Dynamical breaking and restoration of chiral and color symmetries in the static Einstein universe

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We study the behavior of quark and diquark condensates in the framework of the extended Nambu–Jona-Lasinio model under the influence of a gravitational field in the static D-dimensional Einstein universe at finite temperature and quark density. The thermodynamic potential and the gap equations for the model are obtained. Quark and diquark condensates as functions of the chemical potential and temperature at different values of the curvature have been studied. Phase portraits of the system have been constructed.

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I. INTRODUCTION

Effective field theories with four-fermion interaction of the Nambu–Jona-Lasinio (NJL) type [1], which incorporate the phenomenon of dynamical chiral symmetry breaking, are quite useful in describing the physics of light mesons (see e.g. [2–4] and references therein) and diquarks [5,6]. Moreover, on their basis one can study the effects of various external conditions, like temperature and chemical potential [7], and consider the influence of external electromagnetic and gravitational fields. In particular, the role of the NJL approach increases, when detailed numerical lattice calculations are not yet admissible in QCD with nonzero baryon density and in the presence of external gauge fields [8–12]. Moreover, the NJL model has also applications in the nuclear physical and astrophysical researches [13].

It was proposed more than 20 years ago [14–16] that at high baryon densities a colored diquark condensate $\langle qq \rangle$ might appear. In analogy with ordinary superconductivity, this effect was called color superconductivity (CSC). The possibility for the existence of the CSC phase in the region of moderate densities was recently proved (see, e.g., the papers [17–19], as well as the review articles [20,21] and references therein). Since quark Cooper pairing occurs in the color antitriplet channel, a nonzero value of $\langle qq \rangle$ means that, apart from the electromagnetic $U(1)$ symmetry, the color $SU_c(3)$ symmetry should be spontaneously broken inside the CSC phase as well. In the framework of NJL models, the CSC phase formation has generally been considered as a dynamical competition between diquark $\langle qq \rangle$ and usual quark-antiquark condensation $\langle \bar{q}q \rangle$. Special attention has also been paid to the catalyzing influence of color magnetic fields on the condensation of diquarks [12].

In several papers, in the framework of the NJL model, the influence of a gravitational field on the dynamical chiral symmetry breaking (DχSB) due to the creation of a finite quark condensate $\langle \bar{q}q \rangle$ has been investigated at zero values of temperature and chemical potential [22–25]. The study of the combined influences of curvature and temperature has been performed in [26]. Recently, the dynamical chiral symmetry breaking and its restoration for a uniformly accelerated observer due to the thermalization effect of acceleration was studied in [27] at zero chemical potential. Further investigations of the influence of the Unruh temperature on the phase transitions in dense quark matter with a finite chemical potential, and especially on the restoration of the broken color symmetry in CSC, were made in [28]. One of the widely used methods of accounting for the gravitation is based on the expansion of the fermion propagator in powers of small curvature [29,30]. For instance, in [31], the three-dimensional Gross-Neveu model in a space-time with a weakly curved two-dimensional surface was investigated, using an effective potential at finite curvature and nonzero chemical potential. In paper [32], this approximation was used in considering the DχSB at nonvanishing temperature and chemical potential. It should, however, be mentioned that, near the phase transition point, one cannot consider the critical curvature R_c to be small and therefore exact solutions with finite values of R_c should be used. This kind of solution with consideration for the chemical potential and temperature in the gravitational background of a static Einstein universe has recently been considered in [33]. There it was demonstrated that chiral symmetry is restored at large values of the space curvature.

There arises an interesting question, whether similar effects can occur in the study of CSC under the influence of a gravitational field.

For this aim, we study in the present paper the influence of temperature, chemical potential, and a gravitational field on the behavior of the quark and diquark condensates. In particular, we consider the phase structure in an extended NJL model with dynamical breaking of chiral and color symmetries in the case of a gravitational field which is taken into account exactly. This then allows us to study the dependence of the order parameters of symmetry breaking on the gravitational field.

