Right-handed neutrinos at CERN LHC and the mechanism of neutrino mass generation

Jörn Kersten[1,*] and Alexei Yu. Smirnov[1,2,†]

[1]*The Abdus Salam ICTP, Strada Costiera 11, 34014 Trieste, Italy*

[2]*Institute for Nuclear Research, Russian Academy of Sciences, 60th October Anniversary Prospect 7A, 117 312 Moscow, Russia*

We consider the possibility to detect right-handed neutrinos, which are mostly singlets of the standard model gauge group, at future accelerators. Substantial mixing of these neutrinos with the active neutrinos requires a cancellation of different contributions to the light neutrino mass matrix at the level of 10^{-8}. We discuss possible symmetries behind this cancellation and argue that for three right-handed neutrinos they always lead to conservation of total lepton number. Light neutrino masses can be generated by small perturbations violating these symmetries. In the most general case, LHC physics and the mechanism of neutrino mass generation are essentially decoupled; with additional assumptions, correlations can appear between collider observables and features of the neutrino mass matrix.

I. INTRODUCTION

The most appealing and natural mechanism for generating small neutrino masses is the (type-I) seesaw mechanism [1–5]. It relies on the existence of right-handed (RH) neutrinos that are singlets under the standard model (SM) gauge groups and can therefore have large Majorana masses. A direct test of the seesaw mechanism would involve the detection of these neutrinos at a collider and the measurement of their Yukawa couplings with the electroweak doublets. If the Dirac neutrino masses are similar to the other fermion masses, the Majorana masses turn out to be of order (10^8-10^{16}) GeV, so that this test is not possible. In principle, the seesaw mechanism can also be realized with masses as small as 100 GeV, though, which are within the energy reach of the LHC and future colliders. This possibility has attracted renewed interest recently, see e.g. [6–14]. However, given the smallness of the light neutrino masses, small RH masses generically imply tiny Yukawa couplings. Consequently, also the mixing between the singlets and the electroweak doublet neutrinos is tiny, resulting in negligible production cross sections. In order to allow for large mixing, different contributions to the light masses have to cancel. In other words, the leading-order structure of the mass matrices leads to vanishing light neutrino masses [15–27], and nonvanishing masses are generated by small perturbations. Unless this structure can be motivated by some symmetry, it amounts to fine-tuning. The known setups [15,16,23,25,26] contain lepton number conservation, which ensures that the heavy states either form Dirac pairs or decouple from the active neutrinos.

An alternative possibility is that the neutral fermions participating in the seesaw are not singlets of the SM symmetry group or have some other interactions that can lead to their production at future colliders, see e.g. [27–

35]. For instance, if all singlets are relatively light, one can expect that the scale of left-right symmetry is also low. In this case, they have gauge interactions with the W_R and Z'. Then the discovery is possible for masses up to a few TeV. Another example is the type-III seesaw mechanism [36,37], where the heavy neutrinos enter an SU(2) triplet and therefore can be produced by the electroweak interactions even if their mixing with light neutrinos is extremely small.

In this paper we will reconsider from the theoretical perspective the possibility of testing the existence of RH neutrinos at future colliders. We study implications of such a detection for the mechanism of neutrino mass generation. After discussing the generic estimates that lead to the expectation of tiny doublet-singlet mixings, we will consider the cancellation of contributions to light neutrino masses required by large mixings and possible underlying symmetries in Sec. II. Besides the well-known case of lepton number conservation, we will discuss a scenario based on the discrete symmetry A_4 which achieves the same objective in a different way, but ultimately turns out to contain a conserved lepton number, too. We argue that this is a general feature of any symmetry behind the cancellation. In Sec. III, we will systematically study small perturbations of the leading-order mass matrices that yield viable masses for the light neutrinos. In Sec. IV, we will discuss consequences for signatures at colliders. Within the setups relying on a symmetry, lepton number violation is unobservable. Lepton-flavor-violating processes can have sizable amplitudes but are difficult to observe at LHC [14]. Consequently, the discovery of RH neutrinos will probably require a more advanced machine like the ILC.

II. CANCELLATIONS AND SYMMETRIES

A. Mixing of doublet and singlet neutrinos

In the setup we consider, the Lagrangian responsible for neutrino masses is the same as in the type-I seesaw scenario [1–5],

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*jkersten@ictp.it
†smirnov@ictp.it

$$\mathcal{L}_{\text{Mass}}^{\nu} = -\bar{\nu} m_{\text{D}} N - \tfrac{1}{2} \overline{N^C} m_{\text{R}} N + \text{H.c.} \qquad (1)$$

Each RH neutrino,[1] N_i, generates the (rank 1) contribution to the mass matrix of light neutrinos,

$$m_{\nu}^{(i)} = -\frac{1}{M_i} \vec{m}_i \vec{m}_i^T, \qquad (2)$$

where M_i is the mass of N_i and $\vec{m}_i \equiv (m_{ei}, m_{\mu i}, m_{\tau i})^T$. Then the Dirac mass matrix, in the basis where m_{R} is diagonal, is given by $m_{\text{D}} = (\vec{m}_1, \vec{m}_2, \vec{m}_3)$, and the complete mass matrix of the light neutrinos equals

$$m_{\nu} = \sum_i m_{\nu}^{(i)} = -m_{\text{D}} m_{\text{R}}^{-1} m_{\text{D}}^T. \qquad (3)$$

The Dirac mass terms provide the mixing between the light (active) and heavy (singlet) states, described by the mixing matrix elements

$$V_{\alpha i} = (m_{\text{D}} m_{\text{R}}^{-1})_{\alpha i} = \frac{m_{\alpha i}}{M_i} \qquad (\alpha = e, \mu, \tau). \qquad (4)$$

In terms of $V_{\alpha i}$, the elements of the mass matrix in Eq. (2) can be rewritten as

$$(m_{\nu}^{(i)})_{\alpha\beta} = -V_{\alpha i} V_{\beta i} M_i. \qquad (5)$$

Assuming the absence of cancellations, the experimental limits on the light neutrino masses imply that each element is at most of the order $m_{\nu} \sim 0.1$ eV. This yields the upper bound

$$|V_{\alpha i}| \sim \sqrt{\frac{m_{\nu}}{M_i}} \lesssim 10^{-6} \left(\frac{100 \text{ GeV}}{M_i}\right)^{1/2}. \qquad (6)$$

It can be considered the generic bound on the mixing of any heavy Majorana lepton with the light neutrinos.

The limit (6) is much stronger than the direct bound for singlets heavier than the Z, obtained from observations like universality of the weak interactions and the Z width [38,39],

$$\sum_i |V_{\alpha i}|^2 \lesssim 0.01. \qquad (7)$$

If the heavy neutrinos are to be observable at the LHC or the ILC, their mixing angles must not lie far below the upper limit (7) [6,9,10,12,14]:

$$|V_{\alpha i}| \gtrsim 0.01. \qquad (8)$$

Using this value, we obtain from Eq. (5) a contribution to the light neutrino mass,

$$m_{\nu}^{(i)} \sim |V_{\alpha i}|^2 M_i = 10^7 \text{ eV} \left(\frac{|V_{\alpha i}|}{0.01}\right)^2 \left(\frac{M_i}{100 \text{ GeV}}\right). \qquad (9)$$

Thus, to reconcile $m_{\nu} \sim 0.1$ eV with the observability of

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[1]We will call any heavy singlet N that has Yukawa couplings with the usual (active) neutrinos a RH neutrino.

RH neutrinos at the LHC or the ILC, one needs to arrange a cancellation between the contribution from a given RH neutrino and some other contribution at the level of 10^{-8}. The situation improves only slightly if one considers more advanced machines like CLIC or an $e\gamma$ collider, which could increase the reach in the mixing angle by about an order of magnitude compared to Eq. (8) [7,8,10].

