Branching ratios and CP asymmetries of $B \to a_1(1260)\pi$ and $a_1(1260)K$ decays

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We present the studies of the decays $B \to a_1(1260)\pi$ and $a_1(1260)K$ within the framework of QCD factorization. Because of the G-parity, unlike the vector meson, the chiral-odd two-parton light-cone distribution amplitudes of the a_1 are antisymmetric under the exchange of quark and antiquark momentum fractions in the SU(2) limit. The branching ratios for $a_1\pi$ modes are sensitive to tree-penguin interference. The resultant $\mathcal{B}(B^0 \to a_1^\pm \pi^\mp)$ are in good agreement with the data. However, using the current Cabibbo-Kobayashi-Maskawa angles, $\beta = 22.0°$ and $\gamma = 59.0°$, our results for the mixing-induced parameter S and $\alpha_{\rm eff}$ differ from the measurements of the time-dependent CP asymmetries in the decay $B^0 \to a_1^\pm \pi^\mp$ at about the 3.7σ level. This puzzle may be resolved by using a larger $\gamma \gtrsim 80°$. For $a_1 K$ modes, the annihilation topologies give sizable contributions and are sensitive to the first Gegenbauer moment of the leading-twist tensor (chiral-odd) distribution amplitude of the a_1 meson. The $B \to a_1 K$ amplitudes resemble the corresponding $B \to \pi K$ ones very much. Taking the ratios of corresponding CP-averaged $a_1 K$ and πK branching ratios, we can extract information relevant to the electroweak penguins and annihilations. The existence of new physics in the electroweak penguin sector and final-state interactions during decays can thus be explored.

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I. INTRODUCTION

The first charmless hadronic B decay involving a 1^3P_1 axial-vector meson that has been observed is $B^0 \to a_1^\pm(1260)\pi^\mp$ [1–5], which goes through $b \to u\bar{u}d$. The measurements of time-dependent CP asymmetries in hadronic B decays originating from $b \to u\bar{u}d$ can provide the information directly related to the Cabibbo-Kobayashi-Maskawa (CKM) weak phase $\alpha \equiv \arg(-V_{td}V_{tb}^*/V_{ud}V_{ub}^*)$ (or called ϕ_2), for which some results have been given from the data of $B \to \pi^+\pi^-$, $\rho^\pm\pi^\mp$, and $\rho^\pm\rho^\mp$ [6]. The *BABAR* collaboration recently reported the observation of $B^0 \to a_1^\pm(1260)\pi^\mp$, including CP violating parameters, branching fractions, and $\alpha_{\rm eff}$, where the bound on the difference $\Delta\alpha = \alpha - \alpha_{\rm eff}$ can be constrained by using the broken SU(3) flavor symmetry [7,8] or isospin analysis [9–11].

In this paper, we present the phenomenological studies of $B \to a_1\pi$ and $a_1 K$ within the framework of QCD factorization, where the former processes are tree dominated, while the latter are penguin dominated. The $a_1(1260)$, which will be denoted by a_1 for simplicity, is the 1^3P_1 state. Because of the G-parity, the chiral-even two-parton light-cone distribution amplitudes (LCDAs) of the a_1 are symmetric under the exchange of *quark* and *antiquark* momentum fractions in the SU(2) limit, whereas, unlike the vector meson, the chiral-odd two-parton LCDAs are antisymmetric. Reference [12] is the only literature so far for the calculation of LCDAs of 1^3P_1 axial-vector mesons. The large first Gegenbauer moment of the leading-twist tensor distribution amplitude of the a_1 meson [12] could have a sizable impact on the annihilation amplitudes. On the other hand, it is interesting to note that, for the axial-vector mesons with quantum number 1^1P_1, their chiral-even LCDAs are antisymmetric under the exchange of *quark* and *antiquark* momentum fractions in the SU(3) limit, while the chiral-odd two-parton LCDAs are symmetric [12,13]. The hadronic B decays involving such a meson are sensitive to the new-physics search [14,15].

Because the axial-vector and pseudoscalar penguin contributions interfere constructively in the dominant decay amplitudes of $\bar{B} \to a_1\bar{K}$, for which the $\bar{K}$ is emitted and a_1 shares the same spectator quark within the $\bar{B}$ meson, the $\bar{B} \to a_1\bar{K}$ amplitudes resemble very much the corresponding $\bar{B} \to \pi\bar{K}$ ones. Moreover, larger CP asymmetries could be expected in the $a_1^0K^-$ and $a_1^0\bar{K}^0$ modes due to the much larger decay constant of the $a_1(1260)$, as compared with $\pi\bar{K}$ channels.

To resolve the puzzle about the observations of the decays $B \to \pi K$ and $\pi\pi$ within the standard model (SM) [6], some approaches were proposed, including considerations of final-state interactions (FSIs) [16–18], and use of SU(3) flavor symmetry to extract hadronic parameters from the $\pi\pi$ data and then to predict $K\pi$ channels [19–21]. On the other hand, it was argued that new physics with a large CP-violating phase may exist in the electroweak penguin sector [19,20,22]. The present studies for $B \to a_1\pi$ and $a_1 K$ modes can offer further tests for the above theories.

The layout of the present paper is as follows. In Sec. II, we discuss light-cone distribution amplitudes for an axial-vector meson. A brief description for applying QCD factorization to the decays $B \to a_1\pi$ and $a_1 K$ is given in Sec. III, where some relevant formulas are collected in Appendices A and B. In terms of the notations α_i^p and β_i^p, which were given in Ref. [23], one can find that the amplitudes for AP modes have the same expressions with those for PP and VP modes (where $A \equiv$ the axial-vector meson, $P \equiv$ the pseudoscalar meson, and $A \equiv$ the vector meson). Section IV contains the numerical analysis for the branching ratios and CP asymmetries. Our conclusions are summarized in Sec. V.

II. TWO-PARTON LCDAs OF THE a_1 AND PROJECTION OPERATORS ON THE LIGHT CONE

For decays involving an axial-vector meson (denoted as A) in the final state, the QCD corrections can turn the local quark-antiquark operators into a series of nonlocal operators as

$$\langle A(P,\lambda)|\bar{q}_{1\alpha}(y)q_{2\delta}(x)|0\rangle = -\frac{i}{4}\int_0^1 du e^{i(upy+\bar{u}px)}\Big\{f_A m_A\Big(\not{p}\gamma_5\frac{\epsilon^{*(\lambda)}z}{pz}\Phi_\parallel(u) + \not{\epsilon}_\perp^{*(\lambda)}\gamma_5 g_\perp^{(a)}(u) + \epsilon_{\mu\nu\rho\sigma}\epsilon_{(\lambda)}^{*\nu}p^\rho z^\sigma\gamma^\mu\frac{g_\perp^{(v)}(u)}{4}\Big) - f_A^\perp\Big(\not{\epsilon}_\perp^{*(\lambda)}\not{p}\gamma_5\Phi_\perp(u) - i\frac{m_A^2\epsilon^{*(\lambda)}z}{(pz)^2}\sigma_{\mu\nu}\gamma_5 p^\mu z^\nu h_\parallel^{(t)}(u) - im_A^2(\epsilon^{*(\lambda)}z)\frac{h_\parallel^{(p)}(u)}{2}\Big)\Big\}_{\delta\alpha}, \tag{2.1}$$

where the chiral-even LCDAs are given by

$$\langle A(P,\lambda)|\bar{q}_1(y)\gamma_\mu\gamma_5 q_2(x)|0\rangle = if_A m_A\int_0^1 du e^{i(upy+\bar{u}px)}\Big\{p_\mu\frac{\epsilon^{*(\lambda)}z}{pz}\Phi_\parallel(u) + \epsilon_{\perp\mu}^{*(\lambda)}g_\perp^{(a)}(u)\Big\}, \tag{2.2}$$

$$\langle A(P,\lambda)|\bar{q}_1(y)\gamma_\mu q_2(x)|0\rangle = -if_A m_A\epsilon_{\mu\nu\rho\sigma}\epsilon_{(\lambda)}^{*\nu}p^\rho z^\sigma\int_0^1 du e^{i(upy+\bar{u}px)}\frac{g_\perp^{(v)}(u)}{4}, \tag{2.3}$$

with $u(\bar{u}=1-u)$ being the momentum fraction carried by $q_1(\bar{q}_2)$, and the chiral-odd LCDAs are given by

$$\langle A(P,\lambda)|\bar{q}_1(y)\sigma_{\mu\nu}\gamma_5 q_2(x)|0\rangle = f_A^\perp\int_0^1 du e^{i(upy+\bar{u}px)}\Big\{(\epsilon_{\perp\mu}^{*(\lambda)}p_\nu - \epsilon_{\perp\nu}^{*(\lambda)}p_\mu)\Phi_\perp(u) + \frac{m_A^2\epsilon^{*(\lambda)}z}{(pz)^2}(p_\mu z_\nu - p_\nu z_\mu)h_\parallel^{(t)}(u)\Big\}, \tag{2.4}$$

$$\langle A(P,\lambda)|\bar{q}_1(y)\gamma_5 q_2(x)|0\rangle = f_A^\perp m_A^2\epsilon^{*(\lambda)}z\int_0^1 du e^{i(upy+\bar{u}px)}\frac{h_\parallel^{(p)}(u)}{2}. \tag{2.5}$$

