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Hint of a Z' boson from the CERN LEP II data

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The many-parametric fits of the LEP2 data on $e^+e^- \to e^+e^-$, $\mu^+\mu^-$, $\tau^+\tau^-$ processes are performed with the goal to estimate the signals of the Abelian Z' boson. Four independent parameters must be fitted, if the derived already low-energy relations between the Z' couplings to the standard model fermions are taken into consideration. No signals are found when the complete LEP2 data set for these processes is treated. In the fit of the backward bins, the hint at the 1.3σ confidence level is detected. The Z' couplings to the vector and axial-vector lepton currents are constrained. The comparisons with the one-parameter fits and with the corresponding LEP1 experiments are fulfilled.

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I. INTRODUCTION

The precision test of the standard model (SM) at LEP gave a possibility not only to determine all the parameters and particle masses at the level of radiative corrections but also afforded an opportunity for searching for signals of new heavy particles beyond the energy scale of it. On the base of the LEP2 experiments the low bounds on parameters of various models extending the SM have been estimated and the scale of new physics was obtained [1–3]. Although no new particles were discovered, a general belief is that the energy scale of new physics is of order 1 TeV, that may serve as a guide for experiments at the LHC. In this situation, any information about new heavy particles obtained on the base of the present day data is desirable and important.

A lot of extended models include the Z' gauge boson—a massive neutral vector particle associated with the extra $U(1)$ subgroup of an underlying group. The searching for this particle as a virtual state in the either model-dependent or model-independent approaches is widely discussed in the literature (see Ref. [4]). In our papers [5–7] a new approach for the model-independent search for the Z' boson was proposed which, in contrast to other model-independent searches, gives a possibility to pick out uniquely this virtual state and determine its characteristics. The corresponding observables have also been introduced and applied to analyze the LEP2 experiment data. Our consideration is based on two constituents: (1) The relations between the effective low-energy couplings derived from the renormalization group (RG) equation for fermion scattering amplitudes. We called them the RG relations. Because of these relations, a number of unknown Z' parameters entering the amplitudes of different scattering processes considerably decreases. (2) When these relations are accounted for, some kinematics properties of the amplitude become uniquely correlated with this virtual state and the Z' signals exhibit themselves.

The RG relations allow to introduce observables related uniquely with the Z' boson. Comparing the mean values of the observables with the necessary specific values, one could arrive at a conclusion about the Z' existence. The confidence level (CL) of these values will be estimated and adduced in addition. Without taking into consideration the RG relations the determination of the Z' boson requires a supplementary specification due to a larger number of different couplings contributing to the observables. A similar situation takes place in the ''helicity model fits'' of LEP Collaborations [1–3] when different virtual states contribute to each of the specific models (AA, VV, . . .). Therefore these fits had the goal to discover any signals of new physics independently of the particular states which may cause deviations from the SM.

In Refs. [5–7] the one-parametric observables were introduced and the signals of the Z' have been determined at the 1σ CL in the $e^+e^- \to \mu^+\mu^-$ process, and at the 2σ CL in the Bhabha process. The Z' mass was estimated to be 1–1.2 TeV. An increase in statistics could make these signals more pronounced and there is a good chance to discover this particle at the LHC.

Recently the final data of the LEP collaborations DELPHI and OPAL [2,3] were published and new, more precise estimates could be obtained. In the present paper we update the results of the one-parameter fit and perform the complete many-parametric fit of the LEP2 data to estimate a possible signal of the Z' boson. Usually, in a many-parametric fit the uncertainty of the result increases drastically because of extra parameters. On the contrary, in our approach due to the RG relations between the low-energy couplings there are only 2–3 independent parameters for the LEP scattering processes. Therefore, we believe that an inevitable increase of confidence areas (CA) in the many-parametric space could be compensated by accounting for all the accessible experimental information. As it will be shown, the uncertainty of the many-parametric fit can be comparable with the uncertainty of the previous one-parametric fits in Refs. [6,7]. In this approach the combined data fit for all lepton processes is also possible.

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II. THE ABELIAN Z' BOSON AT LOW ENERGIES

Let us adduce a necessary information about the Abelian Z' boson. This particle is predicted by a number of grand unification models. Among them the E_6 and $SO(10)$ based models [8] (for instance, LR, $\chi - \psi$, and so on) are often discussed in the literature. In all the models, the Abelian Z' boson is described by a low-energy $\tilde{U}(1)$ gauge subgroup originated in some symmetry breaking pattern.