In what follows we will discuss cancellations between the contributions from different RH neutrinos, i.e. we will stay within the framework of the type-I seesaw scenario. One could also consider a cancellation with contributions from other mechanisms, for example, involving a Higgs triplet (type-II seesaw [40–43]), a fermion triplet (type-III seesaw [36,37]) or a radiatively generated neutrino mass [44,45]. However, in these cases contributions from different, in general unrelated sources have to cancel, which looks extremely implausible. The left-right symmetric models have been suggested as an exception, since there the type-I and type-II seesaw contributions can be related [46].

B. Cancellation of light neutrino masses

Let us consider first the necessary and sufficient conditions for an exact cancellation of contributions to the light neutrino masses. In the case of two RH neutrinos, two matrices have to cancel,

$$m_{\nu}^{(1)} + m_{\nu}^{(2)} = 0. \qquad (10)$$

Together with Eq. (2) this implies [17,19,20] proportionality of the vectors $\vec{m}_i$,

$$\vec{m}_1 = y_1 \vec{m}_0, \qquad \vec{m}_2 = y_2 \vec{m}_0 \qquad (\vec{m}_0 \equiv m(1, \alpha, \beta)^T), \qquad (11)$$

and

$$\frac{y_1^2}{M_1} + \frac{y_2^2}{M_2} = 0. \qquad (12)$$

Therefore, the Dirac mass matrix has the form

$$m_{\text{D}} = m \begin{pmatrix} y_1 & y_2 \\ \alpha y_1 & \alpha y_2 \\ \beta y_1 & \beta y_2 \end{pmatrix}. \qquad (13)$$

This result can be generalized to the case of three neutrinos [18,21,22]. The light neutrino mass matrix vanishes if and only if the Dirac mass matrix has rank 1,

$$m_{\text{D}} = m \begin{pmatrix} y_1 & y_2 & y_3 \\ \alpha y_1 & \alpha y_2 & \alpha y_3 \\ \beta y_1 & \beta y_2 & \beta y_3 \end{pmatrix}, \qquad (14)$$

and if

$$\frac{y_1^2}{M_1} + \frac{y_2^2}{M_2} + \frac{y_3^2}{M_3} = 0, \qquad (15)$$

where the mass parameters are defined in the basis where

the singlet mass matrix is diagonal. That is, the contributions from the three RH neutrinos to m_ν have to be equal up to a normalization factor in this case as well. Under the conditions (14) and (15), the light neutrino masses vanish exactly, to all orders in $m_D m_R^{-1}$. This can easily be seen by writing down the 6×6 mass matrix $\mathcal{M}$ and verifying that its rank is 3 or smaller. Consequently, the same is true for $\mathcal{M}^\dagger \mathcal{M}$, implying the existence of at least 3 vanishing mass eigenvalues. The νN-mixing relevant for collider physics, as given by Eq. (4), is not restricted by the cancellation condition (15) and hence allowed to be large enough to make the detection of RH neutrinos possible.

In the following, we will show that Eqs. (14) and (15) are also necessary conditions. Let us consider the case of k RH neutrinos coupled with three active neutrinos. (A general consideration of the case with an equal number of left- and right-handed neutrinos has been presented in [22].) We parametrize the contribution of the ith RH neutrino to the light Majorana mass matrix as

$$m_\nu^{(i)} = \mu_i \begin{pmatrix} 1 & \alpha_i & \beta_i \\ \alpha_i & \alpha_i^2 & \alpha_i \beta_i \\ \beta_i & \alpha_i \beta_i & \beta_i^2 \end{pmatrix} \qquad (i = 1 \ldots k). \quad (16)$$

Then the 11-, 12-, and 22-elements of the condition $m_\nu = \sum_i m_\nu^{(i)} = 0$ can be written as

$$\sum_{i=1}^k \mu_i = 0, \qquad \sum_{i=1}^k \alpha_i \mu_i = 0, \qquad \sum_{i=1}^k \alpha_i^2 \mu_i = 0. \quad (17)$$

Introducing $x_i \equiv \alpha_i / \alpha_1$, and subtracting the first equation in (17) from the second and third one (divided by α_1 and α_1^2, respectively), we obtain

$$\sum_{i=2}^k (x_i - 1)\mu_i = 0, \qquad \sum_{i=1}^k (x_i^2 - 1)\mu_i = 0. \quad (18)$$

Equation (18) is a system of linear equations for μ_i. A similar consideration for the 11-, 13-, and 33-elements of the condition $m_\nu = 0$ leads to the same system of equations with $x_i \rightarrow x_i' \equiv \beta_i / \beta_1$.

For $k = 2$ the first equation in (18) gives $\mu_2(x_2 - 1) = 0$ with the unique nontrivial solution $x_2 = 1$ or $\alpha_1 = \alpha_2$. Then the second equation is satisfied automatically. Similarly, one finds $\beta_1 = \beta_2$, and consequently $m_\nu^{(1)} \propto m_\nu^{(2)}$, so that we recover Eqs. (11) and (12).

For $k = 3$ the system

$$\begin{aligned} (x_2 - 1)\mu_2 + (x_3 - 1)\mu_3 &= 0, \\ (x_2^2 - 1)\mu_2 + (x_3^2 - 1)\mu_3 &= 0 \end{aligned} \quad (19)$$

has nontrivial solutions ($\mu_i \neq 0$) only if $(x_2 - 1) \times (x_3 - 1)(x_2 - x_3) = 0$ (zero determinant). If this condition is satisfied with $x_2 \neq 1$ or $x_3 \neq 1$, one μ_i is zero and $\mu_k = -\mu_j$ ($k, j \neq i$) for the two others. This implies that one RH neutrino decouples and the problem is reduced to the case

of two RH neutrinos with canceling contributions, cf. Eqs. (12) and (13). Thus, the only nontrivial case is $x_2 = x_3 = 1$, i.e. $\alpha_1 = \alpha_2 = \alpha_3$. Analogously, $\beta_1 = \beta_2 = \beta_3$, and consequently $m_\nu^{(1)} \propto m_\nu^{(2)} \propto m_\nu^{(3)}$. Then the definition (2) straightforwardly leads to

$$\vec{m}_1 \propto \vec{m}_2 \propto \vec{m}_3, \quad (20)$$

which proves that the rank of the Dirac mass matrix must be 1. Writing m_D as in Eq. (14) and plugging it into the condition $m_\nu = 0$ finally yields Eq. (15).

In the case of $k = 4$, we have two linear equations for three variables μ_2, μ_3, μ_4 and therefore the zero determinant condition does not apply: nontrivial solutions appear even if $x_i \neq \pm 1$. This means that the Majorana matrices generated by different RH neutrinos are not necessarily proportional to each other and nontrivial cancellation conditions appear.

One interesting (and the most symmetric) example is when cancellations occur between two pairs of matrices, for instance

$$m_\nu^{(1)} = -m_\nu^{(2)}, \qquad m_\nu^{(3)} = -m_\nu^{(4)}. \quad (21)$$

In this case two combinations of the light neutrinos couple with RH neutrinos and the latter form two heavy Dirac neutrinos. For $k = 6$, all three combinations of active neutrinos can couple to RH neutrinos. In what follows, we will concentrate mainly on the case of three RH neutrinos.

One can also obtain the cancellation condition using the Casas-Ibarra parametrization [47] for the Dirac mass matrix,

$$m_D = U_{PMNS} \sqrt{m_\nu} R \sqrt{m_R}, \quad (22)$$

where R is an arbitrary orthogonal matrix, $RR^T = \mathbb{1}$. This matrix disappears from the seesaw formula and therefore does not influence the light neutrino masses. On the other hand, R does influence the Dirac mass matrix and therefore the mixing of the RH neutrinos with the active neutrinos. In fact, the elements of R can be arbitrarily large, so that according to (22) one can obtain large m_D (and therefore large mixing) for arbitrarily small m_ν. We will show an example in the Appendix where the limit $m_\nu \rightarrow 0$ but $\sqrt{m_\nu} R = \text{const}$ recovers the cancellation conditions.

