

Self-induced spectral splits in supernova neutrino fluxes

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In the dense-neutrino region above the neutrino sphere of a supernova ($r \lesssim 400$ km), neutrino-neutrino refraction causes collective flavor transformations. They can lead to “spectral splits” where an energy E_{split} splits the transformed spectrum sharply into parts of almost pure but different flavors. Unless there is an ordinary MSW resonance in the dense-neutrino region, E_{split} is determined by flavor-lepton number conservation alone. Spectral splits are created by an adiabatic transition between regions of large and small neutrino density. We solve the equations of motion in the adiabatic limit explicitly and provide analytic expressions for a generic example.

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I. INTRODUCTION

At large densities, neutrino-neutrino refraction causes nonlinear flavor oscillation phenomena with sometimes perplexing results [1–15]. In the region between the neutrino sphere and a radius of about 400 km in core-collapse supernovae (SNe), the neutrino flavor content evolves dramatically [7–13]. The global features of this self-induced transformation are equivalent to the motion of a gyroscopic pendulum in flavor space [11,12]. However, this picture does not explain the “spectral splits” that have been numerically observed in the transformed fluxes [9,10,13]. In a typical case, the primary ν_e flux below a split energy E_{split} emerges from the dense-neutrino region in its original flavor, whereas above E_{split} , it is completely transformed to ν_x (some mixture of ν_μ and ν_τ), the step at E_{split} being very sharp. (To be specific we explore the ν_e - ν_x system with the atmospheric Δm^2 and the small 13-mixing angle.)

It has been suggested that an adiabatic transition from high to low neutrino density is the primary cause for the split [9,12]. Dense neutrinos perform synchronized oscillations: all modes oscillate with a common frequency ω_{synch} , even though their individual frequencies vary as $\omega = |\Delta m^2/2E|$. Flavor oscillations can be visualized as the precession of polarization vectors \mathbf{P}_ω in a “flavor \mathbf{B} field.” The \mathbf{P}_ω “stick together” by the ν - ν -interaction, thus forming a collective object that precesses around \mathbf{B} . The collectivity is lost when the neutrino density decreases. However, if the decrease is slow, all \mathbf{P}_ω align themselves with or against \mathbf{B} in the process of decoupling from each other. Eventually they all precess with their individual ω around \mathbf{B} , but without visible consequences because of their (anti-)alignment with \mathbf{B} .

We extend this interpretation of the split phenomenon in several ways. We (i) show that flavor-lepton number conservation determines E_{split} , (ii) solve the equations of motion explicitly in the adiabatic limit, and (iii) provide an analytic result for a generic case.

II. EQUATIONS OF MOTION

We represent the flavor content of an isotropic ν - $\bar{\nu}$ gas by flavor polarization vectors \mathbf{P}_ω and $\bar{\mathbf{P}}_\omega$, where overbarred quantities correspond to $\bar{\nu}$. We define their global counterparts as $\mathbf{P} = \int_0^\infty d\omega \mathbf{P}_\omega$ and $\bar{\mathbf{P}} = \int_0^\infty d\omega \bar{\mathbf{P}}_\omega$ and introduce $\mathbf{D} \equiv \mathbf{P} - \bar{\mathbf{P}}$, representing the net lepton number. The equations of motion (EOMs) are [11,16]

$$\partial_t \mathbf{P}_\omega = (\omega \mathbf{B} + \lambda \mathbf{L} + \mu \mathbf{D}) \times \mathbf{P}_\omega \quad (1)$$

