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Unparticle physics in DIS

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The unparticle stuff scenario related to the nontrivial IR fixed point in 4D-conformal field theory was recently suggested by Georgi. We illustrate its physical effects in the deep inelastic scattering (DIS) process. The possible signals of the unparticle related to parity violation asymmetry in DIS is investigated. It is found out that the behavior of this parity violation signal is sensitive to the value of the scale dimension $d_{\mathcal{U}}$ of the unparticle.

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I. INTRODUCTION

It is well known that conformal symmetry plays an important role in both critical phenomena and the superstring theory. Nevertheless, in particle physics in four dimensional space-time dimension, conformal symmetry is broken by the masses of the particles explicitly, and even though there is conformal symmetry classically, this symmetry would be broken by the renormalization effect. However, at high energy scale, there could exist stuff with nontrivial scale invariance in the infrared (such as the Banks-Zaks field [1]), which is recently suggested by Howard Georgi as a component of the physics beyond the standard model (SM) above the TeV scale [2].

Since unparticle stuff is completely new to us, we do not have a picture of what the unparticle looks like, and the theories with a nontrivial IR fixed point is very complex, Georgi has used the method of the low energy effective field theory to study the unparticle stuff production [2] and the peculiar virtual effects in high energy processes [3]. Moreover, the unparticle production in $e^+e^- \to \gamma\mathcal{U}$, monojet production, the virtual effects of the unparticle in the Drell-Yan process, and the muon anomaly have been investigated [4], and the fermionic unparticles are introduced as well [5]. Unparticle physics have very interesting and rich phenomenological consequences. In this paper, we shall explore the phenomenological consequences of the unparticle in deep inelastic scattering (DIS) in the framework of the effective field theory.

We follow closely the scenario studied in [2,3]. The fields of the theory with a nontrivial IR fixed point are denoted as $\mathcal{BZ}$ fields, and the SM fields and the $\mathcal{BZ}$ fields interact through the exchanges of particles with a high mass scale $M_{\mathcal{U}}$. Thus, below the high scale $M_{\mathcal{U}}$, the effective nonrenormalizable couplings between them are as follows:

$$\frac{1}{M_{\mathcal{U}}^k}\mathcal{O}_{\rm SM}\mathcal{O}_{\mathcal{BZ}}, \tag{1}$$

where $\mathcal{O}_{\rm SM}$ and $\mathcal{O}_{\mathcal{BZ}}$ are, respectively, the local operators built up of SM fields and the $\mathcal{BZ}$ fields. Furthermore, below the scale $\Lambda_{\mathcal{U}}$ where the scale invariance in the $\mathcal{BZ}$ sector emerges, the $\mathcal{BZ}$ operator $\mathcal{O}_{\mathcal{BZ}}$ is matched onto the unparticle operator $\mathcal{O}_{\mathcal{U}}$, and Eq. (1) is matched onto the effective interactions between the SM fields and the unparticle fields of the following form:

$$\frac{C_{\mathcal{U}}\Lambda_{\mathcal{U}}^{d_{\mathcal{BZ}}-d_{\mathcal{U}}}}{M_{\mathcal{U}}^k}\mathcal{O}_{\rm SM}\mathcal{O}_{\mathcal{U}}, \tag{2}$$

where $d_{\mathcal{BZ}}$ and $d_{\mathcal{U}}$ are, respectively, the scale dimensions of the local operators $\mathcal{O}_{\mathcal{BZ}}$ and $\mathcal{O}_{\mathcal{U}}$, and $C_{\mathcal{U}}$ is the coefficient function which is determined by the matching processes.

The following effective interactions of interesting phenomenologies have been introduced in [2,3],

$$\begin{aligned}
&\frac{C'_{S\mathcal{U}}\Lambda_{\mathcal{U}}^{k-d_{\mathcal{U}}}}{M_{\mathcal{U}}^k}G_{\mu\nu}G^{\mu\nu}\mathcal{O}_{\mathcal{U}},\\
&\frac{C_{T\mathcal{U}}\Lambda_{\mathcal{U}}^{k-d_{\mathcal{U}}}}{M_{\mathcal{U}}^k}G_{\mu\lambda}G_{\nu}^{\lambda}\mathcal{O}_{\mathcal{U}}^{\mu\nu},\\
&\frac{\Lambda_{\mathcal{U}}^{k+1-d_{\mathcal{U}}}}{M_{\mathcal{U}}^k}\bar{f}\gamma_\mu(C_{V\mathcal{U}}+C_{A\mathcal{U}}\gamma_5)f\mathcal{O}_{\mathcal{U}}^{\mu}\\
&\frac{\Lambda_{\mathcal{U}}^{k+1-d_{\mathcal{U}}}}{M_{\mathcal{U}}^k}\bar{f}(C_{S\mathcal{U}}+iC_{P\mathcal{U}}\gamma_5)f\mathcal{O}_{\mathcal{U}},
\end{aligned} \tag{3}$$

where $G_{\mu\nu}$ is the gluon field strength, f denotes the SM fermion fields, and universal coupling between the unparticles and the SM fermionic fields has been assumed. $\mathcal{O}_{\mathcal{U}}$, $\mathcal{O}_{\mathcal{U}}^{\mu}$, and $\mathcal{O}_{\mathcal{U}}^{\mu\nu}$ are, respectively, the scalar, vector, and tensor unparticle operators. They are taken to be Hermitian and the latter two operators are assumed to be transverse. Naively $C_{V\mathcal{U}}$, $C_{A\mathcal{U}}$, $C_{S\mathcal{U}}$, and $C_{P\mathcal{U}}$ should be of the same order. The interactions in Eq. (3) are generally weak, moreover the first two couplings in Eq. (3) are suppressed by $\frac{1}{\Lambda_{\mathcal{U}}}$ in comparison with the latter two, so the latter two effective interactions are dominant in the DIS processes with virtual unparticle exchange. And we should

