Holography and unquenched quark-gluon plasmas

G. Bertoldi,[1,*] F. Bigazzi,[2,†] A. L. Cotrone,[3,‡] and J. D. Edelstein[4,5,§]

[1]*Department of Physics, Swansea University, Swansea, SA28PP, United Kingdom*

[2]*Physique Théorique et Mathématique and International Solvay Institutes, Universit é Libre de Bruxelles, C.P. 231, B-1050 Bruxelles, Belgium*

[3]*Departament ECM, Facultat de Física, Universitat de Barcelona and Institut de Física d'Altes Energies, Diagonal 647, E-08028 Barcelona, Spain*

[4]*Departamento de Física de Partículas and IGFAE, Universidade de Santiago de Compostela, E-15782 Santiago de Compostela, Spain*

[5]*Centro de Estudios Científicos (CECS), Casilla 1469, Valdivia, Chile*

(Received 23 May 2007; published 12 September 2007)

We employ the string/gauge theory correspondence to study properties of strongly coupled quark-gluon plasmas in thermal gauge theories with a large number of colors and flavors. In particular, we analyze noncritical string duals of conformal (S)QCD, as well as ten-dimensional wrapped fivebrane duals of SQCD-like theories. We study general properties of the dual plasmas, including the drag force exerted on a probe quark and the jet quenching parameter. We find that these plasma observables depend on the number of colors and flavors in the "QCD dual"; in particular, we find that the jet quenching parameter increases linearly with N_f/N_c at leading order in the probe limit. In the ten-dimensional case we find a nontrivial drag coefficient but a vanishing jet quenching parameter. We comment on the relation of this result with total screening and argue that the same features are shared by all known plasmas dual to fivebranes in ten dimensions. We also construct new D5 black hole solutions with spherical horizon and show that they exhibit the same features.

DOI: 10.1103/PhysRevD.76.065007 PACS numbers: 11.25.Tq, 11.10.Wx, 12.38.Mh

I. INTRODUCTION

The experimental data collected at RHIC [1], where heavy ions collide at very high energies, indicate that a new state of matter is formed during the collisions: the QCD quark-gluon plasma (QGP). It partially behaves like a fluid with very low shear viscosity and, contrary to the common theoretical expectations, present evidence supports the picture of a strongly interacting medium [2]. Some of its observed features can be fitted with predictions from lattice gauge theory, but this cannot be used to describe its real time dynamical properties.

A powerful tool to study strongly interacting gauge theories, both at finite and at zero temperature, is the string/gauge theory correspondence [3]. At present, it has been used to study the properties of plasmas where gluons and adjoint matter fields (but no quarks) interact. The high temperature gauge theory at strong 't Hooft coupling $g^2_{\rm YM}N_c \gg 1$, and in the limit of a large number of colors $N_c \gg 1$, can be described by a dual classical supergravity model with background metric containing a black hole. The features of the metric depend on those of the dual field theory. The master example is provided by the thermal version of $\mathcal{N}=4$ supersymmetric Yang-Mills (SYM) theory. The dual gravity background is $\mathrm{AdS}_5 \times S^5$ with a large black hole in the center of the space. The above thermal field theory differs from thermal QCD in many aspects, one of the most evident being its lack of dynamical quarks. Nevertheless, the $\mathcal{N}=4$ SYM plasma is nonsupersymmetric and nonconfining as the QCD one, and it is tempting to compare the AdS/CFT predictions on $\mathcal{N}=4$ SYM and the extrapolated data from QCD in order to understand how macroscopic observables are sensitive to the microscopic details of these two theories.

The string/gauge correspondence had been employed to evaluate the shear viscosity of the $\mathcal{N}=4$ plasma [4] some years ago. This was a major success since the resulting value seems to obey a universal law and, furthermore, it is well compatible with the values for the QGP found at RHIC. This heavily supports the use of AdS/CFT to describe this physics. More recently, a lot of explicit calculations of real time dynamical properties, such as the drag force experienced by a quark moving through the plasma [5–7], the jet quenching parameter measuring the average energy loss by the parton per mean-free path [8], the screening length of a moving color charge inside the plasma [9,10], and the rate of photoproduction [11] were given. Generalizations of these results in different directions were pursued, respectively, in [12–15]. See, also, [16–20].

Despite some remarkable and unexpected numerical similarities between observables evaluated in the extrapolation to $N_c = 3$ of large N_c, $\mathcal{N}=4$ thermal SYM theory, and the same observables deduced for the QCD quark-gluon plasma from the RHIC data, there are several reasons to be cautious. Of primary importance, it is mandatory to

*g.bertoldi@swansea.ac.uk
†fbigazzi@ulb.ac.be
‡cotrone@ecm.ub.es
§jedels@usc.es

put the string/gauge theory correspondence at work for field theories closer and closer to real world QCD. In this sense, an unavoidable step to be able to properly talk about a QGP consists in dealing with gauge theories coupled to N_f flavors of dynamical quarks. The first order effects in N_f/N_c of the inclusion of flavors in the thermal string/gauge theory correspondence have been recently studied in [21–24]. In this paper, we aim at considering the full, unquenched influence of dynamical quarks on the plasmas, by analyzing known supergravity solutions encoding these effects.

In Sec. II, we shall study plasmas having a noncritical string theory dual description. The relevant gravity backgrounds we consider are AdS_5 and $AdS_5 \times S^1$ black hole solutions found in [25]. They are the nonextremal deformations of the zero temperature solutions found, respectively, in [26,27]. These are conjectured to be dual to conformal QCD and $\mathcal{N} = 1$ supersymmetric quantum chromodynamics (SQCD) in the Seiberg conformal window, in the so-called Veneziano limit [28], where N_f, $N_c \to \infty$ but $\rho \equiv N_f/N_c$ is kept finite. The gravity solutions are strongly curved and α' corrections are expected to be relevant. Nevertheless, it is worth exploring their properties, as they give a hint on the qualitative behavior of plasma observables in relation with N_f. The holographic expressions for plasma properties like the screening length, the drag force coefficient, the jet quenching parameter, and the diffusion coefficient for heavy quarks, can be immediately taken from the AdS critical cases, as they are not sensitive to the transverse directions. Interestingly enough, when expressed in terms of gauge theory quantities, these observables turn out to be independent on the number of colors and flavors in the "SQCD" model. This is not so for the "QCD" model, where, apart for the screening length which remains insensitive to the degrees of freedom of the plasma, there is an explicit dependence on N_c, N_f. In particular, the drag force coefficient and the jet quenching parameter, which are the quantities on which we will mainly focus, are monotonic growing functions of ρ. This could be connected to a similar behavior of the entropy density. This qualitative behavior would reinstate the discussion about the use of the jet quenching parameter as a direct measure of the temperature of the plasma, since it would explicitly depend on the number of effective degrees of freedom. Finally, we make the (trivial) observation that the shear viscosity to entropy density ratio is still $1/4\pi$ in these backgrounds, in the gravity approximation.

In Sec. III, we analyze a ten-dimensional one-parameter family of black hole solutions found in [29] and conjectured to be dual to the high temperature regime of an $\mathcal{N} = 1$ SQCD-like theory with $N_f = 2N_c$ flavors and adjoint matter. The backgrounds describe the low energy dynamics of a stack of a large number of both wrapped "color" D5-branes and smeared "flavor" branes in the Veneziano limit. We study again the energy loss of a probe moving into the dual plasmas by evaluating the drag force and the jet quenching parameter. Moreover, we study the screening properties of the plasmas. We find that the drag force has analogous functional form as the one evaluated for the $\mathcal{N} = 4$ SYM plasma, that the jet quenching parameter vanishes and that probe quarks and antiquarks are always screened. These results seem related to the critical thermodynamical properties of the model, in particular, the fact that the black hole temperature, and so the temperature of the gauge theory, coincides with the Hagedorn temperature of little string theory (LST). We argue that the same properties are common to all known models, with or without flavors (as the thermal Chamseddine-Volkov solution [30–32]), having an UV LST completion. In particular, as shown explicitly in Appendix B for the backgrounds found in [32], the LST plasmas are well expected to totally screen quark-antiquark interactions. The screening property of these plasmas should be responsible for the vanishing of the jet quenching parameter.

We also present in Appendix A a new family of black hole solutions which arise as compactifications on S^3 of the $N_f = 2N_c$ configuration of [29]. Despite the inclusion of a new scale in the theory (the compactification radius), that naively might have been expected to modify the temperature away from the Hagedorn point, their holographic duals have the same thermodynamic problems and screening properties of the other LST plasmas.

We conclude in Sec. IV with a brief discussion.

