Lepton masses, mixings, and flavor-changing neutral currents in a minimal S_3-invariant extension of the standard model

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The mass matrices of the charged leptons and neutrinos, previously derived in a minimal S_3-invariant extension of the standard model, were reparametrized in terms of their eigenvalues. We obtained explicit, analytical expressions for all entries in the neutrino mixing matrix, V_{PMNS}, the neutrino mixing angles, and the Majorana phases as functions of the masses of charged leptons and neutrinos in excellent agreement with the latest experimental values. The resulting V_{PMNS} matrix is very close to the tribimaximal form of the neutrino mixing matrix. We also derived explicit, analytical expressions for the matrices of the Yukawa couplings and computed the branching ratios of some selected flavor-changing neutral current processes as functions of the masses of the charged leptons and the neutral Higgs bosons. We find that the $S_3 \times Z_2$ flavor symmetry and the strong mass hierarchy of the charged leptons strongly suppress the FCNC processes in the leptonic sector well below the present experimental upper bounds by many orders of magnitude.

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I. INTRODUCTION

The recent discovery of flavor oscillations of solar, atmospheric, reactor, and accelerator neutrinos have irrefutably established that neutrinos have nonvanishing masses and mix among themselves much like the quarks, thereby providing the first conclusive evidence of new physics beyond the standard model [1,2]. Neutrino oscillation observations and experiments, made in the past eight years, have allowed the determination of the differences of the neutrino masses squared and the flavor mixing angles in the leptonic sector. The solar [3–6], atmospheric [7,8], and reactor [9,10] experiments produced the following results:

$$7.1 \times 10^{-5}\ (\mathrm{eV})^2 \le \Delta^2 m_{12} \le 8.9 \times 10^{-5}\ (\mathrm{eV})^2, \quad (1)$$

$$0.24 \le \sin^2\theta_{12} \le 0.40, \quad (2)$$

$$1.4 \times 10^{-3}\ (\mathrm{eV})^2 \le \Delta^2 m_{13} \le 3.3 \times 10^{-3}\ (\mathrm{eV})^2, \quad (3)$$

$$0.34 \le \sin^2\theta_{23} \le 0.68, \quad (4)$$

at 90% confidence level [11,12]. For a recent review on the phenomenology of massive neutrinos, see [13]. The CHOOZ experiment [14] determined an upper bound for the flavor mixing angle between the first and the third generation:

$$\sin^2\theta_{13} \le 0.046. \quad (5)$$

However, neutrino oscillation data are insensitive to the absolute value of neutrino masses and also to the fundamental issue of whether neutrinos are Dirac or Majorana particles. Hence, the importance of the upper bounds on neutrino masses provided by the searches that probe the neutrino mass values at rest: beta decay experiments [15], neutrinoless double beta decay [16], and precision cosmology [17–19].

In the standard model, the Higgs and Yukawa sectors, which are responsible for the generation of the masses of quarks and charged leptons, do not give mass to the neutrinos. Furthermore, the Yukawa sector of the standard model already has too many parameters whose values can only be determined from experiment. These two facts point to the necessity and convenience of extending the standard model in order to make a unified and systematic treatment of the observed hierarchies of masses and mixings of all fermions, as well as the presence or absence of CP violating phases in the mixing matrices. At the same time, we would also like to reduce drastically the number of free parameters in the theory. These two seemingly contradictory demands can be met by means of a flavor symmetry under which the families transform in a nontrivial fashion.

Recently, we introduced a minimal S_3-invariant extension of the standard model [20] in which we argued that such a flavor symmetry unbroken at the Fermi scale, is the permutational symmetry of three objects S_3. In this model, we imposed S_3 as a fundamental symmetry in the matter sector. This assumption led us necessarily to extend the concept of flavor and generations to the Higgs sector. Hence, going to the irreducible representations of S_3, we added to the Higgs $SU(2)_L$ doublet in the S_3-singlet representation two more Higgs $SU(2)_L$ doublets, which can only belong to the two components of the S_3-doublet representation. In this way, all the matter fields in the minimal S_3-invariant extension of the standard model—Higgs, quark, and lepton fields, including the right-handed

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neutrino fields—belong to the three-dimensional representation $\mathbf{1} \oplus \mathbf{2}$ of the permutational group S_3. The leptonic sector of the model was further constrained by an Abelian Z_2 symmetry. We found that the $S_3 \times Z_2$ symmetry predicts the tribimaximal mixing and an inverted mass hierarchy of the left-handed neutrinos in good agreement with experiment [20]. More recently, we reparametrized the mass matrices of the charged leptons and neutrinos, previously derived in [20], in terms of their eigenvalues and derived explicit analytical expressions for the entries in the neutrino mixing matrix, $V_{\rm PMNS}$, and the neutrino mixing angles and Majorana phases as functions of the masses of charged leptons and neutrinos, in excellent agreement with the latest experimental values [21].

The group S_3 [22–31] and the product groups $S_3 \times S_3$ [31–34] and $S_3 \times S_3 \times S_3$ [35,36] have been considered by many authors to explain successfully the hierarchical structure of quark masses and mixings in the standard model. However, in these works, the S_3, $S_3 \times S_3$ and $S_3 \times S_3 \times S_3$ symmetries are explicitly broken at the Fermi scale to give mass to the lighter quarks and charged leptons, while neutrinos are left massless. Some other interesting models based on the S_3, S_4, A_4, and D_5 flavor symmetry groups, unbroken at the Fermi scale, have also been proposed [37–44], but in those models, equality of the number of fields and the irreducible representations is not obtained. The generic properties of mass textures of quarks and leptons derived in the standard model and in supersymmetric models with a Higgs sector with nontrivial flavors and an S_3 flavor symmetry have been discussed in [45,46]. Recent flavor symmetry models are reviewed in [47–50], see also the references therein.

In this paper, we consider the flavor-changing neutral current (FCNC) processes in the minimal S_3-invariant extension of the standard model [20]. After a short review of some relevant results on lepton masses and mixings, we derive exact, explicit expressions for the matrices of the Yukawa couplings in the leptonic sector expressed as functions of the masses of the charged leptons and neutral Higgs bosons. With the help of the Yukawa matrices we compute the branching ratios of some selected FCNC processes as functions of the masses of charged leptons and neutral Higgs bosons. We find that the interplay of the $S_3 \times Z_2$ flavor symmetry and the strong mass hierarchy of charged leptons strongly suppresses the FCNC processes in the leptonic sector well below the experimental upper bounds by many orders of magnitude.

II. THE MINIMAL S_3-INVARIANT EXTENSION OF THE STANDARD MODEL

In the standard model analogous fermions in different generations have identical couplings to all gauge bosons of the strong, weak, and electromagnetic interactions. Prior to the introduction of the Higgs boson and mass terms, the Lagrangian is chiral and invariant with respect to permutations of the left and right fermionic fields.

The six possible permutations of three objects (f_1, f_2, f_3) are elements of the permutational group S_3. This is the discrete, non-Abelian group with the smallest number of elements. The three-dimensional real representation is not an irreducible representation of S_3. It can be decomposed into the direct sum of a doublet f_D and a singlet f_s, where

$$f_s = \frac{1}{\sqrt{3}}(f_1 + f_2 + f_3), \qquad f_D^T = \left(\frac{1}{\sqrt{2}}(f_1 - f_2), \frac{1}{\sqrt{6}}(f_1 + f_2 - 2f_3)\right). \tag{6}$$

The direct product of two doublets $\mathbf{p_D}^T = (p_{D1}, p_{D2})$ and $\mathbf{q_D}^T = (q_{D1}, q_{D2})$ may be decomposed into the direct sum of two singlets $\mathbf{r_s}$ and $\mathbf{r_{s'}}$, and one doublet $\mathbf{r_D}^T$ where

$$\mathbf{r_s} = p_{D1}q_{D1} + p_{D2}q_{D2}, \qquad \mathbf{r_{s'}} = p_{D1}q_{D2} - p_{D2}q_{D1}, \tag{7}$$

$$\mathbf{r_D}^T = (r_{D1}, r_{D2}) = (p_{D1}q_{D2} + p_{D2}q_{D1}, p_{D1}q_{D1} - p_{D2}q_{D2}). \tag{8}$$

The antisymmetric singlet $\mathbf{r_{s'}}$ is not invariant under S_3.

