Nonleptonic two-body B decays including axial-vector mesons in the final state

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We present a systematic study of exclusive charmless nonleptonic two-body B decays including axial-vector mesons in the final state. We calculate branching ratios of $B \to PA$, VA, and AA decays, where A, V, and P denote an axial vector, a vector, and a pseudoscalar meson, respectively. We assume a naive factorization hypothesis and use the improved version of the nonrelativistic Isgur-Scora-Grinstein-Wise quark model for form factors in $B \to A$ transitions. We include contributions that arise from the effective $\Delta B = 1$ weak Hamiltonian $H_{\rm eff}$. The respective factorized amplitudes of these decays are explicitly shown and their penguin contributions are classified. We find that decays $B^- \to a_1^0 \pi^-$, $\bar{B}^0 \to a_1^\pm \pi^\mp$, $B^- \to a_1^- \bar{K}^0$, $\bar{B}^0 \to a_1^+ K^-$, $\bar{B}^0 \to f_1 \bar{K}^0$, $B^- \to f_1 K^-$, $B^- \to K_1^-(1400)\eta^{(\prime)}$, $B^- \to b_1^- \bar{K}^0$, and $\bar{B}^0 \to b_1^+ \pi^-(K^-)$ have branching ratios of the order of 10^{-5}. We also study the dependence of branching ratios for $B \to K_1 P(V, A)$ decays [$K_1 = K_1(1270)$, $K_1(1400)$] with respect to the mixing angle between K_{1A} and K_{1B}.

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I. INTRODUCTION

Recent experimental results for $B \to a_1 \pi$ and $B \to K_1(1270)\gamma$ decays obtained by *BABAR*, Belle, and CLEO [1] have opened an interesting area of research about production of axial-vector mesons in B decays. Two-body B decays have been considered one of the premier places to understand the interplay of QCD and electroweak interactions, to look for CP violation, and overconstrain the Cabibbo-Kobayashi-Maskawa (CKM) parameters in the standard model. Indeed, exclusive modes $B \to PP$, PV, and VV, which have been extensively discussed in the literature, have committed such expectations.

In search of alternative modes to those traditionally studied, we consider processes which include an axial-vector meson in the final state. It is expected that some of these decay channels have large branching ratios [2] and can be within the reach of future experiments. Moreover, they are an additional scenario for understanding QCD and electroweak penguin effects in the standard model. These modes give additional and complementary information about exclusive nonleptonic weak decays of B mesons.

The two most important penguin contributions correspond to a_4 and a_6 QCD coefficients. These coefficients have different sign in the amplitude $\mathcal{M}(B \to VP)$, making their contribution small. For $B \to AP$ decays they have equal sign, thus we have a bigger contribution in the penguin sector. Branching ratios of these decays are good candidates to be measured.

Our purpose is to present a systematic analysis about charmless modes $B \to AP$, $B \to AV$, and $B \to AA$, similar in completeness to previous studies about channels $B \to PP$, $B \to PV$, and $B \to VV$ which have been extensively considered in the literature [3,4].

There are two types of axial-vector mesons [5]. In spectroscopic notation ${}^{2s+1}L_J$, these p-wave mesons are ${}^3P_1$ and ${}^1P_1$, with $J^{PC} = 1^{++}$ and 1^{+-}, respectively. Under $SU(3)$ flavor symmetry, the ${}^3P_1$-nonet is composed by $a_1(1260)$, $f_1(1285)$, $f_1(1420)$, and K_{1A} and the ${}^1P_1$-nonet is integrated by $b_1(1235)$, $h_1(1170)$, $h_1(1380)$, and K_{1B}. However, physical strange axial-vector mesons $K_1(1270)$ and $K_1(1400)$ are a mixture of K_{1A} and K_{1B}:

$$\begin{aligned} K_1(1270) &= K_{1A}\sin\theta + K_{1B}\cos\theta \\ K_1(1400) &= K_{1A}\cos\theta - K_{1B}\sin\theta, \end{aligned} \tag{1}$$

where θ is the mixing angle.

At the theoretical level, some authors have worked with production of axial-vector mesons in nonleptonic B decays. Katoch-Verma [6] studied $B \to PA$ decays at tree level using the factorization hypothesis and the nonrelativistic Isgur-Scora-Grinstein-Wise (ISGW) quark model [7]. Nardulli-Pham in Ref. [2] did an analysis of two-body B decays with an axial-vector meson in the final state using factorization and the $B \to K_1$ form factors obtained from measured radiative decays. They calculated the branching ratio for $B \to J/\psi K_1$ and derived some predictions for a few nonleptonic decay channels involving light strange or nonstrange axial-vector mesons in the final state using naive factorization and relations from heavy quark effective theory. Recently, Laporta-Nardulli-Pham [8] presented an analysis about some charmless $B \to PA$ decays including contributions of the effective weak Hamiltonian $H_{\rm eff}$, assuming factorization approach and employing as inputs a limited number of experimental data. They did not use

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predictions from theoretical models for form factors. In Ref. [9], the authors investigated $B \to K_1 \phi$ decays employing the generalized factorization hypothesis and light-front approach for form factors.

Cheng in Ref. [10] studied Cabibbo-allowed hadronic B decays at tree level containing an even-parity charmed meson in the final state. In this work, the author predicted branching ratios for some decays of type $B \to AP(V)$ where A is, in this case, a charmed axial-vector meson. Calculation was performed within the framework of generalized factorization. Form factors for $B \to A$ transition were calculated with the improved version of the Isgur-Scora-Grinstein-Wise quark model, called ISGW2 [11]. For $B \to P$ and $B \to V$ form factors, the author used the Melikhov-Stech model [12]. Recently, Cheng-Chua [13] continue studying even-parity charmed meson production in B decays, calculating $B \to D^{**}$ (D^{**} denotes a p-wave charmed meson) form factors within the covariant light-front quark model.

Other authors have been interested in radiative $B \to K_1$ decays (see, for example, Ref. [2]). Recently, Lee [14] revisited the $B \to K_1$ form factors in the light-cone sum rules, and reduced the discrepancy between theoretical prediction and experimental data reported by the Belle Collaboration [1] for $B \to K_1 \gamma$. Lee claims that more information is necessary about the mixing angle between K_{1A} and K_{1B} to reduce theoretical uncertainties. In fact, this mixing angle has been estimated by some different methods [15]. However, there is not yet a consensus about its value [16].

CP violation effects have also been investigated in nonleptonic B decays with axial-vector mesons in the final state. For example, in Ref. [17], time-dependent *CP* asymmetries in $\bar{B}^0 \to D^{*-} a_1^+$ are studied in order to learn about the linear combination of weak phases $(2\beta + \gamma)$. More recently (see Ref. [18]), in an analysis of $B^0 \to a_1^{\pm} \pi^{\mp}$ modes, they determined the phase $\alpha_{\rm eff}$, which include the weak phase α and effects due to penguin contribution. Moreover, applying $SU(3)$ symmetry to these decays and to $B \to a_1 K$ and $B \to K_1 \pi$, they obtained bounds on $(\alpha - \alpha_{\rm eff})$.

In this paper we are interested in studying exclusive charmless nonleptonic two-body B decays including axial-vector mesons in the final state. We present an overview and a systematic study about these types of processes. For this, we compute branching ratios for exclusive channels $B \to AP$, AV, AA that are allowed by the CKM factors, including contributions of the effective weak Hamiltonian $H_{\rm eff}$ (tree and penguin), assuming the naive factorization hypothesis and using the improved version of the ISGW [11] quark model for calculating the respective form factors related with $B \to A$ transitions. Form factors for $B \to P$ and $B \to V$ transitions have been taken from the relativistic Wirbel-Stech-Bauer (WSB) quark model [19] and from light-cone sum rules (LCSR) [20].

This paper is organized as follows: In Sec. II we discuss the effective weak Hamiltonian, effective Wilson coefficients, and naive factorization hypothesis. Input parameters and mixing schemes are discussed in Sec. III. In Sec. IV we present form factors for $B \to P(V)$ transitions taken from the WSB model and the LCSR approach, and $B \to A$ transitions calculated in the ISGW2 model. In Sec. V is discussed the amplitudes and manner to calculate branching ratios for processes considered. Numerical results for branching ratios are presented in Sec. VI. We conclude in Sec. VII with a summary. Amplitudes for all charmless $B \to AP$, AV, and AA processes are given explicitly in the appendices.

II. EFFECTIVE HAMILTONIAN AND FACTORIZATION

The basis for the study of two-body charmless hadronic B-decays is the effective weak Hamiltonian $H_{\rm eff}$ [21]. For $\Delta B = 1$ transitions it can be written as

$$\begin{aligned} H_{\rm eff} = \frac{G_F}{\sqrt{2}}\Big[& V_{ub}V_{uq}^*(C_1(\mu)O_1^u(\mu) + C_2(\mu)O_2^u(\mu)) \\ & + V_{cb}V_{cq}^*(C_1(\mu)O_1^c(\mu) + C_2(\mu)O_2^c(\mu)) \\ & - V_{tb}V_{tq}^*\Big(\sum_{i=3}^{10} C_i(\mu)O_i(\mu) + C_g(\mu)O_g(\mu)\Big)\Big] \\ & + \text{H.c.}, \end{aligned} \tag{2}$$

where G_F is the Fermi constant and $C_i(\mu)$ are the Wilson coefficients evaluated at the renormalization scale μ. Local operators $O_i(\mu)$ are given below for $b \to q$ transitions:

$$\begin{aligned} O_1^u &= \bar{q}_\alpha \gamma^\mu L u_\alpha \cdot \bar{u}_\beta \gamma_\mu L b_\beta \\ O_2^u &= \bar{q}_\alpha \gamma^\mu L u_\beta \cdot \bar{u}_\beta \gamma_\mu L b_\alpha \\ O_1^c &= \bar{q}_\alpha \gamma^\mu L c_\alpha \cdot \bar{c}_\beta \gamma_\mu L b_\beta \\ O_2^c &= \bar{q}_\alpha \gamma^\mu L c_\beta \cdot \bar{c}_\beta \gamma_\mu L b_\alpha \\ O_{3(5)} &= \bar{q}_\alpha \gamma^\mu L b_\alpha \cdot \sum_{q'} \bar{q}'_\beta \gamma_\mu L(R) q'_\beta \\ O_{4(6)} &= \bar{q}_\alpha \gamma^\mu L b_\beta \cdot \sum_{q'} \bar{q}'_\beta \gamma_\mu L(R) q'_\alpha \\ O_{7(9)} &= \frac{3}{2} \bar{q}_\alpha \gamma^\mu L b_\alpha \cdot \sum_{q'} e_{q'} \bar{q}'_\beta \gamma_\mu R(L) q'_\beta \\ O_{8(10)} &= \frac{3}{2} \bar{q}_\alpha \gamma^\mu L b_\beta \cdot \sum_{q'} e_{q'} \bar{q}'_\beta \gamma_\mu R(L) q'_\alpha \\ O_g &= (g_s/8\pi^2) m_b \bar{q}_\alpha \sigma^{\mu\nu} R(\lambda^A_{\alpha\beta}/2) b_\beta G^A_{\mu\nu}, \end{aligned} \tag{3}$$

where q can be the quarks d or s. L and R stand for left and right projectors defined as $(1 - \gamma_5)$ and $(1 + \gamma_5)$, respectively. The symbols α and β are $SU(3)$ color indices and $\lambda^A_{\alpha\beta}$ $(A = 1, \ldots, 8)$ are the Gell-Mann matrices. The sums run over active quarks at the scale $\mu = \mathcal{O}(m_b)$, i.e. q' runs with the quarks u, d, s, and c.

We use the next to leading order Wilson coefficients for $\Delta B = 1$ transitions obtained in the naive dimensional regularization scheme at the energy scale $\mu = m_b(m_b)$, $\Lambda^{(5)}_{\overline{MS}} = 225$ MeV, and $m_t = 170$ GeV. These values are $c_1 = 1.082$, $c_2 = -0.185$, $c_3 = 0.014$, $c_4 = -0.035$, $c_5 = 0.009$, $c_6 = -0.041$, $c_7/\alpha = -0.002$, $c_8/\alpha = 0.054$, $c_9/\alpha = -1.292$, and $c_{10}/\alpha = 0.263$, where α is the fine structure constant, see Table XXII in Ref. [21].

In order to calculate the amplitude for a nonleptonic two-body $B \to M_1 M_2$ decay, we use the effective weak Hamiltonian $H_{\rm eff}$,

$$\mathcal{M}(B \to M_1 M_2) = \langle M_1 M_2 | H_{\rm eff} | B \rangle = \frac{G_F}{\sqrt{2}} \sum_i C_i(\mu) \langle M_1 M_2 | O_i(\mu) | B \rangle. \quad (4)$$

Hadronic matrix elements $\langle O_i(\mu)\rangle \equiv \langle M_1 M_2 | O_i(\mu) | B\rangle$ can be evaluated under factorization hypothesis, which approximates the hadronic matrix element by a product of two matrix elements of singlet currents. These currents are parametrized by decay constants and form factors. It is necessary to point out that these matrix elements as products of conserved currents are $\Lambda_{\overline{MS}}$, μ scale, and renormalization scheme independent [22,23]. The suggested energy scale to apply factorization for B decays is $\mu_f = \mathcal{O}(m_b)$. Besides this simple approximation, it is well established that nonfactorizable contributions must be present in the matrix elements in order to cancel the scale μ and renormalization scheme dependence of $C_i(\mu)$.

To solve the issue of scale μ dependence, but not the renormalization scheme dependence [24], it is proposed in Refs. [3,4] to isolate from the matrix element $\langle O_i(\mu)\rangle$ the μ dependence, and link with the μ dependence in the Wilson coefficients $C_i(\mu)$ to form $c_i^{\rm eff}$, effective Wilson coefficients independent of μ. Matrix elements $\langle O_i\rangle_{\rm tree}$ and effective $c_i^{\rm eff}$ Wilson coefficients are scale μ independent, so is the amplitude. Thus, we can write

$$\sum_i C_i(\mu)\langle O_i(\mu)\rangle = \sum_i C_i(\mu) g_i(\mu) \langle O_i\rangle_{\rm tree} = \sum_i c_i^{\rm eff} \langle O_i\rangle_{\rm tree}. \quad (5)$$

The formula for effective Wilson coefficients and their numerical values have been given explicitly in Ref. [4]. These values depend on quark masses, CKM parameters, and renormalization scheme. In this article we recalculate the effective Wilson coefficient $c_i^{\rm eff}$, because there have been some changes in the CKM parameters since the authors of Ref. [4] calculated them. We choose a naive dimensional regularization scheme to calculate. We present the effective Wilson coefficients $c_i^{\rm eff}$ in Table I, for $b \to d$ and $b \to s$ transitions evaluated at the factorization scale $\mu_f = m_b$, averaged momentum transfer $k^2 = m_b^2/2$, and current CKM parameters, see Sec. III.

TABLE I. Effective Wilson coefficients $c_i^{\rm eff}$ for $b \to d$ and $b \to s$ transitions, evaluated at $\mu_f = m_b$ and $k^2 = m_b^2/2$, where we use the Wolfenstein parameters $\lambda = 0.2272$, $A = 0.818$, $\rho = 0.227$, and $\eta = 0.349$, see Sec. III.

$c_i^{\rm eff}$	$b \to d$	$b \to s$
$c_1^{\rm eff}$	1.1680	1.1680
$c_2^{\rm eff}$	−0.3652	−0.3652
$c_3^{\rm eff}$	0.0233 + i 0.0036	0.0233 + i 0.0043
$c_4^{\rm eff}$	−0.0481 − i 0.0109	−0.0482 − i 0.0129
$c_5^{\rm eff}$	0.0140 + i 0.0036	0.0140 + i 0.0043
$c_6^{\rm eff}$	−0.0503 − i 0.0109	−0.0504 − i 0.0129
$c_7^{\rm eff}/\alpha$	−0.0310 − i 0.0317	−0.0312 − i 0.0357
$c_8^{\rm eff}/\alpha$	0.0551	0.0551
$c_9^{\rm eff}/\alpha$	−1.4275 − i 0.0317	−1.4277 − i 0.0357
$c_{10}^{\rm eff}/\alpha$	0.4804	0.4804

TABLE II. Effective coefficients a_i for $b \to d$ and $b \to s$ transitions (in units of 10^{-4} for $a_3, \ldots, a_{10}$).

a_i	$b \to d$	$b \to s$
a_1	1.046	1.046
a_2	0.024	0.024
a_3	72	72
a_4	−403 − i 97	−404 − i 115
a_5	−28	−28
a_6	−456 − i 97	−457 − i 115
a_7	−0.92 − i 2.31	−0.94 − i 2.61
a_8	3.26 − i 0.77	3.26 − i 0.87
a_9	−92.5 − i 2.31	−92.5 − i 2.61
a_{10}	0.33 − i 0.77	0.33 − i 0.87

The effective Wilson coefficients appear in decay amplitudes as linear combinations. This allows one to define a_i coefficients, which encode dynamics of the decay, by

$$a_i \equiv c_i^{\rm eff} + \frac{1}{N_c} c_{i+1}^{\rm eff} \quad (i = \text{odd})$$
$$a_i \equiv c_i^{\rm eff} + \frac{1}{N_c} c_{i-1}^{\rm eff} \quad (i = \text{even}), \quad (6)$$

where index $i = 1, \ldots, 10$ and $N_c = 3$ is the color number. In Table II, we give the a_i values for $b \to d$ and $b \to s$ transitions calculated with the effective Wilson coefficients $c_i^{\rm eff}$, given in Table I.

III. INPUT PARAMETERS

We parametrize the CKM matrix in terms of the Wolfenstein parameters λ, A, $\bar{\rho}$, and $\bar{\eta}$ [25],

$$\begin{pmatrix} 1 - \frac{1}{2}\lambda^2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 - \frac{1}{2}\lambda^2 & A\lambda^2 \\ A\lambda^3(1 - \bar{\rho} - i\bar{\eta}) & -A\lambda^2 & 1 \end{pmatrix}, \quad (7)$$

with $\bar{\rho} = \rho(1 - \lambda^2/2)$ and $\bar{\eta} = \eta(1 - \lambda^2/2)$, including $\mathcal{O}(\lambda^5)$ corrections [26].

By a global fit that uses all available measurements and that imposes unitary constraints, the Wolfenstein parameters are precisely determined. There exist two ways to combining experimental data, the frequentist statistics [27] and the Bayesian approach [28], providing similar results. Thus, we take for the Wolfenstein parameters the central values $\lambda = 0.2272$, $A = 0.818$, $\bar{\rho} = 0.221$, and $\bar{\eta} = 0.340$ [5].

The running quark masses are necessary in calculation of penguin terms in the amplitude where appear scalar and pseudoscalar matrix elements which are reduced by the use of the Dirac equation of motion. Running quark masses are given at the scale $\mu \approx m_b$, since energy released in B decays is of order m_b. We use $m_u(m_b) = 3.2$ MeV, $m_d(m_b) = 6.4$ MeV, $m_s(m_b) = 127$ MeV, $m_c(m_b) = 0.95$ GeV, and $m_b(m_b) = 4.34$ GeV, see Ref. [29].

Decay constants of pseudoscalar and vector mesons are well determined experimentally. We use the following values [5]: $f_\pi = 130.7$ MeV, $f_K = 160$ MeV, $f_\rho = 216$ MeV, $f_\omega = 195$ MeV, $f_{K^*} = 221$ MeV, and $f_\phi = 237$ MeV.

The $\omega - \phi$, $\rho^0 - \omega$, $\eta - \eta'$, and $K_{1A} - K_{1B}$ mixing are introduced through mixing in decay constants and form factors. We consider ideal mixing for the system (ω, ϕ), i.e. $\omega = 1/\sqrt{2}(u\bar{u} + d\bar{d})$ and $\phi = s\bar{s}$. In the next section we will discuss mixing in form factors. In the following we describe mixing in decay constants.