At low energies, the Z' boson can manifest itself by means of the couplings to the SM fermions and scalars as a virtual intermediate state. Moreover, the Z-boson couplings are also modified due to a Z-Z' mixing. In principle, arbitrary effective Z' interactions to the SM fields could be considered at low energies. However, the couplings of nonrenormalizable types have to be suppressed by heavy mass scales because of decoupling. Therefore, significant signals beyond the SM can be inspired by the couplings of renormalizable types. Such couplings can be derived by adding the new $\tilde{U}(1)$ terms to the electroweak covariant derivatives D^{ew} in the Lagrangian [4]

$$\mathcal{L} = \left| \left(D^{\text{ew}}_\mu - i \frac{\tilde{y}_\phi}{2} \tilde{Z}_\mu \right) \phi \right|^2 + i \sum_{f = f_L, f_R} \bar{f} \gamma^\mu \left(D^{\text{ew}}_\mu - i \frac{\tilde{y}_f}{2} \tilde{Z}_\mu \right) f, \tag{1}$$

where ϕ is the SM scalar doublet; f_L, f_R are the SM left-handed fermion doublets and right-handed fermion singlets; $\tilde{Z}_\mu$ denotes the $\tilde{U}(1)$ symmetry eigenstate; and $\tilde{y}_\phi$, $\tilde{y}_{f_L}$, and $\tilde{y}_{f_R}$ mean the unknown couplings characterizing the model beyond the SM. Instead of the couplings to the left-handed and right-handed fermion states it is convenient to introduce the couplings to the axial-vector and vector currents: $a_f = (\tilde{y}_{f_R} - \tilde{y}_{f_L})/2$, $v_f = (\tilde{y}_{f_L} + \tilde{y}_{f_R})/2$.

The spontaneous breaking of the electroweak symmetry leads to the Z-Z' mixing. In the case of the Abelian Z' boson, the Z-Z' mixing angle θ_0 is determined by the coupling $\tilde{y}_\phi$ as follows [5]

$$\theta_0 = \frac{\sin\theta_W \cos\theta_W}{\sqrt{4\pi\alpha_{\text{em}}}} \frac{m_Z^2}{m_{Z'}^2} \tilde{y}_\phi + O\left(\frac{m_Z^4}{m_{Z'}^4}\right), \tag{2}$$

where θ_W is the SM Weinberg angle and α_{em} is the electromagnetic fine structure constant. Although the mixing angle is a small quantity of order $m_{Z'}^{-2}$, it contributes to the Z-boson exchange amplitude and cannot be neglected at the LEP energies.

The Lagrangian (1) leads to the following interactions between the fermions and the Z and Z' mass eigenstates:

$$\begin{aligned} \mathcal{L}_{Z\bar{f}f} &= \tfrac{1}{2} i Z_\mu \bar{f} \gamma^\mu [(v^{\text{SM}}_{fZ} + \gamma^5 a^{\text{SM}}_{fZ}) \cos\theta_0 + (v_f + \gamma^5 a_f) \sin\theta_0] f, \\ \mathcal{L}_{Z'\bar{f}f} &= \tfrac{1}{2} i Z'_\mu \bar{f} \gamma^\mu [(v_f + \gamma^5 a_f) \cos\theta_0 - (v^{\text{SM}}_{fZ} + \gamma^5 a^{\text{SM}}_{fZ}) \sin\theta_0] f, \end{aligned} \tag{3}$$

where f is an arbitrary SM fermion state; v^{SM}_{fZ}, a^{SM}_{fZ} are the SM couplings of the Z boson.

In a particular model the couplings v_f and a_f take some specific values. In the case when the model is unknown, these parameters and the mixing angle remain potentially arbitrary numbers. However, this is not the case if one assumes that the underlying extended model is a renormalizable one. As was shown in Ref. [5], some of them have to be correlated due to renormalizability. The corresponding relations are

$$v_f - a_f = v_{f^\star} - a_{f^\star}, \qquad a_f = T_{3,f} \tilde{y}_\phi, \tag{4}$$

where $f^\star$ is the $SU(2)$ partner of a fermion f, and $T_{3,f}$ is the third component of the fermion isospin. They are motivated by the renormalization group equations at the Z' decoupling energies and also connected with the $\tilde{U}(1)$ gauge symmetry of the Lagrangian. These relations cover all the popular models of the Abelian Z' boson allowing the model-independent searches for this particle.

