Sigma model Q-balls and Q-stars

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A new kind of Q-balls is found: Q-balls in a nonlinear sigma model. Their main properties are presented together with those of their self-gravitating generalization, sigma model Q-stars. A simple special limit of solutions which are bound by gravity alone (“sigma stars”) is also discussed briefly. The analysis is based on calculating the mass, global $U(1)$ charge and binding energy for families of solutions parametrized by the central value of the scalar field. Two kinds (differing by the potential term) of the new sigma model Q-balls and Q-stars are analyzed. They are found to share some characteristics while differing in other respects like their properties for weak central scalar fields which depend strongly on the form of the potential term. They are also compared with their ordinary counterparts and although similar in some respects, significant differences are found like the existence of an upper bound on the central scalar field. A special subset of the sigma model Q-stars contains those which do not possess a flat space limit. Their relation with sigma star solutions is discussed.

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I. INTRODUCTION

Q-balls [1] occur in a wide variety of (theoretical) physical contexts. They appear naturally in the minimal supersymmetric standard model [2,3] as condensates of squarks or sleptons. The larger ones ($Q \gtrsim 10^{15}$) can have a crucial cosmological significance as dark matter candidates [4] if they are stable or long living, or (since they carry baryon or lepton number) as a possible explanation for the baryon asymmetry in the Universe [3] and the baryon to dark matter ratio [5]. Small Q-balls [6] can be produced even more easily in high temperatures and may also be found as dark matter. They may also be produced in colliders for direct inspection of their interesting properties [2,7]. See also e.g. Enqvist and Mazumdar [8] and Dine and Kusenko [9] for further reviews. A large number of discussions of various other aspects of Q-balls exists already with different approaches: analytic [6,10–12], mixed—analytic and numerical [13–17], numerical simulations [18,19] for addressing more complicated issues like scattering (not yet in 3 spatial dimensions) and so on.

All the above-mentioned Q-ball studies are based on the “original” flat space Q-balls. However, it is evident that for a large enough mass scale, gravitational effects become important and one needs to study Q-stars [20–22]. The existence of Q-stars was demonstrated by Friedberg *et al.* [23] and by Lynn [24]. Further studies revealed more features like the fact that gravity limits the size of Q-balls [25] or the properties of spinning Q-stars [16], or made generalizations like Q-stars with nonminimal coupling to gravity [26].

Furthermore, unifying theories (typically in higher dimensions) lead frequently to nonlinear sigma models, so Q-balls and Q-stars should be studied in these models as well. Although topological solitons in nonlinear sigma models [27] have been studied for decades, it seems that nontopological solitons of the same models have received very little attention. This work will be devoted to a special type of those: Q-balls and their self-gravitating counterparts, i.e. sigma model Q-stars.

II. GENERAL CONSIDERATIONS

Sigma model Q-balls and Q-stars are spherically symmetric solutions of the field equations derived from the action

$$S = \int d^4x\sqrt{|g|}\left(\frac{1}{2}\mathcal{E}(|\Phi|)(\nabla_\mu\Phi)^*(\nabla^\mu\Phi) - U(|\Phi|) + \frac{1}{16\pi G}R\right), \tag{2.1}$$

where $\mathcal{E}(|\Phi|)$ is a non-negative dimensionless function, which may be interpreted as a Weyl factor of a conformally flat (two-dimensional) target space metric. A particularly simple system which will be studied here is the $O(3)$ sigma model [27] which corresponds to $\mathcal{E}(|\Phi|) = 1/(1 + |\Phi|^2/m^2)^2$. This is the conformal factor of a target space of S^2 with a diameter m.

The function $U(|\Phi|)$ is a non-negative potential which will be chosen such as Q-ball solutions will exist as will be explained below. More notations and conventions: ∇_μ is the covariant derivative, the signature is $(+, -, -, -)$ and $R^\kappa_{\lambda\mu\nu} = \partial_\nu\Gamma^\kappa_{\lambda\mu} - \partial_\mu\Gamma^\kappa_{\lambda\nu} + \cdots$. G is Newton's constant and $G_{\mu\nu}$ will denote the Einstein tensor.