For the $\eta - \eta'$ mixing we use the two-mixing angle formalism proposed in [30,31], which define physical states η and η' in terms of flavor octet and singlet, η_8 and η_0, respectively:

$$
\begin{aligned}
|\eta\rangle &= \cos\theta_8|\eta_8\rangle - \sin\theta_0|\eta_0\rangle, \\
|\eta'\rangle &= \sin\theta_8|\eta_8\rangle + \cos\theta_0|\eta_0\rangle.
\end{aligned} \tag{8}
$$

We introduce decay constants for η_8 and η_0 by $\langle 0|A_\mu^8|\eta^{(\prime)}(p)\rangle = if^8_{\eta^{(\prime)}}p_\mu$ and $\langle 0|A_\mu^0|\eta^{(\prime)}(p)\rangle = if^0_{\eta^{(\prime)}}p_\mu$. Considering that η_8 and η_0 in terms of quarks are

$$
\begin{aligned}
|\eta_8\rangle &= \frac{1}{\sqrt{6}}|\bar{u}u + \bar{d}d - 2\bar{s}s\rangle, \\
|\eta_0\rangle &= \frac{1}{\sqrt{3}}|\bar{u}u + \bar{d}d + \bar{s}s\rangle,
\end{aligned} \tag{9}
$$

induce a two-mixing angle in decay constants $f^q_{\eta^{(\prime)}}$, defined by $\langle 0|\bar{q}\gamma_\mu\gamma_5 q|\eta^{(\prime)}(p)\rangle = if^q_{\eta^{(\prime)}}p_\mu$,

$$
\begin{aligned}
f^u_{\eta'} &= \frac{f_8}{\sqrt{6}}\sin\theta_8 + \frac{f_0}{\sqrt{3}}\cos\theta_0, \\
f^s_{\eta'} &= -2\frac{f_8}{\sqrt{6}}\sin\theta_8 + \frac{f_0}{\sqrt{3}}\cos\theta_0
\end{aligned} \tag{10}
$$

and

$$
\begin{aligned}
f^u_{\eta} &= \frac{f_8}{\sqrt{6}}\cos\theta_8 - \frac{f_0}{\sqrt{3}}\sin\theta_0, \\
f^s_{\eta} &= -2\frac{f_8}{\sqrt{6}}\cos\theta_8 - \frac{f_0}{\sqrt{3}}\cos\theta_0.
\end{aligned} \tag{11}
$$

From a complete phenomenological fit of the $\eta - \eta'$ mixing parameters in Ref. [31] we have $\theta_8 = -21.1°$, $\theta_0 = -9.2°$, $\theta = -15.4°$, $f_8 = 165$ MeV, and $f_0 = 153$ MeV. Replacing values in Eqs. (10) and (11), the decay constants are $f^u_{\eta'} = 61.8$ MeV, $f^s_{\eta'} = 138$ MeV, $f^u_\eta = 76.2$ MeV, and $f^s_\eta = -110.5$ MeV. To include in the mixing scheme the η_c, in the calculation we use decay constants defined by $\langle 0|\bar{c}\gamma_\mu\gamma_5 c|\eta^{(\prime)}\rangle = if^q_{\eta^{(\prime)}}p_\mu$ as are obtained in Ref. [31]: $f^c_\eta = -(2.4 \pm 0.2)$ MeV and $f^c_{\eta'} = -(6.3 \pm 0.6)$ MeV.

In evaluating hadron matrix elements of scalar and pseudoscalar densities in some penguin terms, the anomaly must be included in order to ensure a correct chiral behavior for those matrix elements. The expressions are [32]

$$
\begin{aligned}
\langle \eta^{(\prime)}|\bar{u}\gamma_5 u|0\rangle &= \langle \eta^{(\prime)}|\bar{d}\gamma_5 d|0\rangle = r_{\eta^{(\prime)}}\langle \eta^{(\prime)}|\bar{s}\gamma_5 s|0\rangle, \\
\langle \eta^{(\prime)}|\bar{s}\gamma_5 s|0\rangle &= -i\frac{m^2_{\eta^{(\prime)}}}{2m_s}(f^s_{\eta^{(\prime)}} - f^u_{\eta^{(\prime)}}),
\end{aligned} \tag{12}
$$

where the ratios $r_{\eta'}$ and r_η are defined by

$$
\begin{aligned}
r_{\eta'} &= \frac{\sqrt{2f_0^2 - f_8}}{\sqrt{2f_8^2 - f_0^2}}\frac{\cos\theta + (1/\sqrt{2})\sin\theta}{\cos\theta - \sqrt{2}\sin\theta}, \\
r_\eta &= -\frac{1}{2}\frac{\sqrt{2f_0^2 - f_8}}{\sqrt{2f_8^2 - f_0^2}}\frac{\cos\theta - \sqrt{2}\sin\theta}{\cos\theta + (1/\sqrt{2})\sin\theta},
\end{aligned} \tag{13}
$$

the numerical values obtained are $r_{\eta'} = 0.462$ and $r_\eta = -0.689$.

The physical states $K_1(1270)$ and $K_1(1400)$ result from the mixing of K_{1A} and K_{1B}, $^3P_1$ and $^1P_1$ mesons, respectively, see Eq. (1). From experimental data on masses and partial ratios of $K_1(1270)$ and $K_1(1400)$, two solutions are found for the mixing angle with a two-fold ambiguity, $\theta = \pm 32°$ and $\theta = \pm 58°$. The masses for the states K_{1A} and K_{1B} are $m_{K_{1A}} = 1367$ MeV and $m_{K_{1B}} = 1310$ MeV, respectively. From τ decays, the decay constants of the physical states are determined. The values obtained are $f_{K1}(1270) = 171$ MeV and $f_{K1}(1400) = 126$ MeV [2], using data from Ref. [5].

Thus, we have experimental information to determine decay constants for strange axial-vector mesons. That is not the case for nonstrange axial-vector mesons. Using the mixing angle of the system $K_{1A} - K_{1B}$ and $SU(3)$ symmetry it is derived the decay constant $f_{a_1} = 215$ MeV for $\theta = 32°$ and $f_{a_1} = 223$ MeV for $\theta = 58°$, see Ref. [2]. In the case of the $^1P_1$-nonet, with $J^{PC} = 1^{+-}$, the axial-vector mesons b_1, h_1, and K_{1B} have even G-parity. The axial current which produces a b_1 or a h_1 has odd G-parity. Thus, G-parity conservation does not allow the transition $\langle 0|\bar{u}\gamma_\mu\gamma_5 d|b_1\rangle$, in consequence $f_{b_1} = 0$. Since, f_1 is in the same nonet that a_1, by $SU(3)$ symmetry we consider the equal decay constant. In the calculations of branching ratios we use the values $f_{a_1} = f_{f_1} = 215$ MeV and $f_{b_1} = f_{h_1} = 0$ MeV.

Matrix elements for $B \to K_1$ transitions are calculated in the flavor base $K_{1A} - K_{1B}$. In the calculation of amplitude involving a final physical state as $K_1(1270)$ or $K_1(1400)$, we transform matrix elements from the flavor base to the physical base using Eq. (1).

We use for B meson lifetime $\tau_{B^-} = (1.638 \pm 0.011) \times 10^{-12}$ s and $\tau_{B^0} = (1.530 \pm 0.009) \times 10^{-12}$ s, see Ref. [5], necessary to calculate branching ratios.

IV. FORM FACTORS

As was stated in Sec. II, hadronic matrix elements $\langle O_i\rangle_{\text{tree}}$ are given in the factorization hypothesis in terms of decay constants and form factors. Unfortunately, due to the nonperturbative nature of these matrix elements, there are no complete reliable calculations and only model dependent evaluations are used for them.

We use the WSB model and LCSR approach to determine form factors for $B \to P$ and $B \to V$ transitions. In the WSB model and LCSR approach, the form factors for $B \to A$ transitions have not been calculated. Thus we calculate form factors for $B \to A$ transitions in the ISGW2 model [11]. In the following subsections, we give relevant information to calculate form factors in the respective models.

A. Form factors for $B \to P(V)$ in the WSB model and LCSR approach

In the WSB quark model meson-meson matrix elements of currents are evaluated from the overlap integrals of corresponding wave functions, which are solutions of a relativistic harmonic oscillator potential. For momentum transfer squared q^2 dependence of form factors in the region where q^2 is not too large, we shall use a single pole dominance ansatz, namely

$$f(q^2) = \frac{f(0)}{(1 - q^2/m_*^2)}, \tag{14}$$

where m_* is the pole mass and $f(0)$ the form factor at zero momentum transfer given in Ref. [19]. Note that the original WSB quark model assumes a monopole behavior for all form factors.

The WSB model has been quite successful in accommodating data in an important number of exclusive semileptonic and nonleptonic two-body decays of D and B mesons.

Form factors for $B \to P$ transitions are defined as follows:

$$\langle P(p_P)|V_\mu|B(p_B)\rangle \equiv \left[(p_B + p_P)_\mu - \frac{m_B^2 - m_P^2}{q^2} q_\mu\right] F_1(q^2) + \left[\frac{m_B^2 - m_P^2}{q^2}\right] q_\mu F_0(q^2), \tag{15}$$

where $q = (p_B - p_P)$, as well as form factors for $B \to V$ transitions by

$$\langle V(p_V, \epsilon)|(V_\mu - A_\mu)|B(p_B)\rangle \equiv -\epsilon_{\mu\nu\alpha\beta}\epsilon^{\nu*} p_B^\alpha p_V^\beta \frac{2V(q'^2)}{(m_B + m_V)} - i\left[\left(\epsilon_\mu^* - \frac{\epsilon^*\cdot q'}{q'^2} q'_\mu\right)(m_B + m_V)A_1(q^2) - \left((p_B + p_V)_\mu - \frac{(m_B^2 - m_V^2)}{q'^2} q'_\mu\right)(\epsilon^*\cdot q')\frac{A_2(q'^2)}{(m_B + m_V)} + \frac{2m_V(\epsilon^*\cdot q')}{q'^2} q_\mu A_0(q'^2)\right], \tag{16}$$

where $q' = (p_B - p_V)$ and ϵ is the polarization vector of V. In order to cancel the poles at $q^2 = 0$, we must impose restrictions over form factors

$$F_1(0) = F_0(0),$$
$$2m_V A_0(0) = (m_B + m_V)A_1(0) - (m_B - m_V)A_2(0). \tag{17}$$

In Table III form factors are given for transitions required in calculations: form factors for $B \to \pi$, $B \to K$, $B \to \eta$, $B \to \eta'$, $B \to \rho$, $B \to K^*$, and $B \to \omega$ are evaluated at the $q^2 = 0$ momentum transfer. With respect to $B \to \eta$ and $B \to \eta'$ transitions, the WSB model does not include the $\eta - \eta'$ mixing effect. We better consider $SU(3)$ symmetry and use the relations $F_0^{B\pi}(0) = \sqrt{3}F^{B\eta_0}(0) = \sqrt{6}F^{B\eta_8}(0)$, calculating physical form factors from

$$F^{B\eta} = F^{B\eta_8}\cos\theta - F^{B\eta_0}\sin\theta,$$
$$F^{B\eta'} = F^{B\eta_8}\sin\theta + F^{B\eta_0}\cos\theta, \tag{18}$$

for $F^{B\pi}(0) = 0.333$ and the mixing angle $\theta = -15.4°$ [31], we obtain the values $F^{B\eta}(0) = 0.181$ and $F^{B\eta'}(0) = 0.148$.

The $\rho^0 - \omega$ mixing and isospin breaking effects are introduced in hadronic matrix elements $B \to \rho^0$, following Ref. [33]. In the limit of isospin symmetry isospin eigenstates ρ^I and ω^I expressed in the flavor basis are $\rho^I = (u\bar{u} - d\bar{d})/\sqrt{2}$ and $\omega^I = (u\bar{u} + d\bar{d})/\sqrt{2}$. The physical states ρ^0 and ω are expressed in term of ρ^I and ω^I by

TABLE III. Form factors at zero momentum transfer for $B \to P$ and $B \to V$ transitions, evaluated in the WSB quark model [19] and LCSR [20].

Transition	$F_1 = F_0$	V	A_1	A_2	$A_3 = A_0$
$B \to \pi$	0.333 [0.258]				
$B \to K$	0.379 [0.331]				
$B \to \eta$	0.168 [0.275]				
$B \to \eta'$	0.114 [$\cdots$]				
$B \to \rho$		0.329 [0.323]	0.283 [0.242]	0.283 [0.221]	0.281 [0.303]
$B \to \omega$		0.232 [0.311]	0.199 [0.233]	0.199 [0.181]	0.198 [0.363]
$B \to K^*$		0.369 [0.293]	0.328 [0.219]	0.331 [0.198]	0.321 [0.281]

$$\begin{aligned}|\rho^0\rangle &= |\rho^I\rangle + \epsilon|\omega^I\rangle \\ &= \frac{1}{\sqrt{2}}(1+\epsilon)|u\bar{u}\rangle + \frac{1}{\sqrt{2}}(-1+\epsilon)|d\bar{d}\rangle \\ |\omega\rangle &= |\omega^I\rangle - \epsilon'|\rho^I\rangle \\ &= \frac{1}{\sqrt{2}}(1-\epsilon')|u\bar{u}\rangle + \frac{1}{\sqrt{2}}(1+\epsilon')|d\bar{d}\rangle,\end{aligned} \tag{19}$$

where the numerical values for mixing parameters are $(1+\epsilon) = (0.092 + 0.016i)$ and $(1-\epsilon') = (1.011 + 0.030i)$. The hadronic matrix elements for the $B \to \rho^0$ and $B \to \omega$ transitions including isospin effects change by the factor $(1+\epsilon)$ and $(1-\epsilon')$, respectively. The effect in $B \to \omega$ transitions is negligible and it is not included in branching ratio predictions.

In the LCSR approach form factors for B decays are given in terms of the correlation function of the weak current and the current with quantum numbers of B meson, evaluated between the vacuum and a pseudoscalar or a vector meson. The like cone expansion allows one to calculate in the large virtualities of these currents. In the short virtualities regime, the LCSR approach depends on the factorization of the correlation function into nonperturbative and universal hadron function amplitudes which are convoluted with process dependent amplitudes.

In Ref. [20] form factors for $B \to P$ and $B \to V$ transitions are calculated in the LCSR approach. In Table III form factor values at zero momentum transfer are shown, for the set 2 of parameters, taken from Ref. [20]. For the q^2 dependency of the form factors we use the fit parametrization done in Ref. [20], valid for the full kinematic regime.

B. Form factors for $B \to A$ in the ISGW2 model

The ISGW2 model is based in a nonrelativistic constituent quark representation. In the original ISGW model [7] form factors only depend on the maximum momentum transfer, $q^2 = q_m^2$. In this model form factor dependence is proportional to $\exp[-(q_m^2 - q^2)]$; consequently, the form factors diminish exponentially as a function of $(q_m^2 - q^2)$. This behavior has been improved in the ISGW2 model [11] by expressing the q^2 dependence as a polynomial term which must be multiplied by a factor which depends on the hyperfine mass. In addition, the improved model incorporates constraints imposed by heavy quark symmetry, hyperfine distortions of wave functions, and a more real high recoil behavior.

We have made use of the ISGW2 model [11] to determine form factors for $B \to A$ transitions. The vector and axial parts of the matrix element for these transitions are parametrized as

$$\begin{aligned}\langle A(p_A,\epsilon)|(V_\mu - A_\mu)|B(p_B)\rangle \equiv\; & l\epsilon_\mu + (\epsilon\cdot p_B)[c_+(p_B+p_A)_\mu \\ & + c_-(p_B-p_A)_\mu] \\ & - iq\epsilon_{\mu\nu\alpha\beta}\epsilon^\nu(p_B+p_A)^\alpha \\ & \times (p_B-p_A)^\beta,\end{aligned} \tag{20}$$

where $A(p_A,\epsilon)$ is a $^3P_1$ axial-vector meson. For the $^1P_1$ axial-vector meson we change in the above matrix element l, c_+, c_-, and q by r, s_+, s_-, and v, respectively.

Considering $B \to A$ transitions, at quark level $b \to q_1$, axial-vector meson A has the quark content $q_1\bar{q}_2$, being q_2 the spectator quark. Thus, form factors defined in the ISGW2 model have the following expressions:

$$\begin{aligned} l &= -\tilde{m}_B\beta_B\left[\frac{1}{\mu_-} + \frac{m_2\tilde{m}_A(\tilde{\omega}-1)}{\beta_B^2}\right. \\ &\quad \left.\times\left(\frac{5+\tilde{\omega}}{6m_1} - \frac{1}{2\mu_-}\frac{m_2}{\tilde{m}_A}\frac{\beta_B^2}{\beta_{BA}^2}\right)\right]F_5^{(l)} \\ c_+ + c_- &= -\frac{m_2\tilde{m}_A}{2m_1\tilde{m}_B\beta_B}\left(1 - \frac{m_1m_2}{2\tilde{m}_A\mu_-}\frac{\beta_B^2}{\beta_{BA}^2}\right)F_5^{(c_++c_-)} \\ c_+ - c_- &= -\frac{m_2\tilde{m}_A}{2m_1\tilde{m}_B\beta_B} \\ &\quad \times\left(\frac{\tilde{\omega}+2}{3} - \frac{m_1m_2}{2\tilde{m}_A\mu_-}\frac{\beta_B^2}{\beta_{BA}^2}\right)F_5^{(c_+-c_-)} \\ q &= -\frac{m_2}{2\tilde{m}_A\beta_B}\left(\frac{5+\tilde{\omega}}{6}\right)F_5^{(q)} \end{aligned} \tag{21}$$

for the $^3P_1$ axial-vector meson and

$$r = \frac{\tilde{m}_B\beta_B}{\sqrt{2}}\left[\frac{1}{\mu_+} + \frac{m_2\tilde{m}_A}{3m_1\beta_B^2}(\tilde{\omega}-1)^2\right]F_5^{(r)}$$

$$s_+ + s_- = \frac{m_2}{\sqrt{2}\tilde{m}_B\beta_B}\left(1 - \frac{m_2}{m_1} + \frac{m_2}{2\mu_+}\frac{\beta_B^2}{\beta_{BA}^2}\right)F_5^{(s_+ + s_-)}$$

$$s_+ - s_- = \frac{m_2}{\sqrt{2}m_1\beta_B}\left(\frac{4-\tilde{\omega}}{3} - \frac{m_1 m_2}{2\tilde{m}_A\mu_+}\frac{\beta_B^2}{\beta_{BA}^2}\right)F_5^{(s_+ - s_-)}$$

$$v = \left[\frac{\tilde{m}_B\beta_B}{4\sqrt{2}m_b m_1\tilde{m}_A} + \frac{(\tilde{\omega}-1)}{6\sqrt{2}}\frac{m_2}{\tilde{m}_A\beta_B}\right]F_5^{(v)} \qquad (22)$$

for the ${}^1P_1$ axial-vector meson. The $F_5^{(i)}$ factors in the above expressions are defined by

$$F_5^{(l)} = F_5^{(r)} = F_5\left(\frac{\bar{m}_B}{\tilde{m}_B}\right)^{1/2}\left(\frac{\bar{m}_A}{\tilde{m}_A}\right)^{1/2},$$

$$F_5^{(q)} = F_5^{(v)} = F_5\left(\frac{\bar{m}_B}{\tilde{m}_B}\right)^{-1/2}\left(\frac{\bar{m}_A}{\tilde{m}_A}\right)^{-1/2},$$

$$F_5^{(c_+ + c_-)} = F_5^{(s_+ + s_-)} = F_5\left(\frac{\bar{m}_B}{\tilde{m}_B}\right)^{-3/2}\left(\frac{\bar{m}_A}{\tilde{m}_A}\right)^{1/2},$$

$$F_5^{(c_+ - c_-)} = F_5^{(s_+ - s_-)} = F_5\left(\frac{\bar{m}_B}{\tilde{m}_B}\right)^{-1/2}\left(\frac{\bar{m}_A}{\tilde{m}_A}\right)^{-1/2}, \qquad (23)$$

and the F_n function by

$$F_n = \left(\frac{\tilde{m}_A}{\tilde{m}_B}\right)^{1/2}\left(\frac{\beta_B\beta_A}{\beta_{BA}}\right)^{n/2}\left[1 + \frac{1}{18}r^2(t_m - t)\right]^{-3}, \qquad (24)$$

where

$$r^2 = \frac{3}{4m_b m_1} + \frac{3m_2^2}{2\bar{m}_B\bar{m}_A\beta_{BA}^2} + \frac{1}{\bar{m}_B\bar{m}_A}\left(\frac{16}{33-2n_f}\right) \times \ln\left[\frac{\alpha_s(\mu_{QM})}{\alpha_s(m_1)}\right]. \qquad (25)$$

The parameters m_1 and m_2 are masses of quarks q_1 and q_2, $\bar{m}$ is the hyperfine averaged mass, $\tilde{m}$ is the sum of the masses of constituent quarks, $t_m = (m_B - m_A)^2$ is the maximum momentum transferred, n_f is the number of active flavors at the b scale, and $\alpha_s(\mu)$ is the QCD coupling at the μ scale. The parameters β_B, β_A are obtained from the model, see Ref. [11]. Moreover, we use the definitions

$$\mu_\pm = \left(\frac{1}{m_1} \pm \frac{1}{m_b}\right), \qquad \tilde{\omega} = \frac{t_m - t}{2\bar{m}_B\bar{m}_A} + 1 \qquad (26)$$

and $\beta_{BA}^2 = 1/2(\beta_B^2 + \beta_A^2)$.

