

Chromohydrodynamical instabilities induced by relativistic jets

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We study the properties of the chromohydrodynamical instabilities induced by a relativistic jet that crosses the quark-gluon plasma. Assuming that the jet of particles and the plasma can be described using a hydrodynamical approach, we derive and discuss the dispersion laws for the unstable collective modes. In our analysis, the chromohydrodynamical equations for the collective modes are tackled in the linear response approximation. Such an approximation, valid for short time scales, allows one to study in a straightforward way the dependence of the dispersion laws of the collective modes on the velocity of the jet, on the magnitude of the momentum of the collective mode and on the angle between these two quantities. In the conformal limit, we find that unstable modes arise for velocity of the jet larger than the speed of the sound of the plasma and only modes with momenta smaller than a certain value are unstable. Moreover, for ultrarelativistic velocities of the jet, the longitudinal mode becomes stable and the most unstable modes correspond to relative angles between the velocity of the jet and momentum of the collective mode larger than $\sim\pi/8$. Our results suggest an alternative mechanism for the description of the jet quenching phenomenon, where the jet crossing the plasma loses energy exciting colored unstable modes.

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I. INTRODUCTION

One of the remarkable findings at the Relativistic Heavy Ion Collider, RHIC, is that hydrodynamical models are able to accurately describe the spectra of soft hadrons produced in heavy-ion collisions [1,2]. Hydrodynamics, being an effective macroscopic approach valid at large distances and long time scales, cannot give account of the preequilibrium stage of the reaction. However, hydrodynamics describes correctly elliptic flow at RHIC, and allows to put some bounds on the thermalization time $t < 1$ fm/ c .

An experimental evidence of the production of a thermalized quark-gluon plasma in ultrarelativistic heavy-ion collisions at RHIC [1] is the so-called jet quenching. In this phenomenon, high p_T partons produced in the initial stage of the collision by hard scatterings lose energy and degrade, mainly by radiative processes, while traveling through the hot and dense medium transferring energy and momentum to the plasma (see [3] for reviews). In order to describe these processes and the subsequent modification of the hadronic spectra due to the interaction of the high p_T partons with the medium, various models have been proposed where perturbative QCD is supplemented with medium-induced parton energy loss [4] or where the AdS/CFT correspondence [5] is employed.

It is interesting to analyze whether some aspects of the process of jet quenching can be described employing a hydrodynamical picture [6]. Hydrodynamics describes the behavior of a system in local equilibrium in terms of conservation laws of macroscopic quantities. On the other hand, the jet quenching is related to the modification of high- p_T parton fragmentation processes in the medium. It is then clear that hydrodynamics cannot describe the mi-

croscopic processes related to jet quenching, but it can give information on the macroscopic and collective behavior of the system composed by the plasma and the hard jets.

In Refs. [7,8] the effect of a jet crossing the plasma at high speed has been analyzed. The authors have employed a hydrodynamical approach to describe the process of energy and momentum deposition from the fast jet to the surrounding plasma. In a hydrodynamical picture, a high p_T jet crossing the medium at a velocity higher than the speed of sound forms shock waves with a Mach cone structure. Such shock waves should be detectable in the low p_T parton distributions at angles $\pi \pm 1.2$ with respect to the direction of the trigger particle. A preliminary analysis of the azimuthal dihadron correlation performed by the STAR collaboration [9] and the PHENIX collaboration [10] seems to suggest the formation of such a conical flow. However, it is yet unclear whether those structures are compatible with Mach cone formation and alternative explanations have been proposed [11]. The study performed in Ref. [6] concludes that, for realistic phenomenological values of the hydrodynamical variables, the Mach cone effects are too weak to explain the PHENIX results. It is our aim here to explore whether other hydrodynamical effects, not considered in [6], may enhance the signal.

The study of the interaction of a relativistic stream of particles with an electromagnetic plasma is a topic of interest in different fields of physics, ranging from inertial confinement fusion, astrophysics, and cosmology. When the particles of the stream are charged, plasma instabilities develop, leading to an initial stage of fast growth of the electromagnetic fields. One then talks about filamentation, two-stream or Weibel instabilities, according to which is

the fastest growing collective mode, although the notation is nonuniversal and sometimes confusing. Weibel instabilities usually refer to the case when the plasma is out of equilibrium, and with an anisotropic distribution in the velocities of its constituents, but the name is also used in the context of a jet moving in an equilibrated plasma if the instability appears in the transverse modes. These jet induced instabilities have been studied using a variety of methods, from kinetic theory to hydrodynamics [12,13]. Experimental evidence of the relativistic filamentation instability has also been reported in Ref. [14].

The study of chromo-Weibel instabilities is now a very active field of research [15–30] (see also the recent reviews [31,32] for a more complete list of references). This is so because it was suggested [15,16] that the presence of plasma instabilities could be a natural explanation for the fast equilibration that measurements of the elliptic flow at RHIC seem to imply. In the early stage of a heavy-ion collision, the nonequilibrium anisotropic distributions of the partons should be responsible for the fast growth of the chromomagnetic plasma modes, which in turn would isotropize the system and speed up the thermalization process. Whether the experimental conditions met at RHIC are favoring this thermalization scenario or not is a question that requires hard numerical simulations.

In this paper we study how a relativistic jet crossing an equilibrated quark-gluon plasma induces instabilities using a chromohydrodynamical approach [33]. We will assume that both the plasma and the jet can be described using hydrodynamics. Therefore, regarding the plasma, we consider conditions that can be realized after $t \sim 1$ fm/ c of the moment the heavy-ion collision has taken place and the plasma has thermalized. Regarding the jet, there are some experimental evidences that the particles that constitute the jet that crosses the plasma equilibrate. Indeed, consider a fast parton that traverses the medium in the direction opposite the direction of a high p_T trigger particle. The mean value of the momentum associated with the soft hadrons emitted by the parton that crosses the medium and detected in the hemisphere opposite to the direction of the trigger particle reaches a common value [9] suggesting that the energy lost by the fast parton thermalizes [6].

In studying the evolution of the system composed by the plasma and the jet, we will employ ideal hydrodynamic equations that do not take into account the effect of collisions. Therefore our results will be valid on time scales shorter than the mean-free path time for collisions. We postpone the study of more realistic fluid equations to future work.

The validity of the chromohydrodynamical approach will be our starting assumption, as this allows us to simplify the equations governing the evolution of the system. To the best of our knowledge, only Ref. [34] considers the possibility of the appearance of filamentation instabilities produced by hard jets in heavy-ion collisions. However, the

approach considered there and ours are different. The analysis of Ref. [34] is performed within kinetic theory, and thus relies on the quasiparticle picture and a weak coupling scenario. Instead, we use fluid equations. We note that colored plasma waves are believed to be very quickly damped (and this is the reason to exclude them in the study of Ref. [6]). That conclusion might have to be reviewed, as in the presence of instabilities rather than damping, one finds exponential growth of the fields. Our approach is also different to the color wakefield scenario of Refs. [35,36]. In those references, the authors studied the field response to a single moving colored particle, which produces a color charge density wake. Instead, we consider a color neutral jet, treated hydrodynamically, that moves through the plasma and produces color fluctuations and instabilities.

This paper is structured as follows. In Sec. II we review the chromohydrodynamical equations describing the fluctuations of a plasma around the stationary colorless state in the linear approximation [33]. In Sec. III the equations describing sound colorless fluctuations and plasma colored fluctuations are derived. In Sec. IV we consider the case where a relativistic jet crosses the plasma. Assuming that both the plasma and the jet can be treated using a hydrodynamical approach, we derive the dispersion law for the collective modes. We find that there is one unstable mode if the velocity of the jet is larger than the speed of sound and if the momentum of the collective mode is in modulus smaller than a threshold value. Regarding the orientation of the momentum of the collective mode, we study separately the case where it is collinear with the velocity of the jet, the case where it is orthogonal, and then for a generic angle between the two. Quite interestingly, we find that the unstable mode with momentum parallel to the velocity of the jet is the dominant one for velocity of the jet $v \lesssim 0.8$. For larger values of the jet velocity only the modes with angles larger than $\sim \pi/8$ are significant and the dominant unstable modes correspond to angles $\sim \pi/4$. We draw our conclusions in Sec. V. We work using natural units $\hbar = c = k_B = 1$ and use the metric convention $(1, -1, -1, -1)$.