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In the following, we will choose a simple model of a curved space-time of constant positive curvature, called the static Einstein universe. This model is widely discussed in literature either as a solution of the Einstein equations with a given cosmological constant and a nonvanishing energy-momentum tensor of an ideal fluid as a source, or as an initial state in inflationary cosmology with a scalar field, and the cosmological constant as its vacuum energy (see, for instance, [34]). Note also that the Einstein universe and other suitably generalized compact curved space-times were extensively employed for studying the phenomenon of Bose-Einstein condensation (see, e.g. [35] and references therein). As is well known, one of the possible models for the expanding universe is the closed Friedmann model. Since the formation of quark condensates is expected to take place considerably faster than the expansion rate of the universe, its radius can be considered in our calculations as constant. In this sense, the chosen model space-time of the static Einstein universe might be considered as a simple cosmological model for a space with positive curvature. Note, however, that the same form of the metric does also hold for the interior of a collapsing, spherically symmetric star, thereby admitting also a noncosmological interpretation of the chosen model.

In order to be able to consider mainly the effects of gravitation in a rigorous nonperturbative way, we are investigating here a NJL quark model in curved space-time with maximal simplifications as concerns the quark matter. In particular, our calculations are performed in the most simple case without taking into account additional requirements like charge neutrality, color neutrality constraints, and restricting ourselves, for simplicity, to the flavor $SU(2)$ group. By using the mean field approximation, we will then derive an analytical expression for the thermodynamical potential of the NJL model which contains all necessary information concerning the chiral and color condensates of quarks moving in a space with constant positive curvature. On this basis we finally study the phase portraits of the model.

II. THE EXTENDED NJL MODEL IN CURVED SPACE-TIME

In D-dimensional curved space-time with signature $(+,-,-,-,\cdots)$, the line element is written as

$$ds^2 = \eta_{\hat{a}\hat{b}} e^{\hat{a}}_{\mu} e^{\hat{b}}_{\nu} dx^{\mu} dx^{\nu}.$$

The gamma matrices γ_μ, metric $g_{\mu\nu}$, and the vielbein $e^{\mu}_{\hat{a}}$, as well as the definitions of the spinor covariant derivative ∇_ν and spin connection $\omega^{\hat{a}\hat{b}}_{\nu}$, are given by the following relations [30,36]:

$$\begin{gathered}\{\gamma_\mu(x), \gamma_\nu(x)\} = 2g_{\mu\nu}(x), \qquad \{\gamma_{\hat{a}}, \gamma_{\hat{b}}\} = 2\eta_{\hat{a}\hat{b}},\\ \eta_{\hat{a}\hat{b}} = \mathrm{diag}(1,-1,-1,-1,\ldots), \qquad g_{\mu\nu}g^{\nu\rho} = \delta^{\rho}_{\mu},\\ g^{\mu\nu}(x) = e^{\mu}_{\hat{a}}(x) e^{\nu\hat{a}}(x), \qquad \gamma_\mu(x) = e^{\hat{a}}_{\mu}(x)\gamma_{\hat{a}}.\end{gathered} \tag{1}$$

$$\begin{gathered}\nabla_\mu = \partial_\mu + \Gamma_\mu, \qquad \Gamma_\mu = \frac{1}{2}\omega^{\hat{a}\hat{b}}_{\mu}\sigma_{\hat{a}\hat{b}},\\ \sigma_{\hat{a}\hat{b}} = \tfrac{1}{4}[\gamma_{\hat{a}}, \gamma_{\hat{b}}],\\ \omega^{\hat{a}\hat{b}}_{\mu} = \tfrac{1}{2}e^{\hat{a}\lambda}e^{\hat{b}\rho}[C_{\lambda\rho\mu} - C_{\rho\lambda\mu} - C_{\mu\lambda\rho}],\\ C_{\lambda\rho\mu} = e^{\hat{a}}_{\lambda}\partial_{[\rho}e_{\mu]\hat{a}}.\end{gathered} \tag{2}$$

