Compatibility of radial, Lorenz, and harmonic gauges

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We observe that the radial gauge can be consistently imposed on-shell *together* with the Lorenz gauge in Maxwell theory and with the harmonic traceless gauge in linearized general relativity. This simple observation has relevance for some recent developments in quantum gravity where, as we argue, the radial gauge is implicitly utilized.

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I. INTRODUCTION

The radial, or Fock-Schwinger [1], gauge is defined by

$$x^\mu A_\mu = 0 \tag{1}$$

in Maxwell theory, and by

$$x^\mu h_{\mu\nu} = 0 \tag{2}$$

in linearized general relativity. Here $x = (x^\mu)$ are Lorentzian (or Euclidean) spacetime coordinates in $d + 1$ spacetime dimensions, where $\mu = 0, 1, \ldots, d$; $A_\mu(x)$ is the electromagnetic potential, and $h_{\mu\nu}(x) = g_{\mu\nu}(x) - \eta_{\mu\nu}$ is the perturbation of the metric field $g_{\mu\nu}(x)$ around the background Minkowski (or Euclidean) metric $\eta_{\mu\nu}$ used to raise and lower indices. The radial gauge is commonly considered as an alternative to the Lorenz and harmonic gauges, defined, respectively, by

$$\partial_\mu A^\mu = 0 \tag{3}$$

in Maxwell theory and by

$$\partial_\mu h^\mu{}_\nu - \tfrac{1}{2}\partial_\nu h^\mu{}_\mu = 0 \tag{4}$$

in linearized general relativity, which are largely utilized in the classical and quantum theories. Radial-gauge perturbation theory was studied for instance in [2]. Here we observe, instead, that in the classical theory the radial gauge is *compatible* with the Lorenz and the harmonic gauges. That is, if A_μ and $h_{\mu\nu}$ solve the Maxwell and the linearized Einstein equations, then they can be gauge transformed to fields A'_μ and $h'_{\mu\nu}$ satisfying (1) and (2) *and* (3) and (4). This is analogous to the well-known fact (see for instance [3]) that the Lorenz and the harmonic gauges can be imposed simultaneously with the temporal gauge

$$A_0 = 0, \tag{5}$$

$$h_{0\mu} = 0. \tag{6}$$

This result has relevance for some recent developments in quantum gravity, for the following reason. One of the open problems in the context of the loop approach to quantum gravity [4,5] is to understand the low-energy limit of the theory. A recent development that has raised much attention is the first derivation of the large-distance behavior of some n-point functions from the full nonperturbative and background-independent theory [6–8]. One is interested in comparing the n-point functions derived in this way, and, in particular, the propagator, with the corresponding quantities computed from the conventional perturbative (background-dependent) expansion of quantum general relativity. Agreement at large distance could be taken as evidence that the nonperturbative quantum theory has a good low-energy limit; while the differences at short distance reflect the improved ultraviolet behavior of the nonperturbative theory.

The comparison, however, is complicated by the peculiar gauge in which these n-functions have been derived. This is implicitly a radial gauge, for the following reason. The technique used in the papers [6–8] consists of considering a (Euclidean) quantum field theory defined on a finite spacetime region. With the chosen conditions on the boundary, the (average) geometry of this region turns out to be (hyper-)spherical. The degrees of freedom on its 3d boundary Σ are identified with the degrees of freedom described by Hamiltonian loop quantum gravity. But loop quantum gravity is defined in a "temporal" gauge where the field components in the direction normal to the boundary surface Σ are gauge fixed. Since the direction normal to a sphere is radial, in the low-energy limit this procedure is equivalent to imposing the radial gauge (2). One should therefore compare the quantities computed in [6–8] with corresponding low-energy quantities computed in radial gauge (2). But perturbative quantum gravity is mostly known in harmonic gauge (4) [9].

On the other hand, what is of primary interest is the lowest order in the expansion around flat space. At this order, the n-point functions are essentially free field quantities, or classical quantities that can be directly computed

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from the classical theory. For instance, the two-point function of a free field theory can be obtained in a straightforward way using canonical techniques, in the context of a fully gauge-fixed classical theory. Expectation has been for sometime that classical quantities computed in the harmonic gauge had to be gauge transformed, in order to be compared with quantities derived in the radial gauge. The result presented here that the harmonic gauge (4) and the radial gauge (2) are compatible makes the comparison much more straightforward. This application of our result will be presented elsewhere.

