Higgs-Higgs bound state due to new physics at a TeV

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We examine the effects of new physics on the Higgs sector of the standard model, focusing on the effects on the Higgs self-couplings. We demonstrate that a low mass Higgs, $m_h < 2m_t$, can have a strong effective self-coupling due to the effects of a new interaction at a TeV. We investigate the possibility that the first evidence of such an interaction could be a Higgs-Higgs bound state. To this end, we construct an effective field theory formalism to examine the physics of such a low mass Higgs boson. We explore the possibility of a nonrelativistic bound state of the Higgs field (Higgsium) at CERN's Large Hadron Collider and construct a nonrelativistic effective field theory of the Higgs sector that is appropriate for such studies (NRHET).

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I. INTRODUCTION

Currently, global fits to all precision electroweak give the Higgs mass to be 113^{+56}_{-40} GeV with an upper bound given by $m_h < 241$ GeV at 95% CL (see, e.g., J. Erler and P. Langacker in Sec. 10 of Ref. [1]). LEP has also placed a lower bound limit of $m_h > 114.4$ GeV [2]. Assuming the standard model (SM) of electroweak interactions, one expects that the Higgs will soon be found at CERN's Large Hadron Collider (LHC).

However, there are at least two reasons why the SM with a single Higgs doublet is expected to be incomplete. The first is the triviality problem. This asserts that the Higgs self-interaction, and hence its mass, must vanish unless the theory has a finite cutoff. Triviality has been rigorously established only for simpler models, but it is widely believed to hold for the SM Higgs. The other is the hierarchy problem that quadratic divergences need to be finely tuned to keep the scale of electroweak breaking smaller than the natural cutoff of the theory (which in the absence of new physics would be the Planck scale).

In this paper, we investigate the effects that new physics, invoked to cure these problems, may have on the Higgs sector of the SM. We assume that the scale of the masses of new quanta, $\mathcal{M}$, is sufficiently higher than the scale of electroweak symmetry breaking ($v \sim 246$ GeV) so that the quanta of the unknown new physics can be integrated out. As we want this new physics to address the hierarchy and triviality problems, and for phenomenological reasons, we are interested in new physics where $\mathcal{M} \sim$ TeV. The resulting low energy effective theory is the one Higgs standard model supplemented with nonrenormalizable local operators, of dimension $D > 4$, which are constructed of standard model fields invariant under the $SU(3) \times SU(2) \times U(1)$ gauge symmetry. This approach has been applied to precision electroweak observables [3–8] and has recently been the subject of further investigations [9–15]. The advantage of this approach is that it is model independent: any new physics scheme that results in a low energy spectrum coinciding with the SMs can be described in this way. The disadvantage is that the new physics is parametrized in terms of several arbitrary parameters, the coefficients of higher dimension operators, and nothing is known *a priori* about these coefficients.

For a particular extension to the standard model, consistency requires that fits such as [1] be reconsidered with the new operators, severely relaxing the constraint on the Higgs mass. In fact, it has been shown that the effect of higher dimension operators [16–18] can eliminate the mass limit on the Higgs. While more exotic possibilities are tantalizing, in this paper, we focus on the possibility that the new physics integrated out is strongly interacting and effecting the Higgs sector above the scale $\mathcal{M}$ while the Higgs itself has a relatively low mass $m_h \lesssim 2m_t$.

Various bounds can be placed on $\mathcal{M}$ from low energy experiments. In particular, flavor changing neutral current bounds such as those arising from $K^0 - \bar{K}^0$ mixing impose strong constraints, $\mathcal{M} \geq 10^4$ TeV. These bounds can be relaxed by restricting the higher dimensional operator basis through adopting the minimal flavor violation (MFV) hypothesis [19–28]. This allows one to consider $\mathcal{M}$ to be a few TeV while naturally suppressing flavor changing neutral currents (FCNC).

However, even utilizing the MFV hypothesis to justify new physics at a TeV, higher dimensional corrections to the standard model could exist that modify the relation $m_W = m_Z \cos\theta_W$. This relationship is experimentally required to be respected to a fraction of a percent. The Particle Data Group (PDG) quotes $\rho_0 = 1.0002^{+0.0007}_{-0.0004}$ for the global fit [29] of precision electroweak observables. This fact motivates the consideration of new physics being integrated out that preserves $\rho_0 \approx 1$ naturally, even with possible strong dynamics effecting the Higgs at the scale $\mathcal{M}$.

This can be accomplished assuming an approximate custodial $SU(2)_C$ symmetry [30–32], where the weak SU(2) gauge vector bosons transform as a triplet and the Higgs field transforms as a triplet and a singlet. The Higgs vacuum expectation value is in the singlet representation of $SU(2)_C$, so the approximate symmetry is explicitly realized, and is explicitly broken only by isospin splitting of fermion Yukawa couplings and by hypercharge. We require that the operator extensions to $\mathcal{L}_{\rm SM}$ respect this $SU(2)_C$ symmetry up to hypercharge and Yukawa coupling violations, as in the standard model. Operators that break the custodial symmetry are allowed but their coefficients are taken to be naturally suppressed.

In the SM, the Higgs cubic and quartic couplings are not independent parameters, but given in terms of the Higgs vacuum expectation value v and mass m_h. The obvious immediate effect of $D > 4$ operators is to shift all these quantities in independent ways, so that effectively the Higgs cubic and quartic couplings become independent parameters. Of course, the shift from the SM values is somewhat restricted, of order $(v^2/\mathcal{M}^2)C$, where C is a dimensionless coefficient, $C \sim 1$. The effect on single Higgs production rates of modifications to the coupling of the Higgs to weak vector bosons or to itself was investigated in Ref. [11]. The modification of Higgs decay widths and this general class of models was also examined in [33].

In this paper we address the question of whether a bound state of Higgs particles can form. In the SM a Higgs bound state forms only if the Higgs is very heavy [34,35]. There is a competition between the repulsive interaction of the quartic coupling, λ_1, and the attractive interaction of the Higgs exchange (between Higgs particles) which is determined by the cubic coupling, $\lambda_1 v$. For large enough coupling the exchange interaction is strong enough to produce binding, but since the mass, $\sqrt{\lambda_1}v$, is also given in terms of the coupling, the Higgs mass is large. In the effective theory context the three parameters (mass and cubic and quartic couplings), are independent and a bound state is possible for smaller Higgs mass. The question of detail becomes, how is the bound on the Higgs mass for a bound state to form relaxed by the coefficients of $D = 6$ operators? Can one have a bound state of light Higgses? We find that the effect of these operators can be significant, allowing for bound states for much lighter Higgs particles. Discovery of such bound states would give valuable information on the scale of new physics.

There is no know solution to the bound state problem for identical scalar particles interacting via cubic and quartic interactions. The Higgs bound state problem has been addressed using different approximations, the N/D method is Ref. [34] and a truncated version of the homogeneous Bethe-Salpeter equation in Ref. [35]. Our aim here is to find a necessary condition on the coupling for which a nonrelativistic (NR) bound state may form. To this end we introduce a new method. We propose to study the formation of the bound state in a nonrelativistic effective theory for Higgs-Higgs interactions.

We begin by listing the $D = 6$ operators of the effective theory. We take two approaches. In the first, linear realization, we consider operators that can be built out of the Higgs doublet and the fields in the gauge sector of the SM. Our primary interest here is in the Higgs sector *per se*, so we focus on Higgs self-interactions. The second approach, uses a nonlinear realization of the symmetry. Since the Higgs field is intimately connected to the symmetry breaking of the SM gauge symmetry, it is natural to expect that below the scale of new physics the effects of symmetry breaking are already apparent. Were the Higgs mass as large as the scale of new physics, the SM would be supplemented not with a Higgs doublet but with a triplet of would-be Goldstone bosons that are eaten by the W and Z vector bosons. The Higgs, if somewhat lighter than the scale $\mathcal{M}$, would appear as a singlet under the gauge symmetry.

We then proceed to construct the effective theory at low energies. If $m_h \lesssim 2m_t$, one can incorporate the virtual effects of the top by integrating it out and constructing a topless effective theory. In order to investigate the minimal coupling for which a NR Higgs-Higgs bound state may form we then construct a nonrelativistic Higgs effective theory, and proceed to determine this condition.

II. HIGGS EFFECTIVE FIELD THEORY: LINEAR REALIZATION

A. The $D = 6$ custodial SU(2) Higgs sector

The Lagrangian density of the standard model containing the Higgs field[1] is given by

$$\mathcal{L}_\phi^4 = (D^\mu \phi)^\dagger (D_\mu \phi) - V(\phi) \tag{1}$$

where ϕ is the Higgs scalar doublet. The covariant derivative of the ϕ field is given by

$$D_\mu = 1\partial_\mu - i\frac{g_1}{2}B_\mu - ig_2\frac{\sigma^I}{2}W_\mu^I \tag{2}$$

where σ^I are the Pauli matrices, W_μ^I, B_μ, are the SU(2) and U(1) SM gauge bosons and the hypercharge of $1/2$ has been assigned to the Higgs. The Higgs potential at tree level is given by

$$V(\phi) = -m^2\phi^\dagger\phi + \frac{\lambda_1}{2}(\phi^\dagger\phi)^2. \tag{3}$$

No dimension five operator can be constructed out of Higgs fields and covariant derivatives that satisfies Lorentz symmetry and the standard model's gauge symmetry.[2] Utilizing the equation of motion of the Higgs field and partial integration the number of dimension six operators is reduced. The effective Lagrangian density of the extended standard model is given by

[1]We have omitted Yukawa interactions with fermions here.