Here, throughout the present discussion, we define $z = y - x$ with $z^2 = 0$, and introduce the lightlike vector $p_\mu = P_\mu - m_A^2 z_\mu/(2Pz)$ with the meson's momentum $P^2 = m_A^2$. Moreover, the meson polarization vector ϵ_μ^* has been decomposed into longitudinal ($\epsilon_{\parallel\mu}^*$) and transverse ($\epsilon_{\perp\mu}^*$) *projections* defined as

$$\epsilon_{\parallel\mu}^* \equiv \frac{\epsilon^* z}{Pz}\Big(P_\mu - \frac{m_A^2}{Pz}z_\mu\Big), \qquad \epsilon_{\perp\mu}^* = \epsilon_\mu^* - \epsilon_{\parallel\mu}^*, \tag{2.6}$$

respectively. The LCDAs $\Phi_\parallel$, $\Phi_\perp$ are of twist-2, and $g_\perp^{(v)}$, $g_\perp^{(a)}$, $h_\perp^{(t)}$, $h_\parallel^{(p)}$ of twist-3. For the a_1 meson, due to G-parity, $\Phi_\parallel$, $g_\perp^{(v)}$, and $g_\perp^{(a)}$ are symmetric with the replacement of $u \leftrightarrow 1-u$, whereas $\Phi_\perp$, $h_\parallel^{(t)}$, and $h_\parallel^{(p)}$ are antisymmetric in the SU(2) limit [12]. Here, we restrict ourselves to two-parton LCDAs with twist-3 accuracy.

Assuming that the axial-vector meson moves along the negative z-axis, the derivation for the light-cone projection operator of an axial-vector meson in the momentum space is in complete analogy to the case of the vector meson. We separate the longitudinal and transverse parts for the projection operator:

$$M_{\delta\alpha}^A = M_{\delta\alpha\parallel}^A + M_{\delta\alpha\perp}^A, \tag{2.7}$$

where only the longitudinal part is relevant in the present study and given by

$$M_\parallel^A = -i\frac{f_A}{4}\frac{m_A(\epsilon^* n_+)}{2}\not{n}_-\gamma_5\Phi_\parallel(u) - \frac{if_A^\perp m_A}{4}\frac{m_A(\epsilon^* n_+)}{2E}\times\Big\{\frac{i}{2}\sigma_{\mu\nu}\gamma_5 n_-^\mu n_+^\nu h_\parallel^{(t)}(u) + iE\int_0^u dv(\Phi_\perp(v) - h_\parallel^{(t)}(v))\sigma_{\mu\nu}\gamma_5 n_-^\mu\frac{\partial}{\partial k_{\perp\nu}} - \gamma_5\frac{h_\parallel^{\prime(p)}(u)}{2}\Big\}\Big|_{k=up}, \tag{2.8}$$

with the momentum of the quark q_1 in the A meson being

$$k_1^\mu = uEn_-^\mu + k_\perp^\mu + \frac{k_\perp^2}{4uE}n_+^\mu, \tag{2.9}$$

for which E is the energy of the axial-vector meson and the term proportional to $k_\perp^2$ is negligible. Here, for simplicity, we introduce two lightlike vectors $n_-^\mu \equiv (1,0,0,-1)$, and $n_+^\mu \equiv (1,0,0,1)$. In general, the QCD factorization amplitudes can be written in terms of the form $\int_0^1 du\,\mathrm{Tr}(M_\parallel^A\cdots)$.

In the following, we will give a brief summary for LCDAs of the a_1 mesons, for which the detailed properties can be found in Ref. [12]. $\Phi_{\parallel,\perp}^{a_1}(u)$ can be expanded in

Gegenbauer polynomials:

$$\Phi^{a_1}_{\parallel(\perp)}(u) = 6u\bar{u}\left[\sum_{i=0}^{\infty} a_i^{\parallel(\perp),a_1} C_i^{3/2}(2u-1)\right]. \tag{2.10}$$

For the $\Phi^{a_1}_{\parallel(\perp)}(u)$, due to the G-parity, only terms with even (odd) Gegenbauer moments survive in the SU(2) limit. In the present work, we consider the approximations:

$$\Phi^{a_1}_{\parallel}(u) = 6u\bar{u}\{1 + a_2^{\parallel,a_1}\tfrac{3}{2}[5(2u-1)^2 - 1]\}, \tag{2.11}$$

$$\Phi^{a_1}_{\perp}(u) = 18 a_1^{\perp,a_1} u\bar{u}(2u-1). \tag{2.12}$$

Note that we have defined $f_{a_1}^{\perp} = f_{a_1}$ since the product $f_{a_1}^{\perp} a_1^{\perp,a_1}$ always appears together. Neglecting the three-parton distributions and terms proportional to the light quark masses, we can relate the twist-3 distribution amplitudes to the twist-2 ones by Wandzura-Wilczek relations [12,24] and then obtain:

$$\begin{aligned}
h_{\parallel}^{(t)}(v) &= (2v-1)\left[\int_0^v \frac{\Phi_\perp(u)}{\bar{u}}du - \int_v^1 \frac{\Phi_\perp(u)}{u}du\right] \equiv (2v-1)\Phi_a(v),\\
h_{\parallel}^{\prime(p)}(v) &= -2\left[\int_0^v \frac{\Phi_\perp(u)}{\bar{u}}du - \int_v^1 \frac{\Phi_\perp(u)}{u}du\right] \equiv -2\Phi_a(v),\\
\int_0^v du(\Phi_\perp(u) - h_{\parallel}^{(t)}(u)) &= v\bar{v}\left[\int_0^v \frac{\Phi_\perp(u)}{\bar{u}}du - \int_v^1 \frac{\Phi_\perp(u)}{u}du\right] \equiv v\bar{v}\Phi_a(v).
\end{aligned} \tag{2.13}$$

The normalization conditions for LCDAs are

$$\int_0^1 du\Phi_{\parallel}(u) = 1, \qquad \int_0^1 du\Phi_{\perp}(u) = 0, \tag{2.14}$$

$$\int_0^1 du h_{\parallel}^{(t)}(u) = 0, \qquad \int_0^1 du h_{\parallel}^{(p)}(u) = 0. \tag{2.15}$$

For the pseudoscalar meson (P) with the four-momentum P_μ, the light-cone projection operator in the momentum space reads

$$\begin{aligned}
M^P = {} & i\frac{f_P}{4}E\not{n}_-\gamma_5\Phi_P(u) + \frac{if_P\mu_P}{4}\left\{\frac{i}{2}\sigma_{\mu\nu}\gamma_5 n_-^\mu n_+^\nu\frac{\phi'_\sigma(u)}{6}\right.\\
& \left.- iE\frac{\phi_\sigma}{6}\sigma_{\mu\nu}\gamma_5 n_-^\mu\frac{\partial}{\partial k_{\perp\nu}} - \gamma_5\frac{\phi_p(u)}{2}\right\}\Bigg|_{k=up},
\end{aligned} \tag{2.16}$$

where $\mu_P = m_P^2/(m_1+m_2)$ is proportional to the chiral condensate (with $m_{1,2}$ the masses of quarks) and the approximate forms of LCDAs that we use are

$$\begin{aligned}
\Phi_P(u) &= 6u\bar{u}\left\{1 + 3a_1^P(2u-1) + a_2^P\frac{3}{2}[5(2u-1)^2 - 1]\right\},\\
\Phi_p(u) &= 1, \qquad \frac{\Phi_\sigma(u)}{6} = u(1-u).
\end{aligned} \tag{2.17}$$

III. DECAY AMPLITUDES

Within the framework of QCD factorization, in general the effective weak Hamiltonian matrix elements for $\bar{B} \to M_1M_2$ decays can be expressed in the form [23]

$$\langle M_1M_2|\mathcal{H}_{\rm eff}|\bar{B}\rangle = \frac{G_F}{\sqrt{2}}\sum_{p=u,c}\lambda_p\langle M_1M_2|\mathcal{T}_{\mathcal{A}}^p + \mathcal{T}_{\mathcal{B}}^p|\bar{B}\rangle, \tag{3.1}$$

where $\lambda_p \equiv V_{pb}V_{pq}^*$ with $q \equiv d$ or s, M_2 is the emitted meson, and M_1 shares the same spectator quark within the $\bar{B}$ meson. Considering a generic b-quark decay, $\mathcal{T}_{\mathcal{A}}^p$ describe contributions from naive factorization, vertex corrections, penguin contractions, and spectator scattering, whereas $\mathcal{T}_{\mathcal{B}}^p$ contain the weak annihilation topologies.

For $\bar{B}$ decay processes, the QCD factorization approach advocated in [25,26] allows us to compute the nonfactorizable corrections in the heavy quark limit since only hard interactions between the $(\bar{B}M_1)$ system and M_2 survive in the $m_b \to \infty$ limit. Naive factorization is recovered in the heavy quark limit and to the zeroth order of QCD corrections. In this approach, the LCDAs play an essential role. In the present study using the notations α_i^p and β_i^p given in Ref. [23], the amplitudes for AP modes have the same expressions with those for PP and VP modes; $\bar{B} \to a_1\pi$, $a_1\bar{K}$ decay amplitudes in terms of α_i^p and β_i^p can be obtained from $\bar{B} \to \rho\pi$, $\rho\bar{K}$ [23] by setting $\rho \to a_1$. However, one should note that the determination of the relative signs of the detailed amplitudes behind the coefficients α_i^p and β_i^p is nontrivial.