II. CHROMOHYDRODYNAMIC EQUATIONS FOR THE QUARK-GLUON PLASMA

Hydrodynamical equations are the expressions of the conservation laws of a system when it is in local equilibrium. In the quark-gluon plasma there are conservation laws which concern the baryon current, the color current, and the energy-momentum tensor. In Ref. [37] the local equilibrium state for the quark-gluon plasma has been determined. It is in general described by one singlet four velocity, a baryon density, singlet energy and pressure, and in principle, a nonvanishing color density. However, dynamical processes associated to the existence of Ohmic currents tend to whiten the plasma quickly, on time scales

much shorter than momentum equilibration processes [38]. For this reason, one can expect that only colorless (singlet) fluctuations are relevant at large time and space scales. However, there are situations, as the one considered in the present paper, when color fluctuations grow on short time scales instead of being damped. Therefore in order to describe the short time evolution of the plasma, one needs to include color hydrodynamical fluctuations in the equations. In Ref. [33] such a chromohydrodynamical approach for the short time evolution of the system has been formulated, and here we will review the basic set of equations.

For simplicity, as in Ref. [33], we will only consider the contribution of quarks in the fundamental representation. The inclusion of antiquarks and gluons is straightforward. The fluid approach is based on the covariant continuity equation for the fluid four-flow

$$D_\mu n^\mu = 0, \quad (1)$$

and on the equation that couples the energy-momentum tensor $T^{\mu\nu}$ to the gauge fields

$$D_\mu T^{\mu\nu} - \frac{g}{2} \{F_\mu{}^\nu, n^\mu\} = 0, \quad (2)$$

where the various quantities are 3×3 Hermitian matrices in color space, and we have suppressed color indices. The covariant derivative is defined as

$$D_\mu = \partial_\mu - ig[A_\mu(x), \dots],$$

with $A_\mu = A_\mu^a(x)\tau^a$ for $a = 1, \dots, 8$ and $\tau^a = \lambda^a/2$, where λ^a are the $SU(3)$ Gell-Mann matrices and therefore $\text{Tr}(\tau^a \tau^b) = \frac{1}{2} \delta^{ab}$. The strength tensor appearing in Eq. (2) is given by $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - ig[A_\mu, A_\nu]$.

We further assume that the four-flow and the energy-momentum tensor have the expression valid for an ideal fluid, i.e.

$$n^\mu(x) = n(x)u^\mu(x), \quad (3)$$

and

$$T^{\mu\nu}(x) = \frac{1}{2}(\epsilon(x) + p(x))\{u^\mu(x), u^\nu(x)\} - p(x)g^{\mu\nu}, \quad (4)$$

where the hydrodynamic velocity u^μ , the particle density n , the energy density ϵ , and the pressure p are 3×3 matrices in color space. The quantities defined above have in general both colorless and colored components, as an example the particle density can be written as

$$n_{\alpha\beta} = n_0 I_{\alpha\beta} + \frac{1}{2} n_a \tau_{\alpha\beta}^a, \quad (5)$$

where $\alpha, \beta = 1, 2, 3$ are color indices and I is the identity matrix. In the following equations we will omit the color indices not to overcharge the notation.

The color current due to the flow of the fluid can be expressed in terms of the hydrodynamic velocity and the particle density as

$$j^\mu(x) = -\frac{g}{2} \left(nu^\mu - \frac{1}{3} \text{Tr}[nu^\mu] \right), \quad (6)$$

and it acts as a source term for the gauge fields in the Yang Mills equation

$$D_\mu F^{\mu\nu}(x) = j^\nu(x). \quad (7)$$

Thus, we will assume that all the gauge fields that appear in the fluid equations are only due to the presence of a colored current in the medium.

It is worth remarking that Eqs. (1) and (2) were derived in Ref. [33] from the collisionless transport equation obeyed by the particle distribution function. Thus, the conservation laws expressed by Eqs. (1) and (2) are strictly valid on time scales shorter than the mean-free path time. As an example, for quarks n^0 represents the quark particle density, which fulfills a conservation law that says that the change of the number of particles within a volume element is equal to the flux of particles across the surface of the volume element. On the other hand, for times larger than the mean-free path time, this conservation law is violated by collision processes that change the particle number inside the volume element. Then, only baryon number is conserved, which requires the knowledge of both the quark and antiquark particle densities.

Summarizing, for time scales shorter than the mean-free path time there are more conservation laws than at long time scales, as Eqs. (1) and (2) indicate. Their validity for the short time phenomena that will be studied here is then guaranteed.

Linearization of the chromohydrodynamic equations

We shall consider the fluctuations of density, energy density, pressure, and plasma velocity, around their stationary and colorless state described by \bar{n} , $\bar{\epsilon}$, \bar{p} , and \bar{u}^μ , respectively. In order to study such fluctuations we linearize the chromohydrodynamic equations (1) and (2), assuming the fluctuations to be small. Notice that in the stationary state the color current defined in Eq. (6) vanishes, indeed

$$\bar{j}^\mu = -g(\bar{n}\bar{u}^\mu - \frac{1}{3} \text{Tr}[\bar{n}\bar{u}^\mu]) = 0, \quad (8)$$

and we will assume that in the stationary state no field $F^{\mu\nu}$ is present in the system. We define the space dependent fluctuations of the various quantities around their colorless values as

$$n(x) = \bar{n} + \delta n(x), \quad \epsilon(x) = \bar{\epsilon} + \delta\epsilon(x), \quad (9)$$

$$p(x) = \bar{p} + \delta p(x), \quad u^\mu(x) = \bar{u}^\mu + \delta u^\mu(x). \quad (10)$$

All the fluctuations can contain both colorless and colored components, therefore they can be decomposed as

$$\delta n_{\alpha\beta} = \delta n_0 I_{\alpha\beta} + \frac{1}{2} \delta n_a \tau_{\alpha\beta}^a, \quad (11)$$

where δn_0 is the colorless fluctuation, and δn_a the corresponding colored components.

The state described by \bar{n} , $\bar{\epsilon}$, \bar{p} , and \bar{u}^μ is assumed to be stationary, colorless, and homogeneous on the scale of variation of the fluctuations and therefore we have that

$$D^\mu \bar{n} = 0, \quad D^\mu \bar{\epsilon} = 0, \quad D^\mu \bar{p} = 0, \quad D^\mu \bar{u}^\nu = 0. \quad (12)$$

Moreover, since we will consider only small deviations from the stationary state, we will assume that the following conditions are obeyed:

$$\bar{n} \gg \delta n, \quad \bar{\epsilon} \gg \delta \epsilon, \quad \bar{p} \gg \delta p, \quad \bar{u}^\mu \gg \delta u^\mu. \quad (13)$$

Actually, δn , $\delta \epsilon$, δp , and δu^μ should be diagonalized to be comparable to the \bar{n} , $\bar{\epsilon}$, \bar{p} , and \bar{u}^μ .

