \ell_i \leq 1$) for two different set of quantum numbers. As can be seen, this approximation is excellent for those states whose solution can be expressed exclusively in terms of $(c\bar{c})$ and $(n\bar{n})$ mesons [the ℓ_i are the relative angular momenta associated to the Jacobi coordinates in the coupling $(c\bar{c})\times(n\bar{n})$], as it is the case of $J^{PC}=1^{++}$, while it fails for those states with a more involved structure. Since one does not know *a priori* the behavior of a particular channel it is inadvisable to perform such an approximation and therefore, a full calculation is required for any global analysis of four-quark states. A similar effect related with the restriction of the Hilbert space can also be observed in Ref. [13], where an analysis of the four-quark problem was performed with a variational solution of the BCN model in a harmonic oscillator basis up to $N=8$, obtaining 3409 and 3468 MeV for the $J^{PC}=0^{++}$ and 1^{+-} states, respectively. Being the hyperspherical harmonic basis for $K=8$ roughly equivalent to the $N=8$ harmonic oscillator basis, we have obtained 3380 and 3436 MeV, respectively, for these states when we restrict ourselves to $K=8$. However, once we allow K to take larger values, the energy decreases more than 200 MeV until the lowest threshold for each channel is obtained.

Among all channels presented in Table VII, three of them deserve a careful analysis: $(L,S)=(1,0)$ for CQC, and $(L,S)=(1,2)$ for CQC and BCN. Their energies,

TABLE IX. Energies (in MeV) obtained using the extrapolation Eq. (30) for the states of Table VIII.

$J^{PC} = 2^{++}$		$J^{PC} = 2^{+-}$	
(K_o, K_f)	$E(K = \infty)$	(K_o, K_f)	$E(K = \infty)$
	CQC		
(2,28)	3657	(2,28)	4042
(4,28)	3704	(4,28)	4106
(6,28)	3720	(6,28)	4124
(8,28)	3727	(8,28)	4135
(10,28)	3731	(10,28)	4141
(12,28)	3734	(12,28)	4145
(14,28)	3736	(14,28)	4148
(16,28)	3737	(16,28)	4150
(18,28)	3739	(18,28)	4151
(20,28)	3739	(20,28)	4153
(22,28)	3740	(22,28)	4154
(24,28)	3741	(24,28)	4156
Threshold	3745	Threshold	4160
	BCN		
(2,26)	3816	(2,28)	4242
(4,26)	3848	(4,28)	4268
(6,26)	3859	(6,28)	4277
(8,26)	3864	(8,28)	4282
(10,26)	3867	(10,28)	4286
(12,26)	3869	(12,28)	4288
(14,26)	3871	(14,28)	4290
(16,26)	3872	(16,28)	4291
(18,26)	3872	(18,28)	4292
(20,26)	3873	(20,28)	4293
(22,26)	3874	(22,28)	4294
		(24,28)	4296
Threshold	3874	Threshold	4296

probabilities, and radius are resumed in Tables XI and XII as a function of K. We observe that they have a dominant octet-octet color component that could be interpreted as evidence of a compact four-quark system. However, since the lowest threshold in all three cases corresponds to a $D\bar{D}$ or $D^*\bar{D}^*$ state, the coupling where the numerical calculation is done, $(c\bar{c})(n\bar{n})$, is not the most appropriate one to identify threshold solutions. Once the states are reexpressed in the proper basis, $(c\bar{n})(n\bar{c})$, it can be easily observed how they converge to the lowest available threshold as nicely as the other sets of quantum numbers.

From the discussion above it is clear that no bound state exists either for CQC or for BCN. Let us now address the question if it is possible to generate a bound state by means of reasonable assumptions in the interacting Hamiltonian, i.e., would a different thoughtful two-body quark-quark interaction be able to generate a bound state? The answer is that it does not seem to be possible. The reason for that is simple, whenever one modifies the interacting potential, not only the four-quark energy but also the two-body energy is affected. This has been illustrated in Table XIII for two particular cases, $(L, S) = (0, 0)$ $J^{PC} = 0^{++}$ CQC, and $(L, S) = (1, 0)$ BCN. In both cases the relative strength of the interactions has been tailored in such a way that the lowest threshold is modified in a significant quantum number (spin, color,...). In particular, we have fine tuned the one-gluon exchange interaction by means of the regularization parameter r_0 to modify the threshold from $J/\psi\omega|_S$ to $\eta_c\eta|_S$ (Table XIII upper part) and from $h_c\eta|_P$ to $D\bar{D}|_P$ (Table XIII lower part). In both cases we show the values of energies, radius, and the relevant probabilities for some values of K. It can be clearly seen that the alternative sets of parameters do converge to the new lowest threshold.