A. Decay amplitudes due to $\mathcal{T}_{\mathcal{A}}^p$

In general, $\mathcal{T}_{\mathcal{A}}^p$ can be expressed in terms of $c\alpha_i^p(M_1M_2)X^{(\bar{B}M_1,M_2)}$, where c contains factors of ±1 and $\pm1/\sqrt{2}$ arising from flavor structures of final-state mesons, α_i are functions of the Wilson coefficients [see Eq. (3.7)], and

$$\begin{aligned}
X^{(\bar{B}A,P)} &= \langle P(q)|(V-A)_\mu|0\rangle\langle A(p)|(V-A)^\mu|\bar{B}(p_B)\rangle\\
&= -2if_Pm_AV_0^{BA}(q^2)(\epsilon^*_{(\lambda)}p_B),
\end{aligned} \tag{3.2}$$

$$\begin{aligned}
X^{(\bar{B}P,A)} &= \langle A(q)|(V-A)_\mu|0\rangle\langle P(p)|(V-A)^\mu|\bar{B}(p_B)\rangle\\
&= -2if_Am_AF_1^{BP}(q^2)(\epsilon^*_{(\lambda)}p_B).
\end{aligned} \tag{3.3}$$

Here the decay constants of the pseudoscalar meson P and the axial-vector meson A are defined by [27]

$$\langle P(p)|\bar{q}_2\gamma_\mu\gamma_5 q_1|0\rangle = -if_P p_\mu, \qquad \langle A(p,\lambda)|\bar{q}_2\gamma_\mu\gamma_5 q_1|0\rangle = if_A\epsilon_\mu^{(\lambda)*}. \tag{3.4}$$

The form factors for the $B \to A$ and P transitions are defined as [27]

$$\begin{aligned}
\langle A(p,\lambda)|A_\mu|\bar{B}(p_B)\rangle &= i\frac{2}{m_B+m_A}\epsilon_{\mu\nu\alpha\beta}\epsilon^{*\nu}_{(\lambda)}p_B^\alpha p^\beta A^{BA}(q^2),\\
\langle A(p,\lambda)|V_\mu|\bar{B}(p_B)\rangle &= -\left[(m_B+m_A)\epsilon_\mu^{(\lambda)*}V_1^{BA}(q^2) - (\epsilon^{(\lambda)*}p_B)(p_B+p)_\mu\frac{V_2^{BA}(q^2)}{m_B+m_A}\right]\\
&\quad + 2m_A\frac{\epsilon^{(\lambda)*}p_B}{q^2}q^\mu[V_3^{BA}(q^2)-V_0^{BA}(q^2)],\\
\langle P(p)|V_\mu|\bar{B}(p_B)\rangle &= \left[(p_B+p)_\mu - \frac{m_B^2-m_P^2}{q^2}q_\mu\right]F_1^{BP}(q^2) + \frac{m_B^2-m_P^2}{q^2}q_\mu F_0^{BP}(q^2),
\end{aligned} \tag{3.5}$$

where $q = p_B - p$, $V_3^{BA}(0) = V_0^{BA}(0)$, $F_1^{BP}(0) = F_0^{BP}(0)$, and

$$V_3^{BA}(q^2) = \frac{m_B+m_A}{2m_A}V_1^{BA}(q^2) - \frac{m_B-m_A}{2m_A}V_2^{BA}(q^2). \tag{3.6}$$

The coefficients of the flavor operators α_i^p can be expressed in terms of a_i^p as follows:

$$\begin{aligned}
\alpha_1(M_1M_2) &= a_1(M_1M_2),\\
\alpha_2(M_1M_2) &= a_2(M_1M_2),\\
\alpha_3^p(M_1M_2) &= a_3^p(M_1M_2) - a_5^p(M_1M_2),\\
\alpha_4^p(M_1M_2) &= \begin{cases} a_4^p(M_1M_2) + r_\chi^{M_2}a_6^p(M_1M_2) & \text{for } M_1M_2 = AP,\\ a_4^p(M_1M_2) - r_\chi^{M_2}a_6^p(M_1M_2) & \text{for } M_1M_2 = PA,\end{cases}\\
\alpha_{3,\mathrm{EW}}^p(M_1M_2) &= a_9^p(M_1M_2) - a_7^p(M_1M_2),\\
\alpha_{4,\mathrm{EW}}^p(M_1M_2) &= \begin{cases} a_{10}^p(M_1M_2) + r_\chi^{M_2}a_8^p(M_1M_2) & \text{for } M_1M_2 = AP,\\ a_{10}^p(M_1M_2) - r_\chi^{M_2}a_8^p(M_1M_2) & \text{for } M_1M_2 = PA,\end{cases}
\end{aligned} \tag{3.7}$$

where

$$r_\chi^P(\mu) = \frac{2m_P^2}{m_b(\mu)(m_2+m_1)(\mu)}, \qquad r_\chi^A(\mu) = \frac{2m_A}{m_b(\mu)}. \tag{3.8}$$

The effective parameters α_i^p in Eq. (3.7) to next-to-leading order in α_s can be expressed in forms of [23]:

$$\begin{aligned}
a_i^p(M_1M_2) &= \left(c_i + \frac{c_{i\pm1}}{N_c}\right)N_i(M_2)\\
&\quad + \frac{c_{i\pm1}}{N_c}\frac{C_F\alpha_s}{4\pi}\left[V_i(M_2) + \frac{4\pi^2}{N_c}H_i(M_1M_2)\right]\\
&\quad + P_i^p(M_2),
\end{aligned} \tag{3.9}$$

where c_i are the Wilson coefficients, $p = u$, c, $C_F = (N_c^2-1)/(2N_c)$ with $N_c = 3$, the upper (lower) signs refer to odd (even) i, M_2 is the emitted meson, M_1 shares the same spectator quark within the B meson, and

$$N_i = \begin{cases} 0 & \text{for } i = 6, 8, \text{ and } M_2 = a_1,\\ 1 & \text{for the rest.}\end{cases} \tag{3.10}$$

$V_i(M_2)$ account for vertex corrections, $H_i(M_1M_2)$ for hard-spectator interactions with a hard gluon exchange between the emitted meson and the spectator quark of the $\bar{B}$ meson, and $P_i(M_2)$ for penguin contractions. The detailed results for the above quantities are collected in Appendix A. Note that in the present case, some relative signs change in H_i as compared with the PP and VP modes.

B. Decay amplitudes due to $\mathcal{T}_\mathcal{B}^p$—annihilation topologies

The $\bar{B} \to AP$ amplitudes governed by the annihilation topologies read

$$\begin{aligned}
\frac{G_F}{\sqrt{2}}\sum_{p=u,c}\lambda_p\langle AP|\mathcal{T}_\mathcal{B}^p|\bar{B}\rangle &= -i\frac{G_F}{\sqrt{2}}f_Bf_Af_P\\
&\quad\times\sum_{p=u,c}\lambda_p\left[\sum_{i=1}^4 e_ib_i + e_5b_{3,\mathrm{EW}}\right.\\
&\quad\left. + e_6b_{4,\mathrm{EW}}\right],
\end{aligned} \tag{3.11}$$

where the coefficients e_i are process dependent and weak annihilation contributions are parametrized as

$$
\begin{aligned}
b_1 &= \frac{C_F}{N_c^2} c_1 A_1^i, \\
b_3 &= \frac{C_F}{N_c^2}[c_3 A_1^i + c_5(A_3^i + A_3^f) + N_c c_6 A_3^f], \\
b_2 &= \frac{C_F}{N_c^2} c_2 A_1^i, \\
b_4 &= \frac{C_F}{N_c^2}[c_4 A_1^i + c_6 A_2^f], \\
b_{3,\mathrm{EW}} &= \frac{C_F}{N_c^2}[c_9 A_1^i + c_7(A_3^i + A_3^f) + N_c c_8 A_3^i], \\
b_{4,\mathrm{EW}} &= \frac{C_F}{N_c^2}[c_{10} A_1^i + c_8 A_2^i].
\end{aligned} \tag{3.12}
$$

The subscripts 1, 2, and 3 of $A_n^{i,f}$ denote the annihilation amplitudes induced from $(V-A)(V-A)$, $(V-A)(V+A)$, and $(S-P)(S+P)$ operators, respectively, and the superscripts i and f refer to gluon emission from the initial and final-state quarks, respectively. For decays $B \to AP$, the detailed expressions for $A_n^{i,f}$ are given in Appendix B. $\beta_i^p(M_1 M_2)$ are defined as

$$\beta_i^p(M_1 M_2) = \frac{-i f_B f_{M_1} f_{M_2}}{X^{(\bar{B}M_1, M_2)}} b_i^p.$$

IV. NUMERICAL RESULTS

A. Input parameters

In the numerical analysis, we use the next-to-leading Wilson coefficients in the naive dimensional regularization scheme [28]. The relevant parameters are summarized in Table I [29,30,32–34]. The value of f_B that we use is consistent with the lattice average [35]. The current value of $F^{B\pi}(0)$ becomes a little smaller, and is more suitable to explain the $\pi\pi$ data [6]. We use the light-cone sum rule results for the $B \to \pi$, K [32], and $B \to a_1$ [33] transition form factors, for which the momentum dependence is parametrized as [36]

$$f(q^2) = f(0)\left(\frac{1}{1 - q^2/m_{B^*}^2} + \frac{r_{BZ(Y)} q^2/m_{B^*}^2}{1 - \alpha_{BZ(Y)} q^2/m_B^2}\right), \tag{4.1}$$

where m_{B^*} is the lowest resonance in the corresponding channel. Note that, since the mass of the a_1 meson is not small, we have, for instance, $[F_1^{B\pi}(m_{a_1}^2)/F_1^{B\pi}(0)]^2 \simeq 1.2$. It means that the q^2 dependence of $B \to \pi$, K form factors cannot be ignored in the prediction. As for the $B \to a_1$ form factor, its q^2 dependence can be negligible due to the small mass of pseudoscalar mesons. However, to be consistent, I also consider its q^2 dependence in the analysis. Our light-cone sum rule result for $V_0^{Ba_1}(0)$ is a little larger than the previous QCD sum rule calculation, 0.23 ± 0.05 [37]. It is interesting to compare with other quark model calculations in the literature. The magnitude of $V_0^{Ba_1}(0)$ is

TABLE I. Summary of input parameters.

