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Penguin-mediated $B_{d,s} \to VV$ decays and the B_s-$\bar{B}_s$ mixing angle

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In this paper, we propose three different strategies to extract the weak mixing angle ϕ_s of the B_s system using penguin-mediated decays into vectors, mainly $B_s \to K^{*0}\bar{K}^{*0}$, $B_s \to \phi\bar{K}^{*0}$, and $B_s \to \phi\phi$. We also provide predictions for the longitudinal branching ratio and CP asymmetries of $B_s \to K^{*0}\bar{K}^{*0}$ using a method that combines QCD factorization with flavor symmetries to relate this decay to its B_d counterpart.

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I. INTRODUCTION

The large amount of data collected by *BABAR* and Belle, the progress of CDF and D0, and the advent of LHCb have increased our ability of testing the CP- and flavor-violating structure of the standard model (SM), increasing our chances of discovering new physics. The phenomenology of penguin-dominated hadronic B decays is particularly relevant in this field. At the theoretical level, $b \to s$ penguin transitions are expected to receive a large impact from new physics [1], compared to $b \to d$ penguin transitions. This comparison has driven detailed experimental and theoretical analyses devoted to B decays that proceed through a $b \to s$ transition, such as $B \to \pi K$ [2,3], $B \to (\phi, \eta, \omega, \rho \ldots)K_s$ [4,5], etc. Deviations from theoretical expectations were observed for some of these decays, but it remains unclear whether these results can or cannot be explained within the SM [6,7].

The properties of the B_s meson have attracted a lot of attention recently: the $B_s - \bar{B}_s$ mass difference ΔM_s has been measured [8], with an immediate impact on new physics [9]. Very recently, the first experimental information on $\Delta\Gamma$ and the $\arg(-M_{12}/\Gamma_{12})$ of the B_s system has also been presented [10] (see also [11]). Another important piece of information will come with the measurement of the mixing angle ϕ_s. Its SM value is $\phi_s^{\rm SM} = 2\beta_s = -2\lambda^2 R_b \sin\gamma \simeq -2°$ [12] and thus probes new CP-violating phases with a high sensitivity. As far as decay modes are concerned, a considerable amount of theoretical work has been carried out to understand charmless B_s decays (for instance in $B_s \to KK$ decays [13,14]). The experimental program, focusing initially on $B_s \to \pi K$ and $B_s \to KK$ modes [15], will extend its scope by considering more and more classes of B_s decays. More observables could be studied on such details with the recent prospect of B or super-B factories working at the $\Upsilon(5S)$ and thus producing B_s-$\bar{B}_s$ pairs [16].

On the theoretical side, nonleptonic B_s decays have been extensively studied, with a recent emphasis on the case of two final vector mesons ($B_s \to VV$), within the framework of QCD factorization (QCDF) [17] (see Ref. [18] for related work within the perturbative QCD (pQCD) approach) and in the context of $SU(3)$ flavor symmetries [19]. QCDF and flavor symmetries provide different tools to tackle nonleptonic decays, with their advantages and shortcomings: the former is a systematic expansion in $1/m_b$ but encounters difficulties with phenomenology due to power-suppressed hadronic effects, such as final-state interactions. The latter takes hadronic effects into account but may be affected by large corrections, up to 30% for $SU(3)$ relations.

In Ref. [20] (see also [21,22]), we have developed an intermediate approach that aims at combining $SU(3)$ relations with QCDF-inspired input in a theoretically controlled way, in order to gain precision over the other approaches. In this paper we extend this approach to a larger class of decay modes and apply it mainly to $B_s \to K^{*0}\bar{K}^{*0}$ and partially to $B_s \to \phi\bar{K}^{*0}$ and $B_s \to \phi\phi$ decays. These decays exhibit particularly alluring experimental and theoretical features. For instance, on the experimental side, $B_s \to K^{*0}\bar{K}^{*0}$ is likely to be measured easily in hadronic machines due to its expected high branching ratio $\sim\mathcal{O}(10^{-5})$, and its prospects are being analyzed in detail at present at LHCb. In addition, some decay products of the K^* resonances are charged $(\pi^\pm, K^\mp)$ and are easier to identify experimentally than in the case of $B_s \to K^0\bar{K}^0$. On the theoretical side, these decays allow for an accurate extraction of the $B_s - \bar{B}_s$ mixing angle, with a small direct CP asymmetry in the SM that should be very sensitive to CP-violating new physics.

The aim of this paper is to provide three strategies to extract ϕ_s from certain nonleptonic B decays. It is organized as follows. In Sec. II, we consider longitudinal observables for $B \to VV$ decays, and relate them to the observables usually obtained from an angular analysis of such decays. In Sec. III, we describe a general method to extract the SM hadronic parameters of a B meson decay given the branching ratio and a theoretical quantity called Δ, which is the difference between tree and penguin contributions (see [20]). We also derive useful bounds for the branching ratios and direct CP asymmetries as a function of Δ. Section IV deals with the theoretical computation of Δ for $B_{d,s} \to K^{*0}\bar{K}^{*0}$, $\phi\bar{K}^{*0}$, $\phi\phi$ decay modes: for penguin-dominated modes, this difference is dominated by short distances and can be computed accurately in

QCD factorization. In Sec. V, we exploit this theoretical information to put a bound on the tree pollution affecting the determination of Cabibbo-Kobayashi-Maskawa (CKM) angles through the mixed CP asymmetry. In Sec. VI, we present general expressions to extract the CKM phases α, β, γ, and β_s from hadronic penguin-dominated $B_{d,s}$ decays. These general expressions can be applied to $B_{d,s} \to K^{*0}\bar{K}^{*0}$, $B_{d,s} \to \phi\bar{K}^{*0}$, and $B_s^0 \to \phi\phi$ decay modes. In Sec. VII, we use flavor symmetries and QCD factorization to relate $B_d \to K^{*0}\bar{K}^{*0}$ and $B_s \to K^{*0}\bar{K}^{*0}$ observables, and exploit this U-spin symmetry to constrain the $B_s - \bar{B}_s$ mixing. Finally, we discuss the three strategies developed and we conclude in Sec. VIII.

II. LONGITUDINAL OBSERVABLES IN $B \to VV$ MODES

The amplitude for a B meson decaying into two vector mesons can be written as

$$\begin{aligned} A(B \to V_1V_2) = {} & \left[\frac{4m_1m_2}{m_B^4}(\epsilon_1^* \cdot p_B)(\epsilon_2^* \cdot p_B)\right]A_0 \\ & + \left[\frac{1}{2}(\epsilon_1^* \cdot \epsilon_2^*) - \frac{(p_B \cdot \epsilon_1^*)(p_B \cdot \epsilon_2^*)}{m_B^2} \right. \\ & \left. - \frac{i\epsilon_{\mu\nu\rho\sigma}\epsilon_1^{*\mu}\epsilon_2^{*\nu}p_1^\rho p_2^\sigma}{2p_1 \cdot p_2}\right]A_+ \\ & + \left[\frac{1}{2}(\epsilon_1^* \cdot \epsilon_2^*) - \frac{(p_B \cdot \epsilon_1^*)(p_B \cdot \epsilon_2^*)}{m_B^2} \right. \\ & \left. + \frac{i\epsilon_{\mu\nu\rho\sigma}\epsilon_1^{*\mu}\epsilon_2^{*\nu}p_1^\rho p_2^\sigma}{2p_1 \cdot p_2}\right]A_-, \end{aligned} \tag{1}$$

where $A_{0,+,-}$ correspond to the amplitudes for longitudinal and transversely polarized final vector mesons. It is also customary to use the basis $A_{0,\|,\perp}$, where $A_{\|,\perp} = (A_+ \pm A_-)\sqrt{2}$.