In Table IV, we list values of form factors at momentum transferred $t = m_\pi^2$. Form factors are functions of momentum transferred $t = (p_B - p_A)^2$, see Eqs. (21) and (22). In general, form factors vary from m_π^2 to $m_{K_1(1400)}^2$, in just only 4%. In addition, values for the form factors depend strongly on the parameters $\beta_B = 0.43$ and $\beta = 0.28$ calculated in the model.

We calculate form factors for $B \to K_{1A}$ and $B \to K_{1B}$ transitions in $SU(3)$ base. Branching ratios are calculated with physical form factors, which are obtained from the mixing, see Eq. (1).

TABLE IV. Form factors at momentum transfer $t = m_\pi^2$ for $B \to A$ transitions, evaluated in the ISGW2 model [11].

Transition	q	l	c_+	c_-
$B \to a_1$	−0.0417	−1.7469	−0.0101	−0.0012
$B \to f_1$	−0.0427	−1.7603	−0.0103	−0.0012
$B \to K_{1A}$	−0.0593	−1.8567	−0.0155	−0.0011
	v	r	s_+	s_-
$B \to b_1$	0.0319	0.9404	0.0177	−0.0082
$B \to h_1$	0.0324	0.9214	0.0134	−0.0051
$B \to K_{1B}$	0.0323	0.8956	0.0275	−0.0124

TABLE V. Form factors $V_{0,1,2,3}$ and A at momentum transfer $t = m_\pi^2$ for $B \to A$ transitions, evaluated in the ISGW2 model [11].

Transition	A	V_1	V_2	$V_3 = V_0$
$B \to a_1$	0.271	−0.268	0.068	−0.818
$B \to f_1$	0.280	−0.268	0.068	−0.792
$B \to K_{1A}$	0.389	−0.283	0.102	−0.890
$B \to b_1$	−0.208	0.145	−0.115	0.572
$B \to h_1$	−0.209	0.143	−0.086	0.546
$B \to K_{1B}$	−0.216	0.134	−0.184	0.573

To compare form factor values for $B \to A$ transitions with those of $B \to V$ transitions, we can define $B \to A$ transitions in the same basis as used in the WSB model, see Eq. (17). We change the symbols for the form factors V and $A_{0,1,2}$ by A and $V_{0,1,2}$, respectively. These form factors are related to form factors in the ISGW2 model by

$$A(q'^2) = -(m_B + m_A)q(q'^2),$$

$$V_1(q'^2) = \frac{l(q'^2)}{(m_B + m_A)},$$

$$V_2(q'^2) = -(m_B + m_A)c_+(q'^2)$$

$$V_0(q'^2) = \frac{1}{2m_A}[l(q'^2) + (m_B^2 - m_A^2)c_+(q'^2) + q'^2 c_-(q'^2)]. \qquad (27)$$

In Table V, we show form factor values for $B \to A$ transitions at momentum transferred $t = m_\pi^2$, which correspond to those of Table IV.

V. AMPLITUDES AND BRANCHING RATIOS

Let us present a comparison between $B \to V$ and $B \to A$ transitions, which seems straightforward. First, we can see, from Secs. 2, 4, and 6 in Appendix B of Ref. [7], that parametrizations of $\langle V|J_\mu|B\rangle$ and $\langle A|J_\mu|B\rangle$ are only different by a global sign, with the substitution of the form factors $f \leftrightarrow l$, r, $a_\pm \leftrightarrow c_\pm$, $s_\pm$, $g \leftrightarrow q$, v. This is because behavior of currents V_μ and A_μ is interchanged. Moreover, this implies that expressions for decay amplitudes and decay rates, at tree level, for the processes $B \to VP$, VV and $B \to AP$, AV, and AA, respectively, are identical.

On the other hand, the situation is different when tree and penguin contributions are considered. The expressions of decay amplitudes for processes $B \to AP$ (see Appendix A) and $B \to VP$ (see Appendices in Refs. [3,4]) are equal when only QCD parameters a_3, a_4, a_9, and a_{10} contribute. When QCD parameters a_6 and a_8 contribute then the linear combination $(za_6 + ya_8)$ is affected by a global sign and $1/(m_b - m_q)$, which is a factor of this linear combination, changes by $1/(m_b + m_q)$. Relevant contributions in penguin sector are coefficients a_4 and a_6 (see Appendix A); in $B \to A$ transitions a_6 and a_4 have the same sign. This fact implies that these terms are summed so their contribution increases. In $B \to V$ transitions, these terms have different sign thus their contribution decreases. The contributions corresponding to a_5 and a_7 change sign when the axial-vector or the vector meson arises from vacuum, but they are not affected if the pseudoscalar meson is produced from vacuum.

Now we are going to compare penguin contributions to decay amplitudes $\mathcal{M}(B \to AV)$ (see Appendix B) with the ones $\mathcal{M}(B \to VV)$ (shown in Appendices F and G of Ref. [4]): (i) in both cases the contribution of parameters a_6 and a_8 does not appear; (ii) the sign of contribution given by parameters a_5 and a_7 changes when one goes from $B \to V$, A to $B \to V$, V; (iii) in modes $B \to V$, V^{charged} and $B \to V$, A^{charged} always appears the contribution $(a_4 + a_{10})$.

In Table VI, we have summarized penguin contributions to decay amplitudes for modes $B \to AP$ displayed in Appendix A, without including $P = \eta^{(\prime)}$. These decay amplitudes can be classified in two groups from these contributions. The first group is integrated by decays where a charged meson in the final state is produced from vacuum and penguin contribution is given by the linear combination $(a_4 + a_{10}) + \alpha(a_6 + a_8)R$, i.e., parameters a_{even} only contribute to this group. Additionally, in this group we find two cases with $\alpha = 0$ and $\alpha = 1$, which correspond to modes $B \to P$, A^{charged} and $B \to A$, P^{charged}, respectively. Here the notation $B \to M_1$, M_2 means that meson M_2 can be factorized out under the factorization approximation.

The second group is integrated by decays where a neutral meson is factorized out under factorization approximation independently if it is pseudoscalar or axial vector. Penguin contribution is given by the linear combination $\alpha_1(a_4 - a_{10}/2) + \alpha_2(a_6 - a_8/2)R + \alpha_3(a_7 - a_9) + \alpha_4(a_3 - a_5)$. Pure penguin contributions belong to this group and have contributions of a_{even}. They arise when the axial-vector meson or the pseudoscalar meson is a neutral strange meson and, of course, it is produced from vacuum. QCD parameters a_4, a_6, a_8, and a_{10} contribute when the pseudoscalar K^0 meson is factorized out under factorization approximation, a_4 and a_{10} when the axial-vector K_1^0 meson arises from vacuum. Note, that in general, decays $B \to P, A$ do not have contributions from a_6 and a_8.

In Table VII, we have classified penguin contributions to decay amplitudes $\mathcal{M}(B \to AV)$ which are shown in Appendix B. There are two types: in one of them the linear combination $\alpha_1(a_4 + a_{10}) + \alpha_2(a_7 \pm a_9)$ contributes. It occurs when decays $B \to A$, V^{charged} or $B \to V$, A^{charged} are produced, i.e., when a charged meson in the final state arises from vacuum; in the other case, a neutral meson is factorized out under factorization approximation and the linear combination $\beta_1(a_4 - a_{10}/2) + \beta_2(a_3 \pm a_5) + \beta_3(a_7 \pm a_9)$ contributes. Pure penguin contributions belong to it. Parameters a_{odd} contribute to the decay amplitude of pure penguin modes $\bar{B}^0 \to a_1^0 f_1$ and $\bar{B}^0 \to a_1^0 \phi$. Like decays $B \to P, A$, in general, decays $B \to AV$ do not have contributions from a_6 and a_8.

Penguin contributions of decay amplitudes $\mathcal{M}(B \to AA)$ (see Appendix C) can be classified in a similar way. There are two groups. In one of them a charged meson is factorized out under a factorization scheme and only a_4 and a_{10} parameters contribute by means of the linear combination $(a_4 + a_{10})$. In the other group a neutral meson is produced from vacuum. In this case the linear combination $\zeta_1(a_4 - a_{10}/2) + \zeta_2(a_3 - a_5) + \zeta_3(a_7 - a_9)$ contributes. In Table VIII, we display the respective coefficients ζ_i. Again, pure penguin decays are in this group.

In the appendices we give explicitly the amplitudes to the processes studied in terms of form factors for $B \to P$,

TABLE VI. Coefficients of the linear combinations $(a_4 + a_{10}) + \alpha(a_6 + a_8)R$ and $\alpha_1(a_4 - a_{10}/2) + \alpha_2(a_6 - a_8/2)R + \alpha_3(a_7 - a_9) + \alpha_4(a_3 - a_5)$ corresponding to penguin contribution of decay amplitudes $\mathcal{M}(B \to AP)$ without $P = \eta(')$. The coefficient R is given by $R = 2m_P^2/(m_1 + m_2)(m_b - m_3)$.

Decays	α	α_1	α_2	α_3	α_4
$\bar{B}^0 \to \pi^+, a_1^-$; $B^- \to \pi^0, a_1^-$; $\bar{B}^0 \to \pi^+, K_1^-$; $B^- \to \pi^0, K_1^-$	0				
$\bar{B}^0 \to a_1^+, \pi^-$; $B^- \to a_1^0, \pi^-$; $\bar{B}^0 \to a_1^+, K^-$; $B^- \to a_1^0, K^-$; $B^- \to f_1, \pi^-$; $B^- \to f_1, K^-$	1				
$\bar{B}^0 \to \pi^0, f_1$; $B^- \to \pi^-, f_1$		1	0	$-1/2$	2
$\bar{B}^0 \to a_1^0, \pi^0$; $B^- \to a_1^-, \pi^0$; $\bar{B}^0 \to f_1, \pi^0$		± 1	± 1	$\pm 3/2$	0
$\bar{B}^0 \to \pi^0, a_1^0$; $B^- \to \pi^-, a_1^0$		-1	0	$-3/2$	0
$\bar{B}^0 \to \bar{K}^0, f_1$; $B^- \to K^-, f_1$		0	0	$-1/2$	2
$\bar{B}^0 \to f_1, \bar{K}^0$; $B^- \to a_1^-, \bar{K}^0$; $\bar{B}^0 \to a_1^0, \bar{K}^0$; $\bar{B}^0 \to \bar{K}_1^0, K^0$; $B^- \to K_1^-, K^0$		1	1	0	0
$\bar{B}^0 \to \bar{K}^0, a_1^0$; $B^- \to K^-, a_1^0$; $B^- \to K_1^-, \pi^0$; $\bar{B}^0 \to \bar{K}_1^0, \pi^0$		0	0	$-3/2$	0
$\bar{B}^0 \to \pi^0, K_1^0$; $B^- \to \pi^-, \bar{K}_1^0$; $\bar{B}^0 \to \bar{K}^0, K_1^0$; $B^- \to K^-, K_1^0$		1	0	0	0

TABLE VII. Coefficients of the linear combinations $\alpha_1(a_4 + a_{10}) + 2(a_7 \pm a_9)$ and $\beta_1(a_4 - a_{10}/2) + \beta_2(a_3 \pm a_5) + \beta_3(a_7 \pm a_9)$ corresponding to penguin contribution of decay amplitudes $\mathcal{M}(B \to AV)$.

Decays	α_1	α_2	β_1	β_2	β_3
$\bar{B}^0 \to \rho^+, a_1^-(K_1^-)$; $\bar{B}^0 \to a_1^+, \rho^-$; $\bar{B}^0(B^-) \to a_1^+(a_1^0), K^{*-}$; $B^- \to \omega, a_1^-(K_1^-)$; $B^- \to f_1, \rho^-(K^{*-})$; $B^- \to \rho^0, K_1^-$	1	0			
$B^- \to a_1^-, \rho^0$; $B^- \to \rho^-, a_1^0(\bar{K}_1^0)$; $\bar{B}^0 \to a_1^0, \bar{K}^{*0}$; $B^- \to a_1^-, \bar{K}^{*0}$; $\bar{B}^0 \to f_1(\bar{K}_1^0), \bar{K}^{*0}$; $\bar{B}^0 \to \omega, \bar{K}_1^0$			± 1	0	0
$\bar{B}^0 \to a_1^0, \rho^0$; $\bar{B}^0 \to \rho^0, a_1^0$; $\bar{B}^0 \to \omega, a_1^0$; $\bar{B}^0 \to f_1, \rho^0$			-1	0	$\pm 3/2$
$\bar{B}^0 \to \bar{K}^{*0}, a_1^0$; $B^- \to K^{*-}, a_1^0$; $\bar{B}^0 \to \bar{K}_1^0, \rho^0$; $B^- \to K_1^-, \rho^0$			0	0	$-3/2$
$\bar{B}^0 \to a_1^0, \omega$; $B^- \to a_1^-, \omega$; $\bar{B}^0 \to \rho^0, f_1$; $B^- \to \rho^-, f_1$; $\bar{B}^0 \to f_1, \omega$; $\bar{B}^0 \to \omega, f_1$			1	2	$\pm 1/2$
$\bar{B}^0 \to a_1^0, \phi$; $\bar{B}^0 \to f_1, \phi$			0	1	$-1/2$
$\bar{B}^0 \to K^{*0}, f_1$; $B^- \to K^{*-}, f_1$; $\bar{B}^0 \to \bar{K}_1^0, \omega$; $B^- \to K_1^-, \omega$			0	2	$-1/2$
$\bar{B}^0 \to \bar{K}_1^0, \phi$; $B^- \to K_1^-, \phi$			1	1	$-1/2$

TABLE VIII. Coefficients of the linear combination $\zeta_1(a_4 - a_{10}/2) + \zeta_2(a_3 - a_5) + \zeta_3(a_7 - a_9)$ corresponding to penguin contribution of decay amplitudes $\mathcal{M}(B \to A, A^{\text{neutral}})$.

Decays	ζ_1	ζ_2	ζ_3
$\bar{B}^0 \to a_1^0, a_1^0$; $\bar{B}^0 \to f_1, a_1^0$	-1	0	$-3/2$
$\bar{B}^0 \to a_1^0, f_1$; $B^- \to a_1^-, f_1$	1	2	$-1/2$
$\bar{B}^0 \to a_1^0, \bar{K}_1^0$; $B^- \to a_1^-, \bar{K}_1^0$; $\bar{B}^0 \to f_1, \bar{K}_1^0$	1	0	0
$\bar{B}^0 \to \bar{K}_1^0, a_1^0$; $B^- \to K_1^-, a_1^0$	0	0	$-3/2$
$\bar{B}^0 \to \bar{K}_1^0, f_1$; $B^- \to K_1^-, f_1$	0	2	$-1/2$

$B \to V$ and $B \to A$ transitions. In Appendix A we have a common factor $(\epsilon^* \cdot p_B)$ which is not included in the expressions to simplify. We use the symbol K_1 to indicate the axial-vector mesons $K_1(1270)$ or $K_1(1400)$. In the appendices we have the factor $G_F/\sqrt{2}$ common to all amplitudes.

It is straightforward to calculate the branching ratios from amplitudes and input parameters. However, here we give general expressions which are useful in decay rate estimations. The decay rate formula for $B \to AX$ decays is, in general, given by

$$\Gamma(B \to AX) = \frac{p_c}{8\pi m_B^2} |\mathcal{M}(B \to AX)|^2, \tag{28}$$

where $p_c = \lambda^{1/2}(m_B^2, m_A^2, m_V^2)/2m_B$ is the momentum of the decay particle in the rest frame of the B meson and X can be P, V, or A and $\lambda(a, b, c) \equiv a^2 + b^2 + c^2 - 2(ab + ac + bc)$.

For branching ratios of $B \to AP$ decays, we note that amplitude $\mathcal{M}(B \to AP)$ is proportional to $(\epsilon_A^* \cdot p_B)$. Thus amplitude squared is proportional to $|(\varepsilon_A^* \cdot p_B)|^2$, which is easily calculated. The general decay rate formula for $B \to AV$ decays is more involved, because the amplitude $\mathcal{M}(B \to AV)$ includes an interfering term.

VI. NUMERICAL RESULTS

In this section we present our numerical results. In Tables IX, X, XI, XII, XIII, and XIV, we display branching ratios of $B \to AP$, $B \to AV$, and $B \to AA$ decays, respectively, using the improved version of the ISGW quark model [11] for calculating form factors for $B \to A$ transitions.

Branching ratios for $B \to AP$ decays, where A is a $^3P_1$ nonstrange axial-vector meson (see Table IX), are bigger than ones where A is a $^1P_1$ nonstrange axial-vector meson. The ratio $\text{Br}(B \to A(^3P_1)P)/\text{Br}(B \to A(^1P_1)P)$, where mesons $A(^3P_1)$ and $A(^1P_1)$ have the same quark content, is $\sim$1.6–4.5, except for a small number of them. The mode $B^- \to a_1^- \bar{K}^0$, which is a pure penguin channel, is the most dominant (its branching ratio of 84.1×10^{-6} is the biggest). Penguin contribution to this mode is given by a_{even} parameters. Other dominant decays are $\bar{B}^0 \to a_1^+ \pi^-$ and $\bar{B}^0 \to a_1^+ K^-$, whose branching ratios are 74.3×10^{-6} and 72.2×10^{-6}, respectively. In these decays there is a destructive interference between penguin and W-external or W-internal contributions. On the other hand, a similar

TABLE IX. Branching ratios (in units of 10^{-6}) of $B \to AP$ decays, where A is a nonstrange axial-vector meson, using the ISGW2 form factors for $B \to A$ transitions and WSB [LCSR] for $B \to P$ transitions.

Mode	$\mathcal{B}$	Mode	$\mathcal{B}$
$\bar{B}^0 \to a_1^- \pi^+$	36.7 [23.5]	$\bar{B}^0 \to b_1^- \pi^+$	0.0 [· · ·]
$\bar{B}^0 \to a_1^+ \pi^-$	74.3 [· · ·]	$\bar{B}^0 \to b_1^+ \pi^-$	36.2 [· · ·]
$\bar{B}^0 \to a_1^0 \pi^0$	0.27 [· · ·]	$\bar{B}^0 \to b_1^0 \pi^0$	0.15 [0.14]
$B^- \to a_1^- \pi^0$	13.6 [7.8]	$B^- \to b_1^- \pi^0$	0.29 [· · ·]
$B^- \to a_1^0 \pi^-$	43.2 [· · ·]	$B^- \to b_1^0 \pi^-$	18.6 [19.4]
$\bar{B}^0 \to a_1^0 \eta$	0.54 [· · ·]	$\bar{B}^0 \to b_1^0 \eta$	0.17 [0.20]
$B^- \to a_1^- \eta$	13.4 [9.1]	$B^- \to b_1^- \eta$	0.06 [· · ·]
$\bar{B}^0 \to a_1^0 \eta'$	0.09 [· · ·]	$\bar{B}^0 \to b_1^0 \eta'$	0.02 [0.03]
$B^- \to a_1^- \eta'$	13.6 [10.1]	$B^- \to b_1^- \eta'$	0.58 [· · ·]
$\bar{B}^0 \to a_1^0 \bar{K}^0$	42.3 [· · ·]	$\bar{B}^0 \to b_1^0 \bar{K}^0$	19.3 [· · ·]
$\bar{B}^0 \to a_1^+ K^-$	72.2 [· · ·]	$\bar{B}^0 \to b_1^+ K^-$	35.7 [· · ·]
$B^- \to a_1^0 K^-$	43.4 [· · ·]	$B^- \to b_1^0 K^-$	18.1 [· · ·]
$B^- \to a_1^- \bar{K}^0$	84.1 [· · ·]	$B^- \to b_1^- \bar{K}^0$	41.5 [· · ·]
$\bar{B}^0 \to f_1 \pi^0$	0.47 [· · ·]	$\bar{B}^0 \to h_1 \pi^0$	0.16 [· · ·]
$B^- \to f_1 \pi^-$	34.1 [· · ·]	$B^- \to h_1 \pi^-$	18.6 [17.9]
$\bar{B}^0 \to f_1 \eta$	37.1 [· · ·]	$\bar{B}^0 \to h_1 \eta$	18.2 [· · ·]
$\bar{B}^0 \to f_1 \eta'$	22.1 [· · ·]	$\bar{B}^0 \to h_1 \eta'$	11.2 [· · ·]
$\bar{B}^0 \to f_1 \bar{K}^0$	34.7 [· · ·]	$\bar{B}^0 \to h_1 \bar{K}^0$	19.0 [18.3]
$B^- \to f_1 K^-$	31.1 [· · ·]	$B^- \to h_1 K^-$	19.0 [17.7]

situation is found changing a $^3P_1$ meson by a $^1P_1$ meson with the same quark content (see the fourth column in Table IX). At the experimental level there is not enough information. Our predictions for $\mathrm{Br}(\bar{B}^0 \to a_1^- \pi^+) = 36.7 \times 10^{-6}$ and $\mathrm{Br}(\bar{B}^0 \to a_1^+ \pi^-) = 74.3 \times 10^{-6}$ are consistent with the experimental average value $\mathrm{Br}(\bar{B}^0 \to a_1^{\mp} \pi^{\pm}) = (40.9 \pm 7.6) \times 10^{-6}$ [8]. This average includes *BABAR* and Belle results [1]. Finally, we want to mention that our predictions are at the same order as the ones obtained by Laporta-Nardulli-Pham (see Tables V and VI in Ref. [8]), although our values are in general bigger, except in a few modes.