and the same for $\bar{\mathbf{P}}_\omega$ with $\omega \rightarrow -\omega$. Here $\lambda \equiv \sqrt{2} G_F n_e$ represents the usual matter potential and $\mu \equiv \sqrt{2} G_F n_\nu$ the ν - ν interaction strength, where n_e and n_ν are the electron and neutrino densities. We work in the mass basis where $\mathbf{B} = (0, 0, -1)$ corresponds to the normal and $\mathbf{B} = (0, 0, +1)$ to the inverted mass hierarchies. The interaction direction \mathbf{L} is a unit vector such that $\mathbf{B} \cdot \mathbf{L} = \cos 2\theta$ with θ being the vacuum mixing angle. Unless there is an MSW resonance in the dense-neutrino region, one can eliminate $\lambda \mathbf{L}$ from Eq. (1) by going into a rotating frame, at the expense of a small effective mixing angle [8,11]. The only difference for antineutrinos is that in vacuum they oscillate “the other way round.” Therefore, instead of using $\bar{\mathbf{P}}_\omega$ we may extend \mathbf{P}_ω to negative frequencies such that $\bar{\mathbf{P}}_\omega = \mathbf{P}_{-\omega}$ ($\omega > 0$) and use only \mathbf{P}_ω with $-\infty < \omega < +\infty$. In these terms, $\mathbf{D} = \int_{-\infty}^{+\infty} d\omega s_\omega \mathbf{P}_\omega$, where $s_\omega \equiv \text{sign}(\omega) = \omega/|\omega|$.

After elimination of $\lambda \mathbf{L}$, the EOM for \mathbf{D} can be obtained by integrating Eq. (1) with s_ω :

$$\partial_t \mathbf{D} = \mathbf{B} \times \mathbf{M} \quad \text{where } \mathbf{M} \equiv \int_{-\infty}^{+\infty} d\omega s_\omega \omega \mathbf{P}_\omega. \quad (2)$$

It shows that $\partial_t (\mathbf{D} \cdot \mathbf{B}) = 0$ so that $D_z = \mathbf{B} \cdot \mathbf{D}$ is conserved [11]. The in-medium mixing angle above a SN core is small and therefore the mass and interaction basis almost coincide. Collective effects then only induce pair transformations of the form $\nu_e \bar{\nu}_e \rightarrow \nu_x \bar{\nu}_x$, whereas the excess ν_e flux from deleptonization is conserved.

III. ADIABATIC SOLUTION

We rewrite the EOMs in terms of an “effective Hamiltonian” for the individual modes as

$$\partial_t \mathbf{P}_\omega = \mathbf{H}_\omega \times \mathbf{P}_\omega \quad \text{where } \mathbf{H}_\omega = \omega \mathbf{B} + \mu \mathbf{D}. \quad (3)$$

In the adiabatic limit each \mathbf{H}_ω moves slowly compared to the precession of \mathbf{P}_ω so that the latter follows the former. We assume that initially all \mathbf{P}_ω represent the same flavor and thus are aligned. If initially μ is large, every \mathbf{P}_ω is practically aligned with \mathbf{H}_ω . Therefore, in the adiabatic limit it stays aligned with \mathbf{H}_ω for the entire evolution:

$$\mathbf{P}_\omega(\mu) = \hat{\mathbf{H}}_\omega(\mu) P_\omega, \quad (4)$$

which solves the EOMs. Here $P_\omega \equiv |\mathbf{P}_\omega|$ and $\hat{\mathbf{H}}_\omega \equiv \mathbf{H}_\omega/|\mathbf{H}_\omega|$ is a unit vector. Here and henceforth we assume an excess flux of neutrinos over antineutrinos, implying that initially \mathbf{P}_ω and \mathbf{D} are collinear and $D_z > 0$.

According to Eq. (3) all \mathbf{H}_ω lie in the plane spanned by \mathbf{B} and \mathbf{D} which we call the “corotating plane.” In the adiabatic limit all \mathbf{P}_ω , and consequently \mathbf{M} , also stay in that plane. Therefore, we can decompose

$$\mathbf{M} = b \mathbf{B} + \omega_c \mathbf{D} \quad (5)$$

and rewrite the EOM of Eq. (2) as

$$\partial_t \mathbf{D} = \omega_c \mathbf{B} \times \mathbf{D}. \quad (6)$$

Therefore \mathbf{D} and the corotating plane precess around \mathbf{B} with the common or “corotation frequency” ω_c .

