

Stability of the Einstein static universe in $f(R)$ gravityChristian G. Böhrer,^{*} Lukas Hollenstein,[†] and Francisco S. N. Lobo[‡]*Institute of Cosmology and Gravitation, University of Portsmouth, Portsmouth PO1 2EG, United Kingdom*

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We analyze the stability of the Einstein static universe by considering homogeneous scalar perturbations in the context of $f(R)$ modified theories of gravity. By considering specific forms of $f(R)$, the stability regions of the solutions are parametrized by a linear equation of state parameter $w = p/\rho$. Contrary to classical general relativity, it is found that in $f(R)$ gravity a stable Einstein cosmos with a positive cosmological constant does indeed exist. Thus, we are lead to conclude that, in principle, modifications in $f(R)$ gravity stabilize solutions which are unstable in general relativity.

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I. INTRODUCTION

Independent observations have confirmed that the Universe is presently undergoing a phase of accelerated expansion [1]. Although the introduction of a cosmological constant into the field equations seems to be the simplest theoretical approach to generate a phase of accelerated expansion, several alternative candidates have been proposed in the literature, ranging from dark energy models to modified theories of gravity. Amongst the latter, models generalizing the Einstein-Hilbert action have been proposed. A nonlinear function of the curvature scalar, $f(R)$, is introduced in the action given by

$$S = \frac{1}{2\kappa^2} \int d^4x \sqrt{-g} f(R) + S_m, \quad (1)$$

where S_m is the matter action. We consider $\kappa^2 = 8\pi G = 1$ throughout this work, for notational simplicity. Varying the action with respect to $g_{\mu\nu}$ provides the following field equation

$$FR_{\mu\nu} - \frac{1}{2}fg_{\mu\nu} - \nabla_\mu \nabla_\nu F + g_{\mu\nu} \square F = T_{\mu\nu}^m, \quad (2)$$

where $F \equiv df/dR$. Note that the Ricci scalar is now a fully dynamical degree of freedom, which is transparent from the following relationship

$$FR - 2f + 3\square F = T, \quad (3)$$

obtained from the contraction of the modified field Eq. (2). One may generalize the action (1) by considering an explicit coupling between an arbitrary function of the scalar curvature, R , and the Lagrangian density of matter [2]. Note that these couplings imply the violation of the equivalence principle [3], which is highly constrained by solar system tests. One may also mention alternative approaches, namely, the Palatini formalism [4,5], where the metric and the connections are treated as separate variables; and the metric-affine formalism, where the matter

part of the action now depends on the connection and is varied with respect to it [5].

A fundamental issue extensively addressed in the literature is the viability of the proposed $f(R)$ models [6,7]. In this context, it has been argued that most $f(R)$ models proposed so far in the metric formalism violate weak-field solar system constraints [8], although viable models do exist [7,9–11] (static and spherically symmetric solutions have also been found [12]). The issue of stability [13] also plays an important role in the viability of cosmological solutions [11,14,15]. In Ref. [10] it was argued that the sign of $f_{RR} = d^2f/dR^2$ determines whether the theory approaches the general relativistic limit at high curvatures, and it was shown that for $f_{RR} > 0$ the models are, in fact, stable. The stability of the de Sitter solution in $f(R)$ gravity has also been extensively analyzed in the literature [16]. Recently, $f(R)$ models which have a viable cosmology were analyzed, and it was found that the models satisfying cosmological and local gravity constraints are practically indistinguishable from the Λ CDM model, at least at the background level [11]. Note that to be a viable theory, the proposed model, in addition, to simultaneously account for the four distinct cosmological phases, namely, inflation, the radiation-dominated and matter-dominated epochs, and the late-time accelerated expansion [17,18], should be consistent with cosmological structure formation observations [19]. In the latter context, it has been argued that the inclusion of inhomogeneities is necessary to distinguish between dark energy models and modified theories of gravity, and therefore, the evolution of density perturbations and the study of perturbation theory in $f(R)$ gravity is of considerable importance [20–22]. See Ref. [23] for studies of a parametrized growth factor approach to distinguish between modified gravity and dark energy models using weak lensing.