The vector mesons in the final state decay typically into pairs of pseudoscalar particles. A full angular analysis of vector-vector modes provides the following set of observables: three polarization fractions f_0, $f_\perp$, and $f_\|$ (only two of them are independent) and their CP-conjugate counterparts $\bar{f}_{0,\perp,\|}$, two phases $\phi_{\perp,\|}$ (again, together with $\bar{\phi}_{\perp,\|}$), a total CP-averaged branching ratio BR, and a total direct CP asymmetry $\mathcal{A}_{\rm dir}$. The polarization fractions are defined as

$$\begin{aligned} f_{0,\perp,\|} &\equiv \frac{|A_{0,\perp,\|}|^2}{|A_0|^2 + |A_\perp|^2 + |A_\||^2} \\ \bar{f}_{0,\perp,\|} &\equiv \frac{|\bar{A}_{0,\perp,\|}|^2}{|\bar{A}_0|^2 + |\bar{A}_\perp|^2 + |\bar{A}_\||^2}. \end{aligned} \tag{2}$$

A full angular analysis is available for $B_d \to \phi K^{*0}$ from *BABAR* and Belle [23,24], and the same type of analysis is expected for $B_d \to K^{*0}\bar{K}^{*0}$.

We will focus in this paper on observables for the longitudinal polarization ($BR^{\rm long}$, $\mathcal{A}_{\rm dir}^{\rm long}$, $\mathcal{A}_{\rm mix}^{\rm long}$, and $\mathcal{A}_{\Delta\Gamma}^{\rm long}$), where only A_0 occurs. These observables, free from the positive- and negative-helicity components, can be predicted with a much better accuracy. Indeed the negative-helicity (positive-helicity) component of the amplitude is $1/m_b$ -suppressed ($1/m_b^2$ suppressed) because of the nature of the interactions involved (left-handed weak interaction, helicity-conserving strong interaction at high energies) [17,25]. This suppression makes longitudinal observables better behaved and easier to compute than transverse ones.

Some decay channels exhibit the $1/m_b$ suppression of transverse amplitudes in a very striking way: the longitudinal polarization is very close to 1, e.g., $f_L \simeq 97\%$ for $B \to \rho^+\rho^-$. In such cases, the full observables (where A_0 is replaced by the sum $A = A_0 + A_- + A_+$) coincide with the longitudinal ones to a high degree of accuracy. On the other hand, for penguin-dominated $\Delta S = 1$ decays, f_L can be as low as $\sim$50%, so that the transverse amplitudes (or $\pm$ helicity amplitudes) contribute significantly to the full observables. Therefore, one must determine whether purely longitudinal observables can be extracted from experimental measurements.

We start from the normalized partial decay rate of $B \to V_1V_2$, where the two vector mesons go subsequently into pairs of pseudoscalar mesons. It can be written [26]

$$\begin{aligned} \frac{d^3\Gamma}{\Gamma d\cos\theta_1 d\cos\theta_2 d\phi} = {} & \frac{9}{8\pi}\frac{1}{|A_0|^2 + |A_\||^2 + |A_\perp|^2}\Big[|A_0|^2\cos^2\theta_1\cos^2\theta_2 + |A_\||^2\frac{1}{2}\sin^2\theta_1\sin^2\theta_2\cos^2\phi \\ & + |A_\perp|^2\frac{1}{2}\sin^2\theta_1\sin^2\theta_2\sin^2\phi + {\rm Re}[A_0^*A_\|]\frac{1}{2\sqrt{2}}\sin2\theta_1\sin2\theta_2\cos\phi \\ & + {\rm Im}[A_0^*A_\perp]\frac{-1}{2\sqrt{2}}\sin2\theta_1\sin2\theta_2\sin\phi + {\rm Im}[A_\|^*A_\perp]\frac{-1}{2}\sin^2\theta_1\sin^2\theta_2\sin2\phi\Big], \end{aligned} \tag{3}$$

where $(\theta_1, \theta_2, \phi)$ are angles introduced to describe the kinematics of the decay $B \to V_1V_2$ followed by $V_1 \to P_1P_1'$ and $V_2 \to P_2P_2'$. θ_1 is the angle of one of the V_1 decay products in the rest frame of V_1 relative to the motion of V_1 in the rest frame of the B meson (same for θ_2 with V_2). ϕ is the angle between the two planes formed by the decay products of V_1 and V_2, respectively (see for instance Fig. 1 of Ref. [27] for a representation of the angles).

There are different ways to perform the angular integrations in order to extract the purely longitudinal component from the differential decay rate. A first option consists in computing moments of $\cos\theta_1$ (or equivalently $\cos\theta_2$):

$$\Gamma^{\text{long}} \equiv \int \frac{d^3\Gamma}{d\cos\theta_1 d\cos\theta_2 d\phi}\left(\frac{5}{2}\cos^2\theta_1 - \frac{1}{2}\right) d\cos\theta_1 \times d\cos\theta_2 d\phi = g_{PS}|A_0|^2/\tau_B, \tag{4}$$

where g_{PS} is the product of phase-space and lifetime factors

$$g_{PS} = \frac{\tau_B}{16\pi M_B^3}\sqrt{[M_B^2 - (m_1 + m_2)^2][M_B^2 - (m_1 - m_2)^2]}. \tag{5}$$

A second possibility amounts to performing asymmetric integrations over one angle [28]

$$\Gamma^{\text{long}} \equiv \int_{-1}^{1} d\cos\theta_1 \int_T d\cos\theta_2 \int_0^{2\pi} d\phi \frac{d^3\Gamma}{d\cos\theta_1 d\cos\theta_2 d\phi} = g_{PS}|A_0|^2/\tau_B, \tag{6}$$

with

$$\int_T d\cos\theta_2 = \left(\frac{11}{9}\int_0^{\pi/3} - \frac{5}{9}\int_{\pi/3}^{2\pi/3} + \frac{11}{9}\int_{2\pi/3}^{\pi}\right) \times (-\sin\theta_2) d\theta_2. \tag{7}$$

In the same way we can obtain the CP conjugate $\Gamma^{\text{long}}(\bar{B}_q^0 \to \bar{f})$ from the corresponding CP-conjugate distribution, leading to the CP-averaged branching ratio of the longitudinal component

$$BR^{\text{long}} = \frac{\tau_B}{2}(\Gamma^{\text{long}}(B_q^0 \to f) + \Gamma^{\text{long}}(\bar{B}_q^0 \to \bar{f})) = g_{PS}\frac{|A_0|^2 + |\bar{A}_0|^2}{2}, \tag{8}$$

where $\bar{A}_0$ is the CP-conjugate amplitude of A_0.

If we include the dependence on time in the above expressions, B-$\bar{B}$ mixing modifies the expressions [27]. We will focus on CP eigenstates f_{CP} in the final state $K^{*0}\bar{K}^{*0}$ and $\phi\phi$, as well as ϕK^{*0} with a subsequent decay of K^{*0} into a CP eigenstate ($K_s\pi^0$ or $K_L\pi^0$).

The time evolution of these observables is obtained by considering the time dependence of $A_0(t)$ [29]. Inserting this time dependence one arrives at the usual expression for the longitudinal component of the time-dependent CP asymmetry:

$$\mathcal{A}_{CP}(t) \equiv \frac{\Gamma^{\text{long}}(B_q^0(t) \to f_{CP}) - \Gamma^{\text{long}}(\bar{B}_q^0(t) \to f_{CP})}{\Gamma^{\text{long}}(B_q^0(t) \to f_{CP}) + \Gamma^{\text{long}}(\bar{B}_q^0(t) \to f_{CP})} = \frac{\mathcal{A}_{\text{dir}}^{\text{long}}\cos(\Delta M t) + \mathcal{A}_{\text{mix}}^{\text{long}}\sin(\Delta M t)}{\cosh(\Delta\Gamma t/2) - \mathcal{A}_{\Delta\Gamma}^{\text{long}}\sinh(\Delta\Gamma t/2)}, \tag{9}$$

where the direct and mixing-induced CP asymmetries are defined by

$$\mathcal{A}_{\text{dir}}^{\text{long}} \equiv \frac{|A_0|^2 - |\bar{A}_0|^2}{|A_0|^2 + |\bar{A}_0|^2}, \qquad \mathcal{A}_{\text{mix}}^{\text{long}} \equiv -2\eta_f\frac{\text{Im}(e^{-i\phi_M}A_0^*\bar{A}_0)}{|A_0|^2 + |\bar{A}_0|^2}, \tag{10}$$

together with the asymmetry related to the width difference:

$$\mathcal{A}_{\Delta\Gamma}^{\text{long}} \equiv -2\eta_f\frac{\text{Re}(e^{-i\phi_M}A_0^*\bar{A}_0)}{|A_0|^2 + |\bar{A}_0|^2}. \tag{11}$$

ϕ_M is the mixing angle and $\Delta\Gamma = \Gamma^H - \Gamma^L$. η_f is the CP eigenvalue of the final state f (± 1): $\eta_{K^{*0}K^{*0}} = \eta_{\phi\phi} = 1$, whereas $\eta_{K^{*0}\phi} = 1$ if K^{*0} decays into $K_s\pi^0$ and -1 if it decays into $K_L\pi^0$. In the latter case, the contribution from the strong process $K^{*0} \to K\pi$ is the same for both B and $\bar{B}$ decays and it cancels in the time-dependent CP asymmetry Eq. (10), which depends only on the amplitudes A_0 and $\bar{A}_0$.

