Noether symmetries of Bianchi I, Bianchi III, and Kantowski-Sachs spacetimes in scalar-coupled gravity theories

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We consider some scalar-coupled theories of gravity, including induced gravity, and study the Noether symmetries of Bianchi I, Bianchi III, and Kantowski-Sachs cosmological models for this theory. For various forms of coupling of the scalar field with gravity, some potentials are found in these cosmological models under the assumption that the Lagrangian admits Noether symmetry. The solutions of the field equations for the considered models are presented by using the results obtained from the Noether symmetry. We also find the explicit form of the scalar field in terms of the conformal time for Bianchi I, III, and Kantowski-Sachs models.

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I. INTRODUCTION

In standard cosmology, the evolution of the universe at a macroscopic scale is given by Friedmann-Robertson-Walker (FRW) metrics, which means that the universe is homogeneous and isotropic on large scales. Inflationary models were developed to explain this homogeneity and isotropy. These models require a phase transition, which is most simply achieved by the use of a scalar field [1]. For this reason, scalar-tensor theories of gravity, which generalize in different ways Einstein's general relativity (GR) theory, have captured new attention in recent decades [2].

Another motivation for scalar fields in cosmology comes from an attempt to resolve the apparent contradiction between quantum ideas and GR: The theory of induced gravity seeks to understand the spacetime background as a mean-field approximation of some underlying microscopic degrees of freedom, and leads to an effective theory with a particular coupling of a scalar field to gravity [1].

The general form of action involving gravity nonminimally coupled with a scalar field is given by [3]

$$\mathcal{A} = \int L dt = \int d^4x\sqrt{-g}\left[F(\Phi)R + \frac{\epsilon}{2}g^{ab}\Phi_a\Phi_b - U(\Phi)\right] \quad (1)$$

where R is the Ricci scalar, $F(\Phi)$ is a generic function describing the coupling, $U(\Phi)$ is the potential for the scalar field Φ, L is the Lagrangian density, $\Phi_a = \Phi_{,a}$ stand for the components of the gradient of Φ and the signature of metric is given by the parameter $\epsilon = +1, -1$ for signatures $(+ - - -)$ and $(- + + +)$, respectively, and Planck units are used. For $F(\Phi) = -1/2$, the action reduces to the form of Einstein-Hilbert action minimally coupled with a scalar field. For $F(\Phi) = c\Phi^2$, $(c \neq \epsilon/12)$, it takes the form of the induced theory of gravity. For $F(\Phi) = 1 - \zeta\Phi^2$, it is of the form of standard nonminimally coupled scalar field theory. For $F(\Phi) = \Phi^2/6$, the conformally coupled theory is obtained.

The variation of (1) with respect to g_{ab} provides the field equations

$$F(\Phi)G_{ab} = -\frac{\epsilon}{2}T_{ab} - g_{ab}\Box F(\Phi) + F(\Phi)_{;ab} \quad (2)$$

where $\Box$ is the d'Alembert operator,

$$G_{ab} = R_{ab} - \tfrac{1}{2}Rg_{ab} \quad (3)$$

is the Einstein tensor, and

$$T_{ab} = \Phi_a\Phi_b - \tfrac{1}{2}g_{ab}\Phi_c\Phi^c + \epsilon g_{ab}U(\Phi) \quad (4)$$

is the energy-momentum tensor relative to the scalar field $\Phi = \Phi(x^a)$ which is a real function on the manifold. The variation with respect to Φ gives rise to the Klein-Gordon equation governing the dynamics of the scalar field

$$\epsilon\Box\Phi - RF'(\Phi) + U'(\Phi) = 0, \quad (5)$$

where the prime indicates the derivative with respect to Φ. It is interesting to note that the last equation is equivalent to the contracted Bianchi identity [3].

The choice of the potential $U(\Phi)$ in the above equations is somewhat arbitrary, and this has given rise to objections of fine tuning, the very problem inflationary theories have set out to solve. Therefore it is desirable to have a way to derive the potential, or at least criteria for acceptable potentials.