[2]To satisfy Lorentz invariance an even number of covariant derivatives are required. To be invariant under the $SU(2) \times U(1)$ gauge group the operator must be bilinear in $\phi^\dagger$ and ϕ.

$$\mathcal{L}_\phi = \mathcal{L}_\phi^4 + \frac{\mathcal{L}_\phi^6}{\mathcal{M}^2} + \mathcal{O}\left(\frac{v^4}{\mathcal{M}^4}\right), \tag{4}$$

where the dimension six operators that preserve the symmetries of the standard model and custodial $SU(2)_C$ in the Higgs sector are given by

$$\mathcal{L}_\phi^6 = C_\phi^1 \partial^\mu(\phi^\dagger\phi)\partial_\mu(\phi^\dagger\phi) + C_\phi^2(\phi^\dagger\phi)(D_\mu\phi)^\dagger(D^\mu\phi) - \frac{\lambda_2}{3!}(\phi^\dagger\phi)^3. \tag{5}$$

Note that the operators considered here preserve custodial symmetry and can result from tree level topologies in the underlying theory [36]. As such, these operators need not be suppressed by loop factors of $1/16\pi^2$ or proportional to a small custodial symmetry breaking parameter. For these reasons these operators are expected to have the dominant effects on the Higgs self-couplings and we take their coefficients to be $\mathcal{O}(1)$. There is only one operator in the Higgs sector that violates custodial symmetry and could come from an underlying tree topology, $(\phi^\dagger D^\mu \phi)^2$. The underlying topology in this case determines that the symmetry breaking parameter is given by g_1^2. The coefficient of this operator has been determined [37] to be $C < 4 \times 10^{-3}$ where we have used $\Lambda = 1$ TeV. We neglect this operator.[3]

We expand the Higgs field about its vacuum expectation value with $\langle h(x)\rangle = 0$ and treat $v^2/\mathcal{M}^2$ as a small perturbation. We expand the field as usual around a vacuum expectation value v so that

$$\phi(x) = \frac{\mathrm{U(x)}}{\sqrt{2}}\begin{pmatrix} 0 \\ v + h(x)\end{pmatrix}. \tag{6}$$

Here $\mathrm{U(x)} = e^{i\xi^a(x)\sigma_a/v}$ and the would-be Goldstone boson fields of the broken symmetry are ξ^a. In unitary gauge, the gauge transformation can be used to remove the Goldstone boson fields. We then redefine the Higgs field (h) so that the kinetic term is normalized to 1/2, using the field redefinition

$$h \to \frac{h'}{(1 + 2C_h^K)^{1/2}}, \tag{7}$$

where $C_h^K = (v^2/\mathcal{M}^2)(C_\phi^1 + \frac{1}{4}C_\phi^2)$. The effective Lagrangian density is given, in terms of the rescaled field, by

$$\mathcal{L}_\phi^4 + \frac{\mathcal{L}_\phi^6}{\mathcal{M}^2} = \frac{1}{2}\partial^\mu h'\partial_\mu h' - V_{\mathrm{eff}}(h') + C_{h'}^{i,j}O_{h'}^{i,j} + C_{WW}O_{WW} + C_{ZZ}O_{ZZ} + C_{h'WW}^{i,j}O_{h'WW}^{i,j} + C_{h'ZZ}^{i,j}O_{h'ZZ}^{i,j}, \tag{8}$$

summed over i, j such that $i + j = 2$, where

$$O_{h'}^{i,j} = \frac{(h')^i v^j}{\mathcal{M}^2}\partial^\mu h'\partial_\mu h', \qquad O_{WW} = W_\mu^+ W_\mu^-,$$
$$O_{ZZ} = Z_\mu^0 Z_\mu^0, \qquad O_{h'WW}^{i,j} = \frac{(h')^i v^j}{\mathcal{M}^2} W_\mu^+ W_\mu^-, \tag{9}$$
$$O_{h'ZZ}^{i,j} = \frac{(h')^i v^j}{\mathcal{M}^2} Z_\mu^0 Z_\mu^0.$$

The coefficients are given by

$$\begin{aligned}
C_{h'}^{0,2} &= 0,\\
C_{h'}^{1,1} &= \frac{1}{2}(4C_\phi^1 + C_\phi^2),\\
C_{h'}^{2,0} &= \left(C_\phi^1 + \frac{1}{4}C_\phi^2\right),\\
C_{WW} &= m_W^2\left(1 + C_\phi^2\frac{v^2}{2\mathcal{M}^2}\right),\\
C_{ZZ} &= \frac{m_Z^2}{2}\left(1 + C_\phi^2\frac{v^2}{2\mathcal{M}^2}\right),\\
C_{h'WW}^{1,1} &= m_W^2\left[\frac{3}{2}C_\phi^2 - 2C_\phi^1 + \frac{2\mathcal{M}^2}{v^2}\right],\\
C_{h'WW}^{2,0} &= m_W^2\left[\frac{5}{2}C_\phi^2 - 2C_\phi^1 + \frac{\mathcal{M}^2}{v^2}\right],\\
C_{h'WW}^{3,-1} &= 2m_W^2 C_\phi^2,\\
C_{h'WW}^{4,-2} &= \frac{1}{2}m_W^2 C_\phi^2,\\
C_{h'ZZ}^{1,1} &= \frac{m_Z^2}{2}\left[\frac{3}{2}C_\phi^2 - 2C_\phi^1 + \frac{2\mathcal{M}^2}{v^2}\right],\\
C_{h'ZZ}^{2,0} &= \frac{m_Z^2}{2}\left[\frac{5}{2}C_\phi^2 - 2C_\phi^1 + \frac{\mathcal{M}^2}{v^2}\right],\\
C_{h'ZZ}^{3,-1} &= m_Z^2 C_\phi^2,\\
C_{h'ZZ}^{4,-2} &= \frac{m_Z^2}{4}C_\phi^2.
\end{aligned} \tag{10}$$

The effective potential is

$$V_{\mathrm{eff}}(h') = \frac{1}{2}m_h^2 h'^2 + \frac{v\lambda_3^{\mathrm{eff}}}{3!}h'^3 + \frac{\lambda_4^{\mathrm{eff}}}{4!}h'^4 + \frac{30\lambda_2}{5!\mathcal{M}^2}vh'^5 + \frac{30\lambda_2}{6!\mathcal{M}^2}h'^6, \tag{11}$$

which is written in terms of the rescaled mass term and the effective couplings, which are given by

[3] We also note that we could only assume that custodial symmetry breaking effects are small in the bosonic sector as fermion corrections to the operators of interest are present only at one loop.

$$\frac{m_h^2}{v^2} = \lambda_1(1 - 2C_h^K) + \frac{\lambda_2}{2}\frac{v^2}{\mathcal{M}^2} + \mathcal{O}\left(\frac{v^4}{\mathcal{M}^4}\right), \quad (12)$$

$$\lambda_3^{\rm eff} = 3\lambda_1(1 - 3C_h^K) + \frac{5}{2}\lambda_2\frac{v^2}{\mathcal{M}^2} + \mathcal{O}\left(\frac{v^4}{\mathcal{M}^4}\right), \quad (13)$$

$$\lambda_4^{\rm eff} = 3\lambda_1(1 - 4C_h^K) + \frac{15}{2}\lambda_2\frac{v^2}{\mathcal{M}^2} + \mathcal{O}\left(\frac{v^4}{\mathcal{M}^4}\right). \quad (14)$$

We will suppress the prime superscript on the Higgs field for the remainder of the paper.

B. $D = 6$ SM field strength operators

The operators that can be constructed out of the Higgs scalar doublet and the field strengths (or duals) of the standard model are as follows. We restrict our attention to those operators listed in [6,15] that preserve the $SU(2)_C$ custodial symmetry:

$$\begin{aligned}\frac{\mathcal{L}^6_{\phi,V}}{\mathcal{M}^2} = &-\frac{c_G g_3^2}{2\mathcal{M}^2}(\phi^\dagger\phi)G^A_{\mu\nu}G^{A\mu\nu} - \frac{c_W g_2^2}{2\mathcal{M}^2}(\phi^\dagger\phi)W^I_{\mu\nu}W^{I\mu\nu}\\ &-\frac{c_B g_1^2}{2\mathcal{M}^2}(\phi^\dagger\phi)B_{\mu\nu}B^{\mu\nu},\\ &-\frac{\tilde{c}_G g_3^2}{2\mathcal{M}^2}(\phi^\dagger\phi)\tilde{G}^A_{\mu\nu}G^{A\mu\nu} - \frac{\tilde{c}_W g_2^2}{2\mathcal{M}^2}(\phi^\dagger\phi)\tilde{W}^I_{\mu\nu}W^{I\mu\nu}\\ &-\frac{\tilde{c}_B g_1^2}{2\mathcal{M}^2}(\phi^\dagger\phi)\tilde{B}^A_{\mu\nu}B^A_{\mu\nu}. \end{aligned} \quad (15)$$

Here $G^A_{\mu\nu}$, $W^I_{\mu\nu}$ and $B_{\mu\nu}$ stand for the field strength tensors of the $SU(3)\times SU(2)\times U(1)$ gauge bosons, and a tilde denotes the dual field strengths, $\tilde{F}_{\mu\nu} = \epsilon_{\mu\nu\lambda\sigma}F^{\lambda\sigma}/2$. Note that the operator that is proportional to the S parameter given by

$$-\frac{c_{WB}\, g_1 g_2}{\mathcal{M}^2}(\phi^\dagger\sigma^I\phi)B^{\mu\nu}W_{I\mu\nu} \quad (16)$$

violates custodial symmetry and is naturally suppressed in our approach.[4]

C. $D = 6$ Fermion sector

Operators of dimension 5 and higher that couple the Higgs to fermions, or purely fermionic operators, can give rise to unacceptably large FCNC. If the coefficient of such operators are generically of order 1 the scale of new physics must be taken to be $\mathcal{M} \gtrsim 10^4$ TeV in order to suppress FCNC effects. We adopt the minimal flavor violation hypothesis (MFV) to naturally suppress the dangerous operators while maintaining a low scale of new physics, $\mathcal{M} \gtrsim 1$ TeV. In the absence of quark and lepton masses the SM has a large flavor symmetry group, $G_F = SU(3)^5$. The MFV asserts that there is a unique source of breaking of this symmetry. All operators that break the symmetry must transform precisely the same way under G_F. As a result FCNC operators are suppressed by the familiar factors of the Kobayashi-Maskawa (CKM) matrix in the quark sector and by the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix and small neutrino masses in the lepton sector.

Since the effects of fermionic operators are not needed for the rest of this investigation, we do not list the operators. The interested reader can find a complete description of the operators and their effects in [6].

[4]See Appendix A

III. HIGGS EFFECTIVE FIELD THEORY: NONLINEAR REALIZATION

The construction in the previous section assumes that the field content of the effective theory includes a Higgs doublet. This is not necessary. If the electroweak symmetry is spontaneously broken by a strong interaction the spectrum below the scale of this new physics does not have to be described by a Higgs doublet field, beyond the SM fields. Only fields describing the would-be Goldstone bosons need be introduced. Such Higgs-less theories have been discussed in the literature [38]. However, if the Higgs particle is somewhat lighter than the scale of new physics it has to be incorporated in the low energy description and symmetry alone does not dictate that it appears as a member of an isodoublet. It is sufficient to have the Goldstone bosons realize the broken symmetry nonlinearly, and the Higgs field is then a singlet under the symmetry.

The situation is entirely analogous to the case of π's and the σ in QCD. A phenomenological Lagrangian density describing π and σ interactions does not have to be a linear realization of the chiral $SU(2)\times SU(2)$ symmetry. Instead, the π-fields have a better description through a nonlinear chiral Lagrangian. Then the σ can be included through interactions that satisfy the nonlinearly realized symmetry and the usual rules for naive dimensional analysis [39].