The relations (4) incorporate the most common features of the Abelian Z' boson. As it is seen, the axial-vector coupling is universal for all the fermion flavors. So, in what follows we will use the shorthand notation $a = a_e = a_\mu = a_\tau$. The axial-vector coupling determines also the coupling to the scalar doublet and, consequently, the mixing angle. As a result, the number of independent couplings is significantly reduced. Considering the leptonic processes $e^+e^- \to \ell^+\ell^-$ ($\ell = e, \mu, \tau$), one has to keep 4 unknown couplings: a, v_e, v_μ, and v_τ. Moreover, the RG relations serve to uniquely specify a kinematic domain of deviations from the SM predictions due to the virtual Z' boson. Thereof a unique definition of the Z' signal can be done.

In our analysis, as the SM values of the cross sections we use the quantities calculated by the LEP2 collaborations [2,3,9,10]. They account for either the one-loop radiative corrections or initial and final state radiation effects (together with the event selection rules, which are specific for each experiment). As it is reported by the DELPHI Collaboration, there is a theoretical error of the SM values of about 2%. In our analysis this error is added to the statistical and systematic ones for all the collaborations. As it was checked, the fit results are practically insensitive to accounting for this error.

The deviation from the SM is computed in the improved Born approximation. This approximation is sufficient for our analysis leading to the systematic error of the fit results less than 5–10 percent. One may speculate about the possibility that the accounting for of other type radiation corrections could change qualitatively our results. However, this is a typical case when the effect of interest is a correction and one tries to increase the accuracy of estimations. In fact, in searching for deviations, the contributions of other types of diagrams are suppressed by an additional small parameter that gives a correction to the value derived in the approximation used. Moreover, there is a more essential theoretical uncertainty related with the effective low-energy Lagrangian (1) applied in our analysis. Therefore we believe that at the present stage of investigation the chosen accuracy is sufficient. To convince ourselves that this is the case, we have altered the theoretical uncertainty of the deviations for 10–20 percent that did not change qualitatively the obtained results.

The deviation from the SM of the differential cross section for the process $e^+e^- \to \ell^+\ell^-$ can be expressed through various quadratic combinations of couplings a, v_e, v_μ, v_τ. For the Bhabha process it reads

$$\frac{d\sigma}{dz} - \frac{d\sigma^{\rm SM}}{dz} = f_1^{ee}(z)\frac{a^2}{m_{Z'}^2} + f_2^{ee}(z)\frac{v_e^2}{m_{Z'}^2} + f_3^{ee}(z)\frac{av_e}{m_{Z'}^2}, \tag{5}$$

where the factors are known functions of the center-of-mass energy and the cosine of the electron scattering angle z plotted in Fig. 1. The deviation of the cross section for $e^+e^- \to \mu^+\mu^-$ ($\tau^+\tau^-$) processes has a similar form

$$\frac{d\sigma}{dz} - \frac{d\sigma^{\rm SM}}{dz} = f_1^{\mu\mu}(z)\frac{a^2}{m_{Z'}^2} + f_2^{\mu\mu}(z)\frac{v_e v_\mu}{m_{Z'}^2} + f_3^{\mu\mu}(z)\frac{av_e}{m_{Z'}^2} + f_4^{\mu\mu}(z)\frac{av_\mu}{m_{Z'}^2}. \tag{6}$$

Equations (5) and (6) are our definition of the Z' signal.

Since the Z' couplings enter the cross section together with the inverse Z' mass, it is convenient to introduce the dimensionless couplings

$$\bar{a}_f = \frac{m_Z}{\sqrt{4\pi}m_{Z'}}a_f, \qquad \bar{v}_f = \frac{m_Z}{\sqrt{4\pi}m_{Z'}}v_f, \tag{7}$$

which can be constrained by experiments.

Note again that the cross sections in Eqs. (5) and (6) account for the relations (4) through the functions $f_1(z)$, $f_3(z)$, $f_4(z)$, since the coupling $\tilde{y}_\phi$ (the mixing angle θ_0) is substituted by the axial coupling constant a. Usually, when a four-fermion effective Lagrangian is applied to describe physics beyond the SM [11], this dependence on the scalar field coupling is neglected. However, in our case, when we are interested in searching for signals of the Z' boson on the base of the effective low-energy Lagrangian (1), these contributions to the cross section are essential.

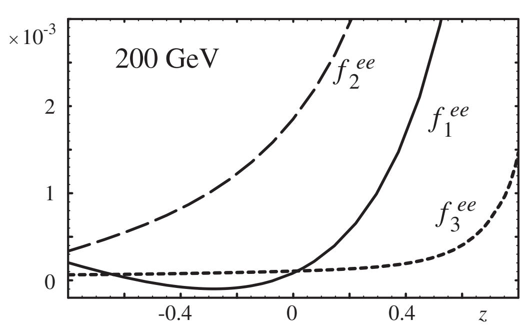

FIG. 1. The factors at the Z' couplings in the differential cross section of the Bhabha process.