This behavior makes evident the main difference between $c\bar{c}n\bar{n}$ and $c\bar{n}c\bar{n}$ systems. For the latter there is a consensus that there would be stable channels against dissociation into two mesons if the ratio of the mass of the heavy to the light quark is large enough [13,42]. One should notice that for these states no other combination than a $(c\bar{n})(c\bar{n})$ two-meson system is allowed for the threshold and therefore, any modification in the interaction between the two-charm quarks or the two-light antiquarks, for instance the ratio of the masses, would not translate into a modification of the energy of the threshold [43].

Once it has been observed that no bound state can be obtained by means of any thoughtful two-body potential, one should reformulate the question we made at the beginning of this section into: Is it possible that $c\bar{c}n\bar{n}$ bound states naturally exist *or other restrictions must be imposed to bind them*? Two main possibilities have been discussed in the literature in order to force four-quark states to be bound. On the one hand three- and four-quark interactions that would not be factorizable as a sum of two-body potentials could be included in the four-quark Hamiltonian. Among these interactions, three alternatives have been thoroughly discussed: color three- and four-quark interactions depending on the color $SU(3)$ quadratic and

TABLE X. Comparison of the energies (MeV) and probability of the dominant components of the four-quark wave function for $J^{PC} = 1^{++}(L, S) = (0, 1)$ $(K = 8)$ and $J^{PC} = (1, 2, 3)^{--}$ $(L, S) = (1, 2)$ $(K = 7)$ states either using $\sum_i \ell_i = \infty$ or $\sum_i \ell_i \leq 1$ using the CQC model.

	$\sum_i \ell_i = \infty$		$\sum_i \ell_i \leq 1$	
	Energy	Probability	Energy	Probability
$L = 0$ $S = 1$	3844	$P_{11}(1, 1) = 0.9871$	3850	$P_{11}(1, 1) = 1.0000$
$L = 1$ $S = 2$	4199	$P_{88}(1, 1) = 0.8779$	4275	$P_{11}(1, 1) = 0.7088$

TABLE XI. Energy (MeV), radius (fm) and probability of the different components of the four-quark wave function as a function of K for the $J^{PC} = 1^{--}(L, S) = (1, 0)$ using the CQC model.

K	E	RMS	$P_{11}(0,0)$	$P_{11}(1,1)$	$P_{88}(0,0)$	$P_{88}(1,1)$
1	4432	0.3939	0.0068	0.1342	0.0068	0.8522
3	4228	0.4242	0.0037	0.0974	0.0108	0.8882
5	4132	0.4600	0.0039	0.1032	0.0175	0.8754
7	4078	0.4969	0.0046	0.1031	0.0264	0.8660
9	4044	0.5341	0.0056	0.1029	0.0363	0.8552
11	4020	0.5714	0.0066	0.1018	0.0464	0.8452
13	4002	0.6086	0.0076	0.1009	0.0556	0.8358
15	3989	0.6458	0.0085	0.1001	0.0637	0.8276
17	3978	0.6828	0.0093	0.0996	0.0708	0.8203
$D\bar{D}\|_P$	3872	0.4396	0.0278	0.0833	0.2222	0.6667

cubic Casimir operators [44], flip-flop confining interactions [45], and string model approaches [46]. On the other hand one could choose to directly restrict the Hilbert space in the few-body problem, selecting *a priori* those components that may favor the binding of the system, the so-called diquarks [47]. A diquark (antidiquark) is an S-wave bound state of two quarks (antiquarks) with particular quantum numbers, i.e., antisymmetric in color, flavor, and spin. This has been explored in recent years not only for four-quark systems, but also for three- and five-quark states [48]. Technically, the effect of both hypothesis is the same, migrating probability from the color singlet-singlet components to the octet-octet ones. This means that the two asymptotically free mesons are no longer a solution of the four-quark Hamiltonian and therefore the interaction can be tailored to create a compact four-quark bound state.