One such approach is based on Noether symmetry and was recently introduced by Capozziello *et al.* [3–8], de Ritis *et al.* [9–11], and others [12–14]. The Noether theorem states that, if there exists a vector field $\mathbf{X}$, for which the Lie derivative of a given Lagrangian L vanishes, i.e.

$$\mathcal{L}_{\mathbf{X}}L = 0, \quad (6)$$

then $\mathbf{X}$ is a symmetry for the dynamics (that is, the Lagrangian admits a Noether symmetry) and thus generates a conserved current. The Noether symmetry approach

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allows one to choose the potential dynamically, restricting the arbitrariness in a suitable way. In these works [3–6,8–11] some exact solutions in the scalar-tensor theories have been presented, including dynamical conserved quantity and solutions of the field equations. In a recent work Sanyal [15] discussed the Noether and dynamical symmetries of the Kantowski-Sachs spacetime. He proved that one can find dynamical symmetries working directly with the field equations and obtain the previous results plus more dynamical symmetries. Hence, for proving Sanyal's original assertion, one has to obtain the continuity equation directly from the field equations to check the existence of the Noether symmetry. In this study we have generalized Sanyal's results additionally including Bianchi I and III spacetimes.

This paper is organized as follows. In the Sec. II, we present the Euler-Lagrange equations of motion and the field equations for Bianchi I, III and Kantowski-Sachs spacetimes, investigate their Noether symmetries and try to find the potential for some particular cases of the coupling function. In Sec. III, we give the solutions of the equations of motion and the fields equations in nonminimally coupling and induced gravity theories. Finally, in Sec. IV, we conclude with a brief summary and discussion of the obtained results.

II. NOETHER SYMMETRIES FOR BIANCHI I, III, AND KANTOWSKI-SACHS MODELS

For the Kantowski-Sachs metrics of the signature $(+2)$, the forms of coupling of the scalar field and the potential with gravity has been found by Sanyal [15] under the assumption that the Lagrangian admits Noether symmetry. Here we shall generalize this work to include Bianchi I and III for both signatures.

The line element of Bianchi I (BI), Bianchi III (BIII) and Kantowski-Sachs (KS) spacetimes has the form

$$ds^2 = \epsilon(dt^2 - A^2dr^2) - \epsilon B^2(d\theta^2 + \Sigma^2(q,\theta)d\phi^2), \quad (7)$$

where A and B are depend on t only, and $\Sigma(q,\theta) = \theta, \sinh\theta, \sin\theta$ for $q = 0, -1, +1$, respectively, covers all the relevant cases. Here $q = 0$ corresponds to the BI, $q = -1$ to BIII and $q = 1$ to KS cosmological models, and the parameter ϵ is assigned as -1 for the signature $(+2)$ and $+1$ for the signature (-2).

The Ricci scalar for this metric is

$$R = -2\epsilon\left[\frac{\ddot{A}}{A} + 2\frac{\ddot{B}}{B} + \frac{\dot{B}^2}{B^2} + 2\frac{\dot{A}\dot{B}}{AB} + \frac{q}{B^2}\right], \quad (8)$$

where the dot indicates the derivation with respect to time. The Lagrangian density coming from (1) becomes a function only of $(A, B, \Phi, \dot{A}, \dot{B}, \dot{\Phi})$:

$$L = 2\epsilon FA\dot{B}^2 + 4\epsilon FB\dot{A}\dot{B} + 2\epsilon F'B^2\dot{A}\dot{\Phi} + 4\epsilon F'AB\dot{B}\dot{\Phi} - 2\epsilon qFA + AB^2\left[\frac{\dot{\Phi}^2}{2} - U(\Phi)\right]. \quad (9)$$