In the nonlinear realization, the Lagrangian density in Eq. (1) is replaced by

$$\mathcal{L}_{\rm NL} = \frac{1}{4}v^2\,{\rm Tr}D_\mu {\rm U}^\dagger D^\mu {\rm U} + \frac{1}{2}\partial_\mu h\partial^\mu h - V(h), \quad (17)$$

where the would-be Goldstone bosons ξ^a appear through the matrix ${\rm U(x)} = e^{i\xi^a(x)\sigma_a/v}$ that transforms under $SU(2)_L\times SU(2)_R$ linearly, ${\rm U}\to L{\rm U}R^\dagger$, and h is a singlet field, describing the Higgs particle. Custodial symmetry $SU(2)_C$ is the diagonal subgroup of $SU(2)_L\times SU(2)_R$ and the Higgs field is invariant under it.[5]

This Lagrangian is supplemented by higher order terms suppressed by powers of $\mathcal{M}$. In the case of the Higgs potential, this can be included simply as

$$V(h) = \mathcal{M}^4 f(h/\mathcal{M}), \quad (18)$$

[5]The custodial symmetry is discussed in more detail in Appendix A.

where $f(x)$ is an arbitrary function with a minimum at zero. The mass and couplings of the Higgs are given in terms of this dimensionless function by

$$m_h^2 = \mathcal{M}^2 f^{(II)}(0), \tag{19}$$

$$v\lambda_3^{\rm eff} = \mathcal{M} f^{(III)}(0), \tag{20}$$

$$\lambda_4^{\rm eff} = f^{(iv)}(0). \tag{21}$$

It is not a surprise that in the nonlinear realization of the symmetry the couplings and mass are completely independent, and that they are all naturally of order 1 times the appropriate power of the dimensionfull scale, $\mathcal{M}$. The natural scale for the Higgs mass is $\mathcal{M}$, and we are considering here the class of theories for which $f^{(II)}(0)$ happens to be small, while higher derivatives may remain of order 1. We stress that the natural scale for the cubic coupling is $\mathcal{M}$. *Unless the mechanism (or numerical accident) that keeps the Higgs mass small compared to* $\mathcal{M}$ *also acts to suppress the cubic coupling, one must naturally expect* $\lambda_3^{\rm eff} \sim \mathcal{M}/v \gg 1$.

We will also need the corrections to the derivative interactions. We write, generally,

$$\mathcal{L} = \frac{1}{2}\left[1 + c_1^{\rm eff}\frac{h}{v} + c_2^{\rm eff}\frac{h^2}{v^2}\right]\partial^\mu h \partial_\mu h - \frac{1}{2}m_h^2 h^2 - \frac{v\lambda_3^{eff}}{3!}h^3 - \frac{\lambda_4^{eff}}{4!}h^4 + \cdots \tag{22}$$

In the linear realization the derivative interaction couplings are related, $\frac{1}{4}c_1^{\rm eff} = \frac{1}{2}c_2^{\rm eff} = C_h^K = (v^2/\mathcal{M}^2)(C_\phi^1 + C_\phi^2/4)$, but in the nonlinear realization they are independent. And, as in the case with $\lambda_3^{\rm eff}$ naive dimensional scaling gives an enhancement of $c_1^{\rm eff}$ that could arise from the nonperturbative dynamics of the symmetry breaking sector. Naively, $c_1^{\rm eff} \sim (v/\mathcal{M})$, which is enhanced over the linear realization value by a power of $(\mathcal{M}/v)$.

As we mentioned earlier, nonlinear realizations have been extensively studied for Higgs-less theories, but have been neglected in studies including a light Higgs. There are two important consequences of the nonlinear realization outside the pure Higgs sector that we point out here. It has been noted that significant corrections to the coupling of a Higgs to gluons are possible from $D > 4$ operators. The modifications can be large because there is no SM contribution at tree level. In the linear realization there is a $D = 6$ operator that contributes at tree level, and therefore competes with the SM one loop, top mediated amplitude:

$$\frac{1}{\mathcal{M}^2}G^a_{\mu\nu}G^a_{\mu\nu}. \tag{23}$$

Note that the linear realization implies a relation between the one and two Higgs couplings to two gluons. However, in the nonlinear realization the two couplings are completely independent,

$$\left(c_1\frac{h}{\mathcal{M}} + c_2\frac{h^2}{\mathcal{M}^2}\right)G^a_{\mu\nu}G^a_{\mu\nu}. \tag{24}$$

In Ref. [40] it was noted that a heavy quark with Yukawa coupling $\lambda \to \infty$ produces a coupling of two gluons to one or more Higgs particles that cannot be described by the effective theory operator in (23). Instead a nonpolynomial interaction was introduced to describe this effect,

$$\frac{\alpha_s}{8\pi}\ln\left(\frac{H^\dagger H}{v^2}\right)G^a_{\mu\nu}G^a_{\mu\nu}.$$

There is no problem accommodating such interactions in the nonlinear realization, by

$$\frac{\alpha_s}{4\pi}\ln(1 + h/v)G^a_{\mu\nu}G^a_{\mu\nu} = \frac{\alpha_s}{4\pi}[h/v + (h/v)^2 + \cdots]G^a_{\mu\nu}G^a_{\mu\nu}. \tag{25}$$

In much of what follows we implicitly assume the linear realization. However, results in terms of the arbitrary parameters m_h, $\lambda_3^{\rm eff}$ and $\lambda_4^{\rm eff}$ can be interpreted readily as arising from the nonlinear realization.

IV. A LOW ENERGY EFFECTIVE THEORY FOR THE HIGGS

In this section we will construct an effective theory for the light Higgs, integrating out momentum modes heavier than the Higgs. This is useful in discussing physical effects with a typical energy of order of the Higgs mass. In particular, we integrate out the top quark. As the coupling of the top quark to the Higgs is fairly large, we would like to estimate the effects of the top quark on the possibility of forming a Higgs bound state. If the top quark mass is much heavier than the Higgs it is appropriate and convenient to describe the Higgs self-interactions in a topless theory. When the top quark has been integrated out, its effects are accounted for through modifications of coupling constants and mass of the Higgs.

While this is clearly appropriate when the top quark is much larger than the Higgs mass, we use this approximation even when the Higgs is slightly heavier than the top. For $m_h < 2m_t$ the approximation is known, *ipso facto*, to work better than one would expect. This is due, in part, to the fact that there is no nonanalytic dependence on the mass since the Higgs is the pseudo-Goldstone boson of spontaneously broken scale invariance [41–43]. It is also known that soft gluon effects are large and correctly reproduced by the effective theory [44].

For single and double Higgs production, comparisons between the full theory calculation and the effective topless theory find that the latter is a good approximation for the total rate for $m_h \lesssim 2m_t$. For example, with the appropriate K factor, the resulting topless effective field theory calculated to two loops is known to accurately describe the full

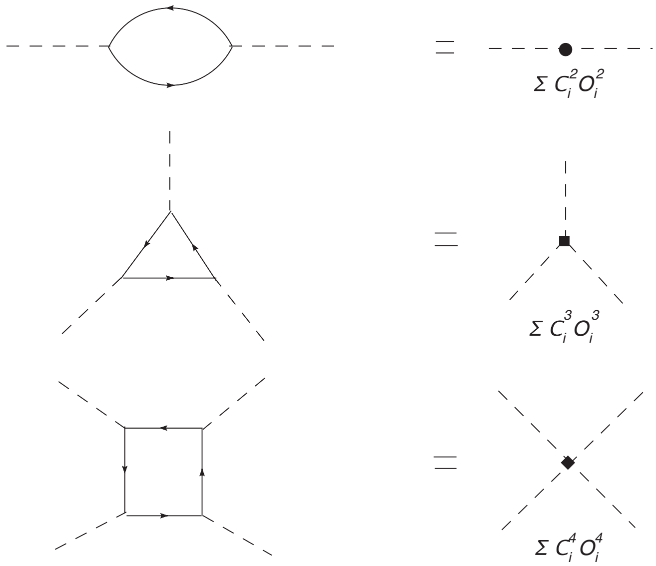

FIG. 1. Integrating out the top quark.

next-to-leading order (NLO) result for $gg \to h$ to better than 5% accuracy in the full range $0 < m_h < 2m_t$ [44].

As another concrete example, consider the Higgs mass dependence in the Higgs IPI self-energy. The first graph of Fig. 1 is the contribution of the top quark to the IPI self-energy which we label $-i\Pi(p^2)$. At 1-loop we find

$$\Pi(p^2) = \frac{N_c}{4\pi^2}\frac{m_t^4}{v^2}\int_0^1 dx\left(1 - x(1-x)\frac{p^2}{m_t^2}\right) \times\left[1 + 3\log\left(\frac{\mu^2}{m_t^2 - x(1-x)p^2}\right)\right], \quad (26)$$

where we have used the $\overline{\mathrm{MS}}$ subtraction scheme. The quantity $1 - \Pi(0)/\Pi(p^2)$ (at $\mu = m_t$) never exceeds 30% when $\sqrt{p^2}$ ranges from zero to $2m_t$.

For these reasons we consider it appropriate to integrate out the top quark for $m_h \lesssim 2m_t$ in this initial study. When $m_h \gg m_t$, these corrections should be taken only as indications of the size of virtual top effects. While the approximation of neglecting higher order terms in the p^2/m_t^2 expansion is known to work better than expected for the applications we will consider, there is no guarantee that it will work well for processes not considered here [45–49].

A. Running to m_t

The coefficients of the $D = 6$ operators at the scale $\mathcal{M}$ are unknown. We are assuming that the new physics couples to the Higgs field and is strongly interacting at the scale $\mathcal{M}$. In this context, it is natural to take

$$C_\phi^i(\mathcal{M}), \lambda_2(\mathcal{M}) \sim 1. \quad (27)$$

Similarly it is natural to assume that the coefficients of the $D > 4$ operators that couple the Higgs to other fields, like those in Eq. (15) or those that couple the Higgs to quarks while satisfying the MFV hypothesis, are all order unity.

The anomalous dimensions of the extended operator basis can be determined systematically. This is beyond the scope of this paper. But the effect of the running is easy to understand. With minimal subtraction the calculation of the running of coefficients of higher dimension operators can be done in the symmetric, massless phase. There is operator mixing among the $D = 6$ operators with common quantum numbers. The anomalous dimension matrix is a function of the relevant couplings (λ_1, g_1, g_2 and the top quark Yukawa, λ_t). The running is always proportional to these coefficients so the effect is roughly of the form

$$C_\phi^i(m_t) \sim C_\phi^i(\mathcal{M})\left(1 + \frac{c_1\alpha}{16\pi^2}\log\left(\frac{m_t}{\mathcal{M}}\right)\right), \quad (28)$$

where mixing is implicit, and $c_1\alpha$ stands for a linear combination of λ_1 and the squares of g_1, g_2 and λ_t.

Since $\log(m_t/\mathcal{M}) \sim 1$ and the coefficients $c_1 \sim 1$ the running produces a small, calculable shift in the unknown coefficients. Hence, we continue to take the unknown Wilson coefficients at the scale m_t to be ~ 1.

At m_t the top quark is integrated out and this produces a different effect, a shift in the $C_\phi^i(m_t)$ by a C_ϕ^i-independent amount. This can be numerically significant, and we estimate this next. Note that once the top is integrated out we continue to run down to the mass of the Higgs scalar m_h. The effect of the running of these coefficients from m_t to m_h is again small, so we take

$$C_\phi^i(m_h), \lambda_2(m_h) \sim 1. \quad (29)$$

B. Integrating out the top quark

Integrating out the top leads to further corrections to the Higgs sector of the standard model. The top mass is a result of symmetry breaking, so the resulting effective theory is better presented in unitary gauge, as in (8) and (11). In unitary gauge, the top mass term and coupling to the Higgs is given by

$$\mathcal{L}_Y = -m_t\bar{q}_t q_t\left(1 + \frac{h}{v}\right). \quad (30)$$

We begin by considering effects on the Higgs self-couplings. Figure 1 shows the Feynman graphs that contribute to modifications of the Higgs self-couplings. The solid line denotes a top quark, the dashed external lines denote the Higgs.

