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Updated constraints on new physics in rare charm decays

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Motivated by recent experimental results on charm physics, we investigate implications of the updated constraints of new physics in rare charm meson decays. We first reconsider effects of the minimal supersymmetric standard model (MSSM) in $c \to u\gamma$ constrained by the recent experimental evidence on Δm_D and find that, due to the dominance of long-distance physics, $D \to V\gamma$ decay rates cannot be modified by MSSM contributions. Then we consider effects of the extra heavy up vectorlike quark models on the decay spectrum of $D^+ \to \pi^+\ell^+\ell^-$ and $D_s^+ \to K^+\ell^+\ell^-$ decays. We find a possibility for the tiny increase of the differential decay rate in the region of large dilepton mass. The R-parity violating supersymmetric model can also modify short distance dynamics in $c \to u\ell^+\ell^-$ decays. We constrain relevant parameters using the current upper bound on the $D^+ \to \pi^+\ell^+\ell^-$ decay rate and investigate the impact of that constraint on the $D_s^+ \to K^+\ell^+\ell^-$ differential decay dilepton distribution. Present bounds still allow a small modification of the standard model differential decay rate distribution.

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I. INTRODUCTION

Recently the evidence for $D^0 - \bar{D}^0$ oscillations has been reported by Belle and *BABAR* collaborations [1,2]. Combining the measured quantities [3] indicates nonzero $\Delta\Gamma$ as well as nonzero Δm_D:

$$x = \Delta m_D/\Gamma_D = 0.0087 \pm 0.003. \qquad (1)$$

These results immediately stimulated many studies (see e.g. [4–9]). The obtained results for the relevant parameters describing $D^0 - \bar{D}^0$ mixing are not in favor of new physics effects. However, they give additional constraints on physics beyond the standard model (SM) as already given in papers [5,6]. On the other hand, the study of rare D meson decays is not considered to be very informative in current searches of physics beyond the standard model [10–17], as it is expected from "b" physics. Namely, most of the charm meson processes, where the flavor changing neutral currents (FCNC) effects might be present like $c \to u$ and $c\bar{u} \leftrightarrow \bar{c}u$ transitions, are dominated by the standard model long-distance contributions [10–18].

Because of the Glashow-Iliopoulos-Maiani mechanism and smallness of the down-type quark masses, the radiative $c \to u\gamma$ decay rate is strongly suppressed at the leading order in the SM [10,14]. The QCD effects enhance it up to the order of 10^{-8} [19]. New bounds on the mass insertion parameters within minimal supersymmetric SM (MSSM) [4,5] are derived using the $D^0 - \bar{D}^0$ oscillations. We include into consideration the possible effect of MSSM with nonuniversal soft-breaking terms on $c \to u\gamma$ along the lines of [20,21]. Although this approach leads to the enhancement of the SM value by a factor 10, it is too small to give any observable effects in $D \to V\gamma$ decays (V is a light vector meson). The dominating long-distance (LD) contributions in the $D \to V\gamma$ decays give the branching ratios of the order Br $\sim 10^{-6}$ [10,14], which makes the search for new physics effects impossible.

Another possibility to search for the effects of new physics in the charm sector is offered in the studies of $D \to X\ell^+\ell^-$ decays which might be a result of the $c \to u\ell^+\ell^-$ FCNC transition [7,11,12,15,16,18]. Here X can be light vector meson V or pseudoscalar meson P. Within SM inclusion of renormalization group improved QCD corrections for the inclusive $c \to u\ell^+\ell^-$ gave an additional significant suppression leading to the rates $\Gamma(c \to ue^+e^-)/\Gamma_{D^0} = 2.4 \times 10^{-10}$ and $\Gamma(c \to u\mu^+\mu^-)/\Gamma_{D^0} = 0.5 \times 10^{-10}$ [22]. These transitions are largely driven by a virtual photon at low dilepton mass $m_{\ell\ell} \equiv \sqrt{(p_+ + p_-)^2}$, while the total rate for $D \to X\ell^+\ell^-$ is dominated by the LD resonant contributions at dilepton masses $m_{\ell\ell} = m_\rho$, m_ω, and m_ϕ [11,16].

New physics could possibly modify the dilepton mass distribution below ρ or distribution above ϕ resonance. In the case of $D \to \pi\ell^+\ell^-$, there is a broad kinematical region of dilepton mass above ϕ resonance which presents a unique possibility to study $c \to u\ell^+\ell^-$ at high $m_{\ell\ell}$ [16].

The leading contribution to $c \to u\ell^+\ell^-$ in general MSSM with the conserved R-parity comes from the one-loop diagram with gluino and squarks in the loop [11,16,23]. It proceeds via the virtual photon and enhances the $c \to u\ell^+\ell^-$ spectrum at small $m_{\ell\ell}$. We find that bounds on the mass insertion parameters make the above-mentioned enhancement in D rare decay [11,12] to be negligible.

Some models of new physics contain an extra uplike heavy quark singlet [24] inducing the FCNCs at tree level for the up-quark sector [13,25–28], while the neutral current for the downlike quarks is the same as in the SM. The

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most stringent bound on these models comes from the recent bound on Δm in the $D^0 - \bar{D}^0$ transition as given in [1,2]. In our calculation, we analyze how these bounds on the FCNC vertex cuZ affect the $D \to P\ell^+\ell^-$ decays. A particular version of the model with tree-level up-quark FCNC transitions is the littlest Higgs model [29]. In this case the magnitude of the relevant $c \to uZ$ coupling is even further constrained by the large scale $f \geq \mathcal{O}(1\ \mathrm{TeV})$ using the precision electroweak data. The smallness implies that the effect of this particular model on $c \to u\ell^+\ell^-$ decay and relevant rare D decays is insignificant [13].