Since the standard model has only one Higgs $SU(2)_L$ doublet, which can only be an S_3 singlet, it can only give mass to the quark or charged lepton in the S_3 singlet representation, one in each family, without breaking the S_3 symmetry.

Hence, in order to impose S_3 as a fundamental symmetry, unbroken at the Fermi scale, we are led to extend the Higgs sector of the theory. The quark, lepton, and Higgs fields are

$$Q^T = (u_L, d_L), u_R, d_R, \qquad L^T = (\nu_L, e_L), e_R, \nu_R \quad \text{and} \quad H, \tag{9}$$

in an obvious notation. All of these fields have three species, and we assume that each one forms a reducible representation $\mathbf{1}_S \oplus \mathbf{2}$. The doublets carry capital indices I and J, which run from 1 to 2, and the singlets are denoted by Q_3, u_{3R}, d_{3R}, L_3, e_{3R}, ν_{3R}, and H_S. Note that the subscript 3 denotes the singlet representation and not the third generation. The most general renormalizable Yukawa interactions of this model are given by

$$\mathcal{L}_Y = \mathcal{L}_{Y_D} + \mathcal{L}_{Y_U} + \mathcal{L}_{Y_E} + \mathcal{L}_{Y_\nu}, \tag{10}$$

where

$$\begin{aligned}\mathcal{L}_{Y_D} = &-Y_1^d \bar{Q}_I H_S d_{IR} - Y_3^d \bar{Q}_3 H_S d_{3R} \\ &- Y_2^d[\bar{Q}_I \kappa_{IJ} H_1 d_{JR} + \bar{Q}_I \eta_{IJ} H_2 d_{JR}] \\ &- Y_4^d \bar{Q}_3 H_I d_{IR} - Y_5^d \bar{Q}_I H_I d_{3R} + \text{H.c.},\end{aligned} \tag{11}$$

$$\begin{aligned}\mathcal{L}_{Y_U} = & -Y_1^u \bar{Q}_I(i\sigma_2)H_S^* u_{IR} - Y_3^u \bar{Q}_3(i\sigma_2)H_S^* u_{3R} \\ & - Y_2^u[\bar{Q}_I\kappa_{IJ}(i\sigma_2)H_1^* u_{JR} + \eta\bar{Q}_I\eta_{IJ}(i\sigma_2)H_2^* u_{JR}] \\ & - Y_4^u\bar{Q}_3(i\sigma_2)H_I^* u_{IR} - Y_5^u\bar{Q}_I(i\sigma_2)H_I^* u_{3R} + \text{H.c.},\end{aligned} \tag{12}$$

$$\begin{aligned}\mathcal{L}_{Y_E} = & -Y_1^e\bar{L}_I H_S e_{IR} - Y_3^e\bar{L}_3 H_S e_{3R} \\ & - Y_2^e[\bar{L}_I\kappa_{IJ}H_1 e_{JR} + \bar{L}_I\eta_{IJ}H_2 e_{JR}] \\ & - Y_4^e\bar{L}_3 H_I e_{IR} - Y_5^e\bar{L}_I H_I e_{3R} + \text{H.c.},\end{aligned} \tag{13}$$

$$\begin{aligned}\mathcal{L}_{Y_\nu} = & -Y_1^\nu\bar{L}_I(i\sigma_2)H_S^*\nu_{IR} - Y_3^\nu\bar{L}_3(i\sigma_2)H_S^*\nu_{3R} \\ & - Y_2^\nu[\bar{L}_I\kappa_{IJ}(i\sigma_2)H_1^*\nu_{JR} + \bar{L}_I\eta_{IJ}(i\sigma_2)H_2^*\nu_{JR}] \\ & - Y_4^\nu\bar{L}_3(i\sigma_2)H_I^*\nu_{IR} - Y_5^\nu\bar{L}_I(i\sigma_2)H_I^*\nu_{3R} + \text{H.c.},\end{aligned} \tag{14}$$

and

$$\kappa = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \text{and} \quad \eta = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \tag{15}$$

Furthermore, we add to the Lagrangian the Majorana mass terms for the right-handed neutrinos

$$\mathcal{L}_M = -M_1\nu_{IR}^T C\nu_{IR} - M_3\nu_{3R}^T C\nu_{3R}. \tag{16}$$

Because of the presence of three Higgs fields, the Higgs potential $V_H(H_S, H_D)$ is more complicated than that of the standard model. This potential was analyzed by Pakvasa and Sugawara [23] who found that in addition to the S_3 symmetry, it has a permutational symmetry S_2': $H_1 \leftrightarrow H_2$, which is not a subgroup of the flavor group S_3. In this communication, we will assume that the vacuum respects the accidental S_2' symmetry of the Higgs potential and that

$$\langle H_1\rangle = \langle H_2\rangle. \tag{17}$$

With these assumptions, the Yukawa interactions, Eqs. (11)–(14) yield mass matrices, for all fermions in the theory, of the general form [20]

$$\mathbf{M} = \begin{pmatrix} \mu_1 + \mu_2 & \mu_2 & \mu_5 \\ \mu_2 & \mu_1 - \mu_2 & \mu_5 \\ \mu_4 & \mu_4 & \mu_3 \end{pmatrix}. \tag{18}$$

The Majorana mass for the left-handed neutrinos ν_L is generated by the seesaw mechanism. The corresponding mass matrix is given by

$$\mathbf{M}_\nu = \mathbf{M}_{\nu_\mathrm{D}}\tilde{\mathbf{M}}^{-1}(\mathbf{M}_{\nu_\mathrm{D}})^T, \tag{19}$$

where $\tilde{\mathbf{M}} = \mathrm{diag}(M_1, M_1, M_3)$.

In principle, all entries in the mass matrices can be complex since there is no restriction coming from the flavor symmetry S_3. The mass matrices are diagonalized by bi-unitary transformations as

$$\begin{aligned} U_{d(u,e)L}^\dagger \mathbf{M}_{d(u,e)} U_{d(u,e)R} &= \mathrm{diag}(m_{d(u,e)}, m_{s(c,\mu)}, m_{b(t,\tau)}), \\ U_\nu^T \mathbf{M}_\nu U_\nu &= \mathrm{diag}(m_{\nu_1}, m_{\nu_2}, m_{\nu_3}). \end{aligned} \tag{20}$$

The entries in the diagonal matrices may be complex, so the physical masses are their absolute values.

The mixing matrices are, by definition,

$$V_{\mathrm{CKM}} = U_{uL}^\dagger U_{dL}, \qquad V_{\mathrm{PMNS}} = U_{eL}^\dagger U_\nu K, \tag{21}$$

where K is the diagonal matrix of the Majorana phase factors.

III. THE MASS MATRICES IN THE LEPTONIC SECTOR AND Z_2 SYMMETRY

A further reduction of the number of parameters in the leptonic sector may be achieved by means of an Abelian Z_2 symmetry. A possible set of charge assignments of Z_2, compatible with the experimental data on masses and mixings in the leptonic sector, is given in Table I.

These Z_2 assignments forbid the following Yukawa couplings

$$Y_1^e = Y_3^e = Y_1^\nu = Y_5^\nu = 0. \tag{22}$$

Therefore, the corresponding entries in the mass matrices vanish, i.e., $\mu_1^e = \mu_3^e = 0$ and $\mu_1^\nu = \mu_5^\nu = 0$.