We stress the fact that our analysis is within the classical theory. We are not concerned here with setting up a quantum field theory in the radial/harmonic gauge. In particular, notice that the compatibility we prove requires the equations of motion to be satisfied, and does not extend off shell. This would raise problems in trying to implement a functional integral formulation of the theory, since such formulation involves off-shell fields. On the corresponding problem for the compatibility of the temporal and Lorenz gauges at the quantum level, and on gauge invariance at the quantum level, see [10]. The result we present is thus relatively simple, but its derivation is not entirely straightforward and requires some care; we have not found it in the literature and our experience has been that it was unexpected for many of our colleagues (and for us). The harmonic/radial gauge might be of use also in other contexts of classical general relativity.

Maxwell theory is discussed in Sec. II. Gravity is discussed in Sec. III. We work in an arbitrary number of dimensions, and we cover the Euclidean and the Lorentzian signatures at the same time. That is, we can take either $(\eta_{\mu\nu}) = \text{diag}[1, 1, 1, 1, \ldots]$ or $(\eta_{\mu\nu}) = \text{diag}[1, -1, -1, -1, \ldots]$. The analysis is local in spacetime and disregards singular points such as the origin. We find it convenient to utilize the language of general covariant tensor calculus. To avoid confusion, let us point out that this does not mean that we work on a curved spacetime. We are only concerned here with Maxwell theory on flat space and with linearized general relativity also on flat space. Tensor calculus is used below only as a tool for dealing in compact form with expressions in the hyperspherical coordinates that simplify the analysis of the radial gauge.

II. MAXWELL THEORY

In this section we show the compatibility between Lorenz and radial gauge in electromagnetism. Maxwell vacuum equations are

$$\partial_\nu F^{\nu\mu} = 0, \tag{7}$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$. That is

$$\Box A_\mu - \partial_\mu \partial_\nu A^\nu = 0, \tag{8}$$

where $\Box = \eta^{\mu\nu}\partial_\mu\partial_\nu$. This equation is of course invariant under the gauge transformation

$$A_\mu \to A'_\mu = A_\mu + \partial_\mu \lambda. \tag{9}$$

A. Temporal and Lorenz gauge

We begin by recalling how one can derive the well-know result that the Lorenz and *temporal* gauges are compatible. This is a demonstration that can be found in most elementary books on electromagnetism; we recall it here in a form that we shall reproduce below for the radial gauge.

Let us write $(x^\mu) = (x^0, x^i) = (t, \vec{x})$, where $i = 1, \ldots, d$. Let A_μ satisfy the Maxwell Eqs. (8). We now show that there is a gauge equivalent field A'_μ satisfying the temporal as well as the Lorenz gauge conditions. That is, there exists a scalar function λ such that A'_μ defined in (9) satisfies (3) and (5). The Eq. (5) for A'_μ defined in (9) gives $A_0 + \partial_0\lambda = 0$, with the general solution

$$\lambda(t, \vec{x}) = -\int_{t_0}^{t} A_0(\tau, \vec{x})\mathrm{d}\tau + \tilde{\lambda}(\vec{x}), \tag{10}$$

where $\tilde{\lambda}(\vec{x})$ is an integration "constant," which is an arbitrary function on the surface Σ defined by $t = t_0$. Can $\tilde{\lambda}(\vec{x})$ (which is a function of d variables) be chosen in such a way that the Lorenz gauge condition (which is a function of $d+1$ variables) is satisfied? To show that this is the case, let us first fix $\tilde{\lambda}(\vec{x})$ in such a way that the Lorenz gauge condition is satisfied *on* Σ. Inserting A'_μ in (3) and using (5) we have

$$\partial_\mu A'^\mu = \partial_i A'^i = \partial_i A^i + \Delta\lambda = 0, \tag{11}$$

where $\Delta = \partial_i \partial^i$ is the Laplace operator[1] on Σ. The restriction of this equation to Σ gives the Poisson equation

$$\Delta\tilde{\lambda}(\vec{x}) = -\partial_i A^i(t_0, \vec{x}), \tag{12}$$

which determines $\tilde{\lambda}(\vec{x})$. With $\tilde{\lambda}(\vec{x})$ satisfying this equation, A'_μ satisfies the temporal gauge condition everywhere and the Lorenz gauge condition on Σ. However, this implies immediately that A'_μ satisfies the Lorenz gauge condition everywhere as well, thanks to the Maxwell equations. In fact, the time component of (8) reads

$$\Box A'_0 - \partial_0\partial_\nu A'^\nu = -\partial_0(\partial_\nu A'^\nu) = 0. \tag{13}$$

That is, for a field in the temporal gauge, the Maxwell equations imply that if the Lorenz gauge is satisfied on Σ then it is satisfied everywhere.

