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# Incompressible fluid inside an astrophysical black hole?

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It is argued that under a natural hypothesis the fermions inside a black hole formed after the collapse of a neutron star could form a noncompressible fluid (well before reaching the Planck scale) leading to some features of the integer quantum Hall effect. The relations with black hole entropy are analyzed. Insights coming from the quantum Hall effect are used to analyze the coupling with the Einstein equations. Connections with some cosmological scenarios and with the higher dimensional quantum Hall effect are briefly pointed out.

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## I. INTRODUCTION

Black hole thermodynamics and Hawking radiation ([1–3]) are some of the few sound results in which general relativity and quantum field theory (henceforth QFT) fit together. On the other hand, there are some important open problems in this field. Mainly, it is still not understood the final stage of black hole evaporation and how to solve the information loss paradox. Black hole entropy is proportional to the area of the horizon and many different ways to deduce such an area law have been proposed (for a review see [4] and references therein). Many such models assume that the effective degrees of freedom of the black hole live on the boundary of the black hole itself and can be described by a conformal theory. Indeed, the success of such proposals has partially inspired the *holographic principle* (after the pioneering ideas of Bekenstein [5], 't Hooft [6], and Susskind [7]) which is believed to play a fundamental role in the yet to be discovered final theory of gravity. However, the questions of *why such effective degrees of freedom should live on the boundary* of the black hole and of *how the bulk degrees of freedom get frozen* are still opened.

Here it is analyzed the case of a black hole formed due to the collapse of a typical neutron star. It is argued that at an energy scale of order $10^{-21} \sim 10^{-18}$ of the Planck scale (after the horizon is formed) many features of quantum Hall effects (QHE) come into play. The classical gravitational force is likely to dominate the other processes of the standard model and the spectrum of the fermions living inside the black hole turns out to be discrete. Because of the gap, the fermions gas inside the black hole cannot be compressed anymore. It is not a scope of the present paper neither to write down an effective action for the effective degrees of freedom of a black hole nor to argue about the quantum degrees of freedom of gravity. The idea is to explain *why*, in a concrete situation, many of the available "conformal" descriptions of the effective degrees of a black hole *should work*.

Based on the AdS/CFT correspondence [8] and on the geometry of the 3-dimensional BTZ black hole [9,10], an analogy between quantum Hall effect and gravity in three dimensions has been pointed out in [11]. It appears that the quantum Hall bulk degrees of freedom as well as the edge excitations are suitable to describe the dynamical features of the BTZ black hole. The perspective in [11] is completely different from the present scheme in which the starting point is the collapse of a neutron star in 4-dimensional general relativity. It is therefore interesting that many connections between so different approaches arise anyway.

The structure of the paper is the following: In Sec. II, the assumptions of the present paper are explained and the standard order of magnitude inside a neutron star are resumed. In Sec. III the arising of features typical of quantum Hall effects is described. In Sec. IV the relations with black hole entropy are analyzed and a simple bound on the entropy is derived. In Sec. V the Einstein equations in the presence of a "quantum Hall" source are solved and the connection with higher dimensional quantum Hall formalism is pointed out. In Sec. VI the relations of the present proposal with some interesting cosmological scenarios are outlined together with the possible weakness of the approach. In Sec. VII some conclusions are drawn.

## II. THE STANDARD APPROXIMATIONS

The first basic assumption of the standard model of a neutron star is that at the energy scale of the standard model of particles physics (up to TeV) the collapsing neutron star can be described very well by QFT coupled to classical general relativity.

The second basic assumption is that the quantum dynamics of the neutrons (or of the quarks) living inside the neutron star is much faster than the dynamics of the gravitational field. This implies that one can compute the

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equation(s) of state of the fermions as usual and then use such equation(s) to solve the Einstein equations in which the source is described by the equation of state itself. The success of this theory initiated by Landau, Chandrasekhar, Tolman, Oppenheimer, Volkoff, Snyder (and many others) tells that such adiabatic approximation is excellent.

The two basic assumptions which will be needed in the following are that the above approximations also hold up to an energy scale of $10^{-21}$–$10^{-15}$ of the Planck scale: it will be assumed that the standard model and general relativity are the correct theories at these scales. It will be also assumed that at this scale of energy the fermions living inside the collapsed neutron star have a dynamic much faster than the dynamics of the gravitational field so that one can compute the equation of state of the fermions and than use the result to solve the Einstein equations coupled with the fermions themselves.

### A. Orders of magnitude inside a neutron star

Let us remind the reader of the typical order of magnitudes of a neutron star (NS). The typical mass $M_{\rm NS}$, radius $R_{\rm NS}$, density $\rho_{\rm NS}$, the baryon number $N_{\rm NS}$, and the Schwarzschild radius $r_G(M_{\rm NS})$ of a NS are

$$M_{\rm NS} \approx 10^{33}\ {\rm g}, \qquad R_{\rm NS} \approx 10^{6}\ {\rm cm},$$
$$\rho_{\rm NS} \approx 10^{15}\ {\rm g/cm^3}, \qquad N_{\rm NS} \approx 10^{54},$$
$$r_G(M_{\rm NS}) \approx 10^{4}\ {\rm cm}.$$

One can compute the strength of the gravitational interaction on fermions living inside a neutron star (which to a very good approximation can be considered as a sphere of constant density) of these characteristics; the result is

$$\hbar\omega_G = \hbar\sqrt{G\rho_{\rm NS}} \approx 10^{-38} E_{\rm Planck}.$$

Therefore, the gravitational interaction is negligible when compared to the strength of the interactions of the standard model (which are of the order of $10^{-21} \sim 10^{-18}$ of the Planck scale). A neutron star with a mass of the above order of magnitude is unstable against the gravitational collapse to a black hole. During the collapse the baryon number is conserved so that the black hole which is eventually formed should have the same parameters $M_{\rm NS}$ and $N_{\rm NS}$ of the parent neutron star (let us forget for a moment Hawking radiation which is, in any case, negligible for black holes of the mass of a neutron star; the issues connected with Hawking radiation will be briefly discussed later on). On the other hand, at first glance, $R_{\rm NS}$ should decrease without bound at least up to the Planck length since, apparently, there is no process which can prevent such a decreasing after the black hole is formed since ‘‘gravity dominates Pauli pressure.’’ In fact, if the radius decreases, at a certain point the strength of the gravitational interaction on fermions inside a blackhole will be comparable and stronger than the other interactions. When the radius decreased up to the following value of the density

$$\hbar\omega_G^* = \hbar\sqrt{G\rho_{\rm NS}^*} \approx (10^{-21}\text{–}10^{-18}) E_{\rm Planck}, \qquad (1)$$

the other interactions among the fermions inside the black hole should be treated as perturbations of the gravitational interaction.

## III. QUANTUM HALL EFFECT AND BLACK HOLES

The principal insight comes from the physics of integer QHE but possible effects related to the interactions of the fermions which could give rise to phenomenology of the fractional QHE should not be excluded *a priori* [12]. In the presence of a strong confining potential a gap opens up in the spectrum. Therefore, when the number of fermions is such that an integer number of levels is full the gas becomes incompressible because of the gap so that its equation of state is simply $\rho = {\rm const}$. If the assumptions made above are correct this also should happen inside a black hole formed after the collapse of a neutron star. The gravitational field inside a neutron star can be well approximated by the Newtonian harmonic oscillator[1] of frequency $\omega_G = \sqrt{G\rho_{\rm NS}}$. As it will be discussed in a moment, the corrections due to general relativity enhance the arising of ‘‘quantum Hall phenomenology’’ so that the essential physics can be understood using the Newtonian expression of the gravitational field. The gap satisfies

$$\hbar\sqrt{G\rho_{\rm NS}^*} = E_{\rm gap} > E_{\rm SM}, \qquad (2)$$

where $E_{\rm SM}$ is the typical energy scale of a process of the standard model.

