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# Electron-positron annihilation into $\phi f_0(980)$ and clues for a new $1^{--}$ resonance

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We study the $e^+e^- \to \phi\pi\pi$ reaction for pions in an isoscalar $s$ wave which is dominated by loop mechanisms. For kaon loops we start from the conventional $R\chi PT$, but use the unitarized amplitude for $K\bar{K} - \pi\pi$ scattering and the full kaon form factor instead of the lowest order terms. We study also effects of vector mesons using $R\chi PT$ supplemented with the conventional anomalous term for $VVP$ interactions and taking into account the effects of heavy vector mesons in the $K^*K$ transition form factor. We find a peak in $m_{\pi\pi}$ around the $f_0(980)$ as in the experiment. Selecting the $\phi f_0(980)$ contribution as a function of the $e^+e^-$ energy we also reproduce the experimental data except for a narrow peak, yielding support to the existence of a $1^{--}$ resonance above the $\phi f_0(980)$ threshold, coupling strongly to this state.

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## I. INTRODUCTION

The initial state radiation $e^+e^- \to \gamma_{\rm ISR} + \gamma^* \to \gamma_{\rm ISR} + X$ in electron-positron machines is being used to study electron-positron annihilation into hadronic states $X$, scanning energies below the original design in the so-called radiative return method. This method has proved to be useful both in the study of the properties of low-lying resonances in $\phi$ factories [1] as well as in the measurement of the cross section for electron-positron annihilation into different hadronic final states in $B$ factories [2]. In the latter case it is possible to study electron-positron annihilation into hadronic states over the range from 1 up to 5 GeV with a clean identification of the desired final states over the hadronic background. Detailed analysis of some of these processes shows enhancements of the corresponding cross sections whose proper description seems to require the existence of new resonances. Indeed, a broad structure was found in the $e^+e^- \to \gamma_{\rm ISR}J/\psi\pi^+\pi^-$ cross section showing the existence of a resonance with a mass of about 4.26 GeV [3]. More recently, in studying the cross section as a function of the center of mass for $e^+e^- \to \gamma_{\rm ISR}\phi\pi\pi$ with the dipion mass close to the $f_0(980)$, another structure was found around 2.2 GeV indicating the existence of a new resonance with a mass of about 2.175 GeV and a width of 58 MeV [4].

For final pions in a $C$ even state, the leading electromagnetic contributions to the $e^+e^- \to \phi\pi\pi$ process come from the exchange of a virtual photon. The quark lines of the $\phi$ and $\pi\pi$ final states are disconnected thus at tree level the $\gamma^* \to \phi\pi\pi$ can only be induced by sequential decays like $\gamma^* \to \omega\pi\pi \to \phi\pi\pi$ which are suppressed by the small $\omega - \phi$ mixing. We explored this possibility finding this contribution rather small. The natural mechanisms appear at one loop level. In particular for a dipion mass close to the $f_0(980)$ this process involves the $\gamma^*\phi f_0$ vertex function with a photon with a virtuality above 2 GeV. The very same vertex function appears also in one of the mechanisms (dominant in the case of neutral pions) for the radiative decay $\phi \to \pi\pi\gamma$ recently measured in electron-positron $\phi$ factories [5] but there photons are on-shell. The vertex function at $k^2 = 0$, the $\phi f_0\gamma$ coupling, appearing in these decays is an important piece in the elucidation of the structure of the lowest lying scalar nonet.

The $\phi \to \pi\pi\gamma$ decays have been studied in effective models for nonperturbative QCD [6] incorporating scalar degrees of freedom and in unitarized chiral perturbation theory [7] (see also applications to $\phi \to K^0\bar{K}^0\gamma$ in [8]). In both formalisms, the dynamics is dominated by the chain $\phi \to S\gamma \to \pi\pi\gamma$ where the $\phi \to S\gamma$ decay is induced at one loop level through charged kaon loops which couple to the explicit scalar fields in the former case or generate them dynamically through $K\bar{K} - \pi\pi$ rescattering in the latter case. The very same dynamics must be at work in the case of virtual photons and should be the dominant one for low photon virtualities. The calculation of such effects is the subject of this paper.

Unlike the case of the $\phi \to S\gamma$ decay where the real photon tests only the electric charge, here we have a highly virtual photon which couples to higher multipoles and the way to incorporate systematically the effects of kaon loops is to consider the full kaon form factor $F_{K^+}(k^2)$ in the $\gamma K^+K^-$ interaction. Furthermore, although the contribution of neutral kaons vanishes for real photons, in the case of virtual photons the $\gamma^*K^0\bar{K}^0$ coupling is not null and we must consider also neutral kaon loops with the corresponding form factor. The challenge here is the proper characterization of the kaon form factor at the energy of the reaction. Fortunately we have at our disposal both a theoretical calculation of the neutral and charged kaon form factors in $U\chi PT$ [9] and direct measurements [10] in the energy region of interest. In the former case, the kaon form factor is matched with the perturbative QCD predictions at high energy and to $\chi PT$ at low energy and, although the calculated form factor cannot account for the effects of excited vector mesons lying around 1.6 GeV, it is in agreement with the scarce experimental data above 2 GeV. Concerning the $K\bar{K}-\pi\pi$ scattering, it remains in the same energy range as in $\phi\to\pi\pi\gamma$ decays and we can safely use the amplitudes calculated in unitarized chiral perturbation theory which contains naturally the scalar poles.

The high virtuality of the exchanged photon makes probable the excitation of higher mass hadronic states. The quark structure of the $\phi$ suggests that the $K^*K$ intermediate state can also give important contributions to $e^+e^-\to\phi\pi\pi$ via the production of virtual $K^*\bar{K}$, with the virtual $K^*$ decaying into a $\phi K$ and the final rescattering of kaons into pions. In this concern it is worth mentioning that experimental data on $e^+e^-\to K^0K^\pm\pi^\mp$ at $\sqrt{s}=$ 1400–2180 MeV show that this reaction is dominated by intermediate neutral $K^{*0}K^0$ production with the $K^{*0}$ decaying into $K^\pm\pi^\mp$ [11], hence there is a sizable coupling of a virtual photon to the $K^*K$ system at the mentioned energies. The proper description of this mechanism requires the knowledge of the transition $K^*K$ electromagnetic form factor but, again, it can be extracted from experimental data on $e^+e^-\to K^0K^\pm\pi^\mp$ which shows that, in addition to the contributions from the exchange of lowest lying vectors, this form factor receives also contributions from the exchange of $\phi'$ and $\rho'$. Remarkably there is no evidence for contributions coming from the exchange of $\omega'$ to this form factor.

In this paper we study the above mentioned mechanisms for $e^+e^-\to\phi\pi\pi$ for the dipion system in an isoscalar $s$ wave. The paper is organized as follows: In Sec. II we calculate the $\gamma^*\phi\pi\pi$ vertex function using $U\chi PT$. In Sec. III we calculate intermediate vector meson contributions using $U\chi PT$ supplemented with the anomalous term describing $VVP$ interactions and incorporate contributions from heavy mesons to the $K^*K$ transition form factor. In Sec. IV we analyze the different contributions and our summary and conclusions are given in Sec. V.

## II. UNITARIZED $\chi PT$ PREDICTIONS FOR $e^+e^-\to\phi(\pi\pi)_{I=J=0}$

Following [7], the process $e^+e^-\to\phi\pi\pi$ is induced at one loop level by the kaon loops. In the calculations the vertices are borrowed from resonance chiral perturbation theory ($R\chi PT$) [12]. We follow the conventions in [12] and the relevant interactions in their notation are

$$\mathcal{L}=\mathcal{L}^{(2)}+\mathcal{L}^{(F)}+\mathcal{L}^{(G)}, \tag{1}$$

$$\mathcal{L}^{(2)}=\tfrac{1}{4}f^2\,\mathrm{tr}[(D_\mu U)^\dagger D^\mu U+\chi U^\dagger+\chi^\dagger U], \tag{2}$$

$$\mathcal{L}^{(F)}=\frac{F_V}{2\sqrt{2}}\,\mathrm{tr}(V_{\mu\nu}f_+^{\mu\nu}), \tag{3}$$

$$\mathcal{L}^{(G)}=\frac{iG_V}{\sqrt{2}}\,\mathrm{tr}(V_{\mu\nu}u^\mu u^\nu), \tag{4}$$

where

$$\begin{aligned} u_\mu &= iu^\dagger D_\mu U u^\dagger, \qquad U=u^2,\\ u &= e^{-(i/\sqrt{2})(\Phi/f)}, \qquad \Phi=\frac{1}{\sqrt{2}}\lambda_i\varphi_i, \end{aligned} \tag{5}$$

$$\begin{aligned} f_+^{\mu\nu} &= uF_L^{\mu\nu}u^\dagger+u^\dagger F_R^{\mu\nu}u,\\ D_\mu U &= \partial_\mu U-i[v_\mu,U]. \end{aligned} \tag{6}$$

We introduce the photon field through $v_\mu=eQA_\mu$ and $F_L^{\mu\nu}=F_R^{\mu\nu}=eQF^{\mu\nu}$ $(e>0)$ where $F^{\mu\nu}$ denotes the electromagnetic strength tensor. For further details in the notation we refer the reader to Ref. [12]. The relevant diagrams are shown in Fig. 1, where for simplicity a shaded circle and a dark circle account for the diagrams 1(i) plus 1(j) and 1(k) plus 1(l), respectively, which differentiate the direct photon coupling from the coupling through an intermediate vector meson. We will address the corresponding diagrams as 1(a) and 1(b), when we have the direct photon coupling and 1(a′), 1(b′), when the coupling goes through the exchange of a vector meson. The kaon form factor at lowest order contains the exchange of vector mesons in diagrams 1(a′) and 1(b′) which in $R\chi PT$ are intrinsically gauge invariant.