We conclude that the system evolves simultaneously in two ways: a fast precession around \mathbf{B} determined by $\omega_c = \omega_c(\mu)$ and a drift in the corotating plane caused by the explicit $\mu(t)$ variation. To isolate the latter from the former, we go (following Ref. [8]) into the corotating frame where the individual Hamiltonians become

$$\mathbf{H}_\omega = (\omega - \omega_c) \mathbf{B} + \mu \mathbf{D}. \quad (7)$$

We use the same notation because the relevant components $H_{\omega z}$, $H_{\omega \perp}$, D_z , and D_\perp remain invariant.

Initially ($\mu \rightarrow \infty$) the oscillations are synchronized, $\omega_c^\infty = \omega_{\text{synch}}$, and all \mathbf{P}_ω form a collective \mathbf{P} . As μ decreases, the \mathbf{P}_ω zenith angles spread out while remaining in a single corotating plane. In the end ($\mu \rightarrow 0$) the corotation frequency is ω_c^0 and Eqs. (4) and (7) imply that all final \mathbf{H}_ω and therefore all \mathbf{P}_ω with $\omega > \omega_c^0$ are aligned with \mathbf{B} , the others antialigned: a spectral split is inevitable with $\omega_{\text{split}} \equiv \omega_c^0$ being the split frequency. The lengths $P_\omega = |\mathbf{P}_\omega|$ are conserved and eventually all \mathbf{P}_ω point in the $\pm \mathbf{B}$ directions. Therefore the conservation of flavor-lepton number gives us ω_{split} , for $D_z > 0$, by virtue of

$$D_z = \int_{-\infty}^0 P_\omega d\omega - \int_0^{\omega_{\text{split}}} P_\omega d\omega + \int_{\omega_{\text{split}}}^{+\infty} P_\omega d\omega. \quad (8)$$

In general, $\omega_{\text{split}} = \omega_c^0 \neq \omega_c^\infty = \omega_{\text{synch}}$.

For individual modes the EOMs given by \mathbf{H}_ω are completely solved if we find $\omega_c(\mu)$ and $D_\perp(\mu)$, the component transverse to \mathbf{B} , since D_z is conserved and given by the initial condition. From Eq. (4) we infer $P_{\omega \perp}/P_\omega = H_{\omega \perp}/H_\omega$, from Eq. (7) $H_{\omega \perp} = \mu D_\perp$ and $H_{\omega z} = \omega - \omega_c + \mu D_z$ so that

$$P_{\omega z} = \frac{(\omega - \omega_c + \mu D_z) P_\omega}{\sqrt{(\omega - \omega_c + \mu D_z)^2 + (\mu D_\perp)^2}}, \quad (9)$$

$$P_{\omega \perp} = \frac{\mu D_\perp P_\omega}{\sqrt{(\omega - \omega_c + \mu D_z)^2 + (\mu D_\perp)^2}}. \quad (10)$$

Integration of the second equation over $s_\omega d\omega$ gives us

$$1 = \int_{-\infty}^{+\infty} d\omega s_\omega \frac{P_\omega}{\sqrt{[(\omega - \omega_c)/\mu + D_z]^2 + D_\perp^2}}. \quad (11)$$

Projecting Eq. (5) on the x - y plane we find $\omega_c = M_\perp/D_\perp(\mu)$ or explicitly

$$\omega_c = \frac{\int_{-\infty}^{+\infty} d\omega s_\omega \omega P_{\omega \perp}}{\int_{-\infty}^{+\infty} d\omega s_\omega P_{\omega \perp}} = \frac{\int_{-\infty}^{+\infty} d\omega s_\omega \omega P_{\omega \perp}}{D_\perp}. \quad (12)$$

