PHYSICAL REVIEW D **76,** 094006 (2007)

Charmless three-body decays of *B* mesons

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(Received 10 April 2007; published 2 November 2007)

An exploratory study of charmless 3-body decays of B mesons is presented using a simple model based on the framework of the factorization approach. The nonresonant contributions arising from $B \to P_1 P_2$ transitions are evaluated using heavy meson chiral perturbation theory (HMChPT). The momentum dependence of nonresonant amplitudes is assumed to be in the exponential form $e^{-\alpha_{\rm NR} p_B \cdot (p_i + p_j)}$ so that the HMChPT results are recovered in the soft meson limit $p_i, p_j \to 0$. In addition, we have identified another large source of the nonresonant signal in the matrix elements of scalar densities, e.g. $\langle K\bar{K}|\bar{s}s|0\rangle$, which can be constrained from the decay $\bar{B}^0 \to K_S K_S K_S$ or $B^- \to K^- K_S K_S$. The intermediate vector-meson contributions to 3-body decays are identified through the vector current, while the scalar meson resonances are mainly associated with the scalar density. Their effects are described in terms of the Breit-Wigner formalism. Our main results are: (i) All KKK modes are dominated by the nonresonant background. The predicted branching ratios of $K^+K^-K_{S(L)}$, $K^+K^-K^-$ and $K^-K_SK_S$ modes are consistent with the data within errors. (ii) Although the penguin-dominated $B^0 \to K^+K^-K_S$ decay is subject to a potentially significant tree pollution, its effective $\sin 2\beta$ is very similar to that of the $K_SK_SK_S$ mode. However, direct CP asymmetry of the former, being of order -4%, is more prominent than the latter. (iii) For $B \to K\pi\pi$ decays, we found sizable nonresonant contributions in $K^-\pi^+\pi^-$ and $\bar{K}^0\pi^+\pi^-$ modes, in agreement with the Belle measurements but larger than the *BABAR* result. (iv) Time-dependent CP asymmetries in $K_S\pi^0\pi^0$, a purely CP-even state, and $K_S\pi^+\pi^-$, an admixture of CP-even and CP-odd components, are studied. (v) The $\pi^+\pi^-\pi^0$ mode is found to have a rate larger than $\pi^+\pi^-\pi^-$ even though the former involves a π^0 in the final state. They are both dominated by resonant ρ contributions. (vi) We have computed the resonant contributions to 3-body decays and determined the rates for the quasi-two-body decays $B \to VP$ and $B \to SP$. The predicted $\rho\pi$, $f_0(980)K$ and $f_0(980)\pi$ rates are in agreement with the data, while the calculated ϕK, $K^*\pi$, ρK and $K_0^*(1430)\pi$ are in general too small compared to experiment. (vii) Sizable direct CP asymmetry is found in $K^+K^-K^-$ and $K^+K^-\pi^-$ modes.

DOI: 10.1103/PhysRevD.76.094006 PACS numbers: 13.25.Hw, 11.30.Er

I. INTRODUCTION

Three-body decays of heavy mesons are more complicated than the two-body case as they receive resonant and nonresonant contributions and involve 3-body matrix elements. The three-body meson decays are generally dominated by intermediate vector and scalar resonances, namely, they proceed via quasi-two-body decays containing a resonance state and a pseudoscalar meson. The analysis of these decays using the Dalitz plot technique enables one to study the properties of various resonances. The nonresonant background is usually believed to be a small fraction of the total 3-body decay rate. Experimentally, it is hard to measure the direct 3-body decays as the interference between nonresonant and quasi-two-body amplitudes makes it difficult to disentangle these two distinct contributions and extract the nonresonant one.

The Dalitz plot analysis of 3-body B decays provides a nice methodology for extracting information on the unitarity triangle in the standard model (SM). For example, the Dalitz analysis combined with isospin symmetry allows one to extract the angle α from $B \to \rho\pi \to \pi\pi\pi$ [1]. Recently, a method has been proposed in [2] for determining Cabibbo-Kobayashi-Maskawa (CKM) parameters in 3-body decays $B \to K\pi\pi$ and $B_s \to K\pi\pi$. This method was extended further in [3] to $\Delta I = 1$, $I(K^*\pi) = 1/2$ amplitudes in the above decays and to $I = 1$ amplitudes in $B_s \to K^*\bar{K}$ and $B_s \to \bar{K}^* K$.

Nonresonant 3-body decays of charmed mesons have been measured in several channels and the nonresonant signals in charm decays are found to be less than 10% [4]. In the past few years, some of the charmless B to 3-body decay modes have also been measured at B factories and studied using the Dalitz plot analysis. The measured fractions and the corresponding branching ratios of nonresonant components for some of 3-body B decay modes are listed in Table I. We see that the nonresonant 3-body decays could play an essential role in B decays. It is now well established that the $B \to KKK$ modes are dominated by the nonresonant background. For example, the nonresonant fraction is about 90% in $\bar{B}^0 \to K^+K^-\bar{K}^0$ decay. While this is a surprise in view of the rather small nonresonant contributions in 3-body charm decays, it is not entirely unexpected because the energy release scale in weak B decays is of order 5 GeV, whereas the major resonances lie in the energy region of 0.77 to 1.6 GeV. Consequently, it is likely that 3-body B decays may receive

TABLE I. Branching ratios of various charmless three-body decays of B mesons. The fractions and the corresponding branching ratios of nonresonant (NR) components are included whenever available. The first and second entries are *BABAR* and Belle results, respectively.

Decay	BR (10^{-6})	$\mathrm{BR}_{\mathrm{NR}}(10^{-6})$	NR fraction (%)	Ref.
$B^- \to \pi^+\pi^-\pi^-$	$16.2 \pm 1.2 \pm 0.9$	$2.3 \pm 0.9 \pm 0.5 < 4.6$	13.6 ± 6.1	[5]
	...	...	...	
$B^- \to K^-\pi^+\pi^-$	$64.1 \pm 2.4 \pm 4.0$	$2.87 \pm 0.65 \pm 0.43^{+0.63}_{-0.25}$	4.5 ± 1.5	[6]
	$48.8 \pm 1.1 \pm 3.6$	$16.9 \pm 1.3 \pm 1.3^{+1.1}_{-0.9}$	$34.0 \pm 2.2^{+2.1}_{-1.8}$	[7]
$B^- \to K^+K^-K^-$	$35.2 \pm 0.9 \pm 1.6$[a]	$50 \pm 6 \pm 4$	$141 \pm 16 \pm 9$	[8]
	$32.1 \pm 1.3 \pm 2.4$[b]	$24.0 \pm 1.5 \pm 1.5$[c]	74.8 ± 3.6[c]	[9]
$B^- \to K^-K_SK_S$	$10.7 \pm 1.2 \pm 1.0$			[10]
	$13.4 \pm 1.9 \pm 1.5$			[11]
$\bar{B}^0 \to \bar{K}^0\pi^+\pi^-$	$43.0 \pm 2.3 \pm 2.3$			[12]
	$47.5 \pm 2.4 \pm 3.7$	$19.9 \pm 2.5 \pm 1.6^{+0.7}_{-1.2}$	$41.9 \pm 5.1 \pm 0.6^{+1.4}_{-2.5}$	[13]
$\bar{B}^0 \to K^-\pi^+\pi^0$	$34.9 \pm 2.1 \pm 3.9$	<4.6		[14]
	$36.6^{+4.2}_{-4.3} \pm 3.0$	$5.7^{+2.7+0.5}_{-2.5-0.4} < 9.4$		[15]
$\bar{B}^0 \to K^+K^-\bar{K}^0$	$23.8 \pm 2.0 \pm 1.6$	26.7 ± 4.6	112.0 ± 14.9	[16]
	$28.3 \pm 3.3 \pm 4.0$			[11]
$\bar{B}^0 \to K_SK_SK_S$	$6.9^{+0.9}_{-0.8} \pm 0.6$			[17]
	$4.2^{+1.6}_{-1.3} \pm 0.8$			[11]

[a]When the intrinsic charm contribution is excluded, the charmless branching ratio will become $(33.5 \pm 0.9 \pm 1.6) \times 10^{-6}$.
[b]When the contribution from $B^+ \to \chi_{c0}K^+$ is excluded, the charmless branching ratio will become $(30.6 \pm 1.2 \pm 2.3) \times 10^{-6}$.
[c]Belle found two solutions for the fractions and branching ratios. We follow Belle to use the large solution.

sizable nonresonant contributions. At any rate, it is important to understand and identify the underlying mechanism for nonresonant decays.

The direct nonresonant three-body decays of mesons in general receive two distinct contributions: one from the pointlike weak transition and the other from the pole diagrams that involve three-point or four-point strong vertices. For D decays, attempts of applying the effective SU(4) × SU(4) chiral Lagrangian to describe the $DP \to DP$ and $PP \to PP$ scattering at energies $\sim m_D$ have been made by several authors [18–22] to calculate the nonresonant D decays, though in principle it is not justified to employ the SU(4) chiral symmetry. As shown in [21,22], the predictions of the nonresonant decay rates in chiral perturbation theory are in general too small when compared with experiment. With the advent of heavy quark symmetry and its combination with chiral symmetry [23–25], the nonresonant D decays can be studied reliably at least in the kinematic region where the final pseudoscalar mesons are soft. Some of the direct 3-body D decays were studied based on this approach [26,27].

For the case of B mesons, consider the three-body B decay $B \to P_1P_2P_3$. Under the factorization hypothesis, one of the nonresonant contributions arises from the transitions $B \to P_1P_2$. The nonresonant background in charmless three-body B decays due to the transition $B \to P_1P_2$ has been studied extensively [28–33] based on heavy meson chiral perturbation theory (HMChPT) [23–25]. However, the predicted decay rates are, in general, unexpectedly large. For example, the branching ratio of the nonresonant decay $B^- \to \pi^+\pi^-\pi^-$ is predicted to be of order 10^{-5} in [28,29], which is too large compared to the limit 4.6×10^{-6} set by *BABAR* [5]. Therefore, it is important to reexamine and clarify the existing calculations.

The issue has to do with the applicability of HMChPT. In order to apply this approach, two of the final-state pseudoscalars in $B \to P_1P_2$ transition have to be soft. The momentum of the soft pseudoscalar should be smaller than the chiral-symmetry breaking scale of order 1 GeV. For 3-body charmless B decays, the available phase space where chiral perturbation theory is applicable is only a small fraction of the whole Dalitz plot. Therefore, it is not justified to apply chiral and heavy quark symmetries to a certain kinematic region and then generalize it to the region beyond its validity. In this work we shall assume the momentum dependence of nonresonant amplitudes in the exponential form $e^{-\alpha_{\mathrm{NR}} p_B \cdot (p_i+p_j)}$ so that the HMChPT results are recovered in the soft meson limit $p_i, p_j \to 0$. We shall see that the parameter α_{NR} can be fixed from the tree-dominated decay $B^- \to \pi^+\pi^-\pi^-$.

However, the nonresonant background in $B \to P_1P_2$ transition does not suffice to account for the experimental observation that the penguin-dominated decay $B \to KKK$ is dominated by the nonresonant contributions. This implies that the two-body matrix element e.g. $\langle K\bar{K}|\bar{s}s|0\rangle$ induced by the scalar density should have a large nonresonant component. In the absence of first-principles calculation, we will use the $\bar{B}^0 \to K_SK_SK_S$ mode in conjunction with the mass spectrum in $\bar{B}^0 \to K^+K^-\bar{K}^0$ to fix the nonresonant contribution to $\langle K\bar{K}|\bar{s}s|0\rangle$.

In this work, we shall study the charmless 3-body decays of B mesons using the factorization approach. Besides the nonresonant background as discussed above, we will also study resonant contributions to 3-body decays. Vector-meson and scalar resonances contribute to the two-body matrix elements $\langle P_1P_2|V_\mu|0\rangle$ and $\langle P_1P_2|S|0\rangle$, respectively. They can also contribute to the three-body matrix element $\langle P_1P_2|V_\mu - A_\mu|B\rangle$. Resonant effects are described in terms of the usual Breit-Wigner formalism. In this manner we are able to figure out the relevant resonances which contribute to the 3-body decays of interest and compute the rates of $B \to VP$ and $B \to SP$. In conjunction with the nonresonant contribution, we are ready to calculate the total rates for three-body decays.

It should be stressed from the outset that in this work we take the factorization approximation as a working hypothesis rather than a first-principles starting point. If we start with theories such as QCD factorization (QCDF) [34], or perturbative QCD (pQCD) [35] or soft-collinear effective theory [36], then we can take power corrections seriously and make an estimation. Since factorization has not been proved for three-body B decays, we shall work in the phenomenological factorization model rather than in the established theories such as QCDF. That is, we start with the simple idea of factorization and see if it works for three-body decays, in the hope that it will provide a useful zeroth step for others to try to improve.

The penguin-induced three-body decays $B^0 \to K^+K^-K_S$ and $K_SK_SK_S$ deserve special attention as the current measurements of the deviation of $\sin 2\beta_{\rm eff}$ in KKK modes from $\sin 2\beta_{J/\psi K_S}$ may indicate new physics in $b \to s$ penguin-induced modes. It is of great importance to examine and estimate how much of the deviation of $\sin 2\beta_{\rm eff}$ is allowed in the SM. Owing to the presence of color-allowed tree contributions in $B^0 \to K^+K^-K_S$, this mode is subject to a potentially significant tree pollution and the deviation of the mixing-induced CP asymmetry from that measured in $B \to J/\psi K_S$ could be as large as $\mathcal{O}(0.10)$. Since the tree amplitude is tied to the nonresonant background, it is very important to understand the nonresonant contributions in order to have a reliable estimate of $\sin 2\beta_{\rm eff}$ in KKK modes.

The layout of the present paper is as follows. In Sec. II we shall apply the factorization approach to study $B^0 \to K^+K^-K_S$ and $K_SK_SK_S$ decays and discuss resonant and nonresonant contributions. In order to set up the framework for calculations we will discuss $B \to KKK$ modes in most details. We then turn to $K\pi\pi$ modes in Sec. III, the tree-dominated modes $KK\pi$ in Sec. IV, and $\pi\pi\pi$ in Sec. V. In Sec. VI, we determine the rates for $B \to VP$ and $B \to SP$ and compare our results with the approach of QCD factorization. Section VII contains our conclusions. The factorizable amplitudes of various $B \to P_1P_2P_3$ decays are summarized in Appendix A. The relevant input parameters such as decay constants, form factors, etc. are collected in Appendix B.

II. $B \to KKK$ DECAYS

For 3-body B decays, the $b \to sq\bar{q}$ penguin transitions contribute to the final states with odd number of kaons, namely, KKK and $K\pi\pi$, while $b \to uq\bar{q}$ tree and $b \to dq\bar{q}$ penguin transitions contribute to final states with even number of kaons, e.g. $KK\pi$ and $\pi\pi\pi$. We shall first discuss the $b \to s$ penguin-dominated 3-body decays in detail and then turn to $b \to u$ tree-dominated modes. For $B \to KKK$ modes, we shall first consider the neutral B decays as they involve mixing-induced CP asymmetries.

A. $\bar{B}^0 \to KKK$ decays

We consider the decay $\bar{B}^0 \to K^+K^-\bar{K}^0$ as an illustration. Under the factorization approach, the $\bar{B}^0 \to K^+K^-\bar{K}^0$ decay amplitude consists of three distinct factorizable terms: (i) the current-induced process with a meson emission, $\langle \bar{B}^0 \to K^+\bar{K}^0\rangle \times \langle 0 \to K^-\rangle$, (ii) the transition process, $\langle \bar{B}^0 \to \bar{K}^0\rangle \times \langle 0 \to K^+K^-\rangle$, and (iii) the annihilation process $\langle \bar{B}^0 \to 0\rangle \times \langle 0 \to K^+K^-\bar{K}^0\rangle$, where $\langle A \to B\rangle$ denotes a $A \to B$ transition matrix element. In the factorization approach, the matrix element of the $\bar{B} \to \bar{K}\bar{K}K$ decay amplitude is given by

$$\langle \bar{K}\bar{K}K|\mathcal{H}_{\rm eff}|\bar{B}\rangle = \frac{G_F}{\sqrt{2}}\sum_{p=u,c}\lambda_p^{(s)}\langle \bar{K}\bar{K}K|T_p|\bar{B}\rangle, \quad (2.1)$$

where $\lambda_p^{(s)} \equiv V_{pb}V_{ps}^*$ and the explicit expression of T_p in terms of four-quark operators is given in Eq. (A2). The factorizable $\bar{B}^0 \to K^+K^-\bar{K}^0$ decay amplitude is given in Eq. (A4). Note that the Okubo-Zweig-Iizuka (OZI)-suppressed matrix element $\langle K^+K^-|(\bar{d}d)_{V-A}|0\rangle$ is included in the factorizable amplitude since it could be enhanced through the long-distance pole contributions via the intermediate vector mesons such as ρ^0 and ω. Likewise, the OZI-suppressed matrix elements $\langle K^+K^-|(\bar{d}b)_{V-A}|\bar{B}^0\rangle$ and $\langle K^+K^-|\bar{d}(1-\gamma_5)b|\bar{B}^0\rangle$ are included as they receive contributions from the scalar resonances like $f_0(980)$.

For the current-induced process, the two-meson transition matrix element $\langle \bar{K}^0K^+|(\bar{u}b)_{V-A}|\bar{B}^0\rangle$ has the general expression [37]

$$\begin{aligned}\langle \bar{K}^0(p_1)K^+(p_2)|(\bar{u}b)_{V-A}|\bar{B}^0\rangle = {}& ir(p_B - p_1 - p_2)_\mu + i\omega_+(p_2+p_1)_\mu + i\omega_-(p_2-p_1)_\mu \\ & + h\epsilon_{\mu\nu\alpha\beta}p_B^\nu(p_2+p_1)^\alpha(p_2-p_1)^\beta, \quad (2.2)\end{aligned}$$

where $(\bar{q}_1q_2)_{V-A} \equiv \bar{q}_1\gamma_\mu(1-\gamma_5)q_2$. This leads to

$$A^{\rm HMChPT}_{\rm current\text{-}ind} \equiv \langle K^-(p_3)|(\bar{s}u)_{V-A}|0\rangle\langle \bar{K}^0(p_1)K^+(p_2)|(\bar{u}b)_{V-A}|\bar{B}^0\rangle = -\frac{f_K}{2}[2m_3^2 r + (m_B^2 - s_{12} - m_3^2)\omega_+ + (s_{23} - s_{13} - m_2^2 + m_1^2)\omega_-], \tag{2.3}$$

where $s_{ij} \equiv (p_i + p_j)^2$. To compute the form factors r, $\omega_\pm$ and h, one needs to consider not only the pointlike contact diagram, Fig. 1(a), but also various pole diagrams depicted in Fig. 1. In principle, one can apply HMChPT to evaluate the form factors r, ω_+ and ω_- [37]. However, this will lead to too large decay rates in disagreement with experiment [38]. The heavy meson chiral Lagrangian given in [23–25] is needed to compute the strong B^*BP, B^*B^*P and $BBPP$ vertices. The results for the form factors are [29,37]

$$\begin{aligned}
\omega_+ &= -\frac{g}{f_\pi^2}\frac{f_{B_s^*}m_{B_s^*}\sqrt{m_B m_{B_s^*}}}{s_{23} - m_{B_s^*}^2}\left[1 - \frac{(p_B - p_1)\cdot p_1}{m_{B_s^*}^2}\right] + \frac{f_B}{2f_\pi^2},\\
\omega_- &= \frac{g}{f_\pi^2}\frac{f_{B_s^*}m_{B_s^*}\sqrt{m_B m_{B_s^*}}}{s_{23} - m_{B_s^*}^2}\left[1 + \frac{(p_B - p_1)\cdot p_1}{m_{B_s^*}^2}\right],\\
r &= \frac{f_B}{2f_\pi^2} - \frac{f_B}{f_\pi^2}\frac{p_B\cdot(p_2 - p_1)}{(p_B - p_1 - p_2)^2 - m_B^2} + \frac{2gf_{B_s^*}}{f_\pi^2}\sqrt{\frac{m_B}{m_{B_s^*}}}\frac{(p_B - p_1)\cdot p_1}{s_{23} - m_{B_s^*}^2} - \frac{4g^2 f_B}{f_\pi^2}\frac{m_B m_{B_s^*}}{(p_B - p_1 - p_2)^2 - m_B^2}\\
&\quad\times\frac{p_1\cdot p_2 - p_1\cdot(p_B - p_1)p_2\cdot(p_B - p_1)/m_{B_s^*}^2}{s_{23} - m_{B_s^*}^2},
\end{aligned} \tag{2.4}$$

where $f_\pi = 132$ MeV, g is a heavy-flavor independent strong coupling which can be extracted from the CLEO measurement of the D^{*+} decay width, $|g| = 0.59 \pm 0.01 \pm 0.07$ [39]. We shall follow [23] to fix its sign to be negative. The pointlike diagram Fig. 1(a) characterized by the term $f_B/(2f_\pi^2)$ contributes to the form factors ω_+ and r, while Figs. 1(b) and 1(d) contribute to r and Fig. 1(c) contributes to all the form factors.