One interesting feature of the use of meson-meson chiral amplitudes is that in the different processes one can factorize the amplitude on-shell inside the loops. This is the case in the construction of the unitary meson-meson amplitudes where the factorization can be seen as a consequence of the reabsorption of the off-shell terms into renormalization of elementary couplings [13], or using the $N/D$ method of unitarization that relies upon the imaginary part of the amplitudes which involves the on-shell part [14]. These two methods have been generalized to the case of meson-baryon interaction in [15,16], respectively. More concretely, for the case close to ours in $\phi\to K^0\bar{K}^0\gamma$ it was demonstrated, using arguments of gauge invariance, that only the on-shell part of the meson-meson amplitudes was needed inside the loops [8]. Explicit cancellation of the off-shell terms can be seen in our formalism and we only sketch the derivation since there are basic principles that tell us this factorization should always be possible. The reason is that the off-shell part of the meson-meson amplitude is unphysical and can be changed with a unitary transformation of the fields, that, however, should not change the physical amplitudes. Technically the cancellations in our formalism go as follows. As discussed in [13,17], to lowest order in the chiral expansion the $K\bar{K}-\pi\pi$ amplitude (denoted by $\tilde{V}^0_{K\pi}$) for arbitrary values of the particle momenta $p_i$ has the form

$$\tilde{V}^0_{K\pi}=V^0_{K\pi}+\beta\sum_i(p_i^2-m_i^2), \tag{7}$$

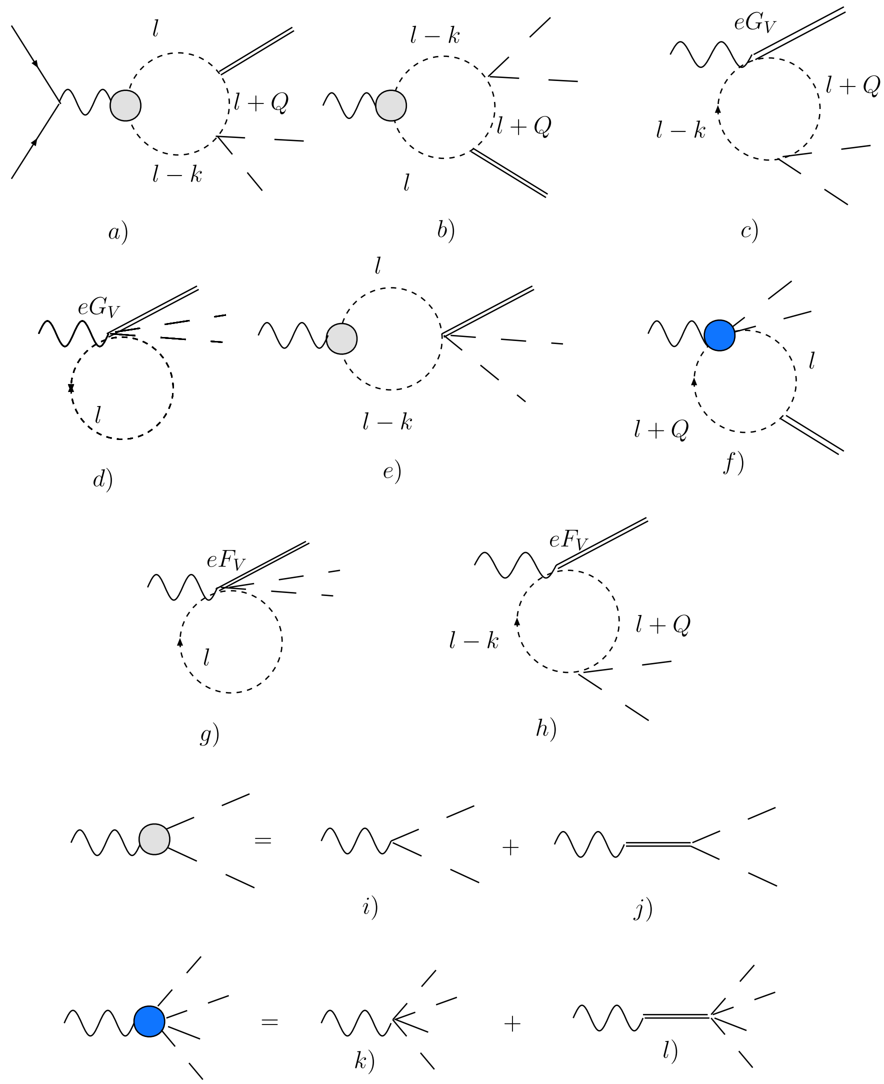


FIG. 1 (color online). Feynman diagrams for $e^+e^- \to \phi\pi\pi$ in $R\chi PT$.

where $V^0_{K\pi}$ denotes the on-shell amplitude. In the following we use the convention that all external particle momenta of the $\gamma^*(k)\phi(Q)\pi(p)\pi(p')$ vertex function flow into the vertices and will change this direction only in the numerical results. Considering the off-shell part of the meson-meson interaction in diagrams 1(a) and 1(b), associated to the line of momentum $l-k$ cancels the corresponding meson propagator and generates a topological structure like the one of diagram 1(f). On the other hand, diagram 1(f) is a genuine diagram that can be calculated by using the Lagrangian $\mathcal{L}^2$ of Eq. (2) expanded to four mesons. When this is done one finds an exact cancellation of the off-shell terms against diagram 1(f). On the other hand, there are similar cancellations between the off-shell part of the meson-meson amplitude associated to the line with momentum $l+Q$ in diagrams 1(a)–1(c) with the genuine contributions in diagrams 1(d) and 1(e). A remnant contribution appears after the cancellations, which vanishes for real photons and involves derivatives in the vector fields. Exact cancellation of this part would require the introduction of counterterm Lagrangians involving derivatives of $V^{\mu\nu}$ and $f_+^{\mu\nu}$, and such Lagrangians are sometimes used for this purpose [18]. Finally the off-shell part of diagram 1(h) which involve charged kaons only cancels exactly diagram 1(g) with charged kaons in the loops. Remaining tadpole contributions from neutral kaons can be canceled by appropriate counterterms. In summary, all one has to do is to evaluate the diagrams 1(a)–1(c) and 1(h) with the meson-meson amplitudes factorized on-shell and omitting the rest of diagrams. This lowest order amplitude is iterated in the coupled channel framework used in [13,17] to obtain the unitarized $K\bar{K} \to \pi\pi$ amplitudes which contain the scalar poles. In a section to come we will study the contributions of loops involving vector meson propagators. In this case we do not have enough information on the higher order Lagrangians to explicitly show the cancellations but we shall equally assume that the meson-meson amplitude can be factorized on-shell out of the loops.

Let us start with the simplest diagrams 1(a) and 1(b) with charged kaons in the loops, pointlike $K^+K^-\gamma$ interaction and charged pions in the final state. A straightforward calculation yields

$$-i\mathcal{M}_{K^+}^{a+b} = -\frac{e^2\sqrt{2}G_V}{f^2}\frac{t^0_{K\pi}}{\sqrt{3}}\frac{L^\mu}{k^2}T_{\mu\nu}Q_\alpha\eta^{\alpha\nu}, \tag{8}$$

where $k^2 = (p^+ + p^-)^2$, $L^\mu \equiv \bar{v}(p^+)\gamma^\mu u(p^-)$, $Q$ denotes the momentum of the $\phi$, and $\eta^{\alpha\nu}$ denotes the polarization tensor of the antisymmetric field $\phi_{\mu\nu}$ used to describe the $\phi$ meson. The on-shell unitarized amplitude for isoscalar $s$-wave $K\bar{K} - \pi\pi$ scattering is denoted as $t^0_{K\pi}$ and it is related to the physical $t_{K^+\pi^+}$ amplitude as $t_{K^+\pi^+} = (t^0_{K\pi}/\sqrt{3})$. It factorizes on-shell out of the loop tensor integral given by

$$T_{\mu\nu} = i\int\frac{d^4l}{(2\pi)^4}\frac{2(2l-k)_\mu l_\nu}{\Box(l+Q)\Box(l)\Box(l-k)}, \tag{9}$$

with $\Box(l) \equiv l^2 - m_K^2 + i\varepsilon$.

The "seagull" diagram 1(c) yields

$$-i\mathcal{M}_{K^+}^{c} = \frac{e^2\sqrt{2}G_V}{f^2}\frac{t^0_{K\pi}}{\sqrt{3}}\frac{L^\mu}{k^2}G_K(m^2_{\pi\pi})g_{\mu\nu}(Q+k)_\alpha\eta^{\alpha\nu}, \tag{10}$$

where $m^2_{\pi\pi} = (Q+k)^2$ and $G_K$ denotes the loop integral

$$G_K(p^2) \equiv \int\frac{d^4l}{(2\pi)^4}\frac{i}{\Box(l)\Box(l+p)}. \tag{11}$$

Using dimensional regularization we get

$$G_K(m^2_{\pi\pi}) = \mu^{2\varepsilon}\int\frac{d^dl}{(2\pi)^d}\frac{i}{\Box(l+Q)\Box(l-k)} = \frac{1}{(4\pi)^2}\left[a(\mu) + \log\frac{m_K^2}{\mu^2} + I_G(m^2_{\pi\pi})\right] \tag{12}$$

with

$$I_G = \int_0^1 dx\log\left(1 - \frac{m^2_{\pi\pi}}{m_K^2}x(1-x) - i\varepsilon\right) = -2 + \sigma\log\frac{\sigma+1}{\sigma-1}, \tag{13}$$

where $\sigma(m^2_{\pi\pi}) = \sqrt{1-(4m_K^2/m^2_{\pi\pi})}$. The substraction constant has been fixed in Ref. [17] to $a(\mu_0) = 1$ for $\mu_0 = 1.2$ GeV matching the cutoff regularized integral for a cutoff $\Lambda = 1$ GeV. It is related at different scales as $a(\mu) = a(\mu_0) + \log\frac{\mu^2}{\mu_0^2}$ in such a way that the loop function is scale independent.

There is no direct coupling of the photon to neutral kaons and adding up all contributions we obtain

$$-i\mathcal{M}_K^{a+b+c} = -\frac{e^2\sqrt{2}G_V}{f^2}\frac{t^0_{K\pi}}{\sqrt{3}}\frac{L^\mu}{k^2}\times[T^{abc}_{\mu\nu}Q_\alpha - G_K(m^2_{\pi\pi})g_{\mu\nu}k_\alpha]\eta^{\alpha\nu}, \tag{14}$$

where

$$T^{abc}_{\mu\nu} = T_{\mu\nu} - G_K(m^2_{\pi\pi})g_{\mu\nu}. \tag{15}$$

Notice that in diagrams 1(a)–1(c) pions appear only through $t^0_{K\pi}$. Since the $K\bar{K} - \pi\pi$ amplitude with neutral pions satisfy $t_{K^+\pi^0} = \frac{t^0_{K\pi}}{\sqrt{3}}$ the amplitude for $e^+e^- \to \phi\pi\pi$ with neutral pions in the final state is also given by Eq. (14).