We now aim to determine the set of equations that the fluctuations δn , $\delta \epsilon$, δp , and δu^μ obey. Employing the equations (9) and (10), we can derive the expression of the fluctuation of n^μ and of $T^{\mu\nu}$ around their colorless and homogeneous values:

$$n^\mu = \bar{n}\bar{u}^\mu + \bar{n}\delta u^\mu + \delta n\bar{u}^\mu, \quad (14)$$

$$T^{\mu\nu} = (\bar{\epsilon} + \bar{p})\bar{u}^\mu\bar{u}^\nu - \bar{p}g^{\mu\nu} + (\delta\epsilon + \delta p)\bar{u}^\mu\bar{u}^\nu + (\bar{\epsilon} + \bar{p})(\bar{u}^\mu\delta u^\nu + \delta u^\mu\bar{u}^\nu) - \delta p g^{\mu\nu}. \quad (15)$$

Upon using these expressions in Eqs. (1) and (2), we obtain that δu^μ , δp , $\delta \epsilon$ and δn must obey the following equations:

$$\begin{aligned} \bar{n}D_\mu\delta u^\mu + (D_\mu\delta n)\bar{u}^\mu &= 0, \\ \bar{u}^\mu D_\mu\delta\epsilon + (\bar{\epsilon} + \bar{p})D_\mu\delta u^\mu &= 0, \\ (\bar{\epsilon} + \bar{p})\bar{u}_\mu D^\mu\delta u^\nu - (D^\nu - \bar{u}^\nu\bar{u}_\mu D^\mu)\delta p - g\bar{n}\bar{u}_\mu F^{\mu\nu} &= 0. \end{aligned} \quad (16)$$

These equations do not form a closed set and one more relation has to be provided. In hydrodynamical treatments, one usually imposes an equation of state (EoS) of the form

$$p = p(\epsilon, n), \quad (17)$$

and in this way one obtains a closed set of equations. For the applications we have in mind, we will consider that the conformal limit is reached, and further the effects of the particle density can be ignored. Thus, we will use an equation of state given by

$$p(x) = c_s^2 \epsilon(x), \quad (18)$$

where c_s is the speed of sound, and in the conformal limit $c_s = 1/\sqrt{3}$. We will however leave c_s as a parameter, and use its value only in the numerical analysis of the equations.

We will study separately colorless and colored fluctuations. In order to obtain the equation governing the color-

less fluctuation we take the trace of Eq. (16) obtaining

$$\begin{aligned} \bar{n}\partial_\mu\delta u_0^\mu + (\partial_\mu\delta n_0)\bar{u}^\mu &= 0, \\ \bar{u}^\mu\partial_\mu\delta\epsilon_0 + (\bar{\epsilon} + \bar{p})\partial_\mu\delta u_0^\mu &= 0, \\ (\bar{\epsilon} + \bar{p})\bar{u}_\mu\partial^\mu\delta u_0^\nu - (\partial^\nu - \bar{u}^\nu\bar{u}_\mu\partial^\mu)\delta p_0 &= 0. \end{aligned} \quad (19)$$

Multiplying Eq. (16) by τ^a and taking the trace, we gather the equations for colored hydrodynamical fluctuations:

$$\begin{aligned} \bar{n}(D_\mu\delta u^\mu)_a + (D_\mu\delta n)_a\bar{u}^\mu &= 0, \\ \bar{u}^\mu(D_\mu\delta\epsilon)_a + (\bar{\epsilon} + \bar{p})(D_\mu\delta u^\mu)_a &= 0, \\ (\bar{\epsilon} + \bar{p})\bar{u}_\mu(D^\mu\delta u^\nu)_a - ((D^\nu - \bar{u}^\nu\bar{u}_\mu D^\mu)\delta p)_a - g\bar{n}\bar{u}_\mu F_a^{\mu\nu} &= 0, \end{aligned} \quad (20)$$

where we have used the notation $(D_\mu X)_a \equiv \partial_\mu X^a + g f^{abc} A_\mu^b X^c$.

III. SOUND AND PLASMA WAVES IN THE QUARK-GLUON PLASMA

In order to analyze the linearized chromohydrodynamical equations derived in the previous section, we will treat separately the colorless and colored fluctuations showing that they describe the propagation of sound and plasma waves, respectively.

A. Sound waves

The colorless fluctuations of plasma velocity, energy density, pressure, and density obey the equations (19) that in momentum space read

$$\begin{aligned} \bar{u}^\mu k_\mu\delta n_0 + \bar{n}k_\mu\delta u_0^\mu &= 0, \\ \bar{u}^\mu k_\mu\delta\epsilon_0 + (\bar{\epsilon} + \bar{p})k_\mu\delta u_0^\mu &= 0, \\ i(\bar{\epsilon} + \bar{p})\bar{u}_\mu k^\mu\delta u_0^\nu + i(\bar{u}^\mu\bar{u}^\nu k_\mu - k^\nu)\delta p_0 &= 0, \end{aligned} \quad (21)$$

where $k^\mu \equiv (\omega, \mathbf{k})$. Since pressure and energy density are related by the EoS (18), their variations are not independent and upon differentiating Eq. (18) we obtain that

$$\delta p_0 = \left(\frac{\partial p}{\partial \epsilon}\right)\delta\epsilon_0 \equiv c_s^2\delta\epsilon_0. \quad (22)$$

Substituting this expression in Eq. (21), we find the following equation for the fluctuations of the pressure:

$$\frac{1}{\bar{u}^\mu k_\mu} \left\{ \left(\frac{1}{c_s^2} - 1\right) (\bar{u}^\mu k_\mu)^2 + k^2 \right\} \delta p_0 = 0, \quad (23)$$

while the fluctuations of the other quantities can be expressed as

$$\delta u_0^\nu = -\frac{1}{\left(\frac{1}{c_s^2} + 1\right)\bar{p}} \frac{k_\mu(\bar{u}^\mu\bar{u}^\nu - g^{\mu\nu})}{\bar{u} \cdot k} \delta p_0, \quad (24)$$

$$\delta n_0 = -\bar{n} \frac{k_\mu\delta u_0^\mu}{\bar{u} \cdot k}. \quad (25)$$

In the plasma rest frame, $\bar{u}^\mu = (1, 0, 0, 0)$, Eq. (23) simplifies to

$$\left(\frac{1}{c_s^2} - \frac{\mathbf{k}^2}{\omega^2}\right)\delta p_0 = 0, \quad (26)$$

which gives the standard expression of the sound waves in a plasma, reflecting the fact that all colorless hydrodynamical fluctuations propagate at the speed of sound.

B. Plasma waves

Regarding the colored components of the fluctuations, Eq. (20), we linearize in the A fields as well. The reason is that we will be interested in the computation of the polarization tensor and expanding at the first order in A is sufficient. One then finds

$$\bar{u}^\mu k_\mu \delta n_a + \bar{n} k_\mu \delta u_a^\mu = 0, \quad (27)$$

$$\bar{u}^\mu k_\mu \delta \epsilon_a + (\bar{\epsilon} + \bar{p}) k_\mu \delta u_a^\mu = 0, \quad (28)$$

$$i(\bar{\epsilon} + \bar{p})\bar{u}_\mu k^\mu \delta u_a^\nu + i(\bar{u}^\mu \bar{u}^\nu k_\mu - k^\nu)\delta p_a - q\bar{n}\bar{u}_\mu F_a^{\mu\nu} = 0, \quad (29)$$

and the fluctuation of the pressure and the fluctuations of the energy density can be related using the EoS (18), that leads to $\delta p_a = c_s^2 \delta \epsilon_a$ [33,39].