Equation (14) implies that only the combinations

$$\tilde{\nu} = \frac{\nu_e + \alpha^* \nu_\mu + \beta^* \nu_\tau}{\sqrt{1 + |\alpha|^2 + |\beta|^2}}, \qquad \tilde{N} = \frac{y_1 N_1 + y_2 N_2 + y_3 N_3}{\sqrt{\sum_i |y_i|^2}} \quad (23)$$

of left- and right-handed neutrinos participate in the Yukawa interactions. Two other combinations of the active neutrinos decouple and therefore remain massless. The mass of $\tilde{\nu}$ is zero because the contributions from the different RH components in $\tilde{N}$ cancel. In the next section we will

elaborate on this cancellation more and give a different interpretation.

The fact that only one combination of the left-handed (LH) neutrinos $\tilde{\nu}$ and one combination of the RH neutrinos $\tilde{N}$ couple can follow from a flavor symmetry. For example, in the basis of LH states that includes $\tilde{\nu}$ and the RH states that includes $\tilde{N}$, a U(1) symmetry with a simple charge assignment can lead to a single coupling. The cancellation condition (15) that involves both the Yukawa couplings and the masses does not show a simple symmetry in the most general case. Without a symmetry motivation, it is a fine-tuning condition and in addition unlikely to be stable against radiative corrections at the required level. In the following we will therefore discuss cases relying on symmetries.

C. Cancellation due to Lepton number conservation

Let us derive a symmetry that leads to the cancellation as well as the additional constraints it implies for the particular realization. According to our consideration in the previous section, only one combination of the active neutrinos has Yukawa interactions with singlets. Therefore, we consider the system of one active neutrino $\tilde{\nu}$ and two or three singlets. We require that all singlets have masses at the electroweak scale or higher or decouple from the system. Since there is only one light neutrino, the only mass that it may have is a Majorana mass. The Majorana mass is forbidden if we assign to $\tilde{\nu}$ a nonzero lepton number, e.g. $L(\tilde{\nu}) = 1$, and require it to be conserved in the whole system.[2] (Notice that in the case of two active components in the system they could form a light Dirac neutrino and our argument would not work.)

Next, we determine the lepton numbers of the singlets which ensure that only one combination couples to $\tilde{\nu}$ and that the singlets are massive. We can rewrite the Dirac mass term in Eq. (1) as $\tilde{m} \, \overline{\tilde{\nu}} \, \tilde{N}$, where

$$\tilde{m} \equiv m \sqrt{\sum_i |y_i|^2 (1 + |\alpha|^2 + |\beta|^2)}. \qquad (24)$$

This mass term implies that $\tilde{N}$ has the lepton number $L(\tilde{N}) = 1$, and all other RH neutrinos have $L \neq 1$.

We first consider the case of two RH neutrinos, denoting by N' the combination of RH components that is orthogonal to $\tilde{N}$. Then the only way to generate a mass for N' and $\tilde{N}$ that is consistent with lepton number conservation is to prescribe $L(N') = -1$ and to introduce the mass term $M \overline{N'^C} \tilde{N}$. Combining the mass terms,

$$-(\tilde{m} \, \overline{\tilde{\nu}} + M \overline{N'^C}) \tilde{N} + \text{H.c.}, \qquad (25)$$

we see that $\tilde{\nu}$ mixes with N'^C to form a heavy Dirac neutrino together with $\tilde{N}$. This Dirac neutrino has mass $\sqrt{M^2 + \tilde{m}^2}$, while the orthogonal combination of $\tilde{\nu}$ and N'^C is massless. In this way we have arrived at the symmetry structure used previously in order to obtain the cancellation in [15,16,23,25,26].

In the case of three singlets, there are two combinations N_1' and N_2' orthogonal to $\tilde{N}$, and consequently, several possibilities to realize lepton number conservation and the cancellation appear [48]:

(1) $L(N_1') = -1$ and $L(N_2') \neq \pm 1$ (or vice versa). In this case a Dirac particle arises as before, whereas N_2' decouples from the system. It can have a Majorana mass if $L(N_2') = 0$.

(2) $L(N_2') = L(N_1') = -1$. Now both N_1' and N_2' can couple to $\tilde{N}$. Then the corresponding combination of $N_1'^C$ and $N_2'^C$ appears in Eq. (25) and forms a Dirac pair with $\tilde{N}$. The orthogonal combination is massless and decouples.

Thus, in all cases with two and three singlets, we arrive at the same conclusion: if the cancellation of active neutrino masses is a consequence of lepton number conservation, this symmetry leads to one decoupled singlet and to the existence of a Dirac fermion formed predominantly by the other two singlets, i.e. the symmetry yields the structure (25). Because of symmetry the structure is stable under radiative corrections. In the flavor basis for the LH neutrinos, $L(\nu_\alpha) = 1$ for all flavors α, and the mass matrices read[3]

$$m_R' = \begin{pmatrix} 0 & M & 0 \\ M & 0 & 0 \\ 0 & 0 & M_3 \end{pmatrix}, \qquad m_D' = m \begin{pmatrix} a & 0 & 0 \\ b & 0 & 0 \\ c & 0 & 0 \end{pmatrix}. \qquad (26)$$

If the decoupled singlet has nonzero lepton number, then $M_3 = 0$. We will refer to Eq. (26) as the cancellation structure hereafter.

Instead of lepton number conservation, we can use some discrete subgroup of U(1)$_L$ to ensure the cancellation. For example, invariance under $\tilde{\nu} \rightarrow i\tilde{\nu}$, $\tilde{N} \rightarrow i\tilde{N}$, $N' \rightarrow -iN'$ does the same job. In models of this type, U(1)$_L$ reappears as an accidental symmetry of the discussed mass terms, but it may be broken in other sectors explicitly to avoid a massless Majoron [49,50]. In the case of 4 and more RH neutrinos, more than one combination of active neutrinos couples with the singlets, cf. Eq. (21), and the arguments presented here do not apply.

The suppression of the masses of the active neutrinos is not due to the seesaw mechanism but due to mixing with additional states and mismatch between the number of left- and right-handed fields [15]. So we can conclude that observation of the RH neutrinos at LHC and other colliders

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[2] More precisely, we impose a global symmetry U(1)$_L$ under which the SM particles have a charge L that equals their lepton number. Thus, it leads to the same consequences as lepton number conservation in the SM, in particular, massless neutrinos and vanishing amplitudes for L-violating processes.

[3] A prime denotes quantities in a basis where the singlet mass matrix is nondiagonal.

would imply that at least those RH neutrinos do not participate in the seesaw mechanism.

D. Three degenerate singlets and the discrete symmetry A_4

In general, the cancellation condition does not require the conservation of lepton number. In the basis $\tilde{\nu}$, $\tilde{N}$, N'_1, N'_2, the sufficient condition for the cancellation is that the determinant of the $N'_1 N'_2$-block of the singlet mass matrix should be zero. This does not forbid entries in the singlet mass matrix that violate lepton number. Hence, one may ask whether symmetries exist which lead to the cancellation, but not to L-conservation.

Let us first assume that such a symmetry produces equal masses for all singlets, $m_R = M \mathbb{1}$, and realizes Eq. (15) in such a way that seemingly all three singlets participate in canceling the light neutrino masses,

$$y_1^2 + y_2^2 + y_3^2 = 0. \tag{27}$$

We will now show that in this case there always exist a Dirac pair of heavy neutrinos and a decoupled singlet. That is, in fact the system does realize lepton number conservation.