B. Radial and Lorenz gauge

We now show that the *radial* and Lorenz gauge are compatible, following steps similar to the ones above. We want to show that there exists a function λ such that A'_μ defined in (9) satisfies (1) and (3), assuming that A_μ satisfies the Maxwell equations.

[1] Minus the Laplace operator in the Lorentzian case.

Because of the symmetry of the problem, it is convenient to use polar coordinates. We write these as $(x^a) = (x^r, x^i) = (r, \vec{x})$, where $r = \sqrt{|\eta_{\mu\nu}x^\mu x^\nu|}$ is the $(d+1)$-dimensional radius and $\vec{x} = (x^i)$ are three angular coordinates. In these coordinates the metric tensor $\eta_{\mu\nu}$ takes the simple form

$$\mathrm{d}s^2 = \gamma_{ab}(r,\vec{x})\mathrm{d}x^a\mathrm{d}x^b = \mathrm{d}r^2 + r^2\xi_{ij}(\vec{x})\mathrm{d}x^i\mathrm{d}x^j, \quad (14)$$

where $\xi_{ij}(\vec{x})$ is independent from r and is the metric of a 3-sphere of unit radius in the Euclidean case, and the metric of a hyperboloid of unit radius in the Lorentzian case. It is easy to see that in these coordinates, the radial gauge condition (1) takes the simple form

$$A'_r = 0. \quad (15)$$

Inserting the definition of A'_μ gives

$$\partial_r\lambda = -A_r, \quad (16)$$

with the general solution

$$\lambda(r,\vec{x}) = -\int_{r_0}^{r} A_r(\rho,\vec{x})\mathrm{d}\rho + \tilde{\lambda}(\vec{x}), \quad (17)$$

where the integration constant $\tilde{\lambda}$ is now a function on the surface Σ defined by $r = r_0$. The surface Σ is a d-sphere in the Euclidean case and a d-dimensional hyperboloid in the Lorentzian case. As in the previous section, we fix $\tilde{\lambda}(\vec{x})$ by requiring the Lorenz condition to be satisfied on Σ. It is convenient to use general covariant tensor calculus in order to simplify the expressions in polar coordinates. In arbitrary coordinates, the Lorenz condition reads

$$\nabla_a A'^a = \frac{1}{\sqrt{\gamma}}\partial_a(\sqrt{\gamma}A'^a) = 0, \quad (18)$$

where ∇_a is the covariant derivative, $A_b = A^a\gamma_{ab}$, and γ is the determinant of γ_{ab}. This determinant has the form $\gamma = r^{2d}\xi$, where ξ is the determinant of ξ_{ij}. When the radial gauge is satisfied, (18) reduces to

$$\partial_i(\sqrt{\xi}A'^i) = 0. \quad (19)$$

Let us now require that A'_μ satisfies this equation on Σ. Using (9), this requirement fixes $\tilde{\lambda}$ to be the solution of a Poisson equation on Σ, that is

$$\Delta\tilde{\lambda} = -\frac{1}{\sqrt{\xi}}\partial_i(\sqrt{\xi}A^i), \quad (20)$$

where the Laplace operator is $\Delta = \nabla_i\xi^{ij}\nabla_j$. In arbitrary coordinates, Maxwell equations read

$$\nabla_a F^{ab} = \frac{1}{\sqrt{\gamma}}\partial_a(\sqrt{\gamma}F^{ab}) = 0, \quad (21)$$

where

$$F^{ab} = \nabla^a A^b - \nabla^b A^a. \quad (22)$$

Consider the radial ($b = r$) component of (21); since $A'_r = 0$, using the form (14) of the metric, we have

$$\begin{aligned}\frac{1}{\sqrt{\gamma}}\partial_a(\sqrt{\gamma}F^{ar}) &= \frac{1}{\sqrt{\gamma}}\partial_a(\sqrt{\gamma}\gamma^{ab}F_{br}) \\ &= \frac{1}{\sqrt{\gamma}}\partial_a(\sqrt{\gamma}\gamma^{ab}(\partial_b A'_r - \partial_r A'_b)) \\ &= -\frac{1}{\sqrt{\xi}}\partial_i\Big(\sqrt{\xi}\frac{\xi^{ij}}{r^2}\partial_r A'_j\Big) \\ &= -\frac{1}{r^2\sqrt{\xi}}\partial_r\partial_i(\sqrt{\xi}\xi^{ij}A'_j) = 0, \end{aligned} \quad (23)$$

which shows that the Lorenz gauge condition (19) is satisfied everywhere if it is satisfied on Σ. This shows that we can find a function λ such that both the radial and the Lorenz gauge are satisfied everywhere.

