Once more about the $K\bar{K}$ molecule approach to the light scalars

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We show that the recent paper [Eur. Phys. J. A **24**, 437 (2005)], claiming that the radiative decays $\phi \to a_0(980)\gamma$ and $\phi \to f_0(980)\gamma$ should be of the same order of magnitude regardless of whether the $a_0(980)$ and $f_0(980)$ are compact four-quark states or extended $K\bar{K}$ molecule states, is misleading.

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Recently Ref. [1] has claimed that the radiative decays of the ϕ meson to the scalar $a_0(980)$ and $f_0(980)$ "should be of the same order of magnitude for a molecular state and for a compact state ...." We show below that this claim is misleading. The authors of Ref. [1] think that their amplitude of the $\phi \to K^+K^- \to \gamma S$ transition (where $S = a_0$ or f_0) is caused by the nonrelativistic kaons in the $K\bar{K}$ molecule S. However, we will show below that this is incorrect.

Equation (14) in Ref. [1], describing the $\phi \to \gamma S$ amplitude, is

$$J_{ik} = 2J_{ik}^{(a)} + J_{ik}^{(c)} + J_{ik}^{(d)} = -\delta_{ik}\frac{i}{4\pi^2}(a-b)I(a,b;\Gamma) + \cdots, \quad (1)$$

where the subscripts ik are the spatial Lorentz indices referring to the ϕ and the photon, $2J_{ik}^{(a)}$ corresponds to the sum of the diagrams of Fig. 1(a) and 1(b), $J_{ik}^{(c)}$ corresponds to the diagram of Fig. 1(c), and $J_{ik}^{(d)}$ corresponds to the diagram of Fig. 1(d) (which is added because "gauge invariance calls for a correction term induced by this additional flow of charge" [1] in an extended molecule case), $a = m_\phi^2/m_K^2$, $b = m_S^2/m_K^2$. "Terms that do not contribute to the process of interest are not shown explicitly" [1]. Note that Fig. 1 of our paper corresponds to Fig. 1 of Ref. [1].

Assuming the nonrelativistic kinematics of kaons in the loop, the authors of Ref. [1] obtain the individual integrals,

$$2J_{ik}^{(a)} = -\frac{i}{m^3}\int\frac{d^3k}{(2\pi)^3} \times \frac{k_ik_j\Gamma(|\mathbf{k}-\mathbf{q}/2|)}{[E_V - \frac{k^2}{m} + i0][E_S - \frac{(\mathbf{k}-\mathbf{q}/2)^2}{m} + i0]},$$
$$J_{ik}^{(c)} = -\frac{i}{2m^2}\delta_{ik}\int\frac{d^3k}{(2\pi)^3}\frac{\Gamma(k)}{E_S - \frac{k^2}{m} + i0},$$
$$J_{ik}^{(d)} = -\frac{i}{2m^2}\int\frac{d^3k}{(2\pi)^3}\frac{k_ik_j}{E_V - \frac{k^2}{m} + i0}\frac{1}{k}\frac{\partial\Gamma(k)}{\partial k}, \quad (2)$$

where $E_V = m_V - 2m > 0$, $E_S = m_S - 2m < 0$, $m = m_K$, $V = \phi$, $\mathbf{q}$ is a photon momentum, $\mathbf{k}$ is a kaon momentum in a molecule, $k = |\mathbf{k}|$, $\Gamma(\mathbf{k}) = \beta^2/(\mathbf{k}^2 + \beta^2)$, $1/\beta$ is a potential range [2]. The range of β is typically $m_\rho \approx 0.8$ GeV, because the ρ-meson exchange in the t-channel "is responsible for the formation of scalars" [1]. The authors of Ref. [1] calculate $(a-b)I(a,b;\Gamma)$ at $\mathbf{q} = 0$:

$$(a-b)I(a,b;\Gamma) = 2(a-b)I^{(a)}(a,b;\Gamma) + (a-b)I^{(c)}(a,b;\Gamma) + (a-b)I^{(d)}(a,b;\Gamma), \quad (3)$$

where

$$2(a-b)I^{(a)}(a,b;\Gamma) = i4\pi^2\frac{1}{3}2J_{ii}^{(a)} = \int\frac{1}{3m^3}\frac{k^2\Gamma(k)}{[E_V - \frac{k^2}{m} + i0][E_S - \frac{k^2}{m} + i0]}\frac{d^3k}{2\pi},$$
$$(a-b)I^{(c)}(a,b;\Gamma) = i4\pi^2\frac{1}{3}J_{ii}^{(c)} = \int\frac{1}{2m^2}\frac{\Gamma(k)}{E_S - \frac{k^2}{m} + i0}\frac{d^3k}{2\pi}, \quad (4)$$

and

$$(a-b)I^{(d)}(a,b;\Gamma) = i4\pi^2\frac{1}{3}J_{ii}^{(d)} = \int\frac{1}{6m^2}\frac{k^2}{E_V - \frac{k^2}{m} + i0}\frac{1}{k}\frac{\partial\Gamma(k)}{\partial k}\frac{d^3k}{2\pi}. \quad (5)$$

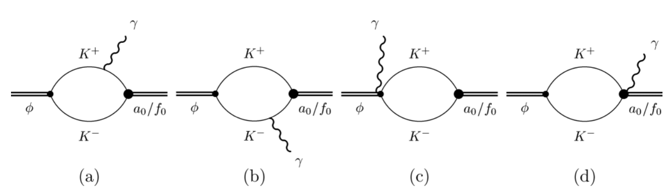

FIG. 1. Diagrams contributing to the radiative decay amplitude (1).

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To reveal what kaon momenta are essential in the real part of the $\phi \to K^+K^- \to \gamma S$ amplitude, we introduce a cutoff k_0 in Eqs. (4) and (5) ($|\mathbf{k}| = k \leq k_0$) and calculate the auxiliary integral,

$$\begin{aligned}(a-b)\,\mathrm{Re}I(a,b;\Gamma;k_0) = 2(a-b)\,\mathrm{Re}I^{(a)}(a,b;\Gamma;k_0)\\ +(a-b)\,\mathrm{Re}I^{(c)}(a,b;\Gamma;k_0)\\ +(a-b)\,\mathrm{Re}I^{(d)}(a,b;\Gamma;k_0)\end{aligned} \tag{6}$$

for $\beta = 0.2$ GeV, 0.3 GeV, and 0.8 GeV [1,3]. When $k_0 \to \infty$ the integral $I(a,b;\Gamma;k_0) \to I(a,b;\Gamma;\infty) \equiv I(a,b;\Gamma)$. We use $m_S = 980$ MeV, $m_K = 495$ MeV [1]. In Fig. 2 is depicted the $\mathrm{Re}I(a,b;\Gamma;k_0)$ dependence on a cutoff k_0 for different β.

As is obvious from Fig. 2, the contribution of the nonrelativistic kaons ($k_0 < 0.3$ GeV) into $\mathrm{Re}I(a,b;\Gamma)$ is small in all instances. What is more, the ultrarelativistic kaons ($k_0 > 2$ GeV) determine the real part of the $\phi \to K^+K^- \to \gamma S$ amplitude in the typical case of $\beta = 0.8$ GeV, see Fig. 2(b) [4]. So the authors of Ref. [1] use a nonrelativistic description beyond its region of applicability [5].

The authors of Ref. [1] in fact evaluate the (d)-contribution by integrating Eq. (5) by parts. This gives a contribution $(a-b)\,\mathrm{Re}\tilde{I}^{(d)}(a,b;\Gamma;k_0)$ which when summed with the $2(a-b)\,\mathrm{Re}I^{(a)}(a,b;\Gamma;k_0)$ and $(a-b)\,\mathrm{Re}I^{(c)}(a,b;\Gamma;k_0)$ contributions gives $(a-b)\,\mathrm{Re}\tilde{I}(a,b;\Gamma;k_0)$ shown in Fig. 3.