From Eq. (14) we know that the Dirac mass terms have the form

$$- m \overline{\tilde{\nu}}(y_1 N_1 + y_2 N_2 + N_3) + \text{H.c.}, \tag{28}$$

where without loss of generality we have set $y_3 = 1$. Now the cancellation condition (27) reads $y_1^2 + y_2^2 = -1$. Recall that y_i are complex parameters and that in general $|y_i|^2 \neq 1$. Performing an orthogonal transformation $N_i \rightarrow N_{ir}$, it is straightforward to check[4] that the mass term (28) can be reduced to

$$- \kappa \overline{\tilde{\nu}}(i N_{1r} + N_{2r}) + \text{H.c.}, \tag{29}$$

where $\kappa = m \sqrt{(\text{Im}y_1)^2 + (\text{Im}y_2)^2}$. Thus, the third singlet decouples. As the singlet mass matrix is proportional to the unit matrix, it does not change under the transformation.

Introducing $\tilde{N} = (i N_{1r} + N_{2r})/\sqrt{2}$ and the orthogonal combination $N'_1 = (-i N_{1r} + N_{2r})/\sqrt{2}$, we find that the Yukawa couplings and the mass term become $\sqrt{2}\kappa \overline{\tilde{\nu}}\tilde{N}$ and $M \overline{N'^C_1} \tilde{N}$, respectively. Consequently, $\tilde{N}$ and N'_1 form a Dirac pair and we reproduce precisely the structure (26) that corresponds to lepton number conservation.

As an interesting special case, let us consider the most symmetric scenario where $|y_1|^2 = |y_2|^2 = |y_3|^2 = 1$ or $y_1 = 1$, $y_2 = \omega$, $y_3 = \omega^2$, where $\omega = e^{(2\pi i)/3}$. This scenario can arise from the discrete flavor symmetry A_4. Suppose that the singlets transform under the representa-

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[4]One has to perform a rotation in the 12-plane that leads to a real Yukawa coupling of the second RH neutrino and afterwards a rotation in the 23-plane that makes the coupling of the third RH neutrino vanish.

tion $\underline{3}$, while all the LH neutrinos transform under $\underline{1}''$ in the notation of [51]. Then $m_R = M \mathbb{1}$, and a Dirac mass matrix is obtained from the interactions

$$\sum_{i=1}^{3} h_i \bar{\nu}_i (N_1 \phi_1 + \omega N_2 \phi_2 + \omega^2 N_3 \phi_3),$$

where h_i are coupling constants and ϕ is a scalar transforming under $\underline{3}$ with vacuum expectation values (vevs) $\phi_k \equiv v_k$. We find

$$m_D = \begin{pmatrix} h_1 v_1 & \omega h_1 v_2 & \omega^2 h_1 v_3 \\ h_2 v_1 & \omega h_2 v_2 & \omega^2 h_2 v_3 \\ h_3 v_1 & \omega h_3 v_2 & \omega^2 h_3 v_3 \end{pmatrix}. \tag{30}$$

In the notation of Eq. (14), this corresponds to $m = h_1 v_1$, $y_1 = 1$, $y_2 = \omega v_2/v_1$, and $y_3 = \omega^2 v_3/v_1$, so that vanishing light neutrino masses are obtained for

$$v_1 = v_2 = v_3 = v \tag{31}$$

(up to phase factors), which is required in most mass models based on A_4. If one did not assign the LH neutrinos to a one-dimensional representation, producing a rank-1 Dirac mass matrix would require tuning or a nontrivial extension of the symmetry.

Transforming the RH fields into $\tilde{N} = U^\dagger_{\text{mag}} N$, where

$$U_{\text{mag}} = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{pmatrix} \tag{32}$$

is the magic matrix, we obtain the Dirac term $\tilde{m} \overline{\tilde{\nu}} \tilde{N}_3$ with $\tilde{m} = \sqrt{3} v \sqrt{|h_1|^2 + |h_2|^2 + |h_3|^2}$, and the mass matrix of the RH neutrinos $\tilde{N}$,

$$\tilde{m}_R = U_{\text{mag}} U_{\text{mag}} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}. \tag{33}$$

That is, $\tilde{\nu}$ mixes with $\tilde{N}^C_2$ and forms a Dirac pair with $\tilde{N}_3$. The decoupled singlet $\tilde{N}_1$ retains the mass M, while the Dirac neutrino is slightly heavier. Thus, lepton number conservation arises as an accidental global symmetry in this A_4 toy model.

E. Cancellation without lepton number conservation?

In the above examples, the singlets contributing to the cancellation mechanism have equal masses. Because of the imposed symmetry the cancellation is stable against radiative corrections. Let us relax the requirement of equal masses and consider the renormalization group evolution of neutrino masses. Suppose that the singlets N_1 and N_2 are relevant for the cancellation and that the condition (15) is imposed by a symmetry at the energy scale M_2, i.e. $m_\nu^{(1)}(M_2) = -m_\nu^{(2)}(M_2)$. Below this scale, the symmetry is broken. We use an effective theory where N_2 is integrated out, so that the contribution $m_\nu^{(2)}$ corresponds to a

dimension-5 operator. In the SM, the running of this operator differs from that of $m_\nu^{(1)}$ [52,53], so that

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\begin{aligned}
\frac{d}{dt} m_\nu \bigg|_{\mu=M_2} &= \frac{d}{dt} m_\nu^{(1)} \bigg|_{\mu=M_2} + \frac{d}{dt} m_\nu^{(2)} \bigg|_{\mu=M_2} \\
&= \frac{1}{16\pi^2} (\alpha_1 m_\nu^{(1)}(M_2) + \alpha_2 m_\nu^{(2)}(M_2)) \\
&= -\frac{1}{16\pi^2} \left(\lambda + \frac{3}{2} g^2 + \frac{3}{2} g'^2 \right) m_\nu^{(1)}(M_2),
\end{aligned}
$$
(34)

where $t \equiv \ln(\mu/\mu_0)$ and μ is the energy scale. The term involving the Higgs self-coupling λ and the gauge couplings g, g' is of order 1. Thus, using the estimate $m_\nu^{(1)} \sim 0.01$ GeV from Eq. (9), we obtain a light neutrino mass of

$$
m_\nu(M_1) \sim \frac{d}{dt} m_\nu \bigg|_{\mu=M_2} \Delta t \sim 10^{-4} \text{ GeV } \ln\frac{M_2}{M_1}
$$
(35)

at M_1, which is unacceptable unless N_1 and N_2 are nearly degenerate. Of course, the problem becomes even worse if also the third singlet contributes to the cancellation, since then there are additional corrections from the running between M_2 and M_3.

As the running is due to diagrams with Higgs fields in the loop, our estimate is not reliable if the Higgs is heavier than M_2. In supersymmetric theories, both $m_\nu^{(1)}$ and $m_\nu^{(2)}$ obey the same renormalization group equation due to the nonrenormalization theorem, so that m_ν remains zero above the mass scale M_{SUSY} of the superparticles. However, as long as $M_1 < M_{\mathrm{SUSY}}$, the SM can be used as an effective theory below this scale, so that the estimate (35) remains valid if we replace M_2 by M_{SUSY}. In any case, further changes of the neutrino mass matrix from threshold corrections [20,54,55] tend to yield too large masses as well, although a simple model-independent estimate is more difficult.

These arguments suggest that the cancellation of light neutrino masses can only be realized without fine-tuning, if the RH neutrinos contributing to the cancellation are nearly degenerate in mass. As we have seen in the previous sections, this implies the existence of a symmetry $U(1)_L$ which guarantees the conservation of total lepton number in the light sector. Consequently, any more complicated

symmetry leading to vanishing neutrino masses has to contain $U(1)_L$ as a subgroup or accidental symmetry.

III. NONZERO NEUTRINO MASSES FROM PERTURBATIONS

We will now discuss small perturbations of the cancellation structure introduced in the previous section that lead to nonvanishing light neutrino masses. We will identify the simplest cases which result in viable neutrino masses and mixings.