Finally, if the direct CP asymmetries of all three helicity components are negligible, the longitudinal branching ratio can be estimated very easily from $BR^{\text{long}} = BR^{\text{total}} f_0$.

III. DETERMINING PENGUIN AND TREE CONTRIBUTIONS

We consider a B meson decaying through $\bar{b} \to \bar{D}q\bar{q}$, with $D = d, s$, and restrict the discussion to the longitudinal component of the amplitude. However, the results are general and can be applied to any $B \to PP, PV, VV$ decay (in the latter case, the relations hold for each helicity amplitude independently). We can parametrize the amplitudes in terms of "tree" and "penguin" contributions,

$$A_0 = \lambda_u^{(D)*}T + \lambda_c^{(D)*}P, \qquad \bar{A}_0 = \lambda_u^{(D)}T + \lambda_c^{(D)}P, \tag{12}$$

where $\lambda_U^{(D)} \equiv V_{UD}^* V_{Ub}$ are a combination of CKM elements, $U = u, c$. The penguin and tree contributions are defined through their associated CKM factor, and not from the topology of the relevant diagrams (even though in many cases, tree contributions correspond to tree diagrams). Such a decomposition is always possible and completely general in the standard model since the unitarity of the CKM matrix allows one to recast contributions proportional to $V_{tD}^* V_{tb}$ into the form of Eq. (12). We will follow the convention of calling "penguin" the piece proportional to $V_{cD}^* V_{cb}$ and "tree" the piece proportional to $V_{uD}^* V_{ub}$. In the particular case of penguin-mediated decays, there is no actual tree diagram and the tree contribution corresponds to penguins containing a u-quark loop (or a t-quark loop).

For both neutral and charged decays, one can define the CP-averaged branching ratio and the direct CP asymmetry as given in (8) and (10). From these two observables we can obtain the magnitudes of the amplitudes

TABLE I. Numerical values for the coefficients $c_i^{(D)}$ and $\mathcal{R}_D$ for $\gamma = 62°$.

$c_0^{(d)}$	$c_1^{(d)}$	$c_2^{(d)}$	$c_0^{(s)}$	$c_1^{(s)}$	$c_2^{(s)}$	$\mathcal{R}_d$	$\mathcal{R}_s$
-3.15×10^{-5}	-0.034	6.93×10^{-5}	3.11×10^{-5}	0.011	1.63×10^{-3}	7.58×10^{-3}	1.54×10^{-3}

$$|A_0|^2 = BR^{\mathrm{long}}(1 + \mathcal{A}_{\mathrm{dir}}^{\mathrm{long}})/g_{PS}$$
$$|\bar{A}_0|^2 = BR^{\mathrm{long}}(1 - \mathcal{A}_{\mathrm{dir}}^{\mathrm{long}})/g_{PS}. \quad (13)$$

We consider the quantity Δ defined as the difference between tree and penguin hadronic contributions [20]

$$\Delta \equiv T - P. \quad (14)$$

The value of Δ might be determined on theoretical grounds, for instance through QCD factorization [30]. In the next sections, we will consider decays where such a computation is particularly clean and free from many long-distance uncertainties. Given the arbitrary common phase for T and P, we can always rotate simultaneously P and Δ and choose Δ to be real positive if we restrict ourselves to a given channel. We will adopt this convention in the following, unless the contrary is explicitly stated.

We can write down the amplitudes (12) in the following way:

$$|A_0|^2 = |\lambda_c^{(D)*} + \lambda_u^{(D)*}|^2 \left| P + \frac{\lambda_u^{(D)*}}{\lambda_c^{(D)*} + \lambda_u^{(D)*}} \Delta \right|^2$$
$$|\bar{A}_0|^2 = |\lambda_c^{(D)} + \lambda_u^{(D)}|^2 \left| P + \frac{\lambda_u^{(D)}}{\lambda_c^{(D)} + \lambda_u^{(D)}} \Delta \right|^2. \quad (15)$$

The previous equations can be solved for P if we know BR^{long}, $\mathcal{A}_{\mathrm{dir}}^{\mathrm{long}}$, Δ, and the CKM parameters $\lambda_u^{(D)}$ and $\lambda_c^{(D)}$. The solutions exhibit a very simple form for Δ real and positive

$$\mathrm{Re}[P] = -c_1^{(D)}\Delta \pm \sqrt{-\mathrm{Im}[P]^2 - \left(\frac{c_0^{(D)}\Delta}{c_2^{(D)}}\right)^2 + \frac{\widetilde{BR}}{c_2^{(D)}}}, \quad (16)$$

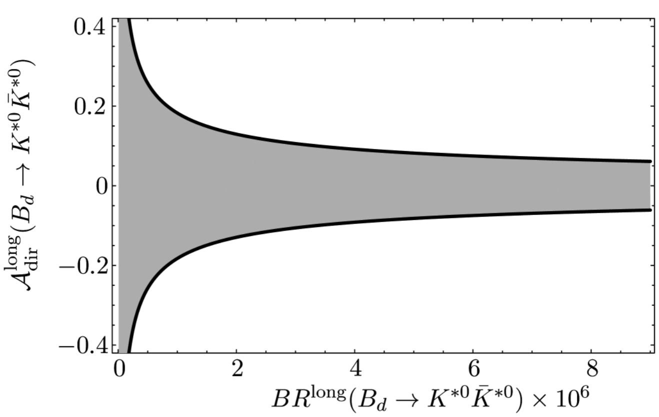

FIG. 1. Allowed region on the $BR^{\mathrm{long}} - \mathcal{A}_{\mathrm{dir}}^{\mathrm{long}}$ plane for $B_d \to K^{*0}\bar{K}^{*0}$, according to the value of $\Delta_{K^*K^*}^d$.

$$\mathrm{Im}[P] = \frac{\widetilde{BR}\mathcal{A}_{\mathrm{dir}}^{\mathrm{long}}}{2c_0^{(D)}\Delta}, \quad (17)$$

where the coefficients $c_i^{(D)}$ are given by

$$c_0^{(D)} = \lambda_c^{(D)}|\lambda_u^{(D)}|\sin\gamma;$$
$$c_1^{(D)} = (|\lambda_u^{(D)}|^2 + \lambda_c^{(D)}|\lambda_u^{(D)}|\cos\gamma)/c_2^{(D)}; \quad (18)$$
$$c_2^{(D)} = |\lambda_u^{(D)} + \lambda_c^{(D)}|^2$$

and $\widetilde{BR} \equiv BR^{\mathrm{long}}/g_{PS}$. The numerical values of these coefficients are collected in Table I. Once P is known, Eqs. (14), (16), and (17) yield the second hadronic parameter T.