For large μ when the oscillations are synchronized, this agrees with the usual expression for ω_{synch} [6], but it changes when the \mathbf{P}_ω spread out in the zenith direction. Inserting Eq. (10) into Eq. (12) we find

$$\omega_c = \int_{-\infty}^{+\infty} d\omega s_\omega \frac{\omega P_\omega}{\sqrt{[(\omega - \omega_c)/\mu + D_z]^2 + D_\perp^2}}. \quad (13)$$

Given D_z and a spectrum P_ω , we can determine ω_c and D_\perp from Eqs. (11) and (13) for any μ . These equations solve the EOMs explicitly in the adiabatic limit.

We have assumed that all \mathbf{P}_ω are initially aligned. One can relax this restriction and allow some \mathbf{P}_ω to have opposite orientation. If different species are emitted from a SN core with equal luminosities but different average energies, the spectra will cross over so that some range of modes is prepared, say, as ν_e and another as ν_x .

IV. NEUTRINOS ONLY

We illustrate the power of our new results with a generic neutrino-only example ($\mathbf{D} = \mathbf{P}$). The spectrum is taken boxlike with $P_\omega = (2\omega_0)^{-1}$ for $0 \leq \omega \leq 2\omega_0$ and 0 otherwise. With P_z being conserved we find from Eq. (8)

$$\omega_c = \omega_0 \times \begin{cases} 1 & \text{for } \mu \rightarrow 0, \\ (1 - P_z) & \text{for } \mu \rightarrow \infty. \end{cases} \quad (14)$$

The case $P_z = 0$ is special because $\omega_c = \omega_0$ remains fixed. For $P_z = 1$ we have $\omega_c^0 = 0$ and no flavor evolution. We use $P_z = 0.5$ to show the initial and final $P_z(\omega)$ in Fig. 1 (left). The dotted line denotes the adiabatic final state where $\omega_{\text{split}} = 0.5\omega_0$. The solid line is from a numerical

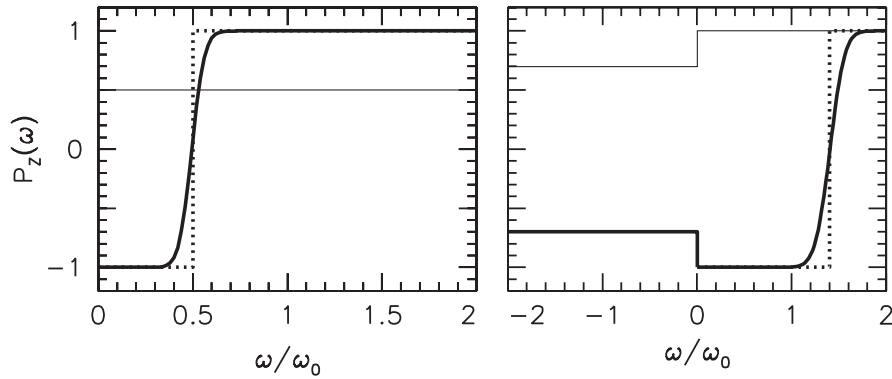


FIG. 1. Spectra of polarization vectors (z -component). Thin line: initial. Thick line: final. Dotted line: fully adiabatic. Solid line: numerical solution as described in the text. *Left*: Box-like initial ν spectrum, large misalignment between \mathbf{B} and \mathbf{P} , and no $\bar{\nu}$. *Right*: Box-like ν and $\bar{\nu}$ spectra, 30% fewer $\bar{\nu}$, small initial misalignment ($\sin 2\theta = 0.05$), and inverted hierarchy.

solution of the EOMs with $\mu(t) = \mu_0 \exp(-t/\tau)$ and $\tau^{-1} = 0.03\omega_0$, typical for a SN. We have checked numerically that the split indeed becomes sharper with increasing τ and thus increasing adiabaticity.