A. Generic perturbations

In the case of the Dirac pair of Sec. II C, the most general possibility for perturbing the cancellation structure (26) is

$$
\begin{aligned}
m'_{\mathrm{R}} &= \begin{pmatrix} \epsilon_1 M & M & \epsilon_{13} M \\ M & \epsilon_2 M & \epsilon_{23} M \\ \epsilon_{13} M & \epsilon_{23} M & M_3 \end{pmatrix}, \\
m'_{\mathrm{D}} &= m \begin{pmatrix} a & \delta_a & \epsilon_a \\ b & \delta_b & \epsilon_b \\ c & \delta_c & \epsilon_c \end{pmatrix} \equiv m(r, r_\delta, r_\epsilon).
\end{aligned}
$$
(36)

Considering the entries in the mass matrices as spurions, we can immediately see which of them are relevant for the light neutrino masses. The latter have a lepton number of $+2$, so that they will receive contributions from combinations of parameters which also have $L = +2$. We have $L(\epsilon_2) = L(r_\delta) = 2$, so that these parameters can contribute directly via terms also involving the large Yukawa couplings r. As $L(\epsilon_{23}) = L(r_\epsilon) = 1$, these quantities will appear quadratically (or in combinations of 2 or more different small parameters). If all perturbations are of the same order of magnitude, these contributions will be subleading. Finally, $L(\epsilon_1)$ and $L(\epsilon_{13})$ are negative, so that terms involving these quantities have to contain at least 2 more small parameters. Consequently, they are almost completely irrelevant for neutrino masses at the tree level.

Explicitly, we obtain

$$
\begin{aligned}
m_\nu = -m^2 [&(m'^{-1}_{\mathrm{R}})_{11} rr^T + (m'^{-1}_{\mathrm{R}})_{12} (rr_\delta^T + r_\delta r^T) \\
&+ (m'^{-1}_{\mathrm{R}})_{13} (rr_\epsilon^T + r_\epsilon r^T) + (m'^{-1}_{\mathrm{R}})_{22} r_\delta r_\delta^T \\
&+ (m'^{-1}_{\mathrm{R}})_{23} (r_\delta r_\epsilon^T + r_\epsilon r_\delta^T) + (m'^{-1}_{\mathrm{R}})_{33} r_\epsilon r_\epsilon^T]
\end{aligned}
$$
(37)

with

$$
m'^{-1}_{\mathrm{R}} = \frac{1}{M_3} \begin{pmatrix} -\frac{M_3}{M}\epsilon_2 + \epsilon_{23}^2 & \frac{M_3}{M} + \frac{M_3}{M}\epsilon_1\epsilon_2 + \epsilon_{13}\epsilon_{23} & -\epsilon_{23} + \epsilon_2\epsilon_{13} \\ \frac{M_3}{M} + \frac{M_3}{M}\epsilon_1\epsilon_2 + \epsilon_{13}\epsilon_{23} & -\frac{M_3}{M}\epsilon_1 + \epsilon_{13}^2 & -\epsilon_{13} + \epsilon_1\epsilon_{23} \\ -\epsilon_{23} + \epsilon_2\epsilon_{13} & -\epsilon_{13} + \epsilon_1\epsilon_{23} & 1 + 2\frac{M}{M_3}\epsilon_{13}\epsilon_{23} \end{pmatrix} + \frac{1}{M}\mathcal{O}(\epsilon^3).
$$

In the following, we will assume that $\max(a, b, c) \sim 1$, $m/M \sim 0.1$, $M \sim 0.1$ TeV (as required by observability of N_i at LHC), that all ϵ_i in m'_{R} are of the same order of magnitude, and that no severe cancellations occur in

Eq. (37). Then neutrino masses $m_\nu \sim 0.1$ eV require each term in square brackets to be of order 10^{-9} TeV^{-1} or smaller. Applying this criterion to the first term and the second one, respectively, we obtain

$$\epsilon_2 \lesssim 10^{-10}, \tag{38}$$

$$\max(\delta_a, \delta_b, \delta_c) \lesssim 10^{-10}. \tag{39}$$

Considering the last term with $M_3 \sim 1$ TeV for concreteness, we obtain

$$\max(\epsilon_a, \epsilon_b, \epsilon_c) \lesssim 10^{-4.5}, \tag{40}$$

i.e. this term would be negligible if also $r_\epsilon \sim r_\delta$. From these constraints on ϵ_2, r_δ, and r_ϵ, it follows that all other terms in Eq. (37) are negligible under our assumptions, so that

$$m_\nu \approx \frac{m^2}{M}[\epsilon_2 r r^T - (r r_\delta^T + r_\delta r^T)] - \frac{m^2}{M_3} r_\epsilon r_\epsilon^T. \tag{41}$$

For completeness, we also list the constraint

$$\epsilon_{23} \lesssim 10^{-4.5} \tag{42}$$

for $M_3 \sim 1$ TeV, which can be obtained from the first term in brackets in Eq. (37). As mentioned, the remaining L-violating parameters are all but irrelevant for neutrino masses and thus unconstrained at the tree level. However, they contribute to one-loop threshold corrections to m_ν [20]. If only ϵ_1 is nonzero, we find, using the result given in [26],

$$\Delta m_\nu \approx \epsilon_1 \frac{g^2}{128\pi^2} \frac{m^2}{M} f(M, M_H) r r^T, \tag{43}$$

where f is of order 1 and depends on M and the Higgs mass. Requiring for simplicity the corrections to be significantly smaller than the tree-level masses, $\Delta m_\nu \lesssim 0.01$ eV, we find

$$\epsilon_1 \lesssim 10^{-8}. \tag{44}$$

The parameter ϵ_{13} violates L by one unit and therefore enters Δm_ν quadratically. Furthermore, its contribution is suppressed by M/M_3. Hence, it is less constrained,

$$\epsilon_{13} \lesssim 10^{-3.5}, \tag{45}$$

again for $M_3 \sim 1$ TeV. All other perturbations yield negligible radiative corrections if they satisfy the above tree-level limits.

B. Special cases
1. Only $\epsilon_2 \neq 0$

Returning to Eq. (41), we see that there are obviously enough free parameters to fit the measured neutrino mass parameters and to prevent any observable imprint of the cancellation structure. Let us therefore look at some more constrained cases. The simplest possibility is that the dominant contribution comes from the first term in brackets,

$$m_\nu \approx \epsilon_2 \frac{m^2}{M}(a, b, c)(a, b, c)^T. \tag{46}$$

Notice that this perturbation generates the singular mass matrix of light neutrinos that is required by data in the first approximation. The condition $b = c$ leads to a maximal atmospheric mixing angle. One also finds

$$\sin\theta_{13} = \frac{|a|}{\sqrt{|a|^2 + |b|^2 + |c|^2}}, \tag{47}$$

so that the experimental 3σ limit $\sin^2\theta_{13} < 0.04$ [56] translates into $|\frac{b}{a}| \gtrsim 3.5$ for $|b| \approx |c|$.

2. $\epsilon_2 \neq 0$, $r_\epsilon \neq 0$, $r_\delta = 0$

Perturbations of the Dirac mass matrix are needed to generate a second nonvanishing mass and the solar mixing angle, since the rank of the product $m_D m_R^{-1} m_D^T$ and thus the number of massive neutrinos is at most as large as the minimum of the rank of m_D and the rank of m_R. Adding nonvanishing entries in either r_ϵ or r_δ is sufficient. If only $r_\epsilon \neq 0$, we can have a scenario where two neutrinos are light due to the U(1)$_L$ symmetry while the third mass is suppressed by the usual seesaw mechanism, i.e. large M_3. If $r_\epsilon \sim 1$, M_3 has to be larger than about 10^{12} GeV here. A relatively simple choice of parameters leading to viable neutrino masses and mixings with a normal mass hierarchy is

$$m = 10 \text{ GeV}, \quad M = 100 \text{ GeV}, \quad M_3 = 10^{12} \text{ GeV},$$
$$\epsilon_2 = 2.5 \times 10^{-11}, \quad a = 0, \quad b = -c = 1,$$
$$\epsilon_a = \epsilon_b = \epsilon_c = 0.17.$$

In [26] an alternative situation was studied where all singlet masses are equal in the leading order, $M_3 = M$. This is enforced by an SO(3) flavor symmetry, which contains U(1)$_L$ as a subgroup and also motivates the smallness of the perturbations in the singlet mass matrix.