II. QGP AND NONCRITICAL HOLOGRAPHY

Noncritical string duals of 4D gauge theories with large N_c, N_f both at zero and at high temperature have appeared in the literature in the past few years. In this section we investigate properties of the quark-gluon plasmas in these contexts by using the string/gauge theory correspondence, though the gravity solutions at our disposal are generically strongly curved and α' corrections are not subleading. Our optimistic prejudice, driven by the unexpected success of bottom-up approaches such as the so-called "AdS/QCD" duality [33,34], is that the α' corrections may be not very large, and that the noncritical solutions should provide at least qualitative information on the dual field theories.

The explicit backgrounds we will consider are an AdS_5 black hole solution which was proposed as the noncritical string dual of the thermal version of conformal QCD and an $AdS_5 \times S^1$ black hole solution conjectured to be dual to thermal $\mathcal{N} = 1$ SQCD in the Seiberg conformal window [25]. The zero temperature models were constructed in [26,27] respectively. In both models the color degrees of freedom are introduced via N_c D3-brane sources and the backreaction of N_f flavor branes on the background is taken into account. The flavor branes are spacetime filling brane-antibrane pairs,[1] so to reproduce the classical $U(N_f) \times U(N_f)$ flavor symmetry expected in the gauge duals with massless fundamental matter (see [36] for a very interesting analysis of this kind of setting).

In order to study the dynamics of hard probes in the quark-gluon plasmas of these theories, we will make the assumption that the mass of the probes is related to the radial distance of a flavor brane from the center of the space, as it happens in the critical $\mathrm{AdS}_5 \times S^5$ example, at least in some effective way.[2] In this case the general results on the drag force and jet quenching parameter obtained for AdS_5 black holes in the critical case extend in a straightforward way to the noncritical setup. Indeed, the general form of the background string frame metric in the noncritical cases under study is

$$ds^2 = \left(\frac{u}{R}\right)^2\left[\left(1 - \frac{u_H^4}{u^4}\right)dt^2 + dx_i dx_i\right] + (Ru)^2\frac{du^2}{u^4 - u_H^4} + e^{2\nu_0}d\Omega_k; \tag{2.1}$$

the dilaton $\phi = \phi_0$ is a constant. Let us now focus on the 5D model, postponing a brief analysis of the 6D one to the end of this section.

A. QCD in the conformal window

In the 5D model "dual to QCD in the conformal window," $k = 0$ and the solution (in $\alpha' = 1$ units) has the same radius, dilaton, and fiveform field strength as the zero temperature one [26]:

$$R^2 = \frac{200}{50 + 7\rho^2 - \rho\sqrt{200 + 49\rho^2}}, \tag{2.2}$$

$$e^{\phi_0} = \frac{\sqrt{200 + 49\rho^2} - 7\rho}{10 Q_c}, \tag{2.3}$$

$$F_{(5)} = Q_c \,\mathrm{Vol(AdS)}, \tag{2.4}$$

where

$$\rho \equiv \frac{Q_f}{Q_c} \sim \frac{N_f}{N_c}. \tag{2.5}$$

Q_c and Q_f are constants proportional to N_c and N_f respectively.[3]

Note that both the AdS radius (2.2) and the dilaton (2.3)—the latter being related to the gauge theory coupling by $g^2_{\mathrm{QCD}} \approx e^{\phi_0}$—depend on the flavor/color ratio ρ. Their dependence is the opposite: while R increases with ρ, the coupling decreases. The latter is consistent with the known fact that the zero temperature theory should be weakly coupled in the upper part of the conformal window, that is when ρ is the largest. Note that the behavior of the coupling is as expected in the Veneziano limit: it is given by $\mathcal{F}(\rho)/N_c$ for some function $\mathcal{F}$ whose behavior is precisely $\mathcal{F}(\rho) \sim 1/\rho$ for large ρ [28].

The black hole temperature and entropy density read[4]

$$T = \frac{u_H}{\pi R^2}, \qquad s = \frac{A_3}{4G_{(5)}} = \frac{\pi^2 R^3 T^3}{e^{2\phi_0}}, \tag{2.6}$$

where A_3 is the horizon area. The free energy can be obtained by suitably renormalizing the Euclidean action

$$I = \frac{1}{16\pi G_{(5)}}\int d^5x\sqrt{g}\left[e^{-2\phi}(R + 4(\partial_\mu\phi)^2 + 5) - \frac{1}{5!}F^2_{(5)} - 2Q_f e^{-\phi}\right], \tag{2.7}$$

where $5 = 10 - d$ is the constant needed to cancel the central charge. Since the dilaton is constant, the Dirac-Born-Infeld (DBI) piece [the last one in (2.7)] plays the role of an additional cosmological constant term and the calculation is then practically the same as in ten dimensions [37]. Performing it, the free energy (density) reads

$$F = TI = -\frac{\pi^2 R^3 T^4}{4e^{2\phi_0}}. \tag{2.8}$$

The energy density can be derived directly as $\epsilon = \partial_\beta I$ ($\beta \equiv 1/T$) or from the ADM definition of the energy

$$E = \frac{1}{8\pi G_{(5)}}\int d^3x\sqrt{|g^{00}|}(K_T - K_0), \tag{2.9}$$

where K_T, K_0 are, respectively, the extrinsic curvatures of the 4D constant time black hole and zero temperature backgrounds. The result is the same:

$$\epsilon = \frac{3\pi^2 R^3 T^4}{4e^{2\phi_0}}, \tag{2.10}$$

so that the thermodynamic relation[5] $F = \epsilon - Ts$ is verified. The heat capacity (density) c_V and the speed of sound v_s are (see, for example, [24])

$$c_V = \partial_T\epsilon = \frac{3\pi^2 R^3 T^3}{e^{2\phi_0}}, \qquad v_s^2 = \frac{s}{c_V} = \frac{1}{3}, \tag{2.11}$$

[1] More precisely, in the six-dimensional model a brane-antibrane pair is just a single D5-brane doubly wrapped on the cigar [35]

[2] Even in the model of [36], where also the branes supporting massive quarks are spacetime filling, one expects a probe quark to be described by a string attached to the brane at finite distance, proportional to its mass, from the bottom of the space.

[3] If we use the same normalizations as in 10D, we have $Q_f = 2k_5 T_4 N_f = N_f/(2\pi)^2$, and $Q_c = 2k_5 T_3 N_c = N_c/(2\pi)$. Even without the form of T_3, T_4 but with only their ratio, one can get $Q_f/Q_c = N_f/(2\pi N_c)$.

[4] Here and in the following we use, for the d-dimensional Newton constant, the expression $16\pi G_d = (2\pi)^{(d-3)} \times (\alpha')^{(d-2)/2} g_s^2$.

[5] Also the relation $s = -\partial_T F$ is verified.

the latter result being consistent with the zero temperature theory being conformal.

The holographic evaluation of the ratio between the shear viscosity η and the entropy density in the plasma dual to this black hole gives the same universal result:

$$\frac{\eta}{s} = \frac{1}{4\pi}, \tag{2.12}$$

as for generic gauge theories equipped with gravity duals. The reason is that this black hole solution and the effective action from which it is deduced have the general symmetries advocated in [38] for this result to be valid. Formula (2.12) is somehow a trivial result in this setting, since we are limiting ourselves to the two derivative action, ignoring α' corrections that are most likely going to modify the ratio. If the latter were not the case, (2.12) would be a proof of the fact that the ubiquitous matter in the adjoint representation of critical string duals does not give any contribution to η/s. Notice, in this respect, that the fact that matter in the fundamental representation does not change this result has already been argued in the ten-dimensional model in [29], that supposedly accounts for the full backreaction of the flavor branes in a SQCD-like theory. The result (2.12) is another example of this fact. From the expressions of the entropy density given above, one can thus immediately deduce the shear viscosity

$$\eta = \frac{\pi R^3 T^3}{4 e^{2\phi_0}}. \tag{2.13}$$

All the thermodynamic quantities F, ϵ, c_V, η have the same qualitative behavior as functions of ρ as the entropy density. The latter is a monotonically increasing function of Q_c^2 (for fixed ρ) and ρ. Its asymptotics in ρ are[6]

$$s \sim 4\pi^2 Q_c^2 T^3 \begin{cases} 1 + \sqrt{2}\rho + \ldots & \text{for } \rho \to 0, \\ \frac{343}{250}\sqrt{\frac{7}{5}}(\rho^2 + \mathcal{O}(\rho^0)) & \text{for } \rho \to \infty. \end{cases} \tag{2.14}$$

The scalings with the number of colors Q_c and the temperature T are the same as in $\mathcal{N} = 4$, as expected. But, most importantly, this solution exhibits a nice dependence on the number of fundamental flavors Q_f. As one increases Q_f, s increases: the flavors contribute positively to the entropy density. At small ρ, (2.14), the correction to the pure glue scaling $s \sim Q_c^2$ is the expected one, namely $Q_c Q_f$. Moreover, noting that the gauge theory 't Hooft coupling goes as $\lambda \equiv e^{\phi_0} Q_c \sim \sqrt{2} - 7\rho/10 + \cdots$ for $\rho \to 0$, it is tempting to write the first ρ correction to the pure glue result as $\lambda Q_c Q_f$. This is the same enhancement effect of the flavor contribution found in the probe setting of $\mathcal{N} = 4$ SYM theory in [24]. On the other hand, the behavior for large ρ, (2.14), is found to be $s \sim Q_f^2$ and the first correction is already of order Q_c^2.