Let us finally notice that when this work was finished Ref. [49] has analyzed the stability of $QQ\bar{q}\,\bar{q}$ and $Q\bar{Q}q\bar{q}$ systems in a simple string model considering only a multiquark confining interaction giving by the minimum of a flip-flop or a butterfly potential. The ground state of systems made of two quarks and two antiquarks of equal masses was found to be below the dissociation threshold. While for the flavor exotic $QQ\bar{q}\,\bar{q}$ the binding increased when increasing the mass ratio m_Q/m_q, for the cryptoexotic $Q\bar{Q}q\bar{q}$ the effect of symmetry breaking is opposite, the system being unbound whenever $m_Q/m_q > 1$. Although more realistic calculations are needed before establishing a definitive conclusion, the findings of Ref. [49] strengthened our results.

C. Isoscalar $J^{PC} = 1^{++}$ and 2^{-+} quantum numbers and the $X(3872)$

Since the $X(3872)$ was first reported by Belle in 2003 [1] it has gradually become the flagship of a new group of states whose properties make their identification as traditional $q\bar{q}$ states unlikely. In this heterogeneous group we could include states like the $Y(2460)$ reported by *BABAR*, and the $D_{sJ}(2317)$ and $D_{sJ}(2460)$ reported by *BABAR* and CLEO. All these states deserve full discussions on their own, however in this section we are going to focus only on the properties of the aforementioned $X(3872)$.

An average mass of 3871.2 ± 0.5 MeV and a narrow width of less than 2.3 MeV have been reported for the $X(3872)$. Note the vicinity of this state to the $D^0\bar{D}^{*0}$ threshold, $M(D^0\bar{D}^{*0}) = 3871.2 \pm 1.2$ MeV [34] (3871.81 ± 0.36 MeV according to the last measurement by CLEO [50]). With respect to the $X(3872)$ quantum numbers, neither D0 nor *BABAR* have been able to offer a clear prediction about its J^{PC}. The isovector nature of this state has been excluded by *BABAR* due to the negative results in the search for a charged partner in the decay $B \rightarrow X(3872)^- K$, $X(3872)^- \rightarrow J/\psi\pi^-\pi^0$ [51]. CDF has per-

TABLE XII. Energy (MeV), radius (fm) and probability of the different components of the four-quark wave function as a function of K for the $J^{PC} = (1, 2, 3)^{--}(L, S) = (1, 2)$ using the CQC and BCN models.

		CQC					BCN		
K	E	RMS	$P_{11}(1,1)$	$P_{88}(1,1)$	K	E	RMS	$P_{11}(1,1)$	$P_{88}(1,1)$
1	4476	0.4063	0.2116	0.7884	1	4518	0.3965	0.2348	0.7652
3	4287	0.4398	0.1488	0.8512	3	4332	0.4442	0.1671	0.8329
5	4199	0.4790	0.1438	0.8562	5	4247	0.4919	0.1374	0.8626
7	4151	0.5193	0.1333	0.8666	7	4199	0.5412	0.1221	0.8779
9	4121	0.5601	0.1270	0.8729	9	4167	0.5909	0.1163	0.8837
11	4101	0.6011	0.1218	0.8782	11	4145	0.6406	0.1134	0.8866
13	4086	0.6424	0.1185	0.8815	13	4128	0.6900	0.1121	0.8879
15	4075	0.6839	0.1162	0.8838	15	4116	0.7389	0.1114	0.8886
17	4066	0.7257	0.1147	0.8853	17	4106	0.7872	0.1111	0.8889
19	4059	0.7675	0.1136	0.8864	19	4099	0.8351	0.1111	0.8889
21	4054	0.8094	0.1129	0.8871	21	4092	0.8824	0.1111	0.8889
$D^*\bar{D}^*\|_P$	4002	0.4684	0.1111	0.8889	$D^*\bar{D}^*\|_P$	4040	0.4794	0.1111	0.8889
$\chi_{cJ}\omega\|_P$	4160	0.6873	1	—	$\chi_{cJ}\omega\|_{S,D}$	4296	0.7259	1	—

TABLE XIII. Energy (MeV), radius (fm) and probability of the dominant component of the four-quark wave function as a function of K for two different parametrizations of the CQC $(L, S) = (0, 0)$ $J^{PC} = 0^{++}$ and BCN $(L, S) = (1, 0)$ $J^{PC} = 1^{--}$. In the bottom part of the table $P_{11(88)}$ stands for the total singlet-singlet (octet-octet) probability, i.e. $P_{11(88)}(1, 1) + P_{11(88)}(0, 0)$.