Interestingly, two consistency conditions exist between BR^{long}, $\mathcal{A}_{\mathrm{dir}}^{\mathrm{long}}$, and Δ, to guarantee the existence of solutions for P (the argument of the square root in Re$[P]$ must be positive):

$$|\mathcal{A}_{\mathrm{dir}}^{\mathrm{long}}| \leq \sqrt{\frac{\mathcal{R}_D^2\Delta^2}{2\widetilde{BR}}\left(2 - \frac{\mathcal{R}_D^2\Delta^2}{2\widetilde{BR}}\right)} \approx \frac{\mathcal{R}_D\Delta}{\sqrt{\widetilde{BR}}}$$
$$\widetilde{BR} \geq \frac{\mathcal{R}_D^2\Delta^2}{4}, \quad (19)$$

with the combination of CKM factors $\mathcal{R}_D = 2|c_0^{(D)}|/\sqrt{c_2^{(D)}}$. The approximation for the upper bound on $|\mathcal{A}_{\mathrm{dir}}^{\mathrm{long}}|$ holds up to very small corrections in the usual situation $\Delta \lesssim \mathcal{O}(10^{-7})$ and $BR^{\mathrm{long}} \sim \mathcal{O}(10^{-6})$.

The relations derived in this section apply to all charmless hadronic B meson decays, and are thus of quite generic nature. As an illustration, we anticipate the results of the next section and assume that we are able to compute Δ accurately for the decay $B_d \to K^{*0}\bar{K}^{*0}$ (denoted by $\Delta_{K^*K^*}^d$). Given a measured value for the longitudinal branching ratio, the quantity $\Delta_{K^*K^*}^d$ in Eq. (20) constrains the direct CP asymmetries according to Eq. (19). The allowed values for the asymmetry are shown in Fig. 1.

IV. THEORETICAL INPUT OF $\Delta = T - P$

The quantity Δ is a hadronic, process-dependent, intrinsically nonperturbative object, and thus difficult to compute theoretically. Such hadronic quantities are usually extracted from data or computed using some factorization-based approach. In the latter case, Δ could suffer from the usual problems related to the factorization ansatz and, in particular, long-distance effects.

However, for penguin-mediated decays, T and P share the same long-distance dynamics: the difference comes from the (u or c) quark running in the loops [20]. Indeed, in such decays, $\Delta = T - P$ is not affected by the breakdown of factorization that affects annihilation and hard-spectator contributions, and it can be computed in a well-controlled way leading to safer predictions and smaller uncertainties.

For vector-vector final states, a Δ is associated to each helicity amplitude, but we focus on longitudinal quantities here. We obtain for the longitudinal Δ of the $B_d \to K^{*0}\bar{K}^{*0}$ ($B_s \to K^{*0}\bar{K}^{*0}$) decay denoted by $\Delta^d_{K^*K^*}$ ($\Delta^s_{K^*K^*}$)

$$|\Delta^d_{K^*K^*}| = A^{d,0}_{K^*K^*}\frac{C_F\alpha_s}{4\pi N_c}C_1|\bar{G}_{K^*}(s_c) - \bar{G}_{K^*}(0)| = (1.85 \pm 0.79) \times 10^{-7}\ \text{GeV}, \tag{20}$$

$$|\Delta^s_{K^*K^*}| = A^{s,0}_{K^*K^*}\frac{C_F\alpha_s}{4\pi N_c}C_1|\bar{G}_{K^*}(s_c) - \bar{G}_{K^*}(0)| = (1.62 \pm 0.69) \times 10^{-7}\ \text{GeV}, \tag{21}$$

where $\bar{G}_V \equiv G_V - r^V_\chi \hat{G}_V$ are the usual penguin functions and $A^{q,0}_{V_1V_2}$ are the naive factorization factors combining decay constants and form factors (see [31] for definitions),

$$A^{q,0}_{V_1V_2} = \frac{G_F}{\sqrt{2}} m^2_{B_q} f_{V_2} A_0^{B_q \to V_1}(0). \tag{22}$$

The numerical values of the used inputs are given in Table II. The contributions to each error from the various sources are detailed in Table III. For the Δ, as well as for the other quantities computed in this paper, we quote as the central value the value obtained from taking the central value of the inputs. To estimate the error, we vary one by one each of the inputs, compute the difference with the central value, then add in quadrature the resulting uncertainties. The main sources of uncertainties are the scale of factorization μ, the mass of the charm quark m_c, and the form factor $A_0^{B\to K*}$.

In a similar way, we can compute the corresponding longitudinal Δ for the decay modes $B_{d,s} \to \phi\bar{K}^{*0}$ and $B_s \to \phi\phi$:

$$|\Delta^d_{\phi K^*}| = A^{d,0}_{K^*\phi}\frac{C_F\alpha_s}{4\pi N_c}C_1|\bar{G}_{\phi}(s_c) - \bar{G}_{\phi}(0)| = (1.02 \pm 1.11) \times 10^{-7}\ \text{GeV}, \tag{23}$$

$$|\Delta^s_{\phi K^*}| = A^{s,0}_{\phi K^*}\frac{C_F\alpha_s}{4\pi N_c}C_1|\bar{G}_{\phi}(s_c) - \bar{G}_{\phi}(0)| = (1.16 \pm 1.05) \times 10^{-7}\ \text{GeV}, \tag{24}$$

$$|\Delta^s_{\phi\phi}| = A^{s,0}_{\phi\phi}\frac{C_F\alpha_s}{4\pi N_c}C_1|\bar{G}_{\phi}(s_c) - \bar{G}_{\phi}(0)| = (2.06 \pm 2.24) \times 10^{-7}\ \text{GeV}. \tag{25}$$

In the following sections we show how to apply the results of Secs. III and IV to the longitudinal contribution of penguin-dominated $B \to VV$ modes. We will see that they can be used to extract the $B_s - \bar{B}_s$ mixing angle and some longitudinal observables like branching ratios and time-dependent CP asymmetries within the standard model. In particular, we outline three different strategies to determine the $B_s - \bar{B}_s$ mixing angle (in the SM and beyond). Indeed, concerning new physics we will see that

TABLE II. Input parameters required in QCD factorization to compute the quantities Δ and δ described in the text. The masses and decay constants are given in GeV.

$m_c(m_b)$	f_B	f_{B_s}	λ_B, λ_{B_s}	$\alpha_1^{(\perp)}(K^*)$	$\alpha_2^{(\perp)}(K^*)$	f_{K^*}
1.3 ± 0.2	0.21 ± 0.02	0.24 ± 0.02	0.35 ± 0.15	0.06 ± 0.06	0.1 ± 0.2	0.218 ± 0.004

$f^\perp_{K^*}(2\ \text{GeV})$	$A_0^{B\to K^*}$	$A_0^{B_s\to K^*}$	f_ϕ	$f^\perp_\phi(2\ \text{GeV})$	$A_0^{B_s\to\phi}$	$\alpha_2^{(\perp)}(\phi)$
0.175 ± 0.025	0.39 ± 0.06	0.33 ± 0.05	0.221 ± 0.003	0.175 ± 0.025	$0.38^{+0.10}_{-0.02}$	0.0 ± 0.3

TABLE III. Relative contributions from the inputs to the errors in Δ for the various decays.

	m_c	$A_0^{B\to K^*}$	f_{K^*}	$f^\perp_{K^*}(2\ \text{GeV})$	μ	$\alpha_1(K^*)$	$\alpha_2(K^*)$	$\alpha_1^\perp(K^*)$	$\alpha_2^\perp(K^*)$
$\Delta^d_{K^*K^*}$	37.3%	13.2%	0.2%	0%	44.2%	0.1%	4.6%	0.1%	0.3%
$\Delta^s_{K^*K^*}$	37.5%	12.9%	0.2%	0%	44.4%	0.1%	4.7%	0.1%	0.3%

	m_c	$A_0^{B\to K^*}$	f_{K^*}	$f^\perp_{K^*}(2\ \text{GeV})$	μ	$\alpha_1(K^*)$	$\alpha_2(K^*)$	$\alpha_2^\perp(K^*)$	$A_0^{B\to\phi}$	$f^\perp_\phi(2\ \text{GeV})$	$\alpha_2(\phi)$	$\alpha_2^\perp(\phi)$
$\Delta^d_{\phi K^*}$	44.2%	2.0%	···	···	52.3%	···	···	···	···	0.4%	0.7%	0.3%
$\Delta^s_{\phi K^*}$	35.0%	···	0.1%	0.7%	58.2%	0.7%	0.1%	0.1%	5.0%	0%	0%	0%
$\Delta^s_{\phi\phi}$	44.1%	···	···	···	52.3%	···	···	···	2.1%	0.4%	0.7%	0.3%

under the assumption of no significant new physics affecting the amplitude, while strategy II can detect the presence of new physics by comparing the obtained ϕ_s with $\phi_s^{SM} = 2\beta_s$, strategies I and III cannot only detect new physics but allow also for the extraction of ϕ_s even in the presence of new physics in the mixing.