A. The mass matrix of the charged leptons

The mass matrix of the charged leptons takes the form

$$M_e = m_\tau\begin{pmatrix} \tilde{\mu}_2 & \tilde{\mu}_2 & \tilde{\mu}_5 \\ \tilde{\mu}_2 & -\tilde{\mu}_2 & \tilde{\mu}_5 \\ \tilde{\mu}_4 & \tilde{\mu}_4 & 0 \end{pmatrix}. \tag{23}$$

The unitary matrix U_{eL} that enters in the definition of the mixing matrix, V_{PMNS}, is calculated from

$$U_{eL}^\dagger M_e M_e^\dagger U_{eL} = \mathrm{diag}(m_e^2, m_\mu^2, m_\tau^2), \tag{24}$$

where m_e, m_μ, and m_τ are the masses of the charged leptons, and

$$\frac{M_e M_e^\dagger}{m_\tau^2} = \begin{pmatrix} 2|\tilde{\mu}_2|^2 + |\tilde{\mu}_5|^2 & |\tilde{\mu}_5|^2 & 2|\tilde{\mu}_2||\tilde{\mu}_4|e^{-i\delta_e} \\ |\tilde{\mu}_5|^2 & 2|\tilde{\mu}_2|^2 + |\tilde{\mu}_5|^2 & 0 \\ 2|\tilde{\mu}_2||\tilde{\mu}_4|e^{i\delta_e} & 0 & 2|\tilde{\mu}_4|^2 \end{pmatrix}. \tag{25}$$

Notice that this matrix has only one nonignorable phase factor.

The parameters $|\tilde{\mu}_2|$, $|\tilde{\mu}_4|$, and $|\tilde{\mu}_5|$ may readily be expressed in terms of the charged lepton masses. From the invariants of $M_e M_e^\dagger$, we get the set of equations

TABLE I. Z_2 assignment in the leptonic sector.

−	+
H_S, ν_{3R}	H_I, L_3, L_I, e_{3R}, e_{IR}, ν_{IR}

$$\mathrm{Tr}\,(M_e M_e^\dagger) = m_e^2 + m_\mu^2 + m_\tau^2 = m_\tau^2[4|\tilde{\mu}_2|^2 + 2(|\tilde{\mu}_4|^2 + |\tilde{\mu}_5|^2)], \quad (26)$$

$$\chi(M_e M_e^\dagger) = m_\tau^2(m_e^2 + m_\mu^2) + m_e^2 m_\mu^2 = 4m_\tau^4[|\tilde{\mu}_2|^4 + |\tilde{\mu}_2|^2(|\tilde{\mu}_4|^2 + |\tilde{\mu}_5|^2) + |\tilde{\mu}_4|^2|\tilde{\mu}_5|^2], \quad (27)$$

$$\det(M_e M_e^\dagger) = m_e^2 m_\mu^2 m_\tau^2 = 4m_\tau^6|\tilde{\mu}_2|^2|\tilde{\mu}_4|^2|\tilde{\mu}_5|^2, \quad (28)$$

where $\chi(M_e M_e^\dagger) = \frac{1}{2}[(\mathrm{Tr}(M_e M_e^\dagger))^2 - \mathrm{Tr}(M_e M_e^\dagger)^2]$.

Solving these equations for $|\tilde{\mu}_2|^2$, $|\tilde{\mu}_4|^2$, and $|\tilde{\mu}_5|^2$, we obtain

$$|\tilde{\mu}_2|^2 = \frac{\tilde{m}_\mu^2}{2}\frac{1+x^4}{1+x^2} + \beta, \quad (29)$$

and

$$|\tilde{\mu}_{4,5}|^2 = \frac{1}{4}\left(1 - \tilde{m}_\mu^2\frac{(1-x^2)^2}{1+x^2} - 4\beta\right) \mp \frac{1}{4}\left[\left(1 - \tilde{m}_\mu^2\frac{(1-x^2)^2}{1+x^2}\right)^2 - 8\tilde{m}_e^2\frac{1+x^2}{1+x^4} + 8\beta\left(1 - \tilde{m}_\mu^2\frac{(1-x^2)^2}{1+x^2} + \frac{x^2}{1+\frac{2\beta(1+x^2)}{\tilde{m}_\mu^2(1+x^4)}}\frac{(1+x^2)^2}{(1+x^4)^2}\right) + 16\beta^2\right]^{1/2}. \quad (30)$$

In these expressions, $x = m_e/m_\mu$, $\tilde{m}_\mu = m_\mu/m_\tau$ and β is the smallest solution of the equation

$$\beta^3 - \frac{1}{2}\left(1 - 2y + 6\frac{z}{y}\right)\beta^2 - \frac{1}{4}\left(y - y^2 - 4\frac{z}{y} + 7z - 12\frac{z^2}{y^2}\right)\beta - \frac{1}{8}yz - \frac{1}{2}\frac{z^2}{y^2} + \frac{3}{4}\frac{z^2}{y} - \frac{z^3}{y^3} = 0, \quad (31)$$

where $y = (m_e^2 + m_\mu^2)/m_\tau^2$ and $z = m_\mu^2 m_e^2/m_\tau^4$.

A good, order of magnitude, estimate for β is obtained from (31)

$$\beta \approx -\frac{m_\mu^2 m_e^2}{2m_\tau^2(m_\tau^2 - (m_\mu^2 + m_e^2))}. \quad (32)$$

Once $M_e M_e^\dagger$ has been reparametrized in terms of the charged lepton masses, it is straightforward to compute M_e and U_{eL} also as functions of the charged lepton masses [21]. The resulting expression for M_e, written to order $(m_\mu m_e/m_\tau^2)^2$ and $x^4 = (m_e/m_\mu)^4$ is

$$M_e \approx m_\tau \begin{pmatrix} \frac{1}{\sqrt{2}}\frac{\tilde{m}_\mu}{\sqrt{1+x^2}} & \frac{1}{\sqrt{2}}\frac{\tilde{m}_\mu}{\sqrt{1+x^2}} & \frac{1}{\sqrt{2}}\sqrt{\frac{1+x^2-\tilde{m}_\mu^2}{1+x^2}} \\ \frac{1}{\sqrt{2}}\frac{\tilde{m}_\mu}{\sqrt{1+x^2}} & -\frac{1}{\sqrt{2}}\frac{\tilde{m}_\mu}{\sqrt{1+x^2}} & \frac{1}{\sqrt{2}}\sqrt{\frac{1+x^2-\tilde{m}_\mu^2}{1+x^2}} \\ \frac{\tilde{m}_e(1+x^2)}{\sqrt{1+x^2-\tilde{m}_\mu^2}}e^{i\delta_e} & \frac{\tilde{m}_e(1+x^2)}{\sqrt{1+x^2-\tilde{m}_\mu^2}}e^{i\delta_e} & 0 \end{pmatrix}. \quad (33)$$

This approximation is numerically exact up to order 10^{-9} in units of the τ mass. Notice that this matrix has no free parameters other than the Dirac phase δ_e.