Among discussed models of new physics, the supersymmetric extension of the SM including the R-parity violation is still not constrained as other new physics models. As noticed by [11,22], one can test some combinations of the R-parity violating contributions in $D^+ \to \pi^+\ell^+\ell^-$ decays. We place new constraints on the relevant parameters and search for the effects of new physics in the $D_s^+ \to K^+\ell^+\ell^-$ decays which might be interesting for the experimental studies.

There are intensive experimental efforts by CLEO [30,31] and FERMILAB [32,33] collaborations to improve the upper limits on the rates for $D \to X\ell^+\ell^-$ decays. Two events in the channel $D^+ \to \pi^+e^+e^-$ with m_{ee} close to m_ϕ have already been observed by CLEO [30]. The other rare D meson decays are not so easily accessible by experimental searches, but the plans make more experimental studies in rare charm decays at CLEO-c, Tevatron, and at charm physics sections at present B-factories and in the future at LHC-b facilities make the study of rare D decays more attractive.

In order to compare effects of new physics and the standard model, we have to determine the size of the LD contributions. As in the case of $D^+ \to \pi^+e^+e^-$ decay, we use experimental data on the D_s nonleptonic decays accompanied by the vector meson dominance as we did in [13].

The paper is organized as follows. In Sec. II we consider the impact of new charm mixing bounds on the $c \to u\gamma$ decay. In Sec. III we study how new physics affects $c \to u\ell^+\ell^-$ and $D \to P\ell^+\ell^-$ decays. We present the framework for calculating SD effects as well as the details of LD calculations. In Sec. IV we discuss our results and in Sec. V we make a short summary.

II. $c \to u\gamma$ DECAY

Given the recent observation of $D^0 - \bar{D}^0$ mixing, we reevaluate the possible effect of MSSM on $c \to u\gamma$. Since the model with universal soft-breaking terms is known to have a negligible effect [21], we consider the model with nonuniversal soft-breaking terms. We consider only the gluino exchange diagrams through $(\delta^u_{12})_{LR,RL}$ mass insertions, since the remaining SUSY contributions cannot have a sizable effect [20,21]. The maximal value of $(\delta^u_{12})_{LR,RL}$ insertion has been constrained by saturating $x = \Delta m_D/\Gamma = (4.8 \pm 2.8) \times 10^{-3}$ with the gluino exchange in [5]. The results corresponding to the measured $x = (8.7 \pm 3) \times 10^{-3}$ [3,9] are shown in the second column of Table I. Another constraint is obtained by requiring the minima of MSSM scalar potential do not break electric charge or color and that they are bounded from above $(\delta^u_{12})_{LR,RL} \leq \sqrt{3}m_c/m_{\tilde{q}}$ [34], with values given in the third column of Table I. The second constraint is obviously stronger for $m_{\tilde{q}} \geq 350$ GeV, while Δm_D gives a more stringent constraint for lighter squarks. Using $(\delta^u_{12})_{LR,RL} \leq \sqrt{3}m_c/m_{\tilde{q}}$, $m_{\tilde{q}} = m_{\tilde{g}} = 350$ GeV, $m_c = 1.25$ GeV and expressions from [21], we get the upper bound

$$\Gamma(c \to u\gamma)/\Gamma_{D^0} \leq 8 \times 10^{-7}, \tag{2}$$

which is 1 order of magnitude larger than the standard model prediction $\Gamma(c \to u\gamma)/\Gamma_{D^0} = 2.5 \times 10^{-8}$ [19].

However, this possible SUSY enhancement by factor 10 would not affect the rate of the $D \to V\gamma$ decays, which are completely dominated by LD contributions with Br $\sim 10^{-6}$ [10–12,14,18]. The only window for probing the $c \to u\gamma$ enhancement remains the $B_c \to B_u^*\gamma$ decay, where LD contributions are strongly suppressed [35].

III. $c \to u\ell^+\ell^-$ AND $D \to P\ell^+\ell^-$ DECAYS

A. SD Effects

The $c \to u\ell^+\ell^-$ transition is driven by the low-energy effective Lagrangian

$$\mathcal{L}^{\mathrm{SD}}_{\mathrm{eff}} = \frac{G_F}{\sqrt{2}}V^*_{cb}V_{ub}\sum_{i=7,9,10} C_iQ_i, \tag{3}$$

given in terms of four-fermion operators

$$Q_7 = \frac{e}{8\pi^2}m_cF_{\mu\nu}\bar{u}\sigma^{\mu\nu}(1+\gamma_5)c, \tag{4}$$

$$Q_9 = \frac{e^2}{16\pi^2}\bar{u}_L\gamma_\mu c_L\bar{\ell}\gamma^\mu\ell, \tag{5}$$

$$Q_{10} = \frac{e^2}{16\pi^2}\bar{u}_L\gamma_\mu c_L\bar{\ell}\gamma^\mu\gamma_5\ell, \tag{6}$$

and the corresponding Wilson coefficients $C_{7,9,10}$. $F_{\mu\nu}$ is the electromagnetic field strength, while $q_L = \frac{1}{2}(1-\gamma_5)q$ are the left-handed quark fields. Wilson coefficients are taken at the scale $\mu = m_c$.

TABLE I. Upper bounds on mass insertions $|(\delta^u_{12})_{LR,RL}|$ from measured Δm_D and stability bound [34].