In Table X, we show branching ratios for $B \to K_1 P$ decays for two values ($\theta = 32°, 58°$) of the mixing angle $K_A - K_B$. The strange axial-vector meson is $K_1(1270)$ or $K_1(1400)$. In this case, the most dominant decays are $B^- \to K_1^-(1400)\eta^{(\prime)}$, with branching ratios of $\mathcal{O}(10^{-5})$. On the other hand, branching ratios of modes $\bar{B}^0 \to K_1^- \pi^+$, $\bar{B}^0 \to \bar{K}_1^0 \pi^0$, $B^- \to K_1^- \pi^0$, $B^- \to \bar{K}_1^0 \pi^-$, $\bar{B}^0 \to K_1^0 \bar{K}^0$, and $B^- \to K_1^0 K^-$, where the K_1 meson can be $K_1(1270)$ or $K_1(1400)$, are not sensitive to the value of the mixing angle. On the contrary, branching ratios of $\bar{B}^0 \to \bar{K}_1^0(1270)\eta^{(\prime)}(K^0)$ and $B^- \to K_1^-(1270)\eta^{(\prime)}(K^0)$ strongly depend on the mixing angle. The same decays but changing $K_1(1270)$ by $K_1(1400)$ are not very sensitive to this angle. Branching ratios of $B \to K_1(1270)P$ and $B \to K_1(1400)P$ are smaller with $\theta = 32°$ and $\theta = 58°$, respectively. Laporta-Nardulli-Pham, in Table IV, Ref. [8], displayed some of the branching ratios that we present in Table X. In general, both predictions agree.

In Table XI, we display branching ratios for $B \to AV$ decays with A being a $^3P_1$ or a $^1P_1$ nonstrange axial-vector meson. Most of these decays are suppressed. In general, $\mathrm{Br}(B \to A(^3P_1)V)$ is bigger than $\mathrm{Br}(B \to A(^1P_1)V)$, where mesons $A(^3P_1)$ and $A(^1P_1)$ have the same quark content. In this case, dominant decays are $B^- \to f_1 \rho^-$, $\bar{B}^0 \to a_1^{\pm} \rho^{\mp}$. Their branching ratios are $\mathcal{O}(10^{-6})$. If we compare Tables IX and XI we found that $\mathrm{Br}(B \to AP(q_1\bar{q}_2)) > \mathrm{Br}(B \to AV(q_1\bar{q}_2))$.

In Table XII, we present branching ratios for $B \to K_1 V$ decays for two values ($\theta = 32°, 58°$) of the mixing angle $K_A - K_B$. The strange axial-vector meson is $K_1(1270)$ or $K_1(1400)$. These decays are, in general, suppressed. The dominant decays are $\bar{B}^0 \to \bar{K}_1^0(1400)K^{*0}$, $B^- \to K_1^-(1400)K^{*0}$, and $B^- \to K_1^0(1270)K^{*-}$. Their branching ratios are $\mathcal{O}(10^{-6})$. On the other hand, the branching ratios of modes $\bar{B}^0 \to K_1^- \rho^+$, $B^- \to \bar{K}_1^0 \rho^-$, $B^- \to K_1^- \omega$, $\bar{B}^0 \to K_1^0 \bar{K}^{*0}$, and $B^- \to K_1^0 K^{*-}$, with $K_1 = K_1(1270)$, $K_1(1400)$, are not sensitive to the value of the mixing angle. On

TABLE XI. Branching ratios (in units of 10^{-6}) of $B \to AV$ decays, where A is a nonstrange axial-vector meson, using the ISGW2 form factors $B \to A$ transitions and WSB [LCSR] for $B \to V$ transitions.

Mode	$\mathcal{B}$	Mode	$\mathcal{B}$
$\bar{B}^0 \to a_1^- \rho^+$	4.7 [3.5]	$\bar{B}^0 \to b_1^- \rho^+$	0.0 [$\cdots$]
$\bar{B}^0 \to a_1^+ \rho^-$	4.3 [$\cdots$]	$\bar{B}^0 \to b_1^+ \rho^-$	1.6 [$\cdots$]
$\bar{B}^0 \to a_1^0 \rho^0$	0.01 [0.009]	$\bar{B}^0 \to b_1^0 \rho^0$	0.002 [$\cdots$]
$B^- \to a_1^- \rho^0$	3.0 [2.3]	$B^- \to b_1^- \rho^0$	0.0005 [$\cdots$]
$B^- \to a_1^0 \rho^-$	2.4 [$\cdots$]	$B^- \to b_1^0 \rho^-$	0.86 [$\cdots$]
$\bar{B}^0 \to a_1^0 \omega$	0.003 [0.02]	$\bar{B}^0 \to b_1^0 \omega$	0.02 [$\cdots$]
$B^- \to a_1^- \omega$	2.2 [5.1]	$B^- \to b_1^- \omega$	0.004 [$\cdots$]
$\bar{B}^0 \to a_1^0 \phi$	0.0005 [$\cdots$]	$\bar{B}^0 \to b_1^0 \phi$	0.0002 [$\cdots$]
$B^- \to a_1^- \phi$	0.001 [$\cdots$]	$B^- \to b_1^- \phi$	0.0004 [$\cdots$]
$\bar{B}^0 \to a_1^0 K^{*0}$	0.64 [0.61]	$\bar{B}^0 \to b_1^0 K^{*0}$	0.15 [$\cdots$]
$\bar{B}^0 \to a_1^+ K^{*-}$	0.92 [$\cdots$]	$\bar{B}^0 \to b_1^+ K^{*-}$	0.32 [$\cdots$]
$\bar{B}^- \to a_1^0 K^{*-}$	0.86 [0.81]	$\bar{B}^- \to b_1^0 K^{*-}$	0.12 [0.17]
$\bar{B}^- \to a_1^- \bar{K}^{*0}$	0.51 [$\cdots$]	$\bar{B}^- \to b_1^- \bar{K}^{*0}$	0.18 [$\cdots$]
$\bar{B}^0 \to f_1 \rho^0$	0.03	$\bar{B}^0 \to h_1 \rho^0$	0.02 [$\cdots$]
$B^- \to f_1 \rho^-$	4.9 [$\cdots$]	$B^- \to h_1 \rho^-$	1.6 [$\cdots$]
$\bar{B}^0 \to f_1 \omega$	0.02 [$\cdots$]	$\bar{B}^0 \to h_1 \omega$	0.005 [$\cdots$]
$\bar{B}^0 \to f_1 \phi$	0.0005 [$\cdots$]	$\bar{B}^0 \to h_1 \phi$	0.0002 [$\cdots$]
$\bar{B}^0 \to f_1 K^{*0}$	0.43 [0.42]	$\bar{B}^0 \to h_1 K^{*0}$	0.16 [$\cdots$]
$B^- \to f_1 K^{*-}$	0.45 [0.48]	$B^- \to h_1 K^{*-}$	0.34 [$\cdots$]

TABLE X. Branching ratios (in units of 10^{-6}) of $B \to AP$ decays, where A is a strange axial-vector meson $K_1(1270)$ or $K_1(1400)$, using the ISGW2 form factors for $B \to A$ transitions and WSB [LCSR] for $B \to P$ transitions.

Mode	$\mathcal{B}$ (32°)	$\mathcal{B}$ (58°)	Mode	$\mathcal{B}$ (32°)	$\mathcal{B}$ (58°)
$\bar{B}^0 \to K_1^-(1270)\pi^+$	4.3 [2.8]	4.3 [2.8]	$\bar{B}^0 \to K_1^-(1400)\pi^+$	2.3 [1.5]	2.3 [1.5]
$\bar{B}^0 \to \bar{K}_1^0(1270)\pi^0$	2.3 [1.5]	2.1 [1.4]	$\bar{B}^0 \to \bar{K}_1^0(1400)\pi^0$	1.7 [1.3]	1.6 [1.3]
$B^- \to K_1^-(1270)\pi^0$	2.5 [1.6]	1.6 [0.9]	$B^- \to K_1^-(1400)\pi^0$	0.67 [0.51]	0.64 [0.55]
$B^- \to \bar{K}_1^0(1270)\pi^-$	4.7 [3.0]	4.7 [3.0]	$B^- \to \bar{K}_1^0(1400)\pi^-$	2.5 [1.7]	2.5 [1.7]
$\bar{B}^0 \to \bar{K}_1^0(1270)\eta$	1.5 [1.1]	10.2 [9.8]	$\bar{B}^0 \to \bar{K}_1^0(1400)\eta$	52.8 [52.5]	46.8 [46.6]
$B^- \to K_1^-(1270)\eta$	0.95 [0.65]	20.7 [19.4]	$B^- \to K_1^-(1400)\eta$	95.1 [93.3]	84.8 [83.1]
$\bar{B}^0 \to \bar{K}_1^0(1270)\eta'$	1.1 [0.8]	9.4 [9.1]	$\bar{B}^0 \to \bar{K}_1^0(1400)\eta'$	51.4 [51.2]	46.0 [45.8]
$B^- \to K_1^-(1270)\eta'$	0.53 [0.4]	16.6 [15.6]	$B^- \to K_1^-(1400)\eta'$	80.0 [78.5]	71.9 [70.5]
$\bar{B}^0 \to K_1^0(1270)\bar{K}^0$	0.20 [0.17]	0.20 [0.17]	$\bar{B}^0 \to K_1^0(1400)\bar{K}^0$	0.11 [0.09]	0.11 [0.09]
$\bar{B}^0 \to \bar{K}_1^0(1270)K^0$	0.02 [$\cdots$]	0.70 [$\cdots$]	$\bar{B}^0 \to \bar{K}_1^0(1400)K^0$	4.1 [$\cdots$]	3.6 [$\cdots$]
$B^- \to K_1^-(1270)K^0$	0.02 [$\cdots$]	0.75 [$\cdots$]	$B^- \to K_1^-(1400)K^0$	4.4 [$\cdots$]	3.9 [$\cdots$]
$B^- \to K_1^0(1270)K^-$	0.22 [0.18]	0.22 [0.18]	$B^- \to K_1^0(1400)K^-$	0.12 [0.10]	0.12 [0.10]

TABLE XII. Branching ratios (in units of 10^{-6}) of $B \to AV$ decays, where A is a strange axial-vector meson $K_1(1270)$ or $K_1(1400)$, using the ISGW2 form factors for $B \to A$ transitions and WSB [LCSR] for $B \to V$ transitions.

Mode	$\mathcal{B}$ (32°)	$\mathcal{B}$ (58°)	Mode	$\mathcal{B}$ (32°)	$\mathcal{B}$ (58°)
$\bar{B}^0 \to K_1^-(1270)\rho^+$	0.62 [0.45]	0.62 [0.45]	$\bar{B}^0 \to K_1^-(1400)\rho^+$	0.45 [0.31]	0.45 [0.31]
$\bar{B}^0 \to \bar{K}_1^0(1270)\rho^0$	0.001 [⋯]	0.02 [⋯]	$\bar{B}^0 \to \bar{K}_1^0(1400)\rho^0$	0.05 [⋯]	0.04 [⋯]
$B^- \to K_1^-(1270)\rho^0$	0.10 [0.03]	0.05 [0.07]	$B^- \to K_1^-(1400)\rho^0$	0.02 [0.01]	0.02 [0.01]
$B^- \to \bar{K}_1^0(1270)\rho^-$	0.001 [⋯]	0.001 [⋯]	$B^- \to \bar{K}_1^0(1400)\rho^-$	0.001 [0.0006]	0.001 [0.0006]
$\bar{B}^0 \to \bar{K}_1^0(1270)\omega$	0.0002 [0.001]	0.001 [0.003]	$\bar{B}^0 \to \bar{K}_1^0(1400)\omega$	0.004 [0.005]	0.003 [0.007]
$B^- \to K_1^-(1270)\omega$	0.06 [0.16]	0.07 [0.15]	$B^- \to K_1^-(1400)\omega$	0.06 [0.07]	0.06 [0.07]
$\bar{B}^0 \to \bar{K}_1^0(1270)\phi$	0.004 [⋯]	0.25 [⋯]	$\bar{B}^0 \to \bar{K}_1^0(1400)\phi$	0.87 [⋯]	0.66 [⋯]
$B^- \to K_1^-(1270)\phi$	0.004 [⋯]	0.27 [⋯]	$B^- \to K_1^-(1400)\phi$	0.93 [⋯]	0.69 [⋯]
$\bar{B}^0 \to K_1^0(1270)\bar{K}^{*0}$	0.96 [0.76]	0.96 [0.76]	$\bar{B}^0 \to K_1^0(1400)\bar{K}^{*0}$	0.67 [0.52]	0.67 [0.52]
$\bar{B}^0 \to \bar{K}_1^0(1270)K^{*0}$	0.0007 [⋯]	0.31 [⋯]	$\bar{B}^0 \to \bar{K}_1^0(1400)K^{*0}$	1.1 [⋯]	0.82 [⋯]
$B^- \to K_1^-(1270)K^{*0}$	0.0007 [⋯]	0.33 [⋯]	$B^- \to K_1^-(1400)K^{*0}$	1.2 [⋯]	0.88 [⋯]
$B^- \to K_1^0(1270)K^{*-}$	1.0 [0.82]	1.0 [0.82]	$B^- \to K_1^0(1400)K^{*-}$	0.73 [0.56]	0.73 [0.56]

TABLE XIII. Branching ratios (in units of 10^{-6}) of $B \to AA$ decays, where A is a nonstrange axial-vector meson, using the ISGW2 form factors.

Mode	$\mathcal{B}$
$\bar{B}^0 \to a_1^- a_1^+$	6.4
$\bar{B}^0 \to a_1^0 a_1^0$	0.1
$B^- \to a_1^- a_1^0$	3.6
$\bar{B}^0 \to a_1^0 f_1$	0.02
$B^- \to a_1^- f_1$	3.7

the contrary, branching ratios of $\bar{B}^0 \to \bar{K}_1^0(1270)K^{*0}$ and $B^- \to K_1^-(1270)K^{*0}$ strongly depend on the value of θ. The predictions obtained in Ref. [9] (see Tables II and III) for $B \to K_1(1270)\phi$ and $B \to K_1(1400)\phi$ and our predictions for these modes do not agree, except for the case with $N_c^{\rm eff} = \infty$ and $\theta = 58°$ (with $\mu = 2.5$ GeV or $\mu = 4.4$ GeV). In this case the respective branching ratios are $\mathcal{O}(10^{-7})$.

Moreover, in Tables IX, X, XI, and XII, between brackets we show values corresponding to the branching ratios $B \to AP$ and $B \to AV$, where $B \to P(V)$ transitions are calculated in the LCSR approach. The values with the symbol [⋯] represent branching ratios which basically have equal value with respect to the calculated with the WSB model. In general, the branching ratios are smaller compared with the calculated with the WSB model. The branching ratios for $B^- \to a_1^- \omega$, $B^- \to b_1^- \omega$, and $B \to K_1 \omega$ decays increase their values, because in the LCSR approach the form factors for $B \to \omega$ transitions are bigger compared with the WSB model.

In Table XIII, we present branching ratios for five $B \to AA$ decays, where A is a nonstrange axial-vector meson. The branching ratio of $\bar{B}^0 \to a_1^- a_1^+$ is $\mathcal{O}(10^{-6})$. In this group, this decay is dominant. From Tables XI (see second column) and XIII we conclude that ${\rm Br}(B \to a_1^- V(q_1\bar{q}_2) \sim {\rm Br}(B \to a_1^- A(q_1\bar{q}_2))$, where V and A are nonstrange mesons.

In Table XIV, we show branching ratios for $B \to K_1 A$ decays for two values ($\theta = 32°, 58°$) of the mixing angle $K_{1A} - K_{1B}$. The strange axial-vector meson is $K_1(1270)$ or $K_1(1400)$. Branching ratios of the decays $\bar{B}^0 \to K_1^- a_1^+$ and $B^- \to \bar{K}_1^0 a_1^-$ are not sensitive to the mixing angle. In this group, ${\rm Br}(\bar{B}^0 \to K_1^-(1270)a_1^+) \sim 10^{-7}$ is the biggest. From Tables XII and XIV we conclude that ${\rm Br}(B \to K_1\rho) \sim {\rm Br}(B \to K_1 a_1)$.

Finally, in Table XV, we present a summary about experimental information given in Ref. [5] for branching ratios of some charmless $B \to AP$, AV, AA decays. In general, bounds for these branching ratios are $<(10^{-3}$–$10^{-4})$. There is a similar situation for charmed and charmonium B decays [5].

TABLE XIV. Branching ratios (in units of 10^{-6}) of $B \to K_1 A$ decays, where A is a nonstrange axial-vector meson, using the ISGW2 form factors. The K_1 axial-vector mesons are $K_1(1270)$ and $K_1(1400)$.

Mode	$\mathcal{B}$ (32°)	$\mathcal{B}$ (58°)	Mode	$\mathcal{B}$ (32°)	$\mathcal{B}$ (58°)
$\bar{B}^0 \to K_1^-(1270)a_1^+$	0.79	0.79	$\bar{B}^0 \to K_1^-(1400)a_1^+$	0.49	0.49
$\bar{B}^0 \to \bar{K}_1^0(1270)a_1^0$	0.002	0.03	$\bar{B}^0 \to \bar{K}_1^0(1400)a_1^0$	0.08	0.06
$B^- \to K_1^-(1270)a_1^0$	0.12	0.06	$B^- \to K_1^-(1400)a_1^0$	0.03	0.03
$B^- \to \bar{K}_1^0(1270)a_1^-$	0.002	0.002	$B^- \to \bar{K}_1^0(1400)a_1^-$	0.001	0.001
$\bar{B}^0 \to \bar{K}_1^0(1270)f_1$	0.44	0.53	$\bar{B}^0 \to \bar{K}_1^0(1400)f_1$	0.48	0.44
$B^- \to K_1^-(1270)f_1$	0.15	0.27	$B^- \to K_1^-(1400)f_1$	0.34	0.29

TABLE XV. Experimental bounds for branching ratios of some charmless $B \to AP$, AV, and AA decays reported in [5].

Mode	$\mathcal{B}^{\rm exp}$
$B^0 \to a_1^{\mp}(1260)\pi^{\pm}$	$<4.9 \times 10^{-4}$
$B^0 \to a_1^0(1260)\pi^0$	$<1.1 \times 10^{-3}$
$B^0 \to a_1^+(1260)\rho^-$	$<3.4 \times 10^{-3}$
$B^0 \to a_1^0(1260)\rho^0$	$<2.4 \times 10^{-3}$
$B^0 \to K_1^+(1400)\pi^-$	$<1.1 \times 10^{-3}$
$B^0 \to a_1^+(1260)K^-$	$<2.3 \times 10^{-4}$
$B^0 \to K_1^0(1400)\rho^0$	$<3.0 \times 10^{-3}$
$B^0 \to K_1^0(1400)\phi$	$<5.0 \times 10^{-3}$
$B^+ \to a_1^+(1260)\pi^0$	$<1.7 \times 10^{-3}$
$B^+ \to a_1^0(1260)\pi^+$	$<9.0 \times 10^{-4}$
$B^+ \to a_1^+(1260)\rho^0$	$<6.2 \times 10^{-4}$
$B^+ \to a_1^+(1260)a_1^0(1260)$	$<1.3 \times 10^{-2}$
$B^+ \to K_1^0(1400)\pi^+$	$<2.6 \times 10^{-3}$
$B^+ \to K_1^+(1400)\rho^0$	$<7.8 \times 10^{-4}$
$B^+ \to K_1^+(1400)\phi$	$<1.1 \times 10^{-3}$

VII. CONCLUSIONS

In this work, we have presented a systematic study of exclusive charmless nonleptonic two-body B decays including axial-vector mesons in the final state. Branching ratios of decays $B \to PA$, $B \to VA$, and $B \to AA$ (where A, V and P denote an axial vector, a vector, and a pseudoscalar meson, respectively) have been calculated assuming the naive factorization hypothesis and using the improved version of the nonrelativistic ISGW quark model in order to obtain form factors required for $B \to A$ transitions. Form factors for $B \to P$ and $B \to V$ transitions were obtained from the WSB model and LCSR approach. We have included contributions that arise from the effective $\Delta B = 1$ weak Hamiltonian $H_{\rm eff}$, i.e., we have considered W-external and W-internal emissions, which have contributions of a_1 and a_2 QCD parameters, respectively, and penguin contributions given by $a_{3,\ldots,10}$ QCD parameters. The respective factorized amplitudes of these decays are explicitly showed in the appendices and their penguin contributions have been classified. We also present a comparison between $B \to A$ and $B \to V$ transitions.