The field equations derived from the action (2.1) are

$$\mathcal{E}(|\Phi|)\nabla_\mu\nabla^\mu\Phi + \frac{\Phi^*}{2|\Phi|}\frac{d\mathcal{E}}{d|\Phi|}\nabla_\mu\Phi\nabla^\mu\Phi + \frac{\Phi}{|\Phi|}\frac{dU}{d|\Phi|} = 0 \tag{2.2}$$

and

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$$-\frac{1}{8\pi\mathcal{G}}G_{\mu\nu}=T_{\mu\nu}$$
$$=\frac{1}{2}\mathcal{E}(|\Phi|)[(\partial_\mu\Phi)^*(\partial_\nu\Phi)+(\partial_\nu\Phi)^*(\partial_\mu\Phi)-(\nabla_\lambda\Phi)^*(\nabla^\lambda\Phi)g_{\mu\nu}]+U(|\Phi|)g_{\mu\nu} \quad (2.3)$$

which is equivalent to

$$\frac{1}{8\pi\mathcal{G}}R_{\mu\nu}+\frac{1}{2}\mathcal{E}(|\Phi|)[(\partial_\mu\Phi)^*(\partial_\nu\Phi)+(\partial_\nu\Phi)^*(\partial_\mu\Phi)]-U(|\Phi|)g_{\mu\nu}=0. \quad (2.4)$$

It is well known that in flat space and a proper choice of potential which "contains any attractive interaction however weak" [21], the "linear" system ($\mathcal{E}(|\Phi|)=1$) has nontopological solitons stabilized by the global $U(1)$ charge Q. The $U(1)$ current density in the general case is given by the slightly modified expression

$$j_\nu=-\frac{i}{2}\mathcal{E}(|\Phi|)(\Phi^*\partial_\nu\Phi-\Phi\partial_\nu\Phi^*). \quad (2.5)$$

The simplest way to obtain nonvanishing $U(1)$ charge is to allow a uniform rotation in target space ("field space"),

$$\Phi=F(x^k)e^{i\omega t} \quad (2.6)$$

and indeed it can be proven [6,21] that this must be the form of the field which minimizes the energy within the sector of a given Q in the linear theory. The generalization to the "nonlinear" case is straightforward. The condition of finite charge leads to the boundary condition $F(x^k)\to 0$ at infinity.

If we assume further spherical symmetry, $F(x^k)$ will depend only on the radial coordinate r and the line element will take the usual form

$$ds^2=A^2(r)dt^2-B^2(r)dr^2-r^2(d\theta^2+\sin^2\theta d\varphi^2). \quad (2.7)$$

Note that the $e^{i\omega t}$ factor indeed modifies the energy-momentum tensor but keeps it static.

Einstein equations (2.4) for Q-stars become

$$\left(\frac{r^2A'}{B}\right)'=8\pi\mathcal{G}ABr^2\left(\frac{\omega^2\mathcal{E}(F)F^2}{A^2}-U(F)\right), \quad (2.8)$$

$$\frac{A''}{A}-\frac{A'B'}{AB}-\frac{2B'}{Br}=-8\pi\mathcal{G}B^2\left(\frac{\mathcal{E}(F)F'^2}{B^2}+U(F)\right), \quad (2.9)$$

$$\frac{1}{r^2}-\frac{1}{B^2r^2}-\frac{1}{B^2r}\left(\frac{A'}{A}-\frac{B'}{B}\right)=8\pi\mathcal{G}U(F), \quad (2.10)$$

and they should be supplemented by the scalar field equation

$$\mathcal{E}(F)\left(\frac{F''}{B^2}+\frac{(Ar^2/B)'F'}{ABr^2}+\frac{\omega^2F}{A^2}\right)+\frac{1}{2}\frac{d\mathcal{E}}{dF}\left(\frac{\omega^2F}{A^2}+\frac{F'}{B^2}\right)-\frac{dU}{dF}=0. \quad (2.11)$$

The charge and mass are given by

$$Q=4\pi\omega\int_0^\infty drr^2(B/A)\mathcal{E}(F)F^2, \quad (2.12)$$

$$M=4\pi\int_0^\infty drr^2\left(\mathcal{E}(F)\left(\frac{\omega^2F^2}{2A^2}+\frac{F'^2}{2B^2}\right)+U(F)\right). \quad (2.13)$$

We note that the sigma Q-star can be viewed as a bound state of $|Q|$ elementary bosons (that is, it is stable against decay into free bosons) if $M/m<|Q|$. Without loss of generality we will assume $\omega>0$ so we will have $Q>0$ as well.

Actually, we have to solve a system of three differential equations: Eq. (2.11) and only two of the three equations (2.8), (2.9), and (2.10). By taking combinations of those three we get two first order equations (which may be obtained directly from the $G_{\mu\nu}$ equations).