In Fig. 2 we show $P_{\omega,z}(\mu)$ for 51 individual modes. They start with the common value $P_{\omega,z} = 0.5(2\omega_0)^{-1}$. Later they spread and eventually split, some of them approaching $+1$ and the others -1 . Some modes first move down and then turn around as ω_c changes. A few modes do not reach ± 1 because of imperfect adiabaticity.

For the box spectrum the integrals Eqs. (11) and (13) are easily performed and one can extract

$$\begin{aligned} \omega_c &= \omega_0 + \omega_0 P_z \left(\frac{1}{\kappa} - \frac{e^\kappa + e^{-\kappa}}{e^\kappa - e^{-\kappa}} \right), \\ P_\perp &= \sqrt{1 - P_z^2} \frac{2\kappa}{e^\kappa - e^{-\kappa}}, \end{aligned} \quad (15)$$

where $\kappa \equiv \omega_0/\mu$. For $\mu \rightarrow \infty$ and $\mu \rightarrow 0$ the limits of ω_c agree with Eq. (14) from lepton number conservation. For $\mu \rightarrow \infty$ we obtain $P_\perp = \sqrt{1 - P_z^2}$, representing the initial condition $P = 1$, and for $\mu \rightarrow 0$ we find $P_\perp = 0$.

With Eq. (9) these results provide analytic solutions for the adiabatic $P_{\omega,z}(\mu)$. We show examples in Fig. 2 (bottom)

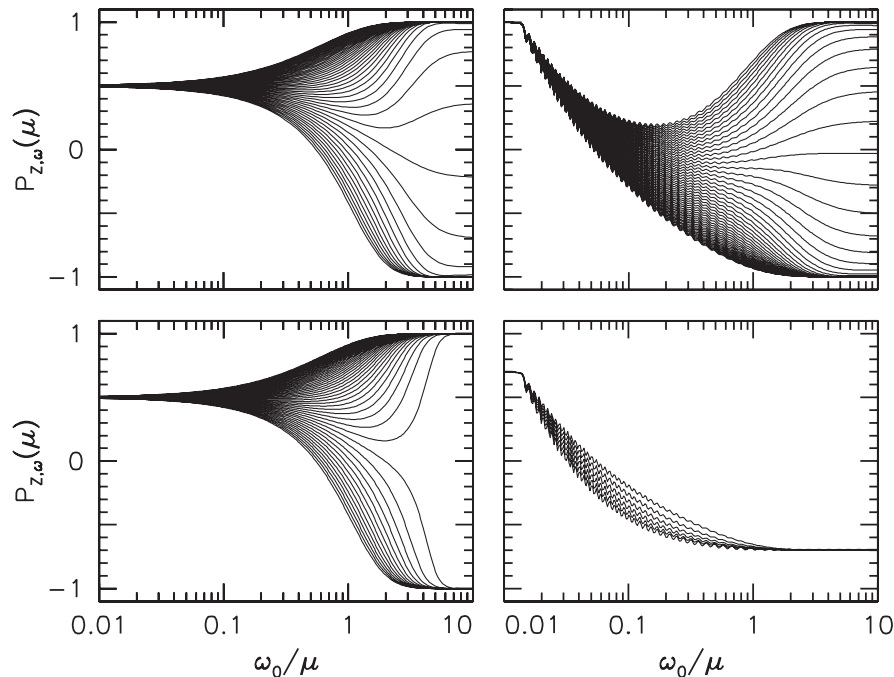


FIG. 2. $P_{z,\omega}(\mu)$ for 51 modes. *Left*: Box-like ν -only spectrum. Numerical solution of EOMs (*top*). Analytic adiabatic solution (*bottom*). *Right*: Box-like ν and $\bar{\nu}$ spectra. Numerical solution for ν (*top*) and $\bar{\nu}$ (*bottom*), here only six modes.

left) for comparison with the numerical solution of the EOMs. The agreement is striking and confirms the picture of adiabatic evolution in the corotating plane. The agreement is poor for modes close to the split ($\omega \approx \omega_c^0$) at low neutrino densities ($\mu < \omega_0$) where the evolution becomes nonadiabatic.