As is seen from Fig. 3, the integral $(a-b)\,\mathrm{Re}\tilde{I}(a,b;\Gamma;k_0)$ converges in the nonrelativistic region ($k_0 < 0.3$ GeV). The authors call this operation "a trick" [1] believing that the rapid convergence of $(a-b)\,\mathrm{Re}\tilde{I}(a,b;\Gamma;k_0)$ justifies their nonrelativistic approximation. But only $(a-b)\,\mathrm{Re}I(a,b;\Gamma;k_0)$ represents the momentum (or space) distribution of kaons and, in particular, the distribution of the charge flow in the $K\bar{K}$-molecule and shows that the decays occur at small distances for the annihilation of the ultrarelativistic kaons and antikaons ($k_0 > 2$ GeV) in the typical case, see Fig. 2(b). The difference between $(a-b)\,\mathrm{Re}I(a,b;\Gamma;k_0)$ and $(a-b)\,\mathrm{Re}\tilde{I}(a,b;\Gamma;k_0)$ equals the slow convergent integral of the total derivative,

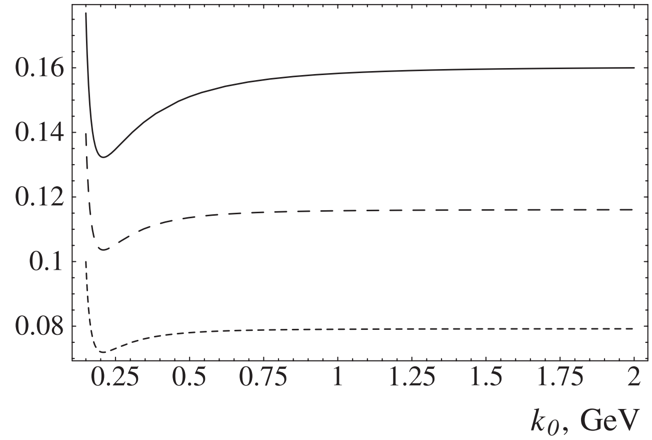

FIG. 3. $(a-b)\,\mathrm{Re}\tilde{I}(a,b;\Gamma;k_0)$, $k_0 \leq 2$ GeV, $\tilde{I}(a,b;\Gamma;\infty) \equiv I(a,b;\Gamma;\infty) \equiv I(a,b;\Gamma)$. The solid line for $\beta = 0.8$ GeV, the dashed line for $\beta = 0.3$ GeV, and the dotted line for $\beta = 0.2$ GeV.

$$\begin{aligned}&(a-b)\,\mathrm{Re}I(a,b;\Gamma;k_0) - (a-b)\,\mathrm{Re}\tilde{I}(a,b;\Gamma;k_0)\\ &= \frac{1}{3m^2}\int_0^{k_0} d\left(\frac{k^3\Gamma(k)}{E_V - \frac{k^2}{m} + i0}\right),\end{aligned} \tag{7}$$

which vanishes at $k_0 \to \infty$. As for the finite k_0, discarding this contribution leads to a loss of physical significance.

So, the real part of the K^+K^- loop is caused by the kaon high virtualities, that is, by a compact four-quark system, which points to the four-quark nature of the $a_0(980)$ and $f_0(980)$ mesons [6].