V. FIRST STRATEGY TO EXTRACT ϕ_s: BOUNDING T/P

The $b \to s$ penguin-dominated decays like $B_s \to K^{*0}\bar{K}^{*0}$ are in principle clean modes to extract the mixing angle ϕ_s. In this section and those following, ϕ_s refers to the same mixing angle that will be measured, for instance, in the mixing-induced CP asymmetry of $B_s \to \psi\phi$, including possible new physics contributions in the mixing. When focusing only on SM we will use the notation $\phi_s = 2\beta_s$.

In an expansion in powers of $\lambda_u^{(s)}/\lambda_c^{(s)}$, the amplitude for the decay $B_s \to K^{*0}\bar{K}^{*0}$ is given by

$$\mathcal{A}_{\rm mix}^{\rm long}(B_s \to K^{*0}\bar{K}^{*0}) \simeq \sin\phi_s + 2\left|\frac{\lambda_u^{(s)}}{\lambda_c^{(s)}}\right| \mathrm{Re}\left(\frac{T^s_{K^*K^*}}{P^s_{K^*K^*}}\right) \times \sin\gamma\cos\phi_s + \cdots. \quad (26)$$

In order to determine the accuracy of this relation, we must assess the size of the CKM-suppressed hadronic contribution T. Notice that this relation is valid even in the presence of new physics in the mixing. In the SM, one can derive from the Wolfenstein parametrization that Eq. (26) is of order λ^2 (with $\lambda = V_{us}$), and both pieces shown on the right-hand side of Eq. (26) are of this same order. However, despite the smallness of the ratio $|\lambda_u^{(s)}/\lambda_c^{(s)}| = 0.044$, a significant value of the hadronic ratio $\mathrm{Re}(T/P)$ could spoil the potentially safe extraction of $\sin\phi_s$ (a similar issue was discussed in Ref. [32] for $B \to \pi\pi$). The deviation from $\sin\phi_s$ is

$$\Delta S(B_s \to K^{*0}\bar{K}^{*0}) \equiv 2\left|\frac{\lambda_u^{(s)}}{\lambda_c^{(s)}}\right| \mathrm{Re}\left(\frac{T^s_{K^*K^*}}{P^s_{K^*K^*}}\right)\sin\gamma\cos\phi_s. \quad (27)$$

We want to set bounds on $\mathrm{Re}(T/P)$, which can be related to the inputs

$$\mathrm{Re}\left(\frac{T}{P}\right) = \mathrm{Re}\left(\frac{P+\Delta}{P}\right) = 1 + \mathrm{Re}\left(\frac{\Delta}{P}\right) = 1 + \frac{\mathrm{Re}(P)\Delta}{\mathrm{Re}(P)^2 + \mathrm{Im}(P)^2}. \quad (28)$$

Equations (16) and (17) show that the maximum of $\mathrm{Re}(T/P)$ is reached for $\mathcal{A}_{\rm dir}^{\rm long} = 0$ together with the positive branch for $\mathrm{Re}(P)$ in Eq. (16). The following bound is obtained:

$$\mathrm{Re}\left(\frac{T}{P}\right) \le 1 + \left(-c_1^{(s)} + \sqrt{-(c_0^{(s)}/c_2^{(s)})^2 + (1/c_2^{(s)})\widetilde{BR}/\Delta^2}\right)^{-1}, \quad (29)$$

where the lower bound for $BR^{\rm long}$ and the upper bound for Δ must be used. In a similar way, the minimum of $\mathrm{Re}(T/P)$ occurs for $\mathcal{A}_{dir}^{\rm long} = 0$, for the negative branch of Eq. (16) for the solution of $\mathrm{Re}(P)$,

$$\mathrm{Re}\left(\frac{T}{P}\right) \ge 1 + \left(-c_1^{(s)} - \sqrt{-(c_0^{(s)}/c_2^{(s)})^2 + (1/c_2^{(s)})\widetilde{BR}/\Delta^2}\right)^{-1}, \quad (30)$$

where the lower bound for $BR^{\rm long}$ and the upper bound for Δ must be used once again. As a conclusion, we obtain a range for $\mathrm{Re}(T/P)$ from two inputs: the branching ratio $BR^{\rm long}(B_s \to K^{*0}\bar{K}^{*0})$ and $\Delta^s_{K^*K^*}$, given in Eq. (21).

Using Eq. (27), these upper and lower bounds on $\mathrm{Re}(T/P)$ are converted into a bound on the pollution $\Delta S(B_s \to K^{*0}\bar{K}^{*0})$. The latter is plotted as a function of the longitudinal $BR^{\rm long}(B_s \to K^{*0}\bar{K}^{*0})$ in Fig. 2.

Once a measurement of $\mathcal{A}_{\rm mix}^{\rm long}(B_s \to K^{*0}\bar{K}^{*0})$ is available, upper and lower bounds for ϕ_s are easily obtained. For instance, if we take as a lower bound for the branching ratio $BR^{\rm long}(B_s \to K^{*0}\bar{K}^{*0}) \gtrsim 5 \times 10^{-6}$, Fig. 2 gives $0.03 < \Delta S(B_s \to K^{*0}\bar{K}^{*0}) < 0.06$. In the case of a moderately large branching ratio $BR^{\rm long}(B_s \to K^{*0}\bar{K}^{*0}) \sim (30\text{–}40) \times 10^{-6}$, the bounds get sharper, with $0.04 < \Delta S(B_s \to K^{*0}\bar{K}^{*0}) < 0.05$ and

$$(\mathcal{A}_{\rm mix}^{\rm long}(B_s \to K^{*0}\bar{K}^{*0}) - 0.05) < \sin\phi_s < (\mathcal{A}_{\rm mix}^{\rm long}(B_s \to K^{*0}\bar{K}^{*0}) - 0.04). \quad (31)$$

The same strategy can be applied to $B_s \to \phi K^{*0}$ and $B_s \to \phi\phi$ decays.

(i) Take the experimental value for the longitudinal branching ratio $BR^{\rm long}$ (once available), and the theoretical value for Δ from Eq. (24) or (25).

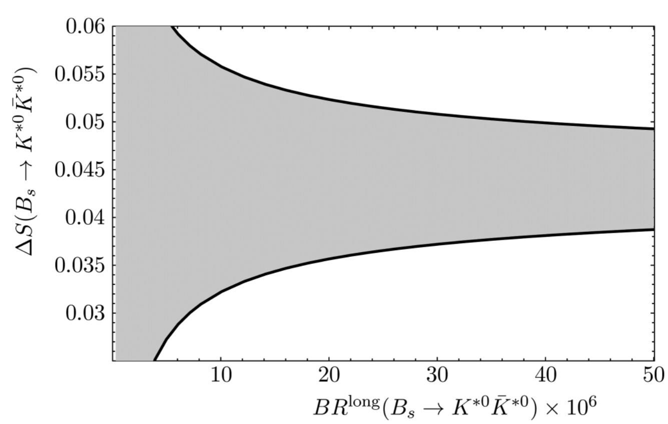

FIG. 2. Absolute bounds on $\Delta S(B_s \to K^{*0}\bar{K}^{*0})$ as a function of $BR^{\rm long}(B_s \to K^{*0}\bar{K}^{*0})$.