We perform the calculation to lowest order in p^2/m_t^2. Some details of the computation are given in Appendix B. The effect of these corrections is to further modify the effective potential of the Higgs scalar field h. The effective couplings and mass term of Eqs. (12)–(14) are shifted by these corrections, and are now given by

$$\frac{m_h^2}{v^2} = \lambda_1(1 - 2C_h^K) + \frac{N_c}{4\pi^2}\left(\frac{m_t^4}{v^4}\right) + \frac{\lambda_2}{2}\frac{v^2}{\mathcal{M}^2} + \mathcal{O}\left(\frac{v^4}{\mathcal{M}^4}, \frac{m_t^2 m_h^2}{v^4}\right), \tag{31}$$

$$\lambda_3^{\rm eff} = 3\lambda_1(1 - 3C_h^K) - \frac{N_c}{\pi^2}\left(\frac{m_t^4}{v^4}\right) + \frac{5}{2}\lambda_2\frac{v^2}{\mathcal{M}^2} + \mathcal{O}\left(\frac{v^4}{\mathcal{M}^4}, \frac{m_t^2 m_h^2}{v^4}\right), \tag{32}$$

$$\lambda_4^{\rm eff} = 3\lambda_1(1 - 4C_h^K) - \frac{4N_c}{\pi^2}\left(\frac{m_t^4}{v^4}\right) + \frac{15}{2}\lambda_2\frac{v^2}{\mathcal{M}^2} + \mathcal{O}\left(\frac{v^4}{\mathcal{M}^4}, \frac{m_t^2 m_h^2}{v^4}\right). \tag{33}$$

As emphasized above, these corrections are not multiplicative, that is, they are present even for $\lambda_2 = C_h^K = 0$. Whether they are important depends on the scale and strength of the new physics. The condition

$$\lambda_2\frac{v^2}{\mathcal{M}^2} \sim \left(\frac{m_t^4}{v^4}\right)\frac{1}{\pi^2} \tag{34}$$

is satisfied for $\lambda_2 \approx 1$ when $\mathcal{M} \approx 2\pi v = 1.6$ TeV. So the corrections are numerically comparable to these new physics terms. Similarly, for $\lambda_1 \sim 1$ the condition

$$C_h^K\lambda_1 = \frac{v^2}{\mathcal{M}^2}\left(C_\phi^1 + \frac{1}{4}C_\phi^2\right)\lambda_1 \sim \left(\frac{m_t^4}{v^4}\right)\frac{1}{\pi^2} \tag{35}$$

still requires $\mathcal{M} \approx 1.6$ TeV for $C_\phi^1 + \frac{1}{4}C_\phi^2 \sim 1$.

C. Corrections to field strength operators

Integrating out the top quark also results in effective operators of the Higgs field and the SM field strengths. The dominant SM production mechanisms for the Higgs at LHC is the gluon fusion process $gg \to h$. We restrict our attention to such operators that effect the production processes of the Higgs through gluon fusion. Figure 2 shows the 1-loop Feynman diagram for the top contribution to $gg \to h$. For a Higgs with $m_h < 2m_t$, the expected production cross section of the $gg \to h$ process has been determined up to next-to-next-to-leading order (NNLO) [50–52]. For SM gluon fusion, the single Higgs production mechanism is given by the $m_t \to \infty$ effective Lagrangian density composed of a dimension five operator

$$\mathcal{L}_{m_t} = C_{GGh}^1(\alpha_s)\frac{h}{v}G_{\mu\nu}^a G_a^{\mu\nu}, \tag{36}$$

where the coefficient is given in the $\overline{\rm MS}$ scheme, in terms of α_s for five active flavors, by [53–58]

$$C_{GGh}^1(\alpha_s) = \frac{\alpha_s}{12\pi} + \frac{11\alpha_s^2}{48\pi^2} + \mathcal{O}(\alpha_s^3). \tag{37}$$

The interactions in the effective Lagrangian of Eq. (15) also contribute to single Higgs production through gluon fusion. Combining results, at the scale m_h, the effective Lagrangian density for single Higgs production is given by

$$\mathcal{L}^{\rm eff} = C_{GGh}^{\rm eff}\frac{h}{v}G_{\mu\nu}^A G^{A\mu\nu} + \tilde{C}_{GGh}^{\rm eff}\frac{h}{v}\tilde{G}_{\mu\nu}^A G^{A\mu\nu}, \tag{38}$$

where

$$C_{GGh}^{\rm eff} = C_{GGh}^1 - 2\pi\alpha_s c_G\frac{v^2}{\mathcal{M}^2}, \tag{39}$$

$$\tilde{C}_{GGh}^{\rm eff} = -2\pi\alpha_s\tilde{c}_G\frac{v^2}{\mathcal{M}^2}. \tag{40}$$

Assuming that the new physics degrees of freedom carry the SU(3) gauge charge, the Wilson coefficients c_G, $\tilde{c}_G$ will be approximately the same size as the coefficients C_ϕ^i, λ_2 we are interested in. If the new physics degrees of freedom are charged under SU(2) × U(1) but not SU(3), below the scales $\mathcal{M}$, m_t effective local operators of this form will still be induced. However, the corresponding Wilson coefficients will be suppressed by factors of $16\pi^2$.

The effect of these interactions on Higgs production rates was examined in [14]. Note that in the standard model, contributions to the operator $\tilde{G}_{\mu\nu}^A G^{A\mu\nu}$ are highly suppressed [59] and therefore neglected.

The production process of two Higgs in the standard model is shown in Fig. 3. In analogy with the single Higgs production case we characterize the process in the effective theory by

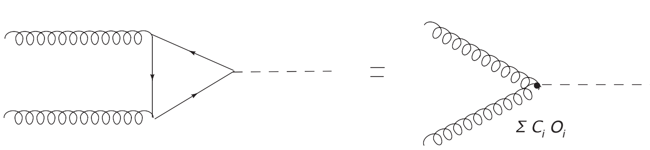

FIG. 2. The gluon fusion $gg \to h$ production process. The production process through the effective local operators is shown in the second column. The effective local operators come from integrating out the top quark and new physics at $\mathcal{M}$.

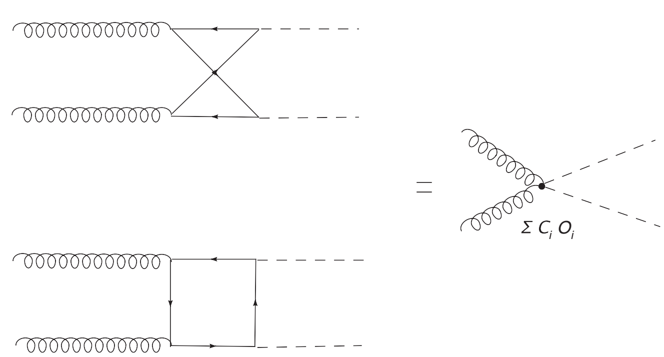

FIG. 3. The gluon fusion $gg \to hh$ production process and the effective local operators.

$$\mathcal{L}^{\text{eff}} = C^{\text{eff}}_{GGhh}\frac{h^2}{v^2}G^A_{\mu\nu}G^{A\mu\nu} + \tilde{C}^{\text{eff}}_{GGhh}\frac{h^2}{v^2}\tilde{G}^A_{\mu\nu}G^{A\mu\nu}, \quad (41)$$

where the coefficients are given by

$$C^{\text{eff}}_{GGhh} = C^1_{GGhh} - \frac{c_G\pi\alpha_s v^2}{\mathcal{M}^2}, \quad (42)$$

$$\tilde{C}^{\text{eff}}_{GGhh} = -\frac{\tilde{c}_G\pi\alpha_s v^2}{\mathcal{M}^2}. \quad (43)$$

Here the top quark contribution is [56]

$$C^1_{GGhh}(\mu^2) = -\frac{\alpha_s(\mu^2)}{12\pi} - \frac{11\alpha_s^2}{48\pi^2} + \mathcal{O}(\alpha_s^3). \quad (44)$$

The Wilson coefficients for two Higgs production in the effective theory is not suppressed relative to the corresponding Wilson coefficient for single Higgs production. Note that unlike the case of single Higgs production the expansion in p^2/m_t^2 does not, in general, have kinematics such that $p^2/m_t^2 \sim m_h^2/m_t^2$. In two Higgs production, higher order terms in p^2/m_t^2 have $p^2 = s, t, u$ and in general $(s, t, u)/m_t^2$ is not small. We calculate the next order in the expansion of p^2/m_t^2 in Appendix B. These terms are neglected, and our application of the expansion is valid for finite values of m_t due to our interest in establishing a necessary condition for a NR bound state to form. The kinematics for the production of a NR bound state at threshold dictate $(s, t, u)/m_t^2 \sim m_h^2/m_t^2$.

V. PHENOMENOLOGY OF HIGGS EFFECTIVE THEORY

A. The magnitude of self-couplings

The effect of the $D = 6$ operators in the effective potential cause corrections to the three and four point contact interactions and m_h. To illustrate that the induced effects on the Higgs sector are under control, consider extending the effective potential with a single dimension eight term. We find the following while neglecting the effects of integrating out the top quark

$$\frac{m_h^2}{v^2} = \lambda_1(1 - 2C_h^K)^2 + 3\times 10^{-2}\lambda_2(1 - 2C_h^K) + 4.5\times 10^{-4}\lambda_3, \quad (45)$$

$$\lambda_3^{\text{eff}} = 3\lambda_1(1 - 3C_h^K + 7.5(C_h^K)^2) + 1.5\times 10^{-1}\lambda_2(1 - 3C_h^K) + 3.1\times 10^{-3}\lambda_3, \quad (46)$$

$$\lambda_4^{\text{eff}} = 3\lambda_1(1 - 4C_h^K + 12(C_h^K)^2) + 4.5\times 10^{-1}\lambda_2(1 - 4C_h^K) + 1.6\times 10^{-2}\lambda_3. \quad (47)$$

From which one sees we are examining the potential of the theory in a controlled expansion, even for $\mathcal{M} \sim 1$ TeV.

Eliminating the self-coupling λ_1 in favor of the Higgs mass, we can write for the effective cubic and quartic Higgs self-couplings,

$$\lambda_3^{\text{eff}} = 3(1 - C_h^K)\frac{m_h^2}{v^2} + \lambda_2\frac{v^2}{\mathcal{M}^2} - \frac{7N_c}{4\pi^2}\frac{m_t^4}{v^4}, \quad (48)$$

and

$$\lambda_4^{\text{eff}} = 3(1 - 2C_h^K)\frac{m_h^2}{v^2} + 6\lambda_2\frac{v^2}{\mathcal{M}^2} - \frac{19N_c}{4\pi^2}\frac{m_t^4}{v^4}. \quad (49)$$

With $v = 246$ GeV, $m_t = 174$ GeV and $\mathcal{M} = 1$ TeV, and taking $m_h = v/2$, these are

$$\lambda_3^{\text{eff}} = 0.62 - 0.05\left(C_\phi^1 + \frac{1}{4}C_\phi^2\right) + 0.06\lambda_2, \quad (50)$$

$$\lambda_4^{\text{eff}} = 0.39 - 0.09\left(C_\phi^1 + \frac{1}{4}C_\phi^2\right) + 0.36\lambda_2. \quad (51)$$

For negative λ_2 of order 1 one can greatly reduce the repulsive contact interaction, λ_4^{eff}, in a putative Higgs-Higgs bound state. Of course, this comes at the price of reducing the attractive interaction, governed by λ_3^{eff}.