We have obtained branching ratios for 141 exclusive channels $B \to AP$, AV, and AA where the axial-vector meson can be a $^3P_1$ or a $^1P_1$ meson. We also studied the dependence of the branching ratios for $B \to K_1 P(V, A)$ decays [$K_1 = K_1(1270)$, $K_1(1400)$ are the physical strange axial-vector mesons] with respect to the mixing angle between K_{1A} and K_{1B}. The best scenarios for determining this mixing angle are the decays $\bar{B}^0 \to \bar{K}_1^0(1270)\eta^{(\prime)} \times (K^0, K^{*0})$ and $B^- \to K_1^-(1270)\eta^{(\prime)}(K^0, K^{*0})$ because their branching ratios strongly depend on the mixing angle.

Our results show that some of these decays can be reached in experiment. In fact, decays $B^- \to a_1^0\pi^-$, $\bar{B}^0 \to a_1^{\pm}\pi^{\mp}$, $B^- \to a_1^-\bar{K}^0$, $\bar{B}^0 \to a_1^+K^-$, $\bar{B}^0 \to f_1\bar{K}^0$, $B^- \to f_1K^-$, $B^- \to K_1^-(1400)\eta^{(\prime)}$, $B^- \to b_1^-\bar{K}^0$, and $\bar{B}^0 \to b_1^+\pi^-(K^-)$ have branching ratios of the order of 10^{-5}.

At the experimental level there is not enough information. In Ref. [5] there are only bounds for branching ratios of some charmless $B \to AP$, AV, AA decays (see Table XV). In general, our results are smaller that these bounds by 2 orders of magnitude. Our predictions ${\rm Br}(\bar{B}^0 \to a_1^-\pi^+) = 36.7 \times 10^{-6}[23.5 \times 10^{-6}]$ and ${\rm Br}(\bar{B}^0 \to a_1^+\pi^-) = 74.3 \times 10^{-6}[\cdots]$, i.e. the CP-averaged branching ratio ${\rm Br}(\bar{B}^0 \to a_1^{\mp}\pi^{\pm}) = 55.5 \times 10^{-6}[48.9 \times 10^{-6}]$ is consistent with the experimental average value ${\rm Br}(\bar{B}^0 \to a_1^{\mp}\pi^{\pm}) = (40.9 \pm 7.6) \times 10^{-6}$ [8]. This average includes *BABAR* and Belle results [1].

In general, we can explain the large branching ratios for $B \to K_1(1400)\eta^{(\prime)}$ as a combination of effects, the constructive interference of the terms a_4 and a_6 which are the bigger coefficients in the penguin sector of the effective Hamiltonian and the two-mixing $K_{1A} - K_{1B}$ and $\eta - \eta'$ involved in the decays.

Finally, we want to mention that our predictions are at the same order as ones obtained by Laporta-Nardulli-Pham (see Tables V and VI in Ref. [8]), although our values are in general bigger, except in a few modes. On the other hand, predictions obtained in Ref. [9] (see Tables II and III) for $B \to K_1(1270)\phi$ and $B \to K_1(1400)\phi$ and our predictions for these modes (see Table XI) do not agree, except for the case with $N_c^{\rm eff} = \infty$ and $\theta = 58°$ (with $\mu = 2.5$ GeV or $\mu = 4.4$ GeV). In this case the respective branching ratios are $\mathcal{O}(10^{-7})$.

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APPENDIX A: MATRIX ELEMENTS FOR B DECAYS TO AN AXIAL AND A PSEUDOSCALAR MESON

$$\mathcal{M}(\bar{B}^0 \to a_1^-\pi^+) = 2m_{a_1}f_{a_1}F_1^{B\to\pi}(m_{a_1}^2)\{V_{ub}V_{ud}^*a_1 - V_{tb}V_{td}^*(a_4 + a_{10})\} \quad \text{(A1)}$$

$$\mathcal{M}(\bar{B}^0 \to a_1^+\pi^-) = 2im_\pi f_\pi V_0^{B\to a_1}(m_\pi^2)\Big\{V_{ub}V_{ud}^*a_1 - V_{tb}V_{td}^* \times\Big[a_4 + a_{10} + 2(a_6 + a_8) \times\frac{m_{\pi^-}^2}{(m_u + m_d)(m_b - m_u)}\Big]\Big\} \quad \text{(A2)}$$

$$\mathcal{M}(\bar{B}^0 \to a_1^0 \pi^0) = 2im_\pi f_\pi V_0^{B\to a_1}(m_\pi^2)\left\{V_{ub}V_{ud}^* a_2 - V_{tb}V_{td}^*\left[-a_4 - (2a_6 - a_8)\frac{m_\pi^2}{2m_d(m_b - m_d)} - \frac{1}{2}(3a_7 - 3a_9 - a_{10})\right]\right\} + 2m_{a_1} f_{a_1} F_1^{B\to\pi}(m_{a_1}^2)\left\{V_{ub}V_{ud}^* a_2 - V_{ub}V_{ud}^*\left[-a_4 - \frac{1}{2}(3a_7 - 3a_9 - a_{10})\right]\right\} \quad \text{(A3)}$$

$$\mathcal{M}(B^- \to a_1^- \pi^0) = 2im_\pi f_\pi V_0^{B\to a_1}(m_\pi^2)\left\{V_{ub}V_{ud}^* a_2 - V_{tb}V_{td}^*\left[-a_4 - \frac{1}{2}(3a_7 - 3a_9 - a_{10}) - (2a_6 - a_8)\frac{m_\pi^2}{2m_d(m_b - m_d)}\right]\right\} + 2m_{a_1} f_{a_1} F_1^{B\to\pi}(m_{a_1}^2)\{V_{ub}V_{ud}^* a_1 - V_{tb}V_{td}^*(a_4 + a_{10})\} \quad \text{(A4)}$$

$$\mathcal{M}(B^- \to a_1^0 \pi^-) = 2im_\pi f_\pi V_0^{B\to a_1}(m_\pi^2)\left\{V_{ub}V_{ud}^* a_1 - V_{tb}V_{td}^*\left[a_4 + a_{10} + 2(a_6 + a_8)\frac{m_{\pi^-}^2}{(m_d + m_u)(m_b - m_u)}\right]\right\} + 2m_{a_1} f_{a_1} F_1^{B\to\pi}(m_{a_1}^2)\left\{V_{ub}V_{ud}^* a_2 - V_{tb}V_{td}^*\left[-a_4 - \frac{1}{2}(3a_7 - 3a_9 - a_{10})\right]\right\} \quad \text{(A5)}$$

$$\mathcal{M}(\bar{B}^0 \to a_1^0 \eta^{(\prime)}) = 2im_{\eta^{(\prime)}} f_{\eta^{(\prime)}}^u V_0^{B\to a_1}(m_{\eta^{(\prime)}}^2)\left\{V_{ub}V_{ud}^* a_2 - V_{tb}V_{td}^*\left[2a_3 + a_4 - 2a_5 + (2a_6 - a_8)\frac{m_{\eta^{(\prime)}}^2}{2m_s(m_b - m_d)}\left(\frac{f_{\eta^{(\prime)}}^s}{f_{\eta^{(\prime)}}^u} - 1\right)r_{\eta^{(\prime)}} - \frac{1}{2}(a_7 - a_9 + a_{10})\right]\right\} + 2m_{a_1} f_{a_1} F_1^{B\to\eta^{(\prime)}}(m_{a_1}^2)\left\{V_{ub}V_{ud}^* a_2 - V_{tb}V_{td}^*\left[-a_4 + \frac{1}{2}a_{10} - \frac{3}{2}(a_7 - a_9)\right]\right\} - 2im_{\eta^{(\prime)}} f_{\eta^{(\prime)}}^s V_0^{B\to a_1}(m_{\eta^{(\prime)}}^2)\left\{V_{tb}V_{td}^*\left[a_3 - a_5 + \frac{1}{2}(a_7 - a_9)\right]\right\} + 2im_{\eta^{(\prime)}} f_{\eta^{(\prime)}}^c V_0^{B\to a_1}(m_{\eta^{(\prime)}}^2)\{V_{cb}V_{cd}^* a_2 - V_{tb}V_{td}^*(a_3 - a_5 - a_7 + a_9)\} \quad \text{(A6)}$$

$$\mathcal{M}(B^- \to a_1^- \eta^{(\prime)}) = 2im_{\eta^{(\prime)}} f_{\eta^{(\prime)}}^u V_0^{B\to a_1}(m_{\eta^{(\prime)}}^2)\left\{V_{ub}V_{ud}^* a_2 - V_{tb}V_{td}^*\left[2a_3 + a_4 - 2a_5 - \frac{1}{2}(a_7 - a_9 + a_{10}) + (2a_6 - a_8)\frac{m_{\eta^{(\prime)}}^2}{2m_s(m_b - m_d)}\left(\frac{f_{\eta^{(\prime)}}^s}{f_{\eta^{(\prime)}}^u} - 1\right)r_{\eta^{(\prime)}}\right]\right\} + 2m_{a_1} f_{a_1} F_1^{B\to\eta^{(\prime)}}(m_{a_1}^2)\{V_{ub}V_{ud}^* a_1 - V_{tb}V_{td}^*(a_4 + a_{10})\} - 2im_{\eta^{(\prime)}} f_{\eta^{(\prime)}}^s V_0^{B\to a_1}(m_{\eta^{(\prime)}}^2)\left\{V_{tb}V_{td}^*\left[2a_3 + a_4 - 2a_5 - \frac{1}{2}(a_7 - a_9)\right]\right\} + 2im_{\eta^{(\prime)}} f_{\eta^{(\prime)}}^c V_0^{B\to a_1}(m_{\eta^{(\prime)}}^2)\{V_{cb}V_{cd}^* a_2 - V_{tb}V_{td}^*(a_3 - a_5 - a_7 + a_9)\} \quad \text{(A7)}$$

$$\mathcal{M}(\bar{B}^0 \to a_1^0 \bar{K}^0) = 2m_{a_1} f_{a_1} F_1^{B\to K}(m_{a_1}^2)\left\{V_{ub}V_{us}^* a_2 - V_{tb}V_{ts}^*\frac{3}{2}(-a_7 + a_9)\right\} - 2im_K f_K V_0^{B\to a_1}(m_K^2) V_{tb}V_{ts}^*\left[a_4 - \frac{1}{2}a_{10} + (2a_6 - a_8)\frac{m_K^2}{(m_s + m_d)(m_b - m_d)}\right] \quad \text{(A8)}$$

$$\mathcal{M}(\bar{B}^0 \to a_1^+ K^-) = 2im_K f_K V_0^{B\to a_1}(m_K^2)\left\{V_{ub}V_{us}^* a_1 - V_{tb}V_{ts}^*\left[a_4 + a_{10} + 2(a_6 + a_8)\frac{m_K^2}{(m_s + m_u)(m_b - m_u)}\right]\right\} \quad \text{(A9)}$$

$$\mathcal{M}(B^- \to a_1^0 K^-) = 2im_K f_K V_0^{B\to a_1}(m_K^2)\left\{V_{ub}V_{us}^* a_1 - V_{tb}V_{ts}^*\left[a_4 + a_{10} + 2(a_6 + a_8)\frac{m_{K^-}^2}{(m_s + m_u)(m_b - m_u)}\right]\right\} + 2m_{a_1} f_{a_1} F_1^{B\to K}(m_{a_1}^2)(\epsilon^* \cdot p_B)\left\{V_{ub}V_{us}^* a_2 - V_{tb}V_{ts}^*\frac{3}{2}(-a_7 + a_9)\right\} \quad \text{(A10)}$$

$$\mathcal{M}(B^- \to a_1^- \bar{K}^0) = -2im_K f_K V_0^{B\to a_1}(m_K^2) V_{tb}V_{ts}^*\left[a_4 - \frac{1}{2}a_{10} + (2a_6 - a_8)\frac{m_K^2}{(m_s + m_d)(m_b - m_d)}\right] \quad \text{(A11)}$$

$$\mathcal{M}(\bar{B}^0 \to f_1\pi^0) = 2im_\pi f_\pi V_0^{B\to f_1}(m_\pi^2)\left\{V_{ub}V_{ud}^* a_2 - V_{tb}V_{td}^*\left[a_4 - \frac{1}{2}(3a_7 - 3a_9 + a_{10}) + (2a_6 - a_8)\frac{m_{\pi^0}^2}{2m_d(m_b - m_d)}\right]\right\} + 2m_{f_1}f_{f_1}F_1^{B\to\pi}(m_{f_1}^2)\left\{V_{ub}V_{ud}^* a_2 - V_{tb}V_{td}^*\left[2a_3 + a_4 - 2a_5 - \frac{1}{2}(a_7 - a_9 + a_{10})\right]\right\} \quad \text{(A12)}$$

$$\mathcal{M}(B^- \to f_1\pi^-) = 2im_\pi f_\pi V_0^{B\to f_1}(m_\pi^2)\left\{V_{ub}V_{ud}^* a_1 - V_{tb}V_{td}^*\left[a_4 + a_{10} + 2(a_6 + a_8)\frac{m_{\pi^-}^2}{(m_d + m_u)(m_b - m_u)}\right]\right\} + 2m_{f_1}f_{f_1}F_1^{B\to\pi}(m_{f_1}^2)\left\{V_{ub}V_{ud}^* a_2 - V_{tb}V_{td}^*\left[2a_3 + a_4 - 2a_5 - \frac{1}{2}(a_7 - a_9 + a_{10})\right]\right\} \quad \text{(A13)}$$

$$\mathcal{M}(\bar{B}^0 \to f_1\eta^{(\prime)}) = 2im_{\eta^{(\prime)}}f_{\eta^{(\prime)}}^u V_0^{B\to f_1}(m_{\eta^{(\prime)}}^2)\left\{V_{ub}V_{ud}^* a_2 - V_{tb}V_{td}^*\left[2a_3 + a_4 - 2a_5 - \frac{1}{2}(a_7 - a_9 + a_{10}) + (2a_6 - a_8)\frac{m_{\eta^{(\prime)}}^2}{2m_d(m_b - m_d)}\left(\frac{f_{\eta^{(\prime)}}^s}{f_{\eta^{(\prime)}}^u} - 1\right)r_{\eta^{(\prime)}}\right]\right\} + 2m_{f_1}f_{f_1}F_1^{B\to\eta^{(\prime)}}(m_{f_1}^2)\left\{V_{ub}V_{ud}^* a_2 - V_{tb}V_{td}^*\left[2a_3 + a_4 - 2a_5 - \frac{1}{2}(a_7 - a_9 + a_{10})\right]\right\} - 2im_{\eta^{(\prime)}}f_{\eta^{(\prime)}}^s V_0^{B\to f_1}(m_{\eta^{(\prime)}}^2)\left\{V_{tb}V_{td}^*\left[a_3 - a_5 + \frac{1}{2}(a_7 - a_9)\right]\right\} + 2im_{\eta^{(\prime)}}f_{\eta^{(\prime)}}^c V_0^{B\to f_1}(m_{\eta^{(\prime)}}^2)\{V_{cb}V_{cd}^* a_2 - V_{tb}V_{td}^*[a_3 - a_5 - a_7 + a_9]\} \quad \text{(A14)}$$

$$\mathcal{M}(\bar{B}^0 \to f_1\bar{K}^0) = 2m_{f_1}f_{f_1}F_1^{B\to K}(m_{f_1}^2)\left\{V_{ub}V_{us}^* a_2 - V_{tb}V_{ts}^*\left[2a_3 - 2a_5 - \frac{1}{2}(a_7 - a_9)\right]\right\} - 2im_K f_K V_0^{B\to f_1}(m_K^2)V_{tb}V_{ts}^*\left[a_4 - \frac{1}{2}a_{10} + (2a_6 - a_8)\frac{m_K^2}{(m_s + m_d)(m_b - m_d)}\right] \quad \text{(A15)}$$

$$\mathcal{M}(B^- \to f_1K^-) = 2im_K f_K V_0^{B\to f_1}(m_K^2)\left\{V_{ub}V_{us}^* a_1 - V_{tb}V_{ts}^*\left[a_4 + a_{10} + 2(a_6 + a_8)\frac{m_{K^-}^2}{(m_s + m_u)(m_b - m_u)}\right]\right\} + 2m_{f_1}f_{f_1}F_1^{B\to K}(m_{f_1}^2)\left\{V_{ub}V_{us}^* a_2 - V_{tb}V_{ts}^*\left[2a_3 - 2a_5 - \frac{1}{2}(a_7 - a_9)\right]\right\} \quad \text{(A16)}$$

$$\mathcal{M}(\bar{B}^0 \to K_1^-\pi^+) = 2im_\pi f_\pi V_0^{B\to K_1}(m_\pi^2)\{V_{ub}V_{us}^* a_1 - V_{tb}V_{ts}^*(a_4 + a_{10})\} \quad \text{(A17)}$$

$$\mathcal{M}(\bar{B}^0 \to \bar{K}_1^0\pi^0) = 2im_\pi f_\pi V_0^{B\to K_1^0}(m_\pi^2)\{V_{ub}V_{us}^* a_2 - V_{tb}V_{ts}^*\tfrac{3}{2}(-a_7 + a_9)\} - 2m_{K_1}f_{K_1}F_1^{B\to\pi}(m_{K_1}^2)V_{tb}V_{ts}^*[a_4 - \tfrac{1}{2}a_{10}] \quad \text{(A18)}$$

$$\mathcal{M}(B^- \to K_1^-\pi^0) = 2im_\pi f_\pi V_0^{B\to K_1}(m_\pi^2)\{V_{ub}V_{us}^* a_2 - V_{tb}V_{ts}^*\tfrac{3}{2}(-a_7 + a_9)\} + 2m_{K_1}f_{K_1}F_1^{B\to\pi}(m_{K_1(1270)}^2)\{V_{ub}V_{us}^* a_1 - V_{tb}V_{ts}^*(a_4 + a_{10})\} \quad \text{(A19)}$$

$$\mathcal{M}(B^- \to \bar{K}_1^0\pi^-) = -2m_{K_1}f_{K_1}F_1^{B\to\pi}(m_{K_1}^2)V_{tb}V_{ts}^*\{(a_4 - \tfrac{1}{2}a_{10})\} \quad \text{(A20)}$$

$$\mathcal{M}(\bar{B}^0 \to \bar{K}_1^0\eta^{(\prime)}) = 2im_{\eta^{(\prime)}}f_{\eta^{(\prime)}}^u V_0^{B\to K_1}(m_{\eta^{(\prime)}}^2)\left\{V_{ub}V_{us}^* a_2 - V_{tb}V_{ts}^*\left[2(a_3 - a_5) - \frac{1}{2}(a_7 - a_9)\right]\right\} - 2m_{K_1}f_{K_1^0}F_1^{B\to\eta^{(\prime)}}(m_{K_1^0}^2)\left\{V_{tb}V_{ts}^*\left(a_4 - \frac{1}{2}a_{10}\right)\right\} - 2im_{\eta^{(\prime)}}f_{\eta^{(\prime)}}^s V_0^{B\to K_1}(m_{\eta^{(\prime)}}^2)\left\{V_{tb}V_{ts}^*\left[a_3 + a_4 - a_5 + \frac{1}{2}(a_7 - a_9 - a_{10}) + (2a_6 - a_8)\frac{m_{\eta^{(\prime)}}^2}{2m_s(m_b - m_s)}\left(1 - \frac{f_{\eta^{(\prime)}}^u}{f_{\eta^{(\prime)}}^s}\right)\right]\right\} + 2im_{\eta^{(\prime)}}f_{\eta^{(\prime)}}^c V_0^{B\to K_1}(m_{\eta^{(\prime)}}^2)\{V_{ub}V_{us}^* a_2 - V_{tb}V_{ts}^*(a_3 - a_5 - a_7 + a_9)\} \quad \text{(A21)}$$

$$\begin{aligned}\mathcal{M}(B^-\to K_1^-\eta^{(\prime)}) &= 2im_{\eta^{(\prime)}}f^u_{\eta^{(\prime)}}V_0^{B\to K_1}(m^2_{\eta^{(\prime)}})\Big\{V_{ub}V^*_{us}a_2 - V_{tb}V^*_{ts}\Big[2a_3-2a_5-\frac{1}{2}(a_7-a_9)\Big]\Big\}\\ &\quad + 2m_{K_1}f_{K_1}F_1^{B\to\eta^{(\prime)}}(m^2_{K_1})\{V_{ub}V^*_{us}a_1 - V_{tb}V^*_{ts}(a_4+a_{10})\} - 2im_{\eta^{(\prime)}}f^s_{\eta^{(\prime)}}V_0^{B\to K_1}(m^2_{\eta^{(\prime)}})\Big\{V_{tb}V^*_{ts}\Big[a_3+a_4\\ &\quad - a_5+\frac{1}{2}(a_7-a_9-a_{10})+(2a_6-a_8)\frac{m^2_{\eta^{(\prime)}}}{2m_s(m_b-m_s)}\Big(1-\frac{f^u_{\eta^{(\prime)}}}{f^s_{\eta^{(\prime)}}}\Big)\Big]\Big\}\\ &\quad + 2im_{\eta^{(\prime)}}f^c_{\eta^{(\prime)}}V_0^{B\to K_1}(m^2_{\eta^{(\prime)}})\{V_{cb}V^*_{cs}a_2 - V_{tb}V^*_{ts}(a_3-a_5-a_7+a_9)\}\end{aligned} \tag{A22}$$

$$\mathcal{M}(\bar{B}^0\to K_1^0\bar{K}^0) = -2m_{K_1}f_{K_1}F_1^{B\to K}(m^2_{K_1})V_{tb}V^*_{td}\{(a_4-\tfrac{1}{2}a_{10})\} \tag{A23}$$

$$\mathcal{M}(\bar{B}^0\to \bar{K}_1^0K^0) = -2im_Kf_KV_0^{B\to K_1}(m^2_K)V_{tb}V^*_{td}\Big\{\Big[a_4-\frac{1}{2}a_{10}+(2a_6-a_8)\frac{m^2_K}{(m_s+m_d)(m_b-m_s)}\Big]\Big\} \tag{A24}$$

$$\mathcal{M}(B^-\to K_1^-K^0) = -2im_Kf_KV_0^{B\to K_1}(m^2_K)V_{tb}V^*_{td}\Big\{\Big[a_4-\frac{1}{2}a_{10}+(2a_6-a_8)\frac{m^2_{K^0}}{(m_s+m_d)(m_b-m_s)}\Big]\Big\} \tag{A25}$$

$$\mathcal{M}(B^-\to K_1^0K^-) = -2m_{K_1}f_{K_1}F_1^{B\to K}(m^2_{K_1})V_{tb}V^*_{td}\{(a_4-\tfrac{1}{2}a_{10})\}. \tag{A26}$$