It is more comfortable and efficient to introduce a dimensionless mass function $\mathcal{M}(r)$ and use it instead of the metric function $B(r)$ following the definition

$$1-\frac{1}{B^2}=\frac{2\mathcal{G}M(r)}{r}=\frac{2\mathcal{M}(r)}{mr}. \quad (2.14)$$

Note that $M(r)$ is the accumulated mass up to radial coordinate r and the total mass of the Q-star is the limit $M(\infty)$ which we simply abbreviate by M where there is no danger for ambiguity. Since the solutions are localized, $M(r)$ becomes essentially constant quite fast (this is one reason for using it instead of $B(r)$), and using the numerical radial end point instead of infinity is accurate enough. It is also simpler and more natural to revert to an angular field Θ defined by

$$|\Phi|=m\tan(\Theta/2). \quad (2.15)$$

It is straightforward to rewrite the field equations in terms of $\mathcal{M}(r)$ and $\Theta(r)$ and actually to cast them in a dimensionless form which is ready for numerical solution. We get

$$\mathcal{M}'=\gamma x^2\left[\frac{\bar{\omega}^2\sin^2(\Theta)}{8A^2}+\left(1-\frac{2\mathcal{M}}{x}\right)\frac{\Theta'^2}{8}+u(\Theta)\right], \quad (2.16)$$

$$\left(1-\frac{2\mathcal{M}}{x}\right)\frac{xA'}{A}=\gamma x^2\left[\frac{\bar{\omega}^2\sin^2(\Theta)}{8A^2}+\left(1-\frac{2\mathcal{M}}{x}\right)\frac{\Theta'^2}{8}-u(\Theta)\right]+\frac{\mathcal{M}}{x}, \quad (2.17)$$

$$\left(1-\frac{2\mathcal{M}}{x}\right)\Theta'' + 2\left(1-\frac{\mathcal{M}}{x}-\gamma x^2 u(\Theta)\right)\frac{\Theta'}{x} + \frac{\bar{\omega}^2\sin(2\Theta)}{2A^2} - 4\frac{du}{d\Theta} = 0, \quad (2.18)$$

where we use a dimensionless potential function $u(\Theta)$ and define $x = mr$, $\bar{\omega} = \omega/m$, and $\gamma = 4\pi G m^2$.

Actually, the flat space limit of this system with a conserved global charge have been studied already to some extent under the title "Q-lumps" [28,29], which refer to solutions which carry additional *topological* charge. This kind of topological solutions exists only in lower dimensionality or at most as stringlike in four dimensions. However, we will be able to find flat space (as well as self-gravitating) spherical solutions (in four spacetime dimensions) since we give up topological nontriviality. Therefore, our solutions may be simply regarded as sigma model Q-balls and Q-stars stabilized by the global charge alone. Within the context of the present discussion, the difference between Q-balls and Q-lumps is just a difference in the boundary conditions: both kinds require $\Theta(\infty) = 0$ (for finite charge) but Q-lumps exist for $\Theta(0) = \pi$ while the sigma Q-balls we find need $\Theta(0) < \pi/2$. These boundary conditions are related to the potential functions and it turns out that Q-balls and Q-stars are easily obtained for a large family of potentials having a global minimum at $\Theta = 0$ and another local one. We will use the simple form $u(\Theta) = \sin^2(\Theta)/8 - \alpha\sin^p(\Theta)/p$ with $p = 3, 4$ so the potentials (whose local minimum is always at $\Theta = \pi/2$) are

$$u_{23}(\Theta) = \frac{\sin^2(\Theta)}{8} - \frac{\alpha\sin^3(\Theta)}{3}, \qquad u_{24}(\Theta) = \frac{\sin^2(\Theta)}{8} - \frac{\alpha\sin^4(\Theta)}{4}. \quad (2.19)$$

We will see that the difference between the corresponding solutions will be analogous to those between the 2-3-4 and the 2-4-6 potentials of the linear system [17].

The first term in both potentials is just a simple mass term which adds up with the ω term as in the linear system. The normalization is such that m is still the mass of the elementary free scalars. We choose representative values of $\alpha = 0.35$ for the 2-3 potential and $\alpha = 0.4$ for the 2-4 one. The potentials are shown for these values in Fig. 1. As usual, one may get the main properties of the Q-balls from the "effective potential" $u_{\rm eff}(\Theta) = u(\Theta) - \bar{\omega}^2\sin^2(\Theta)/8$. The parameter $\bar{\omega}$ will take values between $\bar{\omega}_- = \sqrt{1-8\alpha/p}$ and $\bar{\omega}_+ = 1$ and the corresponding central fields are $\Theta_*(0) = \pi/2$ and $\Theta(0) \to 0$.