(ii) Apply Eqs. (29) and (30) to constrain the range of $\mathrm{Re}(T/P)$.
(iii) Derive the allowed range for ΔS according to the equivalent of (27)
(iv) From the measured value of $\mathcal{A}_{\mathrm{mix}}^{\mathrm{long}}$, determine ϕ_s from

$$(\mathcal{A}_{\mathrm{mix}}^{\mathrm{long}} - \Delta S_{\max}) < \sin\phi_s < (\mathcal{A}_{\mathrm{mix}}^{\mathrm{long}} - \Delta S_{\min}). \tag{32}$$

A weak mixing angle ϕ_s different from ϕ_s^{SM} would signal the presence of new physics.

Interestingly, if the longitudinal direct CP asymmetry becomes available and happens to be inconsistent with zero, the bounds for $\mathrm{Re}(T/P)$ in Eqs. (29) and (30) can be tightened. Equation (28) can be exploited to derive expressions similar to Eqs. (29) and (30) with a nonvanishing $\mathcal{A}_{\mathrm{dir}}^{\mathrm{long}}$, leading to stronger bounds on $\mathrm{Re}(T/P)$ and consequently on $\sin\phi_s$.

VI. SECOND STRATEGY: MEASURING CP ASYMMETRIES AND BRANCHING RATIO

In this section, we show how we can extract mixing angles and related CKM phases in a clean way from experimental data, the length of two sides of the unitarity triangle and the theoretical quantity Δ. The only theoretical requirement is that the decay must allow for a safe way of computing Δ. The approach is general in the same sense as in the previous section, since it can be applied to any B decay into two pseudoscalars or vectors. But it yields different results for the four groups of decays:

(1) B_d decay through a $b \to d$ process, e.g., $B_d \to K^{*0}\bar{K}^{*0}$
(2) B_s decay through a $b \to s$ process, e.g., $B_s \to K^{*0}\bar{K}^{*0}$
(3) B_d decay through a $b \to s$ process, e.g., $B_d \to \phi\bar{K}^{*0}$ (with a subsequent decay into a CP eigenstate)
(4) B_s decay through a $b \to d$ process, e.g., $B_s \to \phi\bar{K}^{*0}$ (with a subsequent decay into a CP eigenstate)

As far as weak interactions are concerned, the difference between B_d and B_s decays consists in the mixing angle, whereas $b \to d$ and $b \to s$ processes differ through the CKM elements $\lambda_{u,c}^{(D)}$, where $D = d$ or s.

In the case of a B_d meson decaying through a $b \to D$ process ($D = d, s$), we can extract the angles α [22] and β from the identities:

$$\sin^2\alpha = \frac{\widetilde{BR}}{2|\lambda_u^{(D)}|^2|\Delta|^2}(1 - \sqrt{1 - (\mathcal{A}_{\mathrm{dir}})^2 - (\mathcal{A}_{\mathrm{mix}})^2}), \tag{33}$$

$$\sin^2\beta = \frac{\widetilde{BR}}{2|\lambda_c^{(D)}|^2|\Delta|^2}(1 - \sqrt{1 - (\mathcal{A}_{\mathrm{dir}})^2 - (\mathcal{A}_{\mathrm{mix}})^2}). \tag{34}$$

In the case of a B_s meson decaying through a $b \to D$ process ($D = d, s$), we can extract the angles β_s [33] and γ, assuming no new physics in the decay, from the following expressions:

$$\sin^2\beta_s = \frac{\widetilde{BR}}{2|\lambda_c^{(D)}|^2|\Delta|^2}(1 - \sqrt{1 - (\mathcal{A}_{\mathrm{dir}})^2 - (\mathcal{A}_{\mathrm{mix}})^2}), \tag{35}$$

$$\sin^2(\beta_s + \gamma) = \frac{\widetilde{BR}}{2|\lambda_u^{(D)}|^2|\Delta|^2}(1 - \sqrt{1 - (\mathcal{A}_{\mathrm{dir}})^2 - (\mathcal{A}_{\mathrm{mix}})^2}). \tag{36}$$

If the obtained β_s differs from its SM value, this would signal the presence of new physics. Notice that this strategy is obtained by combining the definition of Δ with the unitarity of the CKM matrix, so it is designed to work only in the context of the SM. Consequently the previous expressions should be understood as a way of testing the SM. This is an important difference with strategies I and III where one can obtain a value for the weak mixing phase also in the presence of new physics in the mixing (but not in the decay).

While the previous equations are quite general (they can be used for $B \to PP$ decays), it is understood that BR and $A_{\mathrm{dir,mix}}$ refer to the longitudinal branching ratio and longitudinal CP asymmetries, respectively, when they are applied to $B \to VV$ decays.

Equation (35) provides a new way to perform a consistency test for the SM value of $|\sin\beta_s|$ from the measurements of $\mathcal{A}_{\mathrm{mix}}^{\mathrm{long}}(B_s \to K^{*0}\bar{K}^{*0})$, $\mathcal{A}_{\mathrm{dir}}^{\mathrm{long}}(B_s \to K^{*0}\bar{K}^{*0})$, and $BR^{\mathrm{long}}(B_s \to K^{*0}\bar{K}^{*0})$. The same strategy can be applied to $B_s \to \phi\bar{K}^{*0}$ and $B_s \to \phi\phi$ using the corresponding sum rules. This sum rule offers several advantages: it is independent of CKM angles, and all the hadronic input is concentrated on a single well-controlled quantity Δ.

Note that all these equations depend actually on the corresponding branching ratio and $\mathcal{A}_{\Delta\Gamma}^{\mathrm{long}}$. The asymmetry $\mathcal{A}_{\Delta\Gamma}^{\mathrm{long}}$ is indeed related to the direct and mixing-induced CP asymmetries through the equality $(\mathcal{A}_{\mathrm{dir}}^{\mathrm{long}})^2 + (\mathcal{A}_{\mathrm{mix}}^{\mathrm{long}})^2 + (\mathcal{A}_{\Delta\Gamma}^{\mathrm{long}})^2 = 1$. It was already noticed in [14] in the context of $B_s \to K^+K^-$ and in [34] in the context of $B \to J/\psi K^*$, $D_s^{*+}\bar{D}^*$ decays that it is possible to extract $\mathcal{A}_{\Delta\Gamma}^{\mathrm{long}}$ directly from the "untagged" rate:

$$\Gamma^{\mathrm{long}}(B_s(t) \to VV) + \Gamma^{\mathrm{long}}(\bar{B}_s(t) \to VV) \propto R_{\mathrm{H}} e^{-\Gamma_{\mathrm{H}}^{(s)} t} + R_{\mathrm{L}} e^{-\Gamma_{\mathrm{L}}^{(s)} t}. \tag{37}$$

If the time dependence of both exponentials can be separated, one obtains

$$\mathcal{A}^{\text{long}}_{\Delta\Gamma}(B_s \to VV) = \frac{R_{\text{H}} - R_{\text{L}}}{R_{\text{H}} + R_{\text{L}}}. \tag{38}$$

The branching ratio and $\mathcal{A}^{\text{long}}_{\Delta\Gamma}$ are thus the only required observables to extract β_s through this method, which offers the advantage of concentrating in Δ all the hadronic input needed to bound the tree-to-penguin ratio.

VII. THIRD STRATEGY: RELATING $B_s \to K^{*0}\bar{K}^{*0}$ AND $B_d \to K^{*0}\bar{K}^{*0}$

Once an angular analysis of $B_d \to K^{*0}\bar{K}^{*0}$ is performed, it is possible to extract the CP-averaged branching ratio corresponding to the longitudinal helicity final state. Equations (16) and (17) can be used to extract the hadronic parameters, if one assumes that no new physics contributes in an appreciable way. If flavor symmetries are sufficiently accurate for this particular process, this estimate can be converted into a fairly precise determination of hadronic parameters for the $b \to s$ channel $B_s \to K^{*0}\bar{K}^{*0}$. For $B_{d,s} \to KK$ modes [20], we noticed that U-spin analysis combined with QCD factorization led to tight constraints on the ratio of the tree contributions to both decay modes, as well as that for the penguins. In this section we show how to relate $B_d \to K^{*0}\bar{K}^{*0}$ and $B_s \to K^{*0}\bar{K}^{*0}$ decay modes following the same approach.