APPENDIX B: MATRIX ELEMENTS FOR B DECAYS TO AN AXIAL AND A VECTOR MESON

$$\begin{aligned}\mathcal{M}(\bar{B}^0\to a_1^-\rho^+) &= m_{a_1}f_{a_1}\{V_{ub}V^*_{ud}a_1 - V_{tb}V^*_{td}(a_4+a_{10})\}\Big(\frac{2V^{B\to\rho}(m^2_{a_1})}{(m_B+m_\rho)}\epsilon_{\mu\nu\alpha\beta}\epsilon^\mu_{a_1}\epsilon^\nu_\rho p^\alpha_B p^\beta_\rho\\ &\quad - i(m_B+m_\rho)A_1^{B\to\rho}(m^2_{a_1})(\epsilon_\rho\cdot\epsilon_{a_1}) + \frac{iA_2^{B\to\rho}(m^2_{a_1})}{(m_B+m_\rho)}(\epsilon_\rho\cdot p_B)(\epsilon_{a_1}\cdot p_B)\Big)\end{aligned} \tag{B1}$$

$$\begin{aligned}\mathcal{M}(\bar{B}^0\to a_1^+\rho^-) &= -m_\rho f_\rho\{V_{ub}V^*_{ud}a_1 - V_{tb}V^*_{td}(a_4+a_{10})\}\Big(\frac{2A^{B\to a_1}(m^2_\rho)}{(m_B+m_{a_1})}\epsilon_{\mu\nu\alpha\beta}\epsilon^\mu_\rho\epsilon^\nu_{a_1} p^\alpha_B p^\beta_{a_1}\\ &\quad - i(m_B+m_{a_1})V_1^{B\to a_1}(m^2_\rho)(\epsilon_{a_1}\cdot\epsilon_\rho) + \frac{iV_2^{B\to a_1}(m^2_\rho)}{(m_B+m_{a_1})}(\epsilon_{a_1}\cdot p_B)(\epsilon_\rho\cdot p_B)\Big)\end{aligned} \tag{B2}$$

$$\begin{aligned}\mathcal{M}(\bar{B}^0\to a_1^0\rho^0) &= -m_\rho f_\rho\Big\{V_{ub}V^*_{ud}a_2 - V_{tb}V^*_{td}\Big[-a_4+\frac{1}{2}(3a_7+3a_9+a_{10})\Big]\Big\}\Big(\frac{2A^{B\to a_1}(m^2_\rho)}{(m_B+m_{a_1})}\epsilon_{\mu\nu\alpha\beta}\epsilon^\mu_\rho\epsilon^\nu_{a_1}p^\alpha_B p^\beta_{a_1}\\ &\quad - i(m_B+m_{a_1})V_1^{B\to a_1}(m^2_\rho)(\epsilon_{a_1}\cdot\epsilon_\rho) + \frac{iV_2^{B\to a_1}(m^2_\rho)}{(m_B+m_{a_1})}(\epsilon_{a_1}\cdot p_B)(\epsilon_\rho\cdot p_B)\Big)\\ &\quad + m_{a_1}f_{a_1}\Big\{V_{ub}V^*_{ud}a_2 - V_{tb}V^*_{td}\Big[-a_4-\frac{1}{2}(3a_7-3a_9-a_{10})\Big]\Big\}\Big(\frac{2V^{B\to\rho}(m^2_{a_1})}{(m_B+m_\rho)}\epsilon_{\mu\nu\alpha\beta}\epsilon^\mu_{a_1}\epsilon^\nu_\rho p^\alpha_B p^\beta_\rho\\ &\quad - i(m_B+m_\rho)A_1^{B\to\rho}(m^2_{a_1})(\epsilon_\rho\cdot\epsilon_{a_1}) + \frac{iA_2^{B\to\rho}(m^2_{a_1})}{(m_B+m_\rho)}(\epsilon_\rho\cdot p_B)(\epsilon_{a_1}\cdot p_B)\Big)\end{aligned} \tag{B3}$$

$$\mathcal{M}(B^- \to a_1^- \rho^0) = -m_\rho f_\rho \{V_{ub} V_{ud}^* a_2\} \bigg(\frac{2A^{B\to a_1}(m_\rho^2)}{(m_B + m_{a_1})} \epsilon_{\mu\nu\alpha\beta} \epsilon_\rho^\mu \epsilon_{a_1}^\nu p_B^\alpha p_{a_1}^\beta - i(m_B + m_{a_1}) V_1^{B\to a_1}(m_\rho^2)(\epsilon_{a_1} \cdot \epsilon_\rho) + \frac{iV_2^{B\to a_1}(m_\rho^2)}{(m_B + m_{a_1})}(\epsilon_{a_1} \cdot p_B)(\epsilon_\rho \cdot p_B) \bigg) + m_{a_1} f_{a_1} \bigg\{ V_{ub} V_{ud}^* a_1 - V_{tb} V_{td}^* \frac{3}{2}(-a_7 + a_9 + a_{10}) \bigg\} \bigg(\frac{2V^{B\to\rho}(m_{a_1}^2)}{(m_B + m_\rho)} \times \epsilon_{\mu\nu\alpha\beta} \epsilon_{a_1}^\mu \epsilon_\rho^\nu p_B^\alpha p_\rho^\beta - i(m_B + m_\rho) A_1^{B\to\rho}(m_{a_1}^2)(\epsilon_\rho \cdot \epsilon_{a_1}) + \frac{iA_2^{B\to\rho}(m_{a_1}^2)}{(m_B + m_\rho)}(\epsilon_\rho \cdot p_B)(\epsilon_{a_1} \cdot p_B) \bigg) \tag{B4}$$

$$\mathcal{M}(B^- \to a_1^0 \rho^-) = -m_\rho f_\rho \bigg\{ V_{ub} V_{ud}^* a_1 - V_{tb} V_{td}^* \frac{3}{2}(a_7 + a_9 + a_{10}) \bigg\} \bigg(\frac{2A^{B\to a_1}(m_\rho^2)}{(m_B + m_{a_1})} \epsilon_{\mu\nu\alpha\beta} \epsilon_\rho^\mu \epsilon_{a_1}^\nu p_B^\alpha p_{a_1}^\beta - i(m_B + m_{a_1}) V_1^{B\to a_1}(m_\rho^2)(\epsilon_{a_1} \cdot \epsilon_\rho) + \frac{iV_2^{B\to a_1}(m_\rho^2)}{(m_B + m_{a_1})}(\epsilon_{a_1} \cdot p_B)(\epsilon_\rho \cdot p_B) \bigg) + m_{a_1} f_{a_1} V_{ub} V_{ud}^* a_2 \bigg(\frac{2V^{B\to\rho}(m_{a_1}^2)}{(m_B + m_\rho)} \epsilon_{\mu\nu\alpha\beta} \epsilon_{a_1}^\mu \epsilon_\rho^\nu p_B^\alpha p_\rho^\beta - i(m_B + m_\rho) A_1^{B\to\rho}(m_{a_1}^2)(\epsilon_\rho \cdot \epsilon_{a_1}) + \frac{iA_2^{B\to\rho}(m_{a_1}^2)}{(m_B + m_\rho)}(\epsilon_\rho \cdot p_B)(\epsilon_{a_1} \cdot p_B) \bigg) \tag{B5}$$

$$\mathcal{M}(\bar{B}^0 \to a_1^+ K^{*-}) = -m_{K^*} f_{K^*} \{V_{ub} V_{us}^* a_1 - V_{tb} V_{ts}^* (a_4 + a_{10})\} \bigg(\frac{2A^{B\to a_1}(m_{K^*}^2)}{(m_B + m_{a_1})} \epsilon_{\mu\nu\alpha\beta} \epsilon_{K^*}^\mu \epsilon_{a_1}^\nu p_B^\alpha p_{a_1}^\beta - i(m_B + m_{a_1}) V_1^{B\to a_1}(m_{K^*}^2)(\epsilon_{a_1} \cdot \epsilon_{K^*}) + \frac{iV_2^{B\to a_1}(m_{K^*}^2)}{(m_B + m_{a_1})}(\epsilon_{a_1} \cdot p_B)(\epsilon_{K^*} \cdot p_B) \bigg) \tag{B6}$$

$$\mathcal{M}(\bar{B}^0 \to a_1^0 \bar{K}^{*0}) = m_{K^*} f_{K^*} V_{tb} V_{ts}^* \bigg\{ \bigg(a_4 - \frac{1}{2} a_{10} \bigg) \bigg\} \bigg(\frac{2A^{B\to a_1}(m_{K^*}^2)}{(m_B + m_{a_1})} \epsilon_{\mu\nu\alpha\beta} \epsilon_{K^*}^\mu \epsilon_{a_1}^\nu p_B^\alpha p_{a_1}^\beta - i(m_B + m_{a_1}) V_1^{B\to a_1}(m_{K^*}^2)(\epsilon_{a_1} \cdot \epsilon_{K^*}) + \frac{iV_2^{B\to a_1}(m_{K^*}^2)}{(m_B + m_{a_1})}(\epsilon_{a_1} \cdot p_B)(\epsilon_{K^*} \cdot p_B) \bigg) + m_{a_1} f_{a_1} \bigg\{ V_{ub} V_{us}^* a_2 + V_{tb} V_{ts}^* \frac{3}{2}(a_7 - a_9) \bigg\} \times \bigg(\frac{2V^{B\to K^*}(m_{a_1}^2)}{(m_B + m_{K^*})} \epsilon_{\mu\nu\alpha\beta} \epsilon_{a_1}^\mu \epsilon_{K^*}^\nu p_B^\alpha p_{K^*}^\beta - i(m_B + m_{K^*}) A_1^{B\to K^*}(m_{a_1}^2)(\epsilon_{K^*} \cdot \epsilon_{a_1}) + \frac{iA_2^{B\to K^*}(m_{a_1}^2)}{(m_B + m_{K^*})}(\epsilon_{K^*} \cdot p_B)(\epsilon_{a_1} \cdot p_B) \bigg) \tag{B7}$$

$$\mathcal{M}(B^- \to a_1^0 K^{*-}) = -m_{K^*} f_{K^*} \{V_{ub} V_{us}^* a_1 - V_{tb} V_{ts}^* (a_4 + a_{10})\} \bigg(\frac{2A^{B\to a_1}(m_{K^*}^2)}{(m_B + m_{a_1})} \epsilon_{\mu\nu\alpha\beta} \epsilon_{K^*}^\mu \epsilon_{a_1}^\nu p_B^\alpha p_{a_1}^\beta - i(m_B + m_{a_1}) V_1^{B\to a_1}(m_{K^*}^2)(\epsilon_{a_1} \cdot \epsilon_{K^*}) + \frac{iV_2^{B\to a_1}(m_{K^*}^2)}{(m_B + m_{a_1})}(\epsilon_{a_1} \cdot p_B)(\epsilon_{K^*} \cdot p_B) \bigg) + m_{a_1} f_{a_1} \bigg\{ V_{ub} V_{us}^* a_2 + V_{tb} V_{ts}^* \frac{3}{2}(a_7 - a_9) \bigg\} \bigg(\frac{2V^{B\to K^*}(m_{a_1}^2)}{(m_B + m_{K^*})} \epsilon_{\mu\nu\alpha\beta} \epsilon_{a_1}^\mu \epsilon_{K^*}^\nu p_B^\alpha p_{K^*}^\beta - i(m_B + m_{K^*}) A_1^{B\to K^*}(m_{a_1}^2)(\epsilon_{K^*} \cdot \epsilon_{a_1}) + \frac{iA_2^{B\to K^*}(m_{a_1}^2)}{(m_B + m_{K^*})}(\epsilon_{K^*} \cdot p_B)(\epsilon_{a_1} \cdot p_B) \bigg) \tag{B8}$$

$$\mathcal{M}(B^- \to a_1^- \bar{K}^{*0}) = m_{K^*} f_{K^*} \bigg\{ V_{tb} V_{ts}^* \bigg(a_4 - \frac{1}{2} a_{10} \bigg) \bigg\} \bigg(\frac{2A^{B\to a_1}(m_{K^*}^2)}{(m_B + m_{a_1})} \epsilon_{\mu\nu\alpha\beta} \epsilon_{K^*}^\mu \epsilon_{a_1}^\nu p_B^\alpha p_{a_1}^\beta - i(m_B + m_{a_1}) V_1^{B\to a_1}(m_{K^*}^2)(\epsilon_{a_1} \cdot \epsilon_{K^*}) + \frac{iV_2^{B\to a_1}(m_{K^*}^2)}{(m_B + m_{a_1})}(\epsilon_{a_1} \cdot p_B)(\epsilon_{K^*} \cdot p_B) \bigg) \tag{B9}$$

$$\mathcal{M}(\bar{B}^0 \to a_1^0\omega) = -m_\omega f_\omega\bigg\{V_{ub}V_{ud}^* a_2 - V_{tb}V_{td}^*\bigg[2a_3 + a_4 + 2a_5 + \frac{1}{2}(a_7 + a_9 - a_{10})\bigg]\bigg\}\bigg(\frac{2A^{B\to a_1}(m_\omega^2)}{(m_B + m_{a_1})}\epsilon_{\mu\nu\alpha\beta}\epsilon_\omega^\mu\epsilon_{a_1}^\nu p_B^\alpha p_{a_1}^\beta - i(m_B + m_{a_1})V_1^{B\to a_1}(m_\omega^2)(\epsilon_{a_1}\cdot\epsilon_\omega) + \frac{iV_2^{B\to a_1}(m_\omega^2)}{(m_B + m_{a_1})}(\epsilon_{a_1}\cdot p_B)(\epsilon_\omega\cdot p_B)\bigg) + m_{a_1}f_{a_1}\bigg\{V_{ub}V_{ud}^* a_2 - V_{tb}V_{td}^*\bigg[-a_4 - \frac{1}{2}(3a_7 - 3a_9 - a_{10})\bigg]\bigg\}\bigg(\frac{2V^{B\to\omega}(m_{a_1}^2)}{(m_B + m_\omega)}\epsilon_{\mu\nu\alpha\beta}\epsilon_{a_1}^\mu\epsilon_\omega^\nu p_B^\alpha p_\omega^\beta - i(m_B + m_\omega)A_1^{B\to\omega}(m_{a_1}^2)(\epsilon_\omega\cdot\epsilon_{a_1}) + \frac{iA_2^{B\to\omega}(m_{a_1}^2)}{(m_B + m_\omega)}(\epsilon_\omega\cdot p_B)(\epsilon_{a_1}\cdot p_B)\bigg) \tag{B10}$$

$$\mathcal{M}(B^- \to a_1^-\omega) = -m_\omega f_\omega\bigg\{V_{ub}V_{ud}^* a_2 - V_{tb}V_{td}^*\bigg[2a_3 + a_4 + 2a_5 + \frac{1}{2}(a_7 + a_9 - a_{10})\bigg]\bigg\}\bigg(\frac{2A^{B\to a_1}(m_\omega^2)}{(m_B + m_{a_1})}\epsilon_{\mu\nu\alpha\beta}\epsilon_\omega^\mu\epsilon_{a_1}^\nu p_B^\alpha p_{a_1}^\beta - i(m_B + m_{a_1})V_1^{B\to a_1}(m_\omega^2)(\epsilon_{a_1}\cdot\epsilon_\omega) + \frac{iV_2^{B\to a_1}(m_\omega^2)}{(m_B + m_{a_1})}(\epsilon_{a_1}\cdot p_B)(\epsilon_\omega\cdot p_B)\bigg) + m_{a_1}f_{a_1}\{V_{ub}V_{ud}^* a_1 - V_{tb}V_{td}^*(a_4 + a_{10})\}\bigg(\frac{2V^{B\to\omega}(m_{a_1}^2)}{(m_B + m_\omega)}\epsilon_{\mu\nu\alpha\beta}\epsilon_{a_1}^\mu\epsilon_\omega^\nu p_B^\alpha p_\omega^\beta - i(m_B + m_\omega)A_1^{B\to\omega}(m_{a_1}^2)(\epsilon_\omega\cdot\epsilon_{a_1}) + \frac{iA_2^{B\to\omega}(m_{a_1}^2)}{(m_B + m_\omega)}(\epsilon_\omega\cdot p_B)(\epsilon_{a_1}\cdot p_B)\bigg) \tag{B11}$$

$$\mathcal{M}(\bar{B}^0 \to a_1^0\phi) = m_\phi f_\phi V_{tb}V_{td}^*\bigg\{\bigg[a_3 + a_5 - \frac{1}{2}(a_7 + a_9)\bigg]\bigg\}\bigg(\frac{2A^{B\to a_1}(m_\phi^2)}{(m_B + m_{a_1})}\epsilon_{\mu\nu\alpha\beta}\epsilon_\phi^\mu\epsilon_{a_1}^\nu p_B^\alpha p_{a_1}^\beta - i(m_B + m_{a_1})V_1^{B\to a_1}(m_\phi^2)(\epsilon_{a_1}\cdot\epsilon_\phi) + \frac{iV_2^{B\to a_1}(m_\phi^2)}{(m_B + m_{a_1})}(\epsilon_{a_1}\cdot p_B)(\epsilon_\phi\cdot p_B)\bigg) \tag{B12}$$

$$\mathcal{M}(B^- \to a_1^-\phi) = -\sqrt{2}\mathcal{M}(\bar{B}^0 \to a_1^0\phi) \tag{B13}$$