3. Only $r_\delta \neq 0$

If r_δ is not much smaller than r_ϵ, its contribution to m_ν will dominate over that from r_ϵ as mentioned above. We find a particularly interesting case by assuming that the term which involves ϵ_2 is negligible as well.[5] Then the neutrino mass matrix

$$m_\nu \approx -\frac{m^2}{M}(r r_\delta^T + r_\delta r^T) \tag{48}$$

has rank 2, so that we can obtain a realistic mass spectrum

---

[5]A nonzero ϵ_2 can be absorbed into $r_\delta' \equiv r_\delta + \frac{\epsilon_2}{2} r$, so that it does not change the discussion.

with a strong hierarchy from the perturbation r_δ alone. Corrections from the neglected terms yield a tiny mass for the lightest state. In order to verify that this form of m_ν is indeed compatible with the known neutrino masses and mixings, we have determined values of the parameters that lead to tribimaximal mixing [57] and mass squared differences within the experimentally allowed ranges [56]. In this case $\theta_{13} = 0$, which places rather strong restrictions on the form of m_ν. Nevertheless, solutions for r and r_δ can be found. The entries $|a|$, $|b|$, and $|c|$ have to be roughly of the same order, and similarly $|\delta_a|$, $|\delta_b|$, and $|\delta_c|$. One choice leading to an inverted mass hierarchy and a negative CP parity for one mass eigenstate is[6]

$$m = 2.8 \text{ GeV}, \qquad M = 100 \text{ GeV}, \qquad a = 1,$$
$$b = c \approx 0.12, \qquad \delta_a \approx 1.0 \times 10^{-10},$$
$$\delta_b = \delta_c \approx -4.3 \times 10^{-10}.$$

The mass matrix (48) was studied in the context of leptogenesis in [59]. It was found that a normal mass hierarchy with θ_{13} not far below the experimental bound is most natural, if there are no hierarchies or special relations between the parameters in r and r_δ. The branching ratios for the flavor-violating decays $l_i \rightarrow l_j \gamma$ in supersymmetric seesaw models turned out to be related via the observed neutrino masses and mixings and of comparable size.

4. Remarks

Let us conclude the discussion with some comments about variants of the scenario, which may be useful input for the construction of models explaining the perturbations. In order to reduce the number of free parameters, one could impose an "L parity," i.e. a $\mathbb{Z}_2$ symmetry under which all fields with nonzero lepton number change sign. Then only the perturbations ϵ_1, ϵ_2, and r_δ are allowed, which violate L by two units.

If the main goal is avoiding tiny parameters instead, one could use only terms which violate lepton number by one unit, since they appear quadratically in the light neutrino mass matrix as mentioned. This means that only couplings of the singlet N_3 contribute at the tree level, leaving two active neutrinos massless. A second mass can then be generated by radiative corrections if ϵ_{13} is sufficiently large. Alternatively, one could impose the restriction that all perturbations are related to a single more fundamental parameter ε violating L by one unit, i.e. ϵ_{23}, $r_\epsilon \sim \varepsilon$ and ϵ_2, $r_\delta \sim \varepsilon^2$. Then the contributions of all these parameters to the neutrino masses are of similar sizes, if M_3 is not much larger than M. The dependence of $(m_R'^{-1})_{11}$ on ϵ_{23} is non-negligible, and the term proportional to $(m_R'^{-1})_{13}$ in Eq. (37) becomes relevant in general. For $m = 10$ GeV

and $M_3 \sim 10 M \sim 1$ TeV, the value $m_\nu \sim 0.1$ eV requires $\varepsilon \lesssim 10^{-5}$. Finally, one could invoke a cancellation of the leading-order contributions due to ϵ_2 and r_δ, which occurs for $r_\delta = \frac{\epsilon_2}{2} r$ according to Eq. (41), in order to allow larger values for these parameters.

C. A_4 model

One may hope to obtain a connection between the leading-order mass matrices relevant for LHC and the perturbations responsible for nonvanishing neutrino masses in the case of the A_4 toy model discussed in Sec. II D, postulating that the perturbations leave a subgroup of A_4 unbroken. The vevs (31) break A_4 down to a $\mathbb{Z}_3$ subgroup, so that e.g. radiative corrections will generate new couplings that are invariant under $\mathbb{Z}_3$ but not under A_4 [60]. However, we find that these do not change the form of the Dirac mass matrix, similarly to what happens in the models discussed in [60,61]. The form of the singlet mass matrix does change because it obtains nonvanishing off-diagonal entries. As a consequence of $\mathbb{Z}_3$, these entries are all equal and therefore the active neutrinos remain massless.

Consequently, we have to consider additional symmetry breaking. The remaining options are the $\mathbb{Z}_2$ subgroups of A_4. If the A_4-triplet scalar χ responsible for this breaking coupled to the neutrinos via renormalizable interactions, new entries would be generated in every element of the Dirac mass matrix, destroying all predictivity. Let us therefore assume that χ is a SM singlet, so that it can couple to the neutrinos only via the nonrenormalizable operator

$$\sum_{i=1}^3 \kappa_i \bar{\nu}_i (N_2 \phi_3 \chi_1 + \omega N_3 \phi_1 \chi_2 + \omega^2 N_1 \phi_2 \chi_3)$$
$$+ \kappa_i' \bar{\nu}_i (N_3 \phi_2 \chi_1 + \omega N_1 \phi_3 \chi_2 + \omega^2 N_2 \phi_1 \chi_3)$$

with couplings κ_i and κ_i' of dimension (mass)$^{-1}$, and the Yukawa term

$$\lambda[(\overline{N_2^c}N_3 + \overline{N_3^c}N_2)\chi_1 + (\overline{N_3^c}N_1 + \overline{N_1^c}N_3)\chi_2$$
$$+ (\overline{N_1^c}N_2 + \overline{N_2^c}N_1)\chi_3].$$

For concreteness, we assume that χ develops the vev

$$\chi = (v_\chi, 0, 0). \tag{49}$$

Then the corrections to the mass matrices are

$$\Delta m_D = v v_\chi \begin{pmatrix} 0 & \kappa_1 & \kappa_1' \\ 0 & \kappa_2 & \kappa_2' \\ 0 & \kappa_3 & \kappa_3' \end{pmatrix},$$

$$\Delta m_R = \lambda v_\chi \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}. \tag{50}$$

---

[6]The Dirac masses were chosen a bit smaller here in order to satisfy the bound $|\sum_i V_{ei} V_{\mu i}^*| \lesssim 10^{-4}$ from the nonobservation of the decay $\mu \rightarrow e\gamma$ [58].

The complete Dirac mass matrix can have rank 3. To first order in κ_i, κ_i', and $\epsilon \equiv \frac{\lambda v_\chi}{M}$, the elements of the light neutrino mass matrix equal

$$(m_\nu)_{ij} = \frac{v^2}{M}(2h_ih_j\epsilon - h_j\bar{\kappa}_i - h_i\bar{\kappa}_j) \qquad (51)$$

with $\bar{\kappa}_i \equiv \omega v_\chi(\kappa_i + \omega\kappa_i')$. This is the mass matrix of Eq. (41) for $r_\epsilon = 0$, which is compatible with observations, see Sec. III B. We obtain a strong mass hierarchy with the lightest neutrino receiving a mass only from higher-order corrections. Note that including the above-mentioned $\mathbb{Z}_3$-invariant corrections changes only h_i but not ϵ at the considered level of accuracy. If the position of the nonzero entry in $\langle\chi\rangle$ is changed compared to Eq. (49), ϵ and $\bar{\kappa}_i$ will change by factors ω or ω^2, but the form of m_ν will remain unaltered.