Here, the index $\hat{a}$ refers to the flat tangent space defined by the vielbein at space-time point x, and the $\gamma^{\hat{a}}(\hat{a} = 0,1,2,3,\ldots)$ are the usual Dirac gamma matrices of Minkowski space-time. Moreover, γ_5 is defined as usual (see, e.g., [30,37,38]), i.e. to be the same as in flat space-time and thus independent of space-time variables. In the following we shall consider the static Einstein universe with the line element:

$$ds^2 = dt^2 - g_{ij}(\vec{x})dx^i dx^j \qquad (i,j = 1,\ldots,D-1). \tag{3}$$

A conventional model that demonstrates the formation of the color superconducting phase is the extended NJL model with up- and down-quarks. This model may be considered as the low energy limit of QCD. For the color group $SU(3)_c$ its Lagrangian takes the form[1]

$$\begin{aligned}\mathcal{L} = \bar{q}[i\gamma^\mu\nabla_\mu + \mu\gamma^0]q + \frac{G_1}{2N_c}[(\bar{q}q)^2 + (\bar{q}i\gamma^5\vec{\tau}q)^2]\\ + \frac{G_2}{N_c}[i\bar{q}_c\varepsilon\epsilon^b\gamma^5 q][i\bar{q}\varepsilon\epsilon^b\gamma^5 q_c].\end{aligned} \tag{4}$$

Here, μ is the quark chemical potential, $q_c = C\bar{q}^t$, $\bar{q}_c = q^t C$ are charge-conjugated bispinors (t stands for the transposition operation). The charge conjugation operation is defined, as usual (see, e.g., [30]), with the help of the operator $C = i\gamma^{\hat{2}}\gamma^{\hat{0}}$, where the flat-space matrices $\gamma^{\hat{2}}$ and $\gamma^{\hat{0}}$ are used.

The quark field $q \equiv q_{i\alpha}$ is a doublet of flavors and triplet of colors with indices $i = 1,2$; $\alpha = 1,2,3$. Moreover, $\vec{\tau} \equiv (\tau^1, \tau^2, \tau^3)$ denote Pauli matrices in the flavor space; $(\varepsilon)^{ik} \equiv \varepsilon^{ik}$, $(\epsilon^b)^{\alpha\beta} \equiv \epsilon^{\alpha\beta b}$ are the totally antisymmetric tensors in the flavor and color spaces, respectively.

Next, by applying the usual bosonization procedure, we obtain the linearized version of the Lagrangian (4) with collective boson fields σ, $\vec{\pi}$, and Δ,

[1] In this work, we shall consider the particular realistic case of $N_c = 3$. The general case of arbitrary N_c is discussed in [3] with relation to the effective hadron theory of QCD. In what follows, however, we shall keep the general notation N_c for convenience.

$$\tilde{\mathcal{L}} = \bar{q}[i\gamma^\mu\nabla_\mu + \mu\gamma^0]q - \bar{q}(\sigma + i\gamma^5\vec{\tau}\,\vec{\pi})q - \frac{3}{2G_1}(\sigma^2 + \vec{\pi}^2) - \frac{3}{G_2}\Delta^{*b}\Delta^b - \Delta^{*b}[iq^t C\varepsilon\epsilon^b\gamma^5 q] - \Delta^b[i\bar{q}\varepsilon\epsilon^b\gamma^5 C\bar{q}^t]. \quad (5)$$

The Lagrangians (4) and (5) are equivalent, as can be seen by using the equations of motion for the boson fields

$$\Delta^b = -\frac{G_2}{3}iq^t C\varepsilon\epsilon^b\gamma^5 q, \qquad \sigma = -\frac{G_1}{3}\bar{q}q, \qquad \vec{\pi} = -\frac{G_1}{3}\bar{q}i\gamma^5\vec{\tau}q.$$

The fields σ and $\vec{\pi}$ are color singlets, and Δ^b is a color antitriplet and flavor singlet. Therefore, if $\langle\sigma\rangle \neq 0$, the chiral symmetry is broken dynamically, and if $\langle\Delta^b\rangle \neq 0$, the color symmetry is broken.