Because of the "north-south" symmetry which is left in the potential functions there exists of course another family of "mirror" solutions with the boundary condition $\Theta(\infty) = \pi$ instead of the usual $\Theta(\infty) = 0$ that we are using. The second boundary condition will satisfy accordingly $\Theta(0) > \pi/2$. Obviously, our choice does not lead to any loss of generality of the results that will be presented here, but it should be kept in mind that they apply to two different families of solutions.

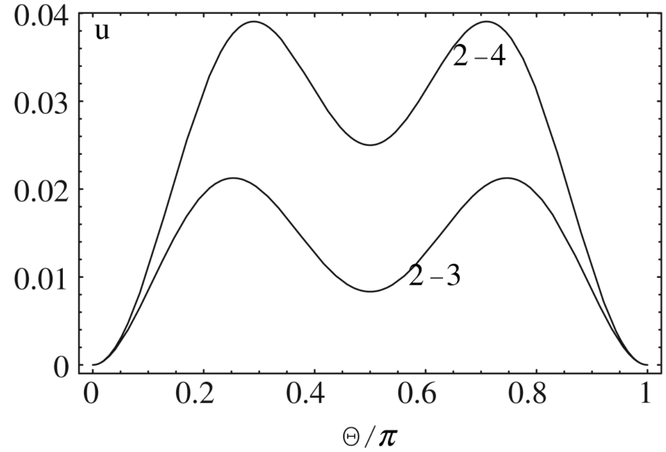

FIG. 1. Plots of the 2-3 potential for $\alpha = 0.35$ and 2-4 potential for $\alpha = 0.4$.

III. PROPERTIES OF THE SOLUTIONS

We study solutions with both potentials for the three values $\gamma = 0, 0.02, 0.2$.

We solve numerically the three field equations and get the mass and charge of the solutions as a function of the central scalar field $\Theta(0)$ which parametrizes the various solutions. The solutions are determined by the $U(1)$ angular frequency parameter, ω only in flat space. In the case of self-gravitating systems the dependence on ω of the mass and charge becomes "spiral" so it ceases to be useful for characterizing the solutions. Actually, the parametrization by the central scalar field is not completely unique either and there are regions where there correspond more than one solution to a given value of the central field [17]. In these regions, and near the limiting value $\Theta_*(0) = \pi/2$ (see below) we use $A(0)/\bar{\omega}$ instead. Note that this is the combination that always appears in Eqs. (2.16), (2.17), and (2.18).

The results were obtained by the MATHEMATICA package using an iterative procedure for solving the three coupled field equations. A first solution was found by the "shooting" method using $\Theta(0)$ as a shooting parameter. Having obtained one solution, the rest can be generated by moving around in sufficiently small steps of $\Theta(0)$, or of $A(0)/\bar{\omega}$ according to the circumstances.

As can be guessed from inspection of the "effective potential," the "thin wall" and "thick wall" solutions exist for both potentials in a way similar to the linear case.

It is obvious that there are no flat space solutions with $\alpha = 0$, that is a mass term only. Gravity changes the situation and allows solutions which are the sigma model analogues of the boson star solutions, so we may call them "sigma stars." One difference with respect to boson stars is that these sigma stars do not enjoy a scaling symmetry and do not fall on a one-parameter curve in parameter space. Therefore the $\gamma = 0.2$ family of pure self-gravitating sigma model solutions which we present is not universal.

The main results are shown in Figs. 2–6. The first, Fig. 2 shows the charge as a function of the central field $\Theta(0)$ in all cases. It covers the region of maximal charge for $\gamma = 0.02$, although the maximum is so narrow that its width cannot be seen it the plots.

It may be expected that curves of the mass as a function of $\Theta(0)$ are superposed in these plots, but they are not easily identified in this resolution. However, the plots for the binding energy per particle, Figs. 3 and 4 give enough information about the masses being larger or smaller than mQ for a given $\Theta(0)$. More insight into the situation is added by the plots of the binding energy per particle vs charge, Figs. 5 and 6.