The field Eqs. (2) and Klein-Gordon Eq. (5) for the metric (7) are found as follows

$$\frac{\dot{B}^2}{B^2} + 2\frac{\dot{A}\dot{B}}{AB} + \frac{q}{B^2} + \frac{F'}{F}\left(\frac{\dot{A}}{A} + 2\frac{\dot{B}}{B}\right)\dot{\Phi} + \frac{\epsilon}{2F}\left[\frac{\dot{\Phi}^2}{2} + U(\Phi)\right] = 0, \quad (10)$$

$$2\frac{\ddot{B}}{B} + \frac{\dot{B}^2}{B^2} + \frac{q}{B^2} + \frac{F'}{F}\left[\ddot{\Phi} + 2\frac{\dot{B}}{B}\dot{\Phi}\right] + \left(\frac{F''}{F} - \frac{\epsilon}{4F}\right)\dot{\Phi}^2 + \frac{\epsilon}{2F}U(\Phi) = 0, \quad (11)$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} + \frac{F'}{F}\left[\ddot{\Phi} + \left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B}\right)\dot{\Phi}\right] + \left(\frac{F''}{F} - \frac{\epsilon}{4F}\right)\dot{\Phi}^2 + \frac{\epsilon}{2F}U(\Phi) = 0, \quad (12)$$

$$\frac{\ddot{A}}{A} + 2\frac{\ddot{B}}{B} + \frac{\dot{B}^2}{B^2} + 2\frac{\dot{A}\dot{B}}{AB} + \frac{q}{B^2} + \frac{\epsilon}{2F'}\left[\ddot{\Phi} + \left(\frac{\dot{A}}{A} + 2\frac{\dot{B}}{B}\right)\dot{\Phi} + U'(\Phi)\right] = 0, \quad (13)$$

where $F' \neq 0$. Using the Lagrangian (9), the obtained Euler-Lagrange equations are the same as Eqs. (11)–(13). The *energy function* E_L associated with the Lagrangian (9) is found as

$$E_L = \frac{\partial L}{\partial \dot{A}}\dot{A} + \frac{\partial L}{\partial \dot{B}}\dot{B} + \frac{\partial L}{\partial \dot{\Phi}}\dot{\Phi} - L = \frac{\dot{B}^2}{B^2} + 2\frac{\dot{A}\dot{B}}{AB} + \frac{q}{B^2} + \frac{F'}{F}\left(\frac{\dot{A}}{A} + 2\frac{\dot{B}}{B}\right)\dot{\Phi} + \frac{\epsilon}{2F}\left[\frac{\dot{\Phi}^2}{2} + U(\Phi)\right]. \quad (14)$$

Therefore, it is obviously seen that the (0,0)-Einstein equation given by (10) is equivalent to $E_L = 0$.

The continuity equation obtained by eliminating $\ddot{A}$ and $\ddot{B}$ from the Eqs. (10)–(13) is given as

$$2(3F'^2 - \epsilon F)\left[\ddot{\Phi} + \frac{\dot{A}}{A}\dot{\Phi} + 2\frac{\dot{B}}{B}\dot{\Phi}\right] + F'(6F'' - \epsilon)\dot{\Phi}^2 + 2\epsilon(2UF' - FU') = 0. \quad (15)$$

To obtain the Noether symmetry one has to choose F and U in such a way that Eq. (15) is identically satisfied. Therefore this equation is required to check the existence of the Noether symmetry (see Refs. [14,15] for detailed information about Noether symmetry).

Now we seek the condition in order that the Lagrangian (9) would admit Noether symmetry. The configuration space of this Lagrangian is $Q = (A, B, \Phi)$, whose tangent space is $TQ = (A, B, \Phi, \dot{A}, \dot{B}, \dot{\Phi})$. The existence of Noether symmetry implies the existence of a vector field $\mathbf{X}$ such that

$$\mathbf{X} = \alpha\frac{\partial}{\partial A} + \beta\frac{\partial}{\partial B} + \gamma\frac{\partial}{\partial \Phi} + \dot{\alpha}\frac{\partial}{\partial \dot{A}} + \dot{\beta}\frac{\partial}{\partial \dot{B}} + \dot{\gamma}\frac{\partial}{\partial \dot{\Phi}}, \tag{16}$$