A direct calculation indicates that the branching ratio of $\bar{B}^0 \to K^+K^-\bar{K}^0$ arising from the current-induced process alone is already at the level of 77×10^{-6} which exceeds the measured total branching ratio of 25×10^{-6} (see Table I). The issue has to do with the applicability of HMChPT. In order to apply this approach, two of the final-state pseudoscalars (K^+ and $\bar{K}^0$ in this example) have to be soft. The momentum of the soft pseudoscalar should be smaller than the chiral-symmetry breaking scale Λ_χ of order $0.83 - 1.0$ GeV. For 3-body charmless B decays, the available phase space where chiral perturbation theory is applicable is only a small fraction of the whole Dalitz plot. Therefore, it is not justified to apply chiral and heavy quark symmetries to a certain kinematic region and then generalize it to the region beyond its validity. If the soft meson result is assumed to be the same in the whole Dalitz plot, the decay rate will be greatly overestimated.

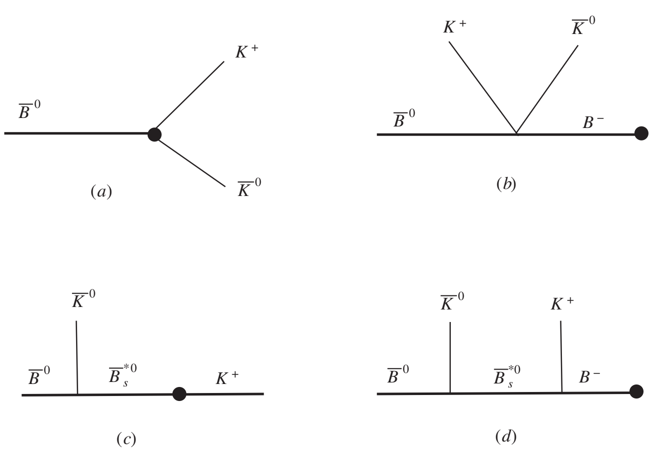

FIG. 1. Pointlike and pole diagrams responsible for the $\bar{B}^0 \to K^+\bar{K}^0$ matrix element induced by the current $\bar{u}\gamma_\mu(1-\gamma_5)b$, where the symbol ● denotes an insertion of the current.

In [38,40] we have tried to circumvent the aforementioned problem by applying HMChPT only to the strong vertex and use the form factors to describe the weak vertex. Moreover, we introduced a form factor to take care of the off-shell effect. For example, Fig. 1(c) can be evaluated by considering the strong interaction $\bar{B}^0 \to \bar{K}^0\bar{B}_s^*$ followed by the weak transition $\bar{B}_s^* \to K^+$ and the result is [38]

$$\begin{aligned}
A_{\rm Fig.\,1(c)} &= \frac{f_K}{f_\pi}\frac{g\sqrt{m_B m_{B_s^*}}}{s_{23} - m_{B_s^*}^2}F(s_{23}, m_{B_s^*})F_1^{B_sK}(m_3^2)\left[m_B + \frac{s_{23}}{m_B}\right.\\
&\quad\left. - m_B\frac{m_B^2 - s_{23}}{m_3^2}\left(1 - \frac{F_0^{B_sK}(m_3^2)}{F_1^{B_sK}(m_3^2)}\right)\right]\left[m_1^2 + m_3^2\right.\\
&\quad\left. - s_{13} + \frac{(s_{23} - m_2^2 + m_3^2)(m_B^2 - s_{23} - m_1^2)}{2m_{B_s^*}^2}\right],
\end{aligned} \tag{2.5}$$

where $F_{0,1}^{B_sK}$ are the $B_s \to K$ weak transition from factors in the standard convention [41] and we have introduced a form factor $F(s_{23}, m_{B_s^*})$ to take into account the off-shell effect of the B_s^* pole [40]. It is parametrized as $F(s_{23}, m_{B_s^*}) = (\Lambda^2 - m_{B_s^*}^2)/(\Lambda^2 - s_{23})$ with the cutoff parameter Λ chosen to be $\Lambda = m_{B_s^*} + \Lambda_{\rm QCD}$. Needless to say, this parametrization of the form factor is somewhat arbitrary. Moreover, the nonresonant contribution thus calculated is too small compared to experiment.

The Dalitz plot analysis of $\bar{B}^0 \to K^+K^-\bar{K}^0$ has been recently performed by *BABAR* [16]. In the *BABAR* analysis, a phenomenological parametrization of the nonresonant amplitudes is described by

$$A_{\rm NR} = (c_{12}e^{i\phi_{12}}e^{-\alpha s_{12}^2} + c_{13}e^{i\phi_{13}}e^{-\alpha s_{13}^2} + c_{23}e^{i\phi_{23}}e^{-\alpha s_{23}^2})(1 + b_{\rm NR}e^{i(\beta+\delta_{\rm NR})}), \tag{2.6}$$

and resonant terms are described by

$$A_{\rm R} = \sum_r c_r(1+b_r)f_re^{i(\phi_r+\delta_r+\beta)}, \qquad \bar{A}_{\rm R} = \sum_r c_r(1-b_r)f_re^{i(\phi_r-\delta_r+\beta)}. \tag{2.7}$$

The *BABAR* results for isobar amplitudes, phases and fractions from the fit to the $B^0 \to K^+K^-K^0$ are summarized in Table II. It is evident that this decay is dominated by the nonresonant background. For our purpose, we will parametrize the current-induced nonresonant amplitude Eq. (2.3) as

$$A_{\text{current-ind}} = A_{\text{current-ind}}^{\rm HMChPT}e^{-\alpha_{\rm NR}p_B\cdot(p_1+p_2)}e^{i\phi_{12}}, \tag{2.8}$$

so that the HMChPT results are recovered in the chiral limit $p_1, p_2 \to 0$. That is, the nonresonant amplitude in the soft meson region is described by HMChPT, but its energy dependence beyond the chiral limit is governed by the exponential term $e^{-\alpha_{\rm NR}p_B\cdot(p_1+p_2)}$. In what follows, we shall use the tree-dominated $B^- \to \pi^+\pi^-\pi^-$ decay data to fix $\alpha_{\rm NR}$, which turns out to be

$$\alpha_{\rm NR} = 0.103^{+0.018}_{-0.011}\ {\rm GeV}^{-2}. \tag{2.9}$$

This is very close to the naive expectation of $\alpha_{\rm NR} \sim \mathcal{O}[1/(2m_B\Lambda_\chi)]$ based on the dimensional argument. The phase ϕ_{12} of the nonresonant amplitude in the $(K^+\bar{K}^0)$ system will be set to zero for simplicity.

For the transition amplitude, we need to evaluate the 2-kaon creation matrix element which can be expressed in terms of timelike kaon current form factors as

$$\langle K^+(p_{K^+})K^-(p_{K^-})|\bar{q}\gamma_\mu q|0\rangle = (p_{K^+}-p_{K^-})_\mu F_q^{K^+K^-},$$
$$\langle K^0(p_{K^0})\bar{K}^0(p_{\bar{K}^0})|\bar{q}\gamma_\mu q|0\rangle = (p_{K^0}-p_{\bar{K}^0})_\mu F_q^{K^0\bar{K}^0}. \tag{2.10}$$

The weak vector form factors $F_q^{K^+K^-}$ and $F_q^{K^0\bar{K}^0}$ can be related to the kaon electromagnetic (e.m.) form factors $F_{em}^{K^+K^-}$ and $F_{em}^{K^0\bar{K}^0}$ for the charged and neutral kaons, respectively. Phenomenologically, the e.m. form factors receive resonant and nonresonant contributions and can be expressed by

$$F_{\rm em}^{K^+K^-} = F_\rho + F_\omega + F_\phi + F_{\rm NR}, \qquad F_{\rm em}^{K^0\bar{K}^0} = -F_\rho + F_\omega + F_\phi + F'_{\rm NR}. \tag{2.11}$$

It follows from Eqs. (2.10) and (2.11) that

$$F_u^{K^+K^-} = F_d^{K^0\bar{K}^0} = F_\rho + 3F_\omega + \frac{1}{3}(3F_{\rm NR} - F'_{\rm NR}),$$
$$F_d^{K^+K^-} = F_u^{K^0\bar{K}^0} = -F_\rho + 3F_\omega, \tag{2.12}$$
$$F_s^{K^+K^-} = F_s^{K^0\bar{K}^0} = -3F_\phi - \frac{1}{3}(3F_{\rm NR} + 2F'_{\rm NR}),$$

where use of isospin symmetry has been made.

The resonant and nonresonant terms in Eq. (2.11) can be parametrized as

$$F_h(s_{23}) = \frac{c_h}{m_h^2 - s_{23} - im_h\Gamma_h}, \qquad F_{\rm NR}^{(\prime)}(s_{23}) = \left(\frac{x_1^{(\prime)}}{s_{23}} + \frac{x_2^{(\prime)}}{s_{23}^2}\right)\left[\ln\left(\frac{s_{23}}{\tilde{\Lambda}^2}\right)\right]^{-1}, \tag{2.13}$$

with $\tilde{\Lambda} \approx 0.3$ GeV. The expression for the nonresonant

TABLE II. *BABAR* results for isobar amplitudes, phases, and fractions from the fit to the $B^0 \to K^+K^-K^0$ [16]. Three rows for nonresonant contribution correspond to coefficients of exponential functions in Eq. (2.6), while the fraction is given for the combined amplitude. For the nonresonant decay mode in K^+K^-, the amplitude c_{12} and the phase ϕ_{12} in Eq. (2.6) are fixed to be one and zero, respectively. Errors are statistical only.

Decay	Amplitude c_r	Phase ϕ_r	Fraction (%)
$\phi(1020)K^0$	0.0085 ± 0.0010	-0.016 ± 0.234	12.5 ± 1.3
$f_0(980)K^0$	0.622 ± 0.046	-0.14 ± 0.14	40.2 ± 9.6
$X_0(1550)K^0$	0.114 ± 0.018	-0.47 ± 0.20	4.1 ± 1.3
$(K^+K^-)_{\rm NR}K^0$	1 (fixed)	0 (fixed)	
$(K^+K^0)_{\rm NR}K^-$	0.33 ± 0.07	1.95 ± 0.27	112.0 ± 14.9
$(K^-K^0)_{\rm NR}K^+$	0.31 ± 0.08	-1.34 ± 0.37	
$\chi_{c0}(1P)K^0$	0.0306 ± 0.00649	$^{0.81}_{-2.33} \pm 0.54$	3.0 ± 1.2
D^+K^-	1.11 ± 0.17	$\cdots$	3.6 ± 1.5
$D_s^+K^-$	0.76 ± 0.14	$\cdots$	1.8 ± 0.6

form factor is motivated by the asymptotic constraint from pQCD, namely, $F(t) \to (1/t)[\ln(t/\tilde{\Lambda}^2)]^{-1}$ in the large t limit [42]. The unknown parameters c_h, x_i and x_i' are fitted from the kaon e.m. data, giving the best fit values (in units of GeV^2 for c_h) [43]:

$$\begin{aligned} c_\rho = 3c_\omega = c_\phi = 0.363, \qquad c_{\rho(1450)} = 7.98\times 10^{-3}, \\ c_{\rho(1700)} = 1.71\times 10^{-3}, \qquad c_{\omega(1420)} = -7.64\times 10^{-2}, \\ c_{\omega(1650)} = -0.116, \qquad c_{\phi(1680)} = -2.0\times 10^{-2}, \end{aligned} \tag{2.14}$$

and

$$\begin{aligned} x_1 = -3.26\ \text{GeV}^2, \qquad x_2 = 5.02\ \text{GeV}^4, \\ x_1' = 0.47\ \text{GeV}^2, \qquad x_2' = 0. \end{aligned} \tag{2.15}$$

Note that the form factors $F_{\rho,\omega,\phi}$ in Eqs. (2.11) and (2.12) include the contributions from the vector mesons $\rho(770)$, $\rho(1450)$, $\rho(1700)$, $\omega(782)$, $\omega(1420)$, $\omega(1650)$, $\phi(1020)$ and $\phi(1680)$. It is interesting to note that (i) the fitted values of c_V are very close to the vector-meson dominance expression $g_{V\gamma}g_{VKK}$ for $V = \rho, \omega, \phi$ [4,44], where $g_{V\gamma}$ is the e.m. coupling of the vector meson defined by $\langle V|j_{\rm em}|0\rangle = g_{V\gamma}\varepsilon_V^*$ and g_{VKK} is the $V \to KK$ strong coupling with $-g_{\phi K^+K^-} \simeq g_{\rho K^+K^-}/\sqrt{2} = g_{\omega K^+K^-}/\sqrt{2} \simeq 3.03$, and (ii) the vector-meson pole contributions alone yield $F_{u,s}^{K^+K^-}(0) \approx 1, -1$ and $F_d^{K^+K^-}(0) \approx 0$ as the charged kaon does not contain the valence d quark. The matrix element for the current-induced decay process then has the expression

$$\begin{aligned} &\langle \bar{K}^0(p_1)|(\bar{s}b)_{V-A}|\bar{B}^0\rangle\langle K^+(p_2)K^-(p_3)|(\bar{q}q)_{V-A}|0\rangle \\ &\quad = (s_{12}-s_{13})F_1^{BK}(s_{23})F_q^{K^+K^-}(s_{23}). \end{aligned} \tag{2.16}$$

We also need to specify the 2-body matrix elements $\langle K^+K^-|\bar{s}s|0\rangle\langle\bar{K}^0|\bar{s}b|\bar{B}^0\rangle$ induced from the scalar densities. The use of the equation of motion leads to

$$\langle \bar{K}^0(p_1)|\bar{s}b|\bar{B}^0(p_B)\rangle = \frac{m_B^2 - m_K^2}{m_b - m_s}F_0^{BK}(s_{23}). \tag{2.17}$$

The matrix element $\langle K^+K^-|\bar{s}s|0\rangle$ receives resonant and nonresonant contributions:

$$\begin{aligned} \langle K^+(p_2)K^-(p_3)|\bar{s}s|0\rangle &\equiv f_s^{K^+K^-}(s_{23}) \\ &= \sum_i \frac{m_{f_{0i}}\bar{f}^s_{f_{0i}}g^{f_{0i}\to K^+K^-}}{m_{f_{0i}}^2 - s_{23} - im_{f_{0i}}\Gamma_{f_{0i}}} + f_s^{\rm NR}, \\ f_s^{\rm NR} &= \frac{v}{3}(3F_{\rm NR} + 2F'_{NR}) + \sigma_{\rm NR}e^{-\alpha s_{23}}, \end{aligned} \tag{2.18}$$

where f_{0i} denote the generic f_0-type scalar mesons, $f_{0i} = f_0(980), f_0(1370), f_0(1500), X_0(1550), \cdots$, the scalar decay constant $\bar{f}^s_{f_{0i}}$ is defined by $\langle f_{0i}|\bar{s}s|0\rangle = m_{f_{0i}}\bar{f}^s_{f_{0i}}$ [see Eq. (B1)], $g^{f_{0i}\to K^+K^-}$ is the $f_{0i} \to K^+K^-$ strong coupling, and the nonresonant terms are related to those in $F_s^{K^+K^-}$ through the equation of motion. The presence of the nonresonant $\sigma_{\rm NR}$ term will be explained shortly. The main scalar meson pole contributions are those that have dominant $s\bar{s}$ content and large coupling to $K\bar{K}$. We consider the scalar mesons $f_0(980)$ and $X_0(1550)$ [denoted as $f_X(1500)$ by Belle] which are supposed to have the largest couplings with the $K\bar{K}$ pair. Note that the nature of the broad state $X_0(1550)$ observed by *BABAR* and Belle, for example, what is its relation with $f_0(1500)$, is not clear. To proceed with the numerical calculations, we shall use $g^{f_0(980)\to K^+K^-} = 4.3$ GeV,[1] $g^{X_0(1550)\to K^+K^-} = 1.4$ GeV, $\Gamma_{f_0(980)} = 80$ MeV, $\Gamma_{X_0(1550)} = 0.257$ GeV [8], $\bar{f}_{f_0(980)}(\mu = m_b/2) \simeq 0.46$ GeV [46] and $\bar{f}_{f_0(1530)} \simeq 0.30$ GeV. The sign of the resonant terms is fixed by $f_s^{K^+K^-}(0) = v$ from a chiral perturbation theory calculation (see, for example, [47]). It should be stressed that although the nonresonant contributions to f_s^{KK} and F_s^{KK} are related through the equation of motion, the resonant ones are different and not related *a priori*. As stressed in [40], to apply the equation of motion, the form factors should be away from the resonant region. In the presence of the resonances, we thus need to introduce a nonresonant term characterized by the parameter $\sigma_{\rm NR}$ in Eq. (2.18) which will be specified later. The parameter α appearing in the same equation should be close to the value of $\alpha_{\rm NR}$ given in Eq. (2.9). We will use the experimental measurement $\alpha = (0.14 \pm 0.02)$ GeV^{-2} [16].

As noticed before, the matrix elements $\langle K^+K^-|(\bar{d}b)_{V-A}|\bar{B}^0\rangle$ and $\langle K^+K^-|\bar{d}(1-\gamma_5)b|\bar{B}^0\rangle$ are included in Eq. (A4) as they receive intermediate scalar pole contributions. More explicitly,

$$\begin{aligned} &\langle K^+(p_2)K^-(p_3)|(\bar{d}b)_{V-A}|\bar{B}^0\rangle^R \\ &\quad = \sum_i \frac{g^{f_{0i}\to K^+K^-}}{m_{f_{0i}}^2 - s_{23} - im_{f_{0i}}\Gamma_{f_{0i}}}\langle f_{0i}|(\bar{d}b)_{V-A}|\bar{B}^0\rangle. \end{aligned} \tag{2.19}$$

Hence,

$$\begin{aligned} &\langle \bar{K}^0(p_1)|(\bar{s}d)_{V-A}|0\rangle\langle K^+(p_2)K^-(p_3)|(\bar{d}b)_{V-A}|\bar{B}^0\rangle^R \\ &\quad = \sum_i \frac{g^{f_{0i}\to K^+K^-}}{m_{f_{0i}}^2 - s_{23} - im_{f_{0i}}\Gamma_{f_{0i}}} f_K F_0^{Bf_{0i}^d}(m_K^2)(m_B^2 - m_{f_{0i}}^2). \end{aligned} \tag{2.20}$$

The superscript u of the form factor $F_0^{Bf_{0i}^u}$ reminds us that it is the $u\bar{u}$ quark content that gets involved in the B to f_{0i}

[1] This is different from the coupling $g^{f_0(980)\to K^+K^-} = 1.5$ GeV originally employed in [40]. The coupling $g^{f_0(980)\to\pi^+\pi^-} \sim 1.33$ GeV can be fixed from a recent Belle measurement of $\Gamma(f_0(980) \to \pi^+\pi^-)$ [see Eq. (3.18)]. Using the BES result $(g^{f_0(980)\to KK}/g^{f_0(980)\to\pi\pi})^2 = 4.21 \pm 0.25 \pm 0.21$ [45], one can deduce that $g^{f_0(980)\to KK} = 2.7 \pm 0.6$ GeV. In this work, we found that a slightly large coupling $g^{f_0(980)\to KK}$ will give better numerical results.

form factor transition. In short, the relevant $f_0(980)$ pole contributions to $\bar{B}^0 \to K^+K^-\bar{K}^0$ are

$$\langle \bar{K}^0 K^+ K^- | T_p | \bar{B}^0 \rangle_{f_0} = \frac{g^{f_0(980)\to K^+K^-}}{m_{f_0}^2 - s_{23} - i m_{f_0}\Gamma_{f_0}} \times \left\{ 2\frac{m_{f_0}}{m_b} \bar{f}_{f_0}^s F_0^{BK}(m_{f_0}^2)(m_B^2 - m_K^2) \times \left(a_6^p - \frac{1}{2} a_8^p \right) + f_K F_0^{Bf_0^d}(m_K^2)(m_B^2 - m_{f_0}^2) \times \left[a_4^p - \frac{1}{2} a_{10}^p - \left(a_6^p - \frac{1}{2} a_8^p\right) r_\chi^K \right] \right\}, \tag{2.21}$$

where we have employed Eq. (2.18) and applied equations of motion to the matrix elements $\langle \bar{K}^0 | \bar{s}\gamma_5 d | 0 \rangle \times \langle K^+K^- | \bar{d}\gamma_5 b | \bar{B}^0 \rangle$. Comparing this equation with Eq. (A6) of [48], we see that the expression inside $\{\cdots\}$ is identical to that of $\bar{B}^0 \to f_0(980)\bar{K}^0$, as it should be.