A new characteristic with respect to the linear solutions is the limiting value of the central scalar field $\Theta(0)$ which is the same for both potentials: $\Theta_*(0) = \pi/2$. Moreover, now the same limiting $\Theta(0)$ appears also for the self-gravitating solutions and not for Q-balls only. The reason for this is the special form of the potential functions that we chose, which yields the minimum of the effective potential to be at $\Theta(0) = \pi/2$ for all values of ω. We stress also that $\Theta = \pi/2$ is the equator of S^2 and does not correspond to $|\Phi| \rightarrow \infty$—see Eq. (2.15). Therefore, it is possible that a different choice of potential function will allow larger values of $\Theta_*(0)$.

Another new characteristic is the existence of mirror solutions with different boundary conditions. The possibility of coexistence of solutions of both type and their interaction deserves further study.

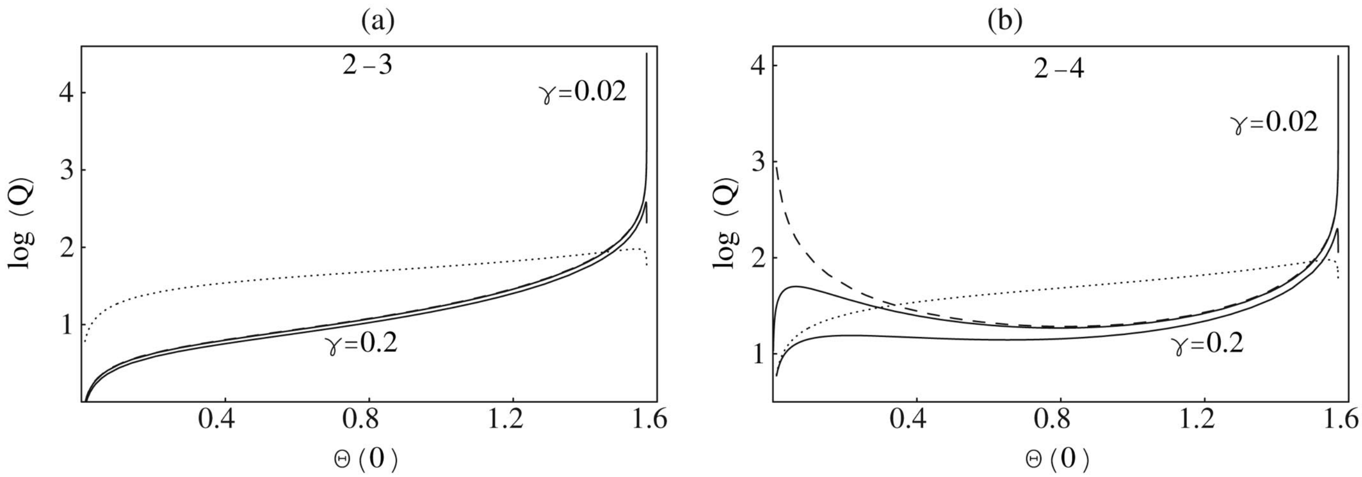

FIG. 2. Plots of $\log(Q)$ vs $\Theta(0)$ for $\gamma = 0$ (sigma model Q-balls—dashed line), $\gamma = 0.02$ and $\gamma = 0.2$. (a) 2-3 sigma model Q-stars; (b) 2-4 sigma model Q-stars. The $\gamma = 0$ line cannot be resolved from the $\gamma = 0.02$ one in the 2-3 potential. The dotted lines correspond to sigma stars with $\gamma = 0.2$.