Let us now consider diagrams 1(a′) and 1(b′) with charged kaons in the loops and charged pions in the final state. These diagrams involve the propagation of vector particles. The propagator for a vector meson in the tensor formalism is given by

$$\Pi_{\alpha\beta\mu\nu}(p) = \frac{i\Delta_{\alpha\beta\mu\nu}(p)}{p^2 - M_V^2 + i\varepsilon}, \tag{16}$$

where

$$\Delta_{\mu\nu\rho\sigma}(p) = \frac{1}{M_V^2}[(p^2 - M_V^2)g_{\mu\rho}g_{\nu\sigma} - g_{\mu\rho}p_\nu p_\sigma + g_{\mu\sigma}p_\nu p_\rho - (\mu \leftrightarrow \nu)]. \tag{17}$$

This tensor is antisymmetric under the exchange $\mu \leftrightarrow \nu$ or $\rho \leftrightarrow \sigma$, symmetric under the exchange $\mu\nu \leftrightarrow \rho\sigma$, and satisfies

$$\begin{aligned} p^\mu\Delta_{\mu\nu\rho\sigma}(p) &= g_{\nu\rho}p_\sigma - g_{\nu\sigma}p_\rho, \\ \Delta_{\mu\nu\rho\sigma}(p)p^\sigma &= g_{\nu\rho}p_\mu - g_{\mu\sigma}p_\nu. \end{aligned} \tag{18}$$

The Lagrangian in Eq. (1) yields the following vertices for the $\gamma(k,\mu)V(k,\alpha\beta)$ and $V(Q,\alpha\beta)P(p)P'(p')$ interactions

$$\begin{aligned} \Gamma^{\gamma V}_{\mu\alpha\beta} &= \frac{eF_V}{3}k_\alpha g_{\mu\beta}C_V, \\ \Gamma^{VPP'}_{\alpha\beta} &= -\frac{\sqrt{2}G_V C_{VPP'}}{f^2}p_\alpha p'_\beta, \end{aligned} \tag{19}$$

with the $SU(3)$ factors given by

$$C_\phi = -\sqrt{2}, \qquad C_\omega = 1, \qquad C_\rho = 3; \tag{20}$$

$$C_{\phi K^+K^-} = C_{\phi K^0\bar{K}^0} = 1, \qquad C_{\omega K^+K^-} = C_{\omega K^0\bar{K}^0} = -\frac{1}{\sqrt{2}}, \qquad C_{\rho K^+K^-} = -\frac{1}{\sqrt{2}}, \qquad C_{\rho K^0\bar{K}^0} = \frac{1}{\sqrt{2}}. \tag{21}$$

The amplitude for diagrams 1(a′) and 1(b′) with charged kaons in the loops is

$$-i\mathcal{M}_{K^+}^{a'+b'} = \frac{\sqrt{2}e^2G_V}{f^2}\frac{L^\mu}{k^2}\frac{t^0_{K\pi}}{\sqrt{3}}\tilde{F}_{K^+}(k^2)T^{a'b'}_{\mu\alpha\nu}\eta^{\alpha\nu} \tag{22}$$

where $\tilde{F}_{K^+}(k^2)$ stands for the vector meson contributions to the charged kaon form factor

$$\tilde{F}_{K^+}(k^2) = \frac{1}{2}\sum_{V=\rho,\phi,\omega}\frac{F_V}{3}\frac{\sqrt{2}G_V C_V C_{VK^+K^-}}{f^2}\frac{k^2}{k^2 - M_V^2} = \frac{G_V F_V}{2f^2}\left(\frac{k^2}{m_\rho^2 - k^2} + \frac{1}{3}\frac{k^2}{m_\omega^2 - k^2} + \frac{2}{3}\frac{k^2}{m_\phi^2 - k^2}\right). \quad (23)$$

and the loop tensor integral is given by

$$T^{a'b'}_{\mu\alpha\nu} = \frac{1}{k^2}k^\sigma \Delta_{\sigma\mu\gamma\delta}(k) i \int \frac{d^4 l}{(2\pi)^4} \times \frac{4(l-k)^\gamma l^\delta l_\alpha (l+Q)_\nu}{\Box_K(l)\Box_K(l+Q)\Box_K(l-k)}. \quad (24)$$

This is an explicitly gauge invariant tensor due to the antisymmetry of $\Delta_{\sigma\mu\gamma\delta}(k)$ under $\sigma \leftrightarrow \mu$. Using $\Delta_{\sigma\mu\gamma\delta}(k)k^\gamma k^\delta = 0$ and $\eta^{\alpha\nu} = -\eta^{\nu\alpha}$ it can be rewritten to

$$T^{a'b'}_{\mu\alpha\nu} = -\left[T^{abc}_{\mu\nu} + \frac{G_K(m^2_{\pi\pi})}{k^2}(k^2 g_{\mu\nu} - k_\mu k_\nu)\right]Q_\alpha. \quad (25)$$

The amplitude for diagrams 1(a′) and 1(b′) can in turn be rewritten as

$$-i\mathcal{M}^{a'+b'}_{K^+} = \frac{-e^2\sqrt{2}G_V}{f^2}\frac{t^0_{K\pi}}{\sqrt{3}}\frac{L^\mu}{k^2}\tilde{F}_{K^+}(k^2) \times \left[T^{abc}_{\mu\nu} + \frac{G_K(m^2_{\pi\pi})}{k^2}(k^2 g_{\mu\nu} - k_\mu k_\nu)\right] \times Q_\alpha \eta^{\alpha\nu}. \quad (26)$$

There are also contributions of neutral kaons in the loops. The calculation of these contributions is similar to the charged kaon loops due to the related $SU(3)$ factors in Eq. (21). The only difference comes from the sign of the $\rho$ factors in Eq. (21) which changes from the charged to the neutral case. The total amplitude is obtained from Eq. (26) just replacing $\tilde{F}_{K^+}$ by $\tilde{F}_{K^+} + \tilde{F}_{K^0}$ where the intermediate $\rho$ contributions cancel. Including neutral and charged kaon contribution we obtain

$$-i\mathcal{M}^{a'+b'}_{K} = \frac{-e^2\sqrt{2}G_V}{f^2}\frac{t^0_{K\pi}}{\sqrt{3}}\frac{L^\mu}{k^2}\tilde{F}_{\rm iso}(k^2) \times \left[T^{abc}_{\mu\nu} + \frac{G_K(m^2_{\pi\pi})}{k^2}(k^2 g_{\mu\nu} - k_\mu k_\nu)\right] \times Q_\alpha \eta^{\alpha\nu}, \quad (27)$$

with

$$\tilde{F}_{\rm iso}(k^2) = \tilde{F}_{K^+}(k^2) + \tilde{F}_{K^0}(k^2) = \frac{F_V G_V}{3f^2}\left(\frac{k^2}{m_\omega^2 - k^2} + \frac{2k^2}{m_\phi^2 - k^2}\right). \quad (28)$$

For neutral pions in the final state we obtain the same result due to relations $t_{K^+\pi^0} = (t^0_{K\pi}/\sqrt{3})$ and $t_{K^0\pi^0} = (t^0_{K\pi}/\sqrt{3})$.

The calculation of diagram 1(h) requires that we work out the $\gamma(k,\mu)\phi(Q,\alpha\nu)K(p)\bar{K}(p')$ vertex contained in $\mathcal{L}^F$ in Eq. (3). For neutral kaons this vertex vanishes and for charged kaons we obtain

$$\Gamma_{\mu\alpha\nu} = \frac{eF_V}{\sqrt{2}f^2}g_{\mu\nu}k_\alpha. \quad (29)$$

The amplitude for diagram 1(h) is

$$-i\mathcal{M}^h = -\frac{e^2 F_V}{\sqrt{2}f^2}\frac{L^\mu}{k^2}\frac{t^0_{K\pi}}{\sqrt{3}}G_K(m^2_{\pi\pi})g_{\mu\nu}k_\alpha\eta^{\alpha\nu}. \quad (30)$$

Adding up contributions of all diagrams in Eqs. (14), (27), and (30) we obtain the kaon loop contributions for both final pion charge states as

$$-i\mathcal{M}_K = -\frac{e^2\sqrt{2}G_V}{f^2}\frac{t^0_{K\pi}}{\sqrt{3}}\frac{L^\mu}{k^2}\left[F^0_{VMD}(k^2)T^{abc}_{\mu\nu} + \tilde{F}_{\rm iso}(k^2) \times \frac{G_K(m^2_{\pi\pi})}{k^2}(k^2 g_{\mu\nu} - k_\mu k_\nu)\right]Q_\alpha\eta^{\alpha\nu} + \frac{e^2\sqrt{2}}{f^2} \times \left(G_V - \frac{F_V}{2}\right)\frac{t^0_{K\pi}}{\sqrt{3}}\frac{L^\mu}{k^2}G_K(m^2_{\pi\pi})g_{\mu\nu}k_\alpha\eta^{\alpha\nu}, \quad (31)$$

where

$$F^0_{VMD}(k^2) = 1 + \tilde{F}_{\rm iso}(k^2) = 1 + \frac{F_V G_V}{3f^2}\left(\frac{k^2}{m_\omega^2 - k^2} + \frac{2k^2}{m_\phi^2 - k^2}\right). \quad (32)$$

accounts for the lowest order terms of the kaon isoscalar form factor (the sum of the charged and neutral kaon form factors) in $R\chi PT$ [9] which is valid at low photon virtualities. Notice that the second term in Eq. (31) contains only the vector meson contributions to the kaon form factor but the constant term due to the electric charge is missing. This term should come from Lagrangians with higher derivatives of the fields (specifically from the term $\partial^\alpha V_{\alpha\nu}\partial_\mu f^{\mu\nu}_+$) which is absent in our basic interactions in Eq. (1). We will assume in the following that the constant term due to the charge is provided by such missing interactions and, hence, write $F^0_{VMD}$ instead of $\tilde{F}_{\rm iso}$ in the second term of Eq. (31).

The high virtualities involved in our process requires that we work out the complete $\gamma K\bar{K}$ vertex functions. The calculation of these vertex functions has been done in the context of $U\chi PT$ in Ref. [9]. We use this result and replace in the following the leading order terms so far obtained, $F^0_{VMD}(k^2)$, by the full isoscalar form factor $F^0_K(k^2) = F_{K^+}(k^2) + F_{K^0}(k^2)$.