CQC $J^{PC} = 0^{++}(L, S) = (0, 0)$									
		$r_0^{n\bar{n}} = 0.38$ fm					$r_0^{n\bar{n}} = 0.18$ fm		
K	E	RMS	$P_{11}(1,1)$	$P_{11}(0,0)$	K	E	RMS	$P_{11}(1,1)$	$P_{11}(0,0)$
2	3984	0.3685	0.9685	0.0024	2	3963	0.3649	0.0011	0.9838
4	3909	0.3987	0.9625	0.0014	4	3889	0.3957	0.0010	0.9853
6	3849	0.4282	0.9898	0.0007	6	4414	0.5562	0.0005	0.9948
8	3843	0.4629	0.9766	0.0004	8	3822	0.4608	0.0003	0.9923
10	3826	0.4955	0.9827	0.0002	10	3804	0.4932	0.0002	0.9941
12	3814	0.5280	0.9868	0.0001	12	3791	0.5251	0.0001	0.9957
14	3804	0.5602	0.9898	0.0001	14	3782	0.5564	0.0001	0.9966
$J/\psi\omega\vert_S$	3745	0.5745	1	0	$\eta_c\eta\vert_S$	3718	0.5749	0	1
BCN $J^{PC} = 1^{--}(L, S) = (1, 0)$									
		$r_0 = 2.2$ fm					$r_0 = 0.5$ fm		
K	E	RMS	P_{11}	P_{88}	K	E	RMS	P_{11}	P_{88}
1	4290	0.3563	0.9267	0.0733	1	4467	0.3899	0.5344	0.4656
3	4066	0.3793	0.9363	0.0637	3	4276	0.4341	0.2734	0.7266
5	3954	0.3993	0.9559	0.0441	5	4188	0.4791	0.1817	0.8182
7	3888	0.4185	0.9702	0.0298	7	4137	0.5261	0.1449	0.8551
9	3843	0.4371	0.9791	0.0209	9	4103	0.5735	0.1298	0.8702
11	3811	0.4553	0.9847	0.0153	11	4080	0.6207	0.1220	0.8780
13	3787	0.4731	0.9883	0.0117	13	4062	0.6676	0.1180	0.8820
$h_c\eta\vert_P$	3638	0.5764	1	0	$D\bar{D}\vert_P$	3961	0.4567	0.1111	0.8889

formed a determination of J^{PC} of the $X(3872)$ using dipion invariant mass distribution and angular analysis, obtaining that only the assignments 1^{++} and 2^{-+} are able to describe data [52]. On the other hand, recent studies by Belle combining angular and kinematic properties of the $\pi^+\pi^-$ invariant mass, strongly favor a $J^{PC} = 1^{++}$ state [53], and the observation of the $X(3872) \to D^0\bar{D}^0\pi^0$ also prefers the 1^{++} assignment compared to the 2^{-+} [54]. Therefore, although some caution is still required until better statistic is obtained [55], an isoscalar $J^{PC} = 1^{++}$ state seems to be the best candidate to describe the properties of the $X(3872)$. All these properties have triggered intense theoretical speculations about the nature of this state. Among the possible structures that have been explored, one can find tetraquarks, cusps, hybrids, glueballs, and molecular states, although in most cases these works have been devoted to the study of a limited set of quantum numbers in an attempt to determine the viability of describing its energy together with its width and decay modes [23].

Although our main conclusion also applies for these quantum numbers, i.e., the nonexistence of four-quark bound states, we summarize in detail in this section the results obtained for the isoscalars 1^{++} and 2^{-+} (Table XIV) four-quark states. In this case we illustrate the convergence plotting the energies as a function of K in Fig. 2. It can be observed how the BCN 1^{++} state does not converge to the lowest threshold for small values of K, being affected by the presence of an intermediate $J/\psi\omega|_S$ threshold with an energy of 3874 MeV. Once sufficiently large values of K are considered the system follows the usual convergence to the lowest threshold (see insert in Fig. 2). This behavior can also be observed in the wave function probabilities (right-hand side of Table XIV). This is the only case where this happens and can be traced back to the unique nature of the intermediate threshold, two S-wave mesons in a relative S-wave. Values of K sufficiently large would generate the correct solution.