III. LINEARIZED GENERAL RELATIVITY

We now consider the compatibility between the radial gauge and the harmonic traceless gauge (also known as transverse traceless gauge [3]) in linearized general relativity. Einstein equations in vacuum are given by the vanishing of the Ricci tensor. If $|h_{\mu\nu}(x)| \ll 1$, and we linearize these equations in $h_{\mu\nu}$, we obtain the linearized Einstein equations

$$\partial_\mu\partial_\nu h^\alpha{}_\alpha + \partial_\alpha\partial^\alpha h_{\mu\nu} - \partial_\mu\partial^\alpha h_{\alpha\nu} - \partial_\nu\partial^\alpha h_{\alpha\mu} = 0. \quad (24)$$

Under infinitesimal coordinate transformations,

$$h_{\mu\nu} \to h'_{\mu\nu} = h_{\mu\nu} + \tfrac{1}{2}(\partial_\mu\lambda_\nu + \partial_\nu\lambda_\mu), \quad (25)$$

where the factor $1/2$ is inserted for convenience. These are gauge transformations of the linearized theory. The harmonic gauge is defined by the condition

$$\nabla^\nu\nabla_\nu x^\mu = 0, \quad (26)$$

where ∇_ν is the covariant partial derivative[2]; in the linearized theory (26) reduces to

$$\partial_\nu h^{\nu\mu} - \tfrac{1}{2}\partial^\mu h^\nu{}_\nu = 0, \quad (27)$$

and in this gauge the Einstein Eqs. (24) read simply

$$\Box h_{\mu\nu} = 0. \quad (28)$$

A. Temporal and harmonic gauge

As we did for Maxwell theory, we begin by recalling how the compatibility between *temporal* and harmonic gauge can be proved. Start by searching a gauge parameter λ_μ that takes $h_{\mu\nu}$ to the temporal gauge $h'_{0\nu} = 0$.

[2] Notice that (26) means the covariant Laplacian of $d+1$ scalars ($d+1$ coordinates), not the covariant Laplacian of a $(d+1)$-vector.

Equation (6) gives

$$h_{0\mu} + \tfrac{1}{2}(\partial_0\lambda_\mu + \partial_\mu\lambda_0) = 0 \tag{29}$$

with the general solution

$$\lambda_0(t,\vec{x}) = -\int_{t_0}^{t} h_{00}(\tau,\vec{x})\mathrm{d}\tau + \tilde{\lambda}_0(\vec{x}), \tag{30a}$$

$$\lambda_i(t,\vec{x}) = -\int_{t_0}^{t}(2h_{0i}(\tau,\vec{x}) + \partial_i\lambda_0(\tau,\vec{x}))\mathrm{d}\tau + \tilde{\lambda}_i(\vec{x}), \tag{30b}$$

where the integration constants $\tilde{\lambda}_\mu(\vec{x})$ are functions on the 3d surface Σ defined by $t = t_0$. Next, we fix $\tilde{\lambda}_i$ by imposing the harmonic gauge condition (27) on Σ. Since we are in temporal gauge, this gives

$$\Delta\tilde{\lambda}_j = -2\partial_i h^i{}_j + \partial_j h^i{}_i, \tag{31}$$

which can be clearly solved on Σ. The time-time component of Einstein equations becomes

$$\partial_t^2 h'^i{}_i = 0, \tag{32}$$

whose only well-behaved solution is $h'^i{}_i = 0$; so in the temporal gauge the invariant trace of $h'_{\mu\nu}$ vanishes:

$$h'^\mu{}_\mu = \eta^{\mu\nu}h'_{\mu\nu} = 0, \tag{33}$$

and the harmonic condition (27) takes the simpler form

$$\partial_\nu h'^{\nu\mu} = 0, \tag{34}$$

similar to the Lorenz gauge. Now the (t, i) components of Einstein equations read

$$\partial_t\partial_j h'^j{}_i = 0, \tag{35}$$

which give $\partial_j h'^j{}_i = 0$ everywhere, once imposed on Σ.