Running quark masses (GeV) and the strong coupling constant [23,29]			
$m_c(m_c)$	$m_s(2\ \mathrm{GeV})$	$(m_u + m_d)/(2m_s)$	$\alpha_s(1\ \mathrm{GeV})$
1.3	0.09 ± 0.01	0.0413	0.497

Wolfenstein parameters for the CKM matrix elements [30]			
A	λ	$\bar{\rho}$	$\bar{\eta}$
0.806	0.2272	0.195	0.326

Decay constants for mesons (MeV) [12,29,31]			
f_π	f_K	f_B	f_{a_1}
131	160	195 ± 10	238 ± 10

Form factors and parameters for their q^2 dependence [32,33]			
$F_1^{B\pi}(0) = 0.26 \pm 0.03$	$\alpha_{BZ} = 0.40$	$r_{BZ} = 0.64$	$m_1 = m_{B^*} = 5.32$ GeV
$F_1^{BK}(0) = 0.33 \pm 0.04$	$\alpha_{BZ} = 0.95$	$r_{BZ} = 0.52$	$m_1 = m_{B_s^*} = 5.41$ GeV
$V_0^{Ba_1}(0) = 0.28 \pm 0.03$	$\alpha_Y = 0.90$	$r_Y = 0.65$	$m_1 = m_{B^*} = 5.32$ GeV

Gegenbauer moments for leading-twist LCDAs of mesons at scale 1 GeV [12,34]				
a_2^π	a_1^K	a_2^K/a_2^π	$a_2^{\parallel,a_1}$	$a_1^{\perp,a_1}$
0.25 ± 0.15	0.06 ± 0.03	1.05 ± 0.15	-0.03 ± 0.02	-1.04 ± 0.34

about 0.13 and $1.02 \sim 1.22$ in the quark model calculations in Refs. [38–40], respectively. The magnitude of the former is too small and the latter is too large if using them to compute the branching ratios of $\bar{B}^0 \to a_1^{\pm}\pi^{\mp}$ and then comparing with the data. The values of the Gegenbauer moments of leading-twist LCDAs for the a_1 meson are quoted from Ref. [12]. The integral of the B meson wave function is parametrized as [25]

$$\int_0^1 \frac{d\rho}{1-\rho}\Phi_1^B(\rho) \equiv \frac{m_B}{\lambda_B}, \tag{4.2}$$

where $1-\rho$ is the momentum fraction carried by the light spectator quark in the B meson. Here we use $\lambda_B(1\ \text{GeV}) = (350 \pm 100)$ MeV.

There are three independent renormalization scales for describing the decay amplitudes. The corresponding scale will be specified as follows: (i) the scale $\mu_v = m_b/2$ for loop diagrams contributing to the vertex and penguin contributions to the hard-scattering kernels, (ii) $\mu_H = \sqrt{\mu_v \Lambda_h}$ for hard-spectator scattering, and (iii) $\mu_A = \sqrt{\mu_v \Lambda_h}$ for the annihilation with the hadronic scale $\Lambda_h \approx 500$ MeV. We follow [25] to parametrize the end point divergences $X_A \equiv \int_0^1 dx/\bar{x}$ and $X_H \equiv \int_0^1 dx/\bar{x}$ in the annihilation and hard-spectator diagrams, respectively, as

$$X_{A(H)} = \ln\left(\frac{m_B}{\Lambda_h}\right)(1 + \rho_{A(H)} e^{i\phi_{A(H)}}), \tag{4.3}$$

with the unknown real parameters ρ_A, ρ_H and ϕ_A, ϕ_H. We adopt the moderate value $\rho_{A,H} \lesssim 0.5$ and arbitrary strong phases $\phi_{A,H}$ with $\rho_{A,H} = 0$ by default, i.e., we assign a 50% uncertainty to the default value of $X_{A(H)}$ (with $\rho_{A,H} = 0$) [31,41]; with the allowed ranges of $\rho_{A,H}$, the theoretical predictions for πK modes are consistent with the data. Note that the $a_1 K$ rates could be sensitive to the magnitude of ρ_A.

B. Results

We follow the standard convention for the direct CP asymmetry

$$A_{CP}(\bar{f}) \equiv \frac{\mathcal{B}(\bar{B}^0 \to \bar{f}) - \mathcal{B}(B^0 \to f)}{\mathcal{B}(\bar{B}^0 \to \bar{f}) + \mathcal{B}(B^0 \to f)}. \tag{4.4}$$

The branching ratios given in the present paper are CP-averaged and simply denoted by $\mathcal{B}(\bar{B} \to f)$. The numerical results for CP-averaged branching ratios and direct CP asymmetries are summarized in Tables II and III, respectively. The results for time-dependent CP parameters of the decay $B(t) \to a_1^{\pm}\pi^{\mp}$ are shown in Table IV.

TABLE II. CP-averaged branching fractions for the decays $B \to a_1(1260)\pi$ and $a_1(1260)K$ (in units of 10^{-6}). The theoretical errors correspond to the uncertainties due to variation of (i) Gegenbauer moments, decay constants, (ii) quark masses, form factors, and (iii) λ_B, $\rho_{A,H}$, $\phi_{A,H}$, respectively, added in quadrature.

Mode	Theory	Experiment (*BABAR*) [4,5]	Experiment (Belle) [3]
$\bar{B}^0 \to a_1^+\pi^-$	$8.7^{+0.2+2.4+2.1}_{-0.2-2.0-1.3}$	12.2 ± 4.5	
$\bar{B}^0 \to a_1^-\pi^+$	$25.1^{+2.5+6.5+2.6}_{-2.4-5.8-1.6}$	21.0 ± 5.4	
$\bar{B}^0 \to a_1^{\pm}\pi^{\mp}$	$33.8^{+2.6+8.9+4.7}_{-2.6-7.8-2.9}$	33.2 ± 5.0	48.6 ± 5.6
$\bar{B}^0 \to a_1^0\pi^0$	$0.7^{+0.1+0.2+0.7}_{-0.1-0.1-0.3}$		
$B^- \to a_1^-\pi^0$	$14.9^{+1.9+3.7+2.4}_{-1.7-3.3-2.1}$		
$B^- \to a_1^0\pi^-$	$7.3^{+0.3+1.7+1.3}_{-0.3-1.5-0.9}$		
$\bar{B}^0 \to a_1^+K^-$	$15.1^{+1.2+12.7+21.2}_{-1.2-6.3-7.2}$		
$\bar{B}^0 \to a_1^0\bar{K}^0$	$6.0^{+0.4+5.6+9.7}_{-0.4-2.6-3.1}$		
$B^- \to a_1^-\bar{K}^0$	$19.1^{+1.3+15.5+24.5}_{-1.3-7.8-11.0}$		
$B^- \to a_1^0K^-$	$11.8^{+1.0+8.7+13.1}_{-1.0-4.6-4.8}$		

TABLE III. Direct CP asymmetries for the decays $B \to a_1(1260)\pi$ and $a_1(1260)K$ (in percent). See Table II for errors.

Mode	Theory	*BABAR* [5,6]	Mode	Theory
$\bar{B}^0 \to a_1^+\pi^-$	$-3.2^{+0.1+0.3+20.1}_{-0.0-0.5-19.5}$	$7 \pm 21 \pm 15$	$\bar{B}^0 \to a_1^+K^-$	$2.7^{+0.2+0.9+11.8}_{-0.2-0.8-11.9}$
$\bar{B}^0 \to a_1^-\pi^+$	$-1.7^{+0.1+0.1+13.6}_{-0.1-0.0-13.4}$	$15 \pm 15 \pm 7$	$\bar{B}^0 \to a_1^0\bar{K}^0$	$-7.9^{+0.7+2.1+7.6}_{-0.7-2.2-8.3}$
$\bar{B}^0 \to a_1^0\pi^0$	$69.3^{+5.4+6.9+25.0}_{-6.1-8.9-74.7}$		$B^- \to a_1^-\bar{K}^0$	$0.7^{+0.0+0.1+0.6}_{-0.0-0.1-0.1}$
$B^- \to a_1^-\pi^0$	$-0.4^{+0.4+0.2+11.1}_{-0.4-0.1-11.1}$		$B^- \to a_1^0K^-$	$8.8^{+0.5+1.5+12.1}_{-0.5-1.7-13.4}$
$B^- \to a_1^0\pi^-$	$-0.5^{+0.5+1.5+13.0}_{-0.3-2.4-14.6}$			

TABLE IV. Parameters of the time-dependent $B \to a_1^\pm \pi^\mp$ decay rate asymmetries. S and ΔS are computed for $\beta = 22.0°$, corresponding to $\sin(2\beta) = 0.695$, and $\gamma = 59.0°$. See Table II for errors.