Equations (27) and (28) relate the fluctuations of the density and energy density with the fluctuations of the velocity

$$\delta n_a = -\bar{n} \frac{k_\mu \delta u_a^\mu}{\bar{u} \cdot k}, \quad (30)$$

$$\delta \epsilon_a = -(\bar{\epsilon} + \bar{p}) \frac{k_\mu \delta u_a^\mu}{\bar{u} \cdot k}, \quad (31)$$

and combining these expression with Eq. (22) we find that Eq. (29) can be rewritten as

$$\begin{aligned} & \left[g^{\nu\mu} + \frac{c_s^2}{(\bar{u} \cdot k)^2} (k^\nu k^\mu - \bar{u}^\nu k^\mu (\bar{u} \cdot k)) \right] \delta u_{\mu,a} \\ & = +ig \frac{\bar{n}}{(\bar{\epsilon} + \bar{p})(\bar{u} \cdot k)} \bar{u}_\mu F_a^{\mu\nu}. \end{aligned} \quad (32)$$

The inverse of the operator on the left-hand side of this equation can be determined observing that

$$\begin{aligned} & \left[g^{\nu\mu} + \frac{c_s^2}{(\bar{u} \cdot k)^2} (k^\nu k^\mu - \bar{u}^\nu k^\mu (\bar{u} \cdot k)) \right] \\ & \times [g_{\sigma\nu} - \mathcal{T}(k)(k_\sigma k_\nu - \bar{u}_\sigma k_\nu (\bar{u} \cdot k))] = g_{\sigma}^\mu, \end{aligned} \quad (33)$$

where

$$\mathcal{T}(k) = -\frac{1}{k^2 + (\frac{1}{c_s^2} - 1)(\bar{u} \cdot k)^2}. \quad (34)$$

Then one can solve Eq. (32) and the fluctuation of the hydrodynamic velocity takes the form

$$\begin{aligned} \delta u_{\sigma,a} & = ig \frac{\bar{n}}{(\bar{\epsilon} + \bar{p})(\bar{u} \cdot k)} \\ & \times [g_{\sigma\nu} + \mathcal{T}(k)(k_\sigma k_\nu - \bar{u}_\sigma k_\nu (\bar{u} \cdot k))] \bar{u}_\mu F_a^{\mu\nu}. \end{aligned} \quad (35)$$

The fluctuation of the current induced by the fluctuation of the density and of the hydrodynamic velocity can be derived from Eq. (6) and is given by

$$\delta j_a^\mu = -\frac{g}{2} (\bar{n} \delta u_a^\mu + \delta n_a \bar{u}^\mu). \quad (36)$$

Such fluctuations are related in linear response theory to fluctuations of the gauge fields via

$$\delta j_a^\mu(k) = -\Pi_{ab}^{\mu\nu}(k) A_{\nu,b}(k), \quad (37)$$

where $\Pi_{ab}^{\mu\nu}$ is the polarization tensor. Considering that the linearized strength tensor equals $F_a^{\mu\nu}(k) = -ik^\mu A_a^\nu(k) + ik^\nu A_a^\mu(k)$, and upon substituting the values of the fluctuations of density and energy density in Eq. (36) we obtain that

$$\begin{aligned} \Pi_{ab}^{\mu\nu}(k) & = -\delta_{ab} \frac{g^2}{2} \frac{\bar{n}^2}{(\bar{\epsilon} + \bar{p})} \frac{1}{(\bar{u} \cdot k)^2} \left[(\bar{u} \cdot k)(k^\mu \bar{u}^\nu + k^\nu \bar{u}^\mu) - (\bar{u} \cdot k)^2 g^{\mu\nu} - k^2 \bar{u}^\mu \bar{u}^\nu \right. \\ & \quad \left. - \frac{1}{k^2 + (\frac{1}{c_s^2} - 1)(\bar{u} \cdot k)^2} ((\bar{u} \cdot k)k^2(k^\mu \bar{u}^\nu + k^\nu \bar{u}^\mu) - (\bar{u} \cdot k)^2 k^\mu k^\nu - k^4 \bar{u}^\mu \bar{u}^\nu) \right]. \end{aligned} \quad (38)$$

One can easily check that the polarization tensor (38) is symmetric and transverse [$k_\mu \Pi^{\mu\nu}(k) = 0$].

In the plasma rest frame, $\bar{u}^\mu = (1, 0, 0, 0)$, the polarization tensor simplifies considerably and its components are given by

$$\begin{aligned}\Pi_{ab}^{00}(\omega, \mathbf{k}) &= -\delta_{ab}\omega_p^2 \frac{k^2}{\omega^2 - c_s^2 \mathbf{k}^2}, \\ \Pi_{ab}^{0i}(\omega, \mathbf{k}) &= \Pi_{ab}^{i0}(\omega, \mathbf{k}) = -\delta_{ab}\omega_p^2 \frac{\omega k^i}{\omega^2 - c_s^2 \mathbf{k}^2}, \\ \Pi_{ab}^{ij}(\omega, \mathbf{k}) &= -\delta_{ab}\omega_p^2 \left(\delta^{ij} + \frac{c_s^2 k^i k^j}{\omega^2 - c_s^2 \mathbf{k}^2} \right),\end{aligned}\quad (39)$$

where

$$\omega_p^2 = \frac{g^2}{2} \frac{\bar{n}^2}{(\bar{\epsilon} + \bar{p})} \quad (40)$$

is the plasma frequency.

At the linear order, the Fourier transformed chromodynamic field $A^\mu(k)$ satisfies the equation of motion

$$[k^2 g^{\mu\nu} - k^\mu k^\nu - \Pi^{\mu\nu}(k)]A_\nu(k) = 0, \quad (41)$$

where we have dropped the color indices a, b .

In order to determine the dispersion law for the gauge fields, we define the dielectric tensor $\varepsilon^{ij}(k)$ according to

$$\varepsilon^{ij}(k) = \delta^{ij} + \frac{1}{\omega^2} \Pi^{ij}(k), \quad (42)$$

that in the plasma rest frame is given by

$$\varepsilon_p^{ij}(\omega, \mathbf{k}) = \left(1 - \frac{\omega_p^2}{\omega^2} \right) \delta^{ij} - \frac{\omega_p^2}{\omega^2} \frac{c_s^2 k^i k^j}{\omega^2 - c_s^2 \mathbf{k}^2}. \quad (43)$$

The dispersion equation can now be determined by computing the determinant of the matrix

$$M^{ij} = \mathbf{k}^2 \delta^{ij} - k^i k^j - \omega^2 \varepsilon^{ij}(k), \quad (44)$$

and solving the equation

$$\det[M^{ij}] = 0, \quad (45)$$

where we assumed the Coulomb gauge, $\mathbf{k} \cdot \mathbf{A}(k) = 0$, and $A^0 = 0$, meaning that $\mathbf{E} = i\omega \mathbf{A}$.

Taking the wave vector in a given direction, say $\mathbf{k} = (k, 0, 0)$, one finds the following solutions to the dispersion relation:

$$\omega^2 = \omega_p^2 + c_s^2 k^2, \quad (46)$$

$$\omega^2 = \omega_p^2 + k^2, \quad (47)$$

which correspond to longitudinal and transverse modes, respectively. Notice also that both modes are not damped. The reason is that we have neglected the dissipative terms in the hydrodynamical equations, assuming that the colored plasma on the time scale of interest to the present analysis can be approximated as an ideal fluid. In order to consider the damping of color fluctuations in the hydrodynamical limit, one should include transport coefficients, such as color conductivity or color diffusion.

Let us compare the dispersion laws of the longitudinal and transverse modes obtained above with the correspond-

ing results in kinetic theory. Using the hard thermal loop effective theory, one can determine the polarization tensor for the gauge fields. The dispersion laws in the limit $k \ll \omega$ for the longitudinal and transverse modes are given by [40]

$$\omega^2 = \omega_p^2 + \frac{3}{5}k^2, \quad (48)$$

$$\omega^2 = \omega_p^2 + \frac{6}{5}k^2, \quad (49)$$

respectively. The chromohydrodynamical approach reproduces the short time physics of kinetic theory in the domain $k \ll \omega$. There is a numerical discrepancy in the k^2 terms when comparing the two sets of dispersion laws. The origin of such a discrepancy is that, in order to go from transport theory to a fluid approach, one has to impose a relation between the energy and the pressure of the system that allows one to discard all the higher momenta moments that are contained in the particle distribution function. Such an approximation, typical of a hydrodynamical approach, does not affect in any profound manner the physics described in the domain of applicability of the formalism.