The evaluation of Eq. (31) requires that we work out the loop tensor $T^{abc}_{\mu\nu}$. It can be easily shown that $T^{abc}_{\mu\nu}$ is finite and gauge invariant. The most general form of this tensor is

$$T^{abc}_{\mu\nu} = ag_{\mu\nu} + bQ_\mu Q_\nu + cQ_\mu k_\nu + dk_\mu Q_\nu + ek_\mu k_\nu \tag{33}$$

where $a$, $b$, $c$, $d$, $e$ are form factors. Gauge invariance requires

$$k^\mu T^{abc}_{\mu\nu} = (a + ck\cdot Q + ek^2)k_\nu + (bk\cdot Q + dk^2)Q_\nu = 0, \tag{34}$$

imposing the following relations among the form factors

$$a = -ck\cdot Q - ek^2, \qquad bk\cdot Q = -dk^2, \tag{35}$$

thus $T^{abc}_{\mu\nu}$ has the following explicitly gauge invariant form

$$T^{abc}_{\mu\nu} = -c(Q\cdot kg_{\mu\nu} - Q_\mu k_\nu) - \frac{d}{k\cdot Q}(k^2 Q_\mu - k\cdot Qk_\mu) \times Q_\nu - e(k^2 g_{\mu\nu} - k_\mu k_\nu). \tag{36}$$

The second term vanishes upon contraction with $Q_\alpha \eta^{\alpha\nu}$ and we are left only with two form factors

$$T^{abc}_{\mu\nu} = -c(Q\cdot kg_{\mu\nu} - Q_\mu k_\nu) - e(k^2 g_{\mu\nu} - k_\mu k_\nu). \tag{37}$$

A straightforward calculation using conventional Feynman parametrization yields

$$c = -\frac{1}{4\pi^2 m_K^2} I_P, \qquad e = -\frac{1}{4\pi^2 m_K^2} J_P. \tag{38}$$

where

$$I_P = \int_0^1 dx \int_0^x dy \frac{y(1-x)}{1 - \frac{Q^2}{m_K^2}x(1-x) - \frac{2Q\cdot k}{m_K^2}(1-x)y - \frac{k^2}{m_K^2}y(1-y) - i\varepsilon} \tag{39}$$

$$J_P = \frac{1}{2}\int_0^1 dx \int_0^x dy \frac{y(1-2y)}{1 - \frac{Q^2}{m_K^2}x(1-x) - \frac{2Q\cdot k}{m_K^2}(1-x)y - \frac{k^2}{m_K^2}y(1-y) - i\varepsilon}. \tag{40}$$

In terms of the $a$ and $e$ form factors we get the amplitude for kaon loops contribution to $e^+e^- \to \gamma^* \to \phi[\pi\pi]_{I,J=0}$ as

$$-i\mathcal{M}_P = \frac{e^2\sqrt{2}G_V}{f^2}\frac{t^0_{K\pi}}{\sqrt{3}}\frac{L^\mu}{k^2}F^0_K(k^2)\left[c(Q\cdot kg_{\mu\nu} - Q_\mu k_\nu) + \left(e - \frac{G_K}{k^2}\right)(k^2 g_{\mu\nu} - k_\mu k_\nu)\right]Q_\alpha\eta^{\alpha\nu} + \frac{e^2\sqrt{2}}{f^2}\left(G_V - \frac{F_V}{2}\right)\frac{t^0_{K\pi}}{\sqrt{3}}\frac{L^\mu}{k^2}G_K(m^2_{\pi\pi})g_{\mu\nu}k_\alpha\eta^{\alpha\nu}. \tag{41}$$

The vertex function for $\gamma^*(k)\phi(Q,\alpha\nu)[\pi(q)\pi(q')]_{I,J=0}$ is straightforwardly obtained just removing the factor $-\frac{eL^\mu}{k^2}$ and it is worthy to analyze our results in terms of this vertex function. Notice that in addition to the terms associated to the full kaon form factors we get a contact term which survives in the real photon limit and has been already noticed in the studies of radiative $\phi$ decays [7]. The combination $G_V - \frac{F_V}{2}$ is small and it vanishes in the context of vector meson dominance [19]. We will keep this term and discuss below its impact on the cross section but we must be clear from the beginning that it cannot be taken seriously at high photon virtualities without its dressing by a form factor.

Tensor and vector fields are related as $\partial^\mu V_{\mu\nu} = M_V V_\nu$ and for an on-shell $\phi$ it is convenient to rewrite Eq. (41) in terms of the conventional polarization vector related to the polarization tensor as $\eta^{\alpha\nu}(Q) = \frac{i}{M_\phi}(Q^\alpha\eta^\nu - Q^\nu\eta^\alpha)$ in such a way that

$$Q_\alpha\eta^{\alpha\nu}(Q) = iM_\phi\eta^\nu(Q), \qquad g_{\mu\nu}k_\alpha\eta^{\alpha\nu}(Q) = \frac{i}{M_\phi}(Q\cdot kg_{\mu\nu} - Q_\mu k_\nu)\eta^\nu. \tag{42}$$

Using these relations we get

$$-i\mathcal{M}_P = \frac{ie^2\sqrt{2}M_\phi}{f^2}\frac{t^0_{K\pi}}{\sqrt{3}}\frac{L^\mu}{k^2}\left[\left(G_V F^0_K(k^2)c + \left(G_V - \frac{F_V}{2}\right) \times \frac{G_K(m^2_{\pi\pi})}{M^2_\phi}\right)(Q\cdot kg_{\mu\nu} - Q_\mu k_\nu) + G_V F^0_K(k^2) \times \left(e - \frac{G_K}{k^2}\right)(k^2 g_{\mu\nu} - k_\mu k_\nu)\right]\eta^\nu \tag{43}$$

Using now Eqs. (38) we obtain

$$-i\mathcal{M}_P = \frac{-ie^2}{2\pi^2 m_K^2}\frac{t^0_{K\pi}}{\sqrt{3}}\frac{L^\mu}{k^2}[A_P L^{(1)}_{\mu\nu} + B_P L^{(2)}_{\mu\nu}]\eta^\nu \tag{44}$$

with the Lorentz structures

$$L^{(1)}_{\mu\nu} \equiv Q\cdot kg_{\mu\nu} - Q_\mu k_\nu, \qquad L^{(2)}_{\mu\nu} = k^2 g_{\mu\nu} - k_\mu k_\nu, \tag{45}$$

and

$$A_P = \frac{\sqrt{2}M_\phi}{2f^2}\Big[G_V F_K^0(k^2) I_P - \Big(G_V - \frac{F_V}{2}\Big)\frac{m_K^2}{4M_\phi^2} g_K(m_{\pi\pi}^2)\Big], \tag{46}$$

$$B_P = \frac{\sqrt{2}M_\phi G_V}{2f^2} F_K^0(k^2)\Big(J_P + \frac{m_K^2}{4k^2} g_K\Big), \tag{47}$$

where we defined $g_K(p^2) \equiv (4\pi)^2 G_K(p^2)$.

## III. CONTRIBUTIONS FROM VECTORS IN THE LOOPS

The process $e^+(p^+)e^-(p^-) \to \phi(Q, \eta)\pi(p)\pi(p')$ can also proceed through $e^+(p^+)e^-(p^-) \to K^*(p)\bar{K}(p') \to \phi(Q, \eta)K(p)\bar{K}(p')$ with the kaons rescattering to a pion pair as shown in Fig. 2. The $VV'P$ interaction is dictated by the anomalous Lagrangian which we rewrite in terms of the tensor field as

$$\begin{aligned}\mathcal{L}_{\text{anom}} &= \frac{G}{\sqrt{2}}\epsilon_{\mu\nu\alpha\beta}\,\mathrm{tr}(\partial^\mu V^\nu \partial^\alpha V^\beta \Phi) \\ &= \frac{G_T}{4\sqrt{2}}\epsilon_{\mu\nu\alpha\beta}\,\mathrm{tr}(V^{\mu\nu}V^{\alpha\beta}\Phi),\end{aligned} \tag{48}$$

with $G_T = M_V M_{V'} G$. The required vertex for $V(k, \mu, \nu)V'(q, \alpha, \beta)P$ is

$$\Gamma_{\mu\nu\alpha\beta}(k, q) = \frac{iG_T C_{VV'P}}{4\sqrt{2}}\epsilon_{\mu\nu\alpha\beta} \tag{49}$$

with the $SU(3)$ factors given by

$$\begin{aligned}&C_{\phi K^{*+}K^-} = C_{\phi K^{*0}K^0} = 1, \\ &C_{\rho K^{*+}K^-} = -C_{\rho K^{*0}K^0} = C_{\omega K^{*+}K^-} = C_{\omega K^{*0}K^0} = \frac{1}{\sqrt{2}}.\end{aligned} \tag{50}$$

The amplitude from the diagram in Fig. 2 gets contributions from $K^{*+}K^-$ and $K^{*-}K^+$ in the loops plus $K^{*0}\bar{K}^0$ and $\bar{K}^{*0}K^0$. The first two contributions can be summed to

$$-i\mathcal{M}_+ = -2e^2 F^{\text{lo}}_{K^{*+}K^-}(k^2)\frac{G_T}{\sqrt{2}}\Big(\frac{M_{K^*}}{16}\Big)\frac{t^0_{K\pi}}{\sqrt{3}}\frac{L^\mu}{k^2}T_{\mu\alpha\nu}\eta^{\alpha\nu}. \tag{51}$$

Here the $K^*K$ transition form factor is given as

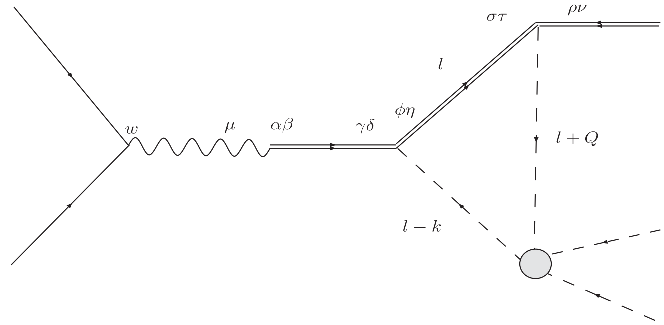


FIG. 2. Feynman diagram for $e^+e^- \to K^*\bar{K} \to \phi K\bar{K} \to \phi\pi\pi$.