$m_{\tilde{q}} = m_{\tilde{g}}$	$\lvert(\delta^u_{12})_{LR,RL}\rvert$ from Δm_D	$\lvert(\delta^u_{12})_{LR,RL}\rvert$ from stability bound
350 GeV	0.007	0.006
500 GeV	0.01	0.004
1000 GeV	0.02	0.002

Since we consider exclusive decay modes $D \to P\ell^+\ell^-$, we have to employ a form factor description of the four-quark operators evaluated between two mesonic states. We use the standard parametrization

$$\langle P(k)|\bar{u}\gamma^\mu(1-\gamma_5)c|D(p)\rangle = (p+k)^\mu f_+(q^2) + (p-k)^\mu f_-(q^2), \quad (7)$$

$$\langle P(k)|\bar{u}\sigma^{\mu\nu}(1\pm\gamma_5)c|D(p)\rangle = is(q^2)[(p+k)^\mu q^\nu - q^\mu(p+k)^\nu \pm i\epsilon^{\mu\nu\alpha\beta}(p+k)_\alpha q_\beta], \quad (8)$$

where $P = \pi^+(K^+)$ in the case of $D = D^+(D_s^+)$. The momentum transfer $q = p - k$ is also the momentum of the lepton pair. For the f_+ form factor we use the double pole parametrization of Ref. [36],

$$f_+(q^2) = \frac{f_+(0)}{(1-x)(1-ax)}, \quad (9)$$

where $x = q^2/m_{D^*}^2$, $f_+(0) = 0.617$, and $a = 0.579$. We approximate $s(q^2)$ by $f_+(q^2)/m_D$, which is valid in the limit of heavy c-quark and zero recoil limit [37]. This relation can be modified as noticed in [38,39]. However, in our case this modification cannot give significant effects. Finally, we arrive at the short distance amplitude for $c \to u\ell^+\ell^-$ decay

$$\mathcal{A}_{\rm SD} = -i\frac{4\pi\alpha G_F}{\sqrt{2}}V_{cb}^*V_{ub}\left[\frac{C_{10}}{16\pi^2}\bar{u}(p_-)\not{p}\gamma_5 v(p_+) + \left(\frac{C_7}{2\pi^2}\frac{m_c}{m_D} + \frac{C_9}{16\pi^2}\right)\bar{u}(p_-)\not{p}v(p_+)\right]f_+(q^2). \quad (10)$$

p, p_+, and p_- are the momenta of the initial D meson and the lepton pair in the final state, respectively.

1. Standard model

The SM rate is dominated by the photon exchange, where $c \to u\gamma$ is a two loop diagram induced by effective weak vertex and a gluon exchange [13,19,22,40]. The effective Wilson coefficient is [13]

$$V_{cb}^*V_{ub}\hat{C}_7^{\rm eff} = V_{cs}^*V_{us}(0.007 + 0.020i)(1 \pm 0.2). \quad (11)$$

The remaining two Wilson coefficients are subdominant in the SM and we ignore them in further analysis. C_9 is small due to the effects of the renormalization group, while the coefficient C_{10} is completely negligible in the SM [13].

2. Models with extra heavy up vectorlike quark singlet

The class of models with an extra uplike quark singlet (EQS) naturally accommodate FCNCs at tree level [13,24],

$$\mathcal{L}_{\rm NC} = \frac{g}{\cos\theta_W}Z_\mu(J^\mu_{W^3} - \sin^2\theta_W J^\mu_{\rm EM}). \quad (12)$$

$J^\mu_{\rm EM}$ is the electromagnetic current, while the weak neutral current,

$$J^\mu_{W^3} = \tfrac{1}{2}\bar{U}_L^m\gamma^\mu\Omega U_L^m - \tfrac{1}{2}\bar{D}_L^m\gamma^\mu D_L^m, \quad (13)$$

mixes up-type quarks [29], where U^m and D^m are the quark mass eigenstates. The transition matrix to the mass eigenbasis for the up-type quarks is 4×4 unitary matrix T_L^U, which causes tree-level FCNCs in the interaction term $J^\mu_{W^3}Z_\mu$ in the up sector. The mixing matrix contains only the elements of the last column of matrix T_L^U:

$$\Omega = \begin{pmatrix} 1-|\Theta_u|^2 & -\Theta_u\Theta_c^* & -\Theta_u\Theta_t^* & -\Theta_u\Theta_T^* \\ -\Theta_c\Theta_u^* & 1-|\Theta_c|^2 & -\Theta_c\Theta_t^* & -\Theta_c\Theta_T^* \\ -\Theta_t\Theta_u^* & -\Theta_t\Theta_c^* & 1-|\Theta_t|^2 & -\Theta_t\Theta_T^* \\ -\Theta_T\Theta_u^* & -\Theta_T\Theta_c^* & -\Theta_T\Theta_t^* & 1-|\Theta_T|^2 \end{pmatrix}. \quad (14)$$