V. ADIABATICITY CONDITION

The speed for the \mathbf{H}_ω evolution in the corotating plane is $d\theta_\omega/dt$, where $\cos\theta_\omega \equiv H_{\omega\perp}/H_\omega$, while \mathbf{P}_ω precesses with speed H_ω . The evolution is adiabatic if the adiabaticity parameter $\gamma_\omega \equiv |d\theta_\omega/dt|H_\omega^{-1} \ll 1$. With Eqs. (7) and (10) we find

$$\gamma_\omega = \frac{(\frac{\omega-\omega_c}{\mu} + D_z)\frac{dD_\perp}{d\mu} + \frac{D_\perp}{\mu}\frac{d\omega_c}{d\mu} + D_\perp\frac{\omega-\omega_c}{\mu^2}}{\tau_\mu[(\frac{\omega-\omega_c}{\mu} + D_z)^2 + D_\perp^2]^{3/2}}, \quad (16)$$

where $\tau_\mu \equiv |d\ln\mu/dt|^{-1}$.

For our neutrino-only ($D_\perp \rightarrow P_\perp$) box spectrum Eqs. (15) give $dP_\perp/d\mu = -P_\perp(\omega_c - \omega_0)/\mu^2$ and $d\omega_c/d\mu = P_z[1 - 4\kappa^2/(e^\kappa - e^{-\kappa})^2]$. For $\mu \gg \omega_0$ we obtain $dP_\perp/d\mu \sim \omega_0^2/\mu^3$ and $d\omega_c/d\mu \sim \omega_0^2/\mu^2$ so that the last term in the numerator of Eq. (16) dominates: $\gamma_\omega \sim P_\perp(\omega - \omega_c)/(h_\mu\mu^2)$. With μ decreasing, γ_ω increases and at $\mu \sim \omega_0$ when $\gamma_\omega \sim 1$, adiabaticity violation begins. For $\mu < \omega_0$ the denominator of Eq. (16) gives the dependence $\gamma_\omega \propto (\omega - \omega_c)^{-3}$, and therefore the closer ω to ω_c the stronger the adiabaticity violation.

VI. INCLUDING ANTINEUTRINOS

As a second generic case we now add antineutrinos. One important difference is that even a very small initial misalignment between \mathbf{D} and \mathbf{B} is enough to cause a strong effect. Consider a single energy mode for ν with $P = 1$ and one for $\bar{\nu}$ with $\bar{P} = \alpha < 1$ that are initially aligned in the flavor direction, now taken very close to the mass direction, and assume an inverted hierarchy. From the dynamics of the flavor pendulum [11,12] we know that in the end $\bar{\mathbf{P}}$ is antialigned with \mathbf{B} , whereas \mathbf{P} retains a large transverse component because $P_z - \bar{P}_z$ is conserved: The system prepares itself for a spectral split.

Assuming box spectra for both ν and $\bar{\nu}$, we show the initial and final $P_{z,\omega}$ in Fig. 1 (right), for the inverted hierarchy, $\sin 2\theta = 0.05$, and $\alpha = 0.7$. From Eq. (12) one infers $\omega_c^\infty = \omega_{\text{synch}} = \omega_0(1 + \alpha)/(1 - \alpha)$. For $\alpha = \frac{7}{10}$ this is $\omega_c^\infty = \frac{17}{3}\omega_0 > 2\omega_0$. Therefore, all modes have negative frequencies in the corotating frame and tilt away from \mathbf{B} (see also the numerical $P_{\omega,z}$ in Fig. 2). The final split frequency is found from flavor-lepton number conservation to be $\omega_{\text{split}} = \omega_0(1 - D_z + \alpha) \approx \omega_0 2\alpha$, using $D_z \approx 1 - \alpha$ for $\sin 2\theta \ll 1$. With $\alpha = \frac{7}{10}$ we find $\omega_{\text{split}} = \frac{14}{10}\omega_0$ in agreement with Fig. 1. For $0 < \alpha < 1$ we have $0 < \omega_{\text{split}} < 2\omega_0$ so that the final split always occurs among the neutrinos. According to Fig. 2 the split starts when the vector