III. MANY-PARAMETER FITS

As the basic observable to fit the LEP2 experiment data on the Bhabha process we propose the differential cross section

$$\frac{d\sigma^{\rm Bhabha}}{dz} - \frac{d\sigma^{\rm Bhabha,SM}}{dz}\bigg|_{z=z_i,\sqrt{s}=\sqrt{s_i}}, \tag{8}$$

where i runs over the bins at various center-of-mass energies $\sqrt{s}$. The final differential cross sections measured by the ALEPH (130–183 GeV, [9]), DELPHI (189–207 GeV, [3]), L3 (183–189 GeV, [10]), and OPAL (130–207 GeV, [2]) collaborations are taken into consideration (299 bins).

As the observables for $e^+e^- \to \mu^+\mu^-$, $\tau^+\tau^-$ processes, we consider the total cross section and the forward-backward asymmetry

$$\sigma_T^{\ell^+\ell^-} - \sigma_T^{\ell^+\ell^-,\rm SM}, \qquad A_{\rm FB}^{\ell^+\ell^-} - A_{\rm FB}^{\ell^+\ell^-,\rm SM}|_{\sqrt{s}=\sqrt{s_i}}, \tag{9}$$

where i runs over 12 center-of-mass energies $\sqrt{s}$ from 130 to 207 GeV. We consider the combined LEP2 data [1] for these observables (24 data entries for each process). These data are more precise as the corresponding differential cross sections. Our analysis is based on the fact that the kinematics of s-channel processes is rather simple and the differential cross section is effectively a two-parameter function of the scattering angle. The total cross section and the forward-backward asymmetry incorporate complete information about the kinematics of the process and therefore are an adequate alternative for the differential cross sections.

The data are analyzed by means of the χ^2 fit. Denoting the observables (8) and (9) by σ_i, one can construct the χ^2 function,

$$\chi^2(\bar{a}, \bar{v}_e, \bar{v}_\mu, \bar{v}_\tau) = \sum_i \left[\frac{\sigma_i^{\rm ex} - \sigma_i^{\rm th}(\bar{a}, \bar{v}_e, \bar{v}_\mu, \bar{v}_\tau)}{\delta\sigma_i}\right]^2, \tag{10}$$

where $\sigma^{\rm ex}$ and $\delta\sigma$ are the experimental values and the uncertainties of the observables, and $\sigma^{\rm th}$ are their theoretical expressions presented in Eqs. (5) and (6). The sum in Eq. (10) refers to either the data for one specific process or the combined data for several processes. By minimizing the χ^2 function, the maximal-likelihood estimate for the Z' couplings can be derived. The χ^2 function is also used to plot the CA in the space of parameters $\bar{a}$, $\bar{v}_e$, $\bar{v}_\mu$, and $\bar{v}_\tau$. Note that in this way of experimental data treating all the possible correlations are neglected. We believe that at the present stage of investigation this is reasonable, because the collaborations have never reported on this possibility.

For all the considered processes, the theoretic predictions $\sigma_i^{\rm th}$ are linear combinations of products of two Z' couplings

$$\sigma_i^{\rm th} = \sum_{j=1}^{7} C_{ij} A_j, \qquad A_j = \{\bar a^2, \bar v_e^2, \bar a\bar v_e, \bar v_e\bar v_\mu, \bar v_e\bar v_\tau, \bar a\bar v_\mu, \bar a\bar v_\tau\}, \tag{11}$$

where C_{ij} are known numbers. In what follows we use the matrix notation $\sigma^{\rm th} = \sigma_i^{\rm th}$, $\sigma^{\rm ex} = \sigma_i^{\rm ex}$, $C = C_{ij}$, $A = A_j$. The uncertainties $\delta\sigma_i$ can be substituted by a covariance matrix D. The diagonal elements of D are experimental errors squared, $D_{ii} = (\delta\sigma_i^{\rm ex})^2$, whereas the nondiagonal elements are responsible for the possible correlations of observables. The χ^2 function can be rewritten as

$$\chi^2(A) = (\sigma^{\rm ex} - \sigma^{\rm th})^{\rm T} D^{-1}(\sigma^{\rm ex} - \sigma^{\rm th}) = (\sigma^{\rm ex} - CA)^{\rm T} D^{-1}(\sigma^{\rm ex} - CA), \tag{12}$$

where the upperscript T denotes the matrix transposition.