	Theory	Experiment (*BABAR*) [5]
$A_{CP}^{a_1\pi}$	$0.01^{+0.00+0.00+0.05}_{-0.00-0.00-0.05}$	$-0.07 \pm 0.07 \pm 0.02$
C	$0.02^{+0.00+0.00+0.13}_{-0.00-0.00-0.13}$	$-0.10 \pm 0.15 \pm 0.09$
S	$-0.55^{+0.02+0.04+0.08}_{-0.02-0.06-0.13}$	$0.37 \pm 0.21 \pm 0.07$
ΔC	$0.48^{+0.04+0.02+0.03}_{-0.04-0.04-0.05}$	$0.26 \pm 0.15 \pm 0.07$
ΔS	$-0.01^{+0.00+0.00+0.03}_{-0.00-0.00-0.03}$	$-0.14 \pm 0.21 \pm 0.06$
$\alpha_{\rm eff}^+$	$(105.1^{+0.3+0.9+4.4}_{-0.3-0.5-2.4})°$	
$\alpha_{\rm eff}^-$	$(113.9^{+0.6+3.2+6.4}_{-0.6-2.1-3.6})°$	
$\alpha_{\rm eff}$	$(109.5^{+0.5+2.1+5.4}_{-0.5-1.3-3.0})°$	$(78.6 \pm 7.3)°$

1. $\bar{B} \to a_1\pi$

The decay of the B^0 meson to $a_1^\pm\pi^\mp$ was recently measured by the *BABAR* and Belle groups [1–5]. A recent updated result by *BABAR* yields [4]

$$\mathcal{B}(B^0 \to a_1^\pm\pi^\mp \to \pi^\mp\pi^\pm\pi^\pm\pi^\mp) = (16.6 \pm 1.9 \pm 1.5) \times 10^{-6}. \tag{4.5}$$

Assuming that $\mathcal{B}(a_1^\pm \to \pi^\mp\pi^\pm\pi^\pm)$ equals $\mathcal{B}(a_1^\pm \to \pi^\mp\pi^0\pi^0)$ and $\mathcal{B}(a_1^\pm \to (3\pi)^\pm)$ equals 100%, they have obtained

$$\mathcal{B}(B^0 \to a_1^\pm\pi^\mp) = (33.2 \pm 3.8 \pm 3.0) \times 10^{-6}. \tag{4.6}$$

Very recently, the measurements of time-dependent *CP* asymmetries in the decay $B^0 \to a_1^\pm\pi^\mp$ have been reported by the *BABAR* collaboration [5]. From the measurements, the individual branching ratios of $\bar{B}^0 \to a_1^+\pi^-$ and $a_1^-\pi^+$ can be obtained. As given in Table II, our theoretical results are in good agreement with experiment. It was shown in Ref. [42] that three-parton Fock states of M_2 can give nonsmall corrections to α_2^p, so that $|\alpha_2^p| \simeq 0.30$, If so, we can expect $\mathcal{B}(\bar{B}^0 \to a_1^0\pi^0) \gtrsim 1.6 \times 10^{-6}$, which can be tested in the future measurement.

The $\bar{B} \to a_1\pi$ amplitudes are analogous to the corresponding $\bar{B} \to \rho\pi$ ones [43]. The tree(T)-penguin(P) interference depends on the sign of $\sin\gamma$ (where $V_{ub} = |V_{ub}|e^{-i\gamma}$) and the relative sign between $\mathrm{Re}(\alpha_1^p)$ and $\mathrm{Re}(\alpha_4^p)$; for $\sin\gamma > 0$, the T-P interference is destructive in $\bar{B}^0 \to a_1^\mp\pi^\pm$, $B^- \to a_1^0\pi^-$, while it is constructive in $B^- \to a_1^-\pi^0$. Because the amplitudes of $a_1\pi$ and $\rho\pi$ modes are dominated by the terms with α_1 and α_4^p, and $\mathrm{Re}[\alpha_4^p(\pi a_1)] \approx \mathrm{Re}[\alpha_4^p(a_1\pi)/3] \approx \mathrm{Re}[\alpha_4^p(\pi\rho)] \approx -\mathrm{Re}[\alpha_4^p(\rho\pi)] \approx -0.034$, one can easily obtain the following relations:

$$\begin{aligned}
&\frac{\mathcal{B}(\bar{B}^0 \to a_1^-\pi^+)}{\mathcal{B}(\bar{B}^0 \to \rho^-\pi^+)} \approx \left(\frac{F_1^{B\pi}(m_{a_1}^2)f_{a_1}}{F_1^{B\pi}(m_\rho^2)f_\rho}\right)^2, \\
&\frac{\mathcal{B}(\bar{B}^0 \to a_1^+\pi^-)}{\mathcal{B}(\bar{B}^0 \to \rho^+\pi^-)} < \left(\frac{V_0^{Ba_1}(m_\pi^2)}{A_0^{B\rho}(m_\pi^2)}\right)^2, \\
&\frac{\mathcal{B}(B^- \to a_1^0\pi^-)}{\mathcal{B}(B^- \to \rho^0\pi^-)} < \left(\frac{V_0^{Ba_1}(m_\pi^2)}{A_0^{B\rho}(m_\pi^2)}\right)^2, \\
&\frac{\mathcal{B}(B^- \to a_1^-\pi^0)}{\mathcal{B}(B^- \to \rho^-\pi^0)} > \left(\frac{F_1^{B\pi}(m_{a_1}^2)f_{a_1}}{F_1^{B\pi}(m_\rho^2)f_\rho}\right)^2 \approx \frac{\mathcal{B}(\bar{B}^0 \to a_1^-\pi^+)}{\mathcal{B}(\bar{B}^0 \to \rho^-\pi^+)},
\end{aligned} \tag{4.7}$$

which can offer constraints on the magnitudes of f_{a_1} and $V_0^{Ba_1}(m_\pi^2)$. Moreover, the ratio $\mathcal{B}(\bar{B}^0 \to a_1^-\pi^+)/\mathcal{B}(\bar{B}^0 \to a_1^+\pi^-)$ is

$$\begin{aligned}
\frac{\mathcal{B}(\bar{B}^0 \to a_1^-\pi^+)}{\mathcal{B}(\bar{B}^0 \to a_1^+\pi^-)} = \left(\frac{F_1^{B\pi}(m_{a_1}^2)f_{a_1}}{V_0^{Ba_1}(m_\pi^2)f_\pi}\right)^2 \bigg\{ 1 + \mathrm{Re}\bigg[\frac{\lambda_t}{\lambda_u}\left(\frac{\alpha_4(\pi a_1) - \alpha_4(a_1\pi) + \beta_3(\pi a_1) - \beta_3(a_1\pi)}{\alpha_1(\pi a_1)}\right) \\
+ 2\left(\frac{V_0^{Ba_1}(m_\pi^2)f_\pi}{F_1^{B\pi}(m_{a_1}^2)f_{a_1}} - 1\right)\mathrm{Re}\left[\frac{\beta_1(\pi a_1)}{\alpha_1(\pi a_1)}\right]\bigg]\bigg\} + \mathcal{O}(\alpha_{4,\rm EW}^p, \beta_4^p, \beta_{3,\rm EW}^p, \beta_{4,\rm EW}^p),
\end{aligned} \tag{4.8}$$

which is not only sensitive to the form factor and decay constant of the a_1 meson but also to the weak phase γ. The measurement of the above ratio allows us to obtain the further constraint on the value of γ.

The large direct *CP* asymmetries may result from the nonzero value of the weak annihilation parameter (ρ_A) and its corresponding phase. See Table III. With default parameters, the direct *CP* asymmetries for $a_1^+\pi^-$, $a_1^-\pi^+$, $a_1^-\pi^0$, $a_1^0\pi^-$ are only at a few percent level, whereas it can be very remarkable for the $a_1^0\pi^0$ mode. At the present time, the large errors in the measurements for *CP* asymmetries do not allow us to draw any particular conclusion in comparison with theoretical predictions. (See Tables III and IV.)