$$\begin{aligned}F^{\text{lo}}_{K^*K}(k^2) &= \sum_{V=\rho,\omega,\phi}\frac{G_T C_{VK^{*+}K^-}}{\sqrt{2}}\frac{F_V C_V}{3M_{K^*}}\frac{1}{k^2 - M_V^2} \\ &= \frac{F_V G}{6}\Big(\frac{M_\omega}{k^2 - M_\omega^2} \pm \frac{3M_\rho}{k^2 - M_\rho^2} - \frac{2M_\phi}{k^2 - M_\phi^2}\Big)\end{aligned} \tag{52}$$

where the upper (lower) sign corresponds to the charged (neutral) case. The explicitly gauge invariant tensor $T_{\mu\alpha\nu}$ is given by

$$T_{\mu\alpha\nu} = i\int\frac{d^4 l}{(2\pi)^4}\frac{k_\alpha \Delta^\alpha{}_\mu{}^{\gamma\delta}(k)\epsilon_{\gamma\delta\phi\eta}\Delta^{\phi\eta\sigma\tau}(l)\epsilon_{\sigma\tau\alpha\nu}}{\Box_K(l+Q)\Box_{K^*}(l)\Box_K(l-k)}. \tag{53}$$

The calculation of this tensor, the separation of the effects at the different scales involved in our reaction and the fixing of the necessary substraction constants is rather involved and for the sake of clarity we deferred it to the appendix. We calculate this tensor in the appendix as

$$\begin{aligned}T_{\mu\alpha\nu} = \frac{16}{M_{K^*}^2}\frac{1}{16\pi^2}\Big[\Big(&2 + I_G - I_2 + \frac{1}{2}\log\frac{m_K^2}{\mu^2} \\ &+ \frac{Q\cdot k}{m_K^2}J_V\Big)g_{\mu\nu}k_\alpha - \frac{1}{m_K^2}J_V(k^2 g_{\mu\nu} - k_\mu k_\nu)Q_\alpha\Big],\end{aligned} \tag{54}$$

with

$$J_V = \int_0^1 dx\int_0^x dy\frac{y(1-x)}{1 - \frac{Q^2}{m_K^2}x(1-x) - \frac{2Q\cdot k}{m_K^2}(1-x)y - \frac{k^2}{m_K^2}y(1-y) - \frac{(m_V^2 - m_K^2)}{m_K^2}(y-x) - i\varepsilon} \tag{55}$$

$$I_2 = \int_0^1 dx\int_0^x dy\log\Big[1 - \frac{Q^2}{m_K^2}x(1-x) - \frac{2Q\cdot k}{m_K^2}(1-x)y - \frac{k^2}{m_K^2}y(1-y) - \frac{(m_V^2 - m_K^2)}{m_K^2}(y-x) - i\varepsilon\Big]. \tag{56}$$

Altogether we obtain the amplitude as

$$-i\mathcal{M}_{+} = \frac{-2e^2}{16\pi^2 m_K^2} F^{\mathrm{lo}}_{K^{*+}K^-}(k^2) \frac{G_T}{\sqrt{2}M_{K^*}} \frac{t^0_{K\pi}}{\sqrt{3}} \frac{L^\mu}{k^2} \times [I_V k_\alpha g_{\mu\nu} - J_V(k^2 g_{\mu\nu} - k_\mu k_\nu)Q_\alpha]\eta^{\alpha\nu} \quad (57)$$

with

$$I_V \equiv m_K^2\left(I_G - I_2 + 2 + \frac{1}{2}\log\frac{m_K^2}{\mu^2}\right) + Q\cdot k J_V. \quad (58)$$

Calculations for the amplitude $\mathcal{M}_0$ corresponding to neutral $K^*$ in the loops are quite similar and can be obtained from $\mathcal{M}_+$ just replacing the charged transition form factor by the neutral one due to $t^0_{K^+\pi^+} = t^0_{K^0\pi^+} \equiv (t^0_{K\pi}/\sqrt{3})$. Adding up these amplitudes we get

$$-i\mathcal{M}_V = \frac{-2e^2}{16\pi^2 m_K^2} \tilde{F}^0_{K^*K}(k^2) \frac{G_T}{\sqrt{2}M_{K^*}} \frac{t^0_{K\pi}}{\sqrt{3}} \frac{L^\mu}{k^2} \times [I_V k_\alpha g_{\mu\nu} - J_V(k^2 g_{\mu\nu} - k_\mu k_\nu)Q_\alpha]\eta^{\alpha\nu}, \quad (59)$$

where the isoscalar transition form factor to lowest order is given by

$$\tilde{F}^0_{K^*K}(k^2) = F^{\mathrm{lo}}_{K^{*+}K^-}(k^2) + F^{\mathrm{lo}}_{K^{*+}K^-}(k^2) = \frac{F_V G}{3}\left(\frac{M_\omega}{k^2 - M_\omega^2} - \frac{2M_\phi}{k^2 - M_\phi^2}\right). \quad (60)$$

This amplitude can be written in terms of the conventional polarization vector for an on-shell $\phi$ using Eqs. (42) and (45) and $G_T = M_\phi M_{K^*} G$. We also replace the lowest order terms in Eq. (60) by the full transition form factor to obtain

$$-i\mathcal{M}_V = \frac{-ie^2}{2\pi^2 m_K^2} \frac{t^0_{K\pi}}{\sqrt{3}} \frac{L^\mu}{k^2} [A_V L^{(1)}_{\mu\nu} + B_V L^{(2)}_{\mu\nu}]\eta^\nu \quad (61)$$

with

$$A_V = \frac{G}{4\sqrt{2}} F^0_{K^*K}(k^2) I_V, \qquad B_V = -\frac{G M_\phi^2}{4\sqrt{2}} F^0_{K^*K}(k^2) J_V. \quad (62)$$

This contribution is proportional to the isoscalar transition form factor $F^0_{K^*K}(k^2)$ and, similarly to the kaon form factor in the case of kaon loops, we need a proper description of this form factor at the energy of the reaction, which could be achieved either by a proper unitarization of this form factor or using experimental data if they exist. At the energy region of interest the unitarization of this form factor would reproduce the poles of known vector resonances coupled to the $K^*K$ system. The lowest order result in Eq. (60) already contains the poles corresponding to the lowest lying vectors. The Particle Data Group lists the $\omega(1650)$, $\phi(1680)$, and $\rho(1700)$ resonances in this energy region, which we will call $\omega'$, $\phi'$, $\rho'$ in the following. In this concern it is remarkable that studies of $e^+e^- \to K^0K^\pm\pi^\mp$ at $\sqrt{s} = 1400$–$2180$ MeV show that this reaction is dominated by intermediate neutral $K^{*0}K^0$ production (with a small contribution of the charged channel and negligible light vector meson contributions) in turn coming from intermediate $\phi'$ and $\rho'$ [11]. There is no evidence for $\omega'$ contributions in these reactions. Furthermore, a direct measurements of the kaon form factors in $e^+e^- \to K^+K^-$, $K^0\bar{K}^0$ [10] at $\sqrt{s} = 1400$–$2200$ MeV shows also evidence for contributions of $\phi'$ and $\rho'$ to the kaon form factors (again no signal for $\omega'$ is found here) around 1700 MeV and there is no signal for contributions of higher vector resonances in the charged case. Although the inclusion of such effects improves the description of the kaon form factor around 1700 MeV the values around 2.2 GeV are roughly the same as those of the unitarized charged kaon form factor [9]. Coming back to the $K^*K$ transition form factor, in Ref. [11] the product

$$\Gamma(\phi' \to e^+e^-)\,\mathrm{BR}(\phi' \to K^*K) = 0.39 \pm 0.11 \text{ KeV} \quad (63)$$

is measured, and assuming that $K^*K$ is the dominant channel for the $\phi'$ meson, it allows us to extract the $\phi'\gamma$ coupling which we write as $g_{\phi'\gamma} = (e m^2_{\phi'}/f_{\phi'})$ from

$$\Gamma(\phi' \to e^+e^-) = \frac{4\pi\alpha^2 m_{\phi'}}{3f^2_{\phi'}} = 0.39 \pm 0.11 \text{ KeV}, \quad (64)$$

which yields $f_{\phi'} = 31$. Similarly the $\phi'K^*K$ coupling can be extracted from the total width

$$\Gamma(V \to V'P) = \frac{g^2_{VV'P}}{4\pi}|\mathbf{p}|^3, \quad (65)$$

which for the case at hand ($|\mathbf{p}| = 462$ MeV, $\Gamma = 150$ MeV) and assuming same coupling of the $\phi'$ to $K^{*+}K^-$ and $K^{*0}K^0$ yields $g_{\phi'K^*K} = 2g_{\phi'K^{*0}K^0} = 2g_{\phi'K^{*+}K^-} = 4.37\times 10^{-3}$ MeV$^{-1}$.