In this work, we explore the stability of the Einstein static universe in $f(R)$ modified theories of gravity. This is motivated by the possibility that the Universe might have started out in an asymptotically Einstein static state, in the inflationary universe context [24]. On the other hand, the Einstein cosmos has always been of great interest in various gravitational theories. In general relativity for instance,

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generalizations with nonconstant pressure have been analyzed in [25]. In the context of brane world models the Einstein static universe was investigated in [26], while its generalization within Einstein-Cartan theory can be found in [27]. Finally, in the context of loop quantum cosmology, we refer the reader to [28,29].

We analyze the stability of the Einstein static universe against homogeneous scalar perturbations in the context of $f(R)$ gravity. In the following section, we provide two specific forms of $f(R)$, and analyze the stability of the solutions, by considering homogeneous scalar perturbations around the Einstein static universe. The stability regions are given in terms of the linear equation of state parameter $w = p/\rho$ and the unperturbed energy density ρ_0 .

II. THE EINSTEIN STATIC UNIVERSE IN $f(R)$ GRAVITY

Consider the metric given by

$$ds^2 = -dt^2 + a^2(t) \left[\frac{dr^2}{1-r^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2) \right]. \quad (4)$$

For the Einstein static universe, $a = a_0 = \text{const}$, the Ricci scalar reduces to $R = 6/a_0^2$, and the field equations take the following form:

$$\rho_0 = \frac{f}{2}, \quad p_0 = \frac{2F}{a_0^2} - \frac{f}{2}, \quad (5)$$

where ρ_0 and p_0 are the unperturbed energy density and isotropic pressure, respectively.

In what follows, we consider specific forms of $f(R)$, and analyze the stability against linear homogeneous scalar perturbations around the Einstein static universe given in Eqs. (5). Thus, we introduce perturbations in the energy density and the metric scale factor which only depend on time:

$$\rho(t) = \rho_0(1 + \delta\rho(t)), \quad a(t) = a_0(1 + \delta a(t)). \quad (6)$$

Subsequently, we consider a linear equation of state, $p(t) = w\rho(t)$, linearize the perturbed field equations, and analyze the dynamics of the solutions.

First, motivated by the possibility that the Universe might have started out in an asymptotically Einstein static state [24], we analyze the case of $f(R) \propto R + R^2$, as in principle R^2 dominates for high curvatures, i.e., the early Universe. Second, we consider the case of $f(R) \propto R + 1/R$, which is known to generate a late-time accelerated expansion phase [30], and has been used in the weak-field limit constraints, as the $1/R$ term dominates for low curvatures.

A. $f(R) \propto R + R^2$ theory

Consider the case of

$$f(R) = R + \frac{\sigma\alpha^2 a_0^4}{6} R^2 - 2\Lambda, \quad (7)$$

where $\sigma = \pm 1$, and α is a positive parameter. We introduced the factor $a_0^4/6$ in the second term to considerably simplify the equations and results in the analysis outlined below.

In this model the unperturbed field Eqs. (5) take the form

$$\rho_0 = \frac{3}{a_0^2} + 3\sigma\alpha^2 - \Lambda, \quad (8)$$

$$p_0 = -\frac{1}{a_0^2} + \sigma\alpha^2 + \Lambda, \quad (9)$$

and yield the following cosmological constant of the Einstein static universe

$$\Lambda = \frac{1}{2}\rho_0(1 + 3w) - 3\sigma\alpha^2. \quad (10)$$

We derive an evolution equation for the scale factor perturbation in the following way. The perturbations defined in Eq. (6) are introduced in the metric and the energy-momentum tensor. Then the perturbed field Eqs. (2) are linearized and the unperturbed Eqs. (8) and (9) are subtracted to end up having only first order terms. The (tt) -component reduces to

$$\delta\rho(t) = -3(1 + w)\delta a(t), \quad (11)$$

which we use to simplify the spatial component to

$$\begin{aligned} & [8\sigma\alpha^2 - \rho_0(1 + w)(1 + 3w)][-4\sigma\alpha^2 + \rho_0(1 + w)]^2 \delta a(t) \\ & - 2[8\sigma\alpha^2 - \rho_0(1 + w)][4\sigma\alpha^2 - \rho_0(1 + w)] \\ & \times \delta a''(t) + 8\sigma\alpha^2 \delta a^{(4)}(t) = 0. \end{aligned} \quad (12)$$