[6]Of course, these expressions must be taken with a grain of salt: since the background is expected to be corrected by order one terms, the numerical coefficients are not trustworthy. Moreover, since the dual field theory should be QCD in the conformal window for definite, finite values of ρ, the limit $\rho \to 0$ ($\rho \to \infty$) is meaningful only as an indication of the behavior for small (large) ρ. The strict $\rho = 0$ case is the finite temperature version of the unflavored Polyakov solution [39], that could be dual to a YM theory without flavors.

B. Drag force and jet quenching

Now we turn to the discussion of the energy loss of partons in this plasma. As it is well known, the phenomenon of jet quenching observed at RHIC demands for a very efficient mechanism of energy loss. Recently, two main ways of accounting for this phenomenon have appeared in the literature. In [8], the main physical process for the energy loss relevant at RHIC, that is gluon radiation, is deduced in the perturbative QCD regime. All the effects of the strongly coupled plasma are encoded in just one quantity, the jet quenching parameter $\hat{q}$, that accounts for the nonperturbative repeated soft interaction of the probe and the emitted gluons with the plasma. So, the problem is reduced to the calculation of $\hat{q}$, which is postulated to be given by the coefficient of the L^2 term of a partially lightlike Wilson loop of dimensions $L^- \gg L$. The calculation of this Wilson loop is then a straightforward exercise in string theory. This description should be the right one for partons with large transverse momentum.

In a second approach initiated in [5,6], the energy loss process is treated completely at strong coupling. The parton of mass m, moving with velocity v, is viewed as the extremum of a macroscopic string attached to a probe flavor brane. In order to keep the velocity fixed, one has to apply a drag force f, giving the parton some energy and momentum, that are ultimately lost in the plasma during the motion. This transfer of energy and momentum is seen as a flow of energy and momentum in the string, from the probe brane to the horizon. The configuration and the momentum flow of this string can be explicitly calculated, allowing to derive the drag coefficient μ from the equation $f = \mu p$, where p is the momentum. Then, of course, the coefficient enters the equation $\dot{p} = -\mu p$, so its inverse is the relaxation time of the parton in the plasma. Being the entire process analyzed in string theory, and so at strong coupling, this description should be suitable for heavy quarks with relatively small transverse momentum.

In this section, instead of calculating the two quantities μ and $\hat{q}$ again (as we will do in the next section), we just note that, indeed, the calculations made for the critical $AdS_5 \times S^5$ black hole can be directly applied to the noncritical backgrounds. Thus, the general expression of the drag force (using the relation $p/m = v/\sqrt{1 - v^2}$) and jet quenching parameter are [5,6,8]

$$\frac{dp}{dt} = -\frac{\pi}{2} R^2 T^2 \frac{v}{\sqrt{1 - v^2}}, \tag{2.15}$$

$$\hat{q} = \frac{\pi^{3/2}\Gamma(\frac{3}{4})}{\Gamma(\frac{5}{4})} R^2 T^3. \tag{2.16}$$

The novelty is that these quantities are now sensitive to the number of colors and flavors in our plasma. The asymptotics for the relaxation time are

$$t \sim \frac{m}{2\pi T^2} \begin{cases} 1 - \frac{\sqrt{2}}{5}\rho + \ldots & \text{for } \rho \to 0, \\ \frac{5}{7} + \mathcal{O}\left(\frac{1}{\rho^2}\right) & \text{for } \rho \to \infty. \end{cases} \tag{2.17}$$

Using as a representative temperature of the plasma $T = 250$ MeV, the relaxation time of a charm quark (taking $m = 1.4$ GeV) turns out to be

$$t \sim 0.7, 0.6, 0.5 \text{ fm} \quad \text{for } \rho = 0, 1, \infty, \tag{2.18}$$

while $t \sim 0.49$, 0.40, 0.35 fm for $T = 300$ MeV and, respectively, the same values of ρ. Notice that the variation with ρ is very small and the values remain in the expected range of RHIC data (between approximately 0.3 and 8 fm [6]).

The jet quenching parameter is a monotonically increasing function of ρ. Thus, it does depend on the number of degrees of freedom once the flavors are included. The asymptotics for the jet quenching parameter are inversely proportional to those of the relaxation time,

$$\hat{q} \sim \frac{4\pi^{3/2}\Gamma(\frac{3}{4})}{\Gamma(\frac{5}{4})} T^3 \begin{cases} 1 + \frac{\sqrt{2}}{5}\rho + \ldots & \text{for } \rho \to 0, \\ \frac{7}{5} + \mathcal{O}\left(\frac{1}{\rho^2}\right) & \text{for } \rho \to \infty. \end{cases} \tag{2.19}$$

If this qualitative behavior remains true in the full α' corrected background, this recasts the possibility of using $\hat{q}$ as a direct measure of the temperature of the quark-gluon plasma, as advocated in [8]. In fact, there is also a dependence on the "effective number of massless quarks" ρ; this is ultimately a function of the temperature too, albeit an unknown one. Putting in the numbers,

$$\hat{q} = 2.4, 2.9, 3.4 \text{ GeV}^2/\text{fm} \quad \text{for } \rho = 0, 1, \infty \quad \text{at } T = 250 \text{ MeV}. \tag{2.20}$$

In order to compare with [8], at $T = 300$ MeV one has

$$\hat{q} = 4.1, 5.0, 5.8 \text{ GeV}^2/\text{fm} \quad \text{for } \rho = 0, 1, \infty \quad \text{at } T = 300 \text{ MeV}, \tag{2.21}$$

so that it remains in the vicinity of the range of values compatible with RHIC data (5–15 GeV^2/fm [8]) for every value of ρ, and it is actually above 5.0 GeV^2/fm for $\rho > 1$ and $T = 300$ MeV. Anyway, aside from the actual numbers, the interesting fact is that, as already noted, the variation of $\hat{q}$ is very small in the whole range of ρ. It signals the fact that the flavor contribution is not drastically changing the properties of the plasma. This is compatible with (and in a sense gives a reason for) the evidence that the actual values of plasma properties computed with gravity duals including only adjoint fields are very similar to the experimental ones. Even if at present the numerical values should not be taken very seriously as phenomenological predictions, their robustness suggests that the string/gauge correspondence provides an avenue that must be pursued.

An interesting comment is in order at this point. It was shown in [20] that, for plasmas having 10D duals of the form AdS_5-black hole times Sasaki-Einstein five manifolds, ratios of jet quenching parameters are proportional to (the square root of) ratios of entropy densities. It was then conjectured that this result might also hold for approximately conformal systems such as the quark-gluon plasma of QCD. The authors of [20], however, ask themselves whether this relation is affected or not by the fact that some of the degrees of freedom in QCD are fundamentals. Let us assume that it is not affected. Then, it would be possible to extract the leading correction in N_f/N_c to the jet quenching parameter first computed in [8] for $\mathcal{N} = 4$ SYM theory, from the corresponding term in the expansion of the entropy density obtained for the D3/D7 system [23]. It gives

$$\hat{q} \approx \hat{q}_{\mathcal{N}=4}\left(1 + \frac{\lambda}{16\pi^2}\frac{N_f}{N_c}\right), \tag{2.22}$$

a result that qualitatively agrees with the leading behavior (2.19) of our noncritical string/gauge dual prediction. Nevertheless, note that the conformal noncritical result, if not drastically modified by α' corrections, has a different behavior with respect to the proposal in [20] in that, say, the ratio of the jet quenching parameter to the one of $\mathcal{N} = 4$ SYM is not proportional to the square root of the ratio of the respective entropy densities at generic ρ.