where α, β, and γ are dependent on A, B, and Φ. Hence the Noether equation (6) yields the following set of equations:

$$2\frac{\partial\beta}{\partial A} + B\frac{F'}{F}\frac{\partial\gamma}{\partial A} = 0, \tag{17}$$

$$\frac{\alpha}{2} + B\frac{\partial\alpha}{\partial B} + A\frac{\partial\beta}{\partial B} + A\frac{F'}{F}\left(\frac{\gamma}{2} + B\frac{\partial\gamma}{\partial B}\right) = 0, \tag{18}$$

$$\frac{\alpha}{2} + \beta\frac{A}{B} + A\frac{\partial\gamma}{\partial\Phi} + 2\epsilon F'\left(\frac{\partial\alpha}{\partial\Phi} + 2\frac{A}{B}\frac{\partial\beta}{\partial\Phi}\right) = 0, \tag{19}$$

$$\beta + B\frac{\partial\alpha}{\partial A} + A\frac{\partial\beta}{\partial A} + B\frac{\partial\beta}{\partial B} + B\frac{F'}{F}\left(\gamma + A\frac{\partial\gamma}{\partial A} + \frac{B}{2}\frac{\partial\gamma}{\partial B}\right) = 0, \tag{20}$$

$$2\frac{\partial\beta}{\partial\Phi} + \frac{F'}{F}\left(2\beta + B\frac{\partial\alpha}{\partial A} + B\frac{\partial\gamma}{\partial\Phi} + 2A\frac{\partial\beta}{\partial A}\right) + \frac{F''}{F}B\gamma + \frac{\epsilon}{2F}AB\frac{\partial\gamma}{\partial A} = 0, \tag{21}$$

$$\frac{\partial\alpha}{\partial\Phi} + \frac{F'}{F}\left(\alpha + \frac{A}{B}\beta + \frac{B}{2}\frac{\partial\alpha}{\partial B} + A\frac{\partial\beta}{\partial B} + A\frac{\partial\gamma}{\partial\Phi}\right) + \frac{A}{B}\frac{\partial\beta}{\partial\Phi} + \frac{F''}{F}A\gamma + \frac{\epsilon}{4F}AB\frac{\partial\gamma}{\partial B} = 0, \tag{22}$$

$$2\epsilon q(F\alpha + F'A\gamma) + (B\alpha + 2A\beta)BU(\Phi) + AB^2\gamma U'(\Phi) = 0. \tag{23}$$

Before tackling the solution of these equations for different choices of $F(\Phi)$, we note that there is a special choice: The Hessian determinant

$$W = \Sigma\left|\frac{\partial^2 L}{\partial\dot{Q}_i\partial\dot{Q}_j}\right| = 16AB^4F(3\epsilon F'^2 - F) \tag{24}$$

vanishes if F is given by

$$F = \frac{\epsilon}{12}(\Phi - \Phi_0)^2 \tag{25}$$

and then the Lagrangian (9) becomes degenerate. The above form is also nonminimal coupling.

For this coupling function, the above set of differential equations (17)–(22) can be solved for α, β, and γ using separation of variables to give

$$\alpha = \frac{\ell}{(1+m)}A^{1+m}B^{n/3}(\Phi - \Phi_0)^n, \tag{26}$$

$$\beta = \frac{\ell}{m}A^m B^{1+(m/2)}(\Phi - \Phi_0)^n, \tag{27}$$

$$\gamma = -\frac{\ell}{m}A^m B^{m/2}(\Phi - \Phi_0)^{(1+n)}, \tag{28}$$

where $n = 3m/2$ and $m \neq 0, -1$. These results generalize the solutions of Eqs. (17)–(22) obtained by Sanyal [15] for the KS model. (Note that Sanyal's paper appears to contain a misprint: In his Eq. (18), the third term should be divided by f.) They reduce to his study for $n = -3$ and $m = -2$.