III. EFFECTIVE ACTION

Following [12], we obtain the partition function of the theory

$$Z = \int [dq][d\bar{q}][d\sigma][d\vec{\pi}][d\Delta^{*b}][d\Delta^b] \times \exp\left\{i\int d^D x\sqrt{-g}\,\tilde{\mathcal{L}}\right\}, \quad (6)$$

where $g = \det|g_{\mu\nu}|$. In what follows, it is convenient to consider the effective action for boson fields, which is expressed through the integral over quark fields

$$\exp\{iS_{\text{eff}}(\sigma, \vec{\pi}, \Delta^b, \Delta^{*b})\} = N'\int [dq][d\bar{q}] \times \exp\left\{i\int d^D x\sqrt{-g}\,\tilde{\mathcal{L}}\right\}, \quad (7)$$

where

$$S_{\text{eff}}(\sigma, \vec{\pi}, \Delta^b, \Delta^{*b}) = -\int d^D x\sqrt{-g}\left[\frac{3(\sigma^2 + \vec{\pi}^2)}{2G_1} + \frac{3\Delta^b\Delta^{*b}}{G_2}\right] + S_q, \quad (8)$$

N' is a normalization constant. The quark contribution to the partition function is given by the expression

$$Z_q = e^{iS_q} = N'\int [dq][d\bar{q}]\exp\Big(i\int d^D x\sqrt{-g}[\bar{q}(i\gamma^\mu\nabla_\mu - \sigma - i\gamma^5\vec{\pi}\,\vec{\tau} + \mu\gamma^0)q - \bar{q}(i\Delta^b\varepsilon\epsilon^b\gamma^5 C)\bar{q}^t - q^t(i\Delta^{*b}C\varepsilon\epsilon^b\gamma^5)q]\Big). \quad (9)$$

In the mean field approximation, the fields σ, $\vec{\pi}$, Δ^b, Δ^{*b} can be replaced by their groundstate averages: $\langle\sigma\rangle$, $\langle\vec{\pi}\rangle$, $\langle\Delta^b\rangle$, and $\langle\Delta^{*b}\rangle$, respectively. Let us choose the following ground state of our model: $\langle\Delta^1\rangle = \langle\Delta^2\rangle = \langle\vec{\pi}\rangle = 0$ and $\langle\sigma\rangle$, $\langle\Delta^3\rangle \neq 0$, denoted by letters σ, Δ. Evidently, this choice breaks the color symmetry down to the residual group $SU_c(2)$.

Let us find the effective potential of the model with the global minimum point that will determine the quantities σ and Δ. By definition $S_{\text{eff}} = -V_{\text{eff}}\int d^D x\sqrt{-g}$, where

$$V_{\text{eff}} = \frac{3\sigma^2}{2G_1} + \frac{3\Delta\Delta*}{G_2} + \tilde{V}_{\text{eff}}; \qquad \tilde{V}_{\text{eff}} = -\frac{S_q}{v}, \qquad v = \int d^D x\sqrt{-g}. \quad (10)$$

In virtue of the assumed vacuum structure ($\langle\Delta^{1,2}\rangle = 0$), the functional integral for S_q in (9) factorizes into two parts:

$$Z_q = \exp iS_q(\sigma, \Delta) = N'\int [d\bar{q}_3][dq_3]\exp\Big(i\int d^D x\sqrt{-g}\bar{q}_3\,\tilde{D}\,q_3\Big) \times \int [d\bar{Q}][dQ]\exp\Big(i\int d^D x\sqrt{-g}[\bar{Q}\,\tilde{D}\,Q + \bar{Q}M\bar{Q}^t + Q^t\bar{M}Q]\Big), \quad (11)$$

where q_3 is the quark field of color 3, and $Q \equiv (q_1, q_2)^t$ is the doublet of quarks of colors 1,2. The following notations are also introduced:

$$\tilde{D} = i\gamma^\mu\nabla_\mu - \sigma + \mu\gamma^0, \qquad \bar{M} = -i\Delta^* C\varepsilon\tilde{\epsilon}\gamma^5, \qquad M = -i\Delta\varepsilon\tilde{\epsilon}\gamma^5 C. \quad (12)$$

Here $\tilde{\epsilon} \equiv (\tilde{\epsilon})^{\alpha\beta}$ denotes an antisymmetric tensor in the color subspace corresponding to the $SU_c(2)$ group.