B. $gg \to hh$ Production

From our results in Sec. IV C, the production of two Higgs in our effective theory framework is straightforward to write down. The contributions to the amplitude are shown in Fig. 4.

The amplitude for two Higgs production, to $\mathcal{O}(\alpha_s)$, is given by

$$\langle hh|iA|A^\alpha(P_1)A^\beta(P_2)\rangle = \langle hh|iA_1|A^\alpha(P_1)A^\beta(P_2)\rangle + \langle hh|iA_2|A^\alpha(P_1)A^\beta(P_2)\rangle \quad (52)$$

where we have

$$\langle iA_1\rangle^{\alpha\beta} = 2iC_F(C^{\text{eff}}_{GGh})\frac{f^{\alpha\beta}(P_1,P_2)}{(P_1+P_2)^2 - m_h^2 + i\epsilon} \times\left(v\lambda_3^{\text{eff}} - \frac{2}{v}C_H^K(P_3^2 + P_4^2 + (P_1+P_2)^2)\right),$$

$$\langle iA_2\rangle^{\alpha\beta} = 4iC_F(C^{\text{eff}}_{GGhh})f^{\alpha\beta}(P_1,P_2), \quad (53)$$

where

$$f^{\alpha\beta}(P_1,P_2) \equiv P_1^\alpha P_2^\beta + P_1^\beta P_2^\alpha - 2g^{\alpha\beta}P_1\cdot P_2. \quad (54)$$

Using two Higgs production as a test of the cubic self-coupling of the Higgs has been examined in [60] where

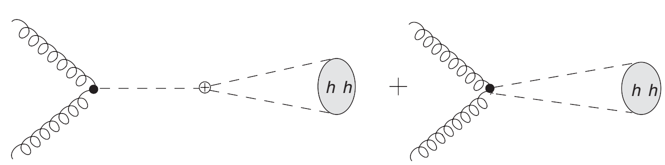

FIG. 4. The two Higgs production process in the effective theory.

testing for the minimal supersymmetric standard model with this signal was investigated. Reference [40] advocated the examination of $gg \to hh$ to compare the one and two Higgs production coefficients in Eq. (24) since the naive relation between the two coefficients could be upset by the presence of novel operators like that in (25). As we have discussed, in the nonlinear realization of broken electroweak symmetry, the relationship between $gg \to hh$ production and $gg \to h$ production is not fixed as in the linear realization. Any deviation from the SM value for $gg \to hh$ must be interpreted with care. The $gg \to hh$ production rate in our effective theory construction (in the linear realization) depends on at least six unknowns, namely, $\mathcal{M}$, λ_2, C^1_ϕ, C^2_ϕ, c_G, $\tilde{c}_G$. The effects of the operator advocated in [40] increase the number of unknown parameters still further.

Clearly two Higgs production is an important signal to test the Higgs mechanism in the standard model. The cross section of $gg \to hh$ is suppressed compared to the cross section of $gg \to h$ by a factor of 1000, due to the effects of parton distribution functions and phase space suppression [60]. The cross section falls from 50 to 10 fb as the Higgs ranges in mass from 100 to 200 GeV. Thus once LHC enters its high luminosity running of 100 fb^{-1}/yr one can expect roughly 1000 events per year. A significant excess or deficit of this signal should be observable. However, the reconstruction of exactly what form of new physics is present requires more information.

One could obtain more information on the unknown parameters involved by further probes of the physics of the self-interaction of the Higgs. In the remainder of the paper we examine the sensitivity of a Higgs bound state (Higgsium) in the appropriate low energy effective field theory on TeV scale physics to these parameters. If a bound state forms, one can use the properties of the bound state such as its binding energy, as a probe of the physics above the scale $\mathcal{M}$.

C. Higgsium: production and decay time

To get some rough understanding of the conditions under which a Higgs-Higgs bound state may form, consider the nonrelativistic Schrödinger equation

$$[-\nabla_r^2 + V(r) - E]\psi(r) = 0, \tag{55}$$

with the potential from a Yukawa exchange and a contact interaction,

$$V(r) = -\frac{g^2}{4\pi}\frac{e^{-m_h r}}{r} + \kappa\delta^3(r). \tag{56}$$

We are interested in the case $g \sim \lambda_3^{\text{eff}}$ and $\kappa \sim \lambda_4^{\text{eff}}$ as a nonrelativistic approximation of the Higgs self-interactions. Neglect for now the contact interaction. It is well known that the Yukawa potential produces bound states provided

$$\frac{g^2}{4\pi} \gtrsim 1.7. \tag{57}$$

Neglecting new physics effects,

$$\lambda_3^{\text{eff}} \approx 3\frac{m_h^2}{v^2}, \tag{58}$$

nonrelativistic bound states could be expected for

$$m_h \gtrsim 1.2v. \tag{59}$$

The effect of TeV scale new physics changes the relationship between the mass and the coupling. The above Yukawa bound state condition is modified to

$$m_h > [1.54 + 0.09(C^1_\phi + \tfrac{1}{4}C^2_\phi) - 0.02\lambda_2]^{1/2}v. \tag{60}$$

This demonstrates the point that if the Higgs self-coupling is significantly stronger due to strong TeV scale new physics that contributes large Wilson coefficients, then a low energy signal of this higher scale physics might be a NR bound state formed by two Higgs.

However, one can see that it is difficult to realize the NR bound state condition when we identify the couplings in this Schrödinger equation with our effective couplings. This identification is in fact incorrect. We will demonstrate in Sec. VI that the correct NR limit of the Higgs sector is described by a Lagrangian containing only contact interactions and higher derivative operators.

The formation time of the bound state can be approximated by the ratio of $4R_0/u$ where R_0 is the characteristic radius of the NR bound state and u is the relative velocity of the two Higgs. This is roughly the period of oscillation for S wave states [61].

For a NR bound state we can approximate the relative momenta of the two Higgs by $p \sim m_h u$ so that

$$\tau_f \sim \frac{4R_0}{u} \sim \frac{4}{m_h u^2}. \tag{61}$$

The SM Higgs decays predominantly via $h \to b\bar{b}$ pairs through Yukawa interactions if $114.4 < m_h \ll 2M_Z$. We take these decays as dictating the decay width of Higgsium.

Neglecting the effects of our new operators, this decay has the decay width

$$\Gamma_b = \frac{m_b^2}{v^2}\frac{3m_h}{4\pi}\left(1 - 4\frac{m_f^2}{m_h^2}\right)^{3/2}. \tag{62}$$

This gives an approximate decay time

$$\tau_b = \frac{4\pi}{3m_h}\frac{v^2}{m_b^2}. \tag{63}$$

The condition that the bound state has time to form is that $\tau_f < \tau_b$ which can be satisfied for

$$u^2 > \frac{3}{\pi}\left(\frac{m_b^2}{v^2}\right). \tag{64}$$

Thus a nonrelativistic bound state has time to form before it decays. Above 135 GeV and below the threshold of W^+W^- production, the dominant decay of the Higgs is through a virtual W pair, $h \to WW^\star$. Above $m_h > 2m_W$ the decay into W^+W^- predominates and the decay width is given by

$$\Gamma_W = \frac{m_h^3}{v^2}\frac{1}{32\pi}\sqrt{1-a_W}(4-4a_W+3a_W^2), \tag{65}$$

where $a_W = 4m_W^2/m_h^2$ using the notation of [29]. Comparing the formation and decay time for a Higgs whose mass is above the threshold of W^+W^- production we find

$$u^2 > \frac{1}{\eta(m_h, m_w)}\frac{3}{8\pi}\left(\frac{m_h^2}{v^2}\right), \tag{66}$$

where $\eta(m_h, m_w) \sim 1$.

The lower bounds on u in either case are compatible with the NR approximation for the full range of Higgs masses we consider. A relativistic bound state is also possible in either case, although an approximation scheme that can estimate its formation time is lacking. In the remainder of the paper we focus on the possibility of a NR bound state being formed by a relatively light Higgs, $m_h < 2m_t$, due to our treatment of the top quark. We also note that a NR bound state may also have observable effects on the spectrum of two Higgs production even if a bound state does not fully form as in the case of top quark pair production near threshold in e^+e^- collisions [61].

VI. NONRELATIVISTIC HIGGS EFFECTIVE THEORY

If the two Higgs are created with small relative velocity and form a nonrelativistic bound state it is appropriate to describe the physics of this state with a nonrelativistic effective field theory of the Higgs sector. We refer to our effective theory derived in Sec. I through Sec. IV C as Higgs effective theory (HET) and now match onto a nonrelativistic version of this theory (NRHET) where we take the $c \to \infty$ limit of the scalar field Lagrangian density of HET. Recall the Lagrangian density is of the form

$$\mathcal{L} = \frac{1}{2}\left[1 + c_1^{\rm eff}\frac{h}{v} + c_2^{\rm eff}\frac{h^2}{v^2}\right]\partial^\mu h \partial_\mu h - \frac{1}{2}m_h^2 h^2 - \frac{v\lambda_3^{\rm eff}}{3!}h^3 - \frac{\lambda_4^{\rm eff}}{4!}h^4 + \mathcal{O}\left(\frac{v^2}{\mathcal{M}^2}\right). \tag{67}$$

We wish to construct the nonrelativistic limit of this Lagrangian density systematically, retaining $\hbar = 1$ and making factors of c explicit with $[c] \sim [x]/[t]$. The dimensionful quantities can be expressed in units of length $[x]$ and time $[t]$. As $\hbar = 1$, we still have $[E] \sim 1/[t]$ and $[p] \sim 1/[x]$. As the action $S = \int dt d^3x \mathcal{L}$ is dimensionless, we have $[\mathcal{L}] \sim [x]^{-3}[t]^{-1}$. For the time and spatial derivatives to have the same units in $\mathcal{L}$ we take

$$\partial_0 = \frac{1}{c}\frac{\partial}{\partial t}, \tag{68}$$

and so $\partial^\mu \sim 1/[x]$. This gives $[h] \sim 1/\sqrt{[x][t]}$. We require $[m_h c^2] \sim [E] \sim 1/[t]$, so that we have $[m_h] \sim [t]/[x]^2$, and choose the electroweak symmetry breaking expectation value to have the same dimensions as the field h, $[v] \sim 1/\sqrt{[x][t]}$. The Lagrangian density with these unit conventions is given by

$$\mathcal{L} = \frac{1}{2}\left[1 + c_1^{\rm eff}\frac{h}{v} + c_2^{\rm eff}\frac{h^2}{v^2}\right]\partial^\mu h \partial_\mu h - \frac{1}{2}m_h^2 c^2 h^2 - \frac{v\lambda_3^{\rm eff}}{3!c}h^3 - \frac{\lambda_4^{\rm eff}}{4!c}h^4 + \mathcal{O}\left(\frac{v^2}{\mathcal{M}^2}\right). \tag{69}$$

Now consider the nonrelativistic limit of this theory. The interaction terms will be determined below by matching. Consider first the theory of a free real scalar field of mass m_h given by

$$\mathcal{L} = \tfrac{1}{2}\partial^\mu\varphi\partial_\mu\varphi - \tfrac{1}{2}m_h^2c^2\varphi^2. \tag{70}$$

The field φ must also be expanded in the $c \to \infty$ limit. We remove a large energy scale $m_h c^2$ from this field with a field redefinition

$$\varphi(x) = e^{-im_h c^2 r\cdot x}\varphi_+(x) + e^{im_h c^2 r\cdot x}\varphi_-(x), \tag{71}$$

where $r = (1, \mathbf{0})$ and $\varphi_+(x)$, $\varphi_-(x)$ correspond to the creation and annihilation components of the scalar field $\varphi(x)$. Expanding the Lagrangian density in terms of $\varphi_\pm(x)$ we neglect terms multiplied by factors of

$$e^{nim_h c^2 t}, \qquad (n \neq 0, n \in \mathcal{I}). \tag{72}$$

These are terms in the Lagrangian density where some of the fields are far off shell. Their effect is only to modify coefficients of local operators in the effective action.