Once all possible quantum numbers of the $X(3872)$ have been analyzed and discarded, very few alternatives remain. If this state is experimentally proved to be a compact four-quark state this will point either to the existence of non two-body forces or to the emergence of strongly bound diquark structures within the tetraquark. Both possibilities are appealing, does the interaction becomes more involved with the number of quark or does the Hilbert space becomes simpler? On the one hand, some lattice QCD collaborations [46] have reported the important role played by three- and four-quark interactions within the confinement (the Y- and H-shape). On the other hand, diquark correlations have been proposed to play a relevant role in several aspects of QCD, from baryon spectroscopy to scaling violation [47]. The spontaneous formation of diquark components can be checked within our formalism. The

TABLE XIV. Energy (MeV), radius (fm) and probability of the dominant components of the four-quark wave function as a function of K for $J^{PC} = 1^{++}(L, S) = (0, 1)$ and $J^{PC} = 2^{-+}(L, S) = (1, 1)$ states both for CQC and BCN models.

$J^{PC} = 1^{++}(L, S) = (0, 1)$									
CQC					BCN				
K	E	RMS	$P_{11}(1,1)$	$P_{88}(1,1)$	K	E	RMS	$P_{11}(1,0)$	$P_{11}(1,1)$
0	4141	0.3418	1.0000	0.0000	0	4196	0.3393	0.0000	1.0000
2	3985	0.3692	0.9822	0.0178	2	4053	0.3766	0.0000	0.9462
4	3911	0.4000	0.9789	0.0211	4	3994	0.4133	0.0000	0.9233
6	3870	0.4322	0.9834	0.0166	6	3963	0.4502	0.0000	0.9236
8	3845	0.4650	0.9871	0.0129	8	3944	0.4883	0.0001	0.9302
10	3827	0.4979	0.9905	0.0095	10	3932	0.5267	0.0002	0.9424
12	3814	0.5305	0.9926	0.0074	12	3920	0.5581	0.9321	0.0605
14	3805	0.5628	0.9943	0.0057	14	3887	0.5829	0.9986	0.0004
16	3797	0.5945	0.9954	0.0046	16	3861	0.6063	0.9993	0.0001
18	3791	0.6255	0.9962	0.0038	18	3840	0.6298	0.9995	0.0000
20	3786	0.6564	0.9968	0.0032	20	3822	0.6520	0.9996	0.0000
22	—	—	—	—	22	3808	0.6736	0.9997	0.0000
$J/\psi\omega\|_S$	3745	0.5745	1	0	$\chi_{cJ}\eta\|_P$	3655	0.5814	1	0
$h_c\omega\|_P$	4157	0.6857	0	0	$J/\psi\omega\|_S$	3874	0.6133	0	1
$\chi_{cJ}f_J\|_D$	4771	0.9706	1	0	$\chi_{cJ}f_J\|_D$	4773	0.8833	0	1
$J^{PC} = 2^{-+}(L, S) = (1, 1)$									
CQC					BCN				
K	E	RMS	$P_{11}(1,1)$	$P_{88}(1,1)$	K	E	RMS	$P_{11}(1,0)$	$P_{88}(1,0)$
1	4311	0.4172	1.0000	0.0000	1	4315	0.3599	0.9627	0.0302
3	4117	0.4521	0.9982	0.0017	3	4088	0.3828	0.9615	0.0343
5	4018	0.4926	0.9986	0.0014	5	3975	0.4029	0.9715	0.0260
7	3958	0.5347	0.9991	0.0009	7	3908	0.4222	0.9800	0.0184
9	3918	0.5768	0.9994	0.0006	9	3862	0.4409	0.9856	0.0134
11	3889	0.6186	0.9996	0.0004	11	3830	0.4591	0.9892	0.0101
13	3868	0.6596	0.9997	0.0003	13	3806	0.4767	0.9915	0.0079
15	3852	0.6999	0.9998	0.0002	15	3787	0.4939	0.9931	0.0065
17	3839	—	—	—	17	3772	0.5106	0.9942	0.0054
19	—	—	—	—	19	3760	0.5270	0.9950	0.0047
$J/\psi\omega\|_P$	3745	0.5745	1	0	$\chi_{cJ}\eta\|_{S,D}$	3655	0.5814	1	0
$h_c\omega\|_{S,D}$	4157	0.6857	0	0	$J/\psi\omega\|_P$	3874	0.6135	0	0

four-quark state can be explicitly written in the $(cn)(\bar{c}\,\bar{n})$ coupling in order to isolate the diquark-antidiquark configurations. In the particular case of the $(L, S) = (0, 1)J^{PC} = 1^{++}$ of all possible components of the wave function, only two of them have the proper quantum numbers to be identified with a diquark, being the total diquark probability less than 3%. Therefore, it is clear that without any further hypothesis two-body potentials do not favor the presence of diquarks and any description of these states in terms of diquark-antidiquark components would be selecting a restricted Hilbert space.