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work to the lowest order in the weak couplings of unparticle fields with the SM fields in the effective theory. Similar to Ref. [3] the following dimensionless coefficients are introduced for convenience:

$$c_{V\mathcal{U}} = \frac{C_{V\mathcal{U}}\Lambda_{\mathcal{U}}^{k+1-d_{\mathcal{U}}}}{M_{\mathcal{U}}^k M_Z^{1-d_{\mathcal{U}}}}, \qquad c_{A\mathcal{U}} = \frac{C_{A\mathcal{U}}\Lambda_{\mathcal{U}}^{k+1-d_{\mathcal{U}}}}{M_{\mathcal{U}}^k M_Z^{1-d_{\mathcal{U}}}}$$
$$c_{S\mathcal{U}} = \frac{C_{S\mathcal{U}}\Lambda_{\mathcal{U}}^{k+1-d_{\mathcal{U}}}}{M_{\mathcal{U}}^k M_Z^{1-d_{\mathcal{U}}}}, \qquad c_{P\mathcal{U}} = \frac{C_{P\mathcal{U}}\Lambda_{\mathcal{U}}^{k+1-d_{\mathcal{U}}}}{M_{\mathcal{U}}^k M_Z^{1-d_{\mathcal{U}}}}. \tag{4}$$

Following Refs. [2,3], by conformal symmetry, we have

$$\langle 0|\mathcal{O}^\mu(0)|P\rangle\langle P|\mathcal{O}^\nu(0)|0\rangle = A_{d_{\mathcal{U}}}\theta(P^0)\theta(P^2) \times \left(-g^{\mu\nu} + \frac{P^\mu P^\nu}{P^2}\right)(P^2)^{d_{\mathcal{U}}-2}, \tag{5}$$

where

$$A_{d_{\mathcal{U}}} = \frac{16\pi^{5/2}}{(2\pi)^{2d_{\mathcal{U}}}}\frac{\Gamma(d_{\mathcal{U}}+1/2)}{\Gamma(d_{\mathcal{U}}-1)\Gamma(2d_{\mathcal{U}})} \tag{6}$$

which is normalized with respect to the phase space of $d_{\mathcal{U}}$ massless particles, and the two-point correlation function of the unparticle operator $\mathcal{O}^\mu$ can be obtained as follows:

$$\int d^4x e^{iP\cdot x}\langle 0|T(\mathcal{O}^\mu(x)\mathcal{O}^\nu(0))|0\rangle = \frac{iA_{d_{\mathcal{U}}}}{2\sin(d_{\mathcal{U}}\pi)}\frac{-g^{\mu\nu}+P^\mu P^\nu/P^2}{(-P^2-i\epsilon)^{2-d_{\mathcal{U}}}}. \tag{7}$$

Similarly,

$$\int d^4x e^{iP\cdot x}\langle 0|T(\mathcal{O}(x)\mathcal{O}(0))|0\rangle = \frac{iA_{d_{\mathcal{U}}}}{2\sin(d_{\mathcal{U}}\pi)}\frac{1}{(-P^2-i\epsilon)^{2-d_{\mathcal{U}}}}. \tag{8}$$

The above propagator factor has been obtained independently by Georgi [3] and Cheung *et al.* [4].

II. DIS PROCESSES AND UNPARTICLE

Since the effective interaction between the unparticle fields and the SM fermions in Eq. (3) is flavor conserving, which is consistent with the suppressed flavor changing neutral current (FCNC) transitions, the unparticle will only affect the neutral current (γ and Z) exchange DIS processes $\ell(\nu)N \to \ell(\nu)X$, which is shown in Fig. 1. For the charged lepton scattering $\ell N \to \ell X$, the differential scattering cross section is

$$\frac{d^2\sigma^\ell}{dxdQ^2} = \frac{4\pi\alpha^2}{xQ^4}\left[xy^2F_1^\ell + (1-y)F_2^\ell + y\left(1-\frac{1}{2}y\right)xF_3^\ell\right], \tag{9}$$

where $F_i^\ell(x, Q^2)$ $(i = 1, 2, 3)$ is the structure functions which measure the structure of the target, and

$$F_1^\ell(x,Q^2) = \sum_q [q(x)+\bar{q}(x)]B_q^\ell(Q^2),$$
$$F_2^\ell(x,Q^2) = \sum_q x[q(x)+\bar{q}(x)]C_q^\ell(Q^2), \tag{10}$$
$$F_3^\ell(x,Q^2) = \sum_q [q(x)-\bar{q}(x)]D_q^\ell(Q^2)$$

with

$$B_q^\ell(Q^2) = \frac{Q_q^2}{2} + \frac{(V_q^2+A_q^2)(V_\ell^2+A_\ell^2)}{32\sin^4\theta_W\cos^4\theta_W}P_Z^2 + \frac{(c_{V\mathcal{U}}^2+c_{A\mathcal{U}}^2)^2}{32\pi^2\alpha^2M_Z^{4(d_{\mathcal{U}}-1)}}P_{\mathcal{U}}^2 - \frac{Q_qV_qV_\ell}{4\sin^2\theta_W\cos^2\theta_W}P_Z - \frac{Q_qc_{V\mathcal{U}}^2}{4\pi\alpha M_Z^{2(d_{\mathcal{U}}-1)}}P_{\mathcal{U}} + \frac{(V_qc_{V\mathcal{U}}-A_qc_{A\mathcal{U}})(V_\ell c_{V\mathcal{U}}-A_\ell c_{A\mathcal{U}})}{16\pi\alpha\sin^2\theta_W\cos^2\theta_WM_Z^{2(d_{\mathcal{U}}-1)}}P_{\mathcal{U}Z} + \frac{(c_{S\mathcal{U}}^2-c_{P\mathcal{U}}^2)^2}{64\pi^2\alpha^2M_Z^{4(d_{\mathcal{U}}-1)}}P_{\mathcal{U}}^2, \tag{11}$$