$$\mathcal{M}(\bar{B}^0 \to f_1\rho^0) = -m_\rho f_\rho\bigg\{V_{ub}V_{ud}^* a_2 - V_{tb}V_{td}^*\bigg[-a_4 + \frac{1}{2}(3a_7 + 3a_9 + a_{10})\bigg]\bigg\}\bigg(\frac{2A^{B\to f_1}(m_\rho^2)}{(m_B + m_{f_1})}\epsilon_{\mu\nu\alpha\beta}\epsilon_\rho^\mu\epsilon_{f_1}^\nu p_B^\alpha p_{f_1}^\beta - i(m_B + m_{f_1})V_1^{B\to f_1}(m_\rho^2)(\epsilon_{f_1}\cdot\epsilon_\rho) + \frac{iV_2^{B\to f_1}(m_\rho^2)}{(m_B + m_{f_1})}(\epsilon_{f_1}\cdot p_B)(\epsilon_\rho\cdot p_B)\bigg) + m_{f_1}f_{f_1}\bigg\{V_{ub}V_{ud}^* a_2 - V_{tb}V_{td}^* \times\bigg[2a_3 + a_4 - 2a_5 - \frac{1}{2}(a_7 - a_9 + a_{10})\bigg]\bigg\}\bigg(\frac{2V^{B\to\rho}(m_{f_1}^2)}{(m_B + m_\rho)}\epsilon_{\mu\nu\alpha\beta}\epsilon_{f_1}^\mu\epsilon_\rho^\nu p_B^\alpha p_\rho^\beta - i(m_B + m_\rho)A_1^{B\to\rho}(m_{f_1}^2)(\epsilon_\rho\cdot\epsilon_{f_1}) + \frac{iA_2^{B\to\rho}(m_{f_1}^2)}{(m_B + m_\rho)}(\epsilon_\rho\cdot p_B)(\epsilon_{f_1}\cdot p_B)\bigg) \tag{B14}$$

$$\mathcal{M}(B^- \to f_1\rho^-) = -m_\rho^- f_\rho\{V_{ub}V_{ud}^* a_1 - V_{tb}V_{td}^*(a_4 + a_{10})\}\bigg(\frac{2A^{B\to f_1}(m_\rho^2)}{(m_B + m_{f_1})}\epsilon_{\mu\nu\alpha\beta}\epsilon_\rho^\mu\epsilon_{f_1}^\nu p_B^\alpha p_{f_1}^\beta - i(m_B + m_{f_1})V_1^{B\to f_1}(m_{\rho^-}^2)(\epsilon_{f_1}\cdot\epsilon_\rho) + \frac{iV_2^{B\to f_1}(m_\rho^2)}{(m_B + m_{f_1})}(\epsilon_{f_1}\cdot p_B)(\epsilon_\rho\cdot p_B)\bigg) + m_{f_1}f_{f_1}\bigg\{V_{ub}V_{ud}^* a_2 - V_{tb}V_{td}^*\bigg[2a_3 + a_4 - 2a_5 - \frac{1}{2}(a_7 - a_9 + a_{10})\bigg]\bigg\}\bigg(\frac{2V^{B\to\rho}(m_{f_1}^2)}{(m_B + m_\rho)}\epsilon_{\mu\nu\alpha\beta}\epsilon_{f_1}^\mu\epsilon_\rho^\nu p_B^\alpha p_\rho^\beta - i(m_B + m_\rho)A_1^{B\to\rho}(m_{f_1}^2)(\epsilon_\rho\cdot\epsilon_{f_1}) + \frac{iA_2^{B\to\rho}(m_{f_1}^2)}{(m_B + m_\rho)}(\epsilon_\rho\cdot p_B)(\epsilon_{f_1}\cdot p_B)\bigg) \tag{B15}$$

$$\mathcal{M}(\bar{B}^0 \to f_1 \bar{K}^{*0}) = m_{K^*} f_{K^*} V_{tb} V_{ts}^* \left\{ \left(a_4 - \frac{1}{2} a_{10} \right) \right\} \left(\frac{2A^{B\to f_1}(m_{K^*}^2)}{(m_B + m_{f_1})} \epsilon_{\mu\nu\alpha\beta} \epsilon_{K^*}^{\mu} \epsilon_{f_1}^{\nu} p_B^{\alpha} p_{f_1}^{\beta} - i(m_B + m_{f_1}) V_1^{B\to f_1}(m_{K^*}^2)(\epsilon_{f_1} \cdot \epsilon_{K^*}) + \frac{iV_2^{B\to f_1}(m_{K^*}^2)}{(m_B + m_{f_1})} (\epsilon_{f_1} \cdot p_B)(\epsilon_{K^*} \cdot p_B) \right) + m_{f_1} f_{f_1} \left\{ V_{ub} V_{us}^* a_2 - V_{tb} V_{ts}^* \left[2a_3 - 2a_5 - \frac{1}{2}(a_7 - a_9) \right] \right\} \times \left(\frac{2V^{B\to K^*(m_{f_1}^2)}}{(m_B + m_{K^*})} \epsilon_{\mu\nu\alpha\beta} \epsilon_{f_1}^{\mu} \epsilon_{K^*}^{\nu} p_B^{\alpha} p_{K^*}^{\beta} - i(m_B + m_{K^*}) A_1^{B\to K^*}(m_{f_1}^2)(\epsilon_{K^*} \cdot \epsilon_{f_1}) + \frac{iA_2^{B\to K^*}(m_{f_1}^2)}{(m_B + m_{K^*})} (\epsilon_{K^*} \cdot p_B)(\epsilon_{f_1} \cdot p_B) \right) \tag{B16}$$

$$\mathcal{M}(B^- \to f_1 K^{*-}) = -m_{K^*} f_{K^*} \{ V_{ub} V_{us}^* a_1 - V_{tb} V_{ts}^* (a_4 + a_{10}) \} \left(\frac{2A^{B\to f_1}(m_{K^*}^2)}{(m_B + m_{f_1})} \epsilon_{\mu\nu\alpha\beta} \epsilon_{K^*}^{\mu} \epsilon_{f_1}^{\nu} p_B^{\alpha} p_{f_1}^{\beta} - i(m_B + m_{f_1}) V_1^{B\to f_1}(m_{K^*}^2)(\epsilon_{f_1} \cdot \epsilon_{K^*}) + \frac{iV_2^{B\to f_1}(m_{K^*}^2)}{(m_B + m_{f_1})} (\epsilon_{f_1} \cdot p_B)(\epsilon_{K^*} \cdot p_B) \right) + m_{f_1} f_{f_1} \left\{ V_{ub} V_{us}^* a_2 - V_{tb} V_{ts}^* \left[2a_3 - 2a_5 - \frac{1}{2}(a_7 - a_9) \right] \right\} \left(\frac{2V^{B\to K^*}(m_{f_1}^2)}{(m_B + m_{K^*})} \epsilon_{\mu\nu\alpha\beta} \epsilon_{f_1}^{\mu} \epsilon_{K^*}^{\nu} p_B^{\alpha} p_{K^*}^{\beta} - i(m_B + m_{K^*}) A_1^{B\to K^*}(m_{f_1}^2)(\epsilon_{K^*} \cdot \epsilon_{f_1}) + \frac{iA_2^{B\to K^*}(m_{f_1}^2)}{(m_B + m_{K^*})} (\epsilon_{K^*} \cdot p_B)(\epsilon_{f_1} \cdot p_B) \right) \tag{B17}$$

$$\mathcal{M}(\bar{B}^0 \to f_1 \omega) = -m_{\omega} f_{\omega} \left\{ V_{ub} V_{ud}^* a_2 - V_{tb} V_{td}^* \left[2a_3 + a_4 + 2a_5 + \frac{1}{2}(a_7 + a_9 - a_{10}) \right] \right\} \left(\frac{2A^{B\to f_1}(m_{\omega}^2)}{(m_B + m_{f_1})} \epsilon_{\mu\nu\alpha\beta} \epsilon_{\omega}^{\mu} \epsilon_{f_1}^{\nu} p_B^{\alpha} p_{f_1}^{\beta} - i(m_B + m_{f_1}) V_1^{B\to f_1}(m_{\omega}^2)(\epsilon_{f_1} \cdot \epsilon_{\omega}) + \frac{iV_2^{B\to f_1}(m_{\omega}^2)}{(m_B + m_{f_1})} (\epsilon_{f_1} \cdot p_B)(\epsilon_{\omega} \cdot p_B) \right) + m_{f_1} f_{f_1} \left\{ V_{ub} V_{ud}^* a_2 - V_{tb} V_{td}^* \left[2a_3 + a_4 - 2a_5 - \frac{1}{2}(a_7 - a_9 + a_{10}) \right] \right\} \left(\frac{2V^{B\to\omega}(m_{f_1}^2)}{(m_B + m_{\omega})} \epsilon_{\mu\nu\alpha\beta} \epsilon_{f_1}^{\mu} \epsilon_{\omega}^{\nu} p_B^{\alpha} p_{\omega}^{\beta} - i(m_B + m_{\omega}) A_1^{B\to\omega}(m_{f_1}^2)(\epsilon_{\omega} \cdot \epsilon_{f_1}) + \frac{iA_2^{B\to\omega}(m_{f_1}^2)}{(m_B + m_{\omega})} (\epsilon_{\omega} \cdot p_B)(\epsilon_{f_1} \cdot p_B) \right) \tag{B18}$$

$$\mathcal{M}(\bar{B}^0 \to f_1 \phi) = m_{\phi} f_{\phi} \left\{ V_{tb} V_{td}^* \left[a_3 + a_5 - \frac{1}{2}(a_7 + a_9) \right] \right\} \left(\frac{2A^{B\to f_1}(m_{\phi}^2)}{(m_B + m_{f_1})} \epsilon_{\mu\nu\alpha\beta} \epsilon_{\phi}^{\mu} \epsilon_{f_1}^{\nu} p_B^{\alpha} p_{f_1}^{\beta} - i(m_B + m_{f_1}) V_1^{B\to f_1}(m_{\phi}^2)(\epsilon_{f_1} \cdot \epsilon_{\phi}) + \frac{iV_2^{B\to f_1}(m_{\phi}^2)}{(m_B + m_{f_1})} (\epsilon_{f_1} \cdot p_B)(\epsilon_{\phi} \cdot p_B) \right) \tag{B19}$$

$$\mathcal{M}(\bar{B}^0 \to K_1 \rho^+) = m_{K_1} f_{K_1} \{ V_{ub} V_{us}^* a_1 - V_{tb} V_{ts}^* (a_4 + a_{10}) \} \left(\frac{2V^{B\to\rho}(m_{K_1}^2)}{(m_B + m_{\rho})} \epsilon_{\mu\nu\alpha\beta} \epsilon_{K_1}^{\mu} \epsilon_{\rho}^{\nu} p_B^{\alpha} p_{\rho}^{\beta} - i(m_B + m_{\rho}) A_1^{B\to\rho}(m_{K_1}^2)(\epsilon_{\rho} \cdot \epsilon_{K_1}) + \frac{iA_2^{B\to\rho}(m_{K_1}^2)}{(m_B + m_{\rho})} (\epsilon_{\rho} \cdot p_B)(\epsilon_{K_1} \cdot p_B) \right) \tag{B20}$$

$$\mathcal{M}(\bar{B}^0 \to \bar{K}_1^0 \rho^0) = -m_{\rho} f_{\rho} \left\{ V_{ub} V_{us}^* a_2 - V_{tb} V_{ts}^* \frac{3}{2}(a_7 + a_9) \right\} \left(\frac{2A^{B\to K_1}(m_{\rho}^2)}{(m_B + m_{K_1})} \epsilon_{\mu\nu\alpha\beta} \epsilon_{\rho}^{\mu} \epsilon_{K_1}^{\nu} p_B^{\alpha} p_{K_1}^{\beta} - i(m_B + m_{K_1}) V_1^{B\to K_1}(m_{\rho}^2)(\epsilon_{K_1} \cdot \epsilon_{\rho}) + \frac{iV_2^{B\to K_1}(m_{\rho}^2)}{(m_B + m_{K_1})} (\epsilon_{K_1} \cdot p_B)(\epsilon_{\rho} \cdot p_B) \right) - m_{K_1} f_{K_1} \left\{ V_{tb} V_{ts}^* \left(a_4 - \frac{1}{2} a_{10} \right) \right\} \times \left(\frac{2V^{B\to\rho}(m_{K_1}^2)}{(m_B + m_{\rho})} \epsilon_{\mu\nu\alpha\beta} \epsilon_{K_1}^{\mu} \epsilon_{\rho}^{\nu} p_B^{\alpha} p_{\rho}^{\beta} - i(m_B + m_{\rho}) A_1^{B\to\rho}(m_{K_1}^2)(\epsilon_{\rho}^0 \cdot \epsilon_{K_1}) + \frac{iA_2^{B\to\rho}(m_{K_1}^2)}{(m_B + m_{\rho})} (\epsilon_{\rho} \cdot p_B)(\epsilon_{K_1} \cdot p_B) \right) \tag{B21}$$

$$\mathcal{M}(B^- \to K_1^- \rho^0) = -m_\rho f_\rho \bigg\{ V_{ub} V_{us}^* a_2 - V_{tb} V_{ts}^* \frac{3}{2}(a_7 + a_9) \bigg\} \bigg(\frac{2A^{B\to K_1}(m_\rho^2)}{(m_B + m_{K_1})} \epsilon_{\mu\nu\alpha\beta} \epsilon_\rho^\mu \epsilon_{K_1}^\nu p_B^\alpha p_{K_1}^\beta - i(m_B + m_{K_1}) V_1^{B\to K_1}(m_\rho^2)(\epsilon_{K_1} \cdot \epsilon_\rho) + \frac{iV_2^{B\to K_1}(m_\rho^2)}{(m_B + m_{K_1})}(\epsilon_{K_1} \cdot p_B)(\epsilon_\rho \cdot p_B) \bigg) + m_{K_1} f_{K_1} \{ V_{ub} V_{us}^* a_1 - V_{tb} V_{ts}^* (a_4 + a_{10}) \} \bigg(\frac{2V^{B\to\rho}(m_{K_1}^2)}{(m_B + m_\rho)} \epsilon_{\mu\nu\alpha\beta} \epsilon_{K_1}^\mu \epsilon_\rho^\nu p_B^\alpha p_\rho^\beta - i(m_B + m_\rho) A_1^{B\to\rho}(m_{K_1})(\epsilon_\rho \cdot \epsilon_{K_1}) + \frac{iA_2^{B\to\rho}(m_{K_1}^2)}{(m_B + m_\rho)}(\epsilon_\rho \cdot p_B)(\epsilon_{K_1} \cdot p_B) \bigg) \tag{B22}$$

$$\mathcal{M}(B^- \to \bar{K}_1^0 \rho^-) = -m_{K_1} f_{K_1} \bigg\{ V_{tb} V_{ts}^* \bigg(a_4 - \frac{1}{2} a_{10} \bigg) \bigg\} \bigg(\frac{2V^{B\to\rho}(m_{K_1^0}^2)}{(m_B + m_\rho)} \epsilon_{\mu\nu\alpha\beta} \epsilon_{K_1}^\mu \epsilon_\rho^\nu p_B^\alpha p_\rho^\beta - i(m_B + m_\rho) A_1^{B\to\rho}(m_{K_1}^2)(\epsilon_\rho \cdot \epsilon_{K_1}) + \frac{iA_2^{B\to\rho}(m_{K_1}^2)}{(m_B + m_\rho)}(\epsilon_\rho \cdot p_B)(\epsilon_{K_1} \cdot p_B) \bigg) \tag{B23}$$

$$\mathcal{M}(\bar{B}^0 \to \bar{K}_1^0 \omega) = -m_\omega f_\omega \bigg\{ V_{ub} V_{us}^* a_2 - V_{tb} V_{ts}^* \bigg[2(a_3 + a_5) + \frac{1}{2}(a_7 + a_9) \bigg] \bigg\} \bigg(\frac{2A^{B\to K_1}(m_\omega^2)}{(m_B + m_{K_1})} \epsilon_{\mu\nu\alpha\beta} \epsilon_\omega^\mu \epsilon_{K_1}^\nu p_B^\alpha p_{K_1}^\beta - i(m_B + m_{K_1}) V_1^{B\to K_1}(m_\omega^2)(\epsilon_{K_1} \cdot \epsilon_\omega) + \frac{iV_2^{B\to K_1}(m_\omega^2)}{(m_B + m_{K_1})}(\epsilon_{K_1} \cdot p_B)(\epsilon_\omega \cdot p_B) \bigg) - m_{K_1} f_{K_1} V_{tb} V_{ts}^* \bigg\{ \bigg(a_4 - \frac{1}{2} a_{10} \bigg) \bigg\} \bigg(\frac{2V^{B\to\omega}(m_{K_1}^2)}{(m_B + m_\omega)} \epsilon_{\mu\nu\alpha\beta} \epsilon_{K_1}^\mu \epsilon_\omega^\nu p_B^\alpha p_\omega^\beta - i(m_B + m_\omega) A_1^{B\to\omega}(m_{K_1}^2)(\epsilon_\omega \cdot \epsilon_{K_1}) + \frac{iA_2^{B\to\omega}(m_{K_1}^2)}{(m_B + m_\omega)}(\epsilon_\omega \cdot p_B)(\epsilon_{K_1} \cdot p_B) \bigg) \tag{B24}$$

$$\mathcal{M}(B^- \to K_1^- \omega) = -m_\omega f_\omega \bigg\{ V_{ub} V_{us}^* a_2 - V_{tb} V_{ts}^* \bigg[2(a_3 + a_5) + \frac{1}{2}(a_7 + a_9) \bigg] \bigg\} \bigg(\frac{2A^{B\to K_1}(m_\omega^2)}{(m_B + m_{K_1})} \epsilon_{\mu\nu\alpha\beta} \epsilon_\omega^\mu \epsilon_{K_1}^\nu p_B^\alpha p_{K_1}^\beta - i(m_B + m_{K_1}) V_1^{B\to K_1}(m_\omega^2)(\epsilon_{K_1} \cdot \epsilon_\omega) + \frac{iV_2^{B\to K_1}(m_\omega^2)}{(m_B + m_{K_1})}(\epsilon_{K_1} \cdot p_B)(\epsilon_\omega \cdot p_B) \bigg) + m_{K_1} f_{K_1} \{ V_{ub} V_{us}^* a_1 - V_{tb} V_{ts}^* (a_4 + a_{10}) \} \bigg(\frac{2V^{B\to\omega}(m_{K_1}^2)}{(m_B + m_\omega)} \epsilon_{\mu\nu\alpha\beta} \epsilon_{K_1}^\mu \epsilon_\omega^\nu p_B^\alpha p_\omega^\beta - i(m_B + m_\omega) A_1^{B\to\omega}(m_{K_1}^2)(\epsilon_\omega \cdot \epsilon_{K_1}) + \frac{iA_2^{B\to\omega}(m_{K_1}^2)}{(m_B + m_\omega)}(\epsilon_\omega \cdot p_B)(\epsilon_{K_1} \cdot p_B) \bigg) \tag{B25}$$

$$\mathcal{M}(\bar{B}^0 \to \bar{K}_1^0 \phi) = m_\phi f_\phi \bigg\{ V_{tb} V_{ts}^* \bigg[a_3 + a_4 + a_5 - \frac{1}{2}(a_7 + a_9 + a_{10}) \bigg] \bigg\} \bigg(\frac{2A^{B\to K_1}(m_\phi^2)}{(m_B + m_{K_1})} \epsilon_{\mu\nu\alpha\beta} \epsilon_\phi^\mu \epsilon_{K_1}^\nu p_B^\alpha p_{K_1}^\beta - i(m_B + m_{K_1}) V_1^{B\to K_1}(m_\phi^2)(\epsilon_{K_1} \cdot \epsilon_\phi) + \frac{iV_2^{B\to K_1}(m_\phi^2)}{(m_B + m_{K_1})}(\epsilon_{K_1} \cdot p_B)(\epsilon_\phi \cdot p_B) \bigg) \tag{B26}$$

$$\mathcal{M}(B^0 \to K_1^0 \bar{K}^{*0}) = m_{K^*} f_{K^*} \bigg\{ V_{tb} V_{td}^* \bigg(a_4 - \frac{1}{2} a_{10} \bigg) \bigg\} \bigg(\frac{2A^{B\to K_1}(m_{K^*}^2)}{(m_B + m_{K_1})} \epsilon_{\mu\nu\alpha\beta} \epsilon_{K^*}^\mu \epsilon_{K_1}^\nu p_B^\alpha p_{K_1}^\beta - i(m_B + m_{K_1}) V_1^{B\to K_1}(m_{K^*}^2)(\epsilon_{K_1} \cdot \epsilon_{K*}) + \frac{iV_2^{B\to K_1}(m_{K^*}^2)}{(m_B + m_{K_1})}(\epsilon_{K_1} \cdot p_B)(\epsilon_{K^*} \cdot p_B) \bigg). \tag{B27}$$