Note that in the general relativistic limit, $\alpha \rightarrow 0$, Eq. (12) reduces to

$$2\delta a''(t) - \rho_0(1 + w)(1 + 3w)\delta a(t) = 0, \quad (13)$$

which provides the solution

$$\delta a(t) = C_1 e^{\bar{\omega}t} + C_2 e^{-\bar{\omega}t}, \quad (14)$$

where C_1 and C_2 are constants of integration, and $\bar{\omega}$ is defined by

$$\bar{\omega} = \sqrt{\frac{1}{2}\rho_0(1 + w)(1 + 3w)}. \quad (15)$$

In order to avoid the blow-up, due to the exponential increase in the scale factor, or the collapse, the solution is stable in the range

$$-1 < w < -1/3. \quad (16)$$

Note that this interval violates the strong energy condition which stipulates that $\rho + 3p \geq 0$.

Since the cosmological constant of the classical Einstein universe is given by

$$\Lambda = \frac{1}{2}\rho_0(1 + 3w), \quad (17)$$

and we are only considering positive energy densities, it turns out that it is *negative* in the region of stability.

Now, the full modified perturbation differential equation, Eq. (12), provides the following solution:

$$\delta a(t) = C_1 e^{\omega_1 t} + C_2 e^{-\omega_1 t} + C_3 e^{\omega_2 t} + C_4 e^{-\omega_2 t}, \quad (18)$$

where C_i (with $i = 1 \dots 4$) are constants, and the parameters ω_1 and ω_2 are given by

$$\omega_{1,2} = \left\{ \frac{\rho_0(1+w) - 4\sigma\alpha^2}{8\sigma\alpha^2} \left[8\sigma\alpha^2 - \rho_0(1+w) \pm \sqrt{\rho_0(1+w)[\rho_0(1+w) + 8\sigma\alpha^2(3w-1)]} \right] \right\}^{1/2}, \quad (19)$$

respectively.

In the following, we require the cosmological constant to be positive. For $\sigma = -1$ and $\Lambda > 0$ no stable solutions can be found. Considering $\sigma = +1$ and $\Lambda > 0$, we have the following three stability regions: First,

$$\text{AI: } 8\alpha^2 < \rho_0 < \frac{3(7 + \sqrt{17})\alpha^2}{2}, \quad (20)$$

$$\frac{8\alpha^2 - \rho_0}{24\alpha^2 + \rho_0} \leq w < \frac{1}{3} \left(-2 + \sqrt{\frac{24\alpha^2 + \rho_0}{\rho_0}} \right). \quad (21)$$

Second,

$$\text{AII: } \rho_0 = \frac{3(7 + \sqrt{17})\alpha^2}{2}, \quad (22)$$

$$-\frac{5 + 3\sqrt{17}}{3(23 + \sqrt{17})} < w < \frac{1}{3} \left(-2 + \sqrt{\frac{23 + \sqrt{17}}{7 + \sqrt{17}}} \right). \quad (23)$$

The latter inequality reduces to $-0.213 < w < -0.146$.