Thus, we have constructed a pattern of symmetry breaking that produces perturbations leading to a viable light neutrino mass matrix, which we had found in the previous section by introducing all possible small perturbations and assuming some of them to dominate. The smallness of the perturbations in the Dirac mass matrix can be motivated by the fact that they arise from nonrenormalizable interactions.

IV. COLLIDER SIGNATURES

In this section, we turn to the consequences of the discussed scenarios for processes involving RH neutrinos at colliders. Their charged-current gauge interactions are given by

$$\mathcal{L}_{cc} = -\frac{g}{\sqrt{2}}\bar{l}_\alpha V_{\alpha i}\gamma^\mu W_\mu \frac{1-\gamma_5}{2}N_i^0 + \text{H.c.}, \qquad (52)$$

where l_α is a charged lepton and N_i^0 is a heavy neutrino mass eigenstate. The Feynman diagrams for the most important processes at LHC [10] are shown in Fig. 1.

A. Lepton number violation

As a promising signal for the production of singlet neutrinos, L-violating processes with like-sign leptons in the final state have been suggested. Their amplitudes are
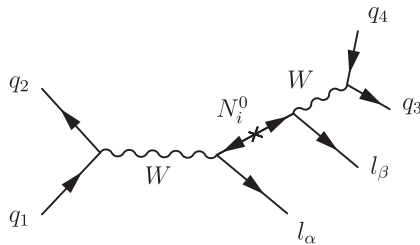
proportional to the combination

$$A_{\text{LNV}} \equiv V_{\alpha i}\frac{M_i}{p^2 - M_i^2 + iM_i\Gamma_i}V_{\beta i}, \qquad (53)$$

where Γ_i is the width of N_i^0. For $M_i \sim 100$ GeV and $|V_{\alpha i}| \sim 0.1$, one finds $\Gamma_i \sim 0.01$ GeV [10]. The quantity A_{LNV} also controls the contribution of the RH neutrinos to neutrinoless double beta decay. The amplitude is proportional to A_{LNV} with $p^2 \to 0$ and $\alpha = \beta = e$.

All the scenarios for the suppression of the light neutrino masses discussed above involve the conservation of lepton number, so that A_{LNV} vanishes. As an explicit example, consider the case of a heavy Dirac pair. Then $M_1 = M_2 = M$, and the mixing matrix of the light and the heavy neutrinos reads

$$V \approx m_D m_R^{-1} = \frac{m}{\sqrt{2}M}\begin{pmatrix} ai & a & 0 \\ bi & b & 0 \\ ci & c & 0 \end{pmatrix} \qquad (54)$$

in the basis where m_R is diagonal and in the usual seesaw approximation $m \ll M$. In order to check the accuracy of the approximation, we have diagonalized the 6×6 mass matrix $\mathcal{M}$ exactly in the special case $a = b = c$, finding no significant changes. Obviously, A_{LNV} vanishes for all flavors α, β.

If L violation is introduced, A_{LNV} will be proportional to the corresponding couplings, which are restricted to be tiny by the smallness of neutrino masses. Hence, the suppression of the cross section emerges in a very similar way as in the usual seesaw scenario. Sizable lepton number violation would require the perturbations of the cancellation structure to split the masses of the singlets forming the Dirac pair by an amount ΔM significantly larger than their decay width. In this case, only one singlet would be produced on-shell and dominate A_{LNV}, resulting in a nonzero amplitude. If, for instance, $p^2 = M_1^2$, then

$$A_{\text{LNV}} = \frac{1}{i\Gamma_1}V_{\alpha 1}V_{\beta 1} + \frac{M_2}{M_1^2 - M_2^2 + iM_2\Gamma_2}V_{\alpha 2}V_{\beta 2}$$

$$\approx \frac{1}{i\Gamma_1}V_{\alpha 1}V_{\beta 1} - \frac{1}{2\Delta M}V_{\alpha 2}V_{\beta 2}. \qquad (55)$$
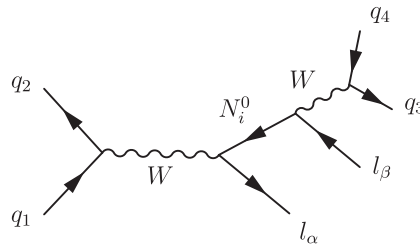

FIG. 1. Feynman diagrams for lepton-number-violating (left) and lepton-flavor-violating processes (right) involving heavy neutrinos at the LHC.

For example, the mass splitting caused by ϵ_1 is roughly $\Delta M \approx \epsilon_1 M$. Consequently, for $\Delta M \sim 1$ GeV $\gg \Gamma_i$, we need $\epsilon_1 \sim 0.01$ (again in the case $M \sim 100$ GeV). This is still a small perturbation but orders of magnitude above the bound (44), so that we cannot avoid unacceptable active neutrino masses without fine-tuning. The parameter ϵ_{13} enters the mass splitting quadratically and therefore has to be larger than ϵ_1 to achieve the same splitting $\Delta M \sim 1$ GeV, e.g. $\epsilon_{13} \sim 0.3$ for $M \sim 100$ GeV and $M_3 \sim 1$ TeV. On the other hand, the bound (45) is weaker than Eq. (44) and can be further relaxed if one allows the one-loop correction to the neutrino masses to be of the same order of magnitude as the tree-level terms. Leaving aside the problem of explaining the large hierarchy between the perturbations ϵ_i, lepton number violation via a large ϵ_{13} may then be achievable with tuning at the percent level.

B. Lepton flavor violation

If L-violating effects are too small to be observable, one can still hope to detect events with different lepton flavors such as $e^- \mu^+$ in the final state, since these have a relatively small SM background as well. According to [14], such signals are unlikely to be observable at LHC, however. Now the amplitude is proportional to

$$A_{\text{LFV}} \equiv V_{\alpha i} \frac{\not{p}}{p^2 - M_i^2} V_{\beta i}^*. \qquad (56)$$

In the considered scenarios, the mechanism leading to the cancellation of A_{LNV} causes the different terms in A_{LFV} to add up constructively. Again considering the example of Eq. (54), we obtain

$$A_{\text{LFV}} = \frac{\not{p}}{p^2 - M^2} (V_{\alpha 1} V_{\beta 1}^* + V_{\alpha 2} V_{\beta 2}^*)$$
$$= \frac{\not{p}}{p^2 - M^2} \frac{m^2}{M^2} (a, b, c)_\alpha (a^*, b^*, c^*)_\beta. \qquad (57)$$

Hence, lepton-flavor-violating (LFV) amplitudes can be sizable. This also means that bounds from low-energy searches for rare decays cannot be avoided by cancellations. The most stringent limit,

$$\left| \sum_i V_{ei} V_{\mu i}^* \right| = \frac{m^2}{M^2} |ab^*| \lesssim 10^{-4}, \qquad (58)$$

comes from the nonobservation of the decay $\mu \to e\gamma$ [58]. In order to have at least a small chance of observing events at LHC, this condition has to be satisfied with either a or b being very small and the other parameter of order 1. In the most minimal examples for perturbations, we have seen that the large atmospheric mixing angle implies $|b| \sim |c|$. In this case, all amplitudes would be suppressed if b were small. Therefore, making a tiny is the more favorable option. Then flavor-violating processes with electrons in the final state are not observable, leaving processes with

the final state $\tau^\pm \mu^\mp$ as the best candidate for observing singlets.