The χ^2 function has a minimum, $\chi^2_{\rm min}$, at

$$\hat A = (C^{\rm T} D^{-1} C)^{-1} C^{\rm T} D^{-1} \sigma^{\rm ex} \tag{13}$$

corresponding to the maximum-likelihood values of Z' couplings. From Eqs. (12) and (13) we obtain

$$\chi^2(A) - \chi^2_{\rm min} = (\hat A - A)^{\rm T} \hat D^{-1}(\hat A - A), \qquad \hat D = (C^{\rm T} D^{-1} C)^{-1}. \tag{14}$$

Usually, the experimental values $\sigma^{\rm ex}$ are normal-distributed quantities with the mean values $\sigma^{\rm th}$ and the covariance matrix D. The quantities $\hat A$, being the superposition of $\sigma^{\rm ex}$, also have the same distribution. It is easy to show that $\hat A$ has the mean values A and the covariance matrix $\hat D$.

The inverse matrix $\hat D^{-1}$ is symmetric and can be diagonalized. The number of nonzero eigenvalues is determined by the rank (denoted M) of $\hat D^{-1}$. The rank M equals the number of linear-independent terms in the observables $\sigma^{\rm th}$. So, the right-hand side of Eq. (14) is a quantity distributed as χ^2 with M degrees of freedom (d.o.f.). Since this random value is independent of A, the CA in the parameter space $(\bar a, \bar v_e, \bar v_\mu, \bar v_\tau)$ corresponding to the probability β can be defined as [12]:

$$\chi^2 \leq \chi^2_{\rm min} + \chi^2_{{\rm CL},\beta}(M), \tag{15}$$

where $\chi^2_{{\rm CL},\beta}(M)$ is the β level of the χ^2 distribution with M d.o.f.

In the Bhabha process, the Z' effects are determined by 3 linear-independent contributions coming from $\bar a^2$, $\bar v_e^2$, and $\bar a\bar v_e$ ($M = 3$). As for the $e^+e^- \to \mu^+\mu^-$, $\tau^+\tau^-$ processes, the observables depend on 4 linear-independent terms for each process: $\bar a^2$, $\bar v_e\bar v_\mu$, $\bar v_e\bar a$, $\bar a\bar v_\mu$ for $e^+e^- \to \mu^+\mu^-$; and $\bar a^2$, $\bar v_e\bar v_\tau$, $\bar v_e\bar a$, $\bar a\bar v_\tau$ for $e^+e^- \to \tau^+\tau^-$ ($M = 4$). Note that some terms in the observables for different processes are the same. Therefore, the number of χ^2 d.o.f. in the combined fits is less than the sum of d.o.f. for separate processes. Hence, the predictive power of the larger set of data is not drastically spoiled by the increased number of d.o.f. In fact, combining the data of the Bhabha and $e^+e^- \to \mu^+\mu^-$ ($\tau^+\tau^-$) processes together we have to treat 5 linear-independent terms. The complete data set for all the lepton processes is ruled by 7 d.o.f. As a consequence, the combination of the data for all the lepton processes is possible.

The parametric space of couplings $(\bar a, \bar v_e, \bar v_\mu, \bar v_\tau)$ is four dimensional. However, for the Bhabha process it is reduced to the plane $(\bar a, \bar v_e)$, and to the three-dimensional volumes $(\bar a, \bar v_e, \bar v_\mu)$, $(\bar a, \bar v_e, \bar v_\tau)$ for the $e^+e^- \to \mu^+\mu^-$ and $e^+e^- \to \tau^+\tau^-$ processes, correspondingly. The predictive power of data is distributed not uniformly over the parameters. The parameters $\bar a$ and $\bar v_e$ are present in all the considered processes and appear to be significantly constrained. The couplings $\bar v_\mu$ or $\bar v_\tau$ enter when the processes $e^+e^- \to \mu^+\mu^-$ or $e^+e^- \to \tau^+\tau^-$ are accounted for. So, in these processes, we also study the projection of the CA onto the plane $(\bar a, \bar v_e)$.

The origin of the parametric space, $\bar a = \bar v_e = 0$, corresponds to the absence of the Z' signal. This is the SM value of the observables. This point could occur inside or outside of the CA at a fixed CL. When it lays out of the CA, this means the distinct signal of the Abelian Z'. Then the signal probability can be defined as the probability that the data agree with the Abelian Z' boson existence and exclude the SM value. This probability corresponds to the most stringent CL (the largest $\chi^2_{\rm CL}$) at which the point $\bar a = \bar v_e = 0$ is excluded. If the SM value is inside the CA, the Z' boson is indistinguishable from the SM. In this case, upper bounds on the Z' couplings can be determined.