We digress for a moment to discuss the wave function of the $f_0(980)$. What is the quark structure of the light scalar mesons below or near 1 GeV has been quite controversial. In this work we shall consider the conventional $q\bar{q}$ assignment for the $f_0(980)$. In the naive quark model, the flavor wave functions of the $f_0(980)$ and $\sigma(600)$ read

$$\sigma = \frac{1}{\sqrt{2}}(u\bar{u} + d\bar{d}), \qquad f_0 = s\bar{s}, \tag{2.22}$$

where the ideal mixing for f_0 and σ has been assumed. In this picture, $f_0(980)$ is purely an $s\bar{s}$ state. However, there also exists some experimental evidence indicating that $f_0(980)$ is not purely an $s\bar{s}$ state. First, the observation of $\Gamma(J/\psi \to f_0\omega) \approx \frac{1}{2}\Gamma(J/\psi \to f_0\phi)$ [4] clearly indicates the existence of the nonstrange and strange quark content in $f_0(980)$. Second, the fact that $f_0(980)$ and $a_0(980)$ have similar widths and that the f_0 width is dominated by $\pi\pi$ also suggests the composition of $u\bar{u}$ and $d\bar{d}$ pairs in $f_0(980)$; that is, $f_0(980) \to \pi\pi$ should not be OZI suppressed relative to $a_0(980) \to \pi\eta$. Therefore, isoscalars $\sigma(600)$ and f_0 must have a mixing

$$|f_0(980)\rangle = |s\bar{s}\rangle \cos\theta + |n\bar{n}\rangle \sin\theta, \quad |\sigma(600)\rangle = -|s\bar{s}\rangle \sin\theta + |n\bar{n}\rangle \cos\theta, \tag{2.23}$$

with $n\bar{n} \equiv (\bar{u}u + \bar{d}d)/\sqrt{2}$. Experimental implications for the $f_0 - \sigma$ mixing angle have been discussed in detail in [49]. It is found that θ lies in the ranges of $25° < \theta < 40°$ and $-40° < \theta < -15°$ (or $140° < \theta < 165°$). Note that the phenomenological analysis of the radiative decays $\phi \to f_0(980)\gamma$ and $f_0(980) \to \gamma\gamma$ favors a solution of the θ to be negative (or in the second quadrant). In this work, we shall use $\theta = -25°$.

Finally, the matrix elements involving 3-kaon creation are given by [38]

$$\langle \bar{K}^0(p_1) K^+(p_2) K^-(p_3) | (\bar{s}d)_{V-A} | 0 \rangle \langle 0 | (\bar{d}b)_{V-A} | \bar{B}^0 \rangle \approx 0,$$
$$\langle \bar{K}^0(p_1) K^+(p_2) K^-(p_3) | \bar{s}\gamma_5 d | 0 \rangle \langle 0 | \bar{d}\gamma_5 b | \bar{B}^0 \rangle = v \frac{f_B m_B^2}{f_\pi m_b} \left(1 - \frac{s_{13} - m_1^2 - m_3^2}{m_B^2 - m_K^2} \right) F^{KKK}(m_B^2), \tag{2.24}$$

where

$$v = \frac{m_{K^+}^2}{m_u + m_s} = \frac{m_K^2 - m_\pi^2}{m_s - m_d}, \tag{2.25}$$

characterizes the quark-order parameter $\langle \bar{q}q \rangle$ which spontaneously breaks the chiral symmetry. Both relations in Eq. (2.24) are originally derived in the chiral limit [38] and hence the quark masses appearing in Eq. (2.25) are referred to the scale $\sim$1 GeV. The first relation reflects helicity suppression which is expected to be even more effective for energetic kaons. For the second relation, we introduce the form factor F^{KKK} to extrapolate the chiral result to the physical region. Following [38] we shall take $F^{KKK}(q^2) = 1/[1 - (q^2/\Lambda_\chi^2)]$ with $\Lambda_\chi = 0.83$ GeV being a chiral-symmetry breaking scale.

To proceed with the numerical calculations, we need to specify the input parameters. The relevant CKM matrix elements, decay constants, form factors, the effective Wilson coefficients a_i^p and the running quark masses are collected in Appendix B. As for the parameter $\sigma_{\rm NR}$ in Eq. (2.18), in principle we can set its phase ϕ_σ to zero and use the measured $K_SK_SK_S$ rate, namely, $\mathcal{B}(\bar{B}^0 \to K_SK_SK_S) = (6.2 \pm 0.9) \times 10^{-6}$ [50], to fix the parameter $\sigma_{\rm NR}$ and then use the data obtained from the Dalitz plot analysis to determine the strong phases ϕ_r for resonant amplitudes. However, in doing so one needs the data of invariant mass spectra. In the absence of such information, instead we will treat ϕ_σ as a free parameter and do not assign any other strong phases to the resonant amplitudes except for those arising from the Breit-Wigner formalism. It turns out that if ϕ_σ is small, the K^+K^- mass spectrum in $\bar{B}^0 \to K^+K^-K_S$ will have a prominent hump at the invariant mass $m_{K^+K^-} = 3$ GeV, which is not seen experimentally [see Fig. 2(c)]. We found that $\phi_\sigma \approx \pi/4$ will yield K^+K^- mass spectrum consistent with the data

$$\sigma_{\rm NR} = e^{i\pi/4}(3.36^{+1.12}_{-0.96})\ {\rm GeV}. \tag{2.26}$$

Note that the phase of $\sigma_{\rm NR}$ is consistent with the *BABAR* measurement shown in Table II, namely, $\phi_\sigma^{BABAR} = 1.19 \pm 0.37$.

The calculated branching ratios of resonant and nonresonant contributions to $\bar{B}^0 \to K^+K^-\bar{K}^0$ are summarized in Table III. The theoretical errors shown there are from the uncertainties in (i) the parameter $\alpha_{\rm NR}$ which governs the momentum dependence of the nonresonant amplitude, (ii) the strange quark mass m_s, the form factor F_0^{BK} and the nonresonant parameter $\sigma_{\rm NR}$, and (iii) the unitarity angle γ.

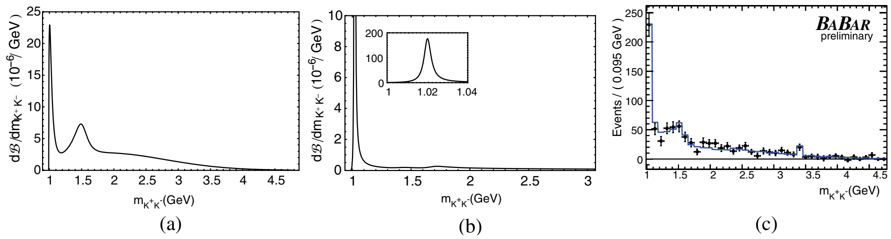

FIG. 2 (color online). The K^+K^- mass spectra for $\bar{B}^0 \to K^+K^-K_S$ decay from (a) CP-even and (b) CP-odd contributions. The inset in (b) is for the ϕ region. The full $K^+K^-K_S$ spectrum, which is the sum of CP-even and CP-odd parts, measured by *BABAR* [16] is depicted in (c).

In QCD calculations based on a heavy quark expansion, one faces uncertainties arising from power corrections such as annihilation and hard-scattering contributions. For example, in QCD factorization, there are large theoretical uncertainties related to the modelling of power corrections corresponding to weak annihilation effects and the chirally enhanced power corrections to hard spectator scattering. Even for two-body B decays, power corrections are of order (10–20)% for tree-dominated modes, but they are usually bigger than the central values for penguin-dominated decays. Needless to say, $1/m_b$ power corrections for three-body decays may well be larger. However, as stressed in the Introduction, in this exploratory work we use the phenomenological factorization model rather than in the established theories based on a heavy quark expansion. Consequently, uncertainties due to power corrections, at this stage, are not included in our calculations, by assumption. In view of such shortcomings we must emphasize that the additional errors due to such model dependent assumptions may be sizable.

From Table III we see that the predicted rates for resonant and nonresonant components are consistent with experiment within errors. The nonresonant contribution arises dominantly from the transition process (88%) via the scalar-density-induced vacuum to $K\bar{K}$ transition, namely, $\langle K^+K^-|\bar{s}s|0\rangle$, and slightly from the current-induced process (3%). Therefore, it is natural to conjecture that nonresonant decays could also play a prominent role in other penguin-dominated 3-body B decays.

TABLE III. Branching ratios (in units of 10^{-6}) of resonant and nonresonant (NR) contributions to $\bar{B}^0 \to K^+K^-\bar{K}^0$. Theoretical errors correspond to the uncertainties in (i) $\alpha_{\rm NR}$, (ii) m_s, F_0^{BK} and $\sigma_{\rm NR}$, and (iii) $\gamma = (59 \pm 7)^\circ$. We do not have $1/m_b$ power corrections within this model. However, systematic errors due to model dependent assumptions may be sizable and are not included in the error estimates that we give. Experimental results are taken from Table II.

Decay mode	*BABAR* [16]	Theory
$\phi\bar{K}^0$	2.98 ± 0.45	$2.6^{+0.0+0.5+0.0}_{-0.0-0.4-0.0}$
$f_0(980)\bar{K}^0$	9.57 ± 2.51	$5.8^{+0.0+0.1+0.0}_{-0.0-0.5-0.0}$
$X_0(1550)K^-$	0.98 ± 0.33	$0.93^{+0.00+0.16+0.00}_{-0.00-0.15-0.00}$
NR	26.7 ± 4.6	$18.1^{+0.6+5.1+0.2}_{-0.7-3.8-0.2}$
Total	$23.8 \pm 2.0 \pm 1.6$	$19.8^{+0.4+0.5+0.1}_{-0.4-0.4-0.2}$

The $K^+K^-K_S$ mode is an admixture of CP-even and CP-odd components. By excluding the major CP-odd contribution from ϕK_S, the 3-body $K^+K^-K_S$ final state is primarily CP even. The K^+K^- mass spectra of the $\bar{B}^0 \to K^+K^-K_S$ decay from CP-even and CP-odd contributions are shown in Fig. 2. For the CP-even spectrum, there are peaks at the threshold and $m_{K^+K^-} = 1.5$ GeV region. The threshold enhancement arises from the $f_0(980)K_S$ and the nonresonant $f_S^{K^+K^-}$ contributions [see Eq. (2.18)].[2] For the CP-odd spectrum, the peak on the lower end corresponds to the ϕK_S contribution, which is also shown in the inset. The $b \to u$ transition is governed by the current-induced process $\langle \bar{B}^0 \to K^+\bar{K}^0\rangle \times \langle 0 \to K^-\rangle$ [see Eq. (A4)]. From Eq. (2.8) it is clear that the $b \to u$ amplitude prefers a small invariant mass of K^+ and $\bar{K}^0$ and hence a large invariant mass of K^+ and K^-. In contrast, the $b \to s$ amplitude prefers a small s_{23}. Consequently, their interference is largely suppressed. The full $K^+K^-K_S$ spectrum, which is the sum of the CP-even and the CP-odd parts, has been measured by *BABAR* [Fig. 2(c)]. It clearly shows the phenomenon of threshold enhancement and the scalar resonances $X_0(1550)$ and χ_{c0}.

The decay $\bar{B}^0 \to K_SK_SK_S$ is a pure penguin-induced mode [cf. Eq. (A7)] and it receives intermediate pole contributions only from the isosinglet scalar mesons such as $f_0(980)$. Just like other KKK modes, this decay is governed by the nonresonant background dominated by the $\sigma_{\rm NR}$ term defined in Eq. (2.18). Hence, this mode is ideal for determining the unknown parameter $\sigma_{\rm NR}$ which is given in Eq. (2.26). Time-dependent CP violation in neu-

[2]In our previous work [40] we have argued that the spectrum should have a peak at the large $m_{K^+K^-}$ end. This is because we have introduced an additional nonresonant contribution to the ω_- parameter parametrized as $\omega_-^{NR} = \kappa \frac{2p_B \cdot p_2}{s_{12}^2}$ and employed the $B^- \to D^0K^0K^-$ data and applied isospin symmetry to the $\bar{B} \to K\bar{K}$ matrix elements to determine the unknown parameter κ. Since this nonresonant term favors a small $m_{K^+K_S}$ region, a peak of the spectrum at large $m_{K^+K^-}$ is thus expected. However, such a bump is not seen experimentally [16]. In this work we will no longer consider this term.

TABLE IV. Branching ratios for $\bar{B}^0 \to K^+K^-K_S$, $K_SK_SK_S$, $K_SK_SK_L$ decays and the fraction of CP-even contribution to $\bar{B}^0 \to K^+K^-K_S$, f_+. The branching ratio of CP-odd $K^+K^-K_S$ with ϕK_S excluded is shown in parentheses. Results for $(K^+K^-K_L)_{CP\pm}$ are identical to those for $(K^+K^-K_S)_{CP\mp}$. For theoretical errors, see Table III. Experimental results are taken from [50].

Final-state	$\mathcal{B}(10^{-6})_{\rm theory}$	$\mathcal{B}(10^{-6})_{\rm expt}$
$K^+K^-K_S$	$9.89^{+0.19+2.28+0.07}_{-0.21-1.81-0.08}$	12.4 ± 1.2
$(K^+K^-K_S)_{CP+}$	$8.33^{+0.10+1.82+0.05}_{-0.12-1.49-0.06}$	
$(K^+K^-K_S)_{CP-}$	$1.57^{+0.09+0.46+0.02}_{-0.10-0.32-0.02}$	
	$(0.14^{+0.06+0.14+0.01}_{-0.06-0.06-0.01})$	
$K_SK_SK_S$	Input	6.2 ± 0.9
$K_SK_SK_L$	$7.63^{+0.01+1.37+0.03}_{-0.01-1.19-0.03}$	<14
	$f_+^{\rm theory}$	$f_+^{\rm expt}$
$K^+K^-K_S$	$0.98^{+0.01+0.01+0.00}_{-0.01-0.02-0.00}$	0.91 ± 0.07
	$f_-^{\rm theory}$	
$K^+K^-K_L$	$0.98^{+0.01+0.01+0.00}_{-0.01-0.02-0.00}$	

tral 3-body decay modes with fixed CP parity was first discussed by Gershon and Hazumi [51].

Results for the decay rates and CP asymmetries in $\bar{B}^0 \to K^+K^-K_{S(L)}$, $K_SK_SK_{S(L)}$ are displayed in Tables IV and V, respectively. (For the decay amplitudes of $\bar{B}^0 \to K_SK_SK_{S(L)}$, see [40] for details.) The mixing-induced CP violations are defined by

$$S_{KKK,CP\pm} = \frac{2\int {\rm Im}(e^{-2i\beta}A_{CP\pm}\bar{A}^*_{CP\pm})ds_{12}ds_{23}}{\int |A_{CP\pm}|^2 ds_{12}ds_{23} + \int |\bar{A}_{CP\pm}|^2 ds_{12}ds_{23}},$$
$$S_{KKK} = \frac{2\int {\rm Im}(e^{-2i\beta}A\bar{A}^*)ds_{12}ds_{23}}{\int |A|^2 ds_{12}ds_{23} + \int |\bar{A}|^2 ds_{12}ds_{23}} = f_+ S_{KKK,CP+} + (1-f_+)S_{KKK,CP-}, \quad (2.27)$$

where A is the decay amplitude of $\bar{B}^0 \to K^+K^-K_{S(L)}$ or $K_SK_SK_{S(L)}$ and $\bar{A}$ is the conjugated B^0 decay amplitude, and f_+ is the CP even fraction defined by

$$f_+ \equiv \frac{\Gamma_{CP+} + \bar{\Gamma}_{CP+}}{\Gamma + \bar{\Gamma}}\Big|_{\phi K_S {\rm excluded}}. \quad (2.28)$$

Generally, it is more convenient to define an effective $\sin 2\beta$ via $S_f \equiv -\eta_f \sin 2\beta_{\rm eff}$ with $\eta_f = 2f_+ - 1$ for $K^+K^-K_S$. The predicted value of f_+ is consistent with the data but it is on the higher end of the experimental measurement because the CP-odd contributions from the vector mesons $\rho, \omega, \ldots,$ are OZI suppressed and the CP-odd nonresonant contribution is constrained by the $\pi^+\pi^-\pi^-$ rate.

The deviation of the mixing-induced CP asymmetry in $B^0 \to K^+K^-K_S$ and $K_SK_SK_S$ from that measured in $B \to \phi_{c\bar{c}}K_S$, i.e. $\sin 2\beta_{\phi_{c\bar{c}}K_S} = 0.681 \pm 0.025$ [50], namely, $\Delta \sin 2\beta_{\rm eff} \equiv \sin 2\beta_{\rm eff} - \sin 2\beta_{\phi_{c\bar{c}}K_S}$, is calculated from

TABLE V. Mixing-induced and direct CP asymmetries $\sin 2\beta_{\rm eff}$ (top) and A_f (in %, bottom), respectively, in $\bar{B}^0 \to K^+K^-K_S$ and $K_SK_SK_S$ decays. Experimental results for $K^+K^-K_S$ and $K^+K^-K_L$ modes are obtained from the data of $\bar{B}^0 \to K^+K^-\bar{K}^0$. Results for $(K^+K^-K_L)_{CP\pm}$ are identical to those for $(K^+K^-K_S)_{CP\mp}$. For theoretical errors, see Table III. Experimental results are taken from [50].

Final state	$\sin 2\beta_{\rm eff}$	Expt.
$(K^+K^-K_S)_{\phi K_S {\rm excluded}}$	$0.728^{+0.001+0.002+0.009}_{-0.002-0.001-0.020}$	0.73 ± 0.10
$(K^+K^-K_S)_{CP+}$	$0.732^{+0.003+0.006+0.009}_{-0.004-0.004-0.020}$	
$(K^+K^-K_L)_{\phi K_L {\rm excluded}}$	$0.728^{+0.001+0.002+0.009}_{-0.002-0.001-0.020}$	0.73 ± 0.10
$K_SK_SK_S$	$0.719^{+0.000+0.000+0.008}_{-0.000-0.000-0.019}$	0.58 ± 0.20
$K_SK_SK_L$	$0.718^{+0.000+0.000+0.008}_{-0.000-0.000-0.019}$	
	A_f (%)	Expt.
$(K^+K^-K_S)_{\phi K_S {\rm excluded}}$	$-4.63^{+1.35+0.53+0.40}_{-1.01-0.54-0.34}$	-7 ± 8
$(K^+K^-K_S)_{CP+}$	$-4.86^{+1.43+0.52+0.42}_{-1.09-0.55-0.35}$	
$(K^+K^-K_L)_{\phi K_L {\rm excluded}}$	$-4.63^{+1.35+0.53+0.40}_{-1.01-0.54-0.34}$	-7 ± 8
$K_SK_SK_S$	$0.69^{+0.01+0.01+0.05}_{-0.01-0.01-0.06}$	14 ± 15
$K_SK_SK_L$	$0.77^{+0.01+0.01+0.05}_{-0.01-0.03-0.07}$	

Table V to be

$$\Delta \sin 2\beta_{K^+K^-K_S} = 0.047^{+0.028}_{-0.033}, \quad \Delta \sin 2\beta_{K_SK_SK_S} = 0.038^{+0.027}_{-0.032}. \quad (2.29)$$

The corresponding experimental values are 0.049 ± 0.10 and -0.101 ± 0.20, respectively. Because of the presence of color-allowed tree contributions in $\bar{B}^0 \to K^+K^-K_S$, it is naively expected that this penguin-dominated mode is subject to a potentially significant tree pollution and hence $\Delta \sin 2\beta_{\rm eff}$ can be as large as $\mathcal{O}(10\%)$. However, our calculation indicates the deviation of the mixing-induced CP asymmetry in $\bar{B}^0 \to K^+K^-K_S$ from that measured in $\bar{B}^0 \to \phi_{c\bar{c}}K_S$ is very similar to that of the $K_SK_SK_S$ mode as the tree pollution effect in the former is somewhat washed out. Nevertheless, direct CP asymmetry of the former, being of order -4%, is more prominent than the latter.[3]

B. $B^- \to KKK$ decays

The $B^- \to K^+K^-K^-$ decay amplitude has a similar expression as Eq. (A4) except that one also needs to add the contributions from the interchange $s_{23} \to s_{12}$ and put a factor of $1/2$ in the decay rate to account for the identical particle effect.