The unitarity of the extended Cabibbo-Kobayashi-Maskawa (CKM) matrix then implies that off-diagonal elements of Ω are nonzero, e.g. $\Omega_{uc} \equiv -\Theta_u\Theta_c^* = V_{ud}V_{cd}^* + V_{us}V_{cs}^* + V_{ub}V_{cb}^* \neq 0$. The low-energy effective description is encoded in Wilson coefficients C_9 and C_{10}. Relative to the negligible SM values, they are modified by the presence of an extra uplike quark:

$$V_{ub}V_{cb}^*\delta C_9 = \frac{4\pi}{\alpha}\Omega_{uc}(4\sin^2\theta_W - 1) \quad (15)$$

$$V_{ub}V_{cb}^*\delta C_{10} = \frac{4\pi}{\alpha}\Omega_{uc}. \quad (16)$$

The element Ω_{uc} of the up-type quark mixing matrix is constrained by the measurements of $D^0 - \bar{D}^0$ mixing [1,2,4] and using expression $\Delta m_D = 2\times10^{-7}|\Omega_{uc}|^2$ GeV [29]:

$$\Omega_{uc} < 2.8\times10^{-4}. \quad (17)$$

3. Minimal supersymmetric SM

The leading contribution to $c \to u\ell^+\ell^-$ in general MSSM with conserved R-parity comes from the gluino exchange diagram via virtual photon and significantly enhances $c \to u\ell^+\ell^-$ at small $m_{\ell\ell}$. This MSSM enhancement cannot be so drastic in hadronic decays, since gauge invariance imposes an additional factor of $m_{\ell\ell}^2$ for $D \to P\ell^+\ell^-$ decays, while $D \to V\ell^+\ell^-$ has a large long-distance contribution at small $m_{\ell\ell}$ just like $D \to V\gamma$.

In the MSSM with broken R-parity (MSSM$\not{R}$), the $c \to u\ell^+\ell^-$ process is mediated by the tree-level exchange of down squarks [11]. Integrating them out leads to the effective four-fermion interaction

$$\mathcal{L}_{\rm eff} = \sum_{i,k=1}^{3}\frac{\tilde{\lambda}'_{i2k}\tilde{\lambda}'_{i1k}}{2M^2_{\tilde{d}_R^k}}(\bar{u}_L\gamma^\mu c_L)(\bar{\ell}_L\gamma_\mu\ell_L). \quad (18)$$

$\tilde{\lambda}'_{ijk}$ are the CKM-rotated couplings between the L, Q, and

D supermultiplets in the superpotential [11]. In our notation (3), the contribution to the Wilson coefficients is [22]

$$V_{cb}^* V_{ub}\delta C_9 = -V_{cb}^* V_{ub}\delta C_{10} = \frac{2\sin\theta_W^2}{\alpha^2}\sum_{k=1}^{3}\left(\frac{m_W}{M_{\tilde{d}_R^k}}\right)^2 \tilde{\lambda}'_{i2k}\tilde{\lambda}'_{i1k}, \qquad (19)$$

where $i = 1(2)$ contributes to the $e^+e^-(\mu^+\mu^-)$ mode. The $\tilde{\lambda}'_{12k}$ and $\tilde{\lambda}'_{11k}$ have been constrained from the charged current universality [22,41], while the strictest constraint on $\sum_k \tilde{\lambda}'_{22k}\tilde{\lambda}'_{21k}$ comes [22] from the experimental limit $\mathrm{Br}(D^+ \to \pi^+\mu^+\mu^-) < 8.8\times 10^{-6}$ [42]. We shall reanalyze the latter case, where LD physics generates the fair amount of experimental branching ratio. It is sensible to use an approach, where one takes into account the interference between LD and MSSM$\not{R}$ part of the amplitude to constrain the couplings of the MSSM$\not{R}$.

B. Long-distance contributions in $D \to P\ell^+\ell^-$

Knowledge of the LD contributions is crucial, if we want to isolate short distance physics in the decays of type $D \to P\ell^+\ell^-$. Following the procedure described in [13], we consider long-distance contributions by employing the resonant decay modes, in which D first decays to P and a virtual neutral vector meson V_0, followed by decay of $V_0 \to \gamma \to \ell^+\ell^-$. The first stage of the decay is controlled by the effective weak nonleptonic Lagrangian

$$\mathcal{L}_{\mathrm{eff}}^{\mathrm{LD}} = -\frac{G_F}{\sqrt{2}}\sum_{q=d,s} V_{uq}V_{cq}^*[a_1\bar{u}\gamma^\mu(1-\gamma_5)q\bar{q}\gamma_\mu(1-\gamma_5)c + a_2\bar{u}\gamma^\mu(1-\gamma_5)c\bar{q}\gamma_\mu(1-\gamma_5)q]. \qquad (20)$$

The effective Wilson coefficients on the scale m_c are [22]

$$a_1 = 1.26, \qquad a_2 = -0.49. \qquad (21)$$

The flavor structure of (20) allows V_0 to be either ρ, ω, or ϕ. Since branching ratios of separate stages in the cascade are well measured, we will not work in a particular theoretical model, but will instead try to make the best use of experimental data currently available. Here we follow the lines of Ref. [13]. For a cascade, we write [43]

$$\frac{d\Gamma}{dq^2}(D_s \to KV_0 \to K\ell^+\ell^-) = \frac{1}{\pi}\Gamma_{D_s\to KV_0}(q^2) \times \frac{\sqrt{q^2}}{(m_{V_0}^2 - q^2)^2 + m_{V_0}^2\Gamma_{V_0}^2} \times \Gamma_{V_0\to\ell^+\ell^-}(q^2). \qquad (22)$$

Here $\Gamma_{D_s\to KV_0}(q^2)$ and $\Gamma_{V_0\to\ell^+\ell^-}$ denote decay rates if V_0 had a mass $\sqrt{q^2}$ and these rates are known experimentally only at $\sqrt{q^2} = m_{V_0}$. Since the resonances $V_0 = \rho, \omega, \phi$ are relatively narrow ($\Gamma_{V_0} \ll m_{V_0}$), the following relation approximately holds:

$$\mathrm{Br}[D \to PV_0 \to P\ell^-\ell^+] = \mathrm{Br}[D \to PV_0] \times \mathrm{Br}[V_0 \to \ell^-\ell^+]. \qquad (23)$$