The 95% CL areas in the $(\bar a, \bar v_e)$ plane for the separate processes are plotted in Fig. 2. As it is seen, the Bhabha process constrains both the axial-vector and vector couplings. As for the $e^+e^- \to \mu^+\mu^-$ and $e^+e^- \to \tau^+\tau^-$ processes, the axial-vector coupling is significantly constrained only. The CAs include the SM point at the meaningful CLs, so the experiment could not pick out clearly the Abelian Z' signal from the SM. An important conclusion from these plots is that the experiment significantly constrains only the couplings entering sign-definite terms in the cross sections.

The combination of all the lepton processes is presented in Fig. 3. There is no visible signal beyond the SM. The

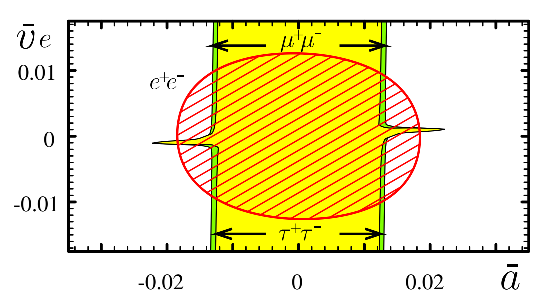

FIG. 2 (color online). The 95% CL areas in the $(\bar a, \bar v_e)$ plane for the Bhabha, $e^+e^- \to \mu^+\mu^-$ and $e^+e^- \to \tau^+\tau^-$ processes.

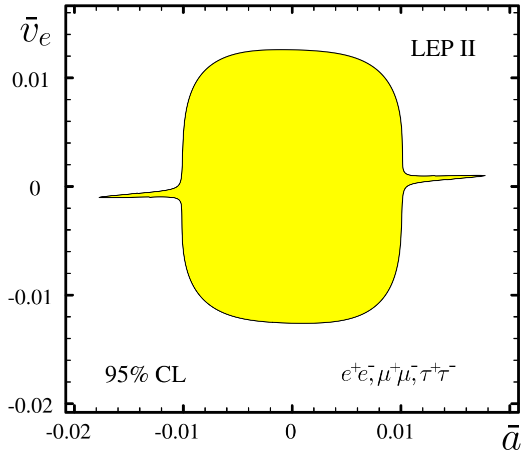

FIG. 3 (color online). The projection of the 95% CL area onto the $(\bar{a}, \bar{v}_e)$ plane for the combination of the Bhabha, $e^+e^- \to \mu^+\mu^-$ and $e^+e^- \to \tau^+\tau^-$ processes.

couplings to the vector and axial-vector electron currents are constrained by the many-parameter fit as $|\bar{v}_e| < 0.013$, $|\bar{a}| < 0.019$ at the 95% CL. If the charge corresponding to the Z' interactions is assumed to be of order of the electromagnetic one, then the Z' mass should be greater than 0.67 TeV. For the charge of order of the SM $SU(2)_L$ coupling constant $m_{Z'} \geq 1.4$ TeV. One can see that the constraint is not too severe to exclude the Z' searches at the LHC.

Let us compare the obtained results with the one-parameter fits in Ref. [7]. Fitting the current data with the one-parameter observable, we find the updated values of the Z' coupling to the electron vector current together with their 1σ uncertainties:

$$\text{ALEPH: } \bar{v}_e^2 = -0.11 \pm 6.53 \times 10^{-4},$$
$$\text{DELPHI: } \bar{v}_e^2 = 1.60 \pm 1.46 \times 10^{-4},$$
$$\text{L3: } \bar{v}_e^2 = 5.42 \pm 3.72 \times 10^{-4},$$
$$\text{OPAL: } \bar{v}_e^2 = 2.42 \pm 1.27 \times 10^{-4},$$
$$\text{Combined: } \bar{v}_e^2 = 2.24 \pm 0.92 \times 10^{-4}.$$

As one can see, the most precise data of DELPHI and OPAL collaborations are resulted in the Abelian Z' hints at 1 and 2 standard deviation level, correspondingly. The combined value shows the 2σ hint, which corresponds to $0.006 \leq |\bar{v}_e| \leq 0.020$.