The unitary matrix U_{eL} that diagonalizes $M_e M_e^\dagger$ and enters in the definition of the neutrino mixing matrix V_{PMNS} may be written as

$$U_{eL} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & e^{i\delta_e} \end{pmatrix}\begin{pmatrix} O_{11} & -O_{12} & O_{13} \\ -O_{21} & O_{22} & O_{23} \\ -O_{31} & -O_{32} & O_{33} \end{pmatrix}, \quad (34)$$

where the orthogonal matrix $\mathbf{O}_{eL}$ in the right-hand side of Eq. (34), written to the same order of magnitude as M_e, is

$$\mathbf{O}_{eL} \approx \begin{pmatrix} \frac{1}{\sqrt{2}}x\frac{(1+2\tilde{m}_\mu^2+4x^2+\tilde{m}_\mu^4+2\tilde{m}_e^2)}{\sqrt{1+\tilde{m}_\mu^2+5x^2-\tilde{m}_\mu^4-\tilde{m}_\mu^6+\tilde{m}_e^2+12x^4}} & -\frac{1}{\sqrt{2}}\frac{(1-2\tilde{m}_\mu^2+\tilde{m}_\mu^4-2\tilde{m}_e^2)}{\sqrt{1-4\tilde{m}_\mu^2+x^2+6\tilde{m}_\mu^4-4\tilde{m}_\mu^6-5\tilde{m}_e^2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}}x\frac{(1+4x^2-\tilde{m}_\mu^4-2\tilde{m}_e^2)}{\sqrt{1+\tilde{m}_\mu^2+5x^2-\tilde{m}_\mu^4-\tilde{m}_\mu^6+\tilde{m}_e^2+12x^4}} & \frac{1}{\sqrt{2}}\frac{(1-2\tilde{m}_\mu^2+\tilde{m}_\mu^4)}{\sqrt{1-4\tilde{m}_\mu^2+x^2+6\tilde{m}_\mu^4-4\tilde{m}_\mu^6-5\tilde{m}_e^2}} & \frac{1}{\sqrt{2}} \\ -\frac{\sqrt{1+2x^2-\tilde{m}_\mu^2-\tilde{m}_e^2}(1+\tilde{m}_\mu^2+x^2-2\tilde{m}_e^2)}{\sqrt{1+\tilde{m}_\mu^2+5x^2-\tilde{m}_\mu^4-\tilde{m}_\mu^6+\tilde{m}_e^2+12x^4}} & -x\frac{(1+x^2-\tilde{m}_\mu^2-2\tilde{m}_e^2)\sqrt{1+2x^2-\tilde{m}_\mu^2-\tilde{m}_e^2}}{\sqrt{1-4\tilde{m}_\mu^2+x^2+6\tilde{m}_\mu^4-4\tilde{m}_\mu^6-5\tilde{m}_e^2}} & \frac{\sqrt{1+x^2}\tilde{m}_e\tilde{m}_\mu}{\sqrt{1+x^2-\tilde{m}_\mu^2}} \end{pmatrix}, \quad (35)$$

where, as before, $\tilde{m}_\mu = m_\mu/m_\tau$, $\tilde{m}_e = m_e/m_\tau$, and $x = m_e/m_\mu$.

B. The mass matrix of the neutrinos

According to the Z_2 selection rule, Eq. (22), the mass matrix of the Dirac neutrinos takes the form

$$\mathbf{M}_{\nu_\mathrm{D}} = \begin{pmatrix} \mu_2^\nu & \mu_2^\nu & 0 \\ \mu_2^\nu & -\mu_2^\nu & 0 \\ \mu_4^\nu & \mu_4^\nu & \mu_3^\nu \end{pmatrix}. \quad (36)$$

Then, the mass matrix for the left-handed Majorana neutrinos, $\mathbf{M}_\nu$, obtained from the seesaw mechanism, $\mathbf{M}_\nu = \mathbf{M}_{\nu_\mathrm{D}}\tilde{\mathbf{M}}^{-1}(\mathbf{M}_{\nu_\mathrm{D}})^T$, is

$$\mathbf{M}_\nu = \begin{pmatrix} 2(\rho_2^\nu)^2 & 0 & 2\rho_2^\nu\rho_4^\nu \\ 0 & 2(\rho_2^\nu)^2 & 0 \\ 2\rho_2^\nu\rho_4^\nu & 0 & 2(\rho_4^\nu)^2 + (\rho_3^\nu)^2 \end{pmatrix}, \quad (37)$$

where $\rho_2^\nu = (\mu_2^\nu)/M_1^{1/2}$, $\rho_4^\nu = (\mu_4^\nu)/M_1^{1/2}$, and $\rho_3^\nu = (\mu_3^\nu)/M_3^{1/2}$; M_1 and M_3 are the masses of the right-handed neutrinos appearing in (16).

The non-Hermitian, complex, symmetric neutrino mass matrix, M_ν, may be brought to a diagonal form by a bi-unitary transformation, as

$$U_\nu^T M_\nu U_\nu = \mathrm{diag}(|m_{\nu_1}|e^{i\phi_1}, |m_{\nu_2}|e^{i\phi_2}, |m_{\nu_3}|e^{i\phi_\nu}), \quad (38)$$

where U_ν is the matrix that diagonalizes the matrix $M_\nu^\dagger M_\nu$.

In order to compute U_ν, we notice that $M_\nu^\dagger M_\nu$ has the same texture zeroes as M_ν

$$M_\nu^\dagger M_\nu = \begin{pmatrix} |A|^2 + |B|^2 & 0 & A^\star B + B^\star D \\ 0 & |A|^2 & 0 \\ AB^\star + BD^\star & 0 & |B|^2 + |D|^2 \end{pmatrix}, \quad (39)$$

where $A = 2(\rho_2^\nu)^2$, $B = 2\rho_2^\nu\rho_4^\nu$, and $D = 2(\rho_4^\nu)^2 + (\rho_3^\nu)^2$.

Furthermore, notice that the entries in the upper right corner and lower left corner are complex conjugates of each other, all other entries are real. Therefore, the matrix $U_{\nu L}$ that diagonalizes $M_\nu^\dagger M_\nu$, takes the form

$$U_\nu = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & e^{i\delta_\nu} \end{pmatrix} \begin{pmatrix} \cos\eta & \sin\eta & 0 \\ 0 & 0 & 1 \\ -\sin\eta & \cos\eta & 0 \end{pmatrix}. \quad (40)$$

If we require that the defining Eq. (38) be satisfied as an identity, we get the following set of equations:

$$\begin{aligned} 2(\rho_2^\nu)^2 &= m_{\nu_3}, \\ 2(\rho_2^\nu)^2 &= m_{\nu_1}\cos^2\eta + m_{\nu_2}\sin^2\eta, \\ 2\rho_2^\nu\rho_4^\nu &= \sin\eta\cos\eta(m_{\nu_2} - m_{\nu_1})e^{-i\delta_\nu}, \\ 2(\rho_4^\nu)^2 + (\rho_3^\nu)^2 &= (m_{\nu_1}\sin^2\eta + m_{\nu_2}\cos^2\eta)e^{-2i\delta_\nu}. \end{aligned} \quad (41)$$

Solving these equations for $\sin\eta$ and $\cos\eta$, we find

$$\sin^2\eta = \frac{m_{\nu_3} - m_{\nu_1}}{m_{\nu_2} - m_{\nu_1}}, \qquad \cos^2\eta = \frac{m_{\nu_2} - m_{\nu_3}}{m_{\nu_2} - m_{\nu_1}}. \quad (42)$$

Hence, the matrices M_ν and U_ν, reparametrized in terms of the complex neutrino masses, take the form [21]

$$M_\nu = \begin{pmatrix} m_{\nu_3} & 0 & \sqrt{(m_{\nu_3} - m_{\nu_1})(m_{\nu_2} - m_{\nu_3})}e^{-i\delta_\nu} \\ 0 & m_{\nu_3} & 0 \\ \sqrt{(m_{\nu_3} - m_{\nu_1})(m_{\nu_2} - m_{\nu_3})}e^{-i\delta_\nu} & 0 & (m_{\nu_1} + m_{\nu_2} - m_{\nu_3})e^{-2i\delta_\nu} \end{pmatrix} \quad (43)$$

and

$$U_\nu = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & e^{i\delta_\nu} \end{pmatrix} \begin{pmatrix} \sqrt{\frac{m_{\nu_2} - m_{\nu_3}}{m_{\nu_2} - m_{\nu_1}}} & \sqrt{\frac{m_{\nu_3} - m_{\nu_1}}{m_{\nu_2} - m_{\nu_1}}} & 0 \\ 0 & 0 & 1 \\ -\sqrt{\frac{m_{\nu_3} - m_{\nu_1}}{m_{\nu_2} - m_{\nu_1}}} & \sqrt{\frac{m_{\nu_2} - m_{\nu_3}}{m_{\nu_2} - m_{\nu_1}}} & 0 \end{pmatrix}. \quad (44)$$

The unitarity of U_ν constrains $\sin\eta$ to be real and thus $|\sin\eta| \leq 1$, this condition fixes the phases ϕ_1 and ϕ_2 as

$$|m_{\nu_1}|\sin\phi_1 = |m_{\nu_2}|\sin\phi_2 = |m_{\nu_3}|\sin\phi_\nu. \quad (45)$$

The only free parameters in these matrices, are the phase ϕ_ν, implicit in m_{ν_1}, m_{ν_2}, and m_{ν_3}, and the Dirac phase δ_ν.