That this should happen well before reaching the Planck scale can be argued as follows. Any cross section $\sigma_{\rm SM}$ computed in the standard model[2] decreases with energy (because of the unitarity of the gauge interactions appearing in the standard model):

$$\sigma_{\rm SM} \sim g(s)s^{-\gamma}, \qquad \gamma > 0,$$

[1]Indeed, the first computations in the theory of the gravitational equilibrium of a neutron star, made in these approximations, were quite successful. The reason is that the neutrons perceive around them a density which is almost uniform. Moreover, the probability for a neutron to escape from the star is negligible already at the level of Newtonian gravity and, as it is well known, ‘‘general relativity is more attractive’’ than Newtonian gravity.

[2]In the case of a neutron star, at the densities at which gravity begins to dominate, it is possible that all the neutrons could be transformed into quarks. In this case, besides gravity, the dominant interaction would be the strong interaction which is asymptotically free.

where $s$ is the typical energy (which in this case is proportional to a negative power of the radius of the collapsed neutron star) and $g(s)$ is the coupling constant at the energy scale of interest.[3] On the other hand, the strength of the gravitational oscillator increases when the radius is decreased so that at a certain point it begins to dominate the other processes. Usually "in vacuum" this happens at the Planck scale when general relativity is not a meaningful theory. In the present case, in the expression of the strength of the gravitational oscillator, also present is the baryon number $N_{\mathrm{NS}}$ of the parent neutron star:

$$\rho_{\mathrm{NS}} = N_{\mathrm{NS}} \frac{m_{\mathrm{Fermions}}}{R_{\mathrm{NS}}^3}$$

(where $m_F$ could be the neutron mass or the quark mass, but this is not relevant as far as the present paper is concerned). This huge number $N_{\mathrm{NS}}$ helps in lowering the critical scale beyond which gravity dominates in a domain in which classical general relativity and QFT can be trusted.

In the case of a black hole formed during the collapse of a nonrotating neutron star one should solve the Dirac equation in a three-dimensional harmonic potential. However, being the mass of the neutrons as well as the mass of the quarks much smaller than $\hbar\omega_G^*$

$$m_{\mathrm{quarks}} c^2 \ll \hbar\omega_G^*$$

the Schrodinger equation can also be used.[4] Therefore, the fermions live in a three-dimensional harmonic oscillator with frequency $\omega_G^*$ in Eq. (1) (an analysis of the quantum Hall behavior of a fermion gas in a three-dimensional harmonic trap can be found in [14]; interesting "harmonic oscillator" features in black hole physics have been stressed in [15–18]). Actually, the problem is more complicated since at densities of the order of the critical density of Eq. (1) the collapsed matter is well inside its Schwarzschild radius. Therefore, the potential should be a harmonic potential up to the end of the collapsed matter and a Schwarzschild potential outside the collapsed matter but inside the horizon:

$$V_G(r) \approx I_1 - \frac{m_{\mathrm{quarks}}}{2}(\omega_G^*)^2 r^2, \qquad r \leq r_M,$$
$$V_G(r) \approx -G\frac{M_{\mathrm{NS}}}{r}, \qquad r_M \leq r \leq r_G(M_{\mathrm{NS}}),$$
$$V_G \to \infty \qquad r > r_G(M_{\mathrm{NS}}),$$

where $I_1$ is a positive constant, $G$ is the Newton constant, $r_M$ is the radius of the collapsed matter, and the Hawking radiation is still neglected[5] (so that it can be assumed that the fermions are confined to be inside the horizon). This complication does not change the main new feature of the model: the energy spectrum is still discrete. The harmonic oscillator part of the potential dominates since in all the successful models of neutron stars the gravitational field outside the neutron star is not important to determine the equation of state of the fermions living inside the star itself.[6] As it is well known, the eigenvalue and the degeneracies of a three-dimensional harmonic oscillator are

$$E_n = \hbar\omega_G^*(n + 3/2) - I_1,$$
$$d(n) = 2\left(\frac{(n+1)(n+2)}{2}\right),$$

where the factor of 2 into the degeneracies is due to the spin degree of freedom, the negative constant $-I_1$ represents negative contributions related to the binding energy of the collapsed matter. Let us first suppose that the Baryon number is such that an integer number of levels is exactly filled

$$N_{\mathrm{NS}} = \sum_n^{n_{\max}} d(n) \approx (n_{\max})^3. \qquad (3)$$

Because of the energy gap, the fermions gas becomes incompressible and its equation of state becomes simply

$$\rho = \mathrm{const},$$

which will be used later on to discuss the coupling with the Einstein equations of such a gas. If the number of baryons does not allow the complete filling of an integer number of levels, one can write

$$N_{\mathrm{NS}} = \left(\sum_n^{n_{\max}} d(n)\right) + \delta_{n_{\max}}, \qquad \delta_{n_{\max}} < d(n_{\max}).$$

In this case (whose physical features will be discussed in more details in the next section), the fermions will form a gas partially compressible. However, such a gas cannot be compressed beyond the "incompressible core" constituted

[3] In the case of the gauge interactions of the standard model the "worst" case could be one in which the coupling constant increases logarithmically with the energy scale. However, this behavior does not change the main qualitative conclusion that at an energy scale well below the Planck scale inside a neutron star gravity begins to dominate. Moreover, the results in [13] indicate that gravitational corrections lower the scale of asymptotic freedom of the gauge interactions.

[4] The corrections due to the Dirac equation are proportional to positive powers of the ratio $\frac{m_{\mathrm{quarks}} c^2}{\hbar\omega_G^*} \ll 1$.

[5] The effects of Hawking radiation are small at this scale. In any case, they will be discussed later on.

[6] The inclusion of such deviations is only a technical problem. The discreteness of the spectrum would not change being, in any case, a confining potential (see, for instance, [19]). Also the order of magnitude of the gap should be dominated by the harmonic part which increases with the decreasing of the radius "strengthening the incompressibility."