On the other hand, our many-parameter fit constrains the Z' coupling to the electron vector current as $|\bar{v}_e| \leq 0.013$ with no evident signal. Why does the one-parameter fit of the Bhabha process show the 2σ CL hint whereas there is no signal in the two-parameter one? Our one-parameter observable accounts mainly for the backward bins. This is in accordance with the kinematic features of the process: the backward bins depend mainly on the vector coupling $\bar{v}_e^2$, whereas the contributions of other couplings are kinematically suppressed (see Fig. 1). Therefore, the difference of the results can be inspired by the data sets used. To check this, we perform the many-parameter fit with the 113 backward bins ($z \leq 0$) only. The χ^2 minimum, $\chi^2_{\min} = 93.0$, is found in the nonzero point $|\bar{a}| = 0.0005$, $\bar{v}_e = 0.015$. This value of the Z' coupling $\bar{v}_e$ is in excellent agreement with the mean value obtained in the one-parameter fit. The 68% CA in the $(\bar{a}, \bar{v}_e)$ plane is plotted in Fig. 4. There is a visible hint of the Abelian Z' boson. The zero point $\bar{a} = \bar{v}_e = 0$ (the absence of the Z' boson) corresponds to $\chi^2 = 97.7$. It is covered by the CA with 1.3σ CL. Thus, the backward bins show the 1.3σ hint of the Abelian Z' boson in the many-parameter fit. So, the many-parameter fit is less pronounced than the analysis of the one-parameter observables.

Of course, the 1.3 or 2σ deviation from the SM is a Z'-boson hint only. One could speculate whether some input quantities could wash it out. Since the experimental data together with the statistical errors are published as the final results, we have no idea how to modify them. So, as a possible source of uncertainty we introduce the errors of theoretical predictions for the SM values of cross -sections. These errors are estimated to be less than 2 percent [3]. Now, let us consider two different cases. First, let the errors be randomly fluctuating over all the bins. They can be accounted for by adding 2% uncertainties to the experimental data. In fact, this case was already considered above. As we have showed, the influence of such errors on the fit results is negligible. So, they cannot spoil the hint. Second, we assume that the SM cross sections were calculated with a number of systematic uncertainties up to 2% accuracy. To estimate the influence of such errors we perform the following calculation. We put the SM values of the cross section to be 1% or 2% higher or lower, and then fit the data to obtain the χ^2 function, the CAs, and possible Z'-boson hints. The summary of calculations is presented in Table I. As it is seen, the considerable increase of the SM value could wash the hint out. On the other hand, the hint becomes more pronounced when the SM value is

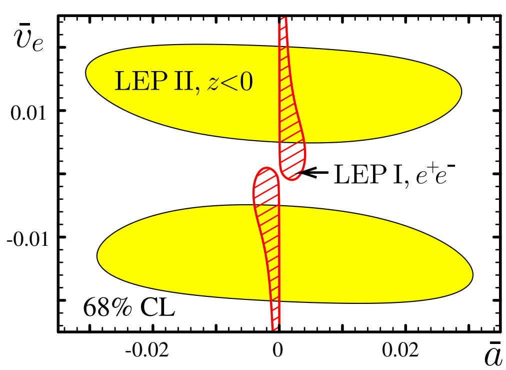

FIG. 4 (color online). The 68% CL area in the $(\bar{a}, \bar{v}_e)$ plane from the backward bins of the Bhabha process in the LEP2 experiments (the shaded area). The hatched area is the 68% CL area from the LEP 1 data on the Bhabha process.

TABLE I. The influence of the changes of the SM values of the cross section on the fit results.

Change of the SM value	-2%	-1%	$+0\%$	$+1\%$	$+2\%$
All the bins (299 bins)					
$\chi^2_{\rm min}$	239.9	237.0	193.5	244.2	258.7
$\chi^2_{\rm SM}$	254.7	242.1	194.5	244.2	258.7
Z' hint	3σ	1.4σ	0.2σ	0σ	0σ
The backward bins (113 bins)					
$\chi^2_{\rm min}$	101.8	102.4	93.0	103.7	104.4
$\chi^2_{\rm SM}$	110.7	109.1	97.7	107.0	106.5
Z' hint	2.1σ	1.7σ	1.3σ	0.9σ	0.6σ

decreased. Hence, we see that the Z' hint could disappear only if the SM values of the observed cross sections are considerably underestimated.

At LEP1 experiments [13] the Z-boson couplings to the vector and axial-vector lepton currents (g_V, g_A) were precisely measured. The Bhabha process shows the 1σ deviation from the SM values for Higgs boson masses $m_H \geq 114$ GeV (see Fig. 7.3 of Ref. [13]). This deviation could be considered as the effect of the Z-Z' mixing. It is interesting to estimate the bounds on the Z' couplings following from these experiments.