C. The neutrino mixing matrix

The neutrino mixing matrix V_{PMNS} is the product $U_{eL}^\dagger U_\nu K$, where K is the diagonal matrix of the Majorana phase factors, defined by

$$\mathrm{diag}\,(m_{\nu_1}, m_{\nu_2}, m_{\nu_3}) = K^\dagger\,\mathrm{diag}(|m_{\nu_1}|, |m_{\nu_2}|, |m_{\nu_3}|)K^\dagger. \quad (46)$$

Except for an overall phase factor $e^{i\phi_1}$, which can be ignored, K is

$$K = \mathrm{diag}(1, e^{i\alpha}, e^{i\beta}), \quad (47)$$

where $\alpha = 1/2(\phi_1 - \phi_2)$ and $\beta = 1/2(\phi_1 - \phi_\nu)$ are the Majorana phases.

Therefore, the theoretical mixing matrix $V_{\mathrm{PMNS}}^{\mathrm{th}}$, is given by

$$V_{\mathrm{PMNS}}^{\mathrm{th}} = \begin{pmatrix} O_{11}\cos\eta + O_{31}\sin\eta e^{i\delta} & O_{11}\sin\eta - O_{31}\cos\eta e^{i\delta} & -O_{21} \\ -O_{12}\cos\eta + O_{32}\sin\eta e^{i\delta} & -O_{12}\sin\eta - O_{32}\cos\eta e^{i\delta} & O_{22} \\ O_{13}\cos\eta - O_{33}\sin\eta e^{i\delta} & O_{13}\sin\eta + O_{33}\cos\eta e^{i\delta} & O_{23} \end{pmatrix} \times K, \quad (48)$$

where $\cos\eta$ and $\sin\eta$ are given in Eq. (42), O_{ij} are given in Eqs. (34) and (35), and $\delta = \delta_\nu - \delta_e$.

To find the relation of our results with the neutrino mixing angles we make use of the equality of the absolute values of the elements of $V_{\mathrm{PMNS}}^{\mathrm{th}}$ and $V_{\mathrm{PMNS}}^{\mathrm{PDG}}$ [51], that is

$$|V_{\rm PMNS}^{\rm th}| = |V_{\rm PMNS}^{\rm PDG}|. \tag{49}$$

This relation allows us to derive expressions for the mixing angles in terms of the charged lepton and neutrino masses

$$|\sin\theta_{13}| = O_{21}, \qquad |\sin\theta_{23}| = \frac{O_{22}}{\sqrt{O_{22}^2 + O_{23}^2}} \tag{50}$$

and

$$|\tan\theta_{12}|^2 = \cot^2\eta \frac{O_{11}^2 \frac{1}{\cot^2\eta} + O_{31}^2 - 2O_{31}O_{11}\frac{1}{\cot\eta}\cos\delta}{O_{11}^2\cot^2\eta + O_{31}^2 + 2O_{31}O_{11}\cot\eta\cos\delta}. \tag{51}$$

The magnitudes of the reactor and atmospheric mixing angles, θ_{13} and θ_{23}, are determined by the masses of the charged leptons only. Keeping terms up to order (m_e^2/m_μ^2) and $(m_\mu/m_\tau)^4$, we get

$$\begin{aligned} \sin\theta_{13} &\approx \frac{1}{\sqrt{2}} x \frac{(1 + 4x^2 - \tilde{m}_\mu^4)}{\sqrt{1 + \tilde{m}_\mu^2 + 5x^2 - \tilde{m}_\mu^4}}, \\ \sin\theta_{23} &\approx \frac{1}{\sqrt{2}} \frac{1 - 2\tilde{m}_\mu^2 + \tilde{m}_\mu^4}{\sqrt{1 - 4\tilde{m}_\mu^2 + x^2 + 6\tilde{m}_\mu^4}}. \end{aligned} \tag{52}$$

Substitution of the small numerical values $\tilde{m}_\mu = 5.94 \times 10^{-2}$, $x = m_e/m_\mu = 4.84 \times 10^{-3}$, and $\tilde{m}_\mu = m_\mu/m_\tau = 5.95 \times 10^{-2}$ for the leptonic mass ratios $\tilde{m}_\mu$ and x in the right-hand side of (52) yields the numerical values of $\sin\theta_{13}$ and $\sin\theta_{23}$

$$\sin\theta_{13} = 0.0034, \qquad \sin\theta_{23} = \tfrac{1}{\sqrt{2}} - 8.4 \times 10^{-6}. \tag{53}$$

From these numbers, it is evident that the theoretical values of $\sin\theta_{13}$ and $\sin\theta_{23}$ are very close to the corresponding tribimaximal mixing values $\sin\theta_{13}^{\rm tri} = 0$ and $\sin\theta_{23}^{\rm tri} = 1/\sqrt{2}$ [52].

The dependence of $\tan\theta_{12}$ on the Dirac phase δ, see (51), is very weak, since $O_{31} \sim 1$ but $O_{11} \sim 1/\sqrt{2}(m_e/m_\mu)$. Hence, we may neglect it when comparing (51) with the data on neutrino mixings.

The dependence of $\tan\theta_{12}$ on the phase ϕ_ν and the physical masses of the neutrinos enters through the ratio of the neutrino mass differences under the square root sign, it can be made explicit with the help of the unitarity constraint on U_ν, Eq. (45),

$$\frac{m_{\nu_2} - m_{\nu_3}}{m_{\nu_3} - m_{\nu_1}} = \frac{(|m_{\nu_2}|^2 - |m_{\nu_3}|^2\sin^2\phi_\nu)^{1/2} - |m_{\nu_3}||\cos\phi_\nu|}{(|m_{\nu_1}|^2 - |m_{\nu_3}|^2\sin^2\phi_\nu)^{1/2} + |m_{\nu_3}||\cos\phi_\nu|}. \tag{54}$$

Similarly, the Majorana phases are given by

$$\begin{aligned} \sin 2\alpha &= \sin(\phi_1 - \phi_2) \\ &= \frac{|m_{\nu_3}|\sin\phi_\nu}{|m_{\nu_1}||m_{\nu_2}|}\Big(\sqrt{|m_{\nu_2}|^2 - |m_{\nu_3}|^2\sin^2\phi_\nu} \\ &\quad + \sqrt{|m_{\nu_1}|^2 - |m_{\nu_3}|^2\sin^2\phi_\nu}\Big), \end{aligned} \tag{55}$$

$$\begin{aligned} \sin 2\beta &= \sin(\phi_1 - \phi_\nu) \\ &= \frac{\sin\phi_\nu}{|m_{\nu_1}|}\Big(|m_{\nu_3}|\sqrt{1 - \sin^2\phi_\nu} \\ &\quad + \sqrt{|m_{\nu_1}|^2 - |m_{\nu_3}|^2\sin^2\phi_\nu}\Big). \end{aligned} \tag{56}$$

A more complete and detailed discussion of the Majorana phases in the neutrino mixing matrix $V_{\rm PMNS}$ obtained in our model is given by J. Kubo [53].