by the first $n_{\max}$ fully filled levels. The largest part of the fermions is in the incompressible core since

$$\frac{\delta_{n_{\max}}}{N_{NS}} < (N_{\rm NS})^{-1/3} \ll 1.$$

### A. The general relativistic corrections

It can be argued that general relativity enhances the arising of quantum Hall phenomenology of the model: the reason is that general relativity is more attractive than Newtonian gravity. For a Newtonian star of uniform density the equilibrium is always possible while in general relativity the central pressure needed for equilibrium diverges when its gravitational radius $(2GM)/c^2$ is greater than $8/9$ of its actual radius while the pressure far from the origin is quite near to the Newtonian counterpart. The corrections due to general relativity are "attractive" and strong at the center of the star. The fermions perceive a modified harmonic potential $V_{\rm GR}$ which schematically can be written as follows:

$$V_{\rm GR} = I_1 - \frac{m_{\rm quarks}}{2}(\omega_G^*)^2 r^2 - f_{\rm GR}(r), \tag{4}$$

where $f_{\rm GR}$ is a positive function small far from the origin of the star but which can be large at the origin representing the increased attraction due to general relativity: for instance

$$f_{\rm GR}(r) = \frac{\kappa^2}{r^\alpha}, \qquad \alpha > 0,$$

($\kappa$ and $\alpha$ being real constants) is a reasonable choice to describe the general relativistic effects on the potential perceived by the fermions inside the black hole [the precise form of $f_{\rm GR}(r)$ is not important, only its qualitative features matter]. The effects of such correction on the wave functions of the fermions can be evaluated with perturbation theory. They are very small on the wave functions belonging to high energy levels since such wave functions are small near the origin where $f_{\rm GR}(r)$ is large (actually, such effects can be neglected for all the wave functions which are not peaked near the origin). The corrections to the eigenvalues of the Hamiltonian are

$$\Delta E_n = -\langle \psi_n | f_{\rm GR}(r) | \psi_n \rangle,$$

where $\psi_n$ is an eigenfunction belonging to the $n$th level of the three dimensional harmonic oscillator. The strength of such corrections decreases with $n$

$$\partial_n |\Delta E_n| < 0,$$

while for small $n$ could be quite strong. The generic result is that the corrections due to general relativity *enhance the gap* between the levels (and, in particular, this fact manifests itself in the eigenfunctions corresponding to the lowest levels). This means that the corrections due to general relativity *strengthen the incompressibility* of the fermions gas.

It is worthwhile to stress an interesting point. The actual potential perceived by the particles is the sum of a harmonic oscillator term plus a further attractive general relativistic correction in which the coupling constants $\omega_G^*(t)$ and $\kappa(t)$ depend adiabatically on time

$$V_{\rm GR} = I_1 - \frac{m_{\rm quarks}}{2}(\omega_G^*(t))^2 r^2 - \frac{(\kappa(t))^2}{r^\alpha}. \tag{5}$$

Because of the assumptions made in this paper, the fermions perceive a static potential. The degeneracies of the highest energy levels are dominated by the harmonic oscillator part and consequently are constant in time. However, in the hypothesis of a spherically symmetric collapse, the whole set of degeneracies is likely not to depend on time. The reason is that any central potential (besides few integrable exceptions like the harmonic oscillator itself, the Coulombian potential and so on) has the same degeneracies related to the spherical symmetry. Therefore, one can roughly divide the energy levels into the most interior levels which feels the general relativistic corrections (but whose degeneracies are constant) and the higher energy levels which are very well approximated by harmonic oscillator states (so that the corresponding degeneracies are constant as well).

## IV. QUANTUM HALL EFFECT AND BLACK HOLE ENTROPY

Many different proposals lead to the same conclusion: the entropy is related to the area of the horizon. This universality could be related to the underlying conformal theory living on the boundary which allows one to use the powerful results in [20]. It is important to stress also a known but important fact. The three-dimensional BTZ black hole [9,10] has entropy as well as Hawking radiation. In such a case, the degrees of freedom to which the BTZ entropy refers are not related in an obvious way to gravitational degrees of freedom since in three dimensions gravity has not local degrees of freedom (this is a highly nontrivial question under active investigation; see [21] and references therein). To get a reasonable physical picture of the situation (in the approximation in which gravity is classical), one can imagine to assign the BTZ entropy to suitable matter degrees of freedom which generate the singularity at the origin. In the analysis of the spherically symmetric collapse of a neutron star in four dimensions there are not propagating gravitational degrees of freedom as well (which would need, at least, a nontrivial quadrupole moment). Therefore, when quantum gravitational effects are neglected, one can assign the black hole entropy to the degrees of freedom living inside (and generating) the black hole itself (in the same way as one actually does in the case of gas fermions living inside a Newtonian star).

In the case analyzed here, QHE (see [12,22]) provides one with a very natural insight into "why the bulk degrees of freedom are frozen" so that only boundary excitations are left. The only ingredients are the presence of a huge number of fermions (related to the Baryon number conservation up to energy scale of $10^{-15}$ of the Planck scale) and the dominance of the classical gravitational attraction.

One possible criticism is that, as the theory of quantum Hall effect clearly stresses [12,22], the assumption that the fermion gas cannot be compressed anymore depends quite strongly on the "exceptional" fact that the number of fermions is such that an integer number of levels is exactly filled. There is a further argument (which does not depend on the above exceptional fact) to see that the present scheme provides with a qualitative explanation both of why the "dynamical" entropy (this term will be explained in a moment) is related to the area of the horizon and of how the bulk degrees of freedom are frozen. However, this argument only works in the case of fermions. In the presence of a strong classical gravitational field, the fermions feel a potential with discrete energy levels. Except the most interior levels, the other levels and the relative degeneracies are well approximated by the corresponding quantities of a harmonic oscillator. The entropy of such a system can be written as follows:

$$S = S_0 + S_{\rm dyn} \qquad S_0 = -\sum_{n}^{n_{\max}-1} p_n \log p_n,$$

$$S_{\rm dyn} = -p_{n_{\max}} \log p_{n_{\max}},$$

where $p_n$ is the probability to be in the $n$th energy level, $n_{\max}$ is the last partially filled energy level, and the reason to split the entropy into two pieces will be explained in a moment. The present fermion gas can be considered as a system at zero temperature since, already for a neutron star, the Fermi level is much higher than the temperature. For this reason, the probabilities to be in a given level only depend on the relative occupation number and on the corresponding degeneracy. It is important in the present context to split the entropy into two terms because such terms play different roles. The first term $S_0$ is the entropy corresponding to the interior fully filled levels. Such a term is likely to be constant in time: due to the gap, even if the whole system could not be in a static situation, the fermions inside the fully filled energy level are frozen. There is no possibility to jump in different energy levels because of the gap. It is also impossible to jump into different places of the same energy level because they are fully filled. The first part of the entropy only depends on the degeneracies of the fully filled levels. In the hypothesis of the present paper, such degeneracies can be assumed to be constant in time[7]:

$$\partial_t S_0 \approx 0.$$

On the other hand, the last term $S_{\rm dyn}$ corresponds to the last partially filled energy level. This part is likely not to be constant in time: the level is only partly filled and the particles living there may interact jumping into different free places of the same energy level. Because of the gap, the fermions inside the partially filled level can only remain in the same level: the interactions are not able to change the fermions' energies. Therefore, the only possible excitations should be low energy excitations. The fermions living in the last partially filled energy level are not frozen and $S_{\rm dyn}$ corresponds to the dynamical part of the entropy which can play an important role during the evolution:

$$\partial_t S_{\rm dyn} \neq 0.$$

One can derive a bound for the dynamical entropy: $N_{\rm last}$ (which is the number of fermions living in the last partially filled energy level) is bounded by the degeneracy of the last level:

$$N_{\rm last} \lesssim d(n_{\max}) = (n_{\max}+1)(n_{\max}+2) \approx (N_{\rm NS})^{2/3}, \quad (6)$$

where it has been taken into account Eq. (3). The total mass of the gas is proportional to the number of particles ($N_{\rm NS}$ in this case) and consequently (being the density constant) the volume also is proportional to $N_{\rm NS}$. Equation (6) tells $N_{\rm last}$ is proportional to the area $A_{\rm NS}$ of the horizon of the collapsed neutron star. To get the bound for $S_{\rm dyn}$ one can write as usual

$$S_{\rm dyn} = \log\Omega,$$

$\Omega$ being the number of possible microscopical configurations corresponding to the partially filled energy level. A reasonable estimate for $\Omega$ is the standard binomial expression

$$\Omega \approx \frac{(d(n_{\max}))!}{(d(n_{\max}) - N_{\rm last})!(N_{\rm last})!}.$$

Eventually, taking into account the Stirling formula which can be applied in this case being $N_{\rm NS}$ a very large number, the dynamical part of the entropy is bounded as follows:

$$\log A_{\rm NS} \lesssim S_{\rm dyn} \lesssim A_{\rm NS}\log A_{\rm NS}, \quad (7)$$

which, because of the simplicity of the argument, appears to be a good order of magnitude estimate strongly suggesting the Bekenstein-Hawking law.