We define the parameters $\delta^P_{K^*K^*}$ and $\delta^T_{K^*K^*}$ as

$$\begin{aligned} P^s_{K^*K^*} &= f P^d_{K^*K^*}(1 + \delta^P_{K^*K^*}), \\ T^s_{K^*K^*} &= f T^d_{K^*K^*}(1 + \delta^T_{K^*K^*}), \end{aligned} \tag{39}$$

where the factor f is given by

$$\frac{m^2_{B_s} A_0^{B_s\to K^*}}{m^2_B A_0^{B\to K^*}} = 0.88 \pm 0.19. \tag{40}$$

We compute $|\delta^{P,T}_{K^*K^*}|$ using QCDF. These parameters are affected by the model-dependent treatment of annihilation and spectator-scattering contributions, so the results should be considered as an estimate. A significant part of long-distance dynamics is common to both decays, and we find the following upper bounds:

$$|\delta^P_{K^*K^*}| \leq 0.12, \qquad |\delta^T_{K^*K^*}| \leq 0.15, \tag{41}$$

where the largest contribution comes from the lower value of λ_B.

We could in principle apply the same strategy to $B_{d,s} \to \phi K^{*0}$, but the corresponding δ's are much larger. Indeed, the computation leads to corrections up to $\delta_{\phi K^*} \sim 50\%$. This shows that U-spin symmetry cannot be expected to hold at a high accuracy for any pair of flavor-related processes. $K^{(*)}K^{(*)}$ offer a much more interesting potential than other final states such as ϕK^{*0}. Moreover, we cannot perform a similar analysis for $\phi\phi$ since $B_d \to \phi\phi$ is a pure weak-annihilation process, contrary to $B_s \to \phi\phi$ mediated through penguins. Therefore we focus on the precise $B_s \to K^{*0}\bar{K}^{*0}$ modes in the remaining part of this section. Notice that the large hadronic uncertainties affecting $B_s \to \phi\phi$ and $B_s \to \phi K^{*0}$ have no impact when we use these modes in the strategies described in Secs. V and VI, since we exploited a quantity Δ where they cancel out.

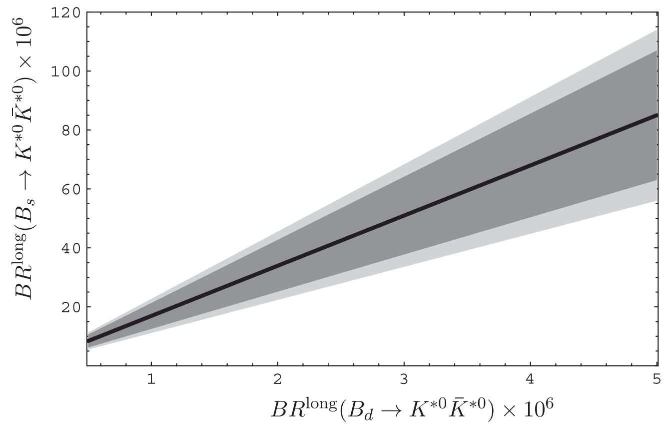

FIG. 3. Longitudinal branching ratio for $B_s \to K^{*0}\bar{K}^{*0}$ in terms of the longitudinal $B_d \to K^{*0}\bar{K}^{*0}$ branching ratio. The light-shaded area corresponds to the uncertainty on the ratio of form factors f, whereas the dark-shaded area comes from varying the various hadronic inputs.

Once the hadronic parameters $P^s_{K^*K^*}$ and $T^s_{K^*K^*}$ have been obtained from Eq. (39), one can give predictions for the $B_s \to K^{*0}\bar{K}^{*0}$ observables. Note that the branching ratio $BR^{\text{long}}(B_d \to K^{*0}\bar{K}^{*0})$ is an experimental input in this analysis, and this piece of information is not available yet. The result for the branching ratio of $B_s \to K^{*0}\bar{K}^{*0}$ is given in terms of the $B_d \to K^{*0}\bar{K}^{*0}$ branching ratio in Fig. 3. Once the branching ratio of $B_d \to K^{*0}\bar{K}^{*0}$ is measured one can use this plot to find the SM prediction for $BR^{\text{long}}(B_s \to K^{*0}\bar{K}^{*0})$.

The ratio of branching ratios $BR^{\text{long}}(B_s \to K^{*0}\bar{K}^{*0})/BR^{\text{long}}(B_d \to K^{*0}\bar{K}^{*0})$ and the asymmetries turn out to be quite insensitive to the exact value of $BR^{\text{long}}(B_d \to K^{*0}\bar{K}^{*0})$ as long as $BR^{\text{long}}(B_d \to K^{*0}\bar{K}^{*0}) \gtrsim 5 \times 10^{-7}$. The numerical values are summarized in Table IV.

TABLE IV. Results for the longitudinal observables related to $B_s \to K^{*0}\bar{K}^{*0}$ according to Sec. VII. These are predictions for the SM contributions under the standard assumption of no new physics in $b \to d$ transition. We used $\phi_s^{\text{SM}} = 2\beta_s = -2°$ for $\mathcal{A}^{\text{long}}_{\text{mix}}$, and we assumed $BR^{\text{long}}(B_d \to K^{*0}\bar{K}^{*0}) \gtrsim 5 \times 10^{-7}$. The quoted uncertainty includes the errors associated to all input parameters including the variation of γ inside the range $56° \leq \gamma \leq 68°$ [35].

$\left(\frac{BR^{\text{long}}(B_s\to K^{*0}\bar{K}^{*0})}{BR^{\text{long}}(B_d\to K^{*0}\bar{K}^{*0})}\right)_{\text{SM}} = 17 \pm 6$
$\mathcal{A}^{\text{long}}_{\text{dir}}(B_s \to K^{*0}\bar{K}^{*0})_{\text{SM}} = 0.000 \pm 0.014$
$\mathcal{A}^{\text{long}}_{\text{mix}}(B_s \to K^{*0}\bar{K}^{*0})_{\text{SM}} = 0.004 \pm 0.018$

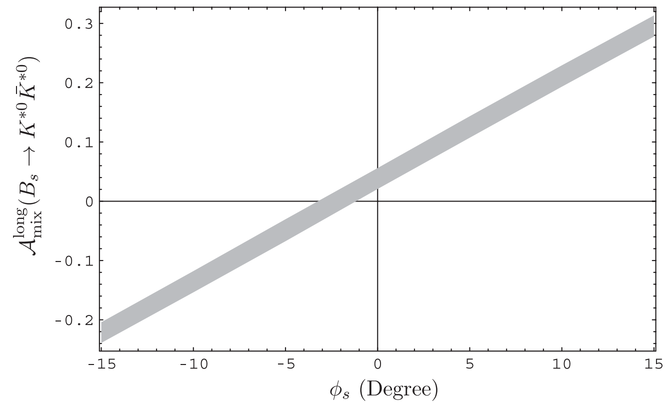

FIG. 4. Relation between $\mathcal{A}_{\rm mix}^{\rm long}(B_s \to K^{*0}\bar{K}^{*0})$ and the $B_s - \bar{B}_s$ mixing angle ϕ_s. We assumed $BR^{\rm long}(B_d \to K^{*0}\bar{K}^{*0}) \gtrsim 5 \times 10^{-7}$ and $\gamma = 62°$. A measurement of this asymmetry leads to a prediction for ϕ_s, which includes hadronic pollution and SU(3) breaking effects, according to Sec. VII.

Under the standard assumption that new physics contribution to $b \to d$ penguins is negligible, and since the experimental input comes entirely from $B_d \to K^{*0}\bar{K}^{*0}$ (a $b \to d$ penguin), the results given in Table IV are SM predictions. In the presence of new physics in $b \to s$ penguins the full prediction can be obtained by adding to the SM piece extra contributions to the amplitude and weak mixing angle as explained in [36,37].