[3]In our previous work [40], $\Delta \sin 2\beta_{\rm eff}$ is found to be

$$\Delta \sin 2\beta_{K^+K^-K_S} = 0.06^{+0.09}_{-0.04}, \qquad \Delta \sin 2\beta_{K_SK_SK_S} = 0.06^{+0.03}_{-0.04},$$

for $\sin 2\beta_{J/\psi K_S} = 0.687 \pm 0.032$, while direct CP asymmetry is less than 1% in both modes. Note that due to an oversight the experimental error bars were not included in our previous paper for the theoretical calculation of $\Delta \sin 2\beta_{\rm eff}$.

TABLE VI. Branching ratios (in units of 10^{-6}) of resonant and nonresonant (NR) contributions to $B^- \to K^+K^-K^-$. For theoretical errors, see Table III.

Decay mode	*BABAR* [8]	Belle [9]	Theory
ϕK^-	$4.14 \pm 0.32 \pm 0.33$	$4.72 \pm 0.45 \pm 0.35^{+0.39}_{-0.22}$	$2.9^{+0.0+0.5+0.0}_{-0.0-0.5-0.0}$
$f_0(980)K^-$	$6.5 \pm 2.5 \pm 1.6$	<2.9	$7.0^{+0.0+0.4+0.1}_{-0.0-0.7-0.1}$
$X_0(1550)K^-$	$43 \pm 6 \pm 3$		$1.1^{+0.0+0.2+0.0}_{-0.0-0.2-0.0}$
$f_0(1710)K^-$	$1.7 \pm 1.0 \pm 0.3$		
NR	$50 \pm 6 \pm 4$	$24.0 \pm 1.5 \pm 1.8^{+1.9}_{-5.7}$	$25.3^{+0.9+4.8+0.3}_{-1.0-4.4-0.3}$
Total	$35.2 \pm 0.9 \pm 1.6$	$32.1 \pm 1.3 \pm 2.4$	$25.5^{+0.5+4.4+0.2}_{-0.6-4.1-0.2}$

TABLE VII. Branching ratios (in units of 10^{-6}) of resonant and nonresonant (NR) contributions to $B^- \to K^-K_SK_S$. For theoretical errors, see Table III.

Decay mode	$f_0(980)K^-$	$X_0(1550)K^-$	NR	Total
Theory	$5.2^{+0.0+0.3+0.1}_{-0.0-0.5-0.1}$	$0.92^{+0.00+0.16+0.00}_{-0.00-0.15-0.00}$	$12.4^{+0.2+2.1+0.1}_{-0.3-2.0-0.1}$	$12.2^{+0.0+1.5+0.0}_{-0.0-1.7-0.0}$
Expt.				11.5 ± 1.3

Branching ratios of resonant and nonresonant contributions to $B^- \to K^+K^-K^-$ are shown in Table VI. It is clear that the predicted rates of resonant and nonresonant components are consistent with the data except for the broad scalar resonance $X_0(1550)$. Both *BABAR* and Belle have seen a large fraction from $X_0(1550)$, $(121 \pm 19 \pm 6)\%$ by *BABAR* [8] and $(63.4 \pm 6.9)\%$ by Belle [9],[4] while our prediction is similar to that in $\bar{B}^0 \to K^+K^-\bar{K}^0$. It is not clear why there is a huge disparity between $B^- \to K^+K^-K^-$ and $\bar{B}^0 \to K^+K^-\bar{K}^0$ as far as the $X_0(1550)$ contribution is concerned. Obviously, a refined measurement of the $X_0(1550)$ contribution to the $K^+K^-K^-$ mode is urgently needed in order to clarify this issue. Our result for the nonresonant contribution is in good agreement with Belle, but disagrees with *BABAR*. Notice that Belle did not see the scalar resonance $f_0(980)$ as Belle employed the E791 result [52] for $g^{f_0\to K\bar{K}}$ which is smaller than $g^{f_0\to\pi\pi}$. In contrast to E791, the ratio $g^{f_0\to K\bar{K}}/g^{f_0\to\pi\pi}$ is measured to be larger than 4 in the existing e^+e^- experiments [45,53]

We next turn to the decay $B^- \to K^-K_SK_S$. Following [54], let us consider the symmetric state of $K^0\bar{K}^0$

$$|K^0\bar{K}^0\rangle_{\rm sym} \equiv [|K^0(p_1)\bar{K}^0(p_2)\rangle + |\bar{K}^0(p_1)K^0(p_2)\rangle]/\sqrt{2} = [|K_S(p_1)K_S(p_2)\rangle - |K_L(p_1)K_L(p_2)\rangle]/\sqrt{2}. \tag{2.30}$$

Hence,

$$\mathcal{B}(B^- \to K^-K_SK_S) = \frac{1}{2}[\mathcal{B}(B^- \to K^-K_SK_S) + \mathcal{B}(B^- \to K^-K_LK_L)] = \frac{1}{2}\mathcal{B}(B^- \to K^-(K^0\bar{K}^0)_{\rm sym}). \tag{2.31}$$

The factorizable amplitude of $B^- \to K^-K^0\bar{K}^0$ is given by Eq. (A8). Just as other KKK modes, this decay is also expected to be dominated by the nonresonant contribution (see Table VII). The calculated total rate is in good agreement with experiment. Just as the pure penguin mode $K_SK_SK_S$, the decay $B^- \to K^-K_SK_S$ also can be used to constrain the nonresonant parameter $\sigma_{\rm NR}$.

As pointed out in [54], isospin symmetry implies the relation

$$A(B^- \to K^-K^0\bar{K}^0) = -A(\bar{B}^0 \to \bar{K}^0K^+K^-). \tag{2.32}$$

This leads to

$$\mathcal{B}(B^- \to K^-(K^0\bar{K}^0)_{\rm sym}) = \frac{\tau(B^-)}{\tau(B^0)}\mathcal{B}(\bar{B}^0 \to K^+K^-\bar{K}^0)_{\phi K\,{\rm excluded}}. \tag{2.33}$$

Experimentally, this relation is well satisfied: lhs $= (23.0 \pm 2.6) \times 10^{-6}$ and rhs $= (22.1 \pm 2.1) \times 10^{-6}$. Hence, the isospin relation Eq. (2.32) is well respected.

III. $B \to K\pi\pi$ DECAYS

In this section we shall consider five $B \to K\pi\pi$ decays, namely, $B^- \to K^-\pi^+\pi^-$, $\bar{K}^0\pi^-\pi^0$, $\bar{B}^0 \to K^-\pi^+\pi^0$, $\bar{K}^0\pi^+\pi^-$ and $\bar{K}^0\pi^0\pi^0$. They are dominated by $b \to s$ penguin transition and consist of three decay processes: (i) the current-induced process, $\langle B \to \pi\pi\rangle \times \langle 0 \to K\rangle$,

[4] Belle [9] actually found two solutions for the fraction of $X_0(1550)K^-$: $(63.4 \pm 6.9)\%$ and $(8.21 \pm 1.94)\%$. The first solution is preferred by Belle.

(ii) the transition processes, $\langle B\to\pi\rangle\times\langle 0\to\pi K\rangle$, and $\langle B\to K\rangle\times\langle 0\to\pi\pi\rangle$, and (iii) the annihilation process $\langle B\to 0\rangle\times\langle 0\to K\pi\pi\rangle$.

The factorizable amplitudes for $B^-\to K^-\pi^+\pi^-$, $\bar K^0\pi^-\pi^0$, $\bar B^0\to K^-\pi^+\pi^0$, $\bar K^0\pi^+\pi^-$ and $\bar K^0\pi^0\pi^0$ are given in Eqs. (A10)–(A14), respectively. All five channels have the three-body matrix element $\langle\pi\pi|(\bar q b)_{V-A}|B\rangle$ which has the similar expression as Eqs. (2.3) and (2.4) except that the pole B_s^* is replaced by B^* and the kaon is replaced by the pion. However, there are additional resonant contributions to this three-body matrix element due to the intermediate vector ρ and scalar f_0 mesons

$$\langle\pi^+(p_2)\pi^-(p_3)|(\bar u b)_{V-A}|B^-\rangle^R=\sum_i\frac{g^{\rho_i^0\to\pi^+\pi^-}}{m_{\rho_i}^2-s_{23}-im_{\rho_i}\Gamma_{\rho_i}}\sum_{\rm pol}\varepsilon^*\cdot(p_2-p_3)\langle\rho_i^0|(\bar u b)_{V-A}|B^-\rangle+\sum_i\frac{g^{f_{0i}\to\pi^+\pi^-}}{m_{f_{0i}}^2-s_{23}-im_{f_{0i}}\Gamma_{f_{0i}}}\langle f_{0i}|(\bar u b)_{V-A}|B^-\rangle, \tag{3.1}$$

where ρ_i denote generic ρ-type vector mesons, e.g. $\rho=\rho(770),\rho(1450),\rho(1700),\ldots$. Applying Eqs. (B1) and (B6) we are led to

$$\begin{aligned}\langle\pi^+(p_2)\pi^-(p_3)|(\bar u b)_{V-A}|B^-\rangle^R\langle K^-(p_1)|(\bar s u)_{V-A}|0\rangle&=\sum_i\frac{f_K}{2}\frac{g^{\rho_i^0\to\pi^+\pi^-}}{m_{\rho_i^2}-s_{23}-im_{\rho_i}\Gamma_{\rho_i}}(s_{12}-s_{13})\\&\quad\times\bigg[(m_B+m_{\rho_i})A_1^{B\rho_i}(q^2)-\frac{A_2^{B\rho_i}(q^2)}{m_B+m_{\rho_i}}(s_{12}+s_{13}-3m_\pi^2)\\&\quad-2m_{\rho_i}[A_3^{B\rho_i}(q^2)-A_0^{B\rho_i}(q^2)]\bigg]\\&\quad+\sum_i\frac{f_K g^{f_{0i}\to\pi^+\pi^-}}{m_{f_{0i}}^2-s_{23}-im_{f_{0i}}\Gamma_{f_{0i}}}(m_B^2-m_{f_{0i}}^2)F_0^{Bf_0^u}(q^2).\end{aligned} \tag{3.2}$$

Likewise, the 3-body matrix element $\langle K^-\pi^+|(\bar s b)_{V-A}|\bar B^0\rangle$ appearing in $\bar B^0\to K^-\pi^+\pi^0$ also receives the following resonant contributions

$$\langle K^-(p_1)\pi^+(p_2)|(\bar s b)_{V-A}|\bar B^0\rangle^R=\sum_i\frac{g^{K_i^*\to K^-\pi^+}}{m_{K_i^*}^2-s_{12}-im_{K_i^*}\Gamma_{K_i^*}}\sum_{\rm pol}\varepsilon^*\cdot(p_1-p_2)\langle\bar K_i^{*0}|(\bar s b)_{V-A}|\bar B^0\rangle, \tag{3.3}$$

with $K_i^*=K^*(892),K^*(1410),K^*(1680),\ldots$.

For the two-body matrix elements $\langle\pi^+K^-|(\bar s d)_{V-A}|0\rangle$, $\langle\pi^+\pi^-|(\bar u u)_{V-A}|0\rangle$ and $\langle\pi^+\pi^-|\bar s s|0\rangle$, we note that

$$\begin{aligned}\langle K^-(p_1)\pi^+(p_2)|(\bar s d)_{V-A}|0\rangle&=\langle\pi^+(p_2)|(\bar s d)_{V-A}|K^+(-p_1)\rangle\\&=(p_1-p_2)_\mu F_1^{K\pi}(s_{12})+\frac{m_K^2-m_\pi^2}{s_{12}}(p_1+p_2)_\mu[-F_1^{K\pi}(s_{12})+F_0^{K\pi}(s_{12})],\end{aligned} \tag{3.4}$$

where we have taken into account the sign flip arising from interchanging the operators $s\leftrightarrow d$. Hence,

$$\begin{aligned}\langle K^-(p_1)\pi^+(p_2)|(\bar s d)_{V-A}|0\rangle\langle\pi^-(p_3)|(\bar d b)_{V-A}|B^-\rangle&=F_1^{B\pi}(s_{12})F_1^{K\pi}(s_{12})\bigg[s_{23}-s_{13}-\frac{(m_B^2-m_\pi^2)(m_K^2-m_\pi^2)}{s_{12}}\bigg]\\&\quad+F_0^{B\pi}(s_{12})F_0^{K\pi}(s_{12})\frac{(m_B^2-m_\pi^2)(m_K^2-m_\pi^2)}{s_{12}}.\end{aligned} \tag{3.5}$$

However, the form factor F_1 also receives resonant contributions

$$\sum_i\bigg(A^\mu_{K_i^*\pi K}\frac{1}{m_{K_i^*}^2-s_{12}-im_{K_i^*}\Gamma_{K_i^*}}m_{K_i^*}f_{K_i^*}+\frac{g^{K_{0i}^*\to K\pi}}{m_{K_{0i}^*}^2-s_{12}-im_{K_{0i}^*}\Gamma_{K_{0i}^*}}f_{K_{0i}^*}(p_1-p_2)_\mu\bigg), \tag{3.6}$$

with

$$\varepsilon^*_\mu A^\mu_{K^*\pi K} = \langle K^-(p_1)\pi^+(p_2)|K^*\rangle = g^{K^*\to\pi K}\varepsilon^*\cdot(p_1-p_2), \tag{3.7}$$

where $K^*_{0i} = K^*_0(1430), \ldots$. Hence, the resonant contributions to the form factor $F_1^{K\pi}$ are

$$F^{K\pi}_{1,R}(s) = \sum_i\left(\frac{m_{K^*_i} f_{K^*_i} g^{K^*_i\to K\pi}}{m^2_{K^*_i} - s - i m_{K^*_i}\Gamma_{K^*_i}} + \frac{f_{K^*_{0i}} g^{K^*_{0i}\to K\pi}}{m^2_{K^*_{0i}} - s - i m_{K^*_{0i}}\Gamma_{K^*_{0i}}}\right). \tag{3.8}$$

In principle, the weak vector form factor $F^{\pi^+\pi^-}$ defined by

$$\langle\pi^+(p_{\pi^+})\pi^-(p_{\pi^-})|\bar u\gamma_\mu u|0\rangle = (p_{\pi^+} - p_{\pi^-})_\mu F^{\pi^+\pi^-}, \tag{3.9}$$

can be related to the timelike pion electromagnetic form factors. However, unlike the kaon case, the timelike e.m. form factors of the pions are not well measured enough allowing us to determine the resonant and nonresonant parts. Therefore, we shall only consider the resonant part which has the expression

$$F^{\pi\pi}_R(s) = \sum_i \frac{m_{\rho_i} f_{\rho_i} g^{\rho_i\to\pi\pi}}{m^2_{\rho_i} - s - i m_{\rho_i}\Gamma_{\rho_i}}. \tag{3.10}$$

Following Eq. (2.18), the relevant matrix elements of scalar densities read

$$\langle\pi^+(p_2)\pi^-(p_3)|\bar s s|0\rangle = \sum_i \frac{m_{f_{0i}} \bar f^s_{f_{0i}} g^{f_{0i}\to\pi^+\pi^-}}{m^2_{f_{0i}} - s_{23} - i m_{f_{0i}}\Gamma_{f_{0i}}} + \langle\pi^+(p_2)\pi^-(p_3)|\bar s s|0\rangle^{\rm NR}, \tag{3.11}$$

and

$$\langle K^-(p_1)\pi^+(p_2)|\bar s d|0\rangle = \sum_i \frac{m_{K^*_{0i}} \bar f_{K^*_{0i}} g^{K^*_{0i}\to K^-\pi^+}}{m^2_{K^*_{0i}} - s_{12} - i m_{K^*_{0i}}\Gamma_{K^*_{0i}}} + \langle K^-(p_1)\pi^+(p_2)|\bar s d|0\rangle^{\rm NR}. \tag{3.12}$$

Note that for the scalar meson, the decay constants f_S and $\bar f_S$ are defined in Eq. (B1) and they are related via Eq. (B2). The nonresonant contribution $\langle\pi^+(p_2)\pi^-(p_3)|\bar s s|0\rangle^{\rm NR}$ vanishes under the OZI rule, while under SU(3) symmetry[5]

$$\langle K^-(p_1)\pi^+(p_2)|\bar s d|0\rangle^{\rm NR} = \langle K^+(p_1)K^-(p_2)|\bar s s|0\rangle^{\rm NR} = f_s^{\rm NR}(s_{12}), \tag{3.13}$$

with the expression of $f_s^{\rm NR}$ given in Eq. (2.18).

It is known that in the narrow width approximation, the 3-body decay rate obeys the factorization relation

$$\Gamma(B\to RP\to P_1P_2P) = \Gamma(B\to RP)\mathcal{B}(R\to P_1P_2), \tag{3.14}$$

with R being a resonance. This means that the amplitudes $A(B\to RP\to P_1P_2P)$ and $A(B\to RP)$ should have the same expressions apart from some factors. Hence, using the known results for quasi-two-body decay amplitude $A(B\to RP)$, one can have a cross check on the three-body decay amplitude of $B\to RP\to P_1P_2P$. For example, from Eq. (A12) we obtain the factorizable amplitude $A(\bar B^0\to K^{*0}_0(1430)\pi^0; K^{*0}_0(1430)\to K^-\pi^+)$ as

$$\langle K^-(p_1)\pi^+(p_2)\pi^0(p_3)|T_p|\bar B^0\rangle_{K^{*0}_0(1430)} = \frac{1}{\sqrt2}\frac{g^{K^{*0}_0(1430)\to K^-\pi^+}}{m^2_{K^*_0} - s_{12} - i m_{K^*_0}\Gamma_{K^*_0}} \times\left\{\left(-a^p_4 + r^{K^*_0}_\chi a^p_6 + \frac12(a^p_{10} - r^{K^*_0}_\chi a^p_8)\right) f_{K^*_0} F^{B\pi}_0(m^2_{K^*_0})(m^2_B - m^2_\pi) - \left[a_2\delta_{pu} + \frac32(a_9 - a_7)\right] f_\pi F^{BK^*_0}_0(m^2_\pi)(m^2_B - m^2_{K^*_0})\right\}, \tag{3.15}$$

where

$$r^{K^*_0}_\chi(\mu) = \frac{2m^2_{K^*_0}}{m_b(\mu)(m_s(\mu) - m_q(\mu))}. \tag{3.16}$$

The expression inside $\{\cdots\}$ is indeed the amplitude of $\bar B^0\to K^{*0}_0(1430)\pi^0$ given in Eq. (A6) of [48].

The strong coupling constants such as $g^{\rho\to\pi^+\pi^-}$ and $g^{f_0(980)\to\pi^+\pi^-}$ are determined from the measured partial widths through the relations

$$\Gamma_S = \frac{p_c}{8\pi m^2_S} g^2_{S\to P_1P_2}, \qquad \Gamma_V = \frac23\frac{p^3_c}{4\pi m^2_V} g^2_{V\to P_1P_2}, \tag{3.17}$$

[5]The matrix elements of scalar densities can be generally decomposed into D-, F- and S (singlet)–type components. Assuming that the singlet component is OZI suppressed, SU(3) symmetry leads to, for example, the relation $\langle K\pi|\bar s q|0\rangle^{\rm NR} = \langle K\bar K|\bar s s|0\rangle^{\rm NR}$.

TABLE VIII. Branching ratios (in units of 10^{-6}) of resonant and nonresonant (NR) contributions to $B^- \to K^- \pi^+ \pi^-$. For theoretical errors, see Table III.