The phenomenological amplitude ansatz that reproduces the above behavior is then [13]

$$\mathcal{A}^{\mathrm{LD}}[D_s(p) \to K(p-q)V_0(q) \to K(p-q)\ell^-(p_-)\ell^+(p_+)] = e^{i\phi_{V_0}}\frac{a_{V_0}}{q^2 - m_{V_0}^2 + im_{V_0}\Gamma_{V_0}}\bar{u}(p_-)\not{p}v(p_+). \qquad (24)$$

The only assumption we made here is that coefficient a_{V_0} is independent of q^2. We included the phase ϕ_{V_0} explicitly, so that a_{V_0} is real and positive number.

1. $D^+ \to \pi^+\ell^+\ell^-$

For the right side of Eq. (23) we use experimental data (Table II), and use ansatz (24) to extract unknown parameters a_{V_0}: $a_\rho = 2.94\times 10^{-9}$, $a_\phi = 4.31\times 10^{-9}$. Decay mode $D^+ \to \pi^+\omega$ has not been measured yet, but we can relate a_ω and its phase to the well-measured contribution of the ρ resonance assuming vector meson dominance as in [13]. Relative phases and magnitudes of the resonances are extracted by considering the decay mechanism, controlled by the weak Lagrangian (20) and electromagnetic coupling of V_0 to photon. Then the flavor structure of the resonances determines relative sizes and phases of resonant amplitudes. Detailed analysis has already been done in Ref. [13]. The relative phases of ρ and ω contributions are found to be opposite in sign, while for the ratio of the magnitudes it was found that $a_\omega/a_\rho = 1/3$. Also the phases of ρ and ϕ are opposite. Thus, the final LD amplitude (up to the phase) becomes

$$\mathcal{A}^{\mathrm{LD}} = \left[a_\rho\left(\frac{1}{q^2 - m_\rho^2 + im_\rho\Gamma_\rho} - \frac{1}{3}\frac{1}{q^2 - m_\omega^2 + im_\omega\Gamma_\omega}\right) - \frac{a_\phi}{q^2 - m_\phi^2 + im_\phi\Gamma_\phi}\right]\bar{u}(p_-)\not{p}v(p_+). \qquad (25)$$

2. $D_s^+ \to K^+\ell^-\ell^+$

Experimental data is not as rich as in the case of nonstrange charmed meson decays (Table III). Contributions of ρ and ω are related like in the case of the D^+ meson,

TABLE II. Branching ratios of decays of D^+ meson to the intermediate resonant states [44].

Mode	$D^+ \to \pi^+\rho$	$D^+ \to \pi^+\omega$	$D^+ \to \pi^+\phi$
$\mathrm{Br}\times 10^3$	1.07 ± 0.11	<0.34	6.50 ± 0.70

TABLE III. Branching ratios of D_s^+ meson to the intermediate resonant state [44].

Mode	$D_s^+ \to K^+\rho$	$D_s^+ \to K^+\omega$	$D_s^+ \to K^+\phi$
Br $\times 10^3$	$2.60 \pm .70$	$\cdots$	<0.50

namely $a_\omega/a_\rho = 1/3$ with opposite relative phase between them. In the same way as for the nonstrange decays, we determine $a_\rho = 6.97 \times 10^{-9}$. However, the contribution of the ϕ resonance is only limited from above by experimental data and we have to rely on a theoretical model. Consequently, the total LD amplitude is a sum of two terms:

$$\mathcal{A}^{\rm LD} = a_\rho\bigg(\frac{1}{q^2 - m_\rho^2 + im_\rho\Gamma_\rho} - \frac{1}{3}\frac{1}{q^2 - m_\omega^2 + im_\omega\Gamma_\omega}\bigg)\bar{u}(p_-)\not{p}v(p_+) + \mathcal{A}_\phi^{\rm LD}. \tag{26}$$

We calculate the ϕ part of (26) using the vector meson dominance (VMD) assumption, where the intermediate ϕ contributes by decaying into a virtual photon, which further decays to the lepton pair. Both a_1 and a_2 parts of the nonleptonic Lagrangian (20) can generate the flavor quantum numbers of ϕ and K^+. The a_1 part connects the initial D_s^+ state to ϕ through a charged current $(\bar{s}c)_{V-A}$, while the $(\bar{u}s)_{V-A}$ creates the K^+ out of vacuum. Neutral currents (the a_2 part) do the opposite: $D_s^+ \to K^+$ and $0 \to \phi$. Utilizing the Feynman rules of (20), and the VMD hypothesis we arrive at the ϕ contribution to the LD amplitude,

$$\mathcal{A}_\phi^{\rm LD} = i\frac{4\pi\sqrt{2}}{3}G_F V_{us}V_{cs}^*\alpha\frac{g_\phi}{q^2(q^2 - m_\phi^2 + im_\phi\Gamma_\phi)} \times [a_1 m_\phi f_K A_0(m_K^2) + a_2 g_\phi f_+(q^2)]\bar{u}(p_-)\not{p}v(p_+). \tag{27}$$

However, the phase of $\mathcal{A}_\phi^{\rm LD}$ relative to the rest of the amplitude (26) remains to be free. The $P \to V$ transition $D_s^+ \to \phi$ is described by the form factor A_0, which we take from [45]. In our calculations, we consider the gauge invariant amplitude for $D_s^+ \to K^+\ell^+\ell^-$ in which $1/q^2$ dependence is canceled.