With this substitution we find

$$\mathcal{L} = \partial_0\varphi_-\partial^0\varphi_+ - \partial_i\varphi_-\partial^i\varphi_+ + im_hc(\varphi_-\partial^0\varphi_+ - \varphi_+\partial^0\varphi_-). \tag{73}$$

The first term, with two time derivatives, is suppressed by $1/c^2$ and is suppressed in the $c \to \infty$ limit. Integrating by parts the remaining kinetic terms and rescaling $h_\pm = \sqrt{2m_h}\varphi_\pm$ gives

$$\mathcal{L}^0_{\rm NR} = h_-\left(i\frac{\partial}{\partial t} + \frac{\nabla^2}{2m_h}\right)h_+. \tag{74}$$

One can extend this effective Lagrangian by adding interactions, including higher order terms suppressed by $|\mathbf{u}|/c$ and $v^2/\mathcal{M}^2$ where $\mathbf{u}$ is the relative velocity of the two Higgs in a nonrelativistic bound state. Scattering is described by a contact interaction which can be parametrized by a coupling $C_{\rm NR}$,

$$\mathcal{L}_{\rm NR} = h_-\left(i\frac{\partial}{\partial t} + \frac{\nabla^2}{2m_h}\right)h_+ + \frac{C_{\rm NR}}{4c}h_-^2h_+^2. \tag{75}$$

There is no cubic interaction because this necessarily involves at least one far off shell particle. The effect of the cubic interaction in the HET is incorporated in the coupling $C_{\rm NR}$, and we will compute this in terms of the parameters of the HET below, in Sec. VI A.

In this effective theory, the energy and the momenta of the system are given by

$$k^0 = \tfrac{1}{2}m_h|\mathbf{u}|^2, \tag{76}$$

$$\mathbf{q} = m_h\mathbf{u}, \tag{77}$$

where $|\mathbf{u}| \ll c$ is the relative velocity of the two Higgs. It is advantageous to have power counting rules in $|\mathbf{u}|$ that are as manifest as possible in the Lagrangian density as demonstrated in [62]. We rescale so that the natural sizes of the coordinates are given by the above energy and momentum and define a new field $\mathrm{H}_\pm(\mathrm{x})$ and new, dimensionless coordinates $\mathbf{X}$ and T by

$$\mathbf{x} = \lambda_x\mathbf{X}, \qquad t = \lambda_t T, \qquad h_\pm(x) = \lambda_h H_\pm(x). \tag{78}$$

To ensure the rescaled energy and momenta are of order unity we have $\lambda_t = m_h\lambda_x^2$ and

$$\lambda_x = \frac{1}{m_h|\mathbf{u}|}, \tag{79}$$

$$\lambda_t = \frac{1}{m_h|\mathbf{u}|^2}, \tag{80}$$

$$\lambda_h = m_h^{3/2}|\mathbf{u}|^{3/2}, \tag{81}$$

$$K^0 = \frac{k^0}{m_h|\mathbf{u}|^2}, \tag{82}$$

$$\mathbf{K} = \frac{\mathbf{q}}{m_h|\mathbf{u}|}. \tag{83}$$

The form of the Lagrangian density when we implement these rescalings and introduce an appropriately rescaled contact coupling, $\hat{C}_{\rm NR} = 4m_h^2 C_{\rm NR}$ is given by

$$\mathcal{L}_{\rm NRH} = H_-\left(i\partial^0 + \frac{\nabla^2}{2}\right)H_+ + \frac{\hat{C}_{NR}}{16}\frac{|\mathbf{u}|}{c}(H_-)^2(H_+)^2. \tag{84}$$

This form of the Lagrangian makes power counting explicit in the small parameter u/c. Physical quantities, such as the energy of bound states, can be equally calculated from the theories in Eqs. (75) or (84). Which is used is a matter of convenience: the former has familiar dimensions while the latter has explicit power counting.[6]

[6]The $c \to \infty$ limit of NR effective field theories was studied in [63]. The reader interested in bound states at threshold in NRHET would also profit from an examination of the treatment of bound states at threshold in NN effective field theory, reviewed in [64].

A. Matching onto NRHET

To determine the matching coefficient $C_{\rm NR}$ we take the nonrelativistic limit of the hh → hh scattering determined in HET. We neglect the running from m_t^2 down to our matching scale $\mu^2 = m_h^2$ in this initial study, and perform the matching at tree level only.

1. Linear realization

The HET contact interaction is given by

$$\mathcal{A}_0^L = -3\lambda_1 + 20\lambda_1 C_h^K - \frac{15}{2}\lambda_2\frac{v^2}{\mathcal{M}^2} + \frac{4N_c}{\pi^2}\left(\frac{m_t^4}{v^4}\right). \tag{85}$$

The Yukawa exchange Feynman diagrams, shown in Fig. 5, give the amplitude

$$i\mathcal{A}_y^L(s,t,u) = i(A_1^L(t) + A_1^L(u) + A_1^L(s)), \tag{86}$$

where s, t, u are the usual Mandelstam variables, and

$$A_1^L(x) = \frac{-3v^2\lambda_1}{x - m_h^2 + i\epsilon}\left(3\lambda_1 + 5\lambda_2\frac{v^2}{\mathcal{M}} - 4(x + 2m_h^2)\frac{C_h^K}{v^2} - \frac{2N_c}{\pi^2}\left(\frac{m_t^4}{v^4}\right) - 18\lambda_1 C_h^K\right). \tag{87}$$

The total amplitude for $hh \to hh$ scattering is given by

$$\mathcal{A}_{hh\to hh}^L(s,t,u) = \mathcal{A}_0^L + \mathcal{A}_y^L(s,t,u). \tag{88}$$

To perform the matching we take the momenta of the Higgs particles to be off-shell by a small residual momenta $\tilde{p}$ with energy and momenta that scale as $\tilde{p}_0 \sim m_h u^2$ and $\tilde{\mathbf{p}} \sim m_h\mathbf{u}$. The momenta of the Higgs are decomposed as [recall $r = (1, \mathbf{0})$]

$$\begin{aligned} p &= m_h r + \tilde{p}, & k &= m_h r + \tilde{k}, \\ p' &= m_h r + \tilde{p}', & k' &= m_h r + \tilde{k}'. \end{aligned} \tag{89}$$

This gives, in the center of mass frame

$$\begin{aligned} s &= 4m_h^2 + 4|\mathbf{q}|^2, \qquad t = -|\mathbf{q}|^2(1-\cos(\theta)), \\ u &= -|\mathbf{q}|^2(1+\cos(\theta)), \end{aligned} \tag{90}$$

with $\mathbf{q} \sim m_h\mathbf{u}$. In the nonrelativistic limit we retain the lowest order in $|\mathbf{u}|$ and we have

$$\begin{aligned} \mathcal{A}_{\rm NR}^L &= \mathcal{A}_0^L + \mathcal{A}_1^L(4m_h^2) + 2\mathcal{A}_1^L(0) \\ &= 12\lambda_1 + 10\lambda_2\frac{v^2}{\mathcal{M}^2} - 64\lambda_1 C_h^K - \frac{39N_c}{4\pi^2}\left(\frac{m_t^4}{v^4}\right). \end{aligned} \tag{91}$$

To determine the coupling $C_{\rm NR}^4$ in the NRHET Lagrangian, Eq. (75), we compute the four point amplitude and insist that it equals $\mathcal{A}_{\rm NR}$. Inserting four factors of $\sqrt{2m_h}$ to account for relativistic normalization of states, we finally arrive at

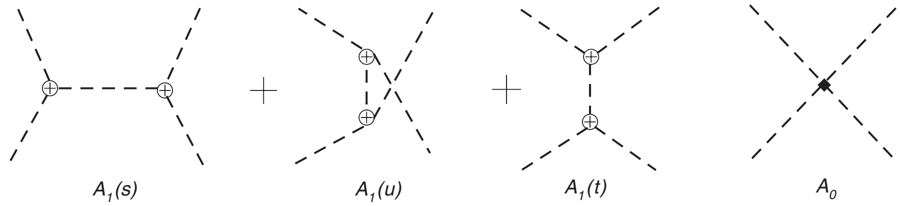

FIG. 5. Tree level $hh \to hh$ scattering in the extended Higgs theory. Time flows left to right.

$$(2m_h)^2 C^L_{\rm NR} = \hat{C}^L_{\rm NR} = 12\lambda_1 + 10\lambda_2 \frac{v^2}{\mathcal{M}^2} - 64\lambda_1 C_h^K - \frac{39N_c}{4\pi^2}\left(\frac{m_t^4}{v^4}\right). \tag{92}$$

2. Nonlinear realization

For a nonlinear realization we find the following for the HET contact interaction

$$\mathcal{A}_0^{\rm NL} = -\lambda_4^{\rm eff} + 4\frac{m_h^2}{v^2}c_2^{\rm eff}. \tag{93}$$

The Yukawa exchange diagrams give

$$A_1^{\rm NL}(x) = \frac{-v^2}{x - m_h^2 + i\epsilon}\left[\lambda_3^{\rm eff} - \frac{c_1^{\rm eff}}{2}\left(\frac{2m_h^2 + x}{v^2}\right)\right]^2. \tag{94}$$

The matching is performed as in a linear realization and we find

$$\mathcal{A}_{\rm NR}^{\rm NL} = \mathcal{A}_0^{\rm NL} + \mathcal{A}_1^{\rm NL}(4m_h^2) + 2\mathcal{A}_1^{\rm NL}(0) = \frac{5}{3}\frac{v^2}{m_h^2}(\lambda_3^{\rm eff})^2 - \lambda_4^{\rm eff} - 2c_1^{\rm eff}\lambda_3^{\rm eff} + (4c_2^{\rm eff} - (c_1^{\rm eff})^2)\frac{m_h^2}{v^2}. \tag{95}$$

This gives the effective HET coupling in the nonlinear realization

$$(2m_h)^2 C^{\rm NL}_{\rm NR} = \hat{C}^{\rm NL}_{\rm NR} = \frac{5}{3}\frac{v^2}{m_h^2}(\lambda_3^{\rm eff})^2 - \lambda_4^{\rm eff} - 2c_1^{\rm eff}\lambda_3^{\rm eff} + (4c_2^{\rm eff} - (c_1^{\rm eff})^2)\frac{m_h^2}{v^2}. \tag{96}$$

B. NRHET bound state energy

To find the approximate bound state energy of the Higgs, we calculate the bubble sum in our NRHET theory and interpret the pole in the resummed bubble chain as the bound state energy of Higgsium. Note that this calculation is formally justified in the large N limit [65] where the Higgs sector is equivalent to an $O(4)$ theory [66]. The Feynman rules for the NRHET Lagrangian in (75) are shown in Fig. 6.