Taking into account the $\phi'$ and $\rho'$ contribution introduces a factor

$$\frac{g_{\phi'K^*K}}{2f_{\phi'}}\left(\pm\frac{3}{2}\frac{m^2_{\rho'}}{k^2 - m^2_{\rho'} + im_{\rho'}\Gamma_{\rho'}} - \frac{m^2_{\phi'}}{k^2 - m^2_{\phi'} + im_{\phi'}\Gamma_{\phi'}}\right) \quad (66)$$

in the transition form factor of charged ( + ) and neutral ( − ) $K^*K$ in the loops. Contributions from $\rho'$ cancel in the sum, thus the isoscalar transition form factor is given by

$$F^0_{K^*K}(k^2) = \frac{F_V G}{3}\left(\frac{M_\omega}{k^2 - M_\omega^2 + im_\omega\Gamma_\omega} - \frac{2M_\phi}{k^2 - M_\phi^2 + im_\phi\Gamma_\phi}\right) - \frac{g_{\phi'K^*K}}{f_{\phi'}} \times \left(\frac{m^2_{\phi'}}{k^2 - m^2_{\phi'} + im_{\phi'}\Gamma_{\phi'}}\right). \quad (67)$$

Finally, taking into account both pseudoscalar and vectors in the loops we obtain the total amplitude as

$$-i\mathcal{M} = \frac{-ie^2}{2\pi^2 m_K^2}\frac{t^0_{K\pi}}{\sqrt{3}}\frac{1}{k^2}\bar{v}(p^+)\gamma^\mu u(p^-) \times [AL^{(1)}_{\mu\nu} + BL^{(2)}_{\mu\nu}]\eta^\nu, \tag{68}$$

where

$$A = A_P + A_V, \tag{69}$$

$$B = B_P + B_V, \tag{70}$$

with the specific functions in Eqs. (46), (47), and (62). Recall these results are valid for ingoing particles. For the numerical computations in the following section we reverse the momenta of the final particles and obtain

$$-i\mathcal{M} = \frac{ie^2}{2\pi^2 m_K^2}\frac{t^0_{K\pi}}{\sqrt{3}}\frac{1}{k^2}\bar{v}(p^+)\gamma^\mu u(p^-)[IL^{(1)}_{\mu\nu} - JL^{(2)}_{\mu\nu}]\eta^\nu \tag{71}$$

with

$$I = \frac{\sqrt{2}M_\phi}{2f^2}\left[G_V F^0_K(k^2)I_P - \left(G_V - \frac{F_V}{2}\right)\frac{m_K^2}{4M_\phi^2}g_K(m^2_{\pi\pi})\right] - \frac{G}{4\sqrt{2}}F^0_{K^*K}(k^2)\left[Q\cdot kJ_V - m_K^2\left(I_G - I_2 + 2 + \frac{1}{2}\log\frac{m_K^2}{\mu^2}\right)\right] \tag{72}$$

$$J = \frac{\sqrt{2}M_\phi G_V}{2f^2}F^0_K(k^2)\left(J_P + \frac{m_K^2}{4k^2}g_K\right) - \frac{GM_\phi^2}{4\sqrt{2}}F^0_{K^*K}(k^2)J_V. \tag{73}$$

and for the integrals $I_P$, $J_P$, $J_V$, and $I_2$ we must use Eqs. (39), (40), (55), and (56) just changing the sign of $Q\cdot k$. Also, since our analysis includes an energy region relatively far from the $\phi'$ peak we use in Eq. (67) an $s$-dependent width given by

$$\Gamma_{\phi'}(s) = \frac{g^2_{\phi' K^* K}}{4\pi}\left(\frac{\lambda^{1/2}(s, m^2_{K^*}, m_K^2)}{2\sqrt{s}}\right)^3, \tag{74}$$

with

$$\lambda(m_1^2, m_2^2, m_3^2) = [m_1^2 - (m_2 - m_3)^2][m_1^2 - (m_2 + m_3)^2]. \tag{75}$$

## IV. NUMERICAL RESULTS

The differential cross section for this process is given as

$$\frac{d\sigma}{dm_{\pi\pi}d\Omega_Q} = \frac{1}{(2\pi)^4}\frac{1}{8s^{3/2}}|\mathbf{Q}||\tilde{\mathbf{p}}||\bar{\mathcal{M}}|^2. \tag{76}$$

Here $\mathbf{Q}$ stands for the trimomentum of the $\phi$ in the center of momentum system of the reaction and $\tilde{\mathbf{p}}$ denotes the momentum of the final charged pion in the dipion center of momentum system

$$|\mathbf{Q}| = \frac{\lambda^{1/2}(s, M_\phi^2, m^2_{\pi\pi})}{2\sqrt{s}}, \qquad |\tilde{\mathbf{p}}| = \frac{\lambda^{1/2}(m^2_{\pi\pi}, m_\pi^2, m_\pi^2)}{2m_{\pi\pi}}, \tag{77}$$

where we neglect terms proportional to $m_e^2$. A straightforward calculation yields

$$|\bar{\mathcal{M}}|^2 = \frac{1}{4}\sum_{\text{pol}}|\mathcal{M}|^2 = |C|^2\left[|I|^2\frac{1}{2}(M_\phi^2 + |\mathbf{Q}|^2x^2 + \omega^2) - 2\,\mathrm{Re}(IJ^*) \times\sqrt{s}\omega + |J|^2\frac{s}{2M_\phi^2}(M_\phi^2 - |\mathbf{Q}|^2x^2 + \omega^2)\right] \tag{78}$$

$$= |C|^2\left\{|I|^2\frac{1}{2}[M_\phi^2(1-x^2) + \omega^2(1+x^2)] - 2\,\mathrm{Re}(IJ^*) \times\sqrt{s}\omega + |J|^2\frac{s}{2M_\phi^2}[M_\phi^2(1+x^2) + \omega^2(1-x^2)]\right\} \tag{79}$$

where $x = \cos\theta$ with $\theta$ the $\phi$-beam angle, $\omega$ the $\phi$ energy

$$\omega = \frac{s + M_\phi^2 - m^2_{\pi\pi}}{2\sqrt{s}}, \tag{80}$$

and $C$ stands for the global factor

$$C = \frac{ie^2}{2\pi^2 m_K^2}\frac{t^0_{K\pi}}{\sqrt{3}}. \tag{81}$$

Integrating the solid angle we get

$$\int|\bar{\mathcal{M}}|^2 d\Omega_Q = \frac{4\pi}{3}|C|^2\left[|I|^2(M_\phi^2 + 2\omega^2) - 6\,\mathrm{Re}(IJ^*)\sqrt{s}\omega + |J|^2\frac{s}{M_\phi^2}(2M_\phi^2 + \omega^2)\right]. \tag{82}$$

The dipion spectrum is finally given as

$$\frac{d\sigma}{dm_{\pi\pi}} = \frac{\alpha^2}{24\pi^5 m_K^4}\frac{|\mathbf{Q}||\tilde{\mathbf{p}}|}{s^{3/2}}\frac{|t^0_{K\pi}|^2}{3}h(s, m_{\pi\pi}), \tag{83}$$

where

$$h(s, m_{\pi\pi}) = |I|^2(M_\phi^2 + 2\omega^2) - 6\,\mathrm{Re}(IJ^*)\sqrt{s}\omega + |J|^2\frac{s}{M_\phi^2}(2M_\phi^2 + \omega^2). \tag{84}$$

We evaluate numerically the integrals and the differential cross section. We are interested in dipion energies $m_{\pi\pi}$ close to the $f_0(980)$ mass in whose case the $K\bar{K}\to\pi\pi$ scattering between the kaons in the loops and the final pions takes place at this energy independently of the value of $\sqrt{s}$ and of the momenta in the loops. As a consequence, when replacing the lowest order terms for this amplitude by the unitarized amplitude as proposed in Sec. II, we can safely use the results of [13,17] and take a renormalization scale $\mu = 1.2$ GeV for the function $G_K$ [17], in spite of the fact that the reaction takes place at a much higher energy $\sqrt{s} \geq 2$ GeV. The unitarized amplitudes naturally contain the scalar poles and there is no need to include explicitly these degrees of freedom in the calculation. For the kaon form factor we use the unitarized version calculated in Ref. [9], the values obtained in this analytic form reproduce the direct measurements of the charged kaon form factor at the energy region of interest [10].

Using the physical masses and coupling constants $m_K = 495$, $m_\phi = 1019.4$, $\alpha = 1/137$, $G_V = 53$ MeV, $F_V = 154$ MeV, $f_\pi = 93$ MeV, and $G = 0.016\ \text{MeV}^{-1}$ in Eq. (83) we obtain the spectrum shown in Figs. 3 where the presence of the $f_0(980)$ is well visible. This is a consequence of the fact that the $f_0(980)$ poles are well reproduced in the unitarization of meson-meson $s$-wave isoscalar amplitudes present in our calculation. The $\sqrt{s}$ dependence in the differential cross section is dominated by the phase space factor in the lower energy region (the opening of the $\phi f_0$ channel) and the lowering beyond the $\phi f_0$ threshold is dictated by the form factors.

Next we integrate $m_{\pi\pi}$ from 850 to 1100 MeV following the cuts implemented in [4]. The obtained cross section is shown in Fig. 4 (solid curve) where we also show the experimental points quoted in Ref. [4]. We must remark that all the parameters in Eqs. (72) and (73) have been fixed in advance and in this sense there are no free parameters in our calculations. We should note that in the loops with pseudoscalars there is a term that has no form factor. At low photon virtualities this term is small and its extrapolation to high $k^2$ requires that we dress it with a form factor which does not come from the Lagrangians that we are using. Thus some uncertainty should be accepted at this point. However, we find numerically that the contributions of the loops with pseudoscalars is far smaller than the contributions of the vector meson loops [by themselves 1 order of magnitude smaller close to the $\phi f_0$ threshold) but through interference with vector meson loops they become more relevant]. The effect of the term with no form factor is shown in Fig. 4 where we plotted the cross section as a function of $\sqrt{s}$ in the case when this term is absent (solid line) and dressed with the kaon form factor (dashed line). As we can see, the effect of this term is negligible when dressed with the kaon form factor.

The elaborate theoretical study carried out in this paper, using standard tools to produce the $\phi f_0(980)$ has succeeded in reproducing the bulk of the experimental data as a function of the energy. Yet, the theory, producing reasonable numbers around $\sqrt{s} = 2000$ MeV and beyond 2300 MeV, fails to provide the right strength in the region around 2150 MeV where a peak appears in the data. There is no way, within our theoretical framework, with reasonable changes of the parameters within existing uncertainties, to obtain this peak. As a consequence of it, we are inclined to conclude, following the lines of Ref. [4], that there is a $1^{--}$ meson resonance around 2150 MeV coupling strongly to $\phi f_0(980)$, as also concluded in [4]. In as much as our theoretical results provide a "background" very similar to the one assumed there, our conclusions about the resonance are the same as in [4] and we refrain from repeating the same analysis leading to the properties of the new resonance. Recalling the result from [4], the resonance has a mass of $M_R = 2175$ MeV, a width of $\Gamma = 58$ MeV, and quantum numbers $1^{--}$ as the photon.