$$C_q^\ell(Q^2) = Q_q^2 + \frac{(V_q^2+A_q^2)(V_\ell^2+A_\ell^2)}{16\sin^4\theta_W\cos^4\theta_W}P_Z^2 + \frac{(c_{V\mathcal{U}}^2+c_{A\mathcal{U}}^2)^2}{16\pi^2\alpha^2M_Z^{4(d_{\mathcal{U}}-1)}}P_{\mathcal{U}}^2 - \frac{Q_qV_qV_\ell}{2\sin^2\theta_W\cos^2\theta_W}P_Z - \frac{Q_qc_{V\mathcal{U}}^2}{2\pi\alpha M_Z^{2(d_{\mathcal{U}}-1)}}P_{\mathcal{U}} + \frac{(V_qc_{V\mathcal{U}}-A_qc_{A\mathcal{U}})(V_\ell c_{V\mathcal{U}}-A_\ell c_{A\mathcal{U}})}{8\pi\alpha\sin^2\theta_W\cos^2\theta_WM_Z^{2(d_{\mathcal{U}}-1)}}P_{\mathcal{U}Z}, \tag{12}$$

$$D_q^\ell(Q^2) = \frac{V_qV_\ell A_qA_\ell}{4\sin^4\theta_W\cos^4\theta_W}P_Z^2 + \frac{c_{V\mathcal{U}}^2c_{A\mathcal{U}}^2}{4\pi^2\alpha^2M_Z^{4(d_{\mathcal{U}}-1)}}P_{\mathcal{U}}^2 - \frac{Q_qA_qA_\ell}{2\sin^2\theta_W\cos^2\theta_W}P_Z - \frac{Q_qc_{A\mathcal{U}}^2}{2\pi\alpha M_Z^{2(d_{\mathcal{U}}-1)}}P_{\mathcal{U}} + \frac{(V_qc_{A\mathcal{U}}-A_qc_{V\mathcal{U}})(V_\ell c_{A\mathcal{U}}-A_\ell c_{V\mathcal{U}})}{8\pi\alpha\sin^2\theta_W\cos^2\theta_WM_Z^{2(d_{\mathcal{U}}-1)}}P_{\mathcal{U}Z}, \tag{13}$$

$$P_Z = \frac{Q^2}{Q^2 + M_Z^2}, \qquad P_{\mathcal{U}} = \frac{A_{d_{\mathcal{U}}}}{2\sin(d_{\mathcal{U}}\pi)}(Q^2)^{(d_{\mathcal{U}}-1)}, \qquad P_{\mathcal{U}Z} = \frac{A_{d_{\mathcal{U}}}}{2\sin(d_{\mathcal{U}}\pi)}\frac{(Q^2)^{d_{\mathcal{U}}}}{Q^2 + M_Z^2}. \tag{14}$$

In Eqs. (11)–(13), θ_W is the Weinberg angle $\sin^2\theta_W \simeq 0.23$ [6], M_Z is the mass of the Z^0 gauge boson $M_Z \simeq 91.19$ GeV [6], and $V_f = T_{3f} - 2Q_f\sin^2\theta_W$, $A_f = T_{3f}$, where Q_f is the electric charge of the fermion f in the unit of the positron electric charge e. Q^2, x, and y are the standard deep inelastic variables, which are defined by

$$Q^2 = -(k-k')^2 = 2k\cdot k', \qquad x = \frac{Q^2}{2p\cdot(k-k')}, \qquad y = \frac{p\cdot(k-k')}{p\cdot k}. \tag{15}$$

The *Callan-Gross* relation $F_2^\ell(x,Q^2) = 2xF_1^\ell(x,Q^2)$ is violated by the term $\frac{(c_{S\mathcal{U}}^2 - c_{P\mathcal{U}}^2)^2}{64\pi^2\alpha^2 M_Z^{4(d_{\mathcal{U}}-1)}}P_{\mathcal{U}}^2$ in Eq. (11) at the present stage, and it will receive additional small corrections if we include the suppressed effective interactions between the unparticle and the gluon in Eq. (3). If we set $c_{V\mathcal{U}} = c_{A\mathcal{U}} = c_{S\mathcal{U}} = c_{P\mathcal{U}} = 0$, (i.e., if the unparticle stuff does not exist in the nature) the differential cross section Eq. (9) coincides with the well-known results. When $c_{i\mathcal{U}} \neq 0$ ($i = V, A, S, P$), the contributions to the structure functions $F_i^\ell(x,Q^2)$ ($i = 1, 2, 3$) due to the unparticle emerge. Since the coupling $c_{V\mathcal{U}}$, $c_{A\mathcal{U}}$, $c_{S\mathcal{U}}$, and $c_{P\mathcal{U}}$ are small, to the lowest nontrivial order, the corrections to the structure functions are the interference terms between the spacelike vector unparticle exchange amplitudes and the standard model amplitudes, whereas the leading corrections to $e^+e^- \to \mu^+\mu^-$ are the interference terms between the timelike vector unparticle exchange amplitudes and the standard model amplitudes [3].

For the neutrino scattering $\nu N \to \nu X$, the corresponding differential scattering cross section is

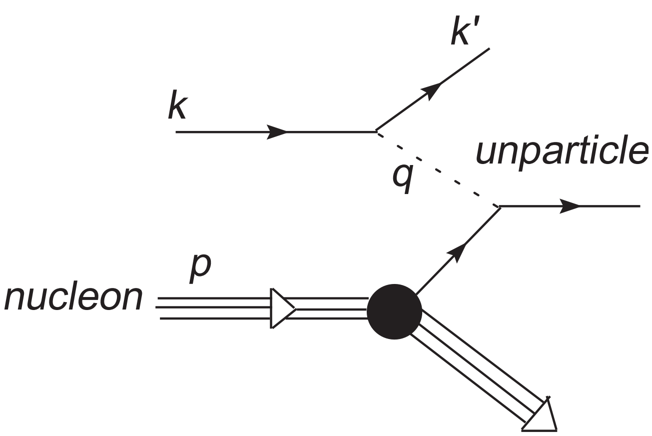

FIG. 1. Deep inelastic scattering with virtual unparticle exchange.