B. Radial and harmonic gauge

Let us finally come to the compatibility between the *radial* and harmonic gauges. We return to the polar coordinates used in the Maxwell case. In these coordinates, the radial gauge condition (2) reads

$$h'_{rr} = h'_{ri} = 0. \tag{36}$$

Inserting the gauge transformation (25) gives

$$\partial_r\lambda_r = -h_{rr}, \tag{37a}$$

$$\partial_r\lambda_i + \partial_i\lambda_r - \frac{2}{r}\lambda_i = -2h_{ri}, \tag{37b}$$

with the general solution

$$\lambda_r(r,\vec{x}) = -\int_{r_0}^{r} h_{rr}(\rho,\vec{x})\mathrm{d}\rho + \tilde{\lambda}_r(\vec{x}), \tag{38a}$$

$$\lambda_i(r,\vec{x}) = -r^2\int_{r_0}^{r}\frac{2h_{ri}(\rho,\vec{x}) + \partial_i\lambda_r(\rho,\vec{x})}{\rho^2}\mathrm{d}\rho + r^2\tilde{\lambda}_i(\vec{x}), \tag{38b}$$

where $\tilde{\lambda}_r$, $\tilde{\lambda}_i$ are functions on the surface Σ given by $r = r_0$. We can then fix $\tilde{\lambda}_i$ by imposing the harmonic condition on Σ precisely as before. In the polar coordinates (14), we have easily the following rules for the Christoffel symbols:

$$\Gamma^a{}_{rr} = 0, \qquad \Gamma^i{}_{jr} = \frac{1}{r}\delta^i{}_j, \qquad \Gamma^r{}_{ra} = 0. \tag{39}$$

We note also that $\Gamma^i{}_{jk}$ is independent of r. Consider the (r, r) component of Einstein equations:

$$\nabla_r\nabla_r h'^a{}_a + \nabla_a\nabla^a h'_{rr} - \nabla_r\nabla^a h'_{ar} - \nabla_r\nabla^a h'_{ar} = 0. \tag{40}$$

Taking into account (14) and (39), it is verified after a little algebra that the previous equation becomes

$$\partial_r^2 h'^a{}_a + \frac{2}{r}\partial_r h'^a{}_a = 0, \tag{41}$$

which is a differential equation for the trace $h'^a{}_a$. Its only solution well behaved at the origin and at infinity is $h'^a{}_a = 0$. Using this, the (r, i) components of Einstein equations read:

$$\nabla_a\nabla^a h'_{ri} - \nabla_r\nabla^a h'_{ai} - \nabla_i\nabla^a h'_{ar} = -\partial_r\nabla_a h'^a{}_i = 0, \tag{42}$$

and the harmonic condition is simply

$$\nabla_a h'^{ab} = 0. \tag{43}$$

Equation (42) shows immediately that the $b = i$ components of the gauge condition (43) hold everywhere if they hold on Σ. The vanishing of the $b = r$ component of (43) follows immediately since, using (39), we have

$$\nabla_a h'^a{}_r = -\frac{1}{r}h'^a{}_a = 0. \tag{44}$$

Therefore, the harmonic gauge condition, the radial gauge condition, and the vanishing of the trace are all consistent with one another.

APPENDIX

We give here for convenience the definition of the coordinate systems in four dimensions we used in this work, followed by the respective line elements (metrics).

Polar hyperspherical coordinates

$$x^0 = r\cos\psi, \tag{A1a}$$

$$x^1 = r\sin\theta\cos\phi\sin\psi, \tag{A1b}$$

$$x^2 = r\sin\theta\sin\phi\sin\psi, \tag{A1c}$$

$$x^3 = r\cos\theta\sin\psi. \tag{A1d}$$

Euclidean line element:

$$\mathrm{d}s^2 = \mathrm{d}r^2 + r^2(\sin^2\psi\mathrm{d}\theta^2 + \sin^2\theta\sin^2\psi\mathrm{d}\phi^2 + \mathrm{d}\psi^2). \tag{A2}$$

Polar hyperbolic coordinates (future light cone)

$$x^0 = \rho \cosh\chi, \tag{A3a}$$

$$x^1 = \rho \sin\theta \cos\phi \sinh\chi, \tag{A3b}$$

$$x^2 = \rho \sin\theta \sin\phi \sinh\chi, \tag{A3c}$$

$$x^3 = \rho \cos\theta \sinh\chi. \tag{A3d}$$

Minkowski line element:

$$\mathrm{d}s^2 = \mathrm{d}\rho^2 - \rho^2(\sinh^2\chi \mathrm{d}\theta^2 + \sin^2\theta \sinh^2\chi \mathrm{d}\phi^2 + \mathrm{d}\chi^2). \tag{A4}$$

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