D. Neutrino masses and mixings

In the present model, $\sin^2\theta_{13}$ and $\sin^2\theta_{23}$ are determined only by the masses of the charged leptons in very good agreement with the experimental values [11,12,54],

$$(\sin^2\theta_{13})^{\rm th} = 1.1 \times 10^{-5}, \qquad (\sin^2\theta_{13})^{\rm exp} \leq 0.046,$$

and

$$(\sin^2\theta_{23})^{\rm th} = 0.499, \qquad (\sin^2\theta_{23})^{\rm exp} = 0.5^{+0.06}_{-0.05}.$$

In this model, the experimental restriction $|\Delta m_{12}^2| < |\Delta m_{13}^2|$ implies an inverted neutrino mass spectrum, $|m_{\nu_3}| < |m_{\nu_1}| < |m_{\nu_2}|$ [20].

As can be seen from Eqs. (51) and (54), the solar mixing angle is sensitive to the neutrino mass differences and the phase ϕ_ν but is only very weakly sensitive to the charged lepton masses. If we neglect the small terms proportional to O_{11} and O_{11}^2 in (51), we get

$$\tan^2\theta_{12} = \frac{(\Delta m_{12}^2 + \Delta m_{13}^2 + |m_{\nu_3}|^2\cos^2\phi_\nu)^{1/2} - |m_{\nu_3}||\cos\phi_\nu|}{(\Delta m_{13}^2 + |m_{\nu_3}|^2\cos^2\phi_\nu)^{1/2} + |m_{\nu_3}||\cos\phi_\nu|}. \tag{57}$$

From this expression, we may readily derive expressions for the neutrino masses in terms of $\tan\theta_{12}$ and ϕ_ν and the differences of the squared masses

$$|m_{\nu_3}| = \frac{\sqrt{\Delta m_{13}^2}}{2\cos\phi_\nu\tan\theta_{12}} \frac{1 - \tan^4\theta_{12} + r^2}{\sqrt{1 + \tan^2\theta_{12}}\sqrt{1 + \tan^2\theta_{12} + r^2}}, \tag{58}$$

in a similar way, we obtain

$$|m_{\nu_1}| = \frac{\sqrt{\Delta m_{13}^2}}{2\cos\phi_\nu t_{12}} \frac{[(1-t_{12}^4)^2 + 4\cos^2\phi_\nu t_{12}^2(1+t_{12}^2)^2 + 2r^2(1-t_{12}^4 + 2\cos^2\phi_\nu t_{12}^2(1+t_{12}^2)) + r^4]^{(1/2)}}{\sqrt{1+t_{12}^2}\sqrt{1+t_{12}^2+r^2}}$$

$$|m_{\nu_2}| = \frac{\sqrt{\Delta m_{13}^2}}{2\cos\phi_\nu t_{12}} \times \frac{[(1-t_{12}^4)^2 + 4\cos^2\phi_\nu t_{12}^2(1+t_{12}^2)^2 + 2r^2(1-t_{12}^4 + 2\cos^2\phi_\nu t_{12}^2(1+t_{12}^2)(2+t_{12}^2)) + r^4(1+4\cos^2\phi_\nu t_{12}^2(1+t_{12}^2))]^{(1/2)}}{\sqrt{1+t_{12}^2}\sqrt{1+t_{12}^2+r^2}}, \tag{59}$$

where $t_{12} = \tan\theta_{12}$, and $r^2 = \Delta m_{12}^2/\Delta m_{13}^2 \approx 3 \times 10^{-2}$. As $r^2 \ll 1$, Eq. (58) reduces to

$$|m_{\nu_3}| \approx \frac{1}{2\cos\phi_\nu} \frac{\sqrt{\Delta m_{13}^2}}{\tan\theta_{12}}(1 - \tan^2\theta_{12}). \tag{60}$$

From these expressions, and setting $r^2 \sim 0$, the sum of the neutrino masses is

$$|m_{\nu_1}| + |m_{\nu_2}| + |m_{\nu_3}| \approx \frac{\sqrt{\Delta m_{13}^2}}{2\cos\phi_\nu \tan\theta_{12}}(1 + 2\sqrt{1 + 2\tan^2\theta_{12}(2\cos^2\phi_\nu - 1) + \tan^4\theta_{12}} - \tan^2\theta_{12}). \tag{61}$$

The most restrictive cosmological upper bound for this sum is [17]

$$\sum |m_\nu| \leq 0.17 \text{ eV}. \tag{62}$$

From this upper bound and the experimentally determined values of $\tan\theta_{12}$ and Δm_{ij}^2, we may derive a lower bound for $\cos\phi_\nu$,

$$\cos\phi_\nu \geq 0.55, \tag{63}$$

or $0 \leq \phi_\nu \leq 57°$. The neutrino masses $|m_{\nu_i}|$ assume their minimal values when $\cos\phi_\nu = 1$. When $\cos\phi_\nu$ takes values in the range $0.55 \leq \cos\phi \leq 1$, the neutrino masses change very slowly with $\cos\phi_\nu$, see Fig. 1. In the absence of experimental information we will assume that ϕ_ν vanishes. Hence, setting $\phi_\nu = 0$ in our formula, we find

$$|m_{\nu_2}| \approx 0.056 \text{ eV}, \qquad |m_{\nu_1}| \approx 0.055 \text{ eV}, \qquad |m_{\nu_3}| \approx 0.022 \text{ eV}, \tag{64}$$

where we used the values $\Delta m_{13}^2 = 2.6 \times 10^{-3}$ eV^2, $\Delta m_{21}^2 = 7.9 \times 10^{-5}$ eV^2, and $\tan\theta_{12} = 0.667$, taken from [13].

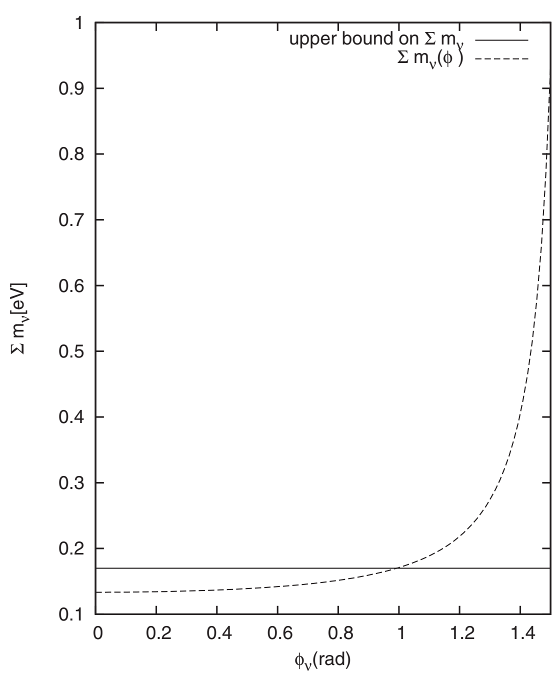

FIG. 1. The dashed line represents the sum of the neutrino masses, $\sum_{i=1}^{3} |m_{\nu_i}|$, as function of ϕ_ν. The horizontal straight line is the cosmological upper bound on $\sum |m_{\nu_i}|$ [17].