$$\frac{d^2\sigma^\nu}{dxdQ^2} = \frac{G_F^2}{2\pi x}\left(\frac{M_Z^2}{Q^2 + M_Z^2}\right)^2\left[xy^2F_1^\nu + (1-y)F_2^\nu + y\left(1 - \frac{1}{2}y\right)xF_3^\nu\right], \tag{16}$$

the structure functions $F_i^\nu(x,Q^2)$ ($i = 1, 2, 3$) are as follows:

$$\begin{aligned} F_1(x,Q^2) &= \sum_q[q(x) + \bar{q}(x)]B_q^\nu(Q^2), \\ F_2^\nu(x,Q^2) &= \sum_q x[q(x) + \bar{q}(x)]C_q^\nu(Q^2), \\ F_3^\nu(x,Q^2) &= \sum_q[q(x) - \bar{q}(x)]D_q^\nu(Q^2) \end{aligned} \tag{17}$$

with

$$\begin{aligned} B_q^\nu(Q^2) = {}& \frac{V_q^2 + A_q^2}{2} + \frac{(c_{V\mathcal{U}}^2 + c_{A\mathcal{U}}^2)^2}{2M_Z^{4d_{\mathcal{U}}}G_F^2}R_{\mathcal{U}Z}^2 \\ &+ \frac{(V_qc_{V\mathcal{U}} - A_qc_{A\mathcal{U}})(c_{V\mathcal{U}} - c_{A\mathcal{U}})}{\sqrt{2}M_Z^{2d_{\mathcal{U}}}G_F}R_{\mathcal{U}Z} \\ &+ \frac{(c_{S\mathcal{U}}^2 - c_{P\mathcal{U}}^2)^2}{4M_Z^{4d_{\mathcal{U}}}G_F^2}R_{\mathcal{U}Z}^2, \end{aligned} \tag{18}$$

$$\begin{aligned} C_q^\nu(Q^2) = {}& V_q^2 + A_q^2 + \frac{(c_{V\mathcal{U}}^2 + c_{A\mathcal{U}}^2)^2}{M_Z^{4d_{\mathcal{U}}}G_F^2}R_{\mathcal{U}Z}^2 \\ &+ \frac{\sqrt{2}(V_qc_{V\mathcal{U}} - A_qc_{A\mathcal{U}})(c_{V\mathcal{U}} - c_{A\mathcal{U}})}{M_Z^{2d_{\mathcal{U}}}G_F}R_{\mathcal{U}Z}, \end{aligned} \tag{19}$$

$$\begin{aligned} D_q^\nu(Q^2) = {}& 2V_qA_q + \frac{4c_{V\mathcal{U}}^2c_{A\mathcal{U}}^2}{M_Z^{4d_{\mathcal{U}}}G_F^2}R_{\mathcal{U}Z}^2 \\ &- \frac{\sqrt{2}(V_qc_{A\mathcal{U}} - A_qc_{V\mathcal{U}})(c_{V\mathcal{U}} - c_{A\mathcal{U}})}{M_Z^{2d_{\mathcal{U}}}G_F}R_{\mathcal{U}Z}, \end{aligned} \tag{20}$$

$$R_{\mathcal{U}Z} = \frac{A_{d_{\mathcal{U}}}}{2\sin(d_{\mathcal{U}}\pi)}(Q^2)^{d_{\mathcal{U}}-2}(Q^2 + M_Z^2), \tag{21}$$

where G_F is the Fermi constant $G_F \simeq 1.166 \times 10^{-5}$ GeV^2 [6]. If we set $c_{V\mathcal{U}} = c_{A\mathcal{U}} = c_{S\mathcal{U}} = c_{P\mathcal{U}} = 0$, the differential cross section reduces to the well-known result. The *Callan-Gross* relation $F_2^\nu(x,Q^2) = 2xF_1^\nu(x,Q^2)$ is violated by the $\frac{(c_{S\mathcal{U}}^2 - c_{P\mathcal{U}}^2)^2}{4M_Z^{4d_{\mathcal{U}}}G_F^2}R_{\mathcal{U}Z}^2$ term in Eq. (18).

In order to see how the unparticle affects the structure functions, it is instructive to first assume $c_{V\mathcal{U}} = 0$, then there is not interference between the unparticle exchange amplitudes and the photon exchange amplitudes. We can easily see that dominant correction is proportional to $c_{A\mathcal{U}}^2$, here we omit the high order terms which contain $c_{A\mathcal{U}}^4$, $c_{S\mathcal{U}}^4$, and $c^4_{P\mathcal{U}}$, and in this case the changes of the functions $B^\ell_q(Q^2)$, $C^\ell_q(Q^2)$, $D^\ell_q(Q^2)$, $B^\nu_q(Q^2)$ $C^\nu_q(Q^2)$, and $D^\nu_q(Q^2)$ caused by unparticle exchanges are, respectively, the following:

$$\begin{aligned}
2\Delta B^\ell_q(Q^2)/c^2_{A\mathcal{U}} &\approx \Delta C^\ell_q(Q^2)/c^2_{A\mathcal{U}} \approx \frac{A_q A_\ell}{16\pi\alpha M_Z^{2(d_\mathcal{U}-1)}\sin^2\theta_W\cos^2\theta_W}\frac{A_{d_\mathcal{U}}}{\sin(d_\mathcal{U}\pi)}\frac{(Q^2)^{d_\mathcal{U}}}{Q^2+M_Z^2},\\
\Delta D^\ell_q(Q^2)/c^2_{A\mathcal{U}} &\approx -\frac{Q_q}{4\pi\alpha M_Z^{2(d_\mathcal{U}-1)}}\frac{A_{d_\mathcal{U}}}{\sin(d_\mathcal{U}\pi)}(Q^2)^{(d_\mathcal{U}-1)} + \frac{V_q V_\ell}{16\pi\alpha\sin^2\theta_W\cos^2\theta_W M_Z^{2(d_\mathcal{U}-1)}}\frac{A_{d_\mathcal{U}}}{\sin(d_\mathcal{U}\pi)}\frac{(Q^2)^{d_\mathcal{U}}}{Q^2+M_Z^2},\\
2\Delta B^\nu_q(Q^2)/c^2_{A\mathcal{U}} &\approx \Delta C^\nu_q(Q^2)/c^2_{A\mathcal{U}} \approx \frac{A_q}{\sqrt{2}M_Z^{2d_\mathcal{U}}G_F}\frac{A_{d_\mathcal{U}}}{\sin(d_\mathcal{U}\pi)}(Q^2)^{(d_\mathcal{U}-2)}(Q^2+M_Z^2),\\
\Delta D^\nu_q(Q^2)/c^2_{A\mathcal{U}} &\approx \frac{V_q}{A_q}(\Delta C^\nu_q(Q^2)/c^2_{A\mathcal{U}}).
\end{aligned} \tag{22}$$

As an illustration the leading order unparticle corrections $\Delta C^e_u(Q^2)/c^2_{A\mathcal{U}}$, $\Delta D^e_u(Q^2)/c^2_{A\mathcal{U}}$, and $\Delta C^\nu_u(Q^2)/c^2_{A\mathcal{U}}$ for various $d_\mathcal{U}$ are, respectively, shown in Figs. 2–4. Since $\Delta D^\nu_q(Q^2)/c^2_{A\mathcal{U}}$ is proportional to $\Delta C^\nu_q(Q^2)/c^2_{A\mathcal{U}}$, we have not shown the profile of $\Delta D^\nu_q(Q^2)/c^2_{A\mathcal{U}}$. For $1 < d_\mathcal{U} < 2$, $\sin(d_\mathcal{U}\pi)$ is negative, so $\Delta C^\nu_u(Q^2)/c^2_{A\mathcal{U}}$ shown in Fig. 4 is negative, while $\Delta C^e_u(Q^2)/c^2_{A\mathcal{U}}$ and $\Delta D^e_u(Q^2)/c^2_{A\mathcal{U}}$ are positive. We note that $\Delta C^e_u(Q^2)/c^2_{A\mathcal{U}}$ and $\Delta D^e_u(Q^2)/c^2_{A\mathcal{U}}$ increase with the increase of Q^2, however, $|\Delta C^\nu_u(Q^2)/c^2_{A\mathcal{U}}|$ decrease first, then begin to increase at $Q^2 = \frac{2-d_\mathcal{U}}{d_\mathcal{U}-1}M_Z^2$.

After discussion on the pure axial vector unparticle couplings, we now turn to the pure vector coupling case, i.e., $c_{V\mathcal{U}}$ is nonzero and $c_{A\mathcal{U}} = 0$. The leading order corrections $\Delta B^\ell_q(Q^2)/c^2_{V\mathcal{U}}$, $\Delta C^\ell_q(Q^2)/c^2_{V\mathcal{U}}$, $\Delta D^\ell_q(Q^2)/c^2_{V\mathcal{U}}$, $\Delta B^\nu_q(Q^2)/c^2_{V\mathcal{U}}$, $\Delta C^\nu_q(Q^2)/c^2_{V\mathcal{U}}$, and $\Delta D^\nu_q(Q^2)/c^2_{V\mathcal{U}}$ induced by the unparticle in this case are as follows:

$$\begin{aligned}
2\Delta B^\ell_q(Q^2)/c^2_{V\mathcal{U}} &\approx \Delta C^\ell_q(Q^2)/c^2_{V\mathcal{U}}\\
&\approx -\frac{Q_q}{4\pi\alpha M_Z^{2(d_\mathcal{U}-1)}}\frac{A_{d_\mathcal{U}}}{\sin(d_\mathcal{U}\pi)}(Q^2)^{(d_\mathcal{U}-1)} + \frac{V_q V_\ell}{16\pi\alpha\sin^2\theta_W\cos^2\theta_W M_Z^{2(d_\mathcal{U}-1)}}\frac{A_{d_\mathcal{U}}}{\sin(d_\mathcal{U}\pi)}\frac{(Q^2)^{d_\mathcal{U}}}{Q^2+M_Z^2},\\
\Delta D^\ell_q(Q^2)/c^2_{V\mathcal{U}} &\approx \frac{A_q A_\ell}{16\pi\alpha\sin^2\theta_W\cos^2\theta_W M_Z^{2(d_\mathcal{U}-1)}}\frac{A_{d_\mathcal{U}}}{\sin(d_\mathcal{U}\pi)}\frac{(Q^2)^{d_\mathcal{U}}}{Q^2+M_Z^2},\\
2\Delta B^\nu_q(Q^2)/c^2_{V\mathcal{U}} &\approx \Delta C^\nu_q(Q^2)/c^2_{V\mathcal{U}} \approx \frac{V_q}{\sqrt{2}M_Z^{2d_\mathcal{U}}G_F}\frac{A_{d_\mathcal{U}}}{\sin(d_\mathcal{U}\pi)}(Q^2)^{(d_\mathcal{U}-2)}(Q^2+M_Z^2),\\
\Delta D^\nu_q(Q^2)/c^2_{V\mathcal{U}} &\approx \frac{A_q}{V_q}(\Delta C^\nu_q(Q^2)/c^2_{V\mathcal{U}}).
\end{aligned} \tag{23}$$