Finally,

$$\text{AIII: } \rho_0 > \frac{3(7 + \sqrt{17})\alpha^2}{2}, \quad (24)$$

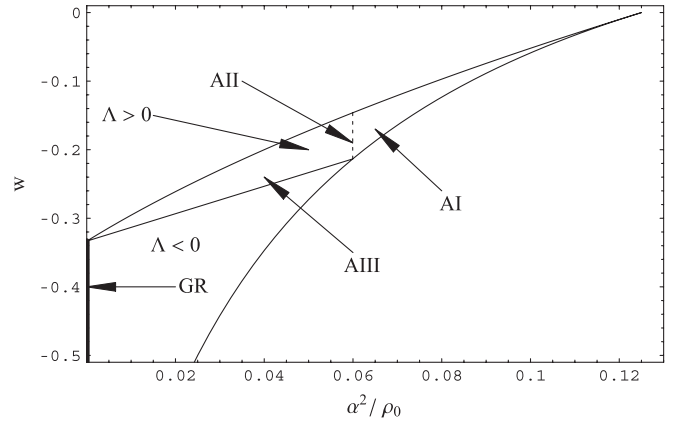


FIG. 1. The stability regions, for the case of $f(R) \propto R + R^2$ theory, are depicted between the curves. The thick solid line represents general relativity. In the lower left region, which tends to general relativity, the cosmological constant is negative. However, in the upper triangularlike shaped region (AI, AII, and AIII) the Einstein static universe is stable and Λ is positive; compare also with Fig. 2. Note that the equation of state parameter w is strictly negative.

$$\frac{6\alpha^2 - \rho_0}{3\rho_0} \leq w < \frac{1}{3} \left(-2 + \sqrt{\frac{24\alpha^2 + \rho_0}{\rho_0}} \right). \quad (25)$$

These stability regions are summarized in Table I, and depicted in Fig. 1. Note that these results are consistent with the stability condition $f_{RR} = d^2 f/dR^2 > 0$ of cosmological models at high curvatures [10].

B. $f(R) \propto R + 1/R$ theory

In this section, we use the specific form of

$$f(R) = R + \frac{\sigma\mu^4}{a_0^2} \frac{1}{R} - 2\Lambda, \quad (26)$$

where $\sigma = \pm 1$, and μ is considered a positive parameter. As in the previous example, we have inserted the factor a_0^2 which simplifies the calculations and the respective notation outlined below.

The choice of the $f(R)$ given by Eq. (26) has been extensively analyzed in the literature since it was first demonstrated to account for the late-time accelerated expansion of the Universe without the need for the introduction of dark energy [30]. It has also been used in the weak-field limit, as now the $1/R$ term dominates for low curva-

TABLE I. Summary of the stability regions in the Einstein static universe for the specific case of $f(R) \propto R + R^2$ theory.

Case	Stability Region	Stability Condition
Case AI	$8\alpha^2 < \rho_0 < \frac{3(7+\sqrt{17})\alpha^2}{2}$	$\frac{8\alpha^2 - \rho_0}{24\alpha^2 + \rho_0} \leq w < \frac{1}{3} \left(-2 + \sqrt{\frac{24\alpha^2 + \rho_0}{\rho_0}} \right)$
Case AII	$\rho_0 = \frac{3(7+\sqrt{17})\alpha^2}{2}$	$-\frac{5+3\sqrt{17}}{3(23+\sqrt{17})} < w < \frac{1}{3} \left(-2 + \sqrt{\frac{23+\sqrt{17}}{7+\sqrt{17}}} \right)$
Case AIII	$\rho_0 > \frac{3(7+\sqrt{17})\alpha^2}{2}$	$\frac{6\alpha^2 - \rho_0}{3\rho_0} \leq w < \frac{1}{3} \left(-2 + \sqrt{\frac{24\alpha^2 + \rho_0}{\rho_0}} \right)$

tures. Unfortunately, it was demonstrated that the originally proposed form suffers from instabilities [31]. However, it was recently shown in [10] that a modification of the sign stabilizes the solution such that $f_{RR} = d^2f/dR^2 > 0$, as emphasized in the introduction.

Considering this case, the unperturbed field Eqs. (5) take the form

$$\rho_0 = \frac{3}{a_0^2} + \frac{\sigma\mu^4}{12} - \Lambda, \quad (27)$$

$$p_0 = \frac{1}{a_0^2} - \frac{5\sigma\mu^4}{36} + \Lambda, \quad (28)$$

which implies that the cosmological constant of the modified Einstein static universe is given by

$$\Lambda = \frac{1}{2}\rho_0(1+3w) + \frac{\sigma\mu^4}{6}. \quad (29)$$

Applying linear perturbation theory, in the procedure outlined in the previous case, we deduce the following differential equation

$$\begin{aligned} [2\sigma\mu^4 - 9(1+w)(1+3w)\rho_0][\sigma\mu^4 + 18(1+w)\rho_0]^2\delta a(t) \\ - 36[2\sigma\mu^4 - 9(1+w)\rho_0][\sigma\mu^4 + 18(1+w)\rho_0] \\ \times \delta a''(t) + 648\sigma\mu^4\delta a^{(4)}(t) = 0. \end{aligned} \quad (30)$$