It is also worth noting the above qualitative reasoning works in any dimension: the reason is that the degeneracies of a $D$-dimensional harmonic oscillator increases with the energy level label $n$ as

$$d_D(n) \approx n^{D-1} \Rightarrow (n_D)_{\max} \approx (N_{\rm NS})^{1/D} \Rightarrow d((n_D)_{\max}) \approx (N_{\rm NS})^{(D-1)/D},$$

where $(n_D)_{\max}$ is the last partially filled energy level in $D$ dimensions and $d_D(n)$ is the degeneracy of the $n$th level in $D$ dimensions. Assuming that the fermions' gas in $D$ dimensions becomes incompressible, one is lead to the

[7]This is obvious in the case of a harmonic oscillator since the collapse simply enhances the gap keeping fixed the degeneracies. However, even if the general relativistic corrections are taken into account, the degeneracies of the energy levels are likely not to depend on time [see the considerations after Eq. (5)].

conclusion that the dynamical part of the entropy should be proportional to the area of the collapsed neutron star.

### A. Some considerations on Hawking radiation

Even if for black holes of the mass of a neutron star Hawking radiation is negligible, it is interesting to note that Hawking radiation should be also affected by the presence of the discrete spectrum inside the collapsed matter. Assuming that the black hole evaporation is a real physical process implies that Hawking particles in some way have to "bring the mass of the black hole to infinity." Until the final stages, the evaporation is not a strong gravitational field phenomenon because the black hole mass decreases slowly with time [23]. The standard Einstein equations with a suitable matter source can describe how the metric evolves during the evaporation. As the standard *semiclassical program* has shown (for a highly incomplete list of references see [24–32] and references therein) the back reaction on the metric due to a null energy-momentum tensor describing the Hawking particles is not able to stop the evaporation.

It is nevertheless worth noting that it can be shown in the spherical symmetric four-dimensional case *without approximation* that if one adds a trace anomaly term to the energy-momentum tensor (allowing, in principle, arbitrary violations of the energy conditions) the evaporation process stops [33]. This conclusion fits quite well with the results obtained in [34,35].

If the energy spectrum of the particles living inside the horizon is gapped, the Hawking weight could be reduced: as originally found by Hawking [3], the expectation value of the operator number of a Bosonic field of spin zero, measured by a static observer in the asymptotic future of Schwarzschild black hole of mass $M$, is

$$n_i(E) = \frac{1}{\exp(E/kT_H) - 1}, \qquad T_H = \frac{\hbar c^3}{8\pi kGM}, \tag{8}$$

where $k$ is the Boltzmann constant and $E$ is the energy of the particle. If the spectrum is gapped, the maximum of the above expression is obtained for

$$n_i(E_{\rm gap}) = \frac{1}{\exp(E_{\rm gap}/kT_H) - 1},$$

where $E_{\rm gap}$ is of order in Eq. (1). This number, as one can expect, turns out to be extremely small in the concrete case of a black hole formed during the collapse of a neutron star. The only possibility for the Hawking particles is to bring outside the black hole the only low energy degrees of freedom available, namely, the gapless boundary excitations. Therefore, the present scheme suggests that the evaporation process, after the Hawking particles "have brought away" the gapless deformations of the boundary, should be highly suppressed by the presence of the gap.

## V. THE COUPLING WITH EINSTEIN EQUATIONS

In [36] a very interesting alternative scenario for the final state of the gravitational collapse has been proposed: the relation of such a proposal with the present scheme will be discussed in the following sections. In this section we will set

$$\hbar = 1, \qquad c = 1$$

while keeping the Newton constant.

One has to describe the interior space time inside the horizon of a black hole. The metric inside the horizon but outside the collapsed matter is the Schwarzschild metric. Inside the collapsed matter before classical gravity begins to dominate the interactions of the standard model, the solution can be represented by the well-known Tolman-Oppenheimer-Volkoff-Snyder solution (which behaves like a Friedman-Robertson-Walker cosmological solution matched with the Schwarzschild metric). Thus, the interior QHE solution, whose equation of state is $\rho = \text{const}$, has two boundaries: one spacelike boundary separating the standard Tolman-Oppenheimer-Volkoff-Snyder interior solution from the interior solution in which the equation of state of the matter changes into an incompressible gas.[8] The timelike boundary separates the interior QHE solution from the exterior Schwarzschild metric.[9] Furthermore, one would like to find an interior solution which as smoothly as possible matches with the standard interior Tolman-Oppenheimer-Volkoff-Snyder (TOVS) solution.

One can analyze this problem with the technique of the junction conditions. Such a technique allows one to match two different solutions provided that the metric is continuous and the discontinuity of the extrinsic curvature is compensated by a suitable energy-momentum tensor $S_\mu^\nu$ living on the junction hypersurface. With a strange enough $S_\mu^\nu$ also quite different metrics could be matched. One has to search for a junction in which $S_\mu^\nu$ is suitable to describe "QHE" features. In the present context, since $S_\mu^\nu$ is the candidate to describe the boundary degrees of freedom of the incompressible gas, it should represent the classical limit of an energy-momentum tensor describing low energy gapless excitations. This means that one should only allow vanishing or traceless $S_\mu^\nu$. This problem can be solved using two results in the cosmological literature found in a different context ([37–40]).

[8] Schematically, one can write the surface representing the spacelike boundary as $\tau = \tau^*$ where $\tau$ is the proper time of the fermions inside the collapsed neutron star and $\tau^*$ can be thought of as the time when gravity begins to dominate the other interactions.

[9] Schematically, one can write the surface representing the timelike boundary as $R = R(\tau)$ where $R(\tau)$ is the radius of the collapsed matter which, in principle, could depend on time.

### A. The spacelike boundary

Here it will be analyzed how to match the interior "quantum Hall metric" with the standard interior solution describing the collapsed neutron star. Actually, the would-be interior metric describing the quantum hall fluid has not been found yet. There is indeed some arbitrariness due to the fact that it is only known that $\rho$ is constant while the pressure is unconstrained. To overcome this problem one can search for an interior solution with $\rho = \text{const}$ such that the matching with the TOVS solution is as smooth as possible. Using the results of [38,39], one can see that this can be done *without introducing any $S^\nu_\mu$ when the interior metric is the de Sitter one* (in which both $\rho$ and $p$ are constant). It is a very welcome fact that along the spacelike boundary no $S^\nu_\mu$ is needed.[10] This implies that the present model is quite natural since the matching can be done in a rather smooth way.