Now α, β, γ, and $F(\Phi)$ obtained here have to satisfy Eq. (23) for the existence of the Noether symmetry. If m is arbitrary and $q = 0$ (BI), the solution of Eq. (23) gives

$$U(\Phi) = \lambda(\Phi - \Phi_0)^{(3m+2/m+1)}. \tag{29}$$

If $q \neq 0$ (BIII, KS), then we must have $m = -2$, and Eq. (23) yields

$$U(\Phi) = \lambda(\Phi - \Phi_0)^4. \tag{30}$$

When we consider induced gravity, that is, choose the coupling function as $F = c\Phi^2$, where $c \neq \epsilon/12$, which means that the Lagrangian (9) is nondegenerate, the solutions of the Noether symmetry equations (17)–(22) are

$$\alpha = 0, \qquad \beta = -kB, \qquad \gamma = k\Phi, \tag{31}$$

where k is a constant. The remaining Eq. (23) implies that we must have $q = 0$ (BI), which gives the potential as

$$U(\Phi) = \lambda\Phi^2. \tag{32}$$

This means that BIII and KS models ($q \neq 0$) have no nontrivial potential for induced gravity.

For $F(\Phi) = 1 - \zeta\Phi^2$, which is of the form of standard nonminimally coupled scalar field theory, there is no Noether symmetry in BI, BIII, and KS spacetimes.

III. FIELD EQUATIONS AND SOLUTION

For the case of the degenerate Lagrangian and $q \neq 0$ (BIII, KS), the coupling function is given by (25) and the potential by (30); so for $\Phi_0 = 0$, the field equations (10)–(13) reduce to

$$\frac{\dot{B}^2}{B^2} + 3\frac{\dot{\Phi}^2}{\Phi^2} + 2\frac{\dot{A}\dot{B}}{AB} + 2\frac{\dot{A}\dot{\Phi}}{A\Phi} + 4\frac{\dot{B}\dot{\Phi}}{B\Phi} + \frac{q}{B^2} + 6\lambda\Phi^2 = 0, \tag{33}$$

$$2\frac{\ddot{B}}{B} + 2\frac{\ddot{\Phi}}{\Phi} + \frac{\dot{B}^2}{B^2} - \frac{\dot{\Phi}^2}{\Phi^2} + 4\frac{\dot{B}\dot{\Phi}}{B\Phi} + \frac{q}{B^2} + 6\lambda\Phi^2 = 0, \tag{34}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + 2\frac{\ddot{\Phi}}{\Phi} - \frac{\dot{\Phi}^2}{\Phi^2} + 2\left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B}\right)\frac{\dot{\Phi}}{\Phi} + \frac{\dot{A}\dot{B}}{AB} + 6\lambda\Phi^2 = 0, \tag{35}$$

$$\frac{\ddot{A}}{A} + 2\frac{\ddot{B}}{B} + 3\frac{\ddot{\Phi}}{\Phi} + \frac{\dot{B}^2}{B^2} + 2\frac{\dot{A}\dot{B}}{AB} + 3\left(\frac{\dot{A}}{A} + 2\frac{\dot{B}}{B}\right)\frac{\dot{\Phi}}{\Phi} + \frac{q}{B^2} + 12\lambda\Phi^2 = 0. \quad (36)$$

For the Cartan one-form

$$\Theta_L = \frac{\partial L}{\partial \dot{A}}dA + \frac{\partial L}{\partial \dot{B}}dB + \frac{\partial L}{\partial \dot{\Phi}}d\Phi, \quad (37)$$

the constant of motion $i_{\mathbf{X}}\Theta_L$ for the Noether symmetry given by (26)–(28) is obtained as

$$\frac{(B\Phi)^{\cdot}}{A\Phi^2} = c_0 \Leftrightarrow \frac{\dot{B}}{B} + \frac{\dot{\Phi}}{\Phi} = c_0\frac{A\Phi}{B}, \quad (38)$$

which is actually the constraint that has to be satisfied by the field equations (33)–(36), where c_0 is a constant of motion.