One may also use this as a strategy to extract the mixing angle ϕ_s. If one assumes no new physics in the decay $B_s \to K^{*0}\bar{K}^{*0}$, this method relates directly $\mathcal{A}_{\rm mix}^{\rm long}(B_s \to K^{*0}\bar{K}^{*0})$ and ϕ_s. Figure 4 shows $\mathcal{A}_{\rm mix}^{\rm long}(B_s \to K^{*0}\bar{K}^{*0})$ vs ϕ_s. Once this asymmetry is measured, this plot can be used as a way to extract ϕ_s, and this result can be compared to the one found in tree decays such as $B \to DK$. A disagreement would point out new physics. Moreover, it is possible to distinguish whether new physics affects the decay or the mixing itself, by confronting $BR^{\rm long}(B_s \to K^{*0}\bar{K}^{*0})$ and $\mathcal{A}_{\rm dir}^{\rm long}(B_s \to K^{*0}\bar{K}^{*0})$ with the SM predictions given in Table IV. If the predictions for the branching ratio and the direct CP asymmetry agree with experiment, but the ϕ_s extracted from $\mathcal{A}_{\rm mix}^{\rm long}(B_s \to K^{*0}\bar{K}^{*0})$ differs from $\phi_s^{\rm SM}$, this will be a clear indication of new physics in $B_s - \bar{B}_s$ mixing. An interesting comparison will be allowed between the value for ϕ_s obtained here and the measurement of ϕ_s from the mixing-induced CP asymmetry of $B_s \to DK$ decay [38].

VIII. DISCUSSION

The increasing list of measured nonleptonic two-body B_d and B_s decays provides many tests of the CKM mechanism of CP violation in the standard model. Of particular interest is the determination of angles through time-dependent CP asymmetries. For instance ϕ_s, related to $B_s - \bar{B}_s$ mixing, should be constrained: it is tiny in the standard model, and can be measured through many - penguin-dominated decays. However, for such determination to be valid, one must assess the size of the various hadronic quantities involved as precisely as possible.

In this paper, we have applied ideas presented in Ref. [20] for $B_{d,s} \to KK$ to vector-vector modes mediated through penguins: $B_{d,s} \to K^{*0}\bar{K}^{*0}$, $\phi\phi$, and ϕK^{*0} (with the condition that K^{*0} decays into a definite CP eigenstate). In order to combine flavor symmetries with QCD factorization, we have restricted our analysis to longitudinal observables, which are under better theoretical control. These observables have been related to the angular analysis performed experimentally in Sec. II. Penguin-mediated modes offer the very interesting feature that the difference between tree and penguin contributions $\Delta = T - P$ should be dominated by short-distance physics. It can be computed fairly accurately using QCD factorization, and it can be used to determine tree and penguin contributions from observables as explained in Secs. III and IV. This theoretical piece of information is used to relate CP asymmetries of $B_{d,s} \to K^{*0}\bar{K}^{*0}$, $\phi\phi$, and ϕK^{*0} to CKM angles according to different strategies. For illustration, we have focused on $B_{d,s} \to K^{*0}\bar{K}^{*0}$, where all three strategies apply.

In Sec. V, we have proposed to use $\Delta = T - P$ to put stringent bounds on the pollution due to hadronic uncertainties. Indeed, even though the ratio $|\lambda_u^{(s)}/\lambda_c^{(s)}| = 0.044$ is small, a large value of the hadronic quantity $\mathrm{Re}(T/P)$ could spoil the naively safe extraction of $\sin\phi_s$ from the mixed asymmetry of $B_s \to K^{*0}\bar{K}^{*0}$. This strategy to control the pollution can be applied to all penguin-mediated processes of interest here.

In Sec. VI, we have suggested a second approach, using $\mathcal{A}_{\rm mix}^{\rm long}(B_s \to K^{*0}\bar{K}^{*0})$, $\mathcal{A}_{\rm dir}^{\rm long}(B_s \to K^{*0}\bar{K}^{*0})$, and $BR^{\rm long}(B_s \to K^{*0}\bar{K}^{*0})$ to extract $|\sin\beta_s|$. In principle, one can also use an alternative set of experimental quantities: the branching ratio together with a direct measurement of the longitudinal untagged rate. The sum rule needed for the $B_s \to K^{*0}\bar{K}^{*0}$ is independent of the CKM angle γ, and the input on hadronic dynamics is limited to a single well-controlled quantity: $\Delta^s_{K^*K^*}$. This strategy can also be applied to extract β_s from $B_s \to \phi\bar{K}^{*0}$ and $B_s \to \phi\phi$ using the corresponding sum rule.

In Sec. VII, we proposed a last method to determine ϕ_s, by relying on the prediction of the mixing-induced CP asymmetry $\mathcal{A}_{\rm mix}^{\rm long}(B_s \to K^{*0}\bar{K}^{*0})$ as a function of the $BR^{\rm long}(B_d \to K^{*0}\bar{K}^{*0})$ and the theoretical input $\Delta^d_{K^*K^*}$. In this strategy, tree pollution is controlled using the hadronic information from flavor symmetry and QCD factorization. The outcome of our analysis is presented in Fig. 4. This strategy requires data on $B_d \to K^{*0}\bar{K}^{*0}$ and on the mixing-induced CP asymmetry $\mathcal{A}_{\rm mix}^{\rm long}(B_s \to K^{*0}\bar{K}^{*0})$. The input from B_s decay is therefore minimal: $\mathcal{A}_{\rm mix}^{\rm long}(B_s \to K^{*0}\bar{K}^{*0})$, while all other inputs can be obtained from B factories.

A comparison among the three different strategies discussed in this paper is given in Table V, where the needed

TABLE V. Comparison between the three strategies for $B_s \to K^{*0}\bar{K}^{*0}$.

	Strategy 1	Strategy 2	Strategy 3
Inputs	$BR^{\rm long}(B_s \to K^{*0}\bar{K}^{*0})$ $\mathcal{A}^{\rm long}_{\rm mix}(B_s \to K^{*0}\bar{K}^{*0})$ $\Delta^s_{K^*K^*}$, γ	$BR^{\rm long}(B_s \to K^{*0}\bar{K}^{*0})$ $\mathcal{A}^{\rm long}_{\rm dir}(B_s \to K^{*0}\bar{K}^{*0})$ $\mathcal{A}^{\rm long}_{\rm mix}(B_s \to K^{*0}\bar{K}^{*0})$ $\Delta^s_{K^*K^*}$	$BR^{\rm long}(B_d \to K^{*0}\bar{K}^{*0})$ $\mathcal{A}^{\rm long}_{\rm mix}(B_s \to K^{*0}\bar{K}^{*0})$ $\Delta^d_{K^*K^*}$, δ_T, δ_P, γ
Outputs	ϕ_s	$\lvert\sin\beta_s\rvert$, γ	$BR^{\rm long}(B_s \to K^{*0}\bar{K}^{*0})_{\rm SM}$ $\mathcal{A}^{\rm long}_{\rm dir}(B_s \to K^{*0}\bar{K}^{*0})_{\rm SM}$ ϕ_s
Advantages	Applies also to $B_s \to \phi K^{*0}$ and $B_s \to \phi\phi$	Applies also to $B_s \to \phi K^{*0}$ and $B_s \to \phi\phi$	It can be easily generalized to include new physics in the decay and mixing.
Limitations	It assumes no new physics in $b \to s$ decay.	It assumes no new physics in $b \to s$ decay.	Does not apply to $B_s \to \phi K^{*0}$ or $B_s \to \phi\phi$ because $\delta_{T,P}$ are big.

inputs are enumerated as well as the predicted observables and the range of validity.

If both hadronic machines and super-B factories [39] running at $\Upsilon(5S)$ provide enough information on B_s decays, it will be interesting to compare the determination from ϕ_s following those methods, which rely on penguin-mediated decays, with the value obtained from tree processes like $B_s \to DK$ [38]. Differences between the values obtained through these two procedures would provide a clear hint of physics beyond the standard model. In such a situation, the different methods presented in this paper would yield very useful cross-checks for the penguin-dominated vector modes.

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