In the limit $\mu \rightarrow 0$ this differential equation also reduces to the general relativistic one, i.e., Eq. (13). Equation (30) provides the following solution:

$$\delta a(t) = C_1 e^{\omega_3 t} + C_2 e^{-\omega_3 t} + C_3 e^{\omega_4 t} + C_4 e^{-\omega_4 t}. \quad (31)$$

The parameters ω_3 and ω_4 are given by

$$\begin{aligned} \omega_{3,4} = \left\{ \frac{\mu^4\sigma + 18\rho_0(1+w)}{36\mu^4\sigma} \left[2\mu^4\sigma - 3 \left[3\rho_0(1+w) \right. \right. \right. \\ \left. \left. \left. \pm \sqrt{\rho_0(1+w)[2\mu^4\sigma(-1+3w) + 9\rho_0(1+w)]} \right] \right\}^{1/2}. \end{aligned} \quad (32)$$

As in the previous example, we verify that for $\sigma = -1$ and a positive cosmological constant, the solutions are unstable. Considering $\sigma = +1$ and $\Lambda > 0$, we have the following stability regions:

$$\text{BI: } \frac{2\mu^4}{9} < \rho_0 < \frac{(5 + \sqrt{41})\mu^4}{12}, \quad (33)$$

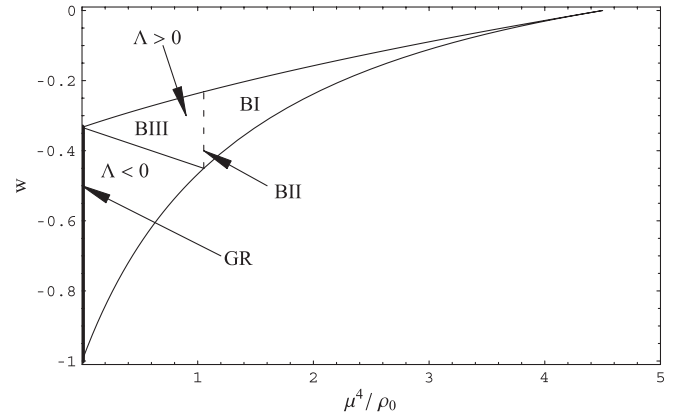


FIG. 2. The stability regions, for the case of $f(R) \propto R + 1/R$ theory, are depicted between the curves. As before, the thick solid line represents general relativity. In the lower left region, which tends to general relativity, the cosmological constant is negative. In the upper triangularlike shaped region (BI, BII, and BIII) the Einstein static universe is stable and Λ is positive. As before, w is strictly negative.

$$\frac{2\mu^4 - 9\rho_0}{3(2\mu^4 + 3\rho_0)} \leq w < \frac{1}{9} \left[-6 + \sqrt{\frac{3(2\mu^4 + 3\rho_0)}{\rho_0}} \right]. \quad (34)$$

Second,

$$\text{BII: } \rho_0 = \frac{(5 + \sqrt{41})\mu^4}{12}, \quad (35)$$

$$-\frac{7 + 3\sqrt{41}}{3(13 + \sqrt{41})} < w < \frac{1}{3} \left(-2 + \sqrt{\frac{13 + \sqrt{41}}{5 + \sqrt{41}}} \right). \quad (36)$$

Finally

$$\text{BIII: } \rho_0 > \frac{(5 + \sqrt{41})\mu^4}{12}, \quad (37)$$

$$-\frac{\mu^4 + 3\rho_0}{9\rho_0} < w < \frac{1}{9} \left[-6 + \sqrt{\frac{3(2\mu^4 + 3\rho_0)}{\rho_0}} \right]. \quad (38)$$

We summarize these stability regions in Table II, and depict the solutions in Fig. 2. Note that, as emphasized above, these results are also consistent with the stability condition $f_{RR} = d^2f/dR^2 > 0$ [10].