Different interior metrics can indeed be considered since the equation of state does not determine uniquely the interior solution. On the other hand, the relations with higher dimensional QHE (discussed in the next sections) strongly suggest that the interior QHE solution should be described by a constant curvature metric.

The metric inside the matter during the collapse is a part of the closed Friedman-Robertson-Walker universe (see, for instance, [41])

$$\begin{aligned} ds^2 &= a_F^2(\tau)(-d\tau^2 + d\chi^2 + \sin^2\chi d\Omega^2), \\ a_F(\tau) &= a_0(1-\cos\tau), \qquad 0 \le \chi \le \chi_0 < \frac{\pi}{2}. \end{aligned} \tag{9}$$

When $\tau = \pi$ the collapsing star reaches the maximum expansion $a_F(\pi) = 2a_0$. The mass $m$ of the collapsing Friedman fluid is constant during the evolution and reads

$$m = \frac{3a_0}{2}(\chi_0 - \sin\chi_0 \cos\chi_0).$$

Because of the gravitational self-energy, the mass of the external Schwarzschild black hole is

$$M = a_0 \sin^3\chi_0.$$

The radius $r(\tau)$ of the collapsing matter evolves as

$$r(\tau) = a_F(\tau)\sin\chi_0. \tag{10}$$

The metric in Eq. (9) has to be matched with the de Sitter metric. The suitable coordinates system to write the de Sitter metric is

$$\begin{aligned} ds^2 &= a_{dS}^2(t)(-dt^2 + d\chi^2 + \sin^2\chi d\Omega^2), \\ a_{dS}(t) &= \frac{l}{\sin t}, \end{aligned}$$

where $l$ is the cosmological length. As it has been already explained, the matching has to be performed on a spacelike hypersurface $\Sigma_0$ in such a way that the energy-momentum tensor $S^\nu_\mu$ of the hypersurface vanishes.[11]

The matching conditions, as formulated in [42], can be introduced as follows. Let $\Sigma$ be the nonnull hypersurface on which the matching has to be performed. Let $\xi^\mu$ be the normal to $\Sigma$, and let

$$h^{\mu\nu} = g^{\mu\nu} \mp \xi^\mu \xi^\nu$$

be the metric induced on $\Sigma$ (in which the minus sign corresponds to a spacelike $\xi^\mu$ and to timelike $\Sigma$ and the plus sign to the other case). The matching conditions are that the metric has to be continuous across $\Sigma$ and

$$\begin{aligned} \gamma^\mu_\nu &= -8\pi G\left(S^\mu_\nu - \frac{1}{2}\delta^\mu_\nu \operatorname{tr} S\right), \\ \gamma_{\mu\nu} &= \lim_{\varepsilon\to 0}[K_{\mu\nu}(\eta = +\varepsilon) - K_{\mu\nu}(\eta = -\varepsilon)], \end{aligned} \tag{11}$$

where $K_{\mu\nu}$ is the extrinsic curvature of $\Sigma$

$$K_{\alpha\beta} = h^\mu_\alpha h^\nu_\beta \nabla_\mu \xi_\nu,$$

$\eta$ is the arc length measured along the geodesic orthogonal to $\Sigma$. and $S^\nu_\mu$ is the energy-momentum tensor living on $\Sigma$ needed to compensate for the discontinuity of the extrinsic curvature

$$S_{\alpha\beta} = h^\mu_\alpha h^\nu_\beta S_{\mu\nu}.$$

The jump conditions at the spacelike hypersurface $\Sigma_0$ (which has parametric equations $t = t_0 = \text{const}$ and $\tau = \tau_0 = \text{const}$ in the de Sitter and Friedman metrics, respectively) are (see [39])

$$a_F(\tau_0) = a_{dS}(t_0), \qquad S^\nu_\mu = -\frac{\lambda}{4\pi}\delta^\nu_\mu, \tag{12}$$

$$\lambda = a_{dS}^{-2}\frac{\partial a_{dS}}{\partial t}\bigg|_{t=t_0} - a_F^{-2}\frac{\partial a_F}{\partial t}\bigg|_{\tau=\tau_0}. \tag{13}$$

Because of the requirements coming from "quantum Hall physics," we have to search for a solution of the equation

[10] A traceless $S^\nu_\mu$ is a welcome feature on the timelike boundary while on the spacelike boundary the interpretation of $S^\nu_\mu$ as the energy-momentum tensor of gapless excitations would be less clear (since such excitations would live on a manifold without a time direction).

[11] The possibility to do the matching without any $S^\nu_\mu$ is a rather nontrivial requirement. The reason is that the equation of state in the standard phase (in which the pressure can be considered small and the density increases) is completely different from the equation of state of the "quantum Hall phase" (in which the density is constant and the pressure can be, in principle, arbitrarily large). The fact that de Sitter space time passes this test is a strong confirmation that the approach proposed here is physically sensible.

$$a_{dS}^{-2}\frac{\partial a_{dS}}{\partial t}\bigg|_{t=t_0}=a_F^{-2}\frac{\partial a_F}{\partial t}\bigg|_{\tau=\tau_0}. \tag{14}$$

Obviously, such a requirement (which is an input of quantum Hall physics) was not present in the main references for this section [38,39]. Explicitly, taking into account Eq. (12), Eq. (14) reads

$$\left(2\frac{a_0}{a_F(\tau_0)}-1\right)^{1/2}=\left(\left(\frac{a_F(\tau_0)}{l}\right)^2-1\right)^{1/2}$$

so that

$$a_F(\tau_0)=(2la_0)^{1/3}, \tag{15}$$

which [when inserted in Eq. (10)] gives a measure of the radius of the collapsed star when the quantum hall regime sets in.

### B. The timelike boundary

One is left with the problem to match the interior de Sitter solution with the exterior Schwarzschild solution with a traceless $S_\mu^\nu$ along the timelike boundary. A traceless $S_\mu^\nu$ is a welcome feature because it would allow the description of the boundary gapless excitations expected on quantum Hall grounds. It is convenient, in this case, to use the following coordinates systems for the interior de Sitter solution $ds_I^2$ and for the exterior Schwarzschild solution $ds_E^2$, respectively,

$$ds_I^2=-(1-k^2R^2)dT^2+\frac{dR^2}{(1-k^2R^2)}+R^2d\Omega^2, \tag{16}$$

$$ds_E^2=-\left(1-\frac{2GM}{R}\right)dT^2+\frac{dR^2}{(1-\frac{2GM}{R})}+R^2d\Omega^2, \tag{17}$$

$$k^2=\frac{8\pi G}{3}\rho_0, \tag{18}$$

where $\rho_0$ is the density of collapsed matter which is of the order in Eq. (2). The results in [37][12] tells that a matching along a timelike boundary can be achieved. Because of the spherical symmetry, it can be assumed that the spatial sections of the timelike matching hypersurface $\Sigma_{(t)}$ are isomorphic to the two sphere so that there exists a coordinates system in which the induced metric $ds^2|_{\Sigma_{(\tau)}}$ and $S_\mu^\nu$, respectively, read:

$$\begin{aligned} ds^2|_{\Sigma_{(\tau)}}&=-d\tau^2+r^2(\tau)d\Omega^2,\\ S^{\mu\nu}&=\sigma(\tau)(U^\mu U^\nu)-\zeta(\tau)(h^{\mu\nu}+U^\mu U^\nu),\\ h^{\mu\nu}&=g^{\mu\nu}-\xi^\mu\xi^\nu,\end{aligned} \tag{19}$$