The bubble sum is straightforward to calculate in NRHET. The leading order term is directly obtained from the Feynman rules, we use the Lagrangian given by Eqs. (75) in the following. The leading bubble graph is given by

$$i\mathcal{A}_{\text{1-loop}} = (iC_{\rm NR})^2 \int \frac{dk^0 d^d k}{(2\pi)^d} \frac{i}{(E + k^0) - \mathbf{k}^2/2m_h + i\epsilon} \cdot \frac{i}{-k^0 - \mathbf{k}^2/2m_h + i\epsilon} \tag{97}$$

We have chosen to work in the center of mass frame, and $E = P_1^0 + P_2^0$ stands for the center of mass energy. Performing the first integral by residues and the remaining integrations with dimensional regularization, we find

$$i\mathcal{A}_{\text{1-loop}} = -i\frac{m_h(C_{\rm NR})^2}{4\pi}(-m_h E)^{1/2}. \tag{98}$$

The terms in the bubble sum of diagrams shown in Fig. 7 are given by the geometric series

$$iC_{\rm NR}\left[1 - \frac{m_h C_{\rm NR}}{4\pi}(-m_h E)^{1/2} + \left(\frac{m_h C_{\rm NR}}{4\pi}(-m_h E)^{1/2}\right)^2 + \cdots\right] = \frac{iC_{\rm NR}}{1 + \frac{m_h C_{\rm NR}}{4\pi}(-m_h E)^{1/2}}.$$

This result agrees with [62,67] and indicates a bound state with a bound state for $C_{\rm NR} > 0$ with binding energy

$$E_b = \frac{1}{m_h}\left(\frac{4\pi}{m_h C_{\rm NR}}\right)^2 = m_h\left(\frac{16\pi}{\hat{C}_{\rm NR}}\right)^2. \tag{99}$$

There is an implicit renormalization condition introduced by dimensional regularization. The integral has no pole as $d \to 3$, so it is interesting to ask what subtraction has been made. This is easily understood by performing the

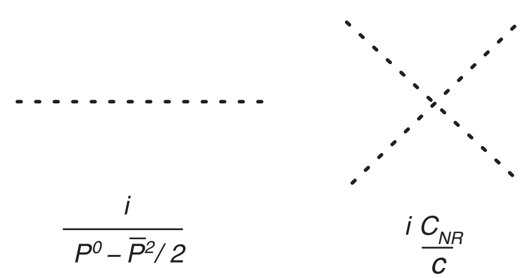

FIG. 6. Feynman rules for NRHET.

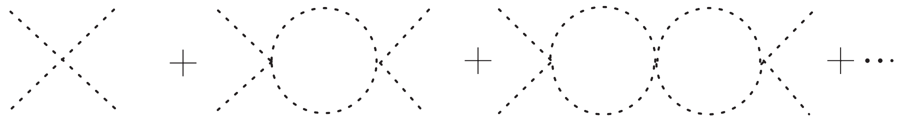

FIG. 7. The bubble sum of graphs leading to the bound state pole in NRHET.

$d = 3$ integration with a momentum cutoff $|\mathbf{k}| < \Lambda$ in terms of the bare coupling $C^0_{\rm NR}$:

$$i\mathcal{A}^{\Lambda}_{\text{1-loop}} = im_h (C^0_{\rm NR})^2 \left[\frac{\Lambda}{2\pi^2} - \frac{1}{4\pi}(-m_h E)^{1/2}\right]. \quad (100)$$

The renormalized coupling $C_{\rm NR}(\mu)$ can be defined as the amplitude at a fixed energy $E = -\mu$ [68]. Then the combination

$$\frac{1}{C_{\rm NR}} \equiv \frac{1}{C^0_{\rm NR}} - \frac{m_h\Lambda}{2\pi^2} = \frac{1}{C_{\rm NR}(\mu)} - m_h\frac{(m_h\mu)^{1/2}}{4\pi} \quad (101)$$

is renormalization group invariant. This is precisely the coupling that appears in (98).

It would appear that for any positive value of $C_{\rm NR}$ we have bound states. However for our NR description to be self-consistent we require that the binding energy of the bound state satisfy $E_b < m_h$, that is,

$$\hat{C}_{\rm NR} > 16\pi. \quad (102)$$

1. Linear realization

In the case of a linear realization a heavy Higgs seems necessary for the bound state to form, but the new physics effects may allow significantly smaller masses for the bound state. If we neglect the effects of new physics (λ_2 and C_h^K) the bound (102) translates into $m_h > 2.0v$. Retaining the effects of λ_2 and C_h^K one determines a condition for the NRHET calculation of the bound state energy to be self-consistent

$$\frac{m_h}{v} > \sqrt{\frac{16\pi - 4\lambda_2\frac{v^2}{\mathcal{M}^2} + \frac{51N_c}{4\pi^2}\left(\frac{m_t^4}{v^4}\right)}{12 - 40C_h^K}}. \quad (103)$$

Alternatively, for a given value of the Higgs mass, say $m_h = \xi v$, the self-consistency condition implies a constraint on the coefficients of the higher dimension operators:

$$12\xi^2 + 4\frac{v^2}{\mathcal{M}^2}\left(\lambda_1 - 10\xi^2\left(C^1_\phi + \frac{1}{4}C^2_\phi\right)\right) > 16\pi + \frac{51N_c}{4\pi^2}\left(\frac{m_t^4}{v^4}\right) \quad (104)$$

Using $\mathcal{M} = 1$ TeV and the PDG value for the top quark mass, this condition simplifies to

$$1.2\xi^2 + 0.024\lambda_1 - 0.24\xi^2(C^1_\phi + \tfrac{1}{4}C^2_\phi) > 5.1 \quad (105)$$

So, for example, for $|C^1_\phi + \frac{1}{4}C^2_\phi| = 1$ or 5 the minimal Higgs mass for a NR bound state is reduced by 6% or 28%, respectively. Near the limit of validity of our calculation $m_h \sim 2m_t$, for negative values of $C^1_\phi + \frac{1}{4}C^2_\phi$ we find that a bound state is possible for $\mathcal{O}(1)$ Wilson coefficients as we illustrate in Fig. 8.

2. Nonlinear realization

In the nonlinear realization this condition is easily satisfied even for a light Higgs, $m_h < v$. Recall that $\lambda_3^{\rm eff}$ and $c_1^{\rm eff}$ are both enhanced by powers of $\mathcal{M}/v$. Taking, for example, $m_h = 120$ GeV and $\mathcal{M} = 1$ TeV, neglecting the contribution of $c_2^{\rm eff}$, the NR bound state condition is

$$\frac{5}{3}\frac{\mathcal{M}^2}{m_h^2}(\tilde{\lambda}_3^{\rm eff})^2 - 2\tilde{c}_1^{\rm eff}\tilde{\lambda}_3^{\rm eff} - \frac{m_h^2}{\mathcal{M}^2}(\tilde{c}_1^{\rm eff})^2 > 16\pi + \lambda_4, \quad (106)$$

where

$$\lambda_3^{\rm eff} = \left(\frac{\mathcal{M}}{v}\right)\tilde{\lambda}_3^{\rm eff}, \qquad c_1^{\rm eff} = \left(\frac{v}{\mathcal{M}}\right)\tilde{c}_1^{\rm eff}. \quad (107)$$

Note that as $\mathcal{M}$ grows larger the region that satisfies the NR bound state condition *grows*. This is due to the fact that the attractive interaction given by $\lambda_3^{\rm eff} \sim \mathcal{M}/v$ is a relevant operator. We find that as m_h grows and as $\mathcal{M}$ is larger the allowed parameter space of the NR bound state condition is significant, demonstrating that a bound state is likely to form in the nonlinear realization. We illustrate the bound state conditions for various parameter choices in Figs. 9 and 10.

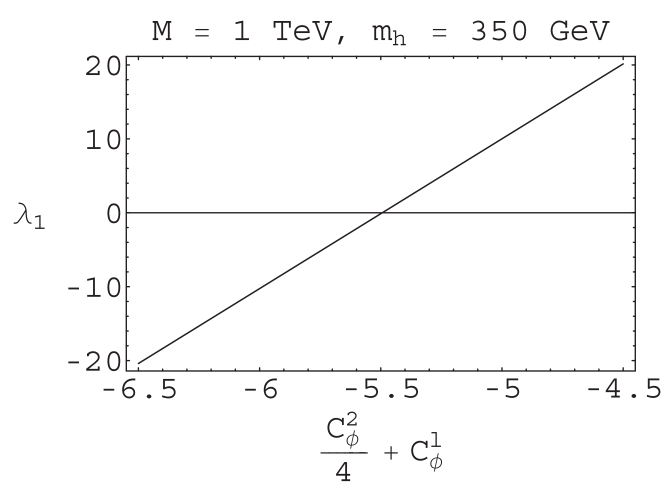

FIG. 8. In the linear realization the allowed parameter space for NR bound state formation is above the line.

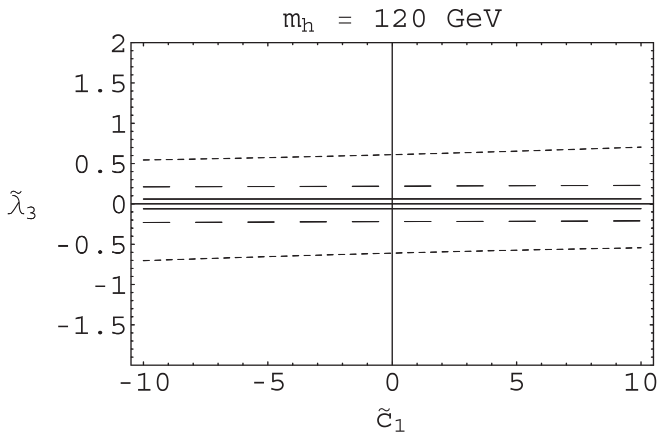

FIG. 9. In the nonlinear realization, holding m_h fixed and set $\lambda_4 = 0$ as it is $\mathcal{O}(1)$ and suppressed by 16π. We vary $\mathcal{M}$ for the values $\mathcal{M} = 1$ TeV (dotted line), $\mathcal{M} = 3$ TeV (dashed line), and $\mathcal{M} = 10$ TeV (solid line). The region above (the upper) and below (the lower) hyperbolic curves satisfy NR the bound state condition.