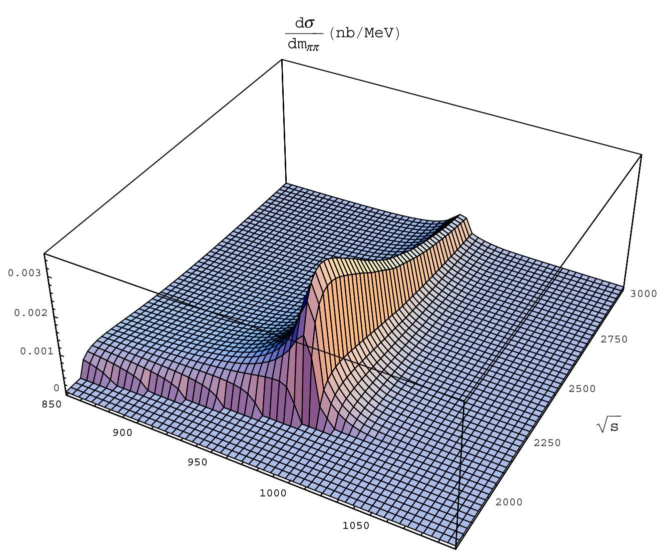


FIG. 3 (color online). Differential cross section as a function of the dipion invariant mass and of the center of mass energy.

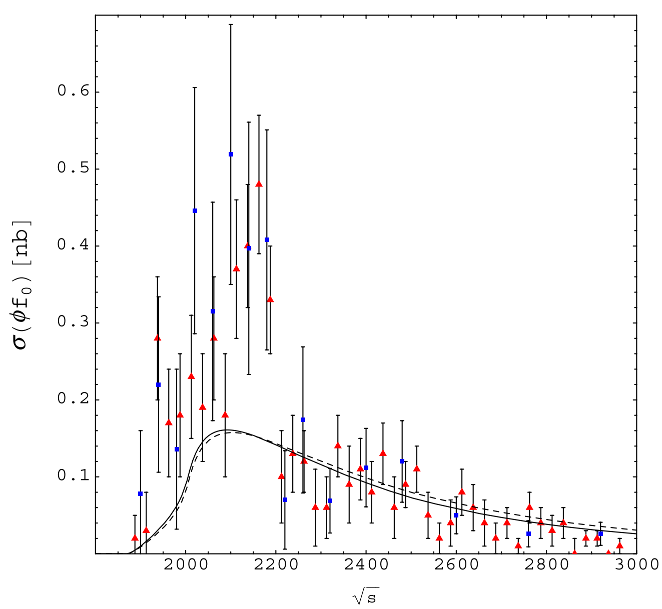


FIG. 4 (color online). Cross section for $e^+e^- \to \phi[\pi\pi]_{I=0}$ integrated in the $m_{\pi\pi} = 850$–$1100$ MeV range as a function of $\sqrt{s}$ including all contributions. Experimental points from Ref. [4], triangled (boxed) points correspond to charged (neutral) pions.

From the theoretical point of view such a resonance is a real challenge since their properties are not predicted by ordinary quark models hinting to a possible exotic character [20].

## V. SUMMARY AND CONCLUSIONS

We studied electron-positron annihilation into $\phi\pi\pi$ for pions in an isoscalar $s$ wave. We find the tree level contributions induced via $\omega - \phi$ mixing negligible. At one loop level, using the vector mesons interactions arising in $R\chi PT$ we show the cancellation of the contributions coming from the off-shell parts of the meson-meson amplitudes in the calculation of the kaon loops. The on-shell parts are iterated to obtain the unitarized meson-meson amplitudes. We obtain contributions proportional to these amplitudes and to the lowest order terms of the kaon form factors. In addition, we find a term with the unitarized meson-meson amplitudes but without the kaon form factors. The effect of the latter is negligible when dressed with the kaon form factor. The photon exchanged in $e^+e^- \to \phi\pi\pi$ is highly virtual and the proper description of this process requires that we use the full kaon form factors. Thus, instead of the lowest order terms arising in the calculation we use the full form factor as calculated in $U\chi PT$ [9].

The high virtuality of the exchanged photon makes the excitation of higher mass states likely. We calculate the excitation of $K^*K$ states with rescattering of kaons into the final pions. This contribution is calculated using $U\chi PT$ supplemented with the anomalous term describing $VVP$ interactions. There are two different energy scales involved in the reaction: $M_\phi$, $m_{\pi\pi} \approx \Lambda$ and $\sqrt{k^2} \gtrsim 2$ GeV and we perform a clear separation of the effects at these scales. It is shown that the only substraction constant required is the one associated to the meson-meson scattering. The formalism naturally yields the contribution from light vector mesons to the $K^*K$ transition form factor. However, the proper description of this form factor at the energy of the reaction requires that we include contributions from heavy mesons, which are extracted from the data on $e^+e^- \to K^0K^\pm\pi^\mp$ at $\sqrt{s} = 1400$–$2180$ MeV [11]. All the parameters entering the calculation have been fixed in advance and there is no freedom in their choice. For the differential cross section we find a peak in $m_{\pi\pi}$ around the $f_0(980)$ as in the experiment [4]. We select the $\phi$ $f_0(980)$ events imposing the cuts used in the analysis of Ref. [4]. The corresponding cross section as a function of the $e^+e^-$ energy describes satisfactorily the experimental data except for a narrow peak around 2150 MeV, yielding support to the existence of a $1^{--}$ resonance above the $\phi f_0(980)$ threshold whose structure started to be debated and seems to be nonconventional [20]. On the other hand, the description of the peaks of $m_{\pi\pi}$ around the $f_0(980)$ resonance, as well as the agreement with data on total cross sections (up to the signal of the new resonance), without the explicit introduction of the $f_0(980)$ state, provides extra support for the $f_0(980)$ as being dynamically generated from the interaction of pseudoscalar mesons in coupled channels.

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## APPENDIX

We use dimensional regularization to calculate the loop tensor in Eq. (53) which in dimension $d = 4 - 2\varepsilon$ reads

$$T_{\mu\alpha\nu} = i\mu^{2\varepsilon}\int\frac{d^dl}{(2\pi)^d} \times \frac{g_{\mu\beta}k_\alpha\Delta^{\alpha\beta\gamma\delta}(k)\epsilon_{\gamma\delta\phi\eta}\Delta^{\phi\eta\sigma\tau}(l)\epsilon_{\sigma\tau\alpha\nu}}{\Box_K(l+Q)\Box_{K^*}(l)\Box_K(l-k)}, \quad \text{(A1)}$$

where $\mu$ stands for the renormalization scale. Using Eq. (18) and the antisymmetry of the Levi-Civita tensors we get

$$g_{\mu\beta}k_\alpha\Delta^{\alpha\beta\gamma\delta}(k)\epsilon_{\gamma\delta\phi\eta} = (g^\gamma_\mu k^\delta - g^\delta_\mu k^\gamma)\epsilon_{\gamma\delta\phi\eta} = 2k^\delta\epsilon_{\mu\delta\phi\eta} \quad \text{(A2)}$$

$$\epsilon_{\mu\delta\phi\eta}\Delta^{\phi\eta\sigma\tau}(l)\epsilon_{\sigma\tau\alpha\nu} = \frac{2}{m^2_{K^*}}[(l^2 - m^2_{K^*})\epsilon_{\mu\delta\phi\eta}\epsilon^{\phi\eta}{}_{\alpha\nu} + 2\epsilon_{\mu\delta\phi\eta}\epsilon_\sigma{}^\phi{}_{\alpha\nu}l^\eta l^\sigma] \quad \text{(A3)}$$

which allows us to split the loop tensor as

$$T_{\mu\alpha\nu} = \frac{4}{m^2_{K^*}}(T^{(1)}_{\mu\alpha\nu} + T^{(2)}_{\mu\alpha\nu}), \quad \text{(A4)}$$

where

$$T^{(1)}_{\mu\alpha\nu} = i\mu^{2\varepsilon}\int\frac{d^dl}{(2\pi)^d}\frac{k^\delta\epsilon_{\mu\delta\phi\eta}\epsilon^{\phi\eta}{}_{\alpha\nu}}{\Box(l+Q)\Box(l-k)} = (d-3)(d-2)(k_\alpha g_{\mu\nu} - k_\nu g_{\mu\alpha})\mu^{2\varepsilon} \times \int\frac{d^dl}{(2\pi)^d}\frac{i}{\Box(l+Q)\Box(l-k)}, \quad \text{(A5)}$$

$$T^{(2)}_{\mu\alpha\nu} = 2i\mu^{2\varepsilon}\int\frac{d^dl}{(2\pi)^d}\frac{k^\delta\epsilon_{\mu\delta\phi\eta}\epsilon_\sigma{}^\phi{}_{\alpha\nu}l^\eta l^\sigma}{\Box_K(l+Q)\Box_{K^*}(l)\Box_K(l-k)}. \quad \text{(A6)}$$

A straightforward calculation yields

$$T^{(1)}_{\mu\alpha\nu} = 4k_\alpha g_{\mu\nu}\left[\frac{3}{16\pi^2} + G_K(m^2_{\pi\pi})\right], \tag{A7}$$

where we used $\eta^{\alpha\nu} = -\eta^{\nu\alpha}$. Notice that we get a constant contribution coming from the contraction of the Levi-Civita tensors in dimension $d$ besides the conventional loop function $G_K$.