IV. A RELATIVISTIC JET TRAVERSING THE PLASMA

We shall now consider a jet of particles propagating across the plasma at a relativistic velocity $v = |\mathbf{v}|$. We assume that both the jet and the plasma are initially in equilibrium but jet and plasma are not in equilibrium with each other. The system formed by the plasma and the jet is one specific example of the so-called two-stream systems, which have been widely studied in magnetohydrodynamics, and extended to the chromohydrodynamical case in Ref. [33].

We are not concerned with the colorless hydrodynamical fluctuations, which have been studied elsewhere [7,8]. We will instead examine the evolution of the colored fluctuations at short time scales, where the equations describing such fluctuations can be linearized.

We consider the frame where the plasma is at rest. The velocity, as well as the pressure, energy density, and particle density of the jet and of the plasma will be different and will be labeled in a different way. Since the plasma is at rest, $\bar{u}_p^\mu = (1, 0, 0, 0)$, whereas the velocity of the jet will have the general expression $\bar{u}_{\text{jet}}^\mu = \gamma(1, \mathbf{v})$, with $\gamma = 1/\sqrt{1 - v^2}$. The polarization tensor associated to the jet can be computed in an analogous way to that of the plasma and will have the expression reported in Eq. (38) with $\bar{u}^\mu \rightarrow \bar{u}_{\text{jet}}^\mu$. In the limit where $\gamma \gg 1$, it is possible to neglect the last term in the square bracket on the right-hand side of Eq. (38). Such an approximation corresponds to solving the fluid equations neglecting the effect of the pressure gradients. One can also see that this approximation corresponds to considering that the distribution function of the constituents of the jet is of the form (see [33] for a discussion of this approximation)

$$f(p) = \bar{n}\bar{u}^0 \delta^{(3)}\left(\mathbf{p} - \frac{\bar{\epsilon} + \bar{p}}{\bar{n}} \mathbf{u}\right), \quad (50)$$

which describes fast moving particles, with a nonthermal distribution.

Employing Eq. (42), we obtain the following expression of the dielectric tensor for the jet:

$$\epsilon_{\text{jet}}^{ij}(\omega, \mathbf{k}) = \left(1 - \frac{\omega_{\text{jet}}^2}{\omega^2}\right) \delta^{ij} - \frac{\omega_{\text{jet}}^2}{\omega^2} \left(\frac{v^i k^j + v^j k^i}{\omega - \mathbf{k} \cdot \mathbf{v}} - \frac{(\omega^2 - \mathbf{k}^2) v^i v^j}{(\omega - \mathbf{k} \cdot \mathbf{v})^2} \right), \quad (51)$$

where

$$\omega_{\text{jet}}^2 = \frac{g^2}{2} \frac{\bar{n}_{\text{jet}}^2}{(\bar{\epsilon}_{\text{jet}} + \bar{p}_{\text{jet}})} \quad (52)$$

is the plasma frequency squared of the jet.

In order to obtain the dielectric tensor of the system composed by the plasma and the jet, we note that the total polarization tensor of the system is given by the sum of the two polarization tensors in linear response theory

$$\Pi_t^{\mu\nu} = \Pi_p^{\mu\nu} + \Pi_{\text{jet}}^{\mu\nu}, \quad (53)$$

where the expression of the components of $\Pi_p^{\mu\nu}$ is given in Eq. (39). The dielectric tensor of the total system turns out to be

$$\epsilon_t^{ij}(\omega, \mathbf{k}) = \left(1 - \frac{\omega_t^2}{\omega^2}\right) \delta^{ij} - \frac{\omega_p^2}{\omega^2} \frac{c_s^2 k^i k^j}{\omega^2 - c_s^2 \mathbf{k}^2} - \frac{\omega_{\text{jet}}^2}{\omega^2} \left(\frac{v^i k^j + v^j k^i}{\omega - \mathbf{k} \cdot \mathbf{v}} - \frac{(\omega^2 - \mathbf{k}^2) v^i v^j}{(\omega - \mathbf{k} \cdot \mathbf{v})^2} \right), \quad (54)$$

where

$$\omega_t^2 = \omega_p^2 + \omega_{\text{jet}}^2. \quad (55)$$

The dispersion laws of the collective modes of the system composed by the plasma and the jet can now be determined solving the equation

$$\det[M_t^{ij}] \equiv \det[\mathbf{k}^2 \delta^{ij} - k^i k^j - \omega^2 \epsilon_t^{ij}(k)] = 0. \quad (56)$$

The solutions of this equation depend on $|\mathbf{k}|$, $|\mathbf{v}|$, $\cos\theta = \hat{\mathbf{k}} \cdot \hat{\mathbf{v}}$, ω_p , ω_{jet} , and c_s . We find that Eq. (56) admits eight solutions, however two of them are trivially given by $\omega = 0$ and six of these solutions are given by the roots of a polynomial of the sixth order. In order to simplify the expression of the dispersion law and to study the dependence of the unstable collective modes on the values of the various parameters, we define the dimensionless quantities,

$$x = \frac{\omega}{\omega_t}, \quad y = \frac{k}{\omega_t}, \quad b = \frac{\omega_{\text{jet}}^2}{\omega_t^2}, \quad (57)$$

meaning that we will measure momenta and energies in units of ω_t . Since the plasma frequency of the jet is

unknown, we will treat b as a parameter and we will analyze values of $b \ll 1$ corresponding to $\omega_{\text{jet}} \ll \omega_p$.

We anticipate that we find that five of the dispersion laws are always stable, whereas one is unstable in a certain range of values of the parameters. In particular, we find that, for values of v smaller than the speed of sound of the plasma, c_s , all the modes are stable independent of the values of the remaining parameters. Let us also note that, when \mathbf{k} and \mathbf{v} are not parallel, it is not possible to decompose the dielectric function (54) with only transverse and longitudinal projectors of \mathbf{k} . We will then not refer to transverse or longitudinal modes in that case.

In the following subsections, we will analyze the dispersion relations of the collective modes for different orientations of \mathbf{k} and \mathbf{v} .

A. \mathbf{k} parallel to \mathbf{v}

Here we consider the case corresponding to $\mathbf{k} \parallel \mathbf{v}$, and choose $\mathbf{k} = (0, 0, k)$ and $\mathbf{v} = (0, 0, v)$. This case can be easily analyzed because for this orientation of \mathbf{k} and \mathbf{v} the dielectric tensor (54) and the matrix M_t^{ij} defined in Eq. (56) are diagonal and the corresponding dispersion equation factorizes. Moreover, since \mathbf{k} is parallel to \mathbf{v} , it is possible to define transverse and longitudinal modes with respect to the orientation of these vectors. We find two stable transverse modes with dispersion law

$$\omega^2 = \omega_t^2 + k^2. \quad (58)$$

Notice that this dispersion law is analogous to the dispersion law that we obtained in Eq. (47) that describes the propagation of the transverse mode in a plasma without a jet. The only difference is that the plasma frequency ω_p has been replaced with ω_t . Therefore the only effect on the transverse modes due to the presence of the jet is to change the plasma frequency of the mode.

Regarding the longitudinal modes, they are given by the solution of the following equation:

$$\omega^2 - \omega_t^2 - \omega_p^2 \frac{c_s^2 k^2}{\omega^2 - c_s^2 k^2} - \omega_{\text{jet}}^2 \frac{2vk\omega - v^2\omega^2 - v^2k^2}{(\omega - kv)^2} = 0. \quad (59)$$

In terms of the dimensionless variables defined in Eq. (57), the equation for the longitudinal modes (59) can be written as

$$x^2[x^4 - 2vyx^3 + x^2(-1 + bv^2 + y^2(v^2 - c_s^2)) + 2vyx(1 - b + c_s^2 y^2) - y^2(bv^2(c_s^2 - 1) - c_s^2 b + v^2(c_s^2 y^2 + 1))] = 0. \quad (60)$$

This equation has two trivial solutions $x = 0$, and four solutions corresponding to the roots of a quartic equation. As already mentioned, we find that at most one mode is unstable. In the present case, one of the longitudinal modes

is unstable for $c_s < v < 1$. Since the cases where $v = c_s$ and $v \rightarrow 1$ will play a special role for these modes, let us first consider the solutions of Eq. (59) for these two values of the velocity.