IV. RESULTS

Using the approach described in Sec. III, we analyze the impact of short distance physics on long-distance resonant background. Since the SD contribution of SM is completely overshadowed by LD, we will only consider the EQS and MSSM$\not{R}$ models of new physics, to see if the experimental searches for them are still viable in $D \to X\ell^+\ell^-$ decays. Current constraints on the EQS model coming from the $D^0 - \bar{D}^0$ mixing already indicate the dominance of LD contributions in the total decay rate. On the other hand, the contribution of MSSM$\not{R}$ is not as constrained and one should still see the deviations from the LD contribution away from the resonant region of the phase space.

TABLE IV. Total branching fractions of the $D^+ \to \pi^+\ell^+\ell^-$ modes. In the first column (LD) are only long-distance branching ratios (BRs). The remaining four columns give maximal contributions of the SD physics models alone and also combined contributions of the SD and LD physics.

Mode	LD	Extra heavy q	LD + extra heavy q
$D^+ \to \pi^+e^+e^-$	2.0×10^{-6}	1.3×10^{-9}	2.0×10^{-6}
$D^+ \to \pi^+\mu^+\mu^-$	2.0×10^{-6}	1.6×10^{-9}	2.0×10^{-6}
Mode	MSSM$\not{R}$	LD + MSSM$\not{R}$	
$D^+ \to \pi^+e^+e^-$	2.1×10^{-7}	2.3×10^{-6}	
$D^+ \to \pi^+\mu^+\mu^-$	6.5×10^{-6}	8.8×10^{-6}	

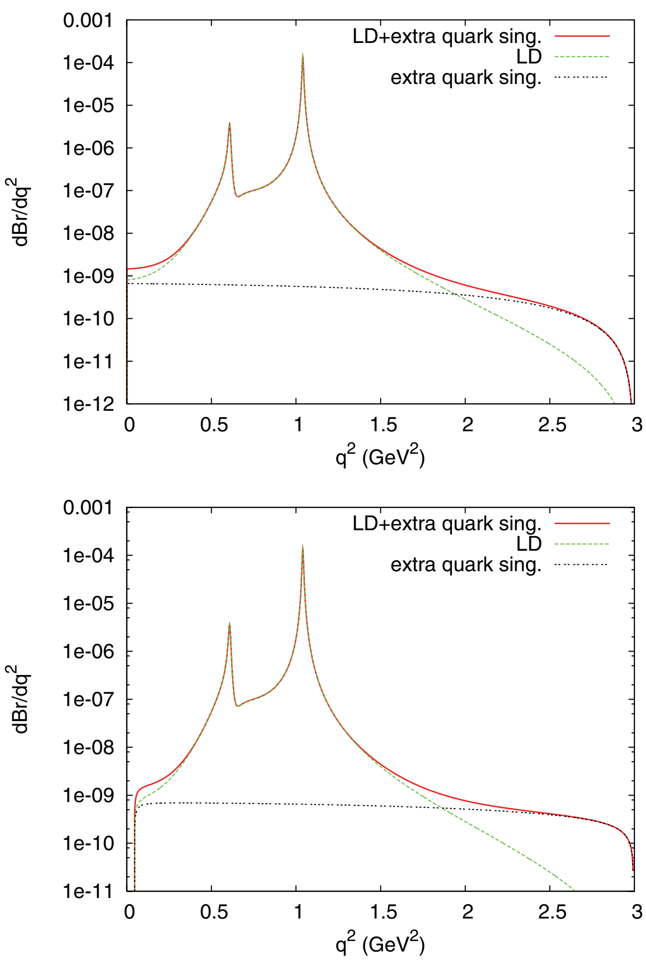

FIG. 1 (color). Distributions of the maximal branching ratios in the model with extra quark singlet for the decay modes $D^+ \to \pi^+e^+e^-$ (upper plot) and $D^+ \to \pi^+\mu^+\mu^-$ (bottom). The solid line represents the combined LD and SD contributions.