Clearly, integration over q_3 in (11) leads to $\det\tilde{D}$. Let us next define $\Psi = (Q^t, \bar{Q})$ and introduce a matrix operator

$$Z = \begin{pmatrix} 2\bar{M}, & -\tilde{D}^t \\ \tilde{D}, & 2M \end{pmatrix}.$$

Then the Gaussian integral in (11) can be written in a compact form:

$$\int [d\Psi]e^{i/2\int d^D x\sqrt{-g}\Psi^t Z\Psi} = \det{}^{1/2}Z. \quad (13)$$

Using the general formula

$$\det\begin{pmatrix} A & B \\ \bar{B} & \bar{A} \end{pmatrix} = \det[-\bar{B}B + \bar{B}A\bar{B}^{-1}\bar{A}] = \det[\bar{A}A - \bar{A}B\bar{A}^{-1}\bar{B}],$$

we obtain

$$\exp(iS_q(\sigma, \Delta)) = N'\det(\tilde{D})\det{}^{1/2}[4M\bar{M} + M\tilde{D}^t M^{-1}\tilde{D}] = N'\det[(i\hat{\nabla} - \sigma + \mu\gamma^0)]\det{}^{1/2}[4|\Delta|^2 + (-i\hat{\nabla} - \sigma + \mu\gamma^0)(i\hat{\nabla} - \sigma + \mu\gamma^0)]. \quad (14)$$

Here, the first determinant is over spinor, flavor, and coordinate spaces, and the second one is over the two-dimensional color space as well, and $\hat{\nabla} = \gamma^\mu \nabla_\mu$.

IV. EVALUATION OF DETERMINANTS IN THE STATIC EINSTEIN UNIVERSE

Let us first calculate the contribution of the determinant:

$$\begin{aligned}\ln\det[(i\hat{\nabla} - \sigma + \mu\gamma^0)] &= \tfrac{1}{2}\ln\det[(i\hat{\nabla} - \sigma + \mu\gamma^0)] \\ &\quad + \tfrac{1}{2}\ln\det[\gamma^5(i\hat{\nabla} - \sigma + \mu\gamma^0)\gamma^5] \\ &= \tfrac{1}{2}\ln\det[\gamma^\mu\gamma^\nu\nabla_\mu\nabla_\nu - 2i\mu\nabla_0 \\ &\quad - \mu^2 + \sigma^2].\end{aligned}$$

In our case, Γ_0 is evidently equal to zero, and $g^{\mu\nu}\nabla_\mu\nabla_\nu = \frac{\partial^2}{\partial t^2} - \nabla^2$, where $\nabla^2 = g^{ij}\nabla_i\nabla_j$ is the spinor Laplacian. Then

$$2\ln\det[(i\hat{\nabla} - \sigma + \mu\gamma^0)] = \ln\det\left[\left(\frac{\partial}{\partial t} - i\mu\right)^2 - \nabla^2 + \frac{1}{4}R + \sigma^2\right]. \tag{15}$$

In (15) we used the identity [30]

$$\gamma^\mu\gamma^\nu\nabla_\mu\nabla_\nu = g^{\mu\nu}\nabla_\mu\nabla_\nu + \tfrac{1}{4}R, \tag{16}$$

where R is the scalar curvature of the space-time. Next, consider the contribution of the second determinant:

$$\begin{aligned}&\det[4|\Delta|^2 + (-i\hat{\nabla} - \sigma + \mu\gamma^0)(i\hat{\nabla} - \sigma + \mu\gamma^0)] \\ &\quad = \det[4|\Delta|^2 + \gamma^\mu\gamma^\nu\nabla_\mu\nabla_\nu + \sigma^2 + \mu^2 - 2\mu\sigma\gamma^0 \\ &\qquad + 2i\mu\gamma^0\gamma^k\nabla_k].\end{aligned} \tag{17}$$