where, as in the previous subsection, $h^{\mu\nu}$ is the metric induced on $\Sigma_{(\tau)}$, $\xi^\mu$ is the (spacelike) normal to $\Sigma_{(\tau)}$, $\tau$ is the arc length measured along the timelike geodesic belonging to $\Sigma_{(\tau)}$, $U^\mu$ is the normalized four velocity of $\Sigma_{(\tau)}$, $\sigma$ is the surface energy density of $\Sigma_{(\tau)}$, $\zeta(\tau)$ is the surface tension, and the conservation of $S_\mu^\nu$ implies

$$\partial_\tau\sigma=-2(\sigma-\zeta)\frac{\partial_\tau r}{r},$$

$r(\tau)$ being the proper circumferential radius of the domain wall $\Sigma_{(\tau)}$. Unlike the dynamics of a false vacuum bubble, here the requirement to be consistent with a ''quantum Hall picture'' tells that $S^{\mu\nu}$ has to be chosen traceless (otherwise it would not correspond to the classical description of boundary gapless degrees of freedom); therefore, one gets

$$\sigma+2\zeta=0\rightarrow\frac{\partial_\tau\sigma}{\sigma}=-3\frac{\partial_\tau r}{r}\Rightarrow \tag{20}$$

$$\sigma=\frac{\sigma_0}{r^3},\qquad \sigma_0>0, \tag{21}$$

where $\sigma_0$ is an integration constant which depends on the microscopic model.

Namely, $\sigma_0$ is related to the surface tension (also related to the energy density of the boundary degrees of freedom) of the incompressible three-dimensional gas (so that it can be assumed to be positive). In the quantum Hall case many tools (related to conformal field theory) would come into play to determine the analogous parameter; in the present case to determine $\sigma_0$ from the microscopic theory appears to be a rather difficult task. One can therefore deal with $\sigma_0$ as a phenomenological parameter: the following results have a nice interpretation if compared with the dynamics of a false vacuum bubble.

The ''equation of motion of the domain wall'' that is, the equation which determines the evolution of $r(\tau)$, can be deduced from the matching condition (11). In particular, the important equation is the angular $\theta\theta$ component of Eq. (11)

$$\gamma_\theta^\theta=-8\pi GS_\theta^\theta. \tag{22}$$

One can see that the $\tau\tau$ component of the matching equations is not independent on the $\theta\theta$ one provided the conservation of energy of $S_\mu^\nu$ is taken into account so that it is enough to deal with the $\theta\theta$ component.

It is interesting to note that at first glance, the right-hand side of Eq. (22) is the same as it appears in the dynamics of a false vacuum bubble (even if in such a case [37] the $S_\mu^\nu$ *is not* traceless)

$$-8\pi GS_\theta^\theta=8\pi G\zeta=-4\pi G\sigma,$$

[12] In which the authors considered the problem to match the de Sitter solution to the exterior Schwarzschild solution along a timelike hypersurface. However, in [37] the authors were not interested in a traceless $S_\mu^\nu$ and considered a different $S_\mu^\nu$ to describe the evolution of false vacuum bubbles.

where Eqs. (19) and (20) have been taken into account, because in the dynamics of a false vacuum bubble the trace of $S^\nu_\mu$ cooperates to give, formally, the same result. The important difference related to the traceless condition will be manifest in a moment.

The standard procedure to compute $\gamma^\theta_\theta$ is as follows: because the domain wall is spherically symmetric, on the exterior Schwarzschild side, the four velocity of any of its point can be written as follows:

$$U^\mu_{(E)} = \left(\frac{\partial_\tau r}{1-\frac{2GM}{r}}, \left(1-\frac{2GM}{r}\right)\partial_\tau t, 0, 0\right), \tag{23}$$

where it has been taken into account that, on the exterior Schwarzschild side (17), the coordinate $R$ approaches to $r$ when approaching the domain wall. On the interior de Sitter side, the four velocity of any of the point of the domain wall is

$$U^\mu_{(I)} = \left(\frac{\partial_\tau r}{1-k^2r^2}, (1-k^2r^2)\partial_\tau t_{(I)}, 0, 0\right), \tag{24}$$

where the notation $t_{(I)}$ has been introduced to stress that $t_{(I)}$ refers to the interior coordinate. To compute the left-hand side of Eq. (22) the four velocity has to have unit norm in both coordinates systems; therefore, taking into account Eq. (23) in the exterior Schwarzschild coordinates system one has

$$\left(1-\frac{2GM}{r}\right)\partial_\tau t = \pm\sqrt{(\partial_\tau r)^2 + 1 - \frac{2GM}{r}},$$

while taking into account Eq. (24) in the interior de Sitter side one gets

$$(1-k^2r^2)\partial_\tau t = \pm\sqrt{(\partial_\tau r)^2 + 1 - k^2r^2}.$$

The $\theta\theta$ component of the extrinsic curvature on the exterior side can be computed

$$K_{\theta\theta}(\text{ext}) = \pm r\sqrt{(\partial_\tau r)^2 + 1 - \frac{2GM}{r}}, \tag{25}$$

while the $\theta\theta$ component of the extrinsic curvature on the interior side is

$$K_{\theta\theta}(\text{int}) = \pm r\sqrt{(\partial_\tau r)^2 + 1 - k^2r^2}. \tag{26}$$

Eventually, Eq. (22) reads

$$r(K_{\theta\theta}(\text{int}) - K_{\theta\theta}(\text{ext})) = 4\pi G\sigma r^2. \tag{27}$$

As it will be shown in a moment, in the dynamical equation for $r(\tau)$ the ambiguity on the relative sign of $K_{\theta\theta}(\text{int})$ and $K_{\theta\theta}(\text{ext})$ is not present. This equation can be written as an ordinary first order equation for $r(\tau)$ bringing on the right-hand side $K_{\theta\theta}(\text{ext})$ and then squaring (obtaining, at a first glance, Eq. (5.1) of [37]). However, now an important difference comes into play. Namely the traceless condition of $S^\nu_\mu$ together with the conservation of energy for $S^\nu_\mu$ itself which imply that $\sigma$ is in Eq. (21) while in the description of the dynamics of a false vacuum bubble $\sigma$ can be assumed to be constant. Thus one obtains the following equation:

$$K_{\theta\theta}(\text{int}) - K_{\theta\theta}(\text{ext}) = \frac{4\pi G\sigma_0}{r^2}. \tag{28}$$

Equation (28) can be written in the standard form

$$\begin{aligned} -1 &= (\partial_\tau r)^2 + V(r), \\ V(r) &= -\left[\frac{2GM}{r} + f(r)^2\right], \\ &V(r)\underset{r\to 0}{\longrightarrow} -\infty, \qquad V(r)\underset{r\to\infty}{\longrightarrow} -\infty, \\ f(r)^2 &= \left[\left(\frac{M}{4\pi\sigma_0}\right)r - \left(\frac{k}{8\pi G\sigma_0}\right)r^4 - \frac{2\pi G\sigma_0}{r^2}\right]^2. \end{aligned} \tag{29}$$

It is suggestive to recall that in the dynamics of a false vacuum bubble the effective potential $V_{BV}$, which reads in normalized units

$$V_{BV}(z) = -\left[\frac{1-z^3}{z^2}\right]^2 - \frac{\gamma^2}{z}, \tag{30}$$

has (no matter the choice of parameters) only one maximum so that all the solutions $r(\tau)$ asymptotically for large $\tau$ approach $+\infty$ or zero. In particular, neither solutions oscillating between two finite values of $r$ nor stable constant solutions exist.