2. Time-dependent CP for $B(t) \to a_1^\pm\pi^\mp$

Following Ref. [8], we define

$$\begin{aligned}
A_+ &\equiv A(B^0 \to a_1^+\pi^-), \qquad & A_- &\equiv A(B^0 \to a_1^-\pi^+), \\
\bar{A}_+ &\equiv A(\bar{B}^0 \to a_1^-\pi^+), \qquad & \bar{A}_- &\equiv A(\bar{B}^0 \to a_1^+\pi^-).
\end{aligned} \tag{4.9}$$

Neglecting *CP* violation in the $B^0 - \bar{B}^0$ mixing and the width difference in the two B^0 mass eigenstates, time-dependent decay rates for initially B^0 decaying into $a_1^\pm \pi^\mp$ can be parametrized by

$$\Gamma(B^0(t) \to a_1^\pm \pi^\mp) = e^{-\Gamma t}\tfrac{1}{2}(|A_\pm|^2 + |\bar{A}_\mp|^2)[1 + (C \pm \Delta C) \times \cos\Delta m t - (S \pm \Delta S)\sin\Delta m t], \tag{4.10}$$

where

$$C \pm \Delta C \equiv \frac{|A_\pm|^2 - |\bar{A}_\mp|^2}{|A_\pm|^2 + |\bar{A}_\mp|^2}, \tag{4.11}$$

and

$$S \pm \Delta S \equiv \frac{2\,\mathrm{Im}(e^{-2i\beta}\bar{A}_\mp A_\pm^*)}{|A_\pm|^2 + |\bar{A}_\mp|^2}. \tag{4.12}$$

Here Δm denotes the neutral B mass difference and Γ is the average B^0 width. For an initial $\bar{B}^0$ the signs of the $\cos\Delta m t$ and $\sin\Delta m t$ terms are reversed. The four decay modes define five asymmetries: C, S, ΔC, ΔS, and the overall CP violating $A_{CP}^{a_1\pi}$:

$$A_{CP}^{a_1\pi} \equiv \frac{|A_+|^2 + |\bar{A}_-|^2 - |A_-|^2 - |\bar{A}_+|^2}{|A_+|^2 + |\bar{A}_-|^2 + |A_-|^2 + |\bar{A}_+|^2}. \tag{4.13}$$

Two α-related phases can be defined by

$$\alpha_{\rm eff}^\pm \equiv \tfrac{1}{2}\arg(e^{-2i\beta}\bar{A}_\pm A_\pm^*), \tag{4.14}$$

which coincide with α in the limit of vanishing penguin amplitudes. The average of $\alpha_{\rm eff}^+$ and $\alpha_{\rm eff}^-$ is called $\alpha_{\rm eff}$:

$$\begin{aligned}\alpha_{\rm eff} &\equiv \frac{\alpha_{\rm eff}^+ + \alpha_{\rm eff}^-}{2}\\ &= \frac{1}{4}\left[\arcsin\left(\frac{S+\Delta S}{\sqrt{1-(C+\Delta C)^2}}\right)\right.\\ &\quad \left. + \arcsin\left(\frac{S-\Delta S}{\sqrt{1-(C-\Delta C)^2}}\right)\right].\end{aligned} \tag{4.15}$$

The numerical results for the time-dependent CP parameters are collected in Table IV. The magnitudes of $A_{CP}^{a_1\pi}$, C and ΔS are small in the QCD factorization calculation, where C is sensitive to the annihilations and can be $\sim 10\%$ in magnitude. ΔC describes the asymmetry between $\mathcal{B}(B^0 \to a_1^+\pi^-) + \mathcal{B}(\bar{B}^0 \to a_1^-\pi^+)$ and $\mathcal{B}(B^0 \to a_1^-\pi^+) + \mathcal{B}(\bar{B}^0 \to a_1^+\pi^-)$, and thus can be read directly from Tables II and III. Neglecting penguin contributions, S and $\alpha_{\rm eff}$, which depend on $\alpha(=\pi-\beta-\gamma)$, coincides with $\sin 2\alpha$ and α, respectively, in the SM. Using $\alpha = 99.0°$, i.e., $\gamma = 59.0°$, the numerical results for S and $\alpha_{\rm eff}$ differ from the experimental values at about the 3.7σ level. This puzzle may be resolved by using a smaller $\alpha = \pi - \beta - \gamma \lesssim 78°$. In Fig. 1, we plot S versus γ (and α), where we parametrize $V_{ub} = 0.003\,68 e^{-i\gamma}$. The best fitted value is $\gamma = (87^{+33}_{-7})°$, corresponding to $\alpha = (71^{+7}_{-33})°$, for $\beta = 22°$.

3. $\bar{B} \to a_1(1260)\bar{K}$ decays

The decays $\bar{B} \to a_1\bar{K}$ are penguin dominated. Because the dominant axial-vector and pseudoscalar penguin coefficients, $a_4^p(a_1\bar{K})$ and $a_6^p(a_1\bar{K})$, are constructive in the $a_1\bar{K}$

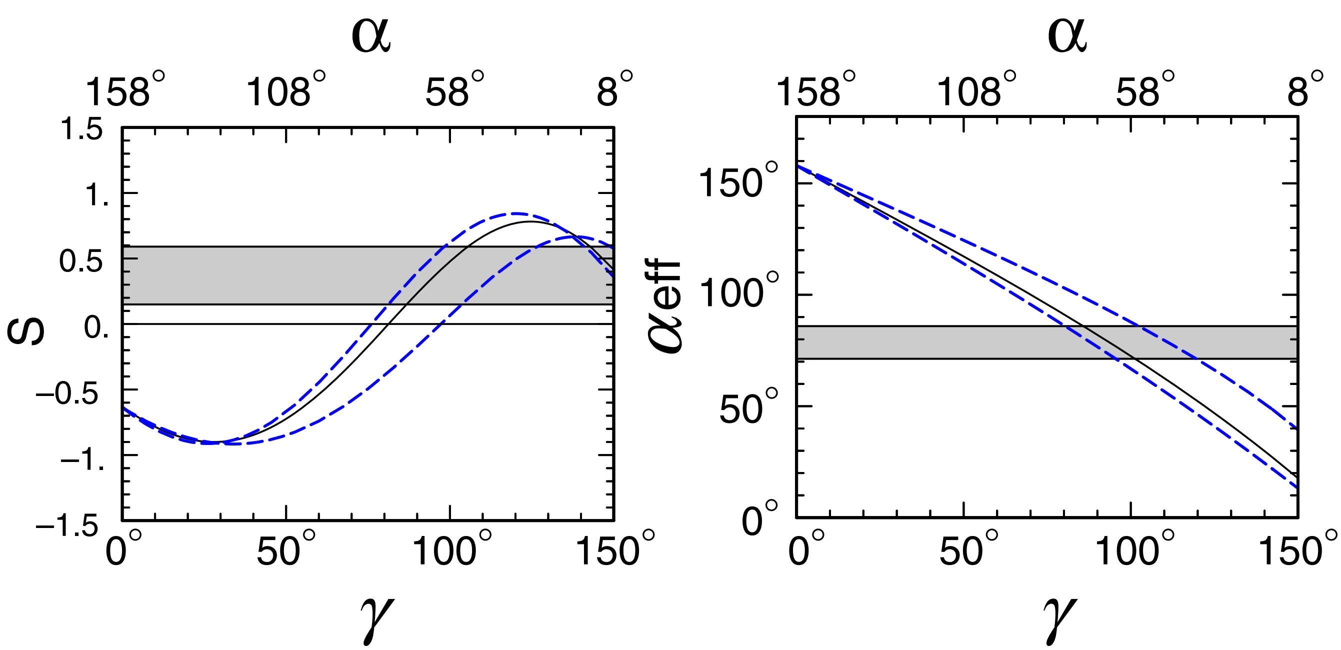

FIG. 1 (color online). S and $\alpha_{\rm eff}$ versus γ (and α) for adopting $\beta = 22°$. The solid curves are obtained by using the central values (default values) of input parameters. The region between two dashed lines is the theoretical variation within the allowed range of input parameters.

modes, $\bar{B} \to a_1\bar{K}$ and the corresponding $\bar{B} \to \pi\bar{K}$ decays should have similar rates. It is instructive to consider the four ratios:

$$\begin{aligned}
R_1 &= \frac{\mathcal{B}(\bar{B}^0 \to a_1^+ K^-)}{\mathcal{B}(\bar{B}^0 \to \pi^+ K^-)} \\
&= \left(\frac{V_0^{Ba_1}(m_K^2)}{F_0^{B\pi}(m_K^2)}\right)^2 \left(\frac{\alpha_4^c(a_1\bar{K})}{\alpha_4^c(\pi\bar{K})}\right)^2 \left[1 + 2\,\mathrm{Re}\left(\frac{\beta_3^c(a_1\bar{K}) - \frac{1}{2}\beta_{3,\mathrm{EW}}^c(a_1\bar{K})}{\alpha_4^c(a_1\bar{K})} - \frac{\beta_3^c(\pi\bar{K}) - \frac{1}{2}\beta_{3,\mathrm{EW}}^c(\pi\bar{K})}{\alpha_4^c(\pi\bar{K})}\right) + \cdots\right], \\
R_2 &= \frac{\mathcal{B}(B^- \to a_1^- \bar{K}^0)}{\mathcal{B}(B^- \to \pi^- \bar{K}^0)} \\
&= \left(\frac{V_0^{Ba_1}(m_K^2)}{F_0^{B\pi}(m_K^2)}\right)^2 \left(\frac{\alpha_4^c(a_1\bar{K})}{\alpha_4^c(\pi\bar{K})}\right)^2 \left[1 + 2\,\mathrm{Re}\left(\frac{\beta_3^c(a_1\bar{K}) + \beta_{3,\mathrm{EW}}^c(a_1\bar{K})}{\alpha_4^c(a_1\bar{K})} - \frac{\beta_3^c(\pi\bar{K}) + \beta_{3,\mathrm{EW}}^c(\pi\bar{K})}{\alpha_4^c(\pi\bar{K})}\right) + \cdots\right], \\
R_3 &= \frac{\mathcal{B}(\bar{B}^0 \to a_1^0 \bar{K}^0)}{\mathcal{B}(\bar{B}^0 \to \pi^0 \bar{K}^0)} \\
&= \left(\frac{V_0^{Ba_1}(m_K^2)}{F_0^{B\pi}(m_K^2)}\right)^2 \left(\frac{\alpha_4^c(a_1\bar{K})}{\alpha_4^c(\pi\bar{K})}\right)^2 \left[1 - 3\,\mathrm{Re}\left[\frac{\alpha_{3,\mathrm{EW}}^c(\bar{K}a_1)}{\alpha_4^c(a_1\bar{K})} r_1 - \frac{\alpha_{3,\mathrm{EW}}^c(\bar{K}\pi)}{\alpha_4^c(\pi\bar{K})} r_2\right]\right. \\
&\quad \left. + 2\,\mathrm{Re}\left(\frac{\beta_3^c(a_1\bar{K}) - \frac{1}{2}\beta_{3,\mathrm{EW}}^c(a_1\bar{K})}{\alpha_4^c(a_1\bar{K})} - \frac{\beta_3^c(\pi\bar{K}) - \frac{1}{2}\beta_{3,\mathrm{EW}}^c(\pi\bar{K})}{\alpha_4^c(\pi\bar{K})}\right) + \cdots\right], \\
R_4 &= \frac{\mathcal{B}(B^- \to a_1^0 K^-)}{\mathcal{B}(B^- \to \pi^0 K^-)} \\
&= \left(\frac{V_0^{Ba_1}(m_K^2)}{F_0^{B\pi}(m_K^2)}\right)^2 \left(\frac{\alpha_4^c(a_1\bar{K})}{\alpha_4^c(\pi\bar{K})}\right)^2 \left[1 + 3\,\mathrm{Re}\left[\frac{\alpha_{3,\mathrm{EW}}^c(\bar{K}a_1)}{\alpha_4^c(a_1\bar{K})} r_1 - \frac{\alpha_{3,\mathrm{EW}}^c(\bar{K}\pi)}{\alpha_4^c(\pi\bar{K})} r_2\right]\right. \\
&\quad \left. + 2\,\mathrm{Re}\left(\frac{\beta_3^c(a_1\bar{K}) + \beta_{3,\mathrm{EW}}^c(a_1\bar{K})}{\alpha_4^c(a_1\bar{K})} - \frac{\beta_3^c(\pi\bar{K}) + \beta_{3,\mathrm{EW}}^c(\pi\bar{K})}{\alpha_4^c(\pi\bar{K})}\right) + \cdots\right],
\end{aligned} \tag{4.16}$$