Because of the RG relations, the Z-Z' mixing angle is completely determined by the axial-vector coupling $\bar{a}$. So, the deviations of g_V, g_A from their SM values are governed by the couplings $\bar{a}$ and $\bar{v}_e$,

$$g_V - g_V^{\rm SM} = -49.06\bar{a}\bar{v}_e, \quad g_A - g_A^{\rm SM} = 49.06\bar{a}^2. \qquad (16)$$

Let us assume that the total deviation of theory from experiments follows due to the Z-Z' mixing. This gives an upper bound on the Z' couplings. In this way one can estimate whether the Z' boson is excluded by the experiments or not.

The 1σ CL area for the Bhabha process from Ref. [13] is converted into the $(\bar{a}, \bar{v}_e)$ plane in Fig. 4. The SM values of the couplings correspond to the top quark mass $m_t =$ 178 GeV and the Higgs scalar mass $m_H = 114$ GeV. As it is seen, the LEP1 data on the Bhabha process is compatible with the Abelian Z' existence at the 1σ CL. The axial-vector coupling is constrained as $|\bar{a}| \leq 0.005$. This bound corresponds to $\bar{a}^2 \leq 2.5 \times 10^{-5}$, which agrees with our previous one-parameter fits of the LEP2 data for $e^+e^- \to \mu^+\mu^-$, $\tau^+\tau^-$ processes [6] ($\bar{a}^2 = 1.3 \pm 3.89 \times 10^{-5}$ at 68% CL). On the other hand, the vector coupling constant $\bar{v}_e$ is practically unconstrained by the LEP1 experiments.

From the analysis carried out we come to the conclusion that, in principle, the LEP experiments were able to detect the Z'-boson signals if the statistics had been sufficient.

IV. DISCUSSION

LEP collaborations have reported about a good agreement between the experimental data and the predictions of the SM [2,3,9,10]. That means that the experiments have not shown any statistically significant deviations from its predictions. The analysis of the leptonic processes based on the same data set and the same SM values of cross sections lead to the conclusion that the existence of the Z' boson with the mass of order 1–1.2 TeV is not excluded at the 1–2σ CL. We observed that in the one-parameter fits in Refs. [6,7] and in the many-parameter fit of the backward bins in the present investigation. The estimated Z' parameters derived by different methods are in good agreement with each other. So, we conclude that there is a discrepancy which needs some explanation. We believe that the reason is in the RG relations playing a crucial role in our treating of experimental data. As it was showed, the RG relations have served to reduce a number of unknown parameters that gave a possibility to extract maximal information about the Z' signals from the experimental data set. If these relations are not taken into account (as this is the case in Refs. [2,3,9,10]), no signals could be found. LEP collaborations performed also model-dependent fits concerning popular Z' models. These models suit the RG relations (4). So, it is interesting to compare their analysis with our results. In the experiments reported in Refs. [1–3,9,10] the low bound on the Z' mass was obtained. It varies from 400 GeV to 800 GeV at the 95% CL dependently on the specific model. These bounds allow the Z' boson with the mass of order 1 TeV that is in agreement with our results. On the other hand, the possibility to select the Z' signals in specific scattering processes was not discussed in the papers mentioned.

In our analysis we treat the data for leptonic processes only. LEP2 collaborations measured also the total cross sections of the electron-positron annihilation into quark-antiquark pairs. The Abelian Z' signal in $e^+e^- \to \bar{q}q$ process is characterized by 5 independent parameters (for example, two Z' couplings to electron, $\bar{a}$ and $\bar{v}_e$, and three Z' couplings to d, s, and b quarks). There are 8 linear-independent terms in the cross section. As one can check, the accumulated statistics of 12 cross sections for different center-of-mass energies reported in [1] is completely insufficient to constrain significantly the parameters of the Z' boson.

As we have shown in Ref. [7], there is the 2σ hint of the Abelian Z' boson in the one-parameter fit of LEP2 data for the Bhabha process. This result is reproduced also in the present paper by fitting the updated experimental data. In the present analysis, we applied the many-parameter fits of the leptonic processes for different sets of bins included. In particular, for the backward bins (responsible for the signal due to the kinematics of the process) the 1.3σ hint of the particle is found. Here we remind that in our many-parameter fits, as well as in the one-parameter ones, the RG relations were used that specify the contributing virtual states to be the Z' bosons only. In the general case, there are no reasons to expect that the mean values of the observables will be in the required range. The fit of the complete set of bins constrains the Z' couplings to vector and axial-vector electron currents allowing the Z' boson with the mass of order 1 TeV. Thus, we have to conclude that the LEP2 data allow the existence of the quite light Z' boson which has a chance to be discovered in the nearest future. We believe that the RG relations used in the present analysis will be also important in searches for the Z' boson at the LHC.

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