V. SUMMARY

In this work we present for the first time a generalization of the hyperspherical harmonic formalism to study systems made of quarks and antiquarks of different flavor. We have focused our analysis on hidden-charm systems, namely, states containing charm quarks and antiquarks with charm quantum number equal to zero. This formalism opens the door to an exact study of several other multiquark systems containing quarks with different masses and/or flavors, like the $s\bar{c}n\bar{n}$ or the $c\bar{c}s\bar{s}$, that up to now have been sparsely analyzed in the literature.

We have performed a systematic analysis of all $c\bar{c}n\bar{n}$ isoscalar ground states. This includes positive parity $L = 0$ and negative parity $L = 1$ systems with $S = 0$, 1, and 2. We have used two standard quark models in the literature, both leading to the same conclusions. The relevance of a careful analysis of the numerical thresholds together with the numerical approximations involved has been emphasized in order to avoid misidentification of bound states.

We have not found any compact four-quark bound state for any set of quantum numbers. We have studied the possibility of generating a bound state by means of a

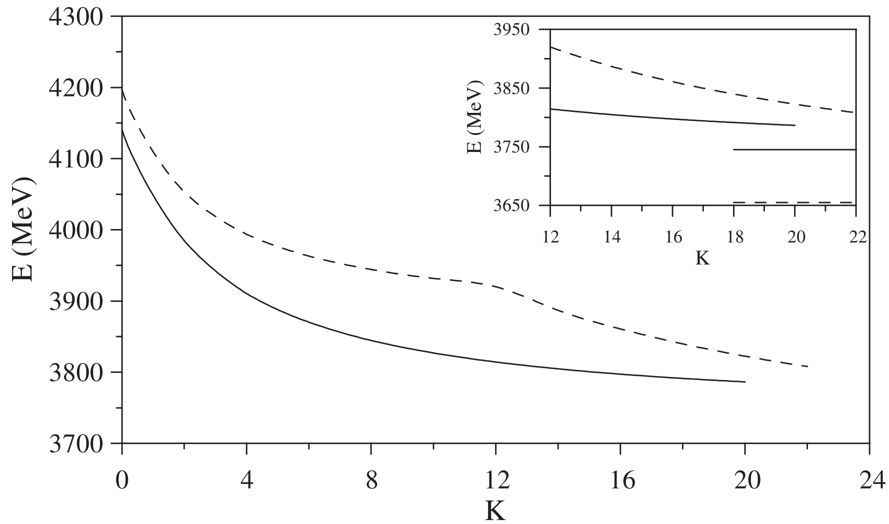

FIG. 2. Energy of the 1^{++} state using the CQC (solid line) and BCN models (dashed line) as a function of K. The insert in the upper-right corner magnifies the large values of K to show the convergence to the corresponding threshold showed by a straight line.

modification of the interacting Hamiltonian. We conclude that no refitting of the models would be able to force a bound state if only two-body color-dependent forces are considered. The reason has been traced back to the particular singlet-singlet decomposition available to the $c\bar{c}n\bar{n}$ states, namely, the possibility of constructing $(c\bar{c})(n\bar{n})$ or $(c\bar{n})(n\bar{c})$ two-meson states in such a way that any modification of the two-body potential in the four-quark problem is automatically translated into the two-meson final state. Concerning the $X(3872)$ we have explicitly discussed the quantum numbers favored by experiment, 1^{++} and 2^{-+}, obtaining that none of them is bound.

It has been said that when you have eliminated the impossible, whatever remains, however improbable, must be the truth [56]. Therefore, the nonexistence of bound $c\bar{c}n\bar{n}$ states together with the experimental observation of suggested non-$q\bar{q}$ states like the $X(3872)$, seems to be clearly emphasizing the need of considering new structures not based in naive two-body interactions, like, for example, diquarks configurations or few-body potentials, in order to improve our understanding of the hadron spectra.

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