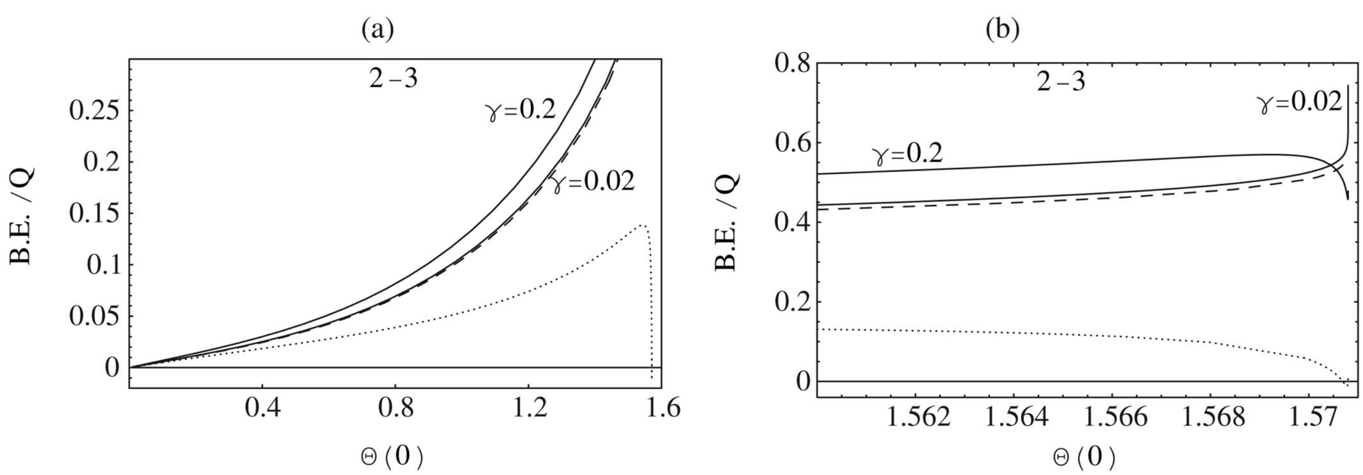

FIG. 3. Plots of binding energy per particle $(mQ - M)/mQ$ vs $\Theta(0)$ for 2-3 sigma model Q-stars with $\gamma = 0$ (sigma model Q-balls—dashed line), $\gamma = 0.02$ and $\gamma = 0.2$. (a) $B.E./Q$ up to 0.30; (b) magnification of the large field region with larger $B.E./Q$. The dotted lines correspond to sigma stars with $\gamma = 0.2$.

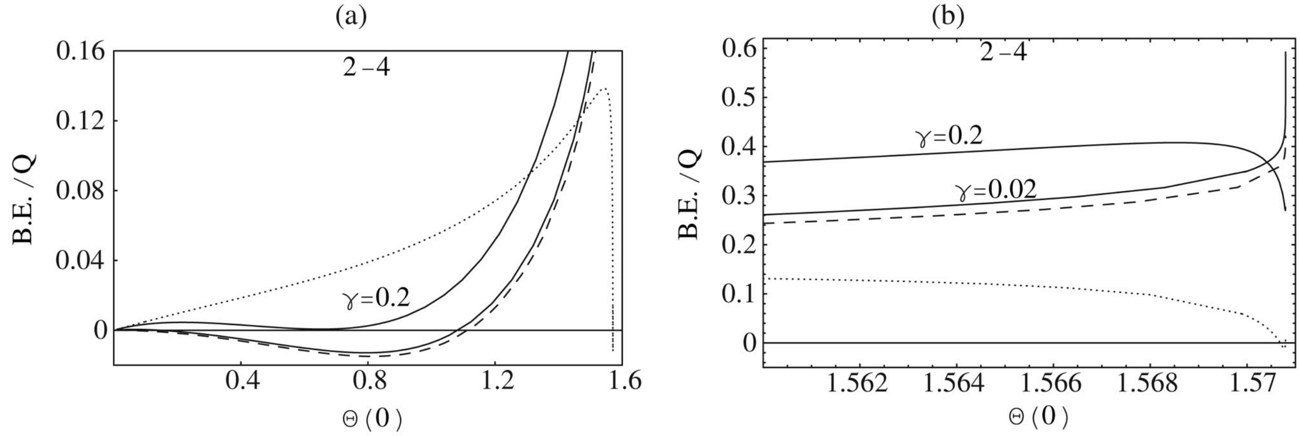

FIG. 4. Plots of binding energy per particle $(mQ - M)/mQ$ vs $\Theta(0)$ for 2-4 sigma model Q-stars with $\gamma = 0$ (sigma model Q-balls—dashed line), $\gamma = 0.02$ and $\gamma = 0.2$. (a) $B.E./Q$ up to 0.15; (b) magnification of the large field region with larger $B.E./Q$. The dotted lines correspond to sigma stars with $\gamma = 0.2$.

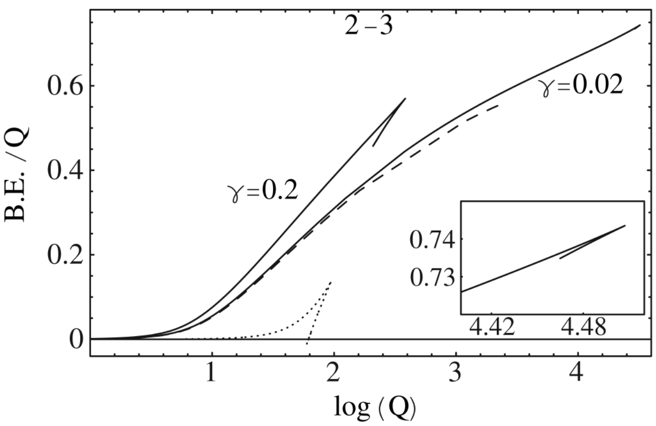

FIG. 5. Plots of binding energy per particle $(mQ - M)/mQ$ vs $\log(Q)$ for $\gamma = 0$ (sigma model Q-balls—dashed line), $\gamma = 0.02$ and $\gamma = 0.2$ and for boson stars with $\gamma = 0.2$ (dotted line). The insert is a magnification of the upper right corner which contains the $\gamma = 0.02$ curve.