Let us point out that the holographic expression of other plasma properties, like the screening length and the diffusion coefficient for heavy quarks, can be immediately taken from the critical AdS cases too, as any other quantity which is not affected by the transverse directions. Thus, in particular, from [9,20], we conclude that the Debye screening length in our noncritical setup will be only dependent on the temperature and the wind velocity, as in the $\mathcal{N} = 4$ plasma case. Since at weak coupling the Debye screening length is known to depend both on the gauge coupling and on the number of colors and flavors, we are led to conclude, as in the $\mathcal{N} = 4$ plasma case, that this could well be a genuine strong coupling effect. As for the abovementioned diffusion coefficient, it is related to the friction coefficient μ by [5] $D = T/\mu m$; it is thus given by $D = 2/(\pi T R^2)$, and, then, it is a decreasing function of ρ in the present 5D setup.

Let us conclude this part by warning the reader about the reliability of this noncritical string model. Aside from α' corrections, three other features have not been established so far [26]. The first one is a complete discussion of the stability: there are no explicitly tachyonic modes, apart from a possible closed string tachyon, whose behavior was not discussed. The second one is the exact spectrum of the D3–D4 system on a linear dilaton background in five dimensions (whose backreaction is supposed to give the AdS_5 solution we have been discussing), that is not *a priori* guaranteed to contain (only) quark modes. Finally, if we take for granted the ten-dimensional expressions for the mass of the open string tachyon ($m^2 = -1/2$) and the mass/conformal dimension relation ($\Delta = 2 \pm \sqrt{4 + m^2R^2}$), the increasing of R^2 with ρ makes the dimension of the quark bilinear decrease in climbing the conformal window, contrary to the *a priori* expected behavior.[7] Of course, the assumption of the validity of the ten-dimensional formulas is far from obvious.

Despite this lack of control, this background seems to encode in a nice way many interesting features in a very simple setting, so we find it worthwhile to study them in a "phenomenologically" inspired way, guided by a similar spirit to that behind the "AdS/QCD" setting [33,34]. In particular, this is the first case where the explicit (non-quenched) dependence on the number of flavors is exhibited for plasma observables in the holographic setting.

C. SQCD in Seiberg's conformal window

Let us briefly discuss the 6D model "dual to SQCD" ($k = 1$ in (2.1)), whose solution reads [27]

$$R^2 = 6, \qquad e^{2\nu_0} = \frac{2}{3\rho^2}, \qquad e^{\phi_0} = \frac{2}{3Q_c\rho}, \qquad \chi = Q_c\theta, \tag{2.23}$$

where χ is the zeroform potential and θ is the angle parametrizing Ω_1. Notice that the AdS radius R is now independent on both Q_c, Q_f, while the dilaton is not. As in the 5D model, the coupling e^{ϕ_0} has the correct behavior with ρ, but this time it is elementary.

As in the 5D model, the temperature T and entropy density s can be computed,

$$T = \frac{u_H}{\pi R^2}, \qquad s = 27\pi^2 Q_c^2 \rho T^3, \tag{2.24}$$

as well as the free energy F and energy densities ϵ,

$$F = -\frac{27\pi^2 Q_c^2 \rho T^4}{4}, \qquad \epsilon = \frac{81\pi^2 Q_c^2 \rho T^4}{4}, \tag{2.25}$$

and the heat capacity c_V, speed of sound v_s, and shear viscosity η,

$$c_V = 81\pi^2 Q_c^2 \rho T^3, \qquad v_s^2 = \frac{1}{3}, \qquad \eta = \frac{27\pi Q_c^2 \rho T^3}{4}. \tag{2.26}$$

In this case, the dependence of the thermodynamics quantities on Q_c^2 and ρ is just linear. Instead, both the drag force coefficient and the jet quenching parameters are independent on Q_c, Q_f, since R^2 is a constant. The values are again slightly different from those in the $\mathcal{N} = 4$ SYM theory. For a charm quark at $T = 250$ MeV, the relaxation time is

$$t = \frac{2m}{\pi R^2 T^2} \sim 0.5 \text{ fm}, \tag{2.27}$$

that is smaller than the $\mathcal{N} = 4$ value but still in the allowed range that results from RHIC data. The jet quenching parameter at the same temperature is $\hat{q} = 3.6 \text{ GeV}^2/\text{fm}$. In order to compare with the values in [8], at $T = 300$ MeV one finds $\hat{q} = 6.2 \text{ GeV}^2/\text{fm}$, that is, higher than the $\mathcal{N} = 4$ one and so more comfortably within the RHIC range.

As in the 5D model, the screening length will be independent on the details of the plasma. The diffusion coefficient D will be independent on N_c, N_f.

III. QGP AND WRAPPED FIVEBRANES

A one-parameter family of black hole solutions in the background sourced by N_c *color* wrapped D5-branes and N_f (smeared) *flavor* D5-branes was presented in [29]. This is conjectured to be the thermal deformation of the string/gauge dual to an $\mathcal{N} = 1$ SQCD with quartic superpotential at the "conformal point" $N_f = 2N_c$ [$\beta = 0$, no breaking of $U(1)_R$], coupled to Kaluza-Klein (KK) adjoint matter. The string frame metric and dilaton read (setting $g_s = 1$)

$$ds^2 = e^{\Phi_0} z^2 \Big[-\mathcal{F} dt^2 + d\vec{x}_3^2 + N_c\alpha' \Big(\frac{4}{z^2\mathcal{F}} dz^2 + \frac{1}{\xi}(d\theta^2 + \sin^2\theta d\varphi^2) + \frac{1}{4-\xi}(d\tilde{\theta}^2 + \sin^2\tilde{\theta} d\tilde{\varphi}^2) + \frac{1}{4}(d\psi + \cos\theta d\varphi + \cos\tilde{\theta} d\tilde{\varphi})^2 \Big) \Big], \tag{3.1}$$

$$e^{\Phi} = z^2 e^{\Phi_0}, \qquad \mathcal{F} = 1 - \frac{z_0^4}{z^4}, \tag{3.2}$$

and ξ is the parameter labeling the family. The horizon is at z_0; for $z_0 = 0$ one recovers the zero temperature solution; t, x_i are dimensional, while the other coordinates are dimensionless. The temperature and entropy density of this black hole (BH) are

$$T = \frac{1}{2\pi\sqrt{\alpha' N_c}}, \qquad s = \frac{A_8}{4G_{(10)}} = \frac{8e^{4\Phi_0} z_0^8 N_c^4}{\xi(4-\xi)} T^3, \tag{3.3}$$

where A_8 is the area of the horizon. The fact that the

[7] We are assuming here that the open string tachyon is the gravity field dual to the quark bilinear, so that approaching the weakly coupled, higher edge of the conformal window its dimension should increase and tend to the free field value, namely, three.

temperature does not depend on z_0 seems to be a common feature of black hole backgrounds of this kind, obtained from NS5 and D5-brane configurations. This also implies that T is independent on the energy density and, therefore, the free energy should be zero at this classical level. Then, the theory is in a Hagedorn phase and, indeed, the temperature above coincides with the Hagedorn temperature of little string theory (LST). This suggests that there could be thermodynamical instabilities in this solution, in the very same way as it happens in the standard LST case [40,41].

The instabilities are related to the UV LST completion of the model and thus cannot be cured by introducing IR cutoffs. In fact, we present in Appendix A a new family of black hole solutions that should be string/gauge duals of the above plasma compactified on S^3, and argue that it should have analogous instabilities (as well as analogous screening properties) as its uncompactified counterpart. In view of these possible problems, it is not clear if these solutions can provide reliable descriptions of 4D finite temperature properties of string/gauge duals. Anyway, it is worth studying their features as long as they are the only known 10D black hole backgrounds where the backreaction of the flavor branes is taken into account.

Concerning the hydrodynamical properties of the $N_f = 2N_c$ plasma, it was already pointed out in [29] that the ratio between the shear viscosity and entropy density has the universal value $1/4\pi$ in the limit where the gravity approximation can be trusted. This result depends only on the symmetries [38] of the black hole solution and, thus, is not affected by the presence of the flavor DBI term in the gravity action (as it happens in [23]), nor by the troublesome thermodynamical behavior mentioned above. This seems to be related to the fact that the η/s ratio depends on the properties of the horizon, while the LST behavior is an UV feature.

A. The $q\bar{q}$ potential

The goal of this section is to extract the potential between a probe quark and a probe antiquark in the $N_f = 2N_c$ QGP dual to the black hole backgrounds presented above. As in the zero temperature case studied in [29], we will find that the quark and antiquark are always screened.