In analogy with the case of a flat Minkowski space-time, consider the Hamiltonian of a massless particle,

$$\hat{H} = \vec{\alpha}\,\hat{\vec{p}}, \tag{18}$$

where $\alpha^k = \gamma^0\gamma^k$, and $(\hat{p})^k = -i\nabla_k$, $k = 1, \ldots, D-1$. One can easily demonstrate that in the case of the static metric (3)

$$\hat{H}^2 = -\nabla^2 + \tfrac{1}{4}R. \tag{19}$$

With consideration of (16) and (19), formula (17) can be rewritten in the form

$$\begin{aligned}&\det[4|\Delta|^2 + (-i\hat{\nabla} - \sigma + \mu\gamma^0)(i\hat{\nabla} - \sigma + \mu\gamma^0)] \\ &\quad = \det[4|\Delta|^2 + \sigma^2 + \mu^2 - \hat{p}_0^2 + \hat{H}^2 - 2\mu(\hat{H} + \sigma\gamma^0)],\end{aligned} \tag{20}$$

where $\hat{p}_0 = i\partial_0$. Finally, let us introduce the Hamiltonian of a massive particle

$$\hat{\mathcal{H}} = \vec{\alpha}\,\hat{\vec{p}} + \sigma\gamma^0, \tag{21}$$

where $\hat{\vec{p}} = -i\vec{\nabla}$. It is related to the massless operator as follows:

$$\hat{\mathcal{H}}^2 = \hat{H}^2 + \sigma^2. \tag{22}$$

Then, (17) is rewritten in the form

$$\begin{aligned}&\det[4|\Delta|^2 + (-i\hat{\nabla} - \sigma + \mu\gamma^0)(i\hat{\nabla} - \sigma + \mu\gamma^0)] \\ &\quad = \det[4|\Delta|^2 + \mu^2 - \hat{p}_0^2 + \hat{\mathcal{H}}^2 - 2\mu\hat{\mathcal{H}}].\end{aligned} \tag{23}$$

Thus, in order to calculate the determinants defining the effective action, one has to find the eigenvalues of the operators $\hat{H}$ and $\hat{\mathcal{H}}$. In particular, they may be found exactly for the case of the static D-dimensional Einstein universe. They are expressed through the corresponding eigenvalues of the Dirac operator on the sphere $\mathbb{S}^{D-1}$ [38,39]. The line element

$$ds^2 = dt^2 - a^2(d\theta^2 + \sin^2\theta d\Omega_{D-2}) \tag{24}$$

gives the global topology $\mathbb{R}\otimes\mathbb{S}^{D-1}$ of the universe, where a is the radius of the universe, related to the scalar curvature by the relation $R = (D-1)(D-2)a^{-2}$. The volume of the universe is determined by the formula

$$V(a) = \frac{2\pi^{D/2}a^{D-1}}{\Gamma(\frac{D}{2})}. \tag{25}$$

Let us denote the absolute values of the eigenvalues of the operators (18) and (21) by ω_l and E_l, respectively. Then, according to [38,39], one has

$$\begin{aligned}&\hat{H}\psi_l = \pm\omega_l\psi_l, \qquad \omega_l = \frac{1}{a}\left(l + \frac{D-1}{2}\right) \\ &\hat{\mathcal{H}}\psi_l = \pm E_l\psi_l, \qquad E_l = \sqrt{\omega_l^2 + \sigma^2}, \qquad l = 0, 1, 2 \ldots.\end{aligned} \tag{26}$$

The degeneracies of ω_l and E_l are equal to

$$d_l = \frac{2^{[(D+1)/2]}\Gamma(D + l - 1)}{l!\Gamma(D-1)}, \tag{27}$$

where $[x]$ is the integer part of x.

V. THERMODYNAMIC POTENTIAL AND GAP EQUATIONS

Next, let us take the complete system of eigenfunctions $\Psi_k(t, \vec{x}) = e^{-ip_0 t}\psi_l(\vec{x})$ as a basis. Then the operators in the two determinants are diagonal and the latter are reduced to products of corresponding eigenvalues $E_l^2 - (p_0 - \mu)^2$ and $4|\Delta|^2 - p_0^2 + (E_l \pm \mu)^2$. Evidently, there appears a gap in the spectrum of the second operator proportional to $|\Delta|$, and as a result, the diquark condensate is formed. Using the relation $\det O = \exp(\mathrm{tr}\ln O)$ and following the standard procedure, one obtains the following expression for the quark contribution to the effective potential:

$$\tilde{V}_{\text{eff}} = -\frac{S_q}{v} = i\frac{N_f}{V}\int\frac{dp_0}{2\pi}\sum_{l=0}^{\infty}\sum_{\pm} d_l\{\ln[(E_l \pm \mu)^2 - p_0^2] + 2\ln[4|\Delta|^2 - p_0^2 + (E_l \pm \mu)^2]\}, \tag{28}$$

where $v = Vt$ and V is the volume of the universe, defined in (25), t is the time interval, and N_f denotes the number of flavors $N_f = 2$.

In the case of finite temperature $T = 1/\beta > 0$, the following substitutions in (28) should be made:

$$\int\frac{dp_0}{2\pi}(\cdots) \to iT\sum_n(\cdots), \qquad p_0 \to i\omega_n,$$
$$\omega_n = \frac{2\pi}{\beta}\left(n + \frac{1}{2}\right), \qquad n = 0, \pm 1 \pm 2, \ldots,$$

where ω_n is the Matsubara frequency. Then the quark contribution to the effective potential (28) becomes the thermodynamic potential Ω_q:

$$\Omega_q = -\frac{N_f}{V\beta}\sum_{n=-\infty}^{n=\infty}\sum_{l=0}^{\infty}\sum_{\pm} d_l\{\ln[(E_l \pm \mu)^2 + \omega_n^2] + 2\ln[4|\Delta|^2 + \omega_n^2 + (E_l \pm \mu)^2]\}. \tag{29}$$

The summation over the Matsubara frequencies can now be performed with the use of the formula

$$\sum_n \ln(\omega_n^2 + \rho^2) = \sum_n\int_{1/\beta^2}^{\rho^2} dx^2\frac{1}{\omega_n^2 + x^2} + \sum_n \ln\left(\omega_n^2 + \frac{1}{\beta^2}\right), \tag{30}$$

where, according to (29), ρ denotes $\sqrt{(E_l \pm \mu)^2}$ or $\sqrt{4|\Delta|^2 + (E_l \pm \mu)^2}$, respectively. In the following, we shall neglect the last term in (30), since it is independent of σ and Δ, and the first term can be rewritten as

$$\sum_n\int_{1/\beta^2}^{\rho^2} dx^2\frac{1}{\omega_n^2 + x^2} = 2\ln\cosh(\rho\beta/2) + \text{const} = \rho\beta + 2\ln(1 + e^{-\rho\beta}) + \text{const}. \tag{31}$$

Then the final form of the thermodynamic potential looks like

$$\Omega(\sigma, \Delta) = 3\left(\frac{\sigma^2}{2G_1} + \frac{|\Delta|^2}{G_2}\right) - \frac{N_f}{V}(N_c - 2)\sum_{l=0}^{\infty} d_l\{E_l + T\ln(1 + e^{-\beta(E_l-\mu)}) + T\ln(1 + e^{-\beta(E_l+\mu)})\} - \frac{N_f}{V}\sum_{l=0}^{\infty} d_l\{\sqrt{(E_l - \mu)^2 + 4|\Delta|^2} + \sqrt{(E_l + \mu)^2 + 4|\Delta|^2} + 2T\ln(1 + e^{-\beta\sqrt{(E_l-\mu)^2+4|\Delta|^2}}) + 2T\ln(1 + e^{-\beta\sqrt{(E_l+\mu)^2+4|\Delta|^2}})\}. \tag{32}$$

In order to find the correspondence of the result obtained with the case of the flat space-time,[2] we consider the zero curvature limit, i.e., $R \to 0$ ($a \to \infty$), of the expression for the thermodynamic potential. In this limit

$$R \to 0, \qquad a \to \infty, \qquad l \to \infty, \qquad \omega_l \to la^{-1},$$
$$\sum_l \to \int dl = a\int d\omega_l = a\int dp$$