where

$$r_1 = \frac{F_0^{BK}(m_{a_1}^2) f_{a_1}}{V_0^{Ba_1}(m_K^2) f_K} \approx 1.9, \tag{4.17}$$

$$r_2 = \frac{F_0^{BK}(m_\pi^2) f_\pi}{F_0^{B\pi}(m_K^2) f_K} \approx 1.1, \tag{4.18}$$

and the dots stand for the neglected terms which are numerically estimated to be less than 1% in magnitude. The ratios $R_{1,2,3,4}$, which are very insensitive to γ, are approximately proportional to $[V_0^{Ba_1}(m_K^2)/(F_0^{B\pi}(m_K^2)]^2$ and receive corrections mainly from the electroweak penguin and annihilation topologies. The value of the annihilation β_3 is sensitive to $a_1^{\perp,a_1}$. The contributions originating from electroweak penguin and annihilation amplitudes can be further explored by taking into account the following measurements for ratios:

$$\frac{R_1}{R_2} \cong 1 - 3\,\mathrm{Re}\left(\frac{\beta_{3,\mathrm{EW}}^c(a_1\bar{K})}{\alpha_4^c(a_1\bar{K})} - \frac{\beta_{3,\mathrm{EW}}^c(\pi\bar{K})}{\alpha_4^c(\pi\bar{K})}\right), \tag{4.19}$$

$$\frac{R_1}{R_2} - \frac{R_3}{R_4} \cong 6\,\mathrm{Re}\left[\frac{\alpha_{3,\mathrm{EW}}^c(\bar{K}a_1)}{\alpha_4^c(a_1\bar{K})} r_1 - \frac{\alpha_{3,\mathrm{EW}}^c(\bar{K}\pi)}{\alpha_4^c(\pi\bar{K})} r_2\right], \tag{4.20}$$

$$\begin{aligned}
\frac{R_1}{R_3} \cong \frac{R_4}{R_2} &\cong 1 + 3\,\mathrm{Re}\left[\frac{\alpha_{3,\mathrm{EW}}^c(\bar{K}a_1)}{\alpha_4^c(a_1\bar{K})} r_1 - \frac{\alpha_{3,\mathrm{EW}}^c(\bar{K}\pi)}{\alpha_4^c(\pi\bar{K})} r_2\right] \\
&\cong 1 + \frac{1}{2}\left(\frac{R_1}{R_2} - \frac{R_3}{R_4}\right).
\end{aligned} \tag{4.21}$$

Although the above ratios are parametrized according to the QCD factorization, they can be treated in a model-independent way. It is worth stressing that, because $\Phi_\perp^{a_1}(u)$ is antisymmetric under interchange of the quark and antiquark momentum fractions in the SU(2) limit, the weak annihilations (and hard-spectator interactions), which could contribute sizable corrections to the decay amplitudes, enter the $\bar{B} \to a_1\bar{K}$ amplitude in a very different pattern compared with $\bar{B} \to \pi\bar{K}$ decays. More relevant information about X_A and $a_1^{\perp,a_1}$ can thus be provided by the measurement of R_1/R_2.

With default parameters, the direct *CP* asymmetries are analogous to the corresponding $\bar{B} \to \pi\bar{K}$ modes; because A_{CP}s are dominated by $\mathrm{Re}(V_{td}^* V_{tb})\,\mathrm{Im}(\alpha_4^c + \beta_3^c)\,\mathrm{Im}(V_{ud}^*(V_{ub})$ times $\mathrm{Re}[\alpha_1 + \alpha_2 F_1^{BK} f_{a_1}/(V_0^{Ba_1} f_K)]$ and $-\mathrm{Re}[\alpha_2 F_1^{BK} f_{a_1}/(V_0^{Ba_1} f_K)]$ terms for $a_1^0 K^-$ and $a_1^0 \bar{K}^0$ modes, respectively, their direct *CP* asymmetries are thus a little larger than the corresponding $\pi\bar{K}$ modes in magnitude due to the decay constant enhancement. Note that the value of β_3 is sensitive to the first Gegenbauer moment of $\Phi_\perp^{a_1}(u)$ and the annihilation parameters ρ_A and ϕ_A. On the other hand, an outstanding problem is the determination of the signs for direct *CP* observations in the $\pi\bar{K}$ modes. The experimental results are $A_{CP}(\bar{B}^0 \to \pi^+ K^-) = -0.095 \pm 0.013$ and $A_{CP}(B^- \to \pi^0 K^-) = 0.046 \pm 0.026$ [6]. Some proposals, for instance the contribution due to new physics in the SM electroweak penguin sector [19,20,22] or due to FSIs [17,18], were advocated for the resolution. The ratio measurements for $R_1/R_2 - R_3/R_4$, R_1/R_3, and R_4/R_2 directly probe the electroweak penguins. Moreover, the approximate relation given in Eq. (4.21) will be violated if the FSI patterns are different between $a_1\bar{K}$ and $\pi\bar{K}$ modes.

V. CONCLUSIONS

We have studied $\bar{B} \to a_1(1260)\pi$, $a_1(1260)\bar{K}$ decays. This paper is the first one in the literature using the QCD factorization approach to study $B \to AP$ decays. Interestingly, due to the G-parity, the leading-twist LCDA $\Phi_\perp^{a_1}$ of the $a_1(1260)$ defined by the nonlocal tensor current is antisymmetric under the exchange of *quark* and *antiquark* momentum fractions in the SU(2) limit, whereas the $\Phi_\parallel^{a_1}$ defined by the nonlocal axial-vector current is symmetric. The large magnitude of the first Gegenbauer moment ($a_1^{\perp,a_1}$) of $\Phi_\perp^{a_1}$ could have a sizable impact on the annihilation amplitudes. If one ignores $\Phi_\perp^{a_1}$, i.e., letting $a_1^{\perp,a_1} = 0$, with default parameters (where $\rho_A = 0$), the branching ratio for $a_1^0\bar{K}^0$ mode becomes 1.8 times smaller, while the changes of branching ratios for $a_1\pi$ and the remaining $a_1\bar{K}$ modes are at the level of 5% and 10%, respectively.

Our main results are summarized as follows.

(i) Our results for $\mathcal{B}(\bar{B}^0 \to a_1^+\pi^-, a_1^-\pi^+)$ are in good agreement with the data. Theoretically, the rates for $\bar{B} \to a_1(1260)\pi$ are close to the corresponding ones for $\bar{B} \to \rho\pi$. The differences between the above two modes are mainly caused by different magnitudes of form factors ($V_0^{Ba_1}$ and $A_0^{B\rho}$) and decay constants (f_{a_1} and f_ρ), and by different patterns of tree-penguin interference. For $\sin\gamma > 0$, the T-P interference is destructive in $\bar{B}^0 \to a_1^\mp\pi^\pm$, $B^- \to a_1^0\pi^-$, but constructive in $B^- \to a_1^-\pi^0$. Because the amplitudes of $a_1\pi$ and $\rho\pi$ modes are dominated by terms with α_1 and α_4^p, and $\mathrm{Re}[\alpha_4^p(\pi a_1)] \approx \mathrm{Re}[\alpha_4^p(a_1\pi)/3] \approx \mathrm{Re}[\alpha_4^p(\pi\rho)] \approx -\mathrm{Re}[\alpha_4^p(\rho\pi)] \approx -0.034$, we obtain the relations as given in Eqs. (4.7) and (4.8). Thus estimates for form factors and decay constants as well as the weak phase γ can thus be made from these ratio measurements.