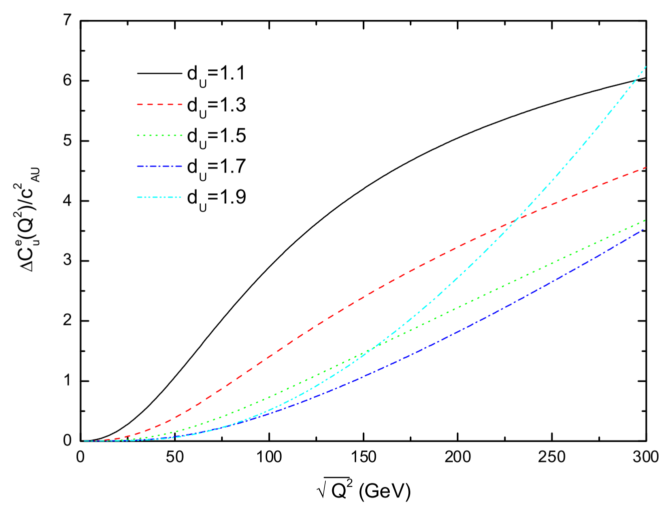

FIG. 2 (color online). The unparticle correction to $C^e_u(Q^2)$ in the unit of $c^2_{A\mathcal{U}}$ for $d_\mathcal{U} = 1.1, 1.3, 1.5, 1.7, 1.9$ in the $c_{V\mathcal{U}} = 0$ and $c_{A\mathcal{U}} \neq 0$ case.

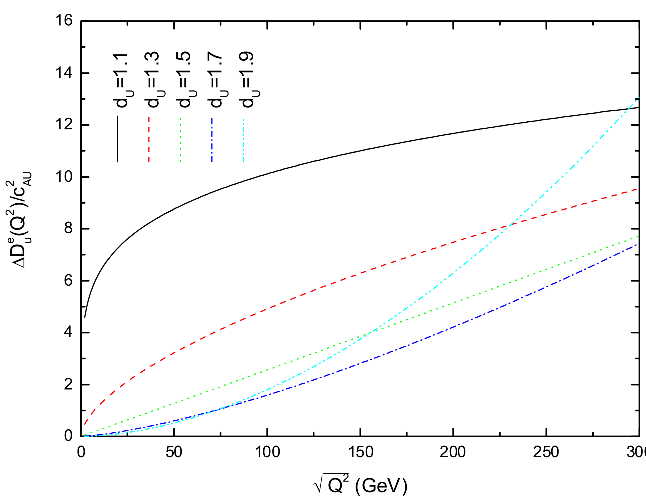

FIG. 3 (color online). The unparticle correction to $D^e_u(Q^2)$ in the unit of $c^2_{A\mathcal{U}}$ for $d_\mathcal{U} = 1.1, 1.3, 1.5, 1.7, 1.9$ in the $c_{V\mathcal{U}} = 0$ and $c_{A\mathcal{U}} \neq 0$ case.

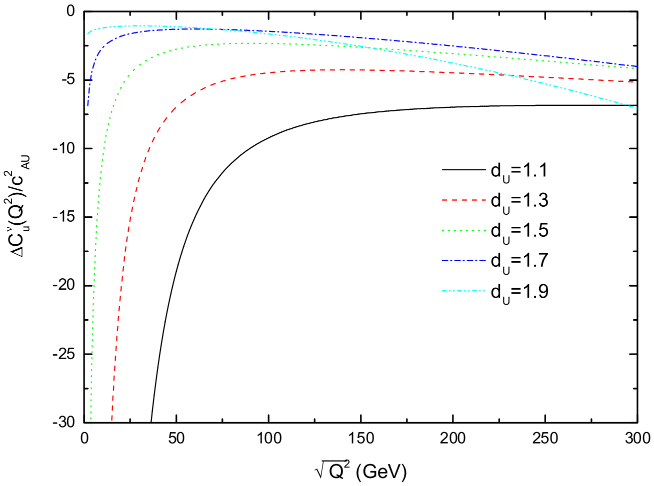

FIG. 4 (color online). The unparticle correction to $C_u^\nu(Q^2)$ in the unit of $c^2_{A\mathcal{U}}$ for $d_\mathcal{U} = 1.1, 1.3, 1.5, 1.7, 1.9$ in the $c_{V\mathcal{U}} = 0$ and $c_{A\mathcal{U}} \neq 0$ case.

From the above equations, we can see that $\Delta B_q^\ell(Q^2)$, $\Delta C_q^\ell(Q^2)$, $\Delta D_q^\ell(Q^2)$, $\Delta B_q^\nu(Q^2)$, $\Delta C_q^\nu(Q^2)$, and $\Delta D_q^\nu(Q^2)$ in the pure vector unparticle coupling case are closely related to those in the pure axial vector coupling case. Thus, we can learn how $\Delta C_u^e(Q^2)/c^2_{V\mathcal{U}}$, $\Delta D_u^e(Q^2)/c^2_{V\mathcal{U}}$, and $\Delta C_u^\nu(Q^2)/c^2_{V\mathcal{U}}$ vary with respect to Q^2 in the pure vector coupling case by making some replacements in Figs. 2–4.

III. ASYMMETRIES IN DEEP INELASTIC POLARIZED ELECTRON-NUCLEON SCATTERING AND UNPARTICLE

There is a strong belief in the physics community that the standard model of particles and interactions is incomplete. Much effort has been paid to search for new physics, and will continue to do so in the upgraded Tevatron, the LHC, and the future Linear Collider (ILC). Direct searches are complemented by precision electroweak experiments that search for the indirect effects of new physics by comparison with expectations calculable in the standard model. Parity violation as an important indirect search for new physics provides precise measurement of electroweak couplings at low Q^2. This measurement is complementary to other existing or planned precision measurement, then yields strong constraints on the possible deviations from the standard model predictions and distinguish various new physics. The Jefferson laboratory has planed to measure precisely the parity violating asymmetry in DIS [7].