Decay mode	*BABAR* [6]	Belle [7]	Theory
$\bar{K}^{*0}\pi^-$	$9.04 \pm 0.77 \pm 0.53^{+0.21}_{-0.37}$	$6.45 \pm 0.43 \pm 0.48^{+0.25}_{-0.35}$	$3.0^{+0.0+0.8+0.0}_{-0.0-0.7-0.0}$
$\bar{K}_0^{*0}(1430)\pi^-$	$34.4 \pm 1.7 \pm 1.8^{+0.1}_{-1.4}$	$32.0 \pm 1.0 \pm 2.4^{+1.1}_{-1.9}$	$10.5^{+0.0+3.2+0.0}_{-0.0-2.7-0.1}$
$\rho^0 K^-$	$5.08 \pm 0.78 \pm 0.39^{+0.22}_{-0.66}$	$3.89 \pm 0.47 \pm 0.29^{+0.32}_{-0.29}$	$1.3^{+0.0+1.9+0.1}_{-0.0-0.7-0.1}$
$f_0(980)K^-$	$9.30 \pm 0.98 \pm 0.51^{+0.27}_{-0.72}$	$8.78 \pm 0.82 \pm 0.65^{+0.55}_{-1.64}$	$7.7^{+0.0+0.4+0.1}_{-0.0-0.8-0.1}$
NR	$2.87 \pm 0.65 \pm 0.43^{+0.63}_{-0.25}$	$16.9 \pm 1.3 \pm 1.3^{+1.1}_{-0.9}$	$18.7^{+0.5+11.0+0.2}_{-0.6-6.3-0.2}$
Total	$64.4 \pm 2.5 \pm 4.6$	$48.8 \pm 1.1 \pm 3.6$	$45.0^{+0.3+16.4+0.1}_{-0.4-10.5-0.1}$

for scalar and vector mesons, respectively, where p_c is the c.m. momentum. The numerical results are

$$g^{\rho\to\pi^+\pi^-} = 6.0, \qquad g^{K^*\to K^+\pi^-} = 4.59,$$
$$g^{f_0(980)\to\pi^+\pi^-} = 1.33^{+0.29}_{-0.26}\ \text{GeV}, \tag{3.18}$$
$$g^{K_0^*\to K^+\pi^-} = 3.84\ \text{GeV}.$$

In determining the coupling of $f_0 \to \pi^+\pi^-$, we have used the partial width

$$\Gamma(f_0(980) \to \pi^+\pi^-) = (34.2^{+13.9+8.8}_{-11.8-2.5})\ \text{MeV} \tag{3.19}$$

measured by Belle [55]. The momentum dependence of the weak form factor $F^{K\pi}(q^2)$ is parametrized as

$$F^{K\pi}(q^2) = \frac{F^{K\pi}(0)}{1 - q^2/\Lambda_\chi^2 + i\Gamma_R/\Lambda_\chi}, \tag{3.20}$$

where $\Lambda_\chi \approx 830$ MeV is the chiral-symmetry breaking scale [47] and Γ_R is the width of the relevant resonance, which is taken to be 200 MeV [38].

The results of the calculation are summarized in Tables VIII, IX, X, XI, and XII. We see that except for $f_0(980)K$, the predicted rates for $K^*\pi$, $K_0^*(1430)\pi$ and ρK are smaller than the data. Indeed, the predictions based on QCD factorization for these decays are also generally smaller than experiment by a factor of $2 \sim 5$. This will be discussed in more detail in Sec. VI.

While Belle has found a sizable fraction of order $(35 \sim 40)\%$ for the nonresonant signal in $K^-\pi^+\pi^-$ and $\bar{K}^0\pi^+\pi^-$ modes (see Table I), *BABAR* reported a small fraction of order 4.5% in $K^-\pi^+\pi^-$. The huge disparity between *BABAR* and Belle is ascribed to the different parametrizations adopted by both groups. *BABAR* [6] used the LASS parametrization to describe the $K\pi$ S-wave and the nonresonant component by a single amplitude suggested by the LASS Collaboration to describe the scalar amplitude in elastic $K\pi$ scattering. As commented in [7], while this approach is experimentally motivated, the use of the LASS parametrization is limited to the elastic region of $M(K\pi) \lesssim 2.0$ GeV, and an additional amplitude is still required for a satisfactory description of the data. In our calculations we have taken into account the nonresonant contributions to the two-body matrix elements of scalar densities, $\langle K\pi|\bar{s}q|0\rangle$. Recall that a large nonresonant contribution from $\langle K\bar{K}|\bar{s}s|0\rangle$ is needed in order to explain

TABLE IX. Same as Table VIII except for the decay $B^- \to \bar{K}^0\pi^-\pi^0$.

Decay mode	Theory	Decay mode	Theory
$K^{*-}\pi^0$	$1.5^{+0.0+0.3+0.2}_{-0.0-0.3-0.2}$	$\bar{K}^{*0}\pi^-$	$1.5^{+0.0+0.4+0.0}_{-0.0-0.3-0.0}$
$K_0^{*-}(1430)\pi^0$	$5.5^{+0.0+1.6+0.1}_{-0.0-1.4-0.1}$	$\bar{K}_0^{*0}(1430)\pi^-$	$5.2^{+0.0+1.6+0.0}_{-0.0-1.4-0.0}$
$\rho^-\bar{K}^0$	$1.3^{+0.0+3.0+0.0}_{-0.0-0.9-0.0}$	NR	$10.0^{+0.2+7.1+0.0}_{-0.2-3.7-0.0}$
Total	$27.0^{+0.3+15.4+0.2}_{-0.2-8.8-0.2}$		

TABLE X. Same as Table VIII except for the decay $\bar{B}^0 \to \bar{K}^0\pi^+\pi^-$.

Decay mode	Belle [13]	Theory
$K^{*-}\pi^+$	$5.6 \pm 0.7 \pm 0.5^{+0.4}_{-0.3}$	$2.1^{+0.0+0.5+0.3}_{-0.0-0.5-0.3}$
$K_0^{*-}(1430)\pi^+$	$30.8 \pm 2.4 \pm 2.4^{+0.8}_{-3.0}$	$10.1^{+0.0+2.9+0.1}_{-0.0-2.5-0.2}$
$\rho^0\bar{K}^0$	$6.1 \pm 1.0 \pm 0.5^{+1.0}_{-1.1}$	$2.0^{+0.0+1.9+0.1}_{-0.0-0.9-0.1}$
$f_0(980)\bar{K}^0$	$7.6 \pm 1.7 \pm 0.7^{+0.5}_{-0.7}$	$7.7^{+0.0+0.4+0.0}_{-0.0-0.7-0.0}$
NR	$19.9 \pm 2.5 \pm 1.6^{+0.7}_{-1.2}$	$15.6^{+0.1+8.3+0.0}_{-0.1-4.9-0.0}$
Total	$47.5 \pm 2.4 \pm 3.7$	$42.0^{+0.3+15.7+0.0}_{-0.2-10.8-0.0}$

TABLE XI. Branching ratios (in units of 10^{-6}) of resonant and nonresonant (NR) contributions to $\bar{B}^0 \to K^-\pi^+\pi^0$. Note that the branching ratios for $K^{*-}\pi^+$ and $\bar{K}^{*0}\pi^0$ given in [14,15] are their absolute ones. We have converted them into the product branching ratios, namely, $\mathcal{B}(B \to Rh) \times \mathcal{B}(R \to hh)$. For theoretical errors, see Table III.

Decay mode	*BABAR* [14]	Belle [15]	Theory
$K^{*-}\pi^+$	$3.6 \pm 0.8 \pm 0.5$	$4.9^{+1.5+0.5+0.8}_{-1.5-0.3-0.3}$	$1.0^{+0.0+0.3+0.1}_{-0.0-0.3-0.1}$
$\bar{K}^{*0}\pi^0$	$2.0 \pm 0.6 \pm 0.3$	<2.3	$1.0^{+0.0+0.3+0.2}_{-0.0-0.2-0.1}$
$K_0^{*-}(1430)\pi^+$	$11.2 \pm 1.5 \pm 3.5$	$5.1 \pm 1.5^{+0.6}_{-0.7}$	$5.0^{+0.0+1.5+0.1}_{-0.0-1.3-0.1}$
$\bar{K}_0^{*0}(1430)\pi^0$	$7.9 \pm 1.5 \pm 2.7$	$6.1^{+1.6+0.5}_{-1.5-0.6}$	$4.2^{+0.0+1.4+0.0}_{-0.0-1.2-0.0}$
ρ^+K^-	$8.6 \pm 1.4 \pm 1.0$	$15.1^{+3.4+1.4+2.0}_{-3.3-1.5-2.1}$	$2.5^{+0.0+3.6+0.2}_{-0.0-1.4-0.2}$
NR	<4.6	$5.7^{+2.7+0.5}_{-2.5-0.4} < 9.4$	$9.6^{+0.3+6.6+0.0}_{-0.2-3.5-0.0}$
Total	$34.9 \pm 2.1 \pm 3.9$	$36.6^{+4.2}_{-4.1} \pm 3.0$	$28.9^{+0.2+16.1+0.2}_{-0.2-9.4-0.2}$

TABLE XII. Same as Table VIII except for the decay $\bar{B}^0 \to \bar{K}^0\pi^0\pi^0$.

Decay mode	$f_0(980)\bar{K}^0$	$\bar{K}^{*0}\pi^0$	$\bar{K}_0^{*0}(1430)\pi^0$	NR	Total
Theory	$3.8^{+0.0+2.0+0.0}_{-0.0-0.4-0.0}$	$0.55^{+0.00+0.16+0.00}_{-0.00-0.13-0.00}$	$2.3^{+0.0+0.8+0.0}_{-0.0-0.6-0.0}$	$5.3^{+0.0+1.8+0.0}_{-0.0-1.1-0.0}$	$12.9^{+0.0+4.0+0.1}_{-0.0-3.0-0.1}$

the observed decay rates of $B^0 \to K_SK_SK_S$ and $B^- \to K^-K_SK_S$. From Tables VIII, IX, X, XI, and XII we see that our predicted nonresonant rates are in agreement with the Belle measurements. The reason why the nonresonant fraction is as large as 90% in KKK decays, but becomes only $(35 \sim 40)\%$ in $K\pi\pi$ channels (see Table I) can be explained as follows. Under SU(3) flavor symmetry, we have the relation $\langle K\pi|\bar{s}q|0\rangle^{\rm NR} = \langle K\bar{K}|\bar{s}s|0\rangle^{\rm NR}$. Hence, the nonresonant rates in the $K^-\pi^+\pi^-$ and $\bar{K}^0\pi^+\pi^-$ modes should be similar to that in $K^+K^-\bar{K}^0$ or $K^+K^-K^-$. Since the KKK channel receives resonant contributions only from ϕ and f_{0_i} mesons, while K_i^*, K_{0i}^*, ρ_i, f_{0i} resonances contribute to $K\pi\pi$ modes, this explains why the nonresonant fraction is of order 90% in the former and becomes of order 40% in the latter. Note that the predicted nonresonant contribution in the $K^-\pi^+\pi^0$ mode is larger than the *BABAR*'s upper bound and barely consistent with the Belle limit. It is conceivable that the SU(3) breaking effect in $\langle K\pi|\bar{s}q|0\rangle^{\rm NR}$ may lead to a result consistent with the Belle limit.

It is interesting to notice that, based on a simple fragmentation model and SU(3) symmetry, Gronau and Rosner [54] found the relations

$$\begin{aligned}\Gamma(B^- \to K^+K^-K^-)_{\rm NR} &= 2\Gamma(\bar{B}^0 \to K^+K^-\bar{K}^0)_{\rm NR}\\ &= 2\Gamma(B^- \to K^-\pi^+\pi^-)_{\rm NR}\\ &= 2\Gamma(\bar{B}^0 \to \bar{K}^0\pi^+\pi^-)_{\rm NR}\\ &= 4\Gamma(\bar{B}^0 \to K^-\pi^+\pi^0)_{\rm NR}. \quad (3.21)\end{aligned}$$

TABLE XIII. Branching ratios, mixing-induced and direct CP asymmetries for $\bar{B}^0 \to K_S\pi^+\pi^-$ decays. Results for $(K_L\pi\pi)_{CP\pm}$ are identical to those for $(K_S\pi\pi)_{CP\mp}$. For theoretical errors, see Table III.

Final state	Branching ratio
$(K_S\pi^+\pi^-)_{CP+}$	$13.52^{+0.02+4.03+0.01}_{-0.03-3.06-0.01}$
$(K_S\pi^+\pi^-)_{CP-}$	$7.45^{+0.10+3.79+0.02}_{-0.08-2.32-0.02}$
f_+	$0.65^{+0.00+0.03+0.00}_{-0.00-0.04-0.00}$
Final state	$\sin 2\beta_{\rm eff}$
$(K_S\pi^+\pi^-)_{CP+}$	$0.693^{+0.000+0.003+0.003}_{-0.000-0.002-0.014}$
$(K_S\pi^+\pi^-)_{\rm full}$	$0.718^{+0.001+0.017+0.008}_{-0.001-0.007-0.018}$
Final state	A_f (%)
$(K_S\pi^+\pi^-)_{CP+}$	$4.27^{+0.00+0.19+0.28}_{-0.00-0.12-0.35}$
$(K_S\pi^+\pi^-)_{\rm full}$	$4.94^{+0.03+0.03+0.32}_{-0.02-0.05-0.40}$

Again, a large nonresonant background in $K^-\pi^+\pi^-$ and $\bar{K}^0\pi^+\pi^-$ is favored by this model.

Although the $\bar{B}^0 \to K_S\pi^0\pi^0$ rate has not been measured, its time-dependent CP asymmetries have been studied by *BABAR* [56] with the results

$$\begin{aligned}\sin 2\beta_{\rm eff} &= -0.72 \pm 0.71 \pm 0.08,\\ A_{CP} &= -0.23 \pm 0.52 \pm 0.13.\end{aligned} \quad (3.22)$$

Note that this mode is a CP-even eigenstate. We found that its branching ratio is not so small, of order 6×10^{-6}, in spite of the presence of two neutral pions in the final state (see Table XII). Theoretically, we obtain

$$\begin{aligned}\sin 2\beta_{\rm eff} &= 0.729^{+0.000+0.001+0.009}_{-0.000-0.001-0.020},\\ A_{\rm CP} &= (0.28^{+0.09+0.07+0.02}_{-0.06-0.06-0.02})\%.\end{aligned} \quad (3.23)$$

Finally, we consider the mode $K_S\pi^+\pi^-$ which is an admixture of CP-even and CP-odd components. Results for the decay rates and CP asymmetries are displayed in Table XIII. We see that the effective $\sin 2\beta$ is of order 0.718 and direct CP asymmetry of order 4.9% for $K_S\pi^+\pi^-$.

IV. $B \to KK\pi$ DECAYS

We now turn to the three-body decay modes dominated by $b \to u$ tree and $b \to d$ penguin transitions, namely, $KK\pi$ and $\pi\pi\pi$. We first consider the decay $B^- \to K^+K^-\pi^-$ whose factorizable amplitude is given by Eq. (A9). Note that we have included the matrix element $\langle K^+K^-|\bar{d}d|0\rangle$. Although its nonresonant contribution vanishes as K^+ and K^- do not contain the valence d or $\bar{d}$ quark, this matrix element does receive a contribution from the scalar f_0 pole

$$\langle K^+(p_2)K^-(p_3)|\bar{d}d|0\rangle^R = \sum_i \frac{m_{f_{0i}}\bar{f}^d_{f_{0i}}g^{f_{0i}\to\pi^+\pi^-}}{m^2_{f_{0i}} - s_{23} - im_{f_{0i}}\Gamma_{f_{0i}}}, \quad (4.1)$$

where $\langle f_0|\bar{d}d|0\rangle = m_{f_0}\bar{f}^d_{f_0}$. In the 2-quark model for $f_0(980)$, $\bar{f}^d_{f_0(980)} = \bar{f}_{f_0(980)}\sin\theta/\sqrt{2}$. Also note that the matrix element $\langle K^-(p_3)|(\bar{s}b)_{V-A}|B^-\rangle \times \langle\pi^-(p_1)K^+(p_2)|(\bar{d}s)_{V-A}|0\rangle$ has a similar expression as Eq. (3.5) except for a sign difference

TABLE XIV. Same as Table VIII except for the decay $B^- \to K^+K^-\pi^-$.

Decay mode	$f_0(980)\pi^-$	$K^{*0}K^-$	$K_0^{*0}(1430)K^-$	NR	Total
Theory	$0.50^{+0.00+0.06+0.02}_{-0.00-0.04-0.02}$	$0.23^{+0.00+0.04+0.02}_{-0.00-0.04-0.02}$	$0.82^{+0.00+0.18+0.09}_{-0.00-0.16-0.08}$	$1.8^{+0.5+0.4+0.2}_{-0.5-0.2-0.2}$	$4.0^{+0.5+0.7+0.3}_{-0.6-0.5-0.3}$
Expt.					<6.3 (*BABAR*) [64] <13 (Belle) [11]

$$\langle K^-(p_3)|(\bar{s}b)_{V-A}|B^-\rangle\langle\pi^-(p_1)K^+(p_2)|(\bar{d}s)_{V-A}|0\rangle = -F_1^{BK}(s_{12})F_1^{K\pi}(s_{12})\left[s_{23} - s_{13} - \frac{(m_B^2 - m_K^2)(m_K^2 - m_\pi^2)}{s_{12}}\right] - F_0^{BK}(s_{12})F_0^{K\pi}(s_{12})\frac{(m_B^2 - m_K^2)(m_K^2 - m_\pi^2)}{s_{12}}. \tag{4.2}$$

As in Eq. (3.8), the form factor $F_1^{K\pi}$ receives a resonant contribution for the K^* pole.

The nonresonant and various resonant contributions to $B^- \to K^+K^-\pi^-$ are shown in Table XVI. The predicted total rate is consistent with upper limits set by *BABAR* and Belle.

V. $B \to \pi\pi\pi$ DECAYS

The factorizable amplitudes of the tree-dominated decay $B^- \to \pi^+\pi^-\pi^-$ and $\bar{B}^0 \to \pi^+\pi^-\pi^0$ are given by Eqs. (A15) and (A16), respectively. We see that the former is dominated by the ρ^0 pole, while the latter receives $\rho^\pm$ and ρ^0 contributions (see Tables XV and XVI). As a consequence, the $\pi^+\pi^-\pi^0$ mode has a rate larger than $\pi^+\pi^-\pi^-$ even though the former involves a π^0 in the final state.

TABLE XV. Same as Table VIII except for $B^- \to \pi^+\pi^-\pi^-$. The nonresonant background is used as an input to fix the parameter $\alpha_{\rm NR}$ defined in Eq. (2.8).

Decay mode	*BABAR* [5]	Theory
$\rho^0\pi^-$	$8.8 \pm 1.0 \pm 0.6^{+0.1}_{-0.7}$	$7.7^{+0.0+1.7+0.3}_{-0.0-1.6-0.2}$
$f_0(980)\pi^-$	$1.2 \pm 0.6 \pm 0.1 \pm 0.4 < 3.0$	$0.39^{+0.00+0.01+0.03}_{-0.00-0.01-0.02}$
NR	$2.3 \pm 0.9 \pm 0.3 \pm 0.4 < 4.6$	Input
Total	$16.2 \pm 1.2 \pm 0.9$	$12.0^{+1.1+2.0+0.4}_{-1.2-1.8-0.3}$

The $\pi^+\pi^-\pi^-$ mode receives nonresonant contributions mostly from the $b \to u$ transition as the nonresonant contribution in the matrix element $\langle\pi^+\pi^-|\bar{d}d|0\rangle$ is suppressed by the smallness of penguin Wilson coefficients a_6 and a_8. Therefore, the measurement of the nonresonant contribution in this decay can be used to constrain the nonresonant parameter $\alpha_{\rm NR}$ in Eq. (2.8).

VI. DIRECT *CP* ASYMMETRIES

Direct *CP* asymmetries for various charmless three-body B decays are collected in Table XVII. Mixing-induced and direct *CP* asymmetries in $B^0 \to K^+K^-K_{S,L}$ and $K_SK_SK_{S,L}$ decays are already shown in Table V. It appears that direct *CP* violation is sizable in $K^+K^-K^-$ and $K^+K^-\pi^-$ modes.

TABLE XVI. Same as Table VIII except for the decay $\bar{B}^0 \to \pi^+\pi^-\pi^0$.