In the limit $v \rightarrow 1$, Eq. (59) simplifies to

$$\begin{aligned} \omega^2 - \omega_t^2 - \omega_p^2 \frac{c_s^2 k^2}{\omega^2 - c_s^2 k^2} + \omega_{\text{jet}}^2 \\ = \omega^2 - \omega_p^2 - \omega_p^2 \frac{c_s^2 k^2}{\omega^2 - c_s^2 k^2} = 0, \end{aligned} \quad (61)$$

with solution, $\omega^2 = 0$ and

$$\omega^2 = \omega_p^2 + c_s^2 k^2. \quad (62)$$

This solution is the same that we have obtained for a plasma without a jet in Eq. (46). Therefore in the limit $v \rightarrow 1$, the longitudinal mode is stable and we see that a jet of particles moving at the speed of light can only affect the transverse modes, but they do not affect at all the longitudinal modes. This is related to the dimensional contraction which occurs in the eikonal limit [41], when it can be shown that the strength field tensor caused by a massless particle that moves at the speed of light is on the plane transverse to its motion.

In the case $v = c_s$, it is possible to find one simple analytical solution [apart from the trivial solution $\omega^2 = 0$ and Eq. (58)] which is given by

$$\omega = c_s k, \quad (63)$$

whereas three more solutions correspond to the roots of

$$\begin{aligned} x^3 - c_s x^2 y + c_s y(1 + b(-2 + c_s^2)) \\ + x(-1 + c_s^2(b - y^2)) + c_s^2 y^2 = 0. \end{aligned} \quad (64)$$

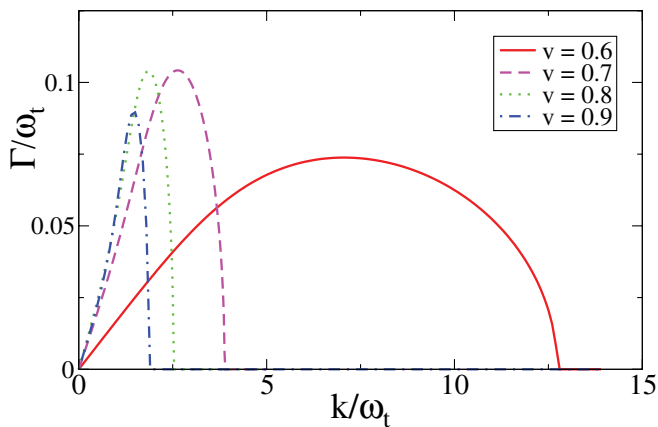


FIG. 1 (color online). Imaginary part of the dispersion law of the unstable longitudinal mode for the system composed by a plasma and a jet as a function of k/ω_t . Here the momentum of the unstable mode is parallel to the velocity of the jet \mathbf{v} , $b = \omega_{\text{jet}}^2/\omega_t^2 = 0.02$ and the four different lines correspond to different values of the velocity of the jet $|\mathbf{v}|$.

The discriminant of this equation is negative for $b < 1$, meaning that the three solutions of this equation are always real and the corresponding modes stable.

We now turn to a numerical study of the nontrivial solutions of Eq. (60) as a function of the parameters b and v . We find a solution with a positive imaginary part for any nonvanishing value of b and for v larger than c_s . Such a solution corresponds to the unstable longitudinal collective mode of the system. In Fig. 1 we have reported the plot of the imaginary part of the frequency, Γ , of this longitudinal mode as a function of k/ω_t , for $b = 0.02$ and for four different values of the velocity of the jet: $v = 0.6$, full (red online) line; $v = 0.7$, dashed (magenta online) line; $v = 0.8$, full (green online) line; $v = 0.9$, dot-dashed (blue online) line. For any value of the velocity smaller than or equal to the speed of sound, we numerically find $\Gamma = 0$. For values of v larger than the speed of sound and less than 1, this collective mode becomes unstable in a range of values of the momentum $k < k_{\text{max}}$. For small values of k , Γ increases with increasing momentum and reaches a peak. Then it decreases and at k_{max} becomes zero. As can be seen in Fig. 1 for $v > c_s$, the value of k_{max} decreases with increasing velocity.

The value of Γ at the peak, that we will indicate with Γ_{max} , depends on v and b . In Fig. 2 we present the plot of Γ_{max} as a function of b for four values of the jet velocity: $v = 0.6$, full (red online) line; $v = 0.7$, dashed (magenta online) line; $v = 0.8$, full (green online) line; $v = 0.9$, dot-dashed (blue online) line. Note that independent of the value of v , for $b = 0$, we obtain that $\Gamma = 0$. In this case the plasma frequency of the jet is zero and corresponds to the case where the system consists of the plasma only. Indeed, for $b = 0$, the dielectric tensor (54) does not have any contribution from the jet and therefore all the collective modes are stable, as shown in the previous

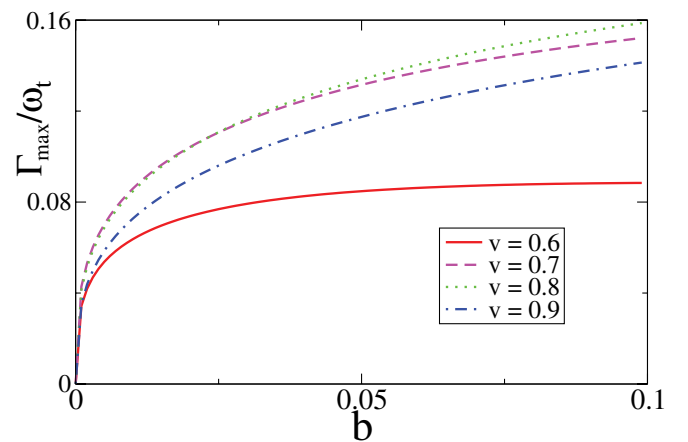


FIG. 2 (color online). Variations of the largest value of the imaginary part of the dispersion law of the unstable longitudinal mode for the system composed by a plasma and a jet in the case $\mathbf{k} \parallel \mathbf{v}$ as a function of b for four different values of the velocity of the jet, $|\mathbf{v}|$.

section. With increasing values of b , i.e. of the plasma frequency of the jet, Γ_{\max} increases, meaning that to larger values of the density of the jet (for fixed values of g and $\bar{\epsilon}_{\text{jet}} + \bar{p}_{\text{jet}}$) correspond larger instabilities. Moreover, for values of $b \lesssim 0.01$ the value of Γ_{\max} increases quickly with increasing b whereas for larger values of b , Γ_{\max} becomes less sensitive to the actual value of b . In particular, for $v = 0.6$, Γ_{\max} saturates at $\sim 0.09\omega_t$. As a function of the velocity we find that for a given value of b the value of Γ_{\max} increases with increasing velocity for $c_s < v \leq 0.7$. For $v \sim 0.7-0.8$ the value of Γ_{\max} reaches its maximum value and then for larger values of the velocity decreases and eventually in the ultrarelativistic case $v \rightarrow 1$ we find that the frequency of the longitudinal mode becomes real for all the values of k .