E. V_{PMNS} and the tribimaximal form

Once the numerical values of the neutrino masses are determined, we may readily verify that the theoretical mixing matrix, V_{PMNS}, is very close to the tribimaximal form of the mixing matrix,

$$V_{\mathrm{PMNS}}^{\mathrm{th}} = \begin{pmatrix} \sqrt{\frac{2}{3}} & \sqrt{\frac{1}{3}} & 0 \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & -\sqrt{\frac{1}{2}} \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & \sqrt{\frac{1}{2}} \end{pmatrix} + \delta V_{\mathrm{PMNS}}^{\mathrm{tri}}, \tag{65}$$

where $\delta V_{\mathrm{PMNS}}^{\mathrm{tri}} = V_{\mathrm{PMNS}}^{\mathrm{th}} - V_{\mathrm{PMNS}}^{\mathrm{tri}}$. From Eqs. (35), (40), (42), (48), (52), and (64), the correction term to the tribimaximal form of the mixing matrix comes out as

$$\delta V_{\rm PMNS}^{\rm tri} \approx \begin{pmatrix} 1.94\times 10^{-2} & -2.84\times 10^{-2} & -3.4\times 10^{-3} \\ 2.21\times 10^{-2} & 1.5\times 10^{-2} & -8.2\times 10^{-6} \\ 1.8\times 10^{-2} & 1.24\times 10^{-2} & 3.1\times 10^{-10} \end{pmatrix}. \tag{66}$$

IV. FLAVOR-CHANGING NEUTRAL CURRENTS (FCNC)

Models with more than one Higgs $SU(2)$ doublet have tree-level flavor-changing neutral currents. In the minimal S_3-invariant extension of the standard model considered here, there is one Higgs $SU(2)$ doublet per generation coupling to all fermions. The flavor-changing Yukawa couplings may be written in a flavor labeled, symmetry adapted weak basis as

$$\begin{aligned} \mathcal{L}_Y^{\rm FCNC} = {}& (\bar{E}_{aL} Y_{ab}^{ES} E_{bR} + \bar{U}_{aL} Y_{ab}^{US} U_{bR} + \bar{D}_{aL} Y_{ab}^{DS} D_{bR}) H_S^0 \\ & + (\bar{E}_{aL} Y_{ab}^{E1} E_{bR} + \bar{U}_{aL} Y_{ab}^{U1} U_{bR} \\ & + \bar{D}_{aL} Y_{ab}^{D1} D_{bR}) H_1^0 + (\bar{E}_{aL} Y_{ab}^{E2} E_{bR} \\ & + \bar{U}_{aL} Y_{ab}^{U2} U_{bR} + \bar{D}_{aL} Y_{ab}^{D2} D_{bR}) H_2^0 + \text{H.c.}, \end{aligned} \tag{67}$$

where the entries in the column matrices $E's$, $U's$, and $D's$ are the left and right fermion fields and $Y_{ab}^{(e,u,d)s}$, $Y_{ab}^{(e,u,d)1,2}$ are 3×3 matrices of the Yukawa couplings of the fermion fields to the neutral Higgs fields H_s^0 and H_I^0 in the S_3-singlet and doublet representations, respectively.

In this basis, the Yukawa couplings of the Higgs fields to each family of fermions may be written in terms of matrices $\mathcal{M}_Y^{(e,u,d)}$, which give rise to the corresponding mass matrices $M^{(e,u,d)}$ when the gauge symmetry is spontaneously broken. From this relation we may calculate the flavor-changing Yukawa couplings in terms of the fermion masses and the vacuum expectation values of the neutral Higgs fields. For example, the matrix $\mathcal{M}_Y^e$ is written in terms of the matrices of the Yukawa couplings of the charged leptons as

$$\mathcal{M}_Y^e = Y_w^{E1} H_1^0 + Y_w^{E2} H_2^0, \tag{68}$$

in this expression, the index w means that the Yukawa matrices are defined in the weak basis,

$$Y_w^{E1} = \frac{m_\tau}{v_1} \begin{pmatrix} 0 & \frac{1}{\sqrt{2}} \frac{\tilde{m}_\mu}{\sqrt{1+x^2}} & \frac{1}{\sqrt{2}} \sqrt{\frac{1+x^2-\tilde{m}_\mu^2}{1+x^2}} \\ \frac{1}{\sqrt{2}} \frac{\tilde{m}_\mu}{\sqrt{1+x^2}} & 0 & 0 \\ \frac{\tilde{m}_e(1+x^2)}{\sqrt{1+x^2-\tilde{m}_\mu^2}} e^{i\delta_e} & 0 & 0 \end{pmatrix} \tag{69}$$

and

$$Y_w^{E2} = \frac{m_\tau}{v_2} \begin{pmatrix} \frac{1}{\sqrt{2}} \frac{\tilde{m}_\mu}{\sqrt{1+x^2}} & 0 & 0 \\ 0 & -\frac{1}{\sqrt{2}} \frac{\tilde{m}_\mu}{\sqrt{1+x^2}} & \frac{1}{\sqrt{2}} \sqrt{\frac{1+x^2-\tilde{m}_\mu^2}{1+x^2}} \\ 0 & \frac{\tilde{m}_e(1+x^2)}{\sqrt{1+x^2-\tilde{m}_\mu^2}} e^{i\delta_e} & 0 \end{pmatrix}. \tag{70}$$

The Yukawa couplings of immediate physical interest in the computation of the flavor-changing neutral currents are those defined in the mass basis, according to $\tilde{Y}_m^{EI} = U_{eL}^\dagger Y_w^{EI} U_{eR}$, where U_{eL} and U_{eR} are the matrices that diagonalize the charged lepton mass matrix defined in Eqs. (20) and (34). We obtain

$$\tilde{Y}_m^{E1} \approx \frac{m_\tau}{v_1} \begin{pmatrix} 2\tilde{m}_e & -\frac{1}{2}\tilde{m}_e & \frac{1}{2}x \\ -\tilde{m}_\mu & \frac{1}{2}\tilde{m}_\mu & -\frac{1}{2} \\ \frac{1}{2}\tilde{m}_\mu x^2 & -\frac{1}{2}\tilde{m}_\mu & \frac{1}{2} \end{pmatrix}_m, \tag{71}$$

and

$$\tilde{Y}_m^{E2} \approx \frac{m_\tau}{v_2} \begin{pmatrix} -\tilde{m}_e & \frac{1}{2}\tilde{m}_e & -\frac{1}{2}x \\ \tilde{m}_\mu & \frac{1}{2}\tilde{m}_\mu & \frac{1}{2} \\ -\frac{1}{2}\tilde{m}_\mu x^2 & \frac{1}{2}\tilde{m}_\mu & \frac{1}{2} \end{pmatrix}_m, \tag{72}$$

where $\tilde{m}_\mu = 5.94\times 10^{-2}$, $\tilde{m}_e = 2.876\times 10^{-4}$, and $x = m_e/m_\mu = 4.84\times 10^{-3}$. All the nondiagonal elements are responsible for tree-level FCNC processes. The actual values of the Yukawa couplings in Eqs. (71) and (72) still depend on the VEV's of the Higgs fields v_1 and v_2, and, hence, on the Higgs potential. If the S_2' symmetry in the Higgs sector is preserved [23], $\langle H_1^0\rangle = \langle H_2^0\rangle = v$. To make an order of magnitude estimate of the coefficient in the Yukawa matrices, m_τ/v, we may further assume that the VEV's for all the Higgs fields are comparable, that is, $\langle H_s^0\rangle = \langle H_1^0\rangle = \langle H_2^0\rangle = \frac{\sqrt{2}}{\sqrt{3}}\frac{M_W}{g_2}$, then, $m_\tau/v = \sqrt{3}/\sqrt{2} g_2 m_\tau/M_W$ and we may estimate the numerical values of the Yukawa couplings from the numerical values of the lepton masses. For instance, the amplitude of the flavor violating process $\tau^- \to \mu^- e^+ e^-$, is proportional to $\tilde{Y}_{\tau\mu}^E \tilde{Y}_{ee}^E$ [55]. Then, the leptonic branching ratio,