At the ILC, the situation is more hopeful, since there the resonant production of RH neutrinos is possible for $|V|_{ei} \gtrsim 0.01$ [6,10], which is allowed by Eq. (58) even if $|a| \sim |b|$. By observing the branching ratios for the subsequent decays into charged leptons, one could determine the mixings with the different flavors directly.

C. Decoupling of collider physics from the light masses

If the observation of RH neutrinos at colliders is to shed light onto the mechanism of neutrino mass generation, the first key question we have to ask is whether the perturbations responsible for neutrino masses could have consequences for signals at colliders. Unfortunately, the smallness of the light neutrino masses immediately tells us that the answer is negative. All perturbations of the couplings of relatively light singlets yielding neutrino masses are restricted to be tiny. Thus, they will not lead to observable collider signatures. Instead, collider experiments are only sensitive to the large Yukawa couplings in Eq. (26), i.e. to the cancellation structure of the mass matrices which does not produce neutrino masses.

This leads to the second key question, whether perturbations can be introduced in such a manner that the light neutrino mass matrix still ''remembers'' in some way the cancellation structure. In other words, can perturbations lead to particular features of the light neutrino mass matrix, so that the cancellation structure is imprinted in the structure of m_ν? As argued above, a light neutrino mass matrix with at least two nonvanishing eigenvalues can only be obtained if the Dirac mass matrix is perturbed. In general, this introduces many new parameters, so that there is little hope to find a simple connection between m_ν and the cancellation structure. Then the answer to the second question is negative, too.

The situation is better in constrained setups where only some of the perturbations are present or dominant. In the cases we discussed, a strong mass hierarchy is expected. The number of free parameters is large enough to reproduce any mixing pattern, so that there are no definite predictions for the mixing angles. However, to the extent that the leading-order Yukawa couplings are fixed by the measured neutrino masses and mixings, correlations between the branching ratios of LFV processes can be obtained, cf. Eq. (57), analogously to what was found for the branching ratios of LFV decays [59]. As we have argued, the severe limit from $\mu \to e\gamma$ probably means that at most one LFV branching ratio will be measurable at LHC. Then the predicted correlations can be falsified by observing a second LFV process. In order to verify them, one has to determine the mixings of RH neutrinos with the different flavors directly, which may be possible at e^+e^- colliders [6,7,10]. In the most optimistic case, collider experiments could even test some predictions of leptogenesis models [26,59].

V. SUMMARY AND DISCUSSION

We have critically reexamined the possibility of a direct test of the seesaw mechanism of neutrino mass generation in collider experiments. We have assumed the existence of right-handed (RH) neutrinos with masses close to the electroweak scale (but no other new particles or interactions). The upper bound on the light neutrino masses immediately leads to the conclusion that these RH neutrinos are much too weakly coupled to the standard model particles to be produced at colliders. This conclusion can only be avoided, if there is a strong cancellation between the contributions from different RH neutrinos to the light neutrino masses, which can be due to lepton number conservation. Then the seesaw mechanism itself plays only a minor role in explaining the smallness of neutrino masses. Light neutrino masses can appear as a result of a small breaking of lepton number. Other effects of this breaking are too small to be tested at accelerators. The only hope to check this possibility is to discover the RH neutrinos and to establish correlations, which may exist in special cases, between properties of the light neutrino mass matrix and processes at accelerators involving the RH neutrinos. Of course, one cannot exclude the existence of additional, very heavy RH neutrinos contributing to neutrino masses via the standard seesaw mechanism, but this cannot be tested directly.

More explicitly, an exact cancellation of light neutrino masses occurs, if only one combination of the active neutrinos, $\tilde{\nu}$, couples with the RH neutrinos, while two others decouple and remain massless. The contributions of the RH neutrinos to the mass of $\tilde{\nu}$ cancel due to a certain correlation between their masses and Yukawa couplings. We have shown that these are necessary conditions in scenarios with two and three RH neutrinos. If there are more than three RH neutrinos, the conditions do not apply. In this case, more than one combination of active neutrinos can couple to the RH neutrinos, and the cancellation can be realized in a more complicated way.

We have discussed examples where the cancellation is due to a symmetry. In the simplest setup, one RH neutrino decouples from the system. Another one mixes with $\tilde{\nu}$ and forms a Dirac pair with the third RH neutrino, and the combination orthogonal to this mixture stays massless. This structure implies conservation of lepton number. We have also presented a simple model based on the discrete symmetry A_4, in which L conservation arises as an accidental symmetry.

If the cancellation is realized by a symmetry that leads to lepton number conservation, it is stable against radiative corrections: three neutrinos remain massless. In all other cases, the cancellation is unstable and therefore requires fine-tuning in several orders of perturbation theory. This is true both for setups without any symmetry motivation and for scenarios relying on a symmetry which does not imply L conservation.

Light neutrino masses are obtained from small perturbations of the leading-order mass matrices. We have systematically considered all possible perturbations of the mass matrices arising in the L-conserving setup. In the A_4 toy model, we have discussed a pattern of symmetry breaking that leaves a $\mathbb{Z}_2$ subgroup unbroken, resulting in a subset of the most general perturbations and partially motivating their smallness.

Thus, both lepton number violation and active neutrino masses arise due to small perturbations, and their magnitudes are related. Therefore, we expect lepton-number-violating signals at colliders to be unobservable in untuned scenarios. The cross sections for lepton-flavor-violating processes are not suppressed, so that LHC might have a chance to observe such reactions. If this is the case, lepton flavor violation should also be observable in upcoming experiments studying the decays of charged leptons.

The flavor pattern of processes with RH neutrinos at colliders depends on the particular combination $\tilde{\nu}$. With a realistic experimental accuracy, a measurement of the perturbations will not be possible. Consequently, in the most general case the theory contains too many free parameters to realize a simple connection between collider observables and the masses and mixings of the active neutrinos. In this sense, the mechanism of neutrino mass generation and collider physics decouple. However, in minimal cases, where only some perturbations of the cancellation structure are present, one can find correlations between features of neutrino masses and accelerator observables. These could be falsified at the LHC and tested at the ILC. Of course, the discovery of standard model singlets close to the electroweak scale would be very interesting by itself in any case.

APPENDIX: CANCELLATION AND CASAS-IBARRA PARAMETRIZATION

Let us demonstrate how the cancellation condition can be derived using the Casas-Ibarra parametrization [47]. For illustration we consider the two-generation case and choose

$$R = \begin{pmatrix} \cosh x & -i\sinh x \\ i\sinh x & \cosh x \end{pmatrix}. \tag{A1}$$

For simplicity we neglect neutrino mixing, $U_{\text{PMNS}} = \mathbb{1}$. Then using Eq. (22) with the diagonal matrices $m_\nu = \text{diag}(m_{\nu 1}, m_{\nu 2})$ and $m_R = \text{diag}(M_1, M_2)$ we can write

$$m_D = \begin{pmatrix} \cosh x \sqrt{m_{\nu 1} M_1} & -i\sinh x \sqrt{m_{\nu 1} M_2} \\ i\sinh x \sqrt{m_{\nu 2} M_1} & \cosh x \sqrt{m_{\nu 2} M_2} \end{pmatrix}. \tag{A2}$$

For $x \gg 1$, $\cosh x \approx \sinh x \approx e^x/2$. If in the limit $m_\nu \to 0$ the products $e^x\sqrt{m_{\nu 1}} \to \sqrt{\mu} = $ const, and $e^x\sqrt{m_{\nu 2}} \to \sqrt{\mu}\alpha = $ const, the Dirac mass matrix becomes

$$m_{\mathrm{D}} = \begin{pmatrix} \sqrt{\mu M_1} & -i\sqrt{\mu M_2} \\ i\sqrt{\mu M_1}\alpha & \sqrt{\mu M_2}\alpha \end{pmatrix}. \qquad (A3)$$

This matrix has rank 1 and satisfies the cancellation condition (12).

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