The general four fermion Lagrangian takes the following form [8], which is responsible for the new physics contribution to the parity violation DIS asymmetry,

$$\mathcal{L}_{\rm PV} = \frac{4\pi\kappa^2}{\Lambda^2}\sum_{q,i,j} h^q_{ij}\bar{e}_i\gamma_\mu e_i\bar{q}_j\gamma^\mu q_j, \tag{24}$$

where κ is the coupling strength of the new interaction, Λ is the characteristic mass of the new degree of freedom, and the h^q_{ij} are the helicity-dependent coupling parameters of the quark q with i and j denoting the handedness of the given fermion. From the effective interaction Eq. (3) between the unparticle and the SM field, we learn that the virtual exchange of the unparticle can result in the following four fermion interaction relevant to the parity violating DIS asymmetry:

$$\begin{aligned}\mathcal{L}^\mathcal{U}_{\rm PV} = \frac{A_{d_\mathcal{U}}}{2\sin(d_\mathcal{U}\pi)}\frac{1}{Q^2}\left(\frac{Q^2}{M_Z^2}\right)^{d_\mathcal{U}-1}\sum_q[&c^2_{R\mathcal{U}}\bar{e}\gamma_\mu P_R e\bar{q}\gamma^\mu P_R q\\ &+ c_{R\mathcal{U}}c_{L\mathcal{U}}\bar{e}\gamma_\mu P_R e\bar{q}\gamma^\mu P_L q\\ &+ c_{R\mathcal{U}}c_{L\mathcal{U}}\bar{e}\gamma_\mu P_L e\bar{q}\gamma^\mu P_R q\\ &+ c^2_{L\mathcal{U}}\bar{e}\gamma_\mu P_L e\bar{q}\gamma^\mu P_L q],\end{aligned} \tag{25}$$

where P_R and P_L are the usual projection operator $P_{R,L} = \frac{1}{2}(1 \pm \gamma_5)$, $c_{R\mathcal{U}}$ and $c_{L\mathcal{U}}$ are expressed in terms of the vector and axial vector coupling constants $c_{V\mathcal{U}}$, $c_{A\mathcal{U}}$,

$$c_{R\mathcal{U}} = c_{V\mathcal{U}} + c_{A\mathcal{U}}, \qquad c_{L\mathcal{U}} = c_{V\mathcal{U}} - c_{A\mathcal{U}}. \tag{26}$$

For an isosinglet target such as the deuteron, the assumption of isospin symmetry is generally made (i.e., all u and d distributions interchanged for the proton and neutron). At sufficiently high x, the relative importance of sea quark contributions approaches zero. And then the parity asymmetry $A_{ed}(x, y)$ is insensitive to parton distribution functions. To the leading order of Q^2/M_Z^2, after lengthy calculation, we have

$$\begin{aligned}A_{ed}(x, y) = \frac{\sigma_R - \sigma_L}{\sigma_R + \sigma_L} = -&\frac{G_F Q^2}{2\sqrt{2}\pi\alpha}\frac{9}{10}\\ \times\Bigg\{&\left[1 - \frac{20}{9}\sin^2\theta_W + \frac{1}{3}\xi(Q^2)c_{V\mathcal{U}}c_{A\mathcal{U}}\right]\\ &+ \left[1 - 4\sin^2\theta_W + \frac{1}{3}\xi(Q^2)c_{V\mathcal{U}}c_{A\mathcal{U}}\right]\\ &\times\frac{1-(1-y)^2}{1+(1-y)^2}\Bigg\},\end{aligned} \tag{27}$$

where $\xi(Q^2) = \frac{\sqrt{2}A_{d_\mathcal{U}}}{\sin(d_\mathcal{U}\pi)}\frac{1}{Q^2 G_F}(\frac{Q^2}{M_Z^2})^{d_\mathcal{U}-1}$, if we set $c_{V\mathcal{U}} = c_{A\mathcal{U}} = 0$, the above asymmetry factor $A_{ed}(x, y)$ reduces to the well-known results in SM [8]. As an illustration, the predicted asymmetries in deep inelastic electron-deuteron scattering are shown in Fig. 5 for $Q^2 = 10\ \mathrm{GeV}^2$ and with different scale dimensions of the unparticle. From this figure, we can see that the asymmetry A_{ed} is very sensitive to the scale dimension $d_\mathcal{U}$: if $1.5 < d_\mathcal{U} < 2$ we almost cannot distinguish the SM from the new physics with the unparticle, however when $1 < d_\mathcal{U} < 1.5$, the situation is changed drastically, i.e., the differences between A_{ed} due to the unparticle and one of SM becomes

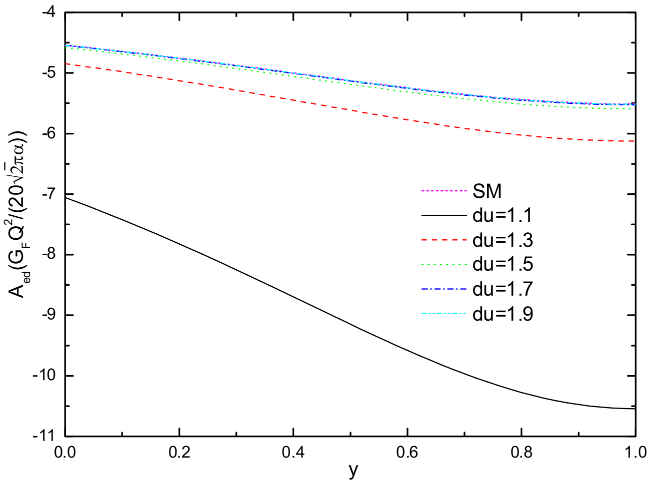

FIG. 5 (color online). The asymmetry $A_{ed} = \frac{\sigma_R - \sigma_L}{\sigma_R + \sigma_L}$ for the deep inelastic polarized electron-deuteron scattering as a function of $y = (E - E')/E$ in SM and the new physics scenario with the unparticle for $d_{\mathcal{U}} = 1.1, 1.3, 1.5, 1.7, 1.9$. The asymmetry is given in the unit of $\frac{G_F Q^2}{20\sqrt{2}\pi\alpha}$, here we assume $Q^2 = 10\ \mathrm{GeV}^2$, $c_{V\mathcal{U}} = c_{A\mathcal{U}} = 0.01$.