(ii) For *CP* asymmetries, the large experimental errors do not allow us to draw any particular conclusion in comparison with theoretical predictions. The time-dependent *CP* asymmetry measurement in $B^0 \to a_1^\pm\pi^\mp$ can lead to the accurate determination of the CKM angle γ. Using the current fitted value $\gamma = 59.0°$, i.e., $\alpha = 99.0°$ corresponding to $\beta = 22.0°$ in the SM, our results show that S and α_{eff} differ from the present data at about the 3.7σ level. This puzzle may be resolved by using a larger $\gamma \gtrsim 80°$. Further measurements can clarify this discrepancy.

(iii) The branching ratios for the decays $B \to a_1\pi$ and a_1K are highly sensitive to the magnitude of $V_0^{Ba_1}(0)$. Using the LC sum rule result, $V_0^{Ba_1}(0) = 0.28 \pm 0.03$ [33], the resultant branching ratios for $a_1^\pm\pi^\mp$ modes show consistency with the data very well. Nevertheless, the value of $V_0^{Ba_1}(0)$ is about 0.13 and $1.02 \sim 1.22$ in the quark model calculations in Refs. [38–40], respectively. If the quark model result is used in the calculation, $\mathcal{B}(\bar{B}^0 \to a_1^\pm\pi^\mp)$ will be too small or large as compared with the data.

(iv) The $\bar{B} \to a_1\bar{K}$ amplitudes resemble the corresponding $\bar{B} \to \pi\bar{K}$ amplitudes very much. Taking the ratios of corresponding *CP*-averaged $a_1\bar{K}$ and $\pi\bar{K}$ branching ratios, we can extract information about the transition form factors, decay constants, electroweak penguin ($\alpha_{3,\mathrm{EW}}^c(\bar{K}a_1)$), and annihilation topology $[\beta_{3,\mathrm{EW}}^c(a_1\bar{K})]$. See Eqs. (4.19), (4.20), and (4.21). Thus, the possibilities for existing new physics in the electroweak penguin sector and for final-state interactions during decays can be explored.

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Note added.—Recently Belle has updated the following measurement [44]: $\mathcal{B}(\bar{B}^0 \to a_1^+\pi^- + a_1^-\pi^+) = (29.8 \pm 3.2 \pm 4.6) \times 10^{-6}$ which is in good agreement with our result. On the other hand, *BABAR* has reported new measurements on $a_1^0\pi^-$, $a_1^-\pi^0$ and $a_1^+K^-$, $a_1^-\bar{K}^0$ modes [45,46], where $\mathcal{B}(\bar{B}^0 \to a_1^+K^-) = (16.3 \pm 2.9 \pm 2.3) \times 10^{-6}$ is also in good agreement with our prediction, whereas the central values of branching ratios for the remaining modes are about $2 \sim 3$ times larger than our predictions. The latter discrepancies should be clarified by the improved measurements in the future.

APPENDIX A: THE COEFFICIENTS a_i^p

In the below discussion, we set $\Phi_\parallel^P \equiv \Phi^P$. In Eq. (3.9), the expressions for effective parameters a_i^p are

$$a_i^p(M_1M_2) = \left(c_i + \frac{c_{i\pm1}}{N_c}\right)N_i(M_2) + \frac{c_{i\pm1}}{N_c}\frac{C_F\alpha_s}{4\pi}\left[V_i(M_2) + \frac{4\pi^2}{N_c}H_i(M_1M_2)\right] + P_i^p(M_2). \tag{A1}$$

N_i is given in Eq. (3.10). The vertex corrections have the same expressions as those for VP modes [23] with LCDAs of the vector meson being replaced by the corresponding ones of the a_1 meson. For the penguin contractions $P_i^p(M_2)$, one can perform the same replacements but needs to add an overall minus sign to $P_6^p(a_1)$ and $P_8^p(a_1)$. $H_i(M_1M_2)$ have the expressions

$$H_i(M_1M_2)=\frac{-if_Bf_{M_1}f_{M_2}}{X^{(\bar{B}M_1,M_2)}}\int_0^1 d\rho\frac{\Phi_1^B(\rho)}{1-\rho}\times\int_0^1 dv\int_0^1 du\left(\frac{\Phi_\parallel^{M_1}(v)\Phi_\parallel^{M_2}(u)}{\bar{u}\,\bar{v}}\pm r_\chi^{M_1}\frac{\Phi_{m_1}(v)\Phi_\parallel^{M_2}(u)}{u\bar{v}}\right), \tag{A2}$$

for $i=1$–4, 9, 10,

$$H_i(M_1M_2)=\frac{if_Bf_{M_1}f_{M_2}}{X^{(\bar{B}M_1,M_2)}}\int_0^1 d\rho\frac{\Phi_1^B(\rho)}{1-\rho}\int_0^1 dv\int_0^1 du\times\left(\frac{\Phi_\parallel^{M_1}(v)\Phi_\parallel^{M_2}(u)}{u\bar{v}}\pm r_\chi^{M_1}\frac{\Phi_{m_1}(v)\Phi_\parallel^{M_2}(u)}{\bar{u}\,\bar{v}}\right), \tag{A3}$$

for $i=5$, 7, and $H_i(M_1M_2)=0$ for $i=6$, 8, where the upper (lower) signs apply when $M_1=P$ ($M_1=A$). Here $\Phi_1^B(\rho)$ is one of the two LCDAs of the $\bar{B}$ meson [25].

APPENDIX B: THE ANNIHILATION AMPLITUDES $A_n^{i,f}$

For $A_n^{i,f}$ [see Eq. (3.12)], some signs change in comparison with the results of $B\to PP$ and PV. We obtain

$$\begin{aligned}
A_1^i &= \pi\alpha_s\int_0^1 dxdy\left\{\Phi_\parallel^{M_2}(x)\Phi_\parallel^{M_1}(y)\left[\frac{1}{y(1-x\bar{y})}+\frac{1}{\bar{x}^2y}\right]-r_\chi^{M_1}r_\chi^{M_2}\Phi_{m_2}(x)\Phi_{m_1}(y)\frac{2}{\bar{x}y}\right\},\\
A_1^f &= A_2^f=0,\\
A_2^i &= \pi\alpha_s\int_0^1 dxdy\left\{\Phi_\parallel^{M_2}(x)\Phi_\parallel^{M_1}(y)\left[\frac{1}{\bar{x}(1-x\bar{y})}+\frac{1}{\bar{x}y^2}\right]-r_\chi^{M_1}r_\chi^{M_2}\Phi_{m_2}(x)\Phi_{m_1}(y)\frac{2}{\bar{x}y}\right\},\\
A_3^i &= \pm\pi\alpha_s\int_0^1 dxdy\left\{r_\chi^{M_1}\Phi_\parallel^{M_2}(x)\Phi_{m_1}(y)\frac{2\bar{y}}{\bar{x}y(1-x\bar{y})}+r_\chi^{M_2}\Phi_\parallel^{M_1}(y)\Phi_{m_2}(x)\frac{2x}{\bar{x}y(1-x\bar{y})}\right\},\\
A_3^f &= \pm\pi\alpha_s\int_0^1 dxdy\left\{r_\chi^{M_1}\Phi_\parallel^{M_2}(x)\Phi_{m_1}(y)\frac{2(1+\bar{x})}{\bar{x}^2y}-r_\chi^{M_2}\Phi_\parallel^{M_1}(y)\Phi_{m_2}(x)\frac{2(1+y)}{\bar{x}y^2}\right\},
\end{aligned} \tag{B1}$$

where the upper (lower) signs apply when $M_1=P$ ($M_1=A$) and the detailed definitions of the distribution amplitudes of the axial mesons have been collected in Sec. II. Again, here we have set $\Phi_\parallel^P\equiv\Phi^P$.

Using the asymptotic distribution amplitudes of $\Phi_\parallel^{a_1}(u)$ and $\Phi_P(u)$, and the approximation $\Phi_\perp^{a_1}(u)=18u\bar{u}(2u-1)a_1^{\perp,a_1}$, we obtain the annihilation amplitudes

$$A_1^i\approx 6\pi\alpha_s\left[3\left(X_A-4+\frac{\pi^2}{3}\right)-a_1^{\perp,a_1}r_\chi^{a_1}r_\chi^PX_A(X_A-3)\right], \tag{B2}$$

$$A_2^i\approx 6\pi\alpha_s\left[3\left(X_A-4+\frac{\pi^2}{3}\right)-a_1^{\perp,a_1}r_\chi^{a_1}r_\chi^PX_A(X_A-3)\right], \tag{B3}$$

$$A_3^i\approx\pm6\pi\alpha_s\left[r_\chi^P\left(X_A^2-2X_A+\frac{\pi^2}{3}\right)+3a_1^{\perp,a_1}r_\chi^{a_1}\left(X_A^2-2X_A-6+\frac{\pi^2}{3}\right)\right], \tag{B4}$$

$$A_3^f\approx 6\pi\alpha_s(2X_A-1)[r_\chi^PX_A-3a_1^{\perp,a_1}r_\chi^{a_1}(X_A-3)]. \tag{B5}$$

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