B. \mathbf{k} orthogonal to \mathbf{v}

We now consider the case corresponding to \mathbf{k} orthogonal to \mathbf{v} , choosing $\mathbf{k} = (k, 0, 0)$ and $\mathbf{v} = (0, 0, v)$. In this case the matrix M_t^{ij} defined in Eq. (56) is block diagonal. One block is one-dimensional and corresponds to the component M_t^{yy} . The other block is two-dimensional and corresponds to the components of M_t^{ij} with indices $i, j = x, z$. Therefore the dispersion equation (56) factorizes into two equations. Regarding the one-dimensional block, it describes a mode orthogonal to both \mathbf{k} and \mathbf{v} . This mode is stable and has dispersion law

$$\omega^2 = \omega_t^2 + k^2. \quad (65)$$

This dispersion law is analogous to the dispersion law for the transverse mode for a plasma without a jet, given in Eq. (47), with the plasma frequency ω_p replaced with ω_t . The remaining collective modes are solutions of the equation

$$\begin{aligned} & \left(\omega^2 - \omega_t^2 - \omega_p^2 \frac{c_s^2 k^2}{\omega^2 - c_s^2 k^2} \right) \\ & \times \left(\omega^2 - k^2 - \omega_t^2 - \omega_{\text{jet}}^2 \frac{v^2(k^2 - \omega^2)}{\omega^2} \right) - \omega_{\text{jet}}^4 \frac{v^2 k^2}{\omega^2} = 0. \end{aligned} \quad (66)$$

Using the variables defined in Eq. (57), one can rewrite this equation as

$$\begin{aligned} & x^2(x^2 - 1 - c_s^2 y^2)(x^2 - 1 - y^2 + v^2 b) + x^2 y^2 b(c_s^2 - v^2) \\ & + b y^2(v^2 - c_s^2 + c_s^2 y^2(v^2 - 1) + c_s^2 v^2 b) - b^2 v^2 y^2 = 0, \end{aligned} \quad (67)$$

where we have already factored out the trivial solution $x^2 = 0$.

Also in this case we find that for $v \leq c_s$ no mode is unstable and for $v > c_s$ only one mode is unstable in a certain range of values of the momentum. We have numerically studied the solutions of Eq. (67) and we have

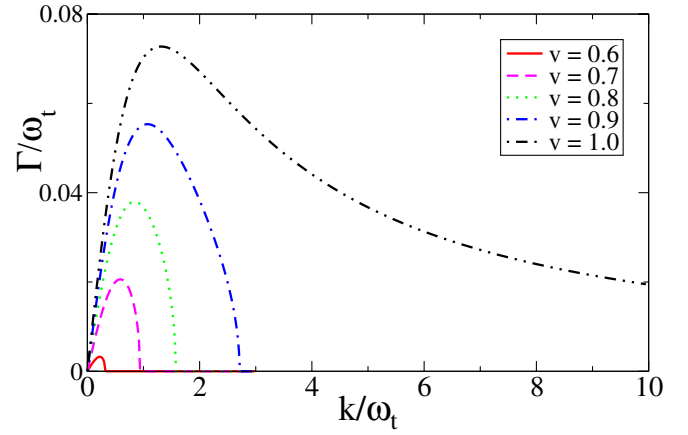


FIG. 3 (color online). Imaginary part of the dispersion law of the unstable mode for the system composed by a plasma and a jet in the case where the momentum of the collective mode, \mathbf{k} , is orthogonal to the velocity of the jet, \mathbf{v} , as a function of the momentum k/ω_t and for $b = \omega_{\text{jet}}^2/\omega_t^2 = 0.02$. The five different lines correspond to different values of the velocity $|\mathbf{v}|$.

reported the results in Figs. 3 and 4. In Fig. 3 it is shown the plot of the imaginary part of ω for $b = 0.02$, as a function of k for five different values of the jet velocity: $v = 0.6$, full (red online) line; $v = 0.7$, dashed (magenta online) line; $v = 0.8$, full (green online) line; $v = 0.9$, dot-dashed (blue online) line; $v = 1$, dot-dot-dashed (black) line.

The largest value of k where the orthogonal mode is unstable is a monotonic increasing function of v and it diverges for $v \rightarrow 1$. Also the value of the imaginary part of Γ at the peak, Γ_{\max} , increases with increasing v , but for $v \rightarrow 1$ reaches a maximum finite value. The behavior of the largest value of the imaginary part of the frequency, Γ_{\max} , as a function of b , for various values of the velocity,

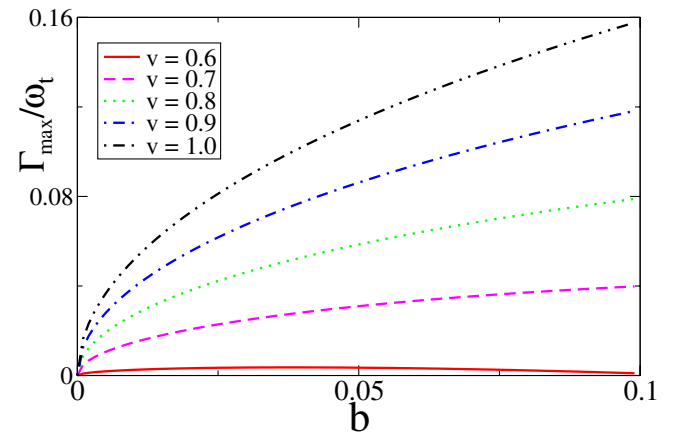


FIG. 4 (color online). Largest value of the imaginary part of the dispersion law of the unstable mode for the system composed by a plasma and a jet in the case where the momentum of the collective mode, \mathbf{k} , is orthogonal to the velocity of the jet, \mathbf{v} , as a function of $b = \omega_{\text{jet}}^2/\omega_t^2$. The five different lines correspond to different values of the velocity $|\mathbf{v}|$.

is reported in Fig. 4. With increasing values of b , i.e. with increasing value of the plasma frequency of the jet, the value of Γ_{\max} increases. Moreover, for a given value of b , the larger the velocity of the jet, the larger is the value of Γ_{\max} .

C. Arbitrary angles

In the previous two subsections, we have analyzed the behavior of the unstable mode of the plasma for two values of the angle θ between \mathbf{k} and \mathbf{v} . In particular, in Sec. IVA we have considered $\theta = 0$, corresponding to modes with $\mathbf{k} \parallel \mathbf{v}$ and in Sec. IV B we have taken $\theta = \pi/2$, corresponding to modes with $\mathbf{k} \perp \mathbf{v}$. A common feature of the two cases is that the value of Γ_{\max} (corresponding to the largest value of the imaginary part of the frequency as a function of $|\mathbf{k}|$), increases with increasing values of the plasma frequency of the jet. Regarding the dependence of Γ_{\max} on the value of the velocity of the jet v one can see, comparing the plots in Fig. 4 with those in Fig. 2, that for

the ultrarelativistic jet the modes with $\mathbf{k} \perp \mathbf{v}$ dominate the modes with $\mathbf{k} \parallel \mathbf{v}$. On the other hand, for velocity larger than the speed of sound, but $v \lesssim 0.9$ the longitudinal modes are dominant.

In this subsection, we analyze the general case, of an arbitrary angle θ and we will determine at which angle corresponds the most unstable mode as a function of v and b . For definitiveness we choose $\mathbf{v} = (0, 0, v)$ and $\mathbf{k} = (0, k \sin\theta, k \cos\theta)$.