APPENDIX C: MATRIX ELEMENTS FOR B DECAYS TO TWO AXIAL MESONS

$$\mathcal{M}(\bar{B}^0 \to a_1^- a_1^+) = m_{a_1} f_{a_1} \{V_{ub} V_{ud}^* a_1 - V_{tb} V_{td}^* (a_4 + a_{10})\} \bigg(\frac{2A^{B\to a_1}(m_{a_1}^2)}{(m_B + m_{a_1})} \epsilon_{\mu\nu\alpha\beta} \epsilon_{a_1^+}^{\mu} \epsilon_{a_1^-}^{\nu} p_B^{\alpha} p_{a_1}^{\beta} - i(m_B + m_{a_1}) V_1^{B\to a_1}(m_{a_1}^2)(\epsilon_{a_1} \cdot \epsilon_{a_1}) + \frac{iV_2^{B\to a_1}(m_{a_1}^2)}{(m_B + m_{a_1})} (\epsilon_{a_1} \cdot p_B)(\epsilon_{a_1} \cdot p_B) \bigg) \tag{C1}$$

$$\mathcal{M}(\bar{B}^0 \to a_1^0 a_1^0) = m_{a_1} f_{a_1} \bigg\{ V_{ub} V_{ud}^* 2a_2 - 2V_{tb} V_{td}^* \bigg[-a_4 - \frac{1}{2}(3a_7 - 3a_9 - a_{10}) \bigg] \bigg\} \bigg(\frac{2A^{B\to a_1}(m_{a_1}^2)}{(m_B + m_{a_1})} \epsilon_{\mu\nu\alpha\beta} \epsilon_{a_1}^{\mu} \epsilon_{a_1^0}^{\nu} p_B^{\alpha} p_{a_1}^{\beta} - i(m_B + m_{a_1}) V_1^{B\to a_1}(m_{a_1}^2)(\epsilon_{a_1} \cdot \epsilon_{a_1}) + \frac{iV_2^{B\to a_1}(m_{a_1}^2)}{(m_B + m_{a_1})} (\epsilon_{a_1} \cdot p_B)(\epsilon_{a_1} \cdot p_B) \bigg) \tag{C2}$$

$$\mathcal{M}(B^- \to a_1^- a_1^0) = m_{a_1} f_{a_1} \bigg\{ V_{ub} V_{ud}^* a_1 - V_{tb} V_{td}^* \frac{3}{2}(-a_7 + a_9 + a_{10}) \bigg\} \bigg(\frac{2A^{B\to a_1}(m_{a_1}^2)}{(m_B + m_{a_1})} \epsilon_{\mu\nu\alpha\beta} \epsilon_{a_1^-}^{\mu} \epsilon_{a_1^+}^{\nu} p_B^{\alpha} p_{a_1}^{\beta} - i(m_B + m_{a_1}) V_1^{B\to a_1}(m_{a_1}^2)(\epsilon_{a_1} \cdot \epsilon_{a_1}) + \frac{iV_2^{B\to a_1}(m_{a_1}^2)}{(m_B + m_{a_1})} (\epsilon_{a_1} \cdot p_B)(\epsilon_{a_1} \cdot p_B) \bigg) + m_{a_1} f_{a_1} \{V_{ub} V_{ud}^* a_2\} \bigg(\frac{2A^{B\to a_1}(m_{a_1}^2)}{(m_B + m_{a_1})} \epsilon_{\mu\nu\alpha\beta} \epsilon_{a_1^+}^{\mu} \epsilon_{a_1^-}^{\nu} p_B^{\alpha} p_{a_1}^{\beta} - i(m_B + m_{a_1}) V_1^{B\to a_1}(m_{a_1}^2)(\epsilon_{a_1} \cdot \epsilon_{a_1}) + \frac{iV_2^{B\to a_1}(m_{a_1}^2)}{(m_B + m_{a_1})} (\epsilon_{a_1} \cdot p_B)(\epsilon_{a_1} \cdot p_B) \bigg) \tag{C3}$$

$$\mathcal{M}(\bar{B}^0 \to a_1^0 f_1) = m_{f_1} f_{f_1} \bigg\{ V_{ub} V_{ud}^* a_2 - V_{tb} V_{td}^* \bigg[2a_3 + a_4 - 2a_5 - \frac{1}{2}(a_7 - a_9 + a_{10}) \bigg] \bigg\} \bigg(\frac{2A^{B\to a_1}(m_{f_1}^2)}{(m_B + m_{a_1})} \epsilon_{\mu\nu\alpha\beta} \epsilon_{f_1}^{\mu} \epsilon_{a_1}^{\nu} p_B^{\alpha} p_{a_1}^{\beta} - i(m_B + m_{a_1}) V_1^{B\to a_1}(m_{f_1}^2)(\epsilon_{a_1} \cdot \epsilon_{f_1}) + \frac{iV_2^{B\to a_1}(m_{f_1}^2)}{(m_B + m_{a_1})} (\epsilon_{a_1} \cdot p_B)(\epsilon_{f_1} \cdot p_B) \bigg) + m_{a_1} f_{a_1} \bigg\{ V_{ub} V_{ud}^* a_2 - V_{tb} V_{td}^* \times \bigg[-a_4 - \frac{3}{2}(a_7 - a_9) + \frac{1}{2} a_{10} \bigg] \bigg\} \bigg(\frac{2A^{B\to f_1}(m_{a_1}^2)}{(m_B + m_{f_1})} \epsilon_{\mu\nu\alpha\beta} \epsilon_{a_1}^{\mu} \epsilon_{f_1}^{\nu} p_B^{\alpha} p_{f_1}^{\beta} - i(m_B + m_{f_1}) V_1^{B\to f_1}(m_{a_1}^2)(\epsilon_{f_1} \cdot \epsilon_{a_1}) + \frac{iV_2^{B\to f_1}(m_{a_1}^2)}{(m_B + m_{f_1})} (\epsilon_{f_1} \cdot p_B)(\epsilon_{a_1} \cdot p_B) \bigg) \tag{C4}$$

$$\mathcal{M}(B^- \to a_1^- f_1) = m_{f_1} f_{f_1} \bigg\{ V_{ub} V_{ud}^* a_2 - V_{tb} V_{td}^* \bigg[2a_3 + a_4 - 2a_5 - \frac{1}{2}(a_7 - a_9 + a_{10}) \bigg] \bigg\} \bigg(\frac{2A^{B\to a_1}(m_{f_1}^2)}{(m_B + m_{a_1})} \epsilon_{\mu\nu\alpha\beta} \epsilon_{f_1}^{\mu} \epsilon_{a_1}^{\nu} p_B^{\alpha} p_{a_1}^{\beta} - i(m_B + m_{a_1}) V_1^{B\to a_1}(m_{f_1}^2)(\epsilon_{a_1} \cdot \epsilon_{f_1}) + \frac{iV_2^{B\to a_1}(m_{f_1}^2)}{(m_B + m_{a_1})} (\epsilon_{a_1} \cdot p_B)(\epsilon_{f_1} \cdot p_B) \bigg) + m_{a_1} f_{a_1} \{V_{ub} V_{ud}^* a_1 - V_{tb} V_{td}^* (a_4 + a_{10})\} \bigg(\frac{2A^{B\to f_1}(m_{a_1}^2)}{(m_B + m_{f_1})} \epsilon_{\mu\nu\alpha\beta} \epsilon_{a_1}^{\mu} \epsilon_{f_1}^{\nu} p_B^{\alpha} p_{f_1}^{\beta} - i(m_B + m_{f_1}) V_1^{B\to f_1}(m_{a_1}^2)(\epsilon_{f_1} \cdot \epsilon_{a_1}) + \frac{iV_2^{B\to f_1}(m_{a_1}^2)}{(m_B + m_{f_1})} (\epsilon_{f_1} \cdot p_B)(\epsilon_{a_1} \cdot p_B) \bigg) \tag{C5}$$

$$\mathcal{M}(\bar{B}^0 \to a_1^+ K_1^-) = m_{K_1} f_{K_1} \{V_{ub} V_{us}^* a_1 - V_{tb} V_{ts}^* (a_4 + a_{10})\} \bigg(\frac{2A^{B\to a_1}(m_{K_1}^2)}{(m_B + m_{a_1})} \epsilon_{\mu\nu\alpha\beta} \epsilon_{K_1}^{\mu} \epsilon_{a_1}^{\nu} p_B^{\alpha} p_{a_1}^{\beta} - i(m_B + m_{a_1}) V_1^{B\to a_1}(m_{K_1}^2)(\epsilon_{a_1} \cdot \epsilon_{K_1}) + \frac{iV_2^{B\to a_1}(m_{K_1}^2)}{(m_B + m_{a_1})} (\epsilon_{a_1} \cdot p_B)(\epsilon_{K_1} \cdot p_B) \bigg) \tag{C6}$$

$$\mathcal{M}(\bar{B}^0 \to a_1^0 \bar{K}_1^0) = m_{a_1} f_{a_1} \left\{ V_{ub} V_{us}^* a_2 - V_{tb} V_{ts}^* \frac{3}{2}(-a_7 + a_9) \right\} \left(\frac{2A^{B\to K_1}(m_{a_1}^2)}{(m_B + m_{K_1})} \epsilon_{\mu\nu\alpha\beta} \epsilon_{a_1}^{\mu} \epsilon_{K_1}^{\nu} p_B^{\alpha} p_{K_1}^{\beta} - i(m_B + m_{K_1}) V_1^{B\to K_1}(m_{a_1}^2)(\epsilon_{K_1} \cdot \epsilon_{a_1}) + \frac{iV_2^{B\to K_1}(m_{a_1}^2)}{(m_B + m_{K_1})}(\epsilon_{K_1 B})(\epsilon_{a_1} \cdot p_B) \right) - m_{K_1} f_{K_1} \left\{ V_{tb} V_{ts}^* \left(a_4 - \frac{1}{2} a_{10} \right) \right\} \left(\frac{2A^{B\to a_1}(m_{K_1}^2)}{(m_B + m_{a_1})} \epsilon_{\mu\nu\alpha\beta} \epsilon_{K_1}^{\mu} \epsilon_{a_1}^{\nu} p_B^{\alpha} p_{a_1}^{\beta} - i(m_B + m_{a_1}) V_1^{B\to a_1}(m_{K_1}^2)(\epsilon_{a_1} \cdot \epsilon_{K_1}) + \frac{iV_2^{B\to a_1}(m_{K_1}^2)}{(m_B + m_{a_1})}(\epsilon_{a_1} \cdot p_B)(\epsilon_{K_1} \cdot p_B) \right) \quad \text{(C7)}$$

$$\mathcal{M}(B^- \to a_1^0 K_1^-) = m_{K_1} f_{K_1} \{ V_{ub} V_{us}^* - V_{tb} V_{ts}^* (a_4 + a_{10}) \} \left(\frac{2A^{B\to a_1}(m_{K_1}^2)}{(m_B + m_{a_1})} \epsilon_{\mu\nu\alpha\beta} \epsilon_{K_1}^{\mu} \epsilon_{a_1}^{\nu} p_B^{\alpha} p_{a_1}^{\beta} - i(m_B + m_{a_1}) V_1^{B\to a_1}(m_{K_1}^2)(\epsilon_{a_1} \cdot \epsilon_{K_1}) + \frac{iV_2^{B\to a_1}(m_{K_1}^2)}{(m_B + m_{a_1})}(\epsilon_{a_1} \cdot p_B)(\epsilon_{K_1} \cdot p_B) \right) + m_{a_1} f_{a_1} \left\{ V_{ub} V_{us}^* + V_{tb} V_{ts}^* \frac{3}{2}(a_7 - a_9) \right\} \left(\frac{2A^{B\to K_1}(m_{a_1}^2)}{(m_B + m_{K_1})} \epsilon_{\mu\nu\alpha\beta} \epsilon_{a_1}^{\mu} \epsilon_{K_1}^{\nu} p_B^{\alpha} p_{K_1}^{\beta} - i(m_B + m_{K_1}) V_1^{B\to K_1}(m_{a_1}^2)(\epsilon_{K_1} \cdot \epsilon_{a_1}) + \frac{iV_2^{B\to K_1}(m_{a_1}^2)}{(m_B + m_{K_1})}(\epsilon_{K_1} \cdot p_B)(\epsilon_{a_1} \cdot p_B) \right) \quad \text{(C8)}$$

$$\mathcal{M}(B^- \to a_0^- \bar{K}_1^0) = -m_{K_1} f_{K_1} \left\{ V_{tb} V_{ts}^* \left(a_4 - \frac{1}{2} a_{10} \right) \right\} \left(\frac{2A^{B\to a_1}(m_{K_1}^2)}{(m_B + m_{a_1})} \epsilon_{\mu\nu\alpha\beta} \epsilon_{K_1}^{\mu} \epsilon_{a_1}^{\nu} p_B^{\alpha} p_{a_1}^{\beta} - i(m_B + m_{a_1}) V_1^{B\to a_1}(m_{K_1}^2)(\epsilon_{a_1} \cdot \epsilon_{K_1}) + \frac{iV_2^{B\to a_1}(m_{K_1}^2)}{(m_B + m_{a_1})}(\epsilon_{a_1} \cdot p_B)(\epsilon_{K_1} \cdot p_B) \right) \quad \text{(C9)}$$

$$\mathcal{M}(\bar{B}^0 \to \bar{K}_1^0 f_1) = m_{f_1} f_{f_1} \left\{ V_{ub} V_{us}^* a_2 - V_{tb} V_{ts}^* \left[2a_3 - 2a_5 - \frac{1}{2}(a_7 - a_9) \right] \right\} \left(\frac{2A^{B\to K_1}(m_{f_1}^2)}{(m_B + m_{K_1})} \epsilon_{\mu\nu\alpha\beta} \epsilon_{f_1}^{\mu} \epsilon_{K_1}^{\nu} p_B^{\alpha} p_{K_1}^{\beta} - i(m_B + m_{K_1}) V_1^{B\to K_1}(m_{f_1}^2)(\epsilon_{K_1} \cdot \epsilon_{f_1}) + \frac{iV_2^{B\to K_1}(m_{f_1}^2)}{(m_B + m_{K_1})}(\epsilon_{K_1} \cdot p_B)(\epsilon_{f_1} \cdot p_B) \right) - m_{K_1} f_{K_1} \left\{ V_{tb} V_{ts}^* \left(a_4 - \frac{1}{2} a_{10} \right) \right\} \left(\frac{2A^{B\to f_1}(m_{K_1}^2)}{(m_B + m_{f_1})} \epsilon_{\mu\nu\alpha\beta} \epsilon_{K_1}^{\mu} \epsilon_{f_1}^{\nu} p_B^{\alpha} p_{f_1}^{\beta} - i(m_B + m_{f_1}) V_1^{B\to f_1}(m_{K_1}^2)(\epsilon_{f_1} \cdot \epsilon_{K_1}) + \frac{iV_2^{B\to f_1}(m_{K_1^0}^2)}{(m_B + m_{f_1})}(\epsilon_{f_1} \cdot p_B)(\epsilon_{K_1} \cdot p_B) \right) \quad \text{(C10)}$$

$$\mathcal{M}(B^- \to K_1^- f_1) = m_{K_1} f_{K_1} \{ V_{ub} V_{us}^* a_1 - V_{tb} V_{ts}^* (a_4 + a_{10}) \} \left(\frac{2A^{B\to f_1}(m_{K_1}^2)}{(m_B + m_{f_1})} \epsilon_{\mu\nu\alpha\beta} \epsilon_{K_1}^{\mu} \epsilon_{f_1}^{\nu} p_B^{\alpha} p_{f_1}^{\beta} - i(m_B + m_{f_1}) V_1^{B\to f_1}(m_{K_1}^2)(\epsilon_{f_1} \cdot \epsilon_{K_1}) + \frac{iV_2^{B\to f_1}(m_{K_1}^2)}{(m_B + m_{f_1})}(\epsilon_{f_1} \cdot p_B)(\epsilon_{K_1} \cdot p_B) \right) + m_{f_1} f_{f_1} \left\{ V_{ub} V_{us}^* a_2 - V_{tb} V_{ts}^* \left[2a_3 - 2a_5 - \frac{1}{2}(a_7 - a_9) \right] \right\} \left(\frac{2A^{B\to K_1}(m_{f_1}^2)}{(m_B + m_{K_1})} \epsilon_{\mu\nu\alpha\beta} \epsilon_{f_1}^{\mu} \epsilon_{K_1}^{\nu} p_B^{\alpha} p_{K_1}^{\beta} - i(m_B + m_{K_1}) V_1^{B\to K_1}(m_{f_1}^2)(\epsilon_{K_1} \cdot \epsilon_{f_1}) + \frac{iV_2^{B\to K_1}(m_{f_1}^2)}{(m_B + m_{K_1})}(\epsilon_{K_1} \cdot p_B)(\epsilon_{f_1} \cdot p_B) \right). \quad \text{(C11)}$$

[1] B. Aubert *et al.* (*BABAR* Collaboration), Phys. Rev. Lett. **97**, 051802 (2006); K. Abe *et al.* (Belle Collaboration), arXiv:hep-ex/0507096; H. Yang *et al.* (Belle Collaboration), Phys. Rev. Lett. **94**, 111802 (2005); K. Abe *et al.* (Belle Collaboration), arXiv:hep-ex/0408138; B. Aubert *et al.* (*BABAR* Collaboration), Phys. Rev. Lett. **90**, 242001 (2003); D. Besson *et al.* (CLEO Collaboration), Phys. Rev. D **68**, 032002 (2003).
[2] G. Nardulli and T. N. Pham, Phys. Lett. B **623**, 65 (2005).
[3] A. Ali, G. Kramer, and C.-D. Lu, Phys. Rev. D **58**, 094009 (1998).
[4] Y.-H. Chen, H.-Y. Cheng, B. Tseng, and K.-C. Yang, Phys. Rev. D **60**, 094014 (1999).
[5] W. M. Yao *et al.* (Particle Data Group), J. Phys. G **33**, 1 (2006).
[6] A. C. Katoch and R. C. Verma, J. Phys. G **22**, 1765 (1996).
[7] N. Isgur, D. Scora, B. Grinstein, and M. B. Wise, Phys. Rev. D **39**, 799 (1989).
[8] V. Laporta, G. Nardulli, and T. N. Pham, Phys. Rev. D **74**, 054035 (2006).
[9] C. H. Chen, C. Q. Geng, Y. K. Hsiao, and Z.-T. Wei, Phys. Rev. D **72**, 054011 (2005).
[10] H. Y. Cheng, Phys. Rev. D **68**, 094005 (2003).
[11] D. Scora and N. Isgur, Phys. Rev. D **52**, 2783 (1995).
[12] D. Melikhov and B. Stech, Phys. Rev. D **62**, 014006 (2000).
[13] H.-Y. Cheng and C.-K. Chua, Phys. Rev. D **74**, 034020 (2006).
[14] J. P. Lee, Phys. Rev. D **74**, 074001 (2006).
[15] See, for example, H.-Y. Cheng, Phys. Rev. D **67**, 094007 (2003); M. Suzuki, Phys. Rev. D **47**, 1252 (1993); N. Isgur and M. B. Wise, Phys. Lett. B **232**, 113 (1989).
[16] D.-M. Li and Z. Li, Eur. Phys. J. A **28**, 369 (2006).
[17] M. Gronau, D. Pirjol, and D. Wyler, Phys. Rev. Lett. **90**, 051801 (2003).
[18] M. Gronau and J. Zupan, Phys. Rev. D **73**, 057502 (2006).
[19] M. Wirbel, B. Stech, and M. Bauer, Z. Phys. C **29**, 637 (1985); M. Bauer and M. Wirbel, Z. Phys. C **42**, 671 (1989).
[20] P. Ball and R. Zwicky, Phys. Rev. D **71**, 014015 (2005); **71**, 014029 (2005).
[21] G. Buchalla, A. J. Buras, and M. E. Lautenbacher, Rev. Mod. Phys. **68**, 1125 (1996).
[22] A. J. Buras, Nucl. Phys. **B434**, 606 (1995).
[23] A. J. Buras, arXiv:hep-ph/9806471.
[24] A. J. Buras and L. Silvestrini, Nucl. Phys. **B548**, 293 (1999).
[25] L. Wolfenstein, Phys. Rev. Lett. **51**, 1945 (1983).
[26] A. J. Buras, M. E. Lautenbacher, and G. Ostermaier, Phys. Rev. D **50**, 3433 (1994).
[27] A. Hocker *et al.*, Eur. Phys. J. C **21**, 225 (2001); J. Charles *et al.*, Eur. Phys. J. C **41**, 1 (2005).
[28] M. Bona *et al.*, J. High Energy Phys. 07 (2005) 028; 03 (2006) 080.
[29] H. Fusaoka and Y. Koide, Phys. Rev. D **57**, 3986 (1998).
[30] H. Leutwyler, Nucl. Phys. B, Proc. Suppl. **64**, 223 (1998).
[31] T. Feldmann, P. Kroll, and B. Stech, Phys. Rev. D **58**, 114006 (1998); Phys. Lett. B **449**, 339 (1999).
[32] H. Y. Cheng and B. Tseng, Phys. Rev. D **58**, 094005 (1998).
[33] J. L. Diaz-Cruz, G. Lopez Castro, and J. H. Munoz, Phys. Rev. D **54**, 2388 (1996).