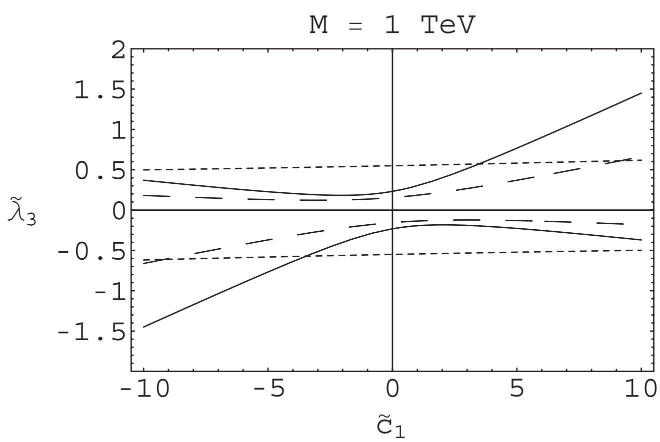

FIG. 10. In the nonlinear realization, holding $\mathcal{M}$ fixed and set $\lambda_4 = 0$ as it is $\mathcal{O}(1)$ and suppressed by 16π. We vary $m_h = 300$ GeV (solid line), $m_h = 200$ GeV (dashed line), and $m_h = 100$ GeV (dotted line). The region above (the upper) and below (the lower) hyperbolic curves satisfy the NR bound state condition.

VII. SUMMARY AND CONCLUSIONS

If a new strong interaction is responsible for electroweak symmetry breaking but a Higgs particle, the pseudo-Goldstone boson of broken scale invariance, remains unnaturally light, the self-interactions of this Higgs particle could be quite strong. If strong enough these self-interactions could bind two Higgs particles.

To study these questions we formulated two different effective theories of the light, self-interacting Higgs below the scale $\mathcal{M}$ of the new physics. In the first, the symmetry is realized linearly and the Higgs field is described as one component of an $SU(2)_L$ doublet, just as in the standard model of electroweak interactions. In the second approach the symmetry is realized nonlinearly: the triplet of would-be Goldstone bosons and the Higgs field are not in a common multiplet. We note that operators of dimension 3 in the effective Lagrangian in the nonlinear realization are naturally expected to be enhanced by a power of $\mathcal{M}/v$ relative to their linear realization counterparts.

In order to study how large these couplings need be, we have studied the case of nonrelativistic bound states. To this end we constructed a nonrelativistic Higgs effective theory (NRHET) describing self-interacting Higgs particles in the rest frame of the bound state, in the nonrelativistic limit.

The effects of the top quark are small but nonnegligible. We estimated them by including the virtual top quark effects as a modification to the couplings in the NRHET.

Our results show, perhaps not surprisingly, that in the nonlinear realization it is quite easy to form light Higgsium, as we call the Higgs-Higgs bound state. For natural couplings in the linear realization a bound state is only likely to form for $m_h \sim v$. Relativistic bound states are possible in both the linear and nonlinear realizations.

There are many questions that we have not addressed. The most immediate one is how to search for Higgsium. Assuming a light Higgs is found, one could imagine strategies involving invariant mass distributions of Higgs-pair production. A dedicated study is required to determine if this or other strategies are viable. Another, related question is whether the effects of a short lived bound state could be seen indirectly, much like would-be toponium affecting the line shape in top quark pair production near threshold in e^+e^- collisions. It would also be interesting to solve the bound state equation in the more general, fully relativistic case. We hope to return to these problems in the future.

ACKNOWLEDGMENTS

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APPENDIX A: CUSTODIAL SYMMETRY AND THE S PARAMETER

There is some confusion in the literature regarding custodial symmetry and the operator

$$-\frac{c_{WB}g_1g_2}{\mathcal{M}^2}(\phi^\dagger\sigma^I\phi)B^{\mu\nu}W_{I\mu\nu} \tag{A1}$$

which corresponds to the S parameter. Consider the matrix representation of this operator [69] where the Higgs doublet field is given by

$$\phi = \begin{pmatrix}\phi^+ \\ \phi^0\end{pmatrix}. \tag{A2}$$

Then $\epsilon\phi^\star$ is also an $SU_L(2)$ doublet with components

$$\epsilon\phi^{\star} = \begin{pmatrix} \phi^{0\star} \\ -\phi^{-} \end{pmatrix}, \tag{A3}$$

where $\phi^{-} = \phi^{+\star}$. The Higgs bidoublet field is given by

$$\Phi = \frac{1}{\sqrt{2}}(\epsilon\phi^{\star}, \phi) = \frac{1}{\sqrt{2}}\begin{pmatrix} \phi^{0\star} & \phi^{+} \\ -\phi^{-} & \phi^{0} \end{pmatrix}. \tag{A4}$$

The $\mathrm{SU_L}(2) \times \mathrm{U_Y}(1)$ gauge symmetry acts on the Higgs bidoublet as

$$\mathrm{SU_L}(2)\colon \Phi \rightarrow L\phi \tag{A5}$$

$$\mathrm{U_Y}(1)\colon \Phi \rightarrow \Phi e^{-i\sigma_3\theta/2}. \tag{A6}$$

In the limit that hyper charge vanishes the Lagrangian also has the following global symmetry

$$\mathrm{SU_R}(2)\colon \Phi \rightarrow \phi R^{\dagger}. \tag{A7}$$

When the Higgs acquires a vacuum expectation value, both $\mathrm{SU_L}(2)$ and $\mathrm{SU_R}(2)$ are broken, however the subgroup $\mathrm{SU_{L=R}}(2)$ is unbroken, i.e.,

$$L\langle\Phi\rangle L^{\dagger} = \langle\Phi\rangle. \tag{A8}$$

This is explicitly the custodial symmetry, and the corresponding transformation of the Higgs bidoublet under this symmetry. It is easy to see that

$$-\frac{c_{WB}g_1g_2}{\mathcal{M}^2}\,\mathrm{Tr}(\Phi^{\dagger}\sigma^I W_{I\mu\nu}\Phi)B^{\mu\nu} \tag{A9}$$

is invariant under this symmetry. The Higgs bidoublet transforms as above and the field strength $\sigma^I W_{I\mu\nu}$ transforms as

$$\sigma^I W_{I\mu\nu} \rightarrow L\sigma^I W_{I\mu\nu}L^{\dagger}. \tag{A10}$$

However, it is also easy to see that this representation of the operator vanishes by explicitly performing the trace; one finds

$$\mathrm{Tr}\,(\Phi^{\dagger}\sigma^I W_{I\mu\nu}\Phi) = 0. \tag{A11}$$

The nontrivial representation of the operator in terms of the bidoublet is given by

$$-\,\mathrm{Tr}(\Phi^{\dagger}\sigma^I\Phi\sigma^3). \tag{A12}$$

With this factor of σ^3, required for a nontrivial representation in terms of the Higgs bidoublet, one finds that this operator violates custodial symmetry.

APPENDIX B: TOP QUARK OPE

As an example of the effect of the neglected terms in the top quark operator product expansion (OPE), consider the OPE corrections to the four point function of the Higgs. The amplitude is given by

$$\begin{aligned} iA_4(s,t,u) = {}& -6N_C\left(\frac{m_t}{v}\right)^4 \int \frac{d^dk}{(2\pi)^d}\,\mathrm{Tr}\left[\frac{(\not{k}+m_t)}{k^2-m_t^2}\right. \\ & \times \frac{(\not{k}+\not{a}+m_t)}{(k+a)^2-m_t^2}\frac{(\not{k}+\not{b}+m_t)}{(k+b)^2-m_t^2} \\ & \left.\times \frac{(\not{k}+\not{c}+m_t)}{(k+c)^2-m_t^2}\right]. \end{aligned}$$

We find the leading order in $p^2/m_t^2 \rightarrow 0$ the amplitude is given by

$$\begin{aligned} iA_4^0(s,t,u) &= -24N_C\left(\frac{m_t}{v}\right)^4 \int \frac{d^dk}{(2\pi)^d}\frac{(m_t^4+6k^2m_t^2+k^4)}{(k^2-m_t^2)^4} \\ &= -\frac{iN_c}{16\pi^2}\left(\frac{m_t}{v}\right)^4\left(\frac{24}{\epsilon} - 64 + 24\log\left[\frac{\mu^2}{m_t^2}\right]\right). \end{aligned} \tag{B1}$$

The leading order matching gives a factor of $-4N_Cm_t^4/v^4$.

Consider performing the top quark OPE to higher orders. We find for the next order in p^2/m_t^2

$$\begin{aligned} iA_4^1(s,t,u) = {}& -\frac{iN_C}{16\pi^2}\left(\frac{m_t}{v}\right)^4\left(\frac{1}{80m_t^2}\right) \\ & \times (a^2+b^2+c^2+a\cdot b+6a\cdot c+b\cdot c). \end{aligned} \tag{B2}$$

The invariants of the external momenta a, b, c averaged over the sum of all A_4 diagrams can be expressed in the Mandelstam variables. We find that our momenta expressed in terms of these variables are

$$\begin{aligned} \langle a^2\rangle &= 4!m_h^2, \\ \langle b^2\rangle &= 8(s+t+u), \\ \langle c^2\rangle &= 4![(s+t+u)-3m_h^2], \\ \langle a\cdot b\rangle &= 4(s+t+u), \\ \langle a\cdot c\rangle &= 8(s+t+u-3m_h^2), \\ \langle b\cdot c\rangle &= 16(s+t+u-3m_h^2). \end{aligned} \tag{B3}$$

With these substitutions, the next order in the expansion gives

$$\begin{aligned} iA_4^1(s,t,u) &= -\frac{iN_C}{16\pi^2}\left(\frac{m_t}{v}\right)^4\left(\frac{m_h^2}{m_t^2}\right)\left(\frac{s+t+u}{4m_h^2} - \frac{3}{5}\right) \\ &= -\frac{iN_C}{16\pi^2}\left(\frac{m_t}{v}\right)^4\left(\frac{m_h^2}{m_t^2}\right)\frac{2}{5}. \end{aligned} \tag{B4}$$

Where in the last expression we simplified with $s+t+u = 4m_h^2$. This term matches onto the operator

$$O_h^{2,0} = \frac{hh}{\mathcal{M}^2}\partial^{\mu}h\partial_{\mu}h, \tag{B5}$$

with a Wilson coefficient that contains contributions from the integrating out TeV scale new physics and the top quark. At the scale $\mu^2 = m_t^2$ the Wilson coefficient is

$$C_h^{2,0}(m_t^2) = \frac{\mathcal{M}^2}{v^2}\left(4C_h^K(m_t^2) + \frac{m_t^2}{v^2}\frac{N_C}{20\pi^2}\right). \quad \text{(B6)}$$

The later term in the Wilson coefficient is an example of a term that is neglected in our calculation. Corrections of this form can be systematically included by taking the top quark OPE to next order in p^2/m_t^2.

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