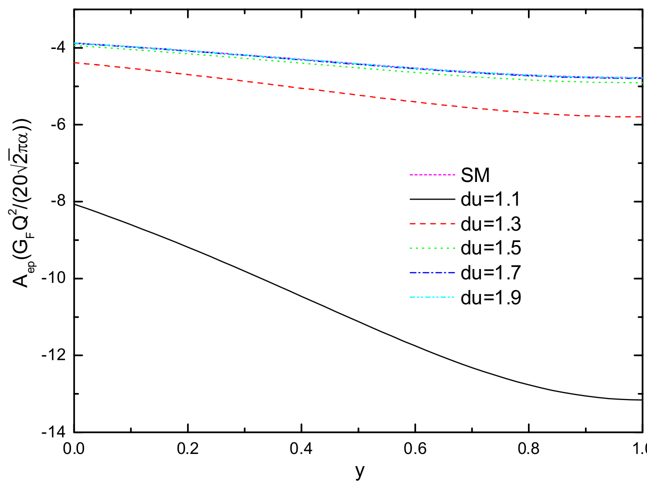

FIG. 6 (color online). The asymmetry $A_{ep}(x = \frac{1}{3}, y) = \frac{\sigma_R - \sigma_L}{\sigma_R + \sigma_L}$ for the deep inelastic polarized electron-proton scattering as a function of $y = (E - E')/E$ in SM and the new physics scenario with the unparticle for $d_{\mathcal{U}} = 1.1, 1.3, 1.5, 1.7, 1.9$. The asymmetry is given in the unit of $\frac{G_F Q^2}{20\sqrt{2}\pi\alpha}$, and we assume $Q^2 = 10\ \mathrm{GeV}^2$, $c_{V\mathcal{U}} = c_{A\mathcal{U}} = 0.01$ as in Fig. 5 for the electron-deuteron case.

rather large. Consequently, by this result, the future precision measurement of A_{ed} would impose strong constrains on the scale dimension $d_{\mathcal{U}}$.

For the sake of completeness we now indicate the result for a proton target, rather than give qualitative results for all x and y, we have chosen $x = \frac{1}{3}$ to give quantitative predictions. Here the "valence" quarks dominate, and furthermore for the proton $u(x = \frac{1}{3}) \approx d(x = \frac{1}{3})$, as in the most naive quark-parton model. Then we find the parity asymmetry $A_{ep}(x = \frac{1}{3}, y)$ in deep inelastic electron-proton scattering,

$$A_{ep}\left(x = \frac{1}{3}, y\right) = \frac{\sigma_R - \sigma_L}{\sigma_R + \sigma_L} = -\frac{G_F Q^2}{2\sqrt{2}\pi\alpha} \times \left\{\left[\frac{5}{6} - 2\sin^2\theta_W + \frac{1}{2}\xi(Q^2)c_{V\mathcal{U}}c_{A\mathcal{U}}\right] + \left[\frac{5}{6}(1 - 4\sin^2\theta_W) + \frac{1}{2}\xi(Q^2)c_{V\mathcal{U}}c_{A\mathcal{U}}\right] \times \frac{1 - (1 - y)^2}{1 + (1 - y)^2}\right\}. \tag{28}$$

The above parity asymmetry $A_{ep}(x = \frac{1}{3}, y)$ becomes the standard result in SM in the case of $c_{V\mathcal{U}} = c_{A\mathcal{U}} = 0$ [8]. The asymmetries predicted in SM and unparticle physics are shown in Fig. 6 for longitudinally polarized electron-proton deep inelastic scattering. Similar to the electron-deuteron case, it is always negative, and the asymmetry $A_{ep}(x = \frac{1}{3}, y)$ in SM almost coincides with that in the unparticle scenario for $1.5 < d_{\mathcal{U}} < 2$, but not as large (in magnitude) as for the electron-deuteron case.

In Figs. 5 and 6, in order to show the $A_{ep}(x = \frac{1}{3}, y)$ curves explicitly and concretely, we have taken $c_{V\mathcal{U}} = c_{A\mathcal{U}} = 0.01$. The values of $c_{V\mathcal{U}}$ and $c_{A\mathcal{U}}$ reflect the strength of the coupling between SM operators and the unparticle's. Since the new physics corrections to SM due to the unparticle must be rather small, we have $|c_{V\mathcal{U}}| \sim |c_{A\mathcal{U}}| < < 1$. When $c_{V\mathcal{U}} = c_{A\mathcal{U}} = 0.01$ is assumed, the corrections to $A_{ep}(x = \frac{1}{3}, y)$ are proportional to the order of $\mathcal{O}(c_{V\mathcal{U}}c_{A\mathcal{U}}) \sim 10^{-4}$.

IV. CONCLUSION

Unparticle stuff with nontrivial scale invariance may exist in our world, and there are a lot of rich phenomenologies associated with the unparticle, such as the unparticle physics effects in fragmentation functions and its contribution to the NuTeV anamoly, etc. [9], which can serve as the experimental tests of the unparticle. In this paper, we have demonstrated the unparticle effects in the neutral current exchange DIS processes $\ell(\nu)N \to \ell(\nu)X$ and the parity violation asymmetry in electron-nucleon deep inelastic scattering in the framework of the effective theory.

The low energy experiment is uniquely sensitive to new physics, which is complementary to direct new physics searches and very useful to distinguish various new physics pictures. If there is really a scale invariant sector, it would manifest itself not only in parity violation asymmetry in electron-nucleon deep inelastic scattering, but also in parity violation Möller scattering, atomic parity violation, parity violation electron-proton scattering, etc. low energy measurements. Parity violating asymmetry in DIS is investigated in this work, together with the precise measurement of other low energy observables (such as the QWeak experiment at JLab), they will yield strong constraints on the unparticle parameter, and will play an important role in distinguishing the unparticle scenario from other extensions of the SM (such as supersymmetry, leptoquarks, and so on).

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