The Wilson loop for these two nondynamical quarks separated by a distance L in the gauge theory coordinates should be evaluated [42,43] by the action of a fundamental string that is parametrized by $t = \tau$, $x_1 = \sigma$, $z = z(\sigma)$. Using the string frame metric (3.1), we find that the length of this string is given by

$$L(z_*) = 4\sqrt{N_c\alpha'}\int_{z_*}^{\infty}\frac{z\sqrt{z_*^4 - z_0^4}dz}{\sqrt{(z^4 - z_0^4)(z^4 - z_*^4)}} = 2\sqrt{N_c\alpha'}\sqrt{1 - \frac{z_0^4}{z_*^4}}\,\mathrm{K}\left[\frac{z_0^4}{z_*^4}\right], \tag{3.4}$$

where z_* is the minimum value of the radial coordinate reached by the string and z_0 is the position of the horizon. By $K[x]$ we denote the complete elliptic integral of the first kind. The length is a monotonic function that goes from a minimum value of 0, attained at $z_* = z_0$, to the limiting value of $\pi\sqrt{N_c\alpha'}$, for $z_* \to \infty$.

The energy of the string, upon subtraction of the infinite masses of the two quarks, reads

$$\begin{aligned} E(z_*) &= \frac{2\sqrt{N_c}}{\pi\sqrt{\alpha'}}\left[\int_{z_*}^{\infty}\frac{e^{2\Phi_0}z^3\mathcal{F}^{1/2}(z)}{\sqrt{e^{2\Phi_0}(z^4\mathcal{F}(z) - z_*^4\mathcal{F}(z_*))}}dz - \int_{z_0}^{\infty}e^{\Phi_0}z dz\right] \\ &= \frac{2\sqrt{N_c}}{\pi\sqrt{\alpha'}}e^{\Phi_0}z_*^2\left[\int_1^{\infty}\left(\frac{x\sqrt{x^4 - \frac{z_0^4}{z_*^4}}}{\sqrt{x^4 - 1}} - x\right)dx - \frac{1}{2}\left(1 - \frac{z_0^2}{z_*^2}\right)\right]. \end{aligned} \tag{3.5}$$

It vanishes for $z_* = z_0 \leftrightarrow L = 0$ but is otherwise positive, which means that the quark and antiquark are always screened, as in the zero temperature case. As a matter of fact, we will argue in the following that total screening is a common feature of all known quark-gluon plasmas having LST as its UV completion. It is worth asking whether this affects or not other properties of the plasma, like the drag force on a probe quark and the jet quenching parameter. Let us address this question in the following subsections.

B. Drag force

Let us now evaluate the energy loss and the drag force of a heavy quark moving through the plasma, following the general procedure described in [5,6,16]. In the string/gauge picture, the quark is a macroscopic string with an extremum attached to a probe flavor D5-brane placed at distance z_m from the black hole horizon. We consider a simple string configuration representing a test quark moving in a given spatial direction x: $t = \tau$, $z = \sigma$, $x = x(\sigma, \tau)$. The static configuration $x =$ constant corresponds to a quark at rest in the plasma. Its mass is given by the energy of the static string hanging down from z_m to the horizon

$$M_{\mathrm{rest}} = \frac{1}{2\pi\alpha'}\int_{z_0}^{z_m}\sqrt{-g_{tt}g_{zz}}dz = \frac{e^{\Phi_0}\sqrt{N_c}}{2\pi\sqrt{\alpha'}}(z_m^2 - z_0^2). \tag{3.6}$$

Thus, to get a very heavy quark we must require that the probe D5-brane is placed very far from the horizon.

Let us now consider the string configuration corresponding to a quark which moves at constant velocity. This is achieved by setting $x(\sigma, \tau) = u(\sigma) + v\tau$. From the Nambu-Goto action on the background (3.1), it follows that the string Lagrangian density is proportional to

$$\mathcal{L} = e^{\Phi_0}\sqrt{4N_c\alpha' z^2 + z^4\mathcal{F}(z)u'^2 - 4N_c\alpha' v^2\frac{z^2}{\mathcal{F}(z)}}, \tag{3.7}$$

from which it follows that the canonical momentum density conjugate to $u(\sigma)$ is

$$\pi_u = \frac{e^{\Phi_0}z^4\mathcal{F}(z)u'}{\sqrt{4N_c\alpha' z^2 + z^4\mathcal{F}(z)u'^2 - 4N_c\alpha' v^2\frac{z^2}{\mathcal{F}(z)}}} \equiv A, \tag{3.8}$$

and, finally,

$$u' = \frac{\sqrt{4N_c\alpha' A^2}}{z^4 - z_0^4}\sqrt{\frac{z^2(z^4 - z_0^4 - v^2 z^4)}{e^{2\Phi_0}(z^4 - z_0^4) - A^2}}. \tag{3.9}$$

For the above square root to be real, at the point $z_c = z_0/(1-v^2)^{1/4}$ at which the numerator changes sign, also the denominator must change sign, this leading to

$$A = \frac{e^{\Phi_0}z_0^4 v^2}{\sqrt{1-v^2}}. \tag{3.10}$$

The momentum loss of the steady moving quark is thus given by

$$\frac{dp}{dt} = \frac{e^{\Phi_0}z_0^2 v}{2\pi\alpha'\sqrt{1-v^2}}. \tag{3.11}$$

Assuming now that the following relation holds, at least for low velocity,

$$\frac{dp}{dt} = -\mu p, \tag{3.12}$$

and that we can trust the relativistic relation

$$p = \frac{M_{\rm kin}v}{\sqrt{1-v^2}}, \tag{3.13}$$

where $M_{\rm kin}$ is the kinetic mass of the quark, one obtains the coefficient

$$\mu M_{\rm kin} = \frac{e^{\Phi_0}z_0^2}{2\pi\alpha'} = 2\pi e^{\Phi_0}z_0^2 N_c T^2, \tag{3.14}$$

or, equivalently, the relaxation time $t = 1/\mu$. Let us notice that $e^{\Phi_0}z_0^2$ is the value of the dilaton at the horizon, e_0^{Φ}. In the unflavored wrapped D5-brane setup of [30], the value of the dilaton at the minimal radial position e_0^{Φ} multiplied by N_c gives the parameter $\lambda \equiv e_0^{\Phi}N_c$ that measures the ratio of the string tension with the KK masses. This is the parameter that shall be fixed in the large N_c limit, and must be much larger than 1 in order for the supergravity approximation to be reliable. Introducing also in our context[8] the parameter $\lambda \equiv e^{\Phi_0}z_0^2 N_c$, we can write

[8]Here λ has not a clear interpretation as some cutoff coupling.

$$\mu M_{\rm kin} = 2\pi\lambda T^2. \tag{3.15}$$

This is independent on the velocity and scales like T^2, just as it happens in the $\mathcal{N} = 4$ SYM plasma [5,6]. The only difference with the latter comes from the dependence on λ, which is enhanced from $\lambda^{1/2}$ to λ (this is a usual phenomenon in passing from AdS to more general backgrounds with threeform Ramond-Ramond fluxes).

It thus seems that the energy loss of a probe quark moving in the QGP dual to (3.1) is not very much affected by the presence of dynamical massless quarks.

Let us end this section by noticing that, in order to have the friction coefficient μ disjointed from the kinematical mass $M_{\rm kin}$, we should analyze, as in [5], the small fluctuations around the trivial static configuration $x = \text{const}$, assuming then a time dependence of the type $x \sim e^{-\mu t}$. We do not pursue this analysis here. Instead we just notice that the limiting value of μ, i.e., the one extrapolated in the extremal case where the probe brane touches the horizon $z_m \to z_0$, is expected to be as the one advocated in [16] for general asymptotically AdS backgrounds. This can be easily deduced from the general results in [19]. For a metric of the form

$$\begin{aligned} ds^2 = &-\Omega(r)^2\Delta(r)^2dt^2 + \Omega(r)^2[g_{\mu\nu}dx^\mu dx^\nu] \\ &+ \Omega(r)^2[g_{rr}dr^2 + g_{ij}dy^i dy^j], \end{aligned} \tag{3.16}$$

where μ, $\nu = 1, \ldots, d$; i, $j = d+2, \ldots, D$, the limiting value of μ is given by

$$\mu_* = \frac{\Delta}{2\sqrt{g_{rr}}}\frac{\partial_r\Omega}{\Omega - \Omega_0}\bigg|_{r=r_0}. \tag{3.17}$$

Applying this formula to the $N_f = 2N_c$ black hole, we find $\mu_* = 2\pi T$ as in the asymptotically AdS cases.[9]

C. Jet quenching parameter

Let us now evaluate the jet quenching parameter in the $N_f = 2N_c$ QGP under study, following the prescriptions given in [8]. What we need to compute is a rectangular lightlike Wilson loop with one of the sides having a much smaller length, L, than that of the lightlike side, L^-. The prescription is that the Wilson loop in the fundamental representation (approximated by the renormalized Nambu-Goto action $S_{{\rm NG},r}$) is related to the jet quenching parameter $\hat{q}$, according to the relation

$$\hat{q} = \frac{8\sqrt{2}}{L^- L^2}S_{{\rm NG},r}. \tag{3.18}$$

Let us first rewrite the relevant part of the metric in lightcone coordinates $x^{\pm} = (t \pm x_1)/\sqrt{2}$:

[9]Incidentally, we notice that the same result is shared by all the other known (wrapped) fivebrane black hole solutions we will refer to in Sec. III D.

$$ds^2 = e^{\Phi_0} z^2 \Big[-(\mathcal{F}(z) + 1) dx^+ dx^- + \frac{1}{2}(1 - \mathcal{F}(z)) \times ((dx^+)^2 + (dx^-)^2) + dx_2^2 + dx_3^2 + N_c \alpha' \Big(\frac{4}{z^2 \mathcal{F}(z)} dz^2 + \ldots \Big) \Big]. \tag{3.19}$$

The string configuration for the loop is just $x^- = \tau$, $x_2 = \sigma$, $z = z(\sigma)$, with the boundary conditions $z(\pm L/2) = \infty$. The Nambu-Goto action for a lightlike length L^- reads, then,

$$S = \frac{2L^- e^{\Phi_0} z_0^2}{2\sqrt{2}\pi\alpha'} \int_0^{L/2} \sqrt{1 + \frac{4\alpha' N_c z'^2}{z^2 \mathcal{F}(z)}} dz. \tag{3.20}$$

The equation of motion for x_2 then gives

$$z' = \sqrt{\frac{z^2 \mathcal{F}(z) \gamma^2}{4 N_c \alpha'}}, \tag{3.21}$$

where γ is an integration constant, that gives

$$\frac{\gamma L}{4\sqrt{N_c \alpha'}} = I \equiv \int_{z_0}^{\infty} \frac{z dz}{\sqrt{z^4 - z_0^4}}. \tag{3.22}$$

In order to evaluate the renormalized action, we have to subtract to this configuration the one corresponding to two separated strings extending from the boundary to the horizon:

$$S_0 = \frac{2L^-}{2\pi\alpha'} \int_{z_0}^{\infty} dz \sqrt{g_{--} g_{zz}} = \frac{2L^- e^{\Phi_0} z_0^2}{2\sqrt{2}\pi\alpha'} \sqrt{4 N_c \alpha'}\, I. \tag{3.23}$$

We see that at small L, that is, large γ, we simply obtain $S - S_0 = 0$, so that the jet quenching parameter $\hat{q}$ is zero[10]:

$$S_{\mathrm{NG},r} = S - S_0 = \frac{2L^- e^{\Phi_0} z_0^2}{2\sqrt{2}\pi\alpha'} \Bigg[\frac{L}{2} \sqrt{1 + \Big(\frac{2\sqrt{4\alpha' N_c z'^2} I}{L} \Big)^2} - \sqrt{4 N_c \alpha'}\, I \Bigg] \to 0. \tag{3.24}$$

As we will point out in what follows, this is a general feature in gauge theory plasmas having little string theory as their UV completion. The $\hat{q} = 0$ result thus shares the same universality of the screening properties in LST plasmas. As such, these properties seem to be much more related to the 6D nonlocal UV theories than to the IR local ones.

[10]This result was obtained in collaboration with Carlos Núñez.

D. On the general behavior of "LST plasmas"

A possible puzzling result has emerged in studying the properties of the $N_f = 2N_c$ plasma obtained from a system of wrapped fivebranes: while the energy loss of a probe quark in the plasma, as described by the drag force calculation, has been found to be nonzero, the jet quenching parameter turns out to be zero. It is known that the two quantities are related to the energy loss of a parton in two opposite regimes of transverse momentum (large momentum for the quenching parameter and small momentum for the drag coefficient), but obtaining such completely different results is an apparent puzzle. So, let us discuss this issue a bit.

It is known that LST has a critical behavior at finite temperature: $T = T_H$ (Hagedorn temperature) and negative specific heat characterize the BH solution related to unwrapped NS5-branes [40]. The same happens [41] to certain black hole backgrounds studied as finite temperature extensions of the wrapped D5-brane solution proposed in [30] as a dual to $\mathcal{N} = 1$ SYM theory (the MN solution). It is not difficult to realize that both in the flat LST case and in the wrapped setups, the drag force is nonzero, while $\hat{q} = 0$. This also happens in the thermalized version [44] of the 3D setup studied in [45,46].

Moreover, there are more general BH versions of the unflavored MN solution, which were studied in [32]. There, a family of finite energy BH was found, whose temperature is above the Hagedorn temperature of LST and that basically have the same problems. Again, zero jet quenching and nonzero drag force are expected to characterize these solutions. Checking the occurrence of total screening is not an immediate task, mainly due to the lack of analytic control on the solutions in [32]. As we show in Appendix B, which can be taken as a reference for analogous calculations in general LST cases, these solutions manifest total screening too. As it transpires from the calculations in Appendix B, it is expected that the same should happen for possible $N_f \neq 2N_c$ plasmas in the framework of [29], even though the corresponding gravity backgrounds are not currently known. A possible way to evade the above critical behavior could have been to put the black hole on S^3. In this manner, another scale is introduced by hand into the system, and it might be expected that this allows one to tune the temperature of the system away from the Hagedorn point. As we show in Appendix A, this is actually not the case and these solutions behave in the same way as the other "LST backgrounds."

These are indications that our "jet quenching puzzle" is not related to the presence of dynamical quarks in the 10D backgrounds.[11] Instead, let us observe that the jet quenching parameter is definitely dependent on the UV behavior of the dual backgrounds, and so on the 6D LST asymptotics, while the same dependence for the drag force is not stringent (indeed we could consider a probe flavor brane "not so far" from the horizon, so to cut off the UV zone).

This fact suggests that the result $\hat{q} = 0$ has to be associated more to nonlocal LST modes than to their claimed local counterparts. Analogous conclusions can be reached for the screening properties of the wrapped fivebrane plasmas, which again appear to be universal. In fact, the total screening result is also sensitive to the UV physics, and so to the LST regime. In conclusion, it seems reasonable to argue that quantities crucially sensitive to the UV behavior of the metric are determined by LST modes. These give total screening and, as a consequence, zero jet quenching parameter. Thus, these backgrounds do not seem to be useful in order to study UV properties of realistic plasmas.

On the other hand, there are properties that seem not to be affected by almost any feature of the plasmas. For example, the η/s ratio has the usual form because it is determined by the universal properties of the horizon. Analogously, the result of the drag force calculations indicates that the form of $\mu M_{\rm kin}$ is exactly the same in all the LST plasmas and practically the same as in $\mathcal{N} = 4$. It is insensitive to the IR (and UV) details of the dual theory[12] and it is not substantially altered by the presence of flavors. In this respect, it seems that the indications it gives for the energy loss process in the dual field theory are very general and universal. The friction that a heavy parton experiences in a strongly coupled plasma is practically always the same, irrespectively of the features of the dual field theory.

IV. CONCLUDING COMMENTS

In this paper we have analyzed some properties of the known backgrounds dual to finite temperature 4D gauge theories with a large number of flavors N_f and colors N_c, in the limit where their ratio $\rho = N_f/N_c$ is kept constant and different from zero. Our aim was to obtain information on the dependence of plasma features on the number of flavors, in order to better understand the connection of the string theory calculations with RHIC physics.

In particular, we have studied plasma features of noncritical gravity backgrounds conjectured to be dual to finite temperature QCD and SQCD in the conformal window [25–27]. We have analyzed the dependence on ρ of thermodynamic quantities such as the entropy, energy, and free energy density and of the jet quenching parameter and drag coefficient for a probe parton moving in the dual plasma. While in the "SQCD dual" the last two features are independent on ρ, in the "QCD dual" all the above quantities have been shown to be monotonically increasing functions of ρ. For the case of the jet quenching parameter, this seems to refine the possibility of using it as a direct measure of the temperature of the RHIC plasma, since it depends on the effective degrees of freedom (that, in turn, are also a function of T, albeit an unknown one). On the other hand, as a general feature, we have found that the dependence of all the observables on ρ is quite mild, so that the numerical results for the jet quenching parameter and the relaxation time are always very similar to the $\mathcal{N} = 4$ ones. This provides a first explanation of why the latter are so similar to the experimental results at RHIC. As far as we know, our results are the first example of unquenched effects of flavors in finite temperature holographic duals of gauge theories.