Generally, we find a close parallelism between the main properties of the sigma Q-balls/Q-stars and the analogous "ordinary" Q-balls/Q-stars. The "damped oscillations" of Q beyond its maximum which exist in the linear model [17] exist also here but they are only partially visible in Fig. 2 due to the $\Theta(0) = \pi/2$ limit. As before the solutions are stable against decay into free bosons only in a limited region of $\Theta(0)$ values which corresponds to positive binding energy, or $M/m < Q$. Note however that charge degeneracy (two masses or more for the same Q) exists, so even in this region a higher mass state may decay into a lower state plus additional free bosons while conserving Q. The other solutions with negative binding energies may either decay completely into free bosons, or will have a smaller stable boson star among their decay products.

The sigma Q-balls with the 2-3 potential are always stable with ever growing charge. Gravity imposes maximal charge values: for $\gamma = 0.2$ the maximum is at $Q = 382.97$ and $\Theta(0) = 1.569\,21$ or $2\Theta(0)/\pi = 0.998\,99$, while for

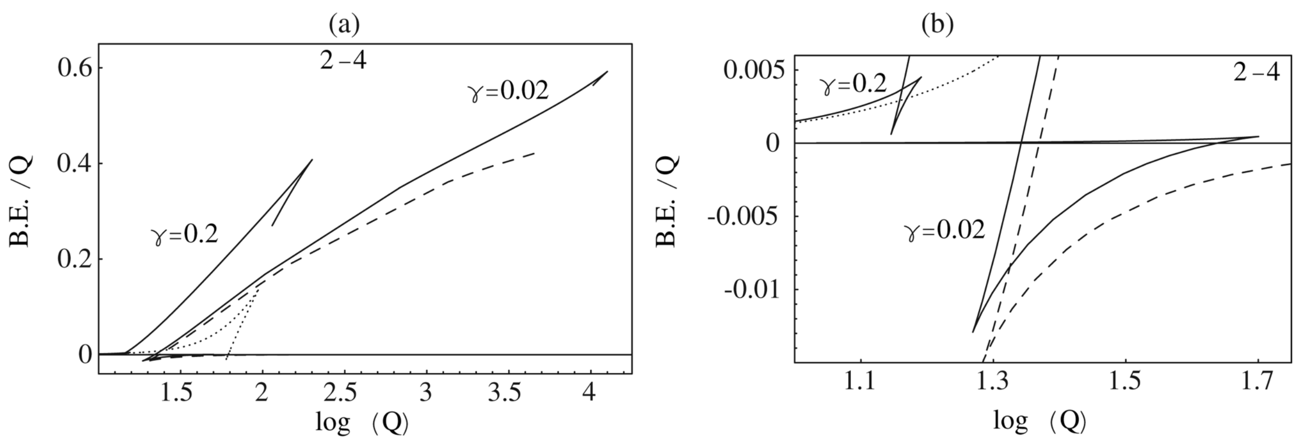

FIG. 6. Plots of binding energy per particle $(mQ - M)/mQ$ vs $\log(Q)$ for 2-4 sigma model Q-stars with $\gamma = 0$ (sigma model Q-balls—dashed line), $\gamma = 0.02$ and $\gamma = 0.2$ and for boson stars with $\gamma = 0.2$ (dotted line). (a) general view; (b) magnification of the small $B.E./Q$ region.

$\gamma = 0.02$ the maximum is at $Q = 32\,155.3$. The corresponding central scalar field is so close to $\pi/2$ that we give it as[1] $1 - 2\Theta(0)/\pi = 1.5543 \times 10^{-15}$.

The sigma star solutions with $\gamma = 0.2$ that appear in Fig. 2 have a maximum at $Q = 95.07$ and $\Theta(0) = 1.543\,55$ or $2\Theta(0)/\pi = 0.982\,65$. These values define the region of stable solutions. As Figs. 3 and 5 show, all the others are either unbound (with negative binding energy), or may decay into the stable solutions while conserving particle number.