$$\mathrm{Br}(\tau\to\mu e^+ e^-) = \frac{\Gamma(\tau\to\mu e^+ e^-)}{\Gamma(\tau\to e\nu\bar{\nu}) + \Gamma(\tau\to\mu\nu\bar{\nu})} \tag{73}$$

and

$$\Gamma(\tau\to\mu e^+ e^-) \approx \frac{m_\tau^5}{3\times 2^{10}\pi^3} \frac{(Y_{\tau\mu}^{1,2} Y_{ee}^{1,2})^2}{M_{H_{1,2}}^4}, \tag{74}$$

which is the dominant term, and the well-known expressions for $\Gamma(\tau\to e\nu\bar{\nu})$ and $\Gamma(\tau\to\mu\nu\bar{\nu})$ [51], give

$$\mathrm{Br}(\tau\to\mu e^+ e^-) \approx \frac{9}{4}\left(\frac{m_e m_\mu}{m_\tau^2}\right)^2 \left(\frac{m_\tau}{M_{H_{1,2}}}\right)^4, \tag{75}$$

taking for $M_{H_{1,2}} \sim 120$ GeV, we obtain

$$\mathrm{Br}(\tau\to\mu e^+ e^-) \approx 3.15\times 10^{-17},$$

well below the experimental upper bound for this process, which is 2.7×10^{-7} [56]. Similar computations give the following estimates

$$\mathrm{Br}\,(\tau \to e\gamma) \approx \frac{3\alpha}{8\pi}\left(\frac{m_\mu}{M_H}\right)^4, \tag{76}$$

$$\mathrm{Br}\,(\tau \to \mu\gamma) \approx \frac{3\alpha}{128\pi}\left(\frac{m_\mu}{m_\tau}\right)^2\left(\frac{m_\tau}{M_H}\right)^4, \tag{77}$$

$$\mathrm{Br}\,(\tau \to 3\mu) \approx \frac{9}{64}\left(\frac{m_\mu}{M_H}\right)^4, \tag{78}$$

$$\mathrm{Br}\,(\mu \to 3e) \approx 18\left(\frac{m_e m_\mu}{m_\tau^2}\right)^2\left(\frac{m_\tau}{M_H}\right)^4, \tag{79}$$

and

$$\mathrm{Br}\,(\mu \to e\gamma) \approx \frac{27\alpha}{64\pi}\left(\frac{m_e}{m_\mu}\right)^4\left(\frac{m_\tau}{M_H}\right)^4. \tag{80}$$

We see that FCNC processes in the leptonic sector are strongly suppressed by the small values of the mass ratios m_e/m_τ, m_μ/m_τ, and m_τ/M_H. The numerical estimates of the branching ratios and the corresponding experimental upper bounds are shown in Table II. It may be seen that, in all cases considered, the numerical values for the branching ratios of the FCNC in the leptonic sector are well below the corresponding experimental upper bounds. The matrices of the quark Yukawa couplings may be computed in a similar way. Numerical values for the Yukawa couplings for u and d-type quarks are given in our previous paper [20]. There it was found that, due to the strong hierarchy in the quark masses and the corresponding small or very small mass ratios, the numerical values of all the Yukawa couplings in the quark sector are small or very small. Kubo, Okada, and Sakamaki [61] have investigated the breaking of the gauge symmetry in the present S_3-invariant extension of the standard model with the S_3-invariant Higgs potential $V_H(H_S, \mathbf{H_2})$ analyzed by Pakvasa and Sugawara [23]. They found that it is possible that all physical Higgs bosons, except one neutral one, could become sufficiently heavy ($M_H \sim 10$ TeV) to suppress all the flavor-changing neutral current processes in the quark sector of the theory without having a problem with triviality.

V. CONCLUSIONS

By introducing three Higgs fields that are $SU(2)_L$ doublets in the theory, we extended the concept of flavor and generations to the Higgs sector and formulated a minimal S_3-invariant extension of the standard model [20]. A well-defined structure of the Yukawa couplings is obtained, which permits the calculation of mass and mixing matrices for quarks and leptons in a unified way. A further reduction of redundant parameters is achieved in the leptonic sector by introducing a Z_2 symmetry. The flavor symmetry group $Z_2 \times S_3$ relates the mass spectrum and mixings. This allowed us to derive explicit, analytical expressions for all entries in the neutrino mixing matrix, V_{PMNS}, as functions of the masses of the charged leptons and neutrinos and two phases δ and ϕ_ν [21]. In this model, the tribimaximal mixing structure of V_{PMNS} and the magnitudes of the three mixing angles are determined by the interplay of the flavor $S_3 \times Z_2$ symmetry, the seesaw mechanism, and the charged lepton mass hierarchy. We also found that V_{PMNS} has three CP violating phases, namely, one Dirac phase $\delta = \delta_\nu - \delta_e$ and two Majorana phases, α and β, which are functions of the neutrino masses and the phase ϕ_ν which is independent of the Dirac phase. The numerical values of the reactor, θ_{13}, and the atmospheric, θ_{23}, mixing angles are determined by the masses of the charged leptons only, in very good agreement with the experiment. The solar mixing angle θ_{12} is almost insensitive to the values of the masses of the charged leptons, but its experimental value allowed us to fix the scale and origin of the neutrino mass spectrum, which has an inverted hierarchy, with the values $|m_{\nu_2}| = 0.056$ eV, $|m_{\nu_1}| = 0.055$ eV, and $|m_{\nu_3}| = 0.022$ eV. In the present work, we obtained explicit expressions for the matrices of the Yukawa couplings of the lepton sector parametrized in terms of the charged lepton masses and the VEV's of the neutral Higgs bosons in the S_3-doublet representation. These Yukawa matrices are closely related to the fermion mass matrices and have a structure of small and very small entries reflecting the observed charged lepton mass hierarchy. With the help of the Yukawa matrices, we computed the branching ratios of a number of FCNC processes and found that the branching ratios of all FCNC processes considered are strongly suppressed by powers of the small mass ratios m_e/m_τ and m_μ/m_τ, and by the ratio $(m_\tau/M_{H_{1,2}})^4$, where $M_{H_{1,2}}$ is the

TABLE II. Leptonic FCNC processes, calculated with $M_{H_{1,2}} \sim 120$ GeV.

FCNC processes	Theoretical BR	Experimental upper bound BR	References
$\tau \to 3\mu$	8.43×10^{-14}	2×10^{-7}	B. Aubert *et al.* [56]
$\tau \to \mu e^+ e^-$	3.15×10^{-17}	2.7×10^{-7}	B. Aubert *et al.* [56]
$\tau \to \mu\gamma$	9.24×10^{-15}	6.8×10^{-8}	B. Aubert *et al.* [57]
$\tau \to e\gamma$	5.22×10^{-16}	1.1×10^{-11}	B. Aubert *et al.* [58]
$\mu \to 3e$	2.53×10^{-16}	1×10^{-12}	U. Bellgardt *et al.* [59]
$\mu \to e\gamma$	2.42×10^{-20}	1.2×10^{-11}	M. L. Brooks *et al.* [60]

mass of the neutral Higgs bosons in the S_3-doublet. Taking for $M_{H_{1,2}}$ a very conservative value ($M_{H_{1,2}} \approx 120$ GeV), we found that the numerical values of the branching ratios of the FCNC in the leptonic sector are well below the corresponding experimental upper bounds by many orders of magnitude. We may add that although the theoretical values of the branching ratios of FCNC processes computed in this work are much smaller than their experimental upper bounds measured in terrestrial laboratories, they still are larger than the vanishing or nearly vanishing values allowed by the standard model, and could be important in astrophysical processes [62]. It has already been argued that small FCNC processes mediating nonstandard quark-neutrino interactions could be important in the theoretical description of the gravitational core collapse and shock generation in the explosion stage of a supernova [63,64].

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