As a second class of backgrounds, we have studied the 10D solution argued to be dual to a SQCD-like theory with $N_f = 2N_c$ flavors found in [29]. Its main features are that it exhibits total screening, the jet quenching parameter is zero, and the drag coefficient is nonvanishing. We have argued that these properties are common to all the finite temperature theories having LST as UV completion. This fact seems to force the interpretation that all these solutions in the UV give a description of the LST physics and so should not be reliable as realistic plasma duals in this regime. On the other hand, the expression of the drag coefficient turns out to be very similar to the $\mathcal{N} = 4$ one, underlying again the fact that dynamical quarks seem not to change drastically the physics of the plasmas.

Both settings we have analyzed have some important problems. The noncritical one is not well under control, while the 10D one is probably thermodynamically unstable. Nevertheless, the results they provide are quite interesting and it should be worth investigating other physical properties of finite ρ physics using these solutions. Of course, finding a finite temperature background which includes the backreaction of the flavors and does not share the problems of the above models would be the best starting point to study such properties.

ACKNOWLEDGMENTS

It is a pleasure to thank R. Argurio, N. Armesto, R. Casero, C. Hoyos, L. Martucci, D. Mateos, C. Núñez, A. Paredes, J. M. Pons, and P. Talavera for useful discus-

[11] There are phenomenological AdS/QCD models like [18] where dynamical quarks are argued to be present and responsible for the breaking of the flux tube connecting two probe quarks beyond some critical distance. The drag force and jet quenching parameter for those models were found to be both different from zero [17] with the jet quenching parameter going like T^4. Together with the results of Sec. II, this indicates that dynamical quarks seem to create no specific problem to the evaluation of $\hat{q}$; a difference of [18] with respect to our 10D case is that we have total screening while, in the former paper, the screening only happens above a certain energy.

[12] It is not sensitive to the fact that the UV LST completion is a nonlocal theory.

sions and comments on the manuscript. The work of F. B. is supported by the Belgian Fonds de la Recherche Fondamentale Collective (Grant No. 2.4655.07), and also by the Belgian Institut Interuniversitaire des Sciences Nucléaires (Grant No. 4.4505.86), the Interuniversity Attraction Poles Programme (Belgian Science Policy), and by the European Commission FP6 program MRTN-CT-2004 v-005104 (in association with V. U. Brussels). The work of J. D. E. is supported in part by MCyT and FEDER (Grant No. FPA2005-00188), and Xunta de Galicia (Consellería de Educación and Grant No. PGIDIT06PXIB206185PR). This work is partially supported by the European Commission Contracts No. MRTN-CT-2004-005104, No. MRTN-CT-2004-503369, No. CYT FPA 2004-04582-C02-01, No. CIRIT GC 2001SGR-00065, and No. MEIF-CT-2006-024173. Institutional support to the Centro de Estudios Científicos (CECS) from Empresas CMPC is gratefully acknowledged. CECS is funded in part by grants from Millennium Science Initiative, Fundación Andes, and the Tinker Foundation.

APPENDIX A: THE BLACK HOLE SOLUTION WITH SPHERICAL HORIZON

In this section, we will introduce certain black hole metrics which generalize the solutions found in [29] for $N_f = 2N_c$, Eq. (3.1). These black holes are characterized by an S^3 horizon and their temperature is always greater than the Hagedorn temperature of LST.

The string frame metric of the compactified black hole solutions is

$$ds^2 = e^{\Phi}\bigg[-\mathcal{F}dt^2 + R^2 d\Omega_3^2 + \frac{R^2 N_c\alpha'}{R^2 + N_c\alpha'}\mathcal{F}^{-1}dr^2 + N_c\alpha'\bigg(\frac{1}{\xi}(d\theta^2 + \sin^2\theta d\varphi^2) + \frac{1}{4-\xi}(d\tilde{\theta}^2 + \sin^2\tilde{\theta}d\tilde{\varphi}^2) + \frac{1}{4}(d\psi + \cos\theta d\varphi + \cos\tilde{\theta}d\tilde{\varphi})^2\bigg)\bigg], \tag{A1}$$

with

$$\mathcal{F} = 1 - e^{2r_0 - 2r}, \tag{A2}$$

so the horizon is placed at $r = r_0$. There is also a corresponding extremal solution which is recovered by setting $\mathcal{F} = 1$. The S_3 metric is written as

$$d\Omega_3^2 = dx_1^2 + \sin^2 x_1 dx_2^2 + \sin^2 x_1 \sin^2 x_2 dx_3^2.$$

The dilaton is linear

$$\Phi = \Phi_0 + r \tag{A3}$$

and $F_{(3)}$ is given by

$$F_{(3)} = N_h \sin^2 x_1 \sin x_2 dx_1 \wedge dx_2 \wedge dx_3 - \frac{N_c}{4}(\sin\tilde{\theta}d\tilde{\theta} \wedge d\tilde{\varphi} + \sin\theta d\theta \wedge d\varphi) \wedge (d\psi + \cos\theta d\varphi + \cos\tilde{\theta}d\tilde{\varphi}). \tag{A4}$$

The radius R of the S_3 horizon is determined by the flux through it:

$$R^2 = \frac{N_h\alpha'}{2}. \tag{A5}$$

Since the flux N_h is quantized, the radius of the horizon takes only discrete values. By taking the formal limit $R \to \infty$ of the above metric, we find the solution of [29].

The temperature and entropy density of the above black holes are

$$T(R) = \frac{1}{2\pi\sqrt{N_c\alpha'}}\sqrt{1 + \frac{N_c\alpha'}{R^2}} = T_0\sqrt{1 + \frac{N_c\alpha'}{R^2}} > T_0, \tag{A6}$$

$$s = \frac{A_8}{4G_{(10)}} = \frac{8e^{4\Phi_0}N_c^4}{\xi(4-\xi)}R^3\bigg(\frac{R^2}{R^2 + N_c\alpha'}\bigg)^{3/2}T^3(R), \tag{A7}$$

where T_0 denotes the temperature of the black hole in [29]. Note that the temperature increases as the radius R of the horizon decreases. There is a hint that the above black holes may be thermodynamically unstable. Let us consider the above family of solutions in the limit $R \to \infty$ and assume that they are a good approximation to black holes with a flat $\mathbf{R}^3$ horizon like the solution in [29]. Then, as the black hole temperature increases as R decreases, (A6), we can see that the entropy $s \sim R^3$ is a decreasing function of the temperature, which is a sign of thermodynamical instability.

1. The $q\bar{q}$ potential

In this section we calculate the potential between a probe quark and a probe antiquark in the above black hole backgrounds, as we have done in Sec. III A. The Wilson loop for two nondynamical quarks separated by a distance L should be evaluated by the action of a fundamental string that is parametrized by $t = \tau$, $x_1 = \sigma$, $r = r(\sigma)$. Using the string frame metric

$$ds^2 = e^{\Phi}\bigg[-\mathcal{F}dt^2 + R^2 d\Omega_3^2 + \frac{R^2 N_c\alpha'}{R^2 + N_c\alpha'}\mathcal{F}^{-1}dr^2 + N_c\alpha'\bigg(\frac{1}{\xi}(d\theta^2 + \sin^2\theta d\varphi^2) + \frac{1}{4-\xi}(d\tilde{\theta}^2 + \sin^2\tilde{\theta}d\tilde{\varphi}^2) + \frac{1}{4}(d\psi + \cos\theta d\varphi + \cos\tilde{\theta}d\tilde{\varphi})^2\bigg)\bigg], \tag{A8}$$

we find that the length L and the energy of this string upon subtraction of the infinite masses of the two quarks are given by

$$L(r_*) = 2\frac{\sqrt{\gamma}}{R}\int_{r_*}^{\infty}\frac{dr}{\sqrt{\mathcal{F}(r)}}\frac{e^{\Phi_*}\mathcal{F}_*^{1/2}}{\sqrt{e^{2\Phi}\mathcal{F}(r) - e^{2\Phi_*}\mathcal{F}_*}} = 2\frac{\sqrt{\gamma}}{R}\sqrt{1 - e^{2r_0-2r_*}}K[e^{2r_0-2r_*}], \quad (A9)$$

$$E(r_*) = \frac{\sqrt{\gamma}}{\pi\alpha'}\left[\int_{r_*}^{\infty}\frac{e^{2\Phi(r)}\mathcal{F}^{1/2}(r)}{\sqrt{e^{2\Phi(r)}\mathcal{F}(r) - e^{2\Phi_*}\mathcal{F}_*}}dr - \int_{r_0}^{\infty}e^{\Phi(r)}dr\right] = \frac{\sqrt{\gamma}}{\pi\alpha'}e^{\Phi_*}\left[\int_0^1\frac{\sqrt{1 - e^{2r_0-2r_*}z^2} - \sqrt{1-z^2}}{z^2\sqrt{1-z^2}} \times dz - (1 - e^{r_0 - r_*})\right], \quad (A10)$$

where r_* is the minimum value of the radial coordinate reached by the string and $\gamma = \frac{R^2 N_c\alpha'}{R^2 + N_c\alpha'}$.