The 2-3 Q-star curve with $\gamma = 0.02$ in Fig. 2 is very similar to the $\gamma = 0$ Q-balls (below the maximum of $Q = 32\,155.3$) and it is impossible to distinguish between them. The difference shows up in the binding energy which is shown in Figs. 3(a) and 5.

The solutions with the 2-4 potential exhibit a more involved structure which may be described again according to the central field value. The small $\Theta(0)$ sigma Q-balls (no gravity: $\gamma = 0$) are large and unstable. The stability region starts at $\Theta(0) = 1.115\,15$ for which $Q = M/m = 23.43$ and extends all the way to $\Theta(0) = \pi/2$ with monotonically increasing charge and mass. For $\gamma = 0.02$ the small $\Theta(0)$ behavior changes completely. The small $\Theta(0)$ solutions have positive binding energies for $\Theta(0) \leq 0.144\,76$ (for which $Q = 43.37$), passing through a local maximum of charge at $\Theta(0) = 0.064\,00$ and $Q = 50.17$. An instability region follows for $\Theta(0) \leq 1.084\,17$. The second range of bound solutions starts at $\Theta(0) = 1.084\,17$ for which $Q = M/m = 21.99$ and goes toward $\Theta(0) \to \pi/2$. There is however a global charge maximum of $Q = 12\,586.03$ very close to $\Theta(0) = \pi/2$. i.e. such that $1 - 2\Theta(0)/\pi = 1.7686 \times 10^{-13}$. We may therefore conclude that for this 2-4 potential there are stable sigma Q-stars for any charge up to a maximal value of $Q = 12\,586.03$ for the parameters we chose. Some charge intervals exhibit charge degeneracy, so the higher mass solutions will decay.

The solutions for $\gamma = 0.2$ have similar behavior with two main quantitative differences: the maximal charge is now much smaller and has a value of $Q = 200.18$ and all solutions have in this case positive binding energy due to the stronger gravitational self-attraction.

As far as the flat space limit is concerned, the 2-3 solutions may be divided into two types in accordance with two regions of the central field interval $0 < \Theta(0) < \pi/2$. The sigma Q-stars in most of this interval are similar to the flat space ones and the limit $\gamma \to 0$ (keeping $\Theta(0)$ fixed) gives well-behaved sigma Q-balls whose existence does not depend upon gravity (although their detailed properties do). The exception is the small region near (and below) $\Theta(0) = \pi/2$ where there are only self-gravitating solutions without a flat space limit. The 2-4 solutions may be divided into three types: in the region of small $\Theta(0)$ and near $\Theta(0) = \pi/2$ gravity gives rise to solutions with no flat space limit, while in the medium $\Theta(0)$ values the solutions may be viewed again as self-gravitating generalizations of the sigma Q-balls.

IV. SUMMARY AND OUTLOOK

We found new Q-ball and Q-star solutions in the sigma model system. We presented the main properties of the solutions in flat spacetime and for two values of the gravitational strength $\gamma = 0.02$ and $\gamma = 0.2$. The corresponding characteristics are quite different for the different values of γ as is evident from the plots of charge vs central field described above, together with the analysis of the $\Theta(0)$ and Q dependence of the binding energy.

We found that the sigma model Q-ball and Q-star properties depend strongly on the form of the potential term and that the Q-star solutions split into two main types: one is a self-gravitating version of the flat space Q-balls, while the other contains solutions which do not have a flat space limit.

There is a close parallelism with the ordinary Q-ball and Q-star solutions [17]: the correspondence is between the sigma model with the 2-3 potential and the linear system with 2-3-4 potential, and sigma model with the 2-4 potential and the linear system with 2-4-6 potential. However, there are some important differences. The most interesting is the existence of two mirror families of solutions differing by their boundary conditions. The possibility of coexistence of solutions of both types and their interaction was not discussed here but deserves further investigation.

Another issue which calls for a further study is that of spinning Q-balls and Q-stars. Although interesting results [15,16] already exist for the linear system, an analogous study for the new sigma model solutions does not exist. A further more systematic analysis is needed in order to clarify questions like the relation between charge, mass, and angular momentum of spinning Q-stars.

Also needed are a better understanding of the dynamics of instability and decay processes of Q-stars and a study of their possible gravitational collapse.

[1] In this region $A(0)/\bar{\omega}$ is used to parametrize the solutions. The central field values are output rather than input.

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