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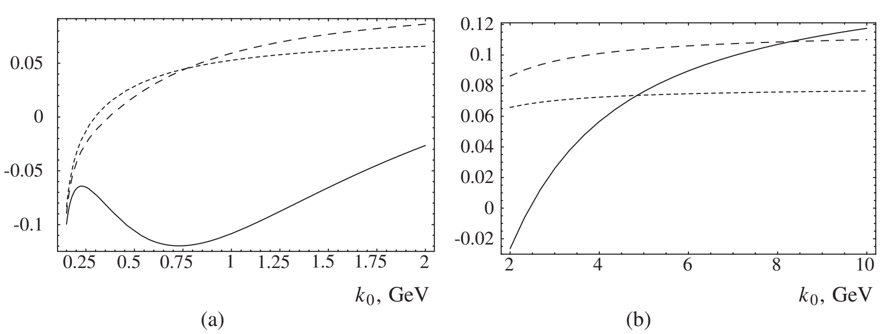

FIG. 2. $(a-b)\,\mathrm{Re}I(a,b;\Gamma;k_0)$ [the definition in the text, see Eq. (6)]. The solid line for $\beta = 0.8$ GeV, the dashed line for $\beta = 0.3$ GeV, the dotted line for $\beta = 0.2$ GeV. (a) $k_0 \leq 2$ GeV, (b) $k_0 \geq 2$ GeV. The limit values of $(a-b)\,\mathrm{Re}I(a,b;\Gamma;\infty) \equiv (a-b)\,\mathrm{Re}I(a,b;\Gamma)$ are 0.16 for $\beta = 0.8$ GeV, 0.116 for $\beta = 0.3$ GeV, and 0.079 for $\beta = 0.2$ GeV.

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[2] The wave function $\sim(1/r)(\exp\{-\alpha r\} - \exp\{-\beta r\})$, where $\alpha = \sqrt{m(2m - m_S)} \approx 0.07$ GeV. The average momentum square in the molecule $\langle k^2\rangle = \alpha\beta$.

[3] As it follows from Ref. [2], this set of β corresponds to the set of the nonrelativistic average momentum square of a K meson in the molecule, $\langle k^2\rangle/m_K^2$: 0.056, 0.084, and 0.224.

[4] Notice that $|\mathrm{Re}I(a,b;\Gamma)/\mathrm{Im}I(a,b;\Gamma)|^2 \ll 1$ at $\beta \leq 0.2$ GeV, so that $|I(a,b;\Gamma)|$ is dominated by $\mathrm{Im}I(a,b;\Gamma)$ which is caused certainly by real intermediate nonrelativistic kaons. But $(a-b)\,\mathrm{Im}I(a,b;\Gamma)$ can explain not greater than 20% of branching ratios of decays under consideration even for the pointlike interaction and leads to an over-narrow resonance structure to fit data [6,7].

[5] Notice that there is another sloppily built place in Ref. [1]. The authors of Ref. [1] calculate $A(\phi(p) \to \gamma(q)S(p'))$ at $q = 0$, $p^2 = m_\phi^2$, and $(p')^2 = m_S^2$ ($m_S - 2m < 0$), that is, at $p \neq p' + q$. The point is that a question of principle for them is an interpretation of $1/(E_S - \frac{k^2}{m} + i0)$ as a nonrelativistic two-particle ($K\bar{K}$) Green function. But gauge invariance forces the authors of Ref. [1] to replace m_S by the invariant mass [$m_{\mathrm{inv}}^2 = (p')^2$] of decay products in the physical region $p = p' + q$, the actual interval of which is quite large [6,8]. The typical values lie in the range -100 MeV $< m_{\mathrm{inv}} - 2m < 30$ MeV which includes the considerable positive region ≤ 30 MeV. So the idea about a nonrelativistic two-particle ($K\bar{K}$) Green function with the bound energy $m_S - 2m = -10$ MeV has no grounds. For the sake of definiteness, we notice that actually the amplitude under consideration is $e^+e^- \to \gamma^*(E) \to \phi(E) \to \gamma(q)S(p') \to \gamma(q)\pi(p_1)\pi(p_2)[\pi(p_1)\eta(p_2)]$, which should be of the order of $O(q)$ at $q = p - p' = p - p_1 - p_2 \to 0$ for gauge invariance [9]. So, m_ϕ should be replaced above by the total energy of beams E.

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