The analysis of the dispersion laws for the modes with $\mathbf{k} \parallel \mathbf{v}$ and $\mathbf{k} \perp \mathbf{v}$ was simplified by the fact that in both cases the matrix M_i^{ij} defined in Eq. (56) becomes block diagonal and the corresponding equation for the dispersion laws factorizes. However, for arbitrary angle θ the matrix M_i^{ij} is not block diagonal. Then we will rely on a numerical solution of Eq. (56). Employing the definitions in Eq. (57) we obtain (apart from the two trivial solutions corresponding to $\omega = 0$) that the six dispersion laws can be obtained from the roots of the equation

$$\begin{aligned}
 0 = & 2x^6 + x^4(-4 + 2bv^2 + (-2 - 2c_s^2 + v^2)y^2) + x^2(2 - 2bv^2 - (-2 - 2(1+b)c_s^2 + (2+b+2bc_s^2)v^2)y^2 \\
 & - (v^2 + c_s^2(-2 + v^2))y^4) + by^2(-v^2(b + y^2) + c_s^2(-2(1 + y^2) + v^2(1 + b + 2y^2))) \\
 & + vy(v(1 + c_s^2y^2)(y + y^3) - 4x(-1 + x^2 - y^2)(-1 + b + x^2 - c_s^2y^2) \cos(\theta) + vy(-b^2(-1 + c_s^2) \\
 & + b(-2 + c_s^2 + x^2 - y^2) + (-1 + x^2 - y^2)(-1 + x^2 - c_s^2y^2)) \cos(2\theta)). \tag{68}
 \end{aligned}$$

Of the six solutions of this equation, only one corresponds to an unstable mode in a certain range of parameters. In particular, for any value of θ , we find an unstable mode only if $v > c_s$ and if the momentum is smaller than a certain value k_{\max} , where k_{\max} depends on θ , v , and b . The largest value of the imaginary part Γ_{\max} depends on θ , v , and b as well and in Fig. 5 we show three plots of Γ_{\max} as a function of b for various values of θ and v . In each plot the full (red online) line corresponds to $\theta = 0$, the dashed (magenta online) line corresponds to $\theta = \pi/8$, the dot-

dashed (green online) line corresponds to $\theta = \pi/4$, the dotted (blue online) line to $\theta = 3\pi/8$, and the dot-dashed (black) line to $\theta = \pi/2$. The left panel corresponds to $v = 0.8$, the central panel to $v = 0.9$, and the right panel corresponds to $v = 1$.

For $v = 0.8$ (left panel), the most unstable modes are those corresponding to small angles θ , i.e. are those modes with \mathbf{k} almost collinear with the velocity of the jet. In this regime these modes dominate the dynamics. With increasing velocity modes with larger values of the angle become

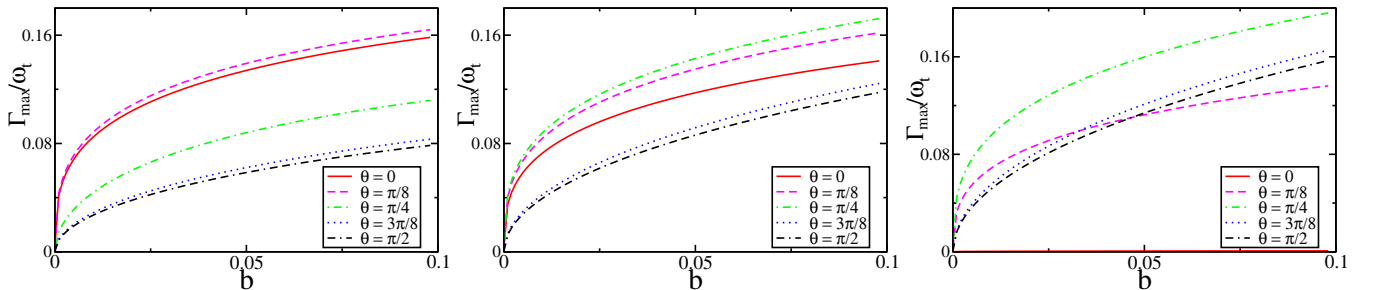


FIG. 5 (color online). Largest value of the imaginary part of the dispersion law for the unstable mode as a function of b for three different values of the velocity of the jet $|\mathbf{v}|$ and five different angles between \mathbf{k} and \mathbf{v} . The left panel corresponds to $v = 0.8$ and the dominant unstable modes correspond to small angles θ , i.e. to modes that are almost collinear with the jet. The central panel corresponds to $v = 0.9$ and the right panel to $v \rightarrow 1$. In both cases the modes corresponding to $\theta \sim \pi/4$ are the dominant one. In the limit $v \rightarrow 1$, modes with momentum collinear with the velocity of the jet are suppressed and the mode with $\theta = 0$ vanishes.

relevant. Indeed for a value of the velocity $v = 0.9$, corresponding to the central panel in Fig. 5, the imaginary part of the modes with $0 < \theta < \pi/4$ is the largest and the corresponding modes dominant. In the ultrarelativistic case $v \rightarrow 1$, right panel in Fig. 5, the dominant modes are those with $\theta \sim \pi/4$. Modes with \mathbf{k} almost collinear with \mathbf{v} , or more precisely all the modes at angles $\theta \lesssim \pi/8$, are suppressed. Notice that the modes with very large values of $\theta \sim \pi/2$ are not the dominant one. This means that modes with \mathbf{k} orthogonal to \mathbf{v} are not the most important: more important are “oblique” modes with $\theta \sim \pi/4$.

In agreement with the analytical results of the previous section, in the limit $v \rightarrow 1$, that is shown on the right panel of Fig. 5, the mode with $\theta = 0$ has a vanishing imaginary part and is stable.

V. CONCLUSIONS

It is well known that hydrodynamics predicts that any object moving in a fluid at a velocity higher than the speed of sound creates shock waves and a Mach cone structure. The conclusion of our study is that a neutral stream of colored particles moving at a velocity higher than the speed of sound in a quark-gluon plasma in the conformal limit also causes gauge field instabilities. It is curious to note that the speed of sound is the threshold value for the appearance of both the Mach cone structure and the gauge field instabilities. Qualitatively one can understand why this happens considering that if an external perturbation to the plasma moves slower than the speed of sound, the system can respond by locally rearranging the values of density, energy density, pressure, and plasma velocity, and then hydrodynamical fluctuations of these quantities may counteract the slow external perturbation. This is not the case when the perturbation propagates faster than the larger velocity of propagation of hydrodynamical fluctuations, i.e. larger than the speed of sound.

Even if we have obtained our results by solving the chromohydrodynamical equation of the system in the linear response approximation, let us stress that the same results are easily translated for electromagnetic relativistic fluids. To the best of our knowledge, the speed of sound bound that we found has not been discussed in the existing literature.

We believe that the mechanism proposed in this article provides one possible effective hydrodynamical description of the jet quenching phenomena, valid at macroscopic scales. Indeed we have shown that the energy and momenta stored in the total system (composed by the plasma and the jet) is effectively converted into (growing) energy and

momenta stored in the gauge fields, which are initially absent. The jet loses energy in its dynamical evolution.

However, our analysis of this mechanism has to be considered as preliminary and therefore we cannot indicate consequences for existing data of jet quenching at RHIC. Indeed, we have neglected several aspects of the system produced in heavy-ion collisions that complicate the treatment of the jet quenching phenomenon, such as the expansion of the plasma, or the transition to a hadronic phase, that takes place when the plasma becomes sufficiently dilute. One of the relevant points here is whether the system has enough time to generate the growth of the gauge fields before hadronization begins. As found in our numerical study, the maximal value of the growth rate, here characterized by Γ_{\max} , runs between $\Gamma_{\max} \sim (0.08-0.15)\omega_I$. Therefore, the plasma instabilities fully develop on time scales of the order $t \sim (6.7-12.5)/\omega_I$. To get an upper bound of that time scale, we evaluate the plasma frequency in a weakly coupling scenario at $T \sim 350$ MeV, finding $t \sim 1-2$ fm/ c .

Another aspect that we have neglected is the existence of dissipative terms in the hydrodynamical equations that at a certain stage of the plasma evolution might be able to damp the hydrodynamical fluctuations.

In any case, assuming that the generated gauge fields have sufficient time to grow, what is their fate? How do they hadronize? The late stage evolution of the chromohydrodynamical instabilities may depend on a different number of facts. One should discuss what is the saturation mechanism of the instabilities, which could be drastically affected by dissipative color damping phenomena. In any case, naively speaking, we would expect that the existence of some gauge field modes at soft values of the momenta less than k_{\max} , and in a given direction in momenta space, should be translated at RHIC into an enhanced production of hadrons with momenta $< k_{\max}$, versus the case without gauge field instabilities. Moreover, if one takes into account the effect of the growth of the gauge fields in the numerical simulations of the conical flow, then this might lead to an enhancement of the number of soft hadrons produced by the jet with respect to the case where the effect of the growth of the gauge fields has not been considered [6].

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