Decay mode	$\rho^+\pi^-$	$\rho^-\pi^+$	$\rho^0\pi^0$	$f_0(980)\pi^0$	NR	Total
Theory	$8.5^{+0.0+1.1+0.2}_{-0.0-1.0-0.1}$	$15.5^{+0.0+4.0+0.3}_{-0.0-3.5-0.3}$	$1.0^{+0.0+0.3+0.0}_{-0.0-0.2-0.0}$	$0.010^{+0.000+0.003+0.000}_{-0.000-0.002-0.000}$	$0.05^{+0.02+0.01+0.00}_{-0.02-0.01-0.00}$	$26.3^{+0.0+5.6+0.2}_{-0.0-5.0-0.2}$

TABLE XVII. Direct *CP* asymmetries (in %) for various charmless three-body B decays. For theoretical errors, see Table III. Experimental results are taken from [50].

Final state	*BABAR*	Belle	Theory
$K^+K^-K^-$	$-2 \pm 3 \pm 2$		$-10.4^{+1.7+0.9+0.9}_{-1.3-1.0-0.8}$
$K^-K_SK_S$	$-4 \pm 11 \pm 2$		$-3.9^{+0.0+0.6+0.3}_{-0.0-0.8-0.3}$
$K^+K^-\pi^-$	$0 \pm 10 \pm 3$		$17.5^{+1.9+2.2+0.0}_{-3.8-3.4-0.2}$
$K^-\pi^+\pi^-$	$-1.3 \pm 3.7 \pm 1.1$	$4.9 \pm 2.6 \pm 2.0$	$-3.3^{+0.7+0.4+0.3}_{-0.5-0.4-0.2}$
$K^-\pi^+\pi^0$		$7 \pm 11 \pm 1$	$6.3^{+0.6+1.4+0.5}_{-0.7-1.4-0.5}$
$\bar{K}^0\pi^+\pi^-$			$4.9^{+0.0+0.0+0.3}_{-0.0-0.1-0.4}$
$\bar{K}^0\pi^0\pi^0$	$-23 \pm 52 \pm 13$	$-17 \pm 24 \pm 6$	$0.28^{+0.09+0.07+0.02}_{-0.06-0.06-0.02}$
$\bar{K}^0\pi^-\pi^0$			$0.4^{+0.0+0.4+0.0}_{-0.0-0.4-0.0}$
$\pi^+\pi^-\pi^-$	$-1 \pm 8 \pm 3$		$4.4^{+0.8+1.2+0.0}_{-0.6-0.9-0.2}$
$\pi^+\pi^-\pi^0$			$-3.0^{+0.1+0.2+0.3}_{-0.1-0.3-0.2}$

The major uncertainty with direct CP violation comes from the strong phases which are needed to induce partial rate CP asymmetries. In this work, the strong phases arise from the effective Wilson coefficients a_i^p listed in (A3) and from the Breit-Wigner formalism for resonances. Since direct CP violation in charmless two-body B decays can be significantly affected by final-state rescattering [57], it is natural to extend the study of final-state rescattering effects to the case of three-body B decays. We will leave this to a future investigation.

VII. TWO-BODY $B \to VP$ AND $B \to SP$ DECAYS

Thus far we have considered the branching ratio products $\mathcal{B}(B \to Rh_1)\mathcal{B}(R \to h_2h_3)$ with the resonance R being a vector meson or a scalar meson. Using the experimental information on $\mathcal{B}(R \to h_2h_3)$ [4]

$$\begin{aligned} \mathcal{B}(K^{*0} \to K^+\pi^-) &= \mathcal{B}(K^{*+} \to K^0\pi^+) \\ &= 2\mathcal{B}(K^{*+} \to K^+\pi^0) = \frac{2}{3}, \\ \mathcal{B}(K_0^{*0}(1430) \to K^+\pi^-) &= 2\mathcal{B}(K_0^{*+}(1430) \to K^+\pi^0) \\ &= \frac{2}{3}(0.93 \pm 0.10), \\ \mathcal{B}(\phi \to K^+K^-) &= 0.492 \pm 0.006. \end{aligned} \tag{7.1}$$

one can extract the branching ratios of $B \to VP$ and $B \to SP$. The results are summarized in Table XVIII.

Two remarks about the experimental branching ratios are in order: (i) The *BABAR* results for the branching ratios of $\bar{B}^0 \to K^{*-}\pi^+$, $\bar{K}^{*0}\pi^0$, $K_0^{*-}(1430)\pi^+$ are inferred from

TABLE XVIII. Branching ratios of quasi-two-body decays $B \to VP$ and $B \to SP$ obtained from the studies of three-body decays based on the factorization approach. Unless specified, the experimental results are obtained from the 3-body Dalitz plot analyses given in previous Tables. Theoretical uncertainties have been added in quadrature. QCD factorization (QCDF) predictions taken from [58] for VP modes and from [48] for SP channels are shown here for comparison.

Decay mode	*BABAR*	Belle	QCDF	This work
ϕK^0	$8.4^{+1.5}_{-1.3} \pm 0.5$ [a]	$9.0^{+2.2}_{-1.8} \pm 0.7$ [b]	$4.1^{+0.4+1.7+1.8+10.6}_{-0.4-1.6-1.9-3.0}$	$5.3^{+1.0}_{-0.9}$
ϕK^-	$8.4 \pm 0.7 \pm 0.7$	$9.60 \pm 0.92^{+1.05}_{-0.84}$	$4.5^{+0.5+1.8+1.9+11.8}_{-0.4-1.7-2.1-3.3}$	$5.9^{+1.1}_{-1.0}$
$\bar{K}^{*0}\pi^-$	$13.5 \pm 1.2^{+0.8}_{-0.9}$	$9.8 \pm 0.9^{+1.1}_{-1.2}$	$3.6^{+0.4+1.5+1.2+7.7}_{-0.3-1.4-1.2-2.3}$	$4.4^{+1.1}_{-1.0}$
$\bar{K}^{*0}\pi^0$	$3.0 \pm 0.9 \pm 0.5$	<3.5	$0.7^{+0.1+0.5+0.3+2.6}_{-0.1-0.4-0.3-0.5}$	$1.5^{+0.5}_{-0.4}$
$K^{*-}\pi^+$	$11.0 \pm 1.5 \pm 0.7$	$8.4 \pm 1.1^{+0.9}_{-0.8}$	$3.3^{+1.4+1.3+0.8+6.2}_{-1.2-1.2-0.8-1.6}$	$3.1^{+0.9}_{-0.9}$
$K^{*-}\pi^0$	$6.9 \pm 2.0 \pm 1.3$ [b]		$3.3^{+1.1+1.0+0.6+4.4}_{-1.0-0.9-0.6-1.4}$	$2.2^{+0.6}_{-0.5}$
$K^{*0}K^-$			$0.30^{+0.11+0.12+0.09+0.57}_{-0.09-0.10-0.09-0.19}$	$0.35^{+0.06}_{-0.06}$
ρ^0K^-	$5.1 \pm 0.8^{+0.6}_{-0.9}$	$3.89 \pm 0.47^{+0.43}_{-0.41}$	$2.6^{+0.9+3.1+0.8+4.3}_{-0.9-1.4-0.6-1.2}$	$1.3^{+1.9}_{-0.7}$
$\rho^0\bar{K}^0$	$4.9 \pm 0.8 \pm 0.9$	$6.1 \pm 1.0 \pm 1.1$	$4.6^{+0.5+4.0+0.7+6.1}_{-0.5-2.1-0.7-2.1}$	$2.0^{+1.9}_{-0.9}$
ρ^+K^-	$8.6 \pm 1.4 \pm 1.0$	$15.1^{+3.4+2.4}_{-3.3-2.6}$	$7.4^{+1.8+7.1+1.2+10.7}_{-1.9-3.6-1.1-3.5}$	$2.5^{+3.6}_{-1.4}$
$\rho^-\bar{K}^0$	$8.0^{+1.4}_{-1.3} \pm 0.5$ [b]		$5.8^{+0.6+7.0+1.5+10.3}_{-0.6-3.3-1.3-3.2}$	$1.3^{+3.0}_{-0.9}$
$\rho^0\pi^-$	$8.8 \pm 1.0^{+0.6}_{-0.9}$	$8.0^{+2.3}_{-2.0} \pm 0.7$ [b]	$11.9^{+6.3+3.6+2.5+1.3}_{-5.0-3.1-1.2-1.1}$	$7.7^{+1.7}_{-1.6}$
$\rho^-\pi^+$			$21.2^{+10.3+8.7+1.3+2.0}_{-8.4-7.2-2.3-1.6}$	$15.5^{+4.0}_{-3.5}$
$\rho^+\pi^-$			$15.4^{+8.0+5.5+0.7+1.9}_{-6.4-4.7-1.3-1.3}$	$8.5^{+1.1}_{-1.0}$
$\rho^0\pi^0$	$1.4 \pm 0.6 \pm 0.3$	$3.1^{+0.9+0.6}_{-0.8-0.8}$	$0.4^{+0.2+0.2+0.9+0.5}_{-0.2-0.1-0.3-0.3}$	$1.0^{+0.3}_{-0.2}$
$f_0(980)K^0$; $f_0 \to \pi^+\pi^-$	$5.5 \pm 0.7 \pm 0.6$	$7.6 \pm 1.7^{+0.8}_{-0.9}$	$6.7^{+0.1+2.1+2.3}_{-0.1-1.5-1.1}$ [c]	$7.7^{+0.4}_{-0.7}$
$f_0(980)K^-$; $f_0 \to \pi^+\pi^-$	$9.3 \pm 1.0^{+0.6}_{-0.9}$	$8.8 \pm 0.8^{+0.9}_{-1.8}$	$7.8^{+0.2+2.3+2.7}_{-0.2-1.6-1.2}$ [c]	$7.7^{+0.4}_{-0.8}$
$f_0(980)K^0$; $f_0 \to K^+K^-$	5.3 ± 2.2			$5.8^{+0.1}_{-0.5}$
$f_0(980)K^-$; $f_0 \to K^+K^-$	$6.5 \pm 2.5 \pm 1.6$	<2.9		$7.0^{+0.4}_{-0.7}$
$f_0(980)\pi^-$; $f_0 \to \pi^+\pi^-$	<3.0		$0.5^{+0.0+0.2+0.1}_{-0.0-0.1-0.0}$ [c]	$0.39^{+0.03}_{-0.02}$
$f_0(980)\pi^-s$; $f_0 \to K^+K^-$				$0.50^{+0.06}_{-0.04}$
$f_0(980)\pi^0$; $f_0 \to \pi^+\pi^-$			$0.02^{+0.01+0.02+0.04}_{-0.01-0.00-0.01}$ [c]	$0.010^{+0.003}_{-0.002}$
$\bar{K}_0^{*0}(1430)\pi^-$	$36.6 \pm 1.8 \pm 4.7$	$51.6 \pm 1.7^{+7.0}_{-7.4}$	$11.0^{+10.3+7.5+49.9}_{-6.0-3.5-10.1}$	$16.9^{+5.2}_{-4.4}$
$\bar{K}_0^{*0}(1430)\pi^0$	$12.7 \pm 2.4 \pm 4.4$	$9.8 \pm 2.5 \pm 0.9$	$6.4^{+5.4+2.2+26.1}_{-3.3-2.1-5.7}$	$6.8^{+2.3}_{-1.9}$
$K_0^{*-}(1430)\pi^+$	$36.1 \pm 4.8 \pm 11.3$	$49.7 \pm 3.8^{+4.0}_{-6.1}$	$11.3^{+9.4+3.7+45.8}_{-5.8-3.7-9.9}$	$16.2^{+4.7}_{-4.0}$
$K_0^{*-}(1430)\pi^0$			$5.3^{+4.7+1.6+22.3}_{-2.8-1.7-4.7}$	$8.9^{+2.6}_{-2.2}$
$K_0^{*0}(1430)K^-$	<2.2 [b]			$1.3^{+0.3}_{-0.3}$

[a]From the Dalitz plot analysis of $B^0 \to K^+K^-K^0$ decay measured by *BABAR* (see Table III), we obtain $\mathcal{B}(B^0 \to \phi K^0) = (6.2 \pm 0.9) \times 10^{-6}$. The experimental value of *BABAR* cited in the table is obtained from a direct measurement of $B^0 \to \phi K^0$.
[b]Not determined directly from the Dalitz plot analysis of three-body decays.
[c]We have assumed $\mathcal{B}(f_0(980) \to \pi^+\pi^-) = 0.50$ for the QCDF calculation.

the three-body decays $\bar{B}^0 \to \bar{K}^0\pi^+\pi^-$ (see Table XI) and Belle results are taken from $\bar{B}^0 \to K^-\pi^+\pi^0$ (see Table X). (ii) Branching ratios of $\bar{B}^0 \to \phi\bar{K}^0$ shown in Table XVIII are not inferred from the Dalitz plot analysis of $B \to KKK$ decays.

For comparison, the predictions of the QCD factorization approach for $B \to VP$ [58] and $B \to SP$ [48] are also exhibited in Table XVIII. In order to compare theory with experiment for $B \to f_0(980)K \to \pi^+\pi^- K$, we need an input for $\mathcal{B}(f_0(980) \to \pi^+\pi^-)$. To do this, we shall use the BES measurement [45]

$$\frac{\Gamma(f_0(980) \to \pi\pi)}{\Gamma(f_0(980) \to \pi\pi) + \Gamma(f_0(980) \to K\bar{K})} = 0.75^{+0.11}_{-0.13}. \tag{7.2}$$

Assuming that the dominance of the $f_0(980)$ width by $\pi\pi$ and $K\bar{K}$ and applying isospin relation, we obtain

$$\mathcal{B}(f_0(980) \to \pi^+\pi^-) = 0.50^{+0.07}_{-0.09}, \quad \mathcal{B}(f_0(980) \to K^+K^-) = 0.125^{+0.018}_{-0.022}. \tag{7.3}$$

At first sight, it appears that the ratio defined by

$$R \equiv \frac{\mathcal{B}(f_0(980) \to K^+K^-)}{\mathcal{B}(f_0(980) \to \pi^+\pi^-)} = 0.25 \pm 0.06 \tag{7.4}$$

is not consistent with the value of 0.69 ± 0.32 inferred from the *BABAR* data (see Tables VI and VIII)

$$R = \frac{\Gamma(B^- \to f_0(980)K^-; f_0(980) \to K^+K^-)}{\Gamma(B^- \to f_0(980)K^-; f_0(980) \to \pi^+\pi^-)} = \frac{6.5 \pm 2.5 \pm 1.6}{9.3 \pm 1.0^{+0.6}_{-0.9}}, \tag{7.5}$$

where we have applied the narrow width approximation Eq. (3.14).

The above-mentioned discrepancy can be resolved by noting that the factorization relation Eq. (3.14) for the resonant three-body decay is applicable only when the two-body decays $B \to RP$ and $R \to P_1P_2$ are kinematically allowed and the resonance is narrow, the so-called narrow width approximation. However, as the decay $f_0(980) \to K^+K^-$ is kinematically barely or even not allowed, the off resonance peak effect of the intermediate resonant state will become important. Therefore, it is necessary to take into account the finite width effect of the $f_0(980)$ which has a width of order 40–100 MeV [4]. In short, one cannot determine the ratio R by applying the narrow width approximation to the three-body decays. That is, one should employ the decays $B \to K\pi\pi$ rather than $B \to KKK$ to extract the experimental branching ratio for $B \to f_0(980)K$ provided $\mathcal{B}(f_0(980) \to \pi\pi)$ is available.

We now compare the present work for $B \to VP$ and $B \to SP$ with the approach of QCD factorization [34,48]. In this work, our calculation of 3-body B decays is similar to the simple generalized factorization approach [59,60] by assuming a set of universal and process independent effective Wilson coefficients a_i^p with $p = u, c$ in Eq. (A3). In QCDF, the calculation of a_i^p is rather sophisticated. They are basically the Wilson coefficients in conjunction with short-distance nonfactorizable corrections such as vertex corrections and hard spectator interactions. In general, they have the expressions [34,58]

$$a_i^p(M_1M_2) = \left(c_i + \frac{c_{i\pm1}}{N_c}\right)N_i(M_2) + \frac{c_{i\pm1}}{N_c}\frac{C_F\alpha_s}{4\pi}\left[V_i(M_2) + \frac{4\pi^2}{N_c}H_i(M_1M_2)\right] + P_i^p(M_2), \tag{7.6}$$

where $i = 1, \ldots, 10$, the upper (lower) signs apply when i is odd (even), c_i are the Wilson coefficients, $C_F = (N_c^2 - 1)/(2N_c)$ with $N_c = 3$, M_2 is the emitted meson and M_1 shares the same spectator quark with the B meson. The quantities $V_i(M_2)$ account for vertex corrections, $H_i(M_1M_2)$ for hard spectator interactions with a hard gluon exchange between the emitted meson and the spectator quark of the B meson and $P_i(M_2)$ for penguin contractions. Hence, the effective Wilson coefficients $a_i^p(M_1M_2)$ depend on the nature of M_1 and M_2; that is, they are process dependent. Moreover, they depend on the order of the argument, namely, $a_i^p(M_2M_1) \neq a_i^p(M_1M_2)$ in general. In the above equation, $N_i(M_2)$ vanishes for $i = 6, 8$ and $M_2 = V$, and equals to unity otherwise. For three-body decays, in principle one should also compute the vertex, gluon and hard spectator-interaction corrections. Of course, these corrections for the three-body case will be more complicated than the two-body decay one. One possible improvement of the present work is to utilize the QCDF results for the effective parameters $a_i^p(M_1M_2)$ in the vicinity of the resonance region.

We next proceed to the comparison of numerical results. For ϕK, $K^*\pi$ and $K^*\bar{K}$ modes, the QCDF and the present work have similar predictions. For the ρ meson in the final states, QCDF predicts slightly small ρK and too large $\rho\pi$ compared to experiment.[6] In contrast, in the present work we obtain reasonable $\rho\pi$ but too small ρK. This is ascribed to the form factor $A_0^{B\rho}(0) = 0.37 \pm 0.06$ employed in [58] that is too large compared to ours $A_0^{B\rho}(0) = 0.28 \pm 0.03$ (see Table XIX). Recall that the recent QCD sum rule calculation also yields a smaller one $A_0^{B\rho}(0) = 0.30^{+0.07}_{-0.03}$ [62].

For $B \to f_0(980)K$ and $B \to f_0(980)\pi$, QCDF [48] and this work are in agreement with experiment. The large rate of the $f_0(980)K$ mode is ascribed to the large $f_0(980)$ decay constant, $\bar{f}_{f_0(980)} \approx 460$ MeV at the renormalization

[6] Recall that the world average of the branching ratio of $B^0 \to \rho^\pm\pi^\mp$ is $(24.0 \pm 2.5) \times 10^{-6}$ [50], while QCDF predicts it to be $\sim 36.6 \times 10^{-6}$ [58].

scale $\mu = 2.1$ GeV [48]. In contrast, the predicted $\bar{K}_0^{*0}(1430)\pi^-$ and $K_0^{*-}(1430)\pi^+$ are too small compared to the data. The fact that QCDF leads to too small rates for ϕK, $K^*\pi s$, ρK and $K_0^*(1430)\pi$ may imply the importance of power corrections due to the nonvanishing ρ_A and ρ_H parameters arising from weak annihilation and hard spectator interactions, respectively, which are used to parametrize the endpoint divergences, or due to possible final-state rescattering effects from charm intermediate states [57]. However, this is beyond the scope of the present work.

VIII. CONCLUSIONS

In this work, an exploratory study of charmless 3-body decays of B mesons is presented using a simple model based on the framework of the factorization approach. The 3-body decay process consists of resonant contributions and the nonresonant signal. Since factorization has not been proved for three-body B decays, we shall work in the phenomenological factorization model rather than in the established theories such as QCD factorization. That is, we start with the simple idea of factorization and see if it works for three-body decays. Our main results are as follows:

(i) If heavy meson chiral perturbation theory (HMChPT) is applied to the three-body matrix elements for $B \to P_1P_2$ transitions and assumed to be valid over the whole kinematic region, then the predicted decay rates for nonresonant 3-body B decays will be too large and even exceed the measured total rate. This can be understood because chiral symmetry has been applied beyond its region of validity. We assume the momentum dependence of nonresonant amplitudes in the exponential form $e^{-\alpha_{\rm NR} p_B\cdot(p_i+p_j)}$ so that the HMChPT results are recovered in the soft meson limit p_i, $p_j \to 0$. The parameter $\alpha_{\rm NR}$ can be fixed from the tree-dominated decay $B^- \to \pi^+\pi^-\pi^-$.