Substracting Eq. (34) from Eq. (33) and using (38) in it, yields

$$\frac{\ddot{B}}{B} + \frac{\ddot{\Phi}}{\Phi} - 2\frac{\dot{\Phi}^2}{\Phi^2} = c_0\frac{\Phi\dot{A}}{B}, \quad (39)$$

which can also be obtained taking the time derivative of Eq. (38) (see Ref. [15] for KS metric). Thus one can eliminate acceleration terms between Eqs. (34) and (39), and then obtain another constraint equation as follows:

$$(A\Phi)^{\cdot} = -\frac{1}{2c_0B}[q + (c_0A\Phi)^2] - \frac{3\lambda}{c_0}B\Phi^2 \quad (40)$$

which can also be found using Eq. (38) in Eq. (33). Therefore, the last equation can be used instead of Eq. (33) or Eq. (34). Further, it follows from the remaining field equations (35) and (36) using Eq. (38) that

$$\frac{\ddot{A}}{A} + \frac{\ddot{\Phi}}{\Phi} - \frac{\dot{\Phi}^2}{\Phi^2} + 2c_0\frac{A\Phi}{B}\left(\frac{\dot{A}}{A} + \frac{\dot{\Phi}}{\Phi}\right) + \frac{\dot{A}\dot{\Phi}}{A\Phi} + 6\lambda\Phi^2 = 0, \quad (41)$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{\Phi}}{\Phi} - 3\frac{\dot{\Phi}^2}{\Phi^2} + \frac{\dot{A}\dot{\Phi}}{A\Phi} + c_0\frac{A\Phi}{B}\left(3\frac{\dot{A}}{A} + 4\frac{\dot{\Phi}}{\Phi}\right) + c_0^2\frac{A^2\Phi^2}{B^2} + \frac{q}{B^2} + 12\lambda\Phi^2 = 0. \quad (42)$$

Then, subtracting Eq. (41) from Eq. (42), and using (40), one gets

$$\frac{\dot{\Phi}^2}{\Phi^2} - \frac{c_0}{2}\frac{A\dot{\Phi}}{B} = \frac{c_0^2}{4}\frac{(A\Phi)^2}{B^2} + \frac{q}{4B^2} + \frac{3\lambda}{2}\Phi^2. \quad (43)$$

In order to find solution of the field equations, we use the transformation of the time coordinate by $dt = B(A\Phi)^{-1}d\tau$ in the above Eqs. (38), (40), and (43). Hence, Eq. (38) can immediately be integrated to get

$$B\Phi = a\exp(c_0\tau), \quad (44)$$

where $a(>0)$ is a constant of integration. Under this time transformation, Eq. (40) becomes

$$A\Phi(A\Phi)_{,\tau} + \frac{c_0}{2}(A\Phi)^2 + \frac{3\lambda a^2}{c_0}\exp(2c_0\tau) + \frac{q}{2c_0} = 0, \quad (45)$$

which has the solution

$$(A\Phi)^2 = -\frac{2\lambda a^2}{c_0^2}\exp(2c_0\tau) + b\exp(-c_0\tau) - \frac{q}{c_0^2}, \quad (46)$$

where b is an another integration constant. Finally, taking into consider the last relation, Eq. (43) can be transformed into the following form

$$\Phi_{,\tau}^2 - \frac{c_0}{2}\Phi\Phi_{,\tau} + g(\tau)\Phi^2 = 0, \quad (47)$$

where the function $g(\tau)$ is given by

$$g(\tau) = -\frac{c_0^2}{4} + \frac{c_0^2[q + 6\lambda a^2\exp(2c_0\tau)]}{4[2\lambda n^2\exp(2c_0\tau) - bc_0^2\exp(-c_0\tau) - q]}. \quad (48)$$

Then, the solution of Eq. (47) is obtained as

$$\Phi(\tau) = ke^{[(c_0/4)\tau(1\pm\sqrt{[5bc_0^2 - q\exp(c_0\tau) + 14\lambda a^2\exp(3c_0\tau)]/\{bc_0^2 - [q + 2\lambda a^2\exp(2c_0\tau)]\exp(c_0\tau)\}}]}, \quad (49)$$

where k is an integration constant. Hence, using this form of Φ, the metric functions A and B can be found from (44) and (46), respectively.