Remarkably enough, in the present case, for suitable choices of the parameters such solutions exist. In particular, sets of parameters which provide the potential in Eq. (29) with two maxima and a minimum in between the two maxima exist. It is not possible to write down an analytic formula for such cases because the potential in Eq. (29) is involved but a numerical graph unravels this important feature. $V(r)$ can be written as

$$V(z) = -\left(\frac{1}{\sigma_0}\right)^2\left[\frac{(\sigma_0)^2}{z} + \left[Bz - Cz^4 - \frac{(\sigma_0)^2 D}{z^2}\right]^2\right], \tag{31}$$

$$B = \frac{(GM)^2}{4\pi G}, \qquad C = \frac{(2GM)^4\rho_0}{3}, \qquad D = \frac{2\pi G}{(2GM)^2}, \tag{32}$$

where Eq. (18) has been taken into account and the dependence on the unknown microscopic parameter $\sigma_0$ has been displayed (see Fig. 1 for a graph of $-V(z)$ in the range of parameters allowing periodic solutions). Taking for simplicity $\sigma_0 = 1$ the choice

$$B = 10, \qquad C = \frac{1}{20}, \qquad D = \frac{1}{80}$$

does the required job [note that the product of coefficient $B$ and $D$ in Eq. (32) is fixed to be $1/8$]. One can understand this feature by comparing $V_{BV}$ in Eq. (30) with the one in

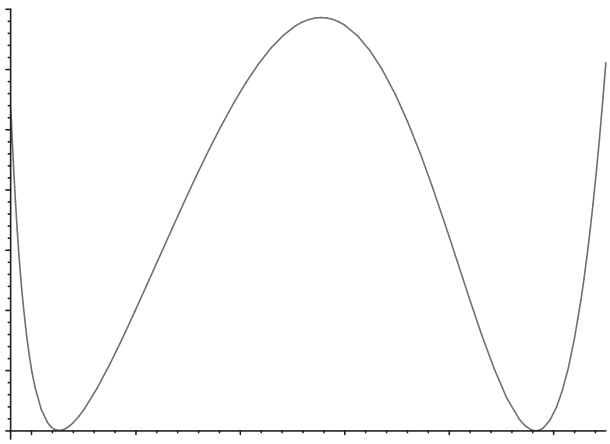

FIG. 1. A graph of (minus) the effective potential for a choice of parameters allowing periodic solutions (a possibility which is not present in the dynamics of false vacuum bubbles). The graph has been rescaled in order to show clearly the "Mexican hat" form of (minus) the potential.

Eq. (31). In the first case the term

$$\left[\frac{1-z^3}{z^2}\right]^2$$

has only one zero for $z = 1$ while the quadratic term in Eq. (31)

$$\left[Bz - Cz^4 - \frac{D}{z^2}\right]^2 = \frac{1}{z^4}[Bz^3 - Cz^6 - D]^2$$

can have two zeros[13] opening the possibility to have oscillating as well as stable static solutions living in the local minimum of $-V(z)$: such a possibility appears to be favored by small values of $\sigma_0$. The fact that, once the quantum Hall regime sets in, solutions for $r(\tau)$ oscillating around a local minimum do indeed exist is an interesting new feature of the present scheme. In Fig. 2 there is a schematic Penrose diagram in the interesting range of parameters.

### C. Connection with the higher dimensional quantum Hall effect

Even if QHE appears as a purely two-dimensional phenomenon, its theoretical structure has been generalized to higher dimensions in [43] (for a review, see [44] and references therein). The basic mathematical structure needed to achieve such generalizations is a manifold endowed with a connection acting on spinors and taking value in a Lie algebra. The curvature of such *quantum Hall connection* has to be "constant": namely, the components of the curvature evaluated in a suitable basis of vielbein have to be constant. The prototype of such manifolds are *coset spaces,* namely, manifolds diffeomorphic to $G/H$ where $G$ is a Lie group and $H$ a compact subgroup of dimension $\geq 1$: the spin connection provides one with a constant background field so that one can choose the background gauge field to be proportional to the spin connection generalizing the concept of a constant magnetic field.[14] Eventually, the Landau problem is expressed in terms of the covariant derivative of such quantum Hall connection and gives rise to a discrete highly degenerate spectrum with a gap [43]. It is a highly nontrivial self-consistency test of the present scheme that from the solutions of the Einstein equations fulfilling the requirements motivated above it naturally emerges the de Sitter metric which has precisely the characteristic giving rise to higher dimensional quantum Hall providing one with a spin connection acting on fermions whose curvature, as it is well known, is constant. This relation between higher dimen-

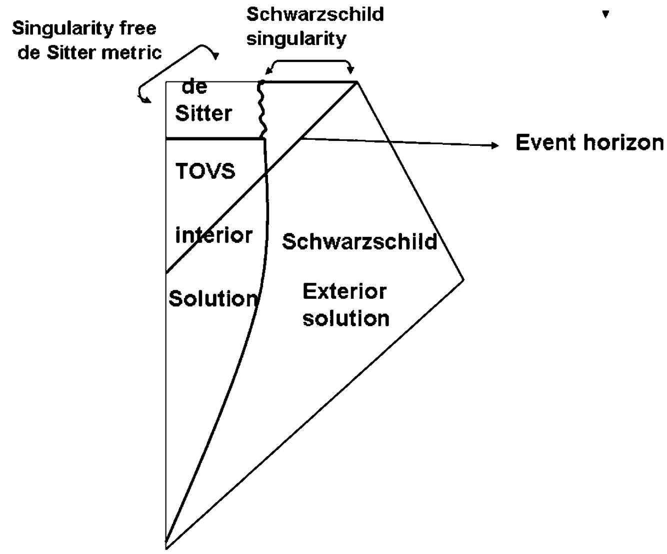


FIG. 2. A schematic Penrose diagram in the range of parameters in which oscillating solutions exist. The line separating the de Sitter from the Schwarzschild solutions is "wavy" to stress the Mexican hat form of $-V(z)$.

[13]So that (at least when the term $-1/z$ in the potential is not taken into account) it is clear that $V(z)$ may have two maxima and a minimum in between.

[14]It was found in [43] that in order to obtain a reasonable thermodynamic limit with a finite spatial density of particles, one has to consider very large $SU(\mathbf{2})$ representations. Each particle is then endowed with an infinite number of $SU(\mathbf{2})$ internal degrees of freedom. Basically, the reason for this choice is that the authors want to find a ground state which already has a macroscopic degeneracy (as it happens in the $2 + 1$ dimensional quantum Hall effect). In the present case, this is not an issue: the fermions live in their own representation (which is not a large representation of the "internal" Lorentz group of the spin connection). This is a welcome feature in the present case: an important ingredient to get the previous entropy bound (7) is that the last partially filled energy level is highly degenerate while the ground state has not a macroscopic degeneracy.

sional quantum Hall effect and the interior of a black hole is worth further investigation. Furthermore, a connection with noncommutative geometry based on the present scheme and the results in [15–18] should not be excluded.