(ii) Besides the nonresonant contributions arising from $B \to P_1P_2$ transitions, we have identified another large source of the nonresonant background in the matrix elements of scalar densities, e.g. $\langle K\bar{K}|\bar{s}s|0\rangle$ which can be constrained from the $K_SK_SK_S$ (or $K^-K_SK_S$) mode in conjunction with the mass spectrum in the decay $\bar{B}^0 \to K^+K^-\bar{K}^0$.

(iii) All KKK modes are dominated by the nonresonant background. The predicted branching ratios of $K^+K^-K_{S(L)}$, $K^+K^-K^-$ and $K^-K_SK_S$ modes are consistent with the data within the theoretical and experimental errors.

(iv) Although the penguin-dominated $B^0 \to K^+K^-K_S$ decay is subject to a potentially significant tree pollution, its effective $\sin 2\beta$ is very similar to that of the $K_SK_SK_S$ mode. However, direct CP asymmetry of the former, being of order -4%, is more prominent than the latter,

(v) The role played by the unknown scalar resonance $X_0(1550)$ in the decay $B^- \to K^+K^-K^-$ should be clarified in order to see if it behaves in the same way as in the $K^+K^-\bar{K}^0$ mode.

(vi) Applying SU(3) symmetry to relate the nonresonant component in the matrix element $\langle K\pi|\bar{s}q|0\rangle$ to that in $\langle K\bar{K}|\bar{s}s|0\rangle$, we found sizable nonresonant contributions in $K^-\pi^+\pi^-$ and $\bar{K}^0\pi^+\pi^-$ modes, in agreement with the Belle measurements but larger than the *BABAR* results. In particular, the predicted nonresonant contribution in the $K^-\pi^+\pi^0$ mode is consistent with the Belle limit and larger than the *BABAR*'s upper bound. It will be interesting to have a refined measurement of the nonresonant contribution to this mode to test our model.

(vii) The $\pi^+\pi^-\pi^0$ mode is predicted to have a rate larger than $\pi^+\pi^-\pi^-$ even though the former involves a π^0 in the final state. This is because the latter is dominated by the ρ^0 pole, while the former receives $\rho^\pm$ and ρ^0 resonant contributions.

(viii) Among the 3-body decays we have studied, the decay $B^- \to K^+K^-\pi^-$ dominated by $b \to u$ tree transition and $b \to d$ penguin transition has the smallest branching ratio of order 4×10^{-6}. It is consistent with the current bound set by *BABAR* and Belle.

(ix) Decay rates and time-dependent CP asymmetries in the decays $K_S\pi^0\pi^0$, a purely CP-even state, and $K_S\pi^+\pi^-$, an admixture of CP-even and CP-odd components, are studied. The corresponding mixing-induced CP violation is found to be of order 0.729 and 0.718, respectively.

(x) Since the decay $f_0(980) \to K^+K^-$ is kinematically barely or even not allowed, it is crucial to take into account the finite width effect of the $f_0(980)$ when computing the decay $B \to f_0(980)K \to KKK$. Consequently, one should employ the Dalitz plot analysis of $K\pi\pi$ mode to extract the experimental branching ratio for $B \to f_0(980)K$ provided $\mathcal{B}(f_0(980) \to \pi\pi)$ is available. The large rate of $B \to f_0(980)K$ is ascribed to the large $f_0(980)$ decay constant, $\bar{f}_{f_0(980)} \approx 460$ MeV.

(xi) The intermediate vector-meson contributions to 3-body decays e.g. ρ, ϕ, K^* are identified through the vector current, while the scalar meson resonances e.g. $f_0(980)$, $X_0(1550)$, $K_0^*(1430)$ are mainly associated with the scalar density. Their effects are described in terms of the Breit-Wigner formalism.

(xii) Based on the factorization approach, we have computed the resonant contributions to 3-body decays and determined the rates for the quasi-two-body decays $B \to VP$ and $B \to SP$. The predicted $\rho\pi$, $f_0(980)K$ and $f_0(980)\pi$ rates are consistent with experiment, while the calculated ϕK,

$K^*\pi$, ρK and $K_0^*(1430)\pi$ are too small compared to the data.

(xiii) Direct CP asymmetries have been computed for the charmless 3-body B decays. We found sizable direct CP violation in $K^+K^-K^-$ and $K^+K^-\pi^-$ modes.

(xiv) In this exploratory work we use the phenomenological factorization model rather than in the established theories based on a heavy quark expansion. Consequently, we do not have $1/m_b$ power corrections within this model. However, systematic errors due to such model dependent assumptions may be sizable and are not included in the error estimates that we give.

ACKNOWLEDGMENTS

This research was supported in part by the National Science Council of R.O.C. under Grant No. NSC95-2112-M-001-013, No. NSC95-2112-M-033-013, and by the U.S. DOE Contract No. DE-AC02-98CH10886 (BNL).

Note added.—After the paper was submitted for publication, [65] has reported the observation of the decay $B^+ \to K^+K^-\pi^+$ with the branching ratio $(5.0 \pm 0.5 \pm 0.5) \times 10^{-6}$. Our prediction for this mode (see Table XIV) is consistent with experiment.

APPENDIX A: DECAY AMPLITUDES OF THREE-BODY B DECAYS

In this appendix we list the factorizable amplitudes of the 3-body decays $B \to KKK$, $KK\pi$, $K\pi\pi$, $\pi\pi\pi$. Under the factorization hypothesis, the decay amplitudes are given by

$$\langle P_1P_2P_3|\mathcal{H}_{\rm eff}|\bar{B}\rangle = \frac{G_F}{\sqrt{2}}\sum_{p=u,c}\lambda_p^{(r)}\langle P_1P_2P_3|T_p|\bar{B}\rangle, \quad \text{(A1)}$$

where $\lambda_p^{(r)} \equiv V_{pb}V_{pr}^*$ with $r = d$, s. For KKK and $K\pi\pi$ modes, $r = s$ and for $KK\pi$ and $\pi\pi\pi$ channels, $r = d$. The Hamiltonian T_p has the expression [34]

$$\begin{aligned} T_p &= a_1\delta_{pu}(\bar{u}b)_{V-A}\otimes(\bar{s}u)_{V-A} + a_2\delta_{pu}(\bar{s}b)_{V-A}\otimes(\bar{u}u)_{V-A} + a_3(\bar{s}b)_{V-A}\otimes\sum_q(\bar{q}q)_{V-A} + a_4^p\sum_q(\bar{q}b)_{V-A}\otimes(\bar{s}q)_{V-A} \\ &\quad + a_5(\bar{s}b)_{V-A}\otimes\sum_q(\bar{q}q)_{V+A} - 2a_6^p\sum_q(\bar{q}b)_{S-P}\otimes(\bar{s}q)_{S+P} + a_7(\bar{s}b)_{V-A}\otimes\sum_q\frac{3}{2}e_q(\bar{q}q)_{V+A} \\ &\quad - 2a_8^p\sum_q(\bar{q}b)_{S-P}\otimes\frac{3}{2}e_q(\bar{s}q)_{S+P} + a_9(\bar{s}b)_{V-A}\otimes\sum_q\frac{3}{2}e_q(\bar{q}q)_{V-A} + a_{10}^p\sum_q(\bar{q}b)_{V-A}\otimes\frac{3}{2}e_q(\bar{s}q)_{V-A}, \end{aligned} \quad \text{(A2)}$$

with $(\bar{q}q')_{V\pm A} \equiv \bar{q}\gamma_\mu(1\pm\gamma_5)q'$, $(\bar{q}q')_{S\pm P} \equiv \bar{q}(1\pm\gamma_5)q'$ and a summation over $q = u$, d, s being implied. For the effective Wilson coefficients, we use

$$\begin{aligned} &a_1 \approx 0.99 \pm 0.037i, \qquad a_2 \approx 0.19 - 0.11i, \qquad a_3 \approx -0.002 + 0.004i, \qquad a_5 \approx 0.0054 - 0.005i, \\ &a_4^u \approx -0.03 - 0.02i, \qquad a_4^c \approx -0.04 - 0.008i, \qquad a_6^u \approx -0.06 - 0.02i, \qquad a_6^c \approx -0.06 - 0.006i, \\ &a_7 \approx 0.54\times10^{-4}i, \qquad a_8^u \approx (4.5 - 0.5i)\times10^{-4}, \qquad a_8^c \approx (4.4 - 0.3i)\times10^{-4}, \qquad a_9 \approx -0.010 - 0.0002i, \\ &a_{10}^u \approx (-58.3 + 86.1i)\times10^{-5}, \qquad a_{10}^c \approx (-60.3 + 88.8i)\times10^{-5}, \end{aligned} \quad \text{(A3)}$$

for typical a_i at the renormalization scale $\mu = m_b/2 = 2.1$ GeV which we are working on.

Various three-body B decay amplitudes are collected below.

$B \to KKK$

$$\begin{aligned} \langle\bar{K}^0K^+K^-|T_p|\bar{B}^0\rangle &= \langle K^+\bar{K}^0|(\bar{u}b)_{V-A}|\bar{B}^0\rangle\langle K^-|(\bar{s}u)_{V-A}|0\rangle[a_1\delta_{pu} + a_4^p + a_{10}^p - (a_6^p + a_8^p)r_\chi^K] \\ &\quad + \langle K^+K^-|(\bar{d}b)_{V-A}|\bar{B}^0\rangle\langle\bar{K}^0|(\bar{s}d)_{V-A}|0\rangle\left(a_4^p - \frac{1}{2}a_{10}^p\right) + \langle\bar{K}^0|(\bar{s}b)_{V-A}|\bar{B}^0\rangle\langle K^+K^-|(\bar{u}u)_{V-A}|0\rangle \\ &\quad \times(a_2\delta_{pu} + a_3 + a_5 + a_7 + a_9) + \langle\bar{K}^0|(\bar{s}b)_{V-A}|\bar{B}^0\rangle\langle K^+K^-|(\bar{d}d)_{V-A}|0\rangle\left[a_3 + a_5 - \frac{1}{2}(a_7 + a_9)\right] \\ &\quad + \langle\bar{K}^0|(\bar{s}b)_{V-A}|\bar{B}^0\rangle\langle K^+K^-|(\bar{s}s)_{V-A}|0\rangle\left[a_3 + a_4^p + a_5 - \frac{1}{2}(a_7 + a_9 + a_{10}^p)\right] \\ &\quad + \langle\bar{K}^0|\bar{s}b|\bar{B}^0\rangle\langle K^+K^-|\bar{s}s|0\rangle(-2a_6^p + a_8^p) + \langle K^+K^-|\bar{d}(1-\gamma_5)b|\bar{B}^0\rangle\langle\bar{K}^0|\bar{s}(1+\gamma_5)d|0\rangle(-2a_6^p + a_8^p) \\ &\quad + \langle K^+K^-\bar{K}^0|(\bar{s}d)_{V-A}|0\rangle\langle0|(\bar{d}b)_{V-A}|\bar{B}^0\rangle\left(a_4^p - \frac{1}{2}a_{10}^p\right) \\ &\quad + \langle K^+K^-\bar{K}^0|\bar{s}\gamma_5d|0\rangle\langle0|\bar{d}\gamma_5b|\bar{B}^0\rangle(-2a_6^p + a_8^p), \end{aligned} \quad \text{(A4)}$$

with $r_\chi^P = \frac{2m_P^2}{m_b(\mu)(m_2+m_1)(\mu)}$.

$$\begin{aligned}\langle K^+K^-K^-|T_p|B^-\rangle &= \langle K^+K^-|(\bar{u}b)_{V-A}|B^-\rangle\langle K^-|(\bar{s}u)_{V-A}|0\rangle[a_1\delta_{pu} + a_4^p + a_{10}^p - (a_6^p + a_8^p)r_\chi^K] \\ &+ \langle K^-|(\bar{s}b)_{V-A}|B^-\rangle\langle K^+K^-|(\bar{u}u)_{V-A}|0\rangle(a_2\delta_{pu} + a_3 + a_5 + a_7 + a_9) \\ &+ \langle K^-|(\bar{s}b)_{V-A}|B^-\rangle\langle K^+K^-|(\bar{d}d)_{V-A}|0\rangle\Big[a_3 + a_5 - \frac{1}{2}(a_7 + a_9)\Big] \\ &+ \langle K^-|(\bar{s}b)_{V-A}|B^-\rangle\langle K^+K^-|(\bar{s}s)_{V-A}|0\rangle\Big[a_3 + a_4^p + a_5 - \frac{1}{2}(a_7 + a_9 + a_{10}^p)\Big] \\ &+ \langle K^-|\bar{s}b|B^-\rangle\langle K^+K^-|\bar{s}s|0\rangle(-2a_6^p + a_8^p) + \langle K^+K^-|\bar{u}(1-\gamma_5)b|\bar{B}^0\rangle\langle K^-|\bar{s}(1+\gamma_5)u|0\rangle \\ &\times(-2a_6^p + a_8^p) + \langle K^+K^-K^-|(\bar{s}u)_{V-A}|0\rangle\langle 0|(\bar{u}b)_{V-A}|B^-\rangle\Big(a_4^p - \frac{1}{2}a_{10}^p\Big) \\ &+ \langle K^+K^-K^-|\bar{s}\gamma_5 u|0\rangle\langle 0|\bar{u}\gamma_5 b|B^-\rangle(-2a_6^p + a_8^p). \end{aligned} \tag{A5}$$

Since there are two identical K^- mesons in this decay, one should take into account the identical particle effects. For example,

$$\begin{aligned}\langle K^+K^-|(\bar{u}b)_{V-A}|B^-\rangle\langle K^-|(\bar{s}u)_{V-A}|0\rangle &= \langle K^+(p_1)K^-(p_2)|(\bar{u}b)_{V-A}|B^-\rangle\langle K^-(p_3)|(\bar{s}u)_{V-A}|0\rangle \\ &+ \langle K^+(p_1)K^-(p_3)|(\bar{u}b)_{V-A}|B^-\rangle\langle K^-(p_2)|(\bar{s}u)_{V-A}|0\rangle, \end{aligned} \tag{A6}$$

and a factor of $\frac{1}{2}$ should be put in the decay rate.

$$\begin{aligned}\langle K^0\bar{K}^0\bar{K}^0|T_p|\bar{B}^0\rangle &= \langle K^0\bar{K}^0|(\bar{d}b)_{V-A}|\bar{B}^0\rangle\langle\bar{K}^0|(\bar{s}d)_{V-A}|0\rangle\Big(a_4^p - \frac{1}{2}a_{10}^p - \Big(a_6^p - \frac{1}{2}a_8^p\Big)r_\chi^K\Big) \\ &+ \langle\bar{K}^0|(\bar{s}b)_{V-A}|\bar{B}^0\rangle\langle K^0\bar{K}^0|(\bar{d}d)_{V-A}|0\rangle\Big[a_3 + a_5 - \frac{1}{2}(a_7 + a_9)\Big] + \langle\bar{K}^0|(\bar{s}b)_{V-A}|\bar{B}^0\rangle\langle K^0\bar{K}^0|(\bar{s}s)_{V-A}|0\rangle \\ &\times\Big[a_3 + a_4^p + a_5 - \frac{1}{2}(a_7 + a_9 + a_{10}^p)\Big] + \langle\bar{K}^0|\bar{s}b|\bar{B}^0\rangle\langle K^0\bar{K}^0|\bar{s}s|0\rangle(-2a_6^p + a_8^p) \\ &+ \langle K^0\bar{K}^0\bar{K}^0|(\bar{s}d)_{V-A}|0\rangle\langle 0|(\bar{d}b)_{V-A}|\bar{B}^0\rangle\Big(a_4^p - \frac{1}{2}(a_7 + a_9 + a_{10}^p)\Big) \\ &+ \langle K^0\bar{K}^0\bar{K}^0|\bar{s}\gamma_5 d|0\rangle\langle 0|\bar{d}\gamma_5 b|\bar{B}^0\rangle(-2a_6^p + a_8^p). \end{aligned} \tag{A7}$$

The second and third terms do not contribute to the purely CP -even decay $\bar{B}^0 \to K_SK_SK_S$.

$$\begin{aligned}\langle K^-K^0\bar{K}^0|T_p|B^-\rangle &= \langle K^0\bar{K}^0|(\bar{u}b)_{V-A}|B^-\rangle\langle K^-|(\bar{s}u)_{V-A}|0\rangle[a_1\delta_{pu} + a_4^p + a_{10}^p - (a_6^p + a_8^p)r_\chi^K] \\ &+ \langle K^0K^-|(\bar{d}b)_{V-A}|B^-\rangle\langle\bar{K}^0|(\bar{s}d)_{V-A}|0\rangle\Big(a_4^p - \frac{1}{2}a_{10}^p - \Big(a_6^p - \frac{1}{2}a_8^p\Big)r_\chi^K\Big) \\ &+ \langle K^-|(\bar{s}b)_{V-A}|B^-\rangle\langle K^0\bar{K}^0|(\bar{d}d)_{V-A}|0\rangle\Big[a_3 + a_5 - \frac{1}{2}(a_7 + a_9)\Big] \\ &+ \langle K^-|(\bar{s}b)_{V-A}|B^-\rangle\langle K^0\bar{K}^0|(\bar{s}s)_{V-A}|0\rangle\Big[a_3 + a_4^p + a_5 - \frac{1}{2}(a_7 + a_9 + a_{10}^p)\Big] \\ &+ \langle K^-|\bar{s}b|B^-\rangle\langle K^0\bar{K}^0|\bar{s}s|0\rangle(-2a_6^p + a_8^p) + \langle K^-K^0\bar{K}^0|(\bar{s}u)_{V-A}|0\rangle\langle 0|(\bar{u}b)_{V-A}|\bar{B}^0\rangle(a_1\delta_{pu} + a_4^p + a_{10}^p) \\ &+ \langle K^-K^0\bar{K}^0|\bar{s}\gamma_5 u|0\rangle\langle 0|\bar{u}(1-\gamma_5)b|B^-\rangle(2a_6^p + 2a_8^p). \end{aligned} \tag{A8}$$

The third and fourth terms do not contribute to the decay $B^- \to K^-K_SK_S$.