For the coupling function (25) and the potential (29) in which $q = 0$ (BI), the constant of motion c_1 corresponding to this Noether symmetry satisfies

$$\frac{\dot{B}}{B} + \frac{\dot{\Phi}}{\Phi} = c_1A^{-1-(2n/3)}B^{-2-(n/3)}\Phi^{-2-n}, \quad (50)$$

where $n = 3m/2$. Then, using the potential (29) in the field equations (10) and (11), and subtracting the resulting equations, we have the same equation with (38). Thus, comparing Eqs. (38) and (50), it is found that $n = -3$ and so $m = -2$. Therefore, for BI metric, we have same solution as (49) taking $q = 0$ in it.

For the potential (32) which is only valid for BI ($q = 0$) metric, the constant of motion corresponding to the Noether symmetry expressed in (31) is now given by

$$\ell \frac{\dot{B}}{B} + \frac{\dot{\Phi}}{\Phi} = \frac{[c_2/(1-8\epsilon c)]}{AB^2\Phi^2}, \tag{51}$$

where $c_2 = \alpha\partial L/\partial\dot{A} + \beta\partial L/\partial\dot{B} + \gamma\partial L/\partial\dot{\Phi}$ is a constant of motion, and $\ell = 4\epsilon c/(1-8\epsilon c)$, $\ell \neq 0, 1$.

IV. CONCLUSIONS

In this work we have studied the Noether symmetries of BI, BIII, and KS cosmological models in some scalar-coupled theories of gravity, including induced gravity. Assigning a parameter ϵ, we have treated our results in both signatures $+2$ ($\epsilon = -1$) and -2 ($\epsilon = +1$). Further, using the relevant coupling function $F(\Phi)$ and the potential $U(\Phi)$, we have solved the equations obtained through the field equations (or the Euler-Lagrange equations of motion) for BI, BIII, and KS spacetimes. For the coupling function (25), we find the potential (30), and the explicit form of the scalar field Φ in terms of the conformal time τ [where we have used $dt = B(A\Phi)^{-1}d\tau$] is obtained which is given by (49). For the latter scalar field, the metric functions A and B for BIII ($q = -1$) and KS ($q = 1$) spacetimes are found from (44) and (46) as

$$A = \frac{1}{\Phi}\left[-\frac{2\lambda a^2}{c_0^2}\exp(2c_0\tau) + b\exp(-c_0\tau) - \frac{q}{c_0^2}\right]^{1/2}, \tag{52}$$

$$B = \frac{a}{\Phi}\exp(c_0\tau). \tag{53}$$

If we use the potential (29) together with the coupling function (25), this case corresponds to the BI ($q = 0$) model and it follows the same potential with (49) and metric functions as given above. For induced gravity, i.e. the potential (32) with nondegenerate coupling function $F = c\Phi^2$ ($c \neq \epsilon/12$), we have a constant of motion given by (51) which is a generic first order differential equation depending upon the values of ℓ.

We note that the continuity Eq. (15) is required to check the existence of the Noether symmetry. Hence, for the potential (29), using the continuity Eq. (15), it is found that m is not arbitrary and it takes the value $m = -2$ and thus the corresponding potential (29) reduces to the form of (30). It is explicitly seen from Eq. (15) that the potential (32) together with the coupling function $F = c\Phi^2$ ($c \neq \epsilon/12$) gives rise to the following constraint equation:

$$\ddot{\Phi} + \frac{\dot{A}}{A}\dot{\Phi} + 2\frac{\dot{B}}{B}\dot{\Phi} + \frac{\dot{\Phi}^2}{\Phi} + \frac{\lambda}{(12c\epsilon - 1)} = 0. \tag{54}$$

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