\mathbf{D} develops a significant transverse component, and it proceeds efficiently in a region $\mu \sim \omega_0$.

The ‘‘wiggles’’ in the curves in the right panels of Fig. 2 stem from the nutation of the flavor pendulum [11,12]. We have chosen a relatively fast $\mu(t)$ evolution ($\tau^{-1} = 0.1\omega_0$), implying poor adiabaticity, to avoid too many nutation periods on the plot. For a very slow $\mu(t)$ the nutations disappear and the corotating frame removes the full global evolution of the system.

VII. DISCUSSION

We have studied the phenomenon of spectral splits that is caused by neutrino-neutrino refraction in the SN dense-neutrino region. We have carried previous explanations of this novel effect [9,12] to the point of explicit solutions in the adiabatic limit.

A spectral split occurs when a neutrino ensemble is prepared such that the common direction of the flavor polarization vectors deviates from the mass direction. An adiabatic density decrease turns all modes below a split energy $E_{\text{split}} \equiv \Delta m^2/2\omega_{\text{split}}$ into the mass direction, and the others in the opposite direction. Remarkably, during this phase all modes remain in a single rotating plane, even after losing full synchronization. E_{split} is determined by lepton number conservation in the mass basis.

The spectral split is a generic feature of the adiabatic evolution when the density changes from large to small values. It can appear even in the absence of neutrino-neutrino interactions. Indeed, in the usual MSW case the evolution to zero density transforms ν_e to ν_2 and $\bar{\nu}_e$ to $\bar{\nu}_1$ for all energies. This corresponds to $\omega_{\text{split}} = 0$. The neutrino-neutrino interactions shift ω_{split} to nonzero values.

A spectral split is caused in the SN neutrino (but not antineutrino) flux by neutrino-neutrino interactions alone, especially during the accretion phase when ordinary MSW resonances occur far outside the dense-neutrino region. Later the matter profile may become so shallow that the H-resonance moves into this region [9,10,12]. The simultaneous action of collective effects and an ordinary MSW resonance may then cause spectral splits for both neutrinos and antineutrinos, leading to a rich phenomenology, perhaps modifying r-process nucleosynthesis [9,12]. Of course, the fluxes will be further processed by ordinary conversion in the SN envelope [17,18], thus modifying observable signatures. Still, observing spectral splits would provide a smoking gun signature both for the relevant neutrino properties and, if it occurs among antineutrinos at late times, for the occurrence of a shallow density profile above the neutrino sphere.

The neutrino flux emitted by a SN is anisotropic so that neutrinos on different trajectories experience different neutrino-neutrino interaction histories [9,12] that would be expected to cause kinematical flavor decoherence of different angular modes [14]. A numerical exploration

reveals, however, that in a typical SN scenario the deleptonization flux suppresses decoherence and the evolution is almost identical to that of an isotropic ensemble [15]. Our treatment of the spectral evolution is apparently applicable in a realistic SN context.

Collective neutrino oscillation phenomena in a SN may well be important for the explosion mechanism, r-process nucleosynthesis and may provide detectable signatures in a high-statistics signal from the next galactic SN. Building on previous ideas, our formalism gives a simple, elegant, and quantitative explanation of seemingly impenetrable numerical results. Our approach provides the basis for

developing a quantitative understanding of realistic consequences of collective neutrino oscillations for SN physics and observational signatures.

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