## VI. RELATIONS WITH COSMOLOGY AND GRAVASTARS

It has been shown that the interior solution describing a phase in which the fermions' gas is incompressible can be chosen to be a de Sitter metric which is smoothly matched with the standard TOVS solution (describing the interior of a collapsed neutron star up to energy of the order $10^{-21}$ of the Planck energy) along the spacelike boundary and with the Schwarzschild solution on the timelike boundary (with a surface energy-momentum tensor describing the boundary gapless excitations expected on quantum Hall grounds). In the cosmological literature, interesting models (see, for instance, [37–39]) propose that "inside a black hole a *baby* universe could be generated." The matching with an interior de Sitter metric is argued to be reasonable on various grounds. In [37] this scheme represents the evolution of a false vacuum bubble separated by the true vacuum bubble by a timelike hypersurface. In [38,39] (assuming that the would-be quantum theory of gravity will regularize the divergence of the curvature invariants of general relativity) the collapsing black hole is matched with a de Sitter interior inside the horizon on a spacelike boundary. In the proposal of [37], part of the Schwarzschild singularity is smoothed while in the proposal of [38,39] the whole Schwarzschild singularity is removed. In both cases, instead of the (partially or fully) removed singularity a "baby universe" is present in which the inflationary evolution would arise in a rather natural way. It is therefore interesting that the conditions to realize such a scenario would arise inside a collapsed neutron star at an energy scale of order $10^{-18}$ of the Planck scale.

Recently, an interesting proposal, called *gravastar*, for the final state of the collapse of a massive star (alternative to the black hole) has been discussed [36] (for a discussion on how it is possible to distinguish phenomenologically a gravastar from a black hole see, for instance, [45] and references therein). The authors propose as a final state of the gravitational collapse an incompressible fluid (which could be a Bose-Einstein condensate) described by an equation of state in which both $\rho$ and $p$ are constant so that their interior solution is described by a de Sitter metric. The ambitious idea is that if this would happen before the formation of the event horizon one could solve the difficult theoretical problems related with black hole entropy and Hawking radiation. A point which in the literature on gravastars has not been solved yet is the precise mechanism giving rise to a Bose-Einstein condensate. On the other hand, once a neutron star is formed, the theory predicts that for masses larger than (more or less) five solar masses the neutron star should collapse to form a black hole: such predictions appear to be quite sound. Therefore, if the would-be mechanism giving rise to gravastar does not come into play preventing the formation of a neutron star, the formation of a black hole seems unavoidable. The main idea of the present paper is that the fermionic nature of the particles living inside a neutron star together with the strong gravitational field could give rise to quantum Hall phenomenology. When the typical order of magnitudes of a neutron star are taken into account, one recognizes that this would-be quantum Hall phase occurs *after the horizon is formed*. The quantum Hall phenomenology tells that the interior solution is well described by a de Sitter metric as in [36]. However, unlike the results in [36], the present scheme suggests that quantum Hall phenomenology together with the fermionic nature of the particles living inside the neutron star confirm that the entropy should be the sum of a frozen constant plus a term proportional to the area.

### A. About the correctness of the assumptions

The arising of an incompressible gas of fermions depend on the assumption that at energy scales up to $10^{-18}$ of the Planck energy quantum gravitational effects can be neglected and that the standard model can be fully trusted (so that, for instance, the Baryon number is conserved). Another key assumption related to the previous one is that at energy scales lower than $10^{-18}$ of the Planck energy the time scale of the dynamics of the gravitational field is greater than the typical time scale of the quantum evolution of the fermions living inside the neutron star so that the fermions reach equilibrium "before the gravitational field changes" (which is the standard assumption in the theory of the evolution of neutron stars). If these assumptions are correct, the results of the present paper can be trusted. About the first assumption, nothing precise can be said since the final theory of quantum gravity is still lacking. Nevertheless, a comparison with some analogous situations suggests that such an assumption could be safe. For instance, if one is studying quantum electrodynamics at an energy scale which is 18 orders of magnitude less than the energy scale at which quantum effects come into play, classical electrodynamics should be enough (unless there are very few photons but this is not the present case in which there should be a huge number of gravitons). Standard dimensional arguments would suggest that the second assumption also could not be incorrect. The time derivatives in the Einstein equations appear together with factors of the Newton constant $G$

$$\frac{1}{G}\frac{\partial^2}{\partial t^2},$$

while the time derivatives in quantum field theory acting on the fermionic operators appear as follows:

$$\frac{1}{\hbar}\frac{\partial}{\partial t};$$

therefore, the two evolutions become comparable at the Planck scale. This standard argument leads one to think that (as it happens inside usual collapsing neutron stars) fermions reach equilibrium before the gravitational field changes relevantly so that one can use the fermions equation of state ($\rho = \text{const}$ in the present case) to solve the Einstein equations. In similar situations (in which the microscopic time scale is 18 orders of magnitude smaller than the macroscopic time scale) one would say that it is safe to assume that the microscopic degrees of freedom reach the equilibrium. On the other hand, quantum gravitational effects could manifest themselves in a subtle way preventing the fermions from reaching equilibrium (and making incorrect the hypothesis made here). If this is the case, it would be a rather novel type of "low energy" quantum gravitational effect worth further investigation (since, to the best of the author's knowledge, no similar effects of "lacking of equilibrium" in the presence of so different time scales have been studied).

A possible mechanism preventing the picture here proposed could be to transform the fermions into Bosons: in this case, the present picture would be incorrect. On the other hand, no obvious way to realize that is available. At an energy scale of $10^{-18}$ of the Planck energy, the leading interactions are the strong interactions among the neutrons (or the quarks) which, as it has been already discussed, are weaker than the classical gravitational field inside the black hole.

Various models leading to superfluidity due to the formation of Bosonic bound states of neutrons via "Bardeen-Cooper-Schrieffer mechanism" inside a neutron star (see, for a review, [46]) have been proposed. On the other hand, at the energy scales at which classical gravity dominates the strong interactions which are responsible for the superfluidity can be considered as small perturbations. Moreover, the strong interactions are weaker the higher the energy scale is so that the "Bardeen-Cooper-Schrieffer mechanism" could not be effective anymore at density of the order in Eq. (1). This interesting question is worthwhile to be further investigated.

## VII. CONCLUSIONS AND PERSPECTIVES

It has been argued that the collapse of a black hole formed during the evolution of a typical neutron star could lead to an incompressible gas of fermion well before reaching the Planck scale. The fermionic nature of the degrees of freedom together with the strong classical gravitational field perceived by the fermions lead to some features typical of quantum Hall effects. The entropy of the gas splits naturally into two terms: a "frozen" constant (corresponding to the fermions living in the fully filled discrete energy levels) and a dynamical term which is bounded by two suitable functions of the area of the horizon strongly suggesting the Bekenstein-Hawking area law. The Einstein equations have been solved with this incompressible fluid and it has been shown that the interior metric describing the incompressible phase is well described by de Sitter space time. The behavior of the matching hypersurface manifests an interesting dependence on the parameters of the model allowing oscillating solutions around a local minimum of the effective potential for $r(\tau)$ (the proper circumferential radius of the domain wall $\Sigma_{(\tau)}$), a peculiar feature of the present model which is related to the gapless nature of the boundary excitations. The relations with higher dimensional quantum Hall effect and with interesting cosmological scenarios have been pointed out and worth further investigation. The case in which the hypothesis of the present paper does not hold has been briefly discussed.

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