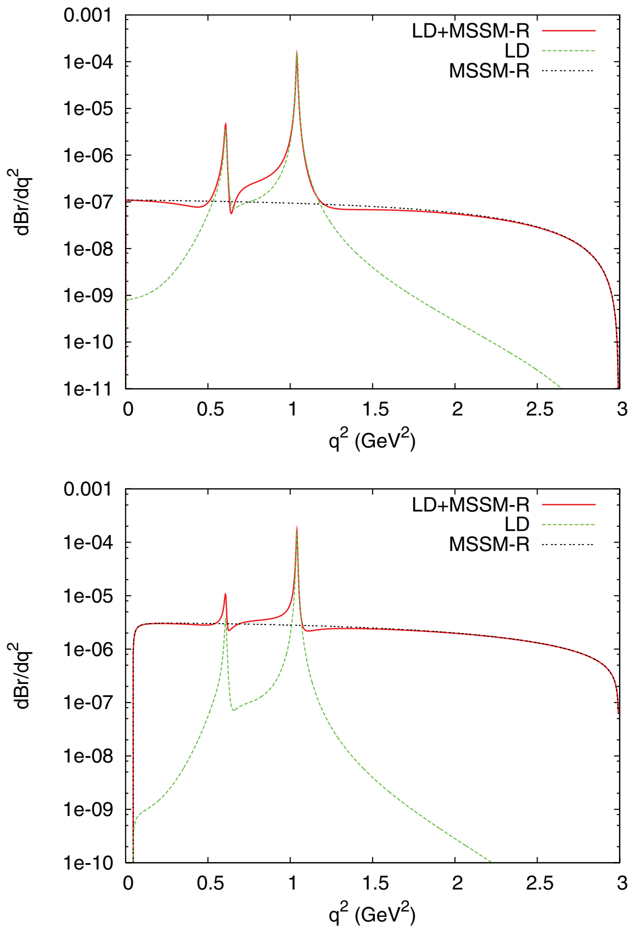

FIG. 2 (color). Distributions of the maximal branching ratios in the MSSM$\not{R}$ model for the decay modes $D^+ \to \pi^+ e^+ e^-$ (upper plot) and $D^+ \to \pi^+ \mu^+ \mu^-$ (bottom). The solid line represents the combined LD and SD contributions, and it corresponds to the experimental upper bound $\mathrm{BR}(D^+ \to \pi^+ \mu^+ \mu^-) = 8.8 \times 10^{-6}$ on the bottom plot.

We shall analyze the dilepton squared mass ($q^2 = m_{\ell\ell}^2$) distribution of the branching ratios. We fix free phases in the amplitude in a way which maximizes branching ratio for the considered decay mode.

A. $D^+ \to \pi^+ \ell^+ \ell^-$

Branching fractions are listed in Table IV. Clearly, the EQS model contribution is too small to be observed (Fig. 1).

On the other hand, the MSSM$\not{R}$ gives a slight increase to the mode with electrons. Deviation from the LD amplitude is pronounced in the region without resonances, where $m_{\ell\ell} < m_\rho$ or $m_{\ell\ell} > m_\phi$ (Fig. 2, left). However, the most promising mode is the channel with muons. The long-distance contribution (2.2×10^{-6}) is a fair share of the experimental upper bound [42] (8.8×10^{-6}) and should be taken into account together with the short distance part, when one is constraining the Wilson coefficients. The

TABLE V. Total branching fractions of the $D_s^+ \to K^+ \ell^+ \ell^-$ modes. In the first column (LD) are only long-distance BRs. The remaining four columns give maximal contributions of the SD physics models alone and also combined contributions of the SD and LD physics.

Mode	LD	Extra heavy q	LD + extra heavy q
$D_s^+ \to K^+ e^+ e^-$	6.0×10^{-7}	5.4×10^{-10}	6.0×10^{-7}
$D_s^+ \to K^+ \mu^+ \mu^-$	6.0×10^{-7}	6.2×10^{-10}	6.0×10^{-7}
Mode	MSSM$\not{R}$	LD + MSSM$\not{R}$	
$D_s^+ \to K^+ e^+ e^-$	9×10^{-8}	7.6×10^{-7}	
$D_s^+ \to K^+ \mu^+ \mu^-$	2.6×10^{-6}	3.6×10^{-6}	

difference is not big, i.e. when we drop the LD part we get for the bound $|V_{cb}^* V_{ub} C_{9,10}^\mu| < 27$, while the analysis with the LD part included gives

$$|V_{cb}^* V_{ub} C_{9,10}^\mu| < 23. \tag{28}$$

The latter bound is a maximum with respect to the free relative phase between the LD and MSSM$\not{R}$ parts of the

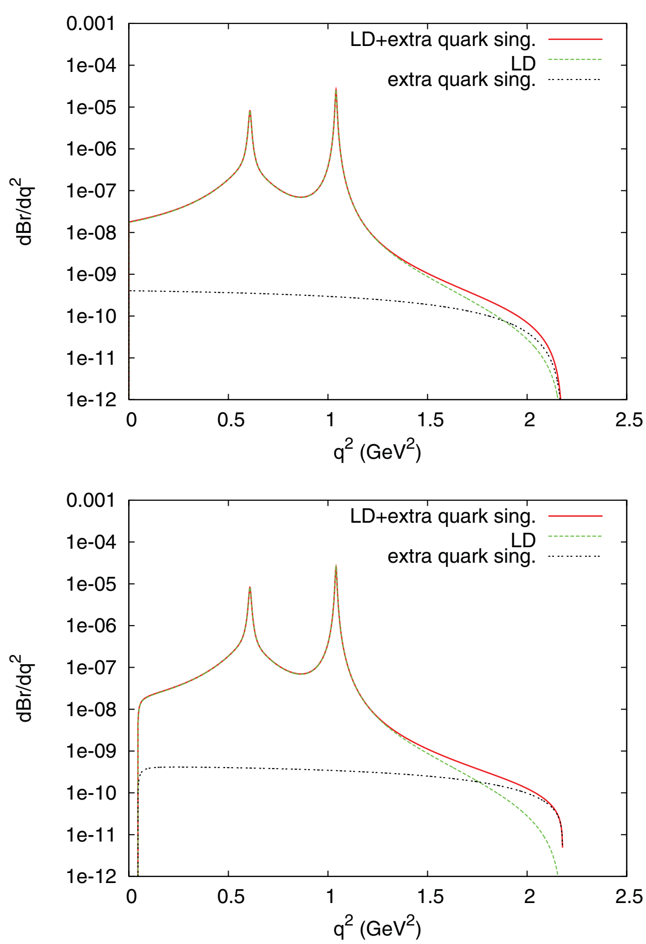

FIG. 3 (color). Distributions of the maximal branching ratios in the model with extra quark singlet for the decay modes $D_s^+ \to K^+ e^+ e^-$ (upper plot) and $D_s^+ \to K^+ \mu^+ \mu^-$ (bottom). The solid line represents the combined LD and SD contributions.