The length $L(r_*)$ is a monotonic function that goes from a minimum value of 0 attained at $r_* = r_0$ to the limit value of $\frac{\pi\sqrt{\gamma}}{R}$ for $r_* \to \infty$.[13] The energy $E(r_*)$ vanishes for $r_* = r_0 \leftrightarrow L = 0$ but is otherwise positive, which means that the quark and antiquark are always screened, as for the black holes (3.1).

APPENDIX B: TOTAL SCREENING IN GUBSER-TSEYTLIN-VOLKOV BLACK HOLES

In this Appendix, we will study the quark-antiquark potential in the black hole backgrounds found in [32]. We will find that, as in the case of the $N_f = 2N_c$ black hole, quark, and antiquarks are always totally screened. Furthermore, this seems to be a generic feature of black hole backgrounds associated to systems of wrapped five-branes. Indeed, one can repeat the analysis for the finite temperature solutions found in [44] and find the same result. As it transpires from the analysis below, this is related to the asymptotics of the solution both close to the horizon and far away from it.

In the notation where the string frame metric reads

$$ds^2_{\text{string}} = -G_{00}(s)dt^2 + G_{x_\parallel x_\parallel}dx_\parallel^2 + G_{ss}(s)ds^2 + G_{x_T x_T}(s)dx_T^2, \quad (B1)$$

define

$$f^2(s(x)) = G_{00}(s(x))G_{x_\parallel x_\parallel}(s(x)), \quad g^2(s(x)) = G_{00}(s(x))G_{ss}(s(x)). \quad (B2)$$

The distance between quark and antiquark is given by the general formula [42,43]

$$L(s_0) = 2\int_{s_0}^{s_1}\frac{g(s)f(s_0)ds}{f(s)\sqrt{f^2(s) - f^2(s_0)}}. \quad (B3)$$

In the case we are interested in, the string frame metric reads [32]

$$ds^2 = e^{\Phi(r)}[-\nu(r)dt^2 + d\vec{x}^2 + \nu(r)^{-1}dr^2 + R^2(r)(d\theta^2 + \sin^2\theta d\varphi^2) + \tilde{\epsilon}_1^2 + \tilde{\epsilon}_2^2 + \tilde{\epsilon}_3^2],$$

so that

$$f^2(s(x)) = G_{00}(s(x))G_{x_\parallel x_\parallel}(s(x)) = \nu(s(x))e^{2\Phi(s(x))}, \quad g^2(s(x)) = G_{00}(s(x))G_{ss}(s(x)) = e^{2\Phi(s(x))}. \quad (B4)$$

Close to the horizon, which we can set at $r = 0$ [32]

$$\nu = \nu_h' r + \mathcal{O}(r^2), \qquad \Phi = \Phi_h + \mathcal{O}(r), \qquad R = R_h + \mathcal{O}(r), \quad (B5)$$

whereas for $r \to \infty$

$$\nu = 1 + \mathcal{O}(e^{-r}), \qquad \Phi = \Phi_\infty + \frac{r}{2} - \frac{1}{4}\log r + \mathcal{O}(r^{-2}), \qquad R \sim \sqrt{2r}. \quad (B6)$$

Because of the asymptotic behavior of the above functions, the integral (B3) is convergent in the limit $s_1 \to \infty$.

We would like to show that $L(s_0)$ vanishes in the limit $s_0 \to 0$. To this end, let us introduce the function

$$\Upsilon(s) \equiv \frac{1}{\sqrt{\nu(s)(\nu(s)e^{2\Phi(s)} - \nu(s_0)e^{2\Phi(s_0)})}}, \quad (B7)$$

that is useful to write $L(s_0)$ in a compact form,

$$L(s_0) = 2\int_{s_0}^{\infty}\sqrt{\nu(s_0)}e^{\Phi(s_0)}\Upsilon(s)ds. \quad (B8)$$

The function $\Upsilon(s)$ is analytic in $[1, \infty[$ and its integral is convergent. Therefore, if we split the integral in (B8) into two integrals from s_0 to 1, and from 1 to ∞, the latter simply vanishes in the limit $s_0 \to 0$, as long as $\sqrt{\nu(s_0)} \sim \sqrt{s_0}$. Then,

$$L(s_0) = 2\int_{s_0}^{1}\sqrt{\nu(s_0)}e^{\Phi(s_0)}\Upsilon(s)ds \sim \frac{2}{\sqrt{\nu_h'}}\sqrt{s_0}\int_{s_0}^{1}\frac{ds}{\sqrt{s(s - s_0)}} \sim -\sqrt{s_0}\log s_0. \quad (B9)$$

In summary, in the limit $s_0 \to 0$,

$$L(s_0) \sim -\sqrt{s_0}\log s_0 \to 0. \quad (B10)$$

This behavior of the length functional is the same for all black holes with LST asymptotics [32,44] and differs from other more familiar cases where $L(s_0)$ actually diverges in the same limit. It is also drastically different from the

[13] For these black hole solutions with spherical horizon, x_1 is an equatorial angle and L is then the angular separation between quark and antiquark along the equator. Note also that the limiting value of L is also less than π.

behavior in the zero temperature solutions [30,46] where $L(s_0)$ actually diverges and there is linear confinement.

We will now study the energy of the Wilson loop in the limit $s_0 \to 0$ and show that it is positive in a neighborhood of zero. The energy of the regularized Wilson loop, which means that we are subtracting the masses of the quark and antiquark, is given by the following formula:

$$E(s_0) = 2\int_{s_0}^{\infty}\left(\frac{f(s)}{\sqrt{f(s)^2 - f(s_0)^2}} - 1\right)g(s)ds - 2\int_0^{s_0} g(s)ds. \tag{B11}$$

Given the asymptotics at infinity of the functions involved (B6), the above integral is actually finite. Since the first integrand is positive

$$E(s_0) > \tilde{E}(s_0) \equiv 2\int_{s_0}^{2s_0}\left(\frac{f(s)}{\sqrt{f(s)^2 - f(s_0)^2}} - 1\right)g(s)ds - 2\int_0^{s_0} g(s)ds. \tag{B12}$$

Thus, in the limit $s_0 \to 0$,

$$\begin{aligned}\tilde{E}(s_0) &= 2\int_{s_0}^{2s_0}\left(\frac{f(s)}{\sqrt{f(s)^2 - f(s_0)^2}} - 1\right)g(s)ds - 2\int_0^{s_0} g(s)ds \\ &= 2\int_{s_0}^{2s_0}(\nu(s)\Upsilon(s)e^{\Phi(s)} - 1)e^{\Phi(s)}ds - 2\int_0^{s_0} e^{\Phi(s)}ds \\ &= 2s_0e^{\Phi(s_0)}\int_1^2\left(\frac{\sqrt{t}}{\sqrt{t-1}} - 1\right)dt - 2s_0e^{\Phi(s_0)} + \mathcal{O}(s_0^2) \\ &= (2\sqrt{2} + 2\log(1+\sqrt{2}) - 4)s_0e^{\Phi(s_0)} + \mathcal{O}(s_0^2) \\ &\approx 0.591\,174 s_0e^{\Phi(s_0)} > 0.\end{aligned} \tag{B13}$$

The above is not enough to show that for this class of black holes quark antiquark pairs are totally screened. In fact, it could be possible that $L(s_0)$ vanishes again and that around this value of s_0 the energy is actually negative. However, numerical analysis shows that the length L is approximately a monotonic increasing function and does not vanish again. Furthermore, one can show that in the limit $s_0 \to \infty$ $L(s_0)$ goes to a constant value determined by the slope of the dilaton

$$L(s_0) \to \frac{\pi}{Z} \leftrightarrow \Phi(s) \sim Zs \qquad s \to \infty, \tag{B14}$$

as we found in the $N_f = 2N_c$ black hole. Numerical analysis also confirms that the energy of the Wilson loops is always positive for $s_0 > 0$.

It would be interesting to check explicitly whether black hole solutions for $N_f \neq 2N_c$ exhibit total screening as well. This fact seems to be determined by and intertwined with the asymptotics of the solution close to the horizon (B5) and also the large r asymptotics (B6). The fact that the energy of the Wilson loop is positive for small enough L and that the length presumably does not vanish again are strong hints that total screening might be a feature of general fivebrane or LST black holes. Indeed, one can perform the same analysis for the black holes found in [44] that have essentially the same asymptotic behavior as [32] and again find total screening.

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