TABLE II. Summary of the stability regions in the Einstein static universe for the specific case of $f(R) \propto R + 1/R$ theory.

Case	Stability Condition	Stability Condition
Case BI	$\frac{2\mu^4}{9} < \rho_0 < \frac{(5 + \sqrt{41})\mu^4}{12}$	$\frac{2\mu^4 - 9\rho_0}{3(2\mu^4 + 3\rho_0)} \leq w < \frac{1}{9} \left[-6 + \sqrt{\frac{3(2\mu^4 + 3\rho_0)}{\rho_0}} \right]$
Case BII	$\rho_0 = \frac{(5 + \sqrt{41})\mu^4}{12}$	$-\frac{7 + 3\sqrt{41}}{3(13 + \sqrt{41})} < w < \frac{1}{3} \left(-2 + \sqrt{\frac{13 + \sqrt{41}}{5 + \sqrt{41}}} \right)$
Case BIII	$\rho_0 > \frac{(5 + \sqrt{41})\mu^4}{12}$	$-\frac{\mu^4 + 3\rho_0}{9\rho_0} < w < \frac{1}{9} \left[-6 + \sqrt{\frac{3(2\mu^4 + 3\rho_0)}{\rho_0}} \right]$

III. SUMMARY AND DISCUSSION

The Einstein static universe has recently been revived as the asymptotic origin of an emergent universe, namely, as an inflationary cosmology without a singularity [24]. The role of positive curvature, negligible at late times, is crucial in the early universe, as it allows these cosmologies to inflate and later reheat to a hot big-bang epoch. An attractive feature of these cosmological models is the absence of a singularity, of an “initial time,” and of the horizon problem, and the quantum regime can even be avoided. Furthermore, the Einstein static universe was found to be neutrally stable against inhomogeneous linear vector and tensor perturbations, and against scalar density perturbations provided that the speed of sound satisfies $c_s^2 > 1/5$ [32]. Further issues related to the stability of the Einstein static universe may be found in Ref. [33]

In this work we have analyzed linear homogeneous scalar perturbations around the Einstein static universe in the context of $f(R)$ modified theories of gravity. We have considered two specific forms of $f(R)$ and found the stability regions of the solutions for the scale factor perturbation. The first case considered, namely, $f(R) \propto R + R^2$, was motivated by the fact that, in principle, R^2 dominates for high curvatures which is expected in the early universe. Secondly, we considered the case of $f(R) \propto R + 1/R$, which is known to generate a late-time accelerated expansion phase [30], and has been used in the weak-field limit, as now the $1/R$ term dominates for low curvatures. The stability regions were parametrized by an equation of state parameter $w = p/\rho$, and it was found that in the context of $f(R)$ modified theories of gravity the range of the parameter is greatly enhanced relatively to the results obtained in general relativity. However, in both cases we analyzed, the equation of state parameter was strictly negative, an issue which we hope to overcome in future

work by considering other modified gravity models. These results are consistent with the condition $f_{RR} = d^2f/dR^2 > 0$, for the stability of cosmological models [10]. Relative to the issue of inhomogeneous perturbations in $f(R)$ gravity, with the closed Einstein static universe as here investigated as a background metric, work is currently underway, and the respective paper is in preparation [34]. However, we would also like to emphasize that the inhomogeneous perturbation case lies outside the scope of this work, as we are mainly interested in the Einstein static universe as an asymptotic origin of an emergent universe, and its respective stability as a static solution to Einstein’s equations.

Concluding, we have found that the modified Einstein static universe, with a positive cosmological constant and matter described by the equation of state, $p = w\rho$, can be stabilized against homogeneous perturbations, contrary to classical general relativity. Therefore, we are led to conclude that, in principle, stable modified gravity solutions, which are unstable in general relativity [34], do indeed exist. That this is actually possible relies on the fact that the perturbations of the metric couple to the matter perturbations; see our Eq. (11). Similar results have been obtained in [29] where the modified Friedmann equations in loop quantum cosmology are considered.

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