The second loop tensor contains two different scales: $M_\phi$, $m_{\pi\pi} \approx \Lambda$ and $\sqrt{k^2} \gtrsim 2$ GeV and we must ensure a clean separation of the effects at these scales and the correct estimate of the corresponding substraction constants. With this aim we perform a decomposition of this tensor in terms of scalar integrals. The tensor integral

$$C^{\eta\sigma} = i\mu^{2\varepsilon}\int \frac{d^d l}{(2\pi)^d}\frac{l^\eta l^\sigma}{\Box_K(l+Q)\Box_{K^*}(l)\Box_K(l-k)} \tag{A8}$$

can be decomposed as

$$C_{\eta\sigma} = C_{00}g_{\eta\sigma} + C_{11}Q_\eta Q_\sigma + C_{12}(Q_\eta k_\sigma + k_\eta Q_\sigma) + C_{22}k_\eta k_\sigma. \tag{A9}$$

We will be interested only in the coefficients of $g_{\eta\sigma}$ and $Q_\eta k_\sigma$ since the remaining terms give vanishing contributions to the process at hand. Contracting with $g^{\eta\sigma}$, $Q^\eta$, and $k^\eta$ we get the following equations for the coefficients:

$$dC_{00} + Q^2C_{11} + 2Q\cdot kC_{12} + k^2C_{22} = G_K(m^2_{\pi\pi}) + M_V^2C_0 \equiv R_{00}, \tag{A10}$$

$$C_{00} + Q^2C_{11} + Q\cdot kC_{12} = \tfrac{1}{2}[\tfrac{1}{2}G_K(m^2_{\pi\pi}) - (Q^2+\Delta^2)C_1] \equiv R_{11} \tag{A11}$$

$$Q^2C_{12} + Q\cdot kC_{22} = \tfrac{1}{2}[V_1(k^2) - \tfrac{1}{2}G_K(m^2_{\pi\pi}) - (Q^2+\Delta^2)C_2] \equiv R_{12} \tag{A12}$$

$$C_{00} + Q\cdot kC_{12} + k^2C_{22} = \tfrac{1}{2}[(k^2+\Delta^2)C_2 + \tfrac{1}{2}G_K(m^2_{\pi\pi})] \equiv R_{22}, \tag{A13}$$

where $\Delta^2 = M_V^2 - m_K^2$, $C_0$ stands for the finite scalar integral

$$C_0 = \mu^{2\varepsilon}\int\frac{d^d l}{(2\pi)^d}\frac{i}{\Box_K(l+Q)\Box_{K^*}(l)\Box_K(l-k)}, \tag{A14}$$

and $C_1$, $C_2$ stand for the coefficients of the decomposition of the vector integral

$$C_\sigma = i\mu^{2\varepsilon}\int\frac{d^d l}{(2\pi)^d}\frac{l_\sigma}{\Box_K(l+Q)\Box_{K^*}(l)\Box_K(l-k)} = C_1Q_\sigma + C_2k_\sigma. \tag{A15}$$

It can be easily shown that $C_1$ and $C_2$ are finite. The functions $V_1$ and $V$ are given by

$$V_1(k^2) = \frac{1}{2}\left\{V(k^2) + \frac{\Delta^2}{k^2}[V(k^2) - V(0)]\right\}, \tag{A16}$$

$$V(k^2) = \mu^{2\varepsilon}\int\frac{d^d l}{(2\pi)^d}\frac{i}{\Box_{K^*}(l)\Box(l-k)}. \tag{A17}$$

The required coefficients read

$$C_{00} = \frac{1}{d-2}(R_{00} - R_{11} - R_{22}), \tag{A18}$$

$$C_{12} = \frac{1}{d-2}\frac{1}{(Q\cdot k)^2 - Q^2k^2}\times[Q\cdot k(-R_{00} + R_{11} + 3R_{22}) - 2k^2R_{12}]. \tag{A19}$$

Explicitly

$$C_{00} = \frac{1}{2(d-2)}[G_K(m^2_{\pi\pi}) + 2M_V^2C_0 + Q^2C_1 - k^2C_2 + \Delta^2(C_1 - C_2)], \tag{A20}$$

$$\begin{aligned}C_{12} = -\frac{1}{4[(Q\cdot k)^2 - Q^2k^2]}\{&2M_V^2Q\cdot kC_0 \\ &+ k^2[V(k^2) - G_K(m^2_{\pi\pi})] + \Delta^2[V(k^2) - V(0)] \\ &+ Q\cdot k(Q^2+\Delta^2)C_1 \\ &- [3Q\cdot k(k^2+\Delta^2) + 2k^2(Q^2+\Delta^2)]C_2\}.\end{aligned} \tag{A21}$$

Notice that the dependence of the integrals on the two different scales ($k^2$ and $Q^2$, $m^2_{\pi\pi}$) involved in the process have been neatly separated. Furthermore, divergences in $V(k^2) - V(0)$ and $V(k^2) - G_K(m^2_{\pi\pi})$ cancel out rendering $C_{12}$ finite as expected. In contrast $C_{00}$ is divergent *but its divergent term appears in* $G_K(m^2_{\pi\pi})$ whose finite part has already been matched to the cutoff regularized integral. As a final result we obtain that effects involving the scale $k^2$ are finite and the only substraction constant required is the one in the loop integral associated to the meson-meson scattering.

Contracting the Levi-Civita tensors (in dimension $d$) we obtain

$$\begin{aligned}T^{(2)}_{\mu\alpha\nu} = &-2[Q\cdot kC_{12} + (d-2)C_{00}](d-3) \\ &\times(g_{\mu\nu}k_\alpha - g_{\mu\alpha}k_\nu) - 2C_{12}(d-3) \\ &\times(k^2g_{\mu\alpha} - k_\mu k_\alpha)Q_\nu \\ &+ 2C_{12}(d-3)(k^2g_{\mu\nu} - k_\mu k_\nu)Q_\alpha.\end{aligned} \tag{A22}$$

The antisymmetry of $\eta^{\alpha\nu}$ allows us to rewrite this tensor as

$$T^{(2)}_{\mu\alpha\nu} = -4[Q\cdot kC_{12} + (d-2)C_{00}](d-3)g_{\mu\nu}k_\alpha + 4C_{12}(d-3)(k^2g_{\mu\nu} - k_\mu k_\nu)Q_\alpha. \tag{A23}$$

For the piece containing the divergent integral $C_{00}$ we obtain

$$-4(d-3)(d-2)C_{00} = -2\Big[\frac{2}{(4\pi)^2} + G_K(m_{\pi\pi}^2) + 2M_V^2 C_0 + Q^2 C_1 - k^2 C_2 + \Delta^2 (C_1 - C_2)\Big]. \tag{A24}$$

The constant term in this equation comes from the dimensional factor $d-3$ which in turn arises from the contraction of the Levi-Civita tensors in dimension $d$.

In the numerical computation it is easier to work with these integrals written in terms of Feynman parameters. In order to calculate $T_{\mu\alpha\nu}^{(2)}$ we use the following Feynman parametrization

$$\frac{1}{\alpha\beta\gamma} = 2\int_0^1 dx \int_0^x dy \frac{1}{[\alpha + (\beta-\alpha)x + (\gamma-\beta)y]^3} \tag{A25}$$

with

$$\alpha = (l+Q)^2 - m_K^2 + i\varepsilon, \qquad \beta = l^2 - m_V^2 + i\varepsilon, \qquad \gamma = (l-k)^2 - m_K^2 + i\varepsilon. \tag{A26}$$

After some algebra we get the term contributing to our process as

$$T_{\mu\alpha\nu}^{(2)} = 4\epsilon_{\mu\delta\phi\eta}\epsilon_\sigma{}^\phi{}_{\alpha\nu} k^\delta \mu^{2\varepsilon} i \int_0^1 dx \int_0^x dy \times \int \frac{d^d r}{(2\pi)^d} \frac{r^\eta r^\sigma - (1-x)y Q^\eta k^\sigma}{[r^2 - \tilde{m}]^3}, \tag{A27}$$

where

$$\tilde{m}^2 = m_K^2 - Q^2 x(1-x) - 2Q\cdot k(1-x)y - k^2 y(1-y) - (m_V^2 - m_K^2)(y-x) - i\varepsilon, \tag{A28}$$

A comparison with

$$T_{\mu\alpha\nu}^{(2)} = 2\epsilon_{\mu\delta\phi\eta}\epsilon_\sigma{}^\phi{}_{\alpha\nu} k^\delta [C_{00} g^{\eta\sigma} + C_{12} Q^\eta k^\sigma] \tag{A29}$$

allows us to identify

$$C_{00} = \frac{2}{d}\mu^{2\varepsilon} i \int_0^1 dx \int_0^x dy \int \frac{d^d r}{(2\pi)^d} \frac{r^2}{[r^2 - \tilde{m}]^3}, \tag{A30}$$

$$C_{12} = -2\mu^{2\varepsilon} i \int_0^1 dx \int_0^x dy \int \frac{d^d r}{(2\pi)^d} \frac{(1-x)y}{[r^2 - \tilde{m}^2]^3}. \tag{A31}$$

The $C_{12}$ integral is finite thus we can set $d=4$ wherever it appears to obtain

$$C_{12} = -\frac{1}{16\pi^2 m_K^2} J_V \tag{A32}$$

with

$$J_V \equiv \int_0^1 dx \int_0^x dy \frac{(1-x)y}{1 - \frac{Q^2}{m_K^2}x(1-x) - \frac{2Q\cdot k}{m_K^2}(1-x)y - \frac{k^2}{m_K^2}y(1-y) - \frac{(m_V^2 - m_K^2)}{m_K^2}(y-x) - i\varepsilon}. \tag{A33}$$

As to the term containing the divergent integral $C_{00}$ we obtain

$$-8(1-\varepsilon)(1-2\varepsilon)C_{00} = -\frac{2}{16\pi^2}\Big[a(\mu) + 3 + \log\frac{m_K^2}{\mu^2} + 2I_2\Big] \tag{A34}$$

with $a(\mu)$ the substraction constant of $G(m_{\pi\pi}^2)$ and

$$I_2 = \int_0^1 dx \int_0^x dy \log\Big[1 - \frac{Q^2}{m_K^2}x(1-x) - \frac{2Q\cdot k}{m_K^2}(1-x)y - \frac{k^2}{m_K^2}y(1-y) - \frac{(m_V^2 - m_K^2)}{m_K^2}(y-x) - i\varepsilon\Big] \tag{A35}$$

Summarizing, the tensors $T_{\mu\nu}^{(1)}$, $T_{\mu\nu}^{(2)}$ are given by

$$T_{\mu\alpha\nu}^{(1)} = \frac{4}{16\pi^2}\Big[4 + \log\frac{m_K^2}{\mu^2} + I_G\Big] k_\alpha g_{\mu\nu} \tag{A36}$$

$$T_{\mu\alpha\nu}^{(2)} = \frac{4}{16\pi^2}\Big\{\Big[\frac{Q\cdot k}{m_K^2}J_V - \frac{1}{2}\Big(4 + \log\frac{m_K^2}{\mu^2}\Big) - I_2\Big] k_\alpha g_{\mu\nu} - \frac{1}{m_K^2} J_V (k^2 g_{\mu\nu} - k_\mu k_\nu) Q_\alpha\Big\}, \tag{A37}$$

thus from Eq. (A4) we get

$$T_{\mu\alpha\nu} = \frac{16}{M_{K^*}^2}\frac{1}{16\pi^2}\Big[\Big(2 + I_G - I_2 + \frac{1}{2}\log\frac{m_K^2}{\mu^2} + \frac{Q\cdot k}{m_K^2}J_V\Big) \times g_{\mu\nu}k_\alpha - \frac{1}{m_K^2}J_V(k^2 g_{\mu\nu} - k_\mu k_\nu)Q_\alpha\Big]. \tag{A38}$$

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