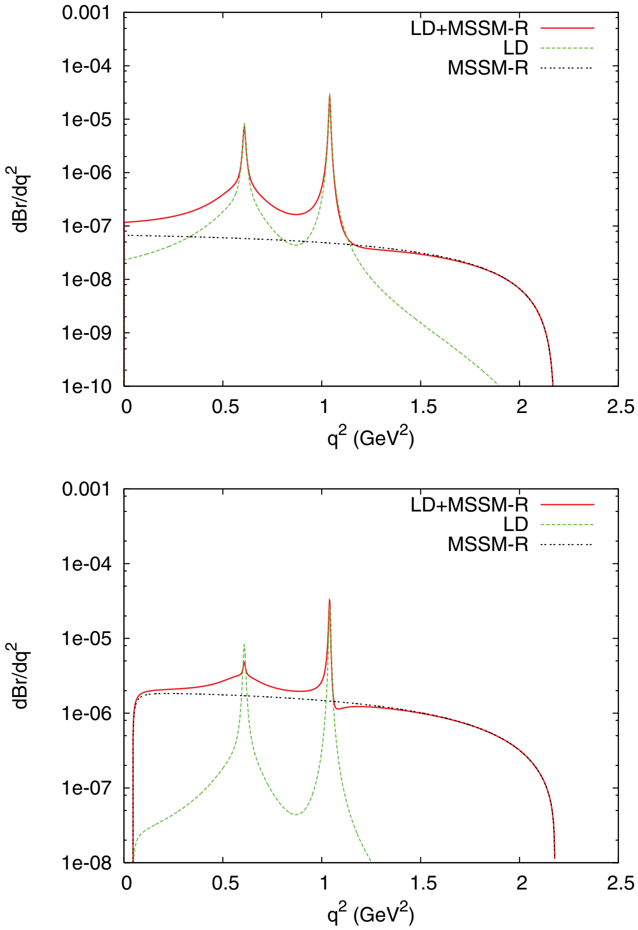

FIG. 4 (color). Distributions of the maximal branching ratios in the MSSM$\not{R}$ model for the decay modes $D_s^+ \to K^+ e^+ e^-$ (upper plot) and $D_s^+ \to K^+ \mu^+ \mu^-$ (bottom). The solid line represents combined LD and SD contributions.

amplitude. Although the inclusion of the LD term does not make a substantial difference, it will grow rapidly as the experimental bound is approaching 2.2×10^{-6}. All the branching ratios concerning MSSM$\not{R}$ and muons in the final state (Tables IV and V) and their kinematical distributions (Figs. 2 and 4) use the bound (28).

B. $D_s^+ \to K^+ \ell^+ \ell^-$

The branching ratio contributions are summarized in Table V. Again, the EQS model has a negligible effect (Fig. 3). MSSM$\not{R}$ has a notable effect, especially in the $\mu^+\mu^-$ mode, where it increases branching ratio by an order of magnitude (Fig. 4). In this case, the MSSM$\not{R}$ overshadows the LD contribution throughout the phase space, except in the close vicinity of the LD resonant peaks.

V. SUMMARY

Recently observed $D^0 - \bar{D}^0$ mass difference constrains the value of tree-level flavor changing neutral coupling $c \to uZ$, which is present in the models with an additional singlet uplike quark. We have studied the impact of this coupling on rare $D^+ \to \pi^+ \ell^+ \ell^-$ and $D_s^+ \to K^+ \ell^+ \ell^-$ decays, where its effects are accompanied by the long-distance contributions. We have determined long-distance contributions in $D_s^+ \to K^+ \ell^+ \ell^-$ following the same phenomenologically inspired model as it has been done previously in the case of $D^+ \to \pi^+ \ell^+ \ell^-$. We find that the effect of the new extra singlet uplike quark is too small to be seen in dilepton mass distributions for both decay modes. In our previous study we have considered forward-backward asymmetry in the $D^0 \to \rho^0 \ell^+ \ell^-$ and found very small effect. The new constraint reduces that asymmetry even more, making it insignificant for the experimental searches.

Present constraints on mass insertions in MSSM with conserved R-parity still allow for an increase of $c \to u\gamma$ rate by 1 order of magnitude. For the same reason, MSSM could significantly increase the $c \to u\ell^+\ell^-$ rate at small $m_{\ell\ell}$. However, this MSSM enhancement is not drastic in D decays, since $D \to V\gamma$ and $D \to V\ell^+\ell^-$ have large long-distance contributions for small $m_{\ell\ell}$, while the $D \to P\ell^+\ell^-$ rate is multiplied by a factor of $m_{\ell\ell}^2$ due to gauge invariance.

The remaining possibility to search for new physics in rare D decays is offered by the MSSM models which contain R-parity violating terms. We reinvestigate bounds on the combinations of these parameters in $D^+ \to \pi^+ \ell^+ \ell^-$ by including the long-distance effects. Using the current upper bound on the rate for $D^+ \to \pi^+ \ell^+ \ell^-$, we derive the new bound:

$$\sum_{k=1}^{3} \left(\frac{100\ \text{GeV}}{M_{\tilde{d}_R^k}} \right)^2 \tilde{\lambda}'_{21k} \tilde{\lambda}'_{22k} < 0.0041. \qquad (29)$$

Since at Tevatron there are plans to investigate $D_s^+ \to K^+ \ell^+ \ell^-$ decay, we use the upper bound (29) and calculate dilepton invariant mass distribution. This bound still gives a small increase of the dilepton invariant mass distribution for the larger invariant dilepton mass, making it attractive for the planned experimental studies.

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