$B \to KK\pi$

$$\begin{aligned}\langle \pi^- K^+ K^- | T_p | B^- \rangle &= \langle K^+ K^- | (\bar{u}b)_{V-A} | B^- \rangle \langle \pi^- | (\bar{d}u)_{V-A} | 0 \rangle [a_1 \delta_{pu} + a_4^p + a_{10}^p - (a_6^p + a_8^p) r_\chi^\pi] \\ &+ \langle \pi^- | (\bar{d}b)_{V-A} | B^- \rangle \langle K^+ K^- | (\bar{u}u)_{V-A} | 0 \rangle (a_2 \delta_{pu} + a_3 + a_5 + a_7 + a_9) \\ &+ \langle \pi^- | \bar{d}b | B^- \rangle \langle K^+ K^- | \bar{d}d | 0 \rangle (-2a_6^p + a_8^p) + \langle \pi^- | (\bar{d}b)_{V-A} | B^- \rangle \langle K^+ K^- | (\bar{s}s)_{V-A} | 0 \rangle \\ &\times \left[a_3 + a_5 - \frac{1}{2}(a_7 + a_9) \right] + \langle K^- | (\bar{s}b)_{V-A} | B^- \rangle \langle K^+ \pi^- | (\bar{d}s)_{V-A} | 0 \rangle \left(a_4^p - \frac{1}{2} a_{10}^p \right) \\ &+ \langle K^- | \bar{s}b | B^- \rangle \langle K^+ \pi^- | \bar{d}s | 0 \rangle (-2a_6^p + a_8^p) + \langle K^+ K^- \pi^- | (\bar{d}u)_{V-A} | 0 \rangle \langle 0 | (\bar{u}b)_{V-A} | B^- \rangle (a_1 \delta_{pu} + a_4^p \\ &+ a_{10}^p) + \langle K^+ K^- \pi^- | \bar{d}\gamma_5 u | 0 \rangle \langle 0 | \bar{u}\gamma_5 b | B^- \rangle (2a_6^p - a_8^p). \end{aligned} \tag{A9}$$

$B \to K\pi\pi$

$$\begin{aligned}\langle K^- \pi^+ \pi^- | T_p | B^- \rangle &= \langle \pi^+ \pi^- | (\bar{u}b)_{V-A} | B^- \rangle \langle K^- | (\bar{s}u)_{V-A} | 0 \rangle [a_1 \delta_{pu} + a_4^p + a_{10}^p - (a_6^p + a_8^p) r_\chi^K] \\ &+ \langle K^- | (\bar{s}b)_{V-A} | B^- \rangle \langle \pi^+ \pi^- | (\bar{u}u)_{V-A} | 0 \rangle \left[a_2 \delta_{pu} + \frac{3}{2}(a_7 + a_9) \right] \\ &+ \langle K^- | \bar{s}b | B^- \rangle \langle \pi^+ \pi^- | \bar{s}s | 0 \rangle (-2a_6^p + a_8^p) + \langle \pi^- | (\bar{d}b)_{V-A} | B^- \rangle \langle K^- \pi^+ | (\bar{s}d)_{V-A} | 0 \rangle \left(a_4^p - \frac{1}{2} a_{10}^p \right) \\ &+ \langle \pi^- | \bar{d}b | B^- \rangle \langle K^- \pi^+ | \bar{s}d | 0 \rangle (-2a_6^p + a_8^p) + \langle K^- \pi^+ \pi^- | (\bar{s}u)_{V-A} | 0 \rangle \langle 0 | (\bar{u}b)_{V-A} | B^- \rangle \\ &\times (a_1 \delta_{pu} + a_4^p + a_{10}^p) + \langle K^- \pi^+ \pi^- | \bar{s}\gamma_5 u | 0 \rangle \langle 0 | \bar{u}\gamma_5 b | B^- \rangle (2a_6^p + 2a_8^p). \end{aligned} \tag{A10}$$

$$\begin{aligned}\langle \bar{K}^0 \pi^+ \pi^- | T_p | \bar{B}^0 \rangle &= \langle \pi^+ \pi^- | (\bar{d}b)_{V-A} | \bar{B}^0 \rangle \langle \bar{K}^0 | (\bar{s}d)_{V-A} | 0 \rangle \left[a_4^p - \frac{1}{2} a_{10}^p - \left(a_6^p - \frac{1}{2} a_8^p \right) r_\chi^K \right] \\ &+ \langle \bar{K}^0 | (\bar{s}b)_{V-A} | \bar{B}^0 \rangle \langle \pi^+ \pi^- | (\bar{u}u)_{V-A} | 0 \rangle \left[a_2 \delta_{pu} + \frac{3}{2}(a_7 + a_9) \right] \\ &+ \langle \bar{K}^0 | \bar{s}b | \bar{B}^0 \rangle \langle \pi^+ \pi^- | \bar{s}s | 0 \rangle (-2a_6^p + a_8^p) + \langle \pi^+ | (\bar{u}b)_{V-A} | \bar{B}^0 \rangle \langle \bar{K}^0 \pi^- | (\bar{s}u)_{V-A} | 0 \rangle (a_1 + a_4^p + a_{10}^p) \\ &+ \langle \pi^+ | \bar{u}b | \bar{B}^0 \rangle \langle \bar{K}^0 \pi^- | \bar{s}u | 0 \rangle (-2a_6^p - 2a_8^p) + \langle \bar{K}^0 \pi^+ \pi^- | (\bar{s}d)_{V-A} | 0 \rangle \langle 0 | (\bar{d}b)_{V-A} | \bar{B}^0 \rangle \\ &\times (a_1 \delta_{pu} + a_4^p + a_{10}^p) + \langle \bar{K}^0 \pi^+ \pi^- | \bar{s}(1 + \gamma_5) d | 0 \rangle \langle 0 | \bar{d}\gamma_5 b | \bar{B}^0 \rangle (2a_6^p - a_8^p). \end{aligned} \tag{A11}$$

$$\begin{aligned}\langle K^- \pi^+ \pi^0 | T_p | \bar{B}^0 \rangle &= \langle \pi^+ \pi^0 | (\bar{u}b)_{V-A} | \bar{B}^0 \rangle \langle K^- | (\bar{s}u)_{V-A} | 0 \rangle [a_1 \delta_{pu} + a_4^p + a_{10}^p - (a_6^p + a_8^p) r_\chi^K] \\ &+ \langle K^- \pi^+ | (\bar{s}b)_{V-A} | \bar{B}^0 \rangle \langle \pi^0 | (\bar{u}u)_{V-A} | 0 \rangle \left[a_2 \delta_{pu} + \frac{3}{2}(-a_7 + a_9) \right] \\ &+ \langle \pi^+ | (\bar{u}b)_{V-A} | \bar{B}^0 \rangle \langle K^- \pi^0 | (\bar{s}u)_{V-A} | 0 \rangle [a_1 \delta_{pu} + a_4^p + a_{10}^p] + \langle \pi^0 | (\bar{d}b)_{V-A} | \bar{B}^0 \rangle \langle K^- \pi^+ | (\bar{s}d)_{V-A} | 0 \rangle \\ &\times \left(a_4^p - \frac{1}{2} a_{10}^p \right) + \langle \pi^+ | \bar{u}b | \bar{B}^0 \rangle \langle K^- \pi^0 | \bar{s}u | 0 \rangle (-2a_6^p - 2a_8^p) + \langle \pi^0 | \bar{d}b | \bar{B}^0 \rangle \langle K^- \pi^+ | \bar{s}d | 0 \rangle (-2a_6^p + a_8^p) \\ &+ \langle K^- \pi^+ \pi^0 | (\bar{s}d)_{V-A} | 0 \rangle \langle 0 | (\bar{d}b)_{V-A} | \bar{B}^0 \rangle \left(a_4^p - \frac{1}{2} a_{10}^p \right) + \langle K^- \pi^+ \pi^0 | \bar{s}(1 + \gamma_5) d | 0 \rangle \langle 0 | \bar{d}\gamma_5 b | \bar{B}^0 \rangle \\ &\times (2a_6^p - a_8^p). \end{aligned} \tag{A12}$$

$$\langle \bar{K}^0\pi^-\pi^0|T_p|B^-\rangle = \langle\pi^0\pi^-|(\bar{d}b)_{V-A}|B^-\rangle\langle\bar{K}^0|(\bar{s}d)_{V-A}|0\rangle\left[a_4^p - \frac{1}{2}a_{10}^p - \left(a_6^p - \frac{1}{2}a_8^p\right)r_\chi^K\right] + \langle\bar{K}^0\pi^-|(\bar{s}b)_{V-A}|B^-\rangle\langle\pi^0|(\bar{u}u)_{V-A}|0\rangle\left[a_2\delta_{pu} + \frac{3}{2}(-a_7 + a_9)\right] + \langle\pi^0|(\bar{u}b)_{V-A}|B^-\rangle\langle\bar{K}^0\pi^-|(\bar{s}u)_{V-A}|0\rangle[a_1\delta_{pu} + a_4^p + a_{10}^p] + \langle\pi^-|(\bar{d}b)_{V-A}|B^-\rangle\langle\bar{K}^0\pi^0|(\bar{s}d)_{V-A}|0\rangle \times\left[a_4^p - \frac{1}{2}a_{10}^p\right] + \langle\pi^0|\bar{u}b|B^-\rangle\langle\bar{K}^0\pi^-|\bar{s}u|0\rangle(-2a_6^p - 2a_8^p) + \langle\pi^-|\bar{d}b|B^-\rangle\langle\bar{K}^0\pi^0|\bar{s}d|0\rangle(-2a_6^p + a_8^p) + \langle\bar{K}^0\pi^-\pi^0|(\bar{s}u)_{V-A}|0\rangle\langle 0|(\bar{u}b)_{V-A}|B^-\rangle(a_1\delta_{pu} + a_4^p + a_{10}^p) + \langle\bar{K}^0\pi^-\pi^0|\bar{s}(1+\gamma_5)u|0\rangle \times\langle 0|\bar{u}\gamma_5 b|B^-\rangle(2a_6^p + 2a_8^p). \tag{A13}$$

$$\langle \bar{K}^0\pi^0\pi^0|T_p|\bar{B}^0\rangle = \langle\pi^0\pi^0|(\bar{d}b)_{V-A}|\bar{B}^0\rangle\langle\bar{K}^0|(\bar{s}d)_{V-A}|0\rangle\left[a_4^p - \frac{1}{2}a_{10}^p - (a_6^p - \frac{1}{2}a_8^p)r_\chi^K\right] + \langle\bar{K}^0\pi^0|(\bar{s}b)_{V-A}|\bar{B}^0\rangle\langle\pi^0|(\bar{u}u)_{V-A}|0\rangle\left[a_2\delta_{pu} + \frac{3}{2}(-a_7 + a_9)\right] + \langle\pi^0|(\bar{d}b)_{V-A}|\bar{B}^0\rangle\langle\bar{K}^0\pi^0|(\bar{s}d)_{V-A}|0\rangle \times\left[a_4^p - \frac{1}{2}a_{10}^p\right] + \langle\pi^0\pi^0|(\bar{u}u)_{V-A}|0\rangle\langle\bar{K}^0|(\bar{s}b)_{V-A}|\bar{B}^0\rangle\left(a_2\delta_{pu} + 2a_3 + 2a_5 + \frac{1}{2}(a_7 + a_9)\right) + \langle\bar{K}^0\pi^0|\bar{s}d|0\rangle\langle\pi^0|\bar{d}b|\bar{B}^0\rangle(-2a_6^p + a_8^p) + \langle\pi^0\pi^0|\bar{s}s|0\rangle\langle\bar{K}^0|\bar{s}b|\bar{B}^0\rangle(-2a_6^p + a_8^p) + \langle\bar{K}^0\pi^0\pi^0|(\bar{s}d)_{V-A}|0\rangle\langle 0|(\bar{d}b)_{V-A}|\bar{B}^0\rangle\left(a_4^p - \frac{1}{2}a_{10}^p\right) + \langle\bar{K}^0\pi^0\pi^0|\bar{s}(1+\gamma_5)d|0\rangle\langle 0|\bar{d}\gamma_5 b|\bar{B}^0\rangle \times(2a_6^p - a_8^p). \tag{A14}$$

$B \to \pi\pi\pi$

$$\langle\pi^-\pi^+\pi^-|T_p|B^-\rangle = \langle\pi^+\pi^-|(\bar{u}b)_{V-A}|B^-\rangle\langle\pi^-|(\bar{d}u)_{V-A}|0\rangle[a_1\delta_{pu} + a_4^p + a_{10}^p - (a_6^p + a_8^p)r_\chi^\pi] + \langle\pi^-|(\bar{d}b)_{V-A}|B^-\rangle\langle\pi^+\pi^-|(\bar{u}u)_{V-A}|0\rangle\left[a_2\delta_{pu} - a_4^p + \frac{3}{2}(a_7 + a_9) + \frac{1}{2}a_{10}^p\right] + \langle\pi^-|\bar{d}b|B^-\rangle\langle\pi^+\pi^-|\bar{d}d|0\rangle(-2a_6^p + a_8^p) + \langle\pi^-\pi^+\pi^-|(\bar{d}u)_{V-A}|0\rangle\langle 0|(\bar{u}b)_{V-A}|B^-\rangle \times(a_1\delta_{pu} + a_4^p + a_{10}^p) + \langle\pi^-\pi^+\pi^-|\bar{d}(1+\gamma_5)u|0\rangle\langle 0|\bar{u}\gamma_5 b|B^-\rangle(2a_6^p + 2a_8^p). \tag{A15}$$

$$\langle\pi^0\pi^+\pi^-|T_p|\bar{B}^0\rangle = \langle\pi^+\pi^0|(\bar{u}b)_{V-A}|\bar{B}^0\rangle\langle\pi^-|(\bar{d}u)_{V-A}|0\rangle[a_1\delta_{pu} + a_4^p + a_{10}^p - (a_6^p + a_8^p)r_\chi^\pi] + \langle\pi^+\pi^-|(\bar{d}b)_{V-A}|\bar{B}^0\rangle\langle\pi^0|(\bar{u}u)_{V-A}|0\rangle\left[a_2\delta_{pu} - a_4^p + \left(a_6^p - \frac{1}{2}a_8^p\right)r_\chi^\pi + \frac{3}{2}(a_7 + a_9) + \frac{1}{2}a_{10}^p\right] + \langle\pi^+|(\bar{u}b)_{V-A}|\bar{B}^0\rangle\langle\pi^-\pi^0|(\bar{d}u)_{V-A}|0\rangle[a_1\delta_{pu} + a_4^p + a_{10}^p] + \langle\pi^0|(\bar{d}b)_{V-A}|\bar{B}^0\rangle\langle\pi^+\pi^-|(\bar{u}u)_{V-A}|0\rangle \times\left[a_2\delta_{pu} - a_4^p + \frac{3}{2}(a_7 + a_9) + \frac{1}{2}a_{10}^p\right] + \langle\pi^0|\bar{d}b|\bar{B}^0\rangle\langle\pi^+\pi^-|\bar{d}d|0\rangle(-2a_6^p + a_8^p). \tag{A16}$$

APPENDIX B: DECAY CONSTANTS, FORM FACTORS AND OTHERS

In this appendix we collect the numerical values of the decay constants, form factors, CKM matrix elements and quark masses needed for the calculations. We first discuss the decay constants of the pseudoscalar meson P and the scalar meson S defined by

$$\langle P(p)|\bar{q}_2\gamma_\mu\gamma_5 q_1|0\rangle = -if_P p_\mu, \quad \langle S(p)|\bar{q}_2\gamma_\mu q_1|0\rangle = f_S p_\mu, \quad \langle S|\bar{q}_2 q_1|0\rangle = m_S\bar{f}_S, \tag{B1}$$

and $\langle V(p,\varepsilon)|V_\mu|0\rangle = f_V m_V \varepsilon_\mu^*$ for the vector meson. For the scalar mesons, the vector decay constant f_S and the scale-dependent scalar decay constant $\bar{f}_S$ are related by equations of motion

$$\mu_S f_S = \bar{f}_S, \quad \text{with} \quad \mu_S = \frac{m_S}{m_2(\mu) - m_1(\mu)}, \tag{B2}$$

where m_2 and m_1 are the running current quark masses. The neutral scalar mesons σ, f_0 and a_0^0 cannot be produced via the vector current owing to charge conjugation invariance or conservation of vector current:

TABLE XIX. Form factors of $B \to \pi$, K, $K_0^*(1430)$, ρ transitions obtained in the covariant light-front model [61].

F	$F(0)$	$F(q^2_{\max})$	a	b	F	$F(0)$	$F(q^2_{\max})$	a	b
$F_1^{B\pi}$	0.25	1.16	1.73	0.95	$F_0^{B\pi}$	0.25	0.86	0.84	0.10
F_1^{BK}	0.35	2.17	1.58	0.68	F_0^{BK}	0.35	0.80	0.71	0.04
$F_1^{BK_0^*}$	0.26	0.70	1.52	0.64	$F_0^{BK_0^*}$	0.26	0.33	0.44	0.05
$V^{B\rho}$	0.27	0.79	1.84	1.28	$A_0^{B\rho}$	0.28	0.76	1.73	1.20
$A_1^{B\rho}$	0.22	0.53	0.95	0.21	$A_2^{B\rho}$	0.20	0.57	1.65	1.05
V^{BK^*}	0.31	0.96	1.79	1.18	$A_0^{BK^*}$	0.31	0.87	1.68	1.08
$A_1^{BK^*}$	0.26	0.58	0.93	0.19	$A_2^{BK^*}$	0.24	0.70	1.63	0.98

$$f_\sigma = f_{f_0} = f_{a_0^0} = 0. \tag{B3}$$

However, the decay constant $\bar f_S$ is nonvanishing. In [48] we have applied the QCD sum rules to estimate this quantity. In this work we follow [48] to use

$$\bar f_{f_0(980)} = 460\ \text{MeV}, \qquad \bar f_{K_0^*(1430)} = 550\ \text{MeV}, \tag{B4}$$

at $\mu = 2.1$ GeV. As for the decay constants of vector mesons, we use (in units of MeV).

$$f_\rho = 216, \qquad f_{K^*} = 218, \qquad f_\phi = 215. \tag{B5}$$

Form factors for $B \to P$ transitions are defined by [41]

$$\begin{aligned}
\langle P(p')|V_\mu|B(p)\rangle &= \left((p+p')_\mu - \frac{m_B^2 - m_P^2}{q^2} q_\mu\right) F_1^{BP}(q^2) + \frac{m_B^2 - m_P^2}{q^2} q_\mu F_0^{BP}(q^2),\\
\langle S(p')|A_\mu|B(p)\rangle &= -i\left[\left((p+p')_\mu - \frac{m_B^2 - m_S^2}{q^2} q_\mu\right) F_1^{BS}(q^2) + \frac{m_B^2 - m_S^2}{q^2} q_\mu F_0^{BS}(q^2)\right],\\
\langle V(p',\varepsilon)|V_\mu|B(p)\rangle &= \frac{2}{m_B + m_V}\epsilon_{\mu\nu\alpha\beta}\varepsilon^{*\nu}p^\alpha p'^\beta V(q^2),\\
\langle V(p',\varepsilon)|A_\mu|B(p)\rangle &= i\left[(m_B+m_V)\varepsilon^*_\mu A_1^{BV}(q^2) - \frac{\varepsilon^*\cdot p}{m_B+m_V}(p+p')_\mu A_2^{BV}(q^2) - 2m_V\frac{\varepsilon^*\cdot p}{q^2} q_\mu [A_3^{BV}(q^2) - A_0^{BV}(q^2)]\right],
\end{aligned} \tag{B6}$$

where $q = p - p'$, $F_1(0) = F_0(0)$, $A_3(0) = A_0(0)$, and

$$A_3(q^2) = \frac{m_P + m_V}{2m_V} A_1(q^2) - \frac{m_P - m_V}{2m_V} A_2(q^2), \tag{B7}$$

where $P_\mu = (p + p')_\mu$, $q_\mu = (p - p')_\mu$. As shown in [61], a factor of $(-i)$ is needed in $B \to S$ transition in order for the $B \to S$ form factors to be positive. This also can be checked from heavy quark symmetry [61].

Various form factors for $B \to S$ transitions have been evaluated in the relativistic covariant light-front quark model [61]. In this model form factors are first calculated in the spacelike region and their momentum dependence is fitted to a 3-parameter form

$$F(q^2) = \frac{F(0)}{1 - a(q^2/m_B^2) + b(q^2/m_B^2)^2}. \tag{B8}$$

The parameters a, b and $F(0)$ are first determined in the spacelike region. This parametrization is then analytically continued to the timelike region to determine the physical form factors at $q^2 \geq 0$. The results relevant for our purposes are summarized in Table XIX. In practical calculations, we shall assign the form factor error to be 0.03. For example, $F_{0,1}^{BK}(0) = 0.35 \pm 0.03$.

The form factor for B to $f_0(980)$ is of order 0.25 at $q^2 = 0$ [48]. In the $q\bar q$ model for the $f_0(980)$, $F^{Bf_0^u} = F^{Bf_0}\sin\theta/\sqrt{2}$.

For the heavy-flavor independent strong coupling g in HMChPT, we use $|g| = 0.59 \pm 0.01 \pm 0.07$ as extracted from the CLEO measurement of the D^{*+} decay width [39]. The sign is fixed to be negative in the quark model [23].

For the CKM matrix elements, we use the Wolfenstein parameters $A = 0.806$, $\lambda = 0.22717$, $\bar\rho = 0.195$ and $\bar\eta = 0.326$ [63]. The corresponding CKM angles are $(\sin 2\beta)_{\rm CKM} = 0.695^{+0.018}_{-0.016}$ and $\gamma = (59 \pm 7)^\circ$ [63]. For the running quark masses we shall use

$$m_b(m_b) = 4.2\ \text{GeV}, \qquad m_b(2.1\ \text{GeV}) = 4.95\ \text{GeV},$$
$$m_b(1\ \text{GeV}) = 6.89\ \text{GeV}, \qquad m_c(m_b) = 1.3\ \text{GeV},$$
$$m_c(2.1\ \text{GeV}) = 1.51\ \text{GeV}, \qquad m_s(2.1\ \text{GeV}) = 90\ \text{MeV},$$
$$m_s(1\ \text{GeV}) = 119\ \text{MeV}, \qquad m_d(1\ \text{GeV}) = 6.3\ \text{MeV},$$
$$m_u(1\ \text{GeV}) = 3.5\ \text{MeV}. \tag{B9}$$

The uncertainty of the strange quark mass is specified as $m_s(2.1\ \text{GeV}) = 90 \pm 20$ MeV.

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