(where we have changed the variables $\omega_l \to p$) and, since

$$\frac{\Gamma(D + l - 1)}{\Gamma(l + 1)} \approx l^{D-2},$$

the degeneracy is equal to

$$d_l = \frac{2^{[(D+1)/2]}\Gamma(D + l - 1)}{\Gamma(l + 1)\Gamma(D - 1)} \approx \frac{2^{[(D+1)/2]}}{(D - 2)!}l^{D-2}.$$

By taking the expression (25) for the volume $V(a)$ into account, we finally obtain for the case $D = 4$

$$\Omega = 3\left(\frac{\sigma^2}{2G_1} + \frac{\Delta^2}{G_2}\right) - 2N_f\int\frac{d^3p}{(2\pi)^3} \times\{E_p + T\ln(1 + e^{-\beta(E_p-\mu)}) + T\ln(1 + e^{-\beta(E_p+\mu)})\} - 2N_f\int\frac{d^3p}{(2\pi)^3}\{\sqrt{(E_p - \mu)^2 + 4|\Delta|^2} + \sqrt{(E_p + \mu)^2 + 4|\Delta|^2} + 2T\ln(1 + e^{-\beta\sqrt{(E_p-\mu)^2+4|\Delta|^2}}) + 2T\ln(1 + e^{-\beta\sqrt{(E_p+\mu)^2+4|\Delta|^2}})\}. \tag{33}$$

This equation coincides with the corresponding result for the ordinary four-dimensional flat space-time (see, e.g., [17–19]).

Now, imposing the condition on the effective potential, $\Omega(0, 0) = 0$, we should subtract a corresponding constant from it. The thermodynamic potential, normalized in this way, is still divergent at large l, and hence, we should

[2]Because of different topologies of the flat space-time and the Einstein universe, the expressions for their line elements do not relate to each other by any simple limiting procedure.

introduce a (soft) cutoff in the summation over l by means of the multiplier [33] $e^{-\omega_l/\Lambda}$, where Λ is the cutoff parameter.[3]

For convenience, we shall divide all the dimensional quantities that enter the thermodynamic potential by the corresponding power of Λ to make them dimensionless, i.e., Ω/Λ^D, σ/Λ, Δ/Λ, $\Lambda^{D-2}G_{1,2}$, $\Lambda^{D-1}V$, R/Λ^2, T/Λ, μ/Λ, ω_l/Λ, and denote them using the same letters as before: Ω, σ, Δ, $G_{1,2}$, V, R, T, μ, ω_l. Then the regularized thermodynamic potential is written as

$$\Omega^{\rm reg}(\sigma,\Delta) = 3\left(\frac{\sigma^2}{2G_1}+\frac{|\Delta|^2}{G_2}\right) - \frac{N_f}{V}(N_c-2)\sum_{l=0}^{\infty} e^{-\omega_l} d_l\{E_l + T\ln(1+e^{-\beta(E_l-\mu)}) + T\ln(1+e^{-\beta(E_l+\mu)})\} - \frac{N_f}{V}\sum_{l=0}^{\infty} e^{-\omega_l} d_l\Big\{\sqrt{(E_l-\mu)^2+4|\Delta|^2} + \sqrt{(E_l+\mu)^2+4|\Delta|^2} + 2T\ln(1+e^{-\beta\sqrt{(E_l-\mu)^2+4|\Delta|^2}}) + 2T\ln(1+e^{-\beta\sqrt{(E_l+\mu)^2+4|\Delta|^2}})\Big\}. \quad (34)$$

In the following section, we shall perform the numerical calculation of the points of the global minimum of the finite regularized thermodynamic potential $\Omega^{\rm reg}(\sigma,\Delta) - \Omega^{\rm reg}(0,0)$ [they should of course coincide with the minima of the potential $\Omega^{\rm reg}(\sigma,\Delta)$], and with the use of them, consider phase transitions in the Einstein universe. To this end, we have to consider the following gap equations:

$$\frac{\partial\Omega^{\rm reg}}{\partial\sigma} = 0, \qquad \frac{\partial\Omega^{\rm reg}}{\partial\Delta} = 0. \quad (35)$$

VI. PHASE TRANSITIONS

As it is seen from (32), the thermodynamic potential depends on two independent coupling constants G_1 and G_2. In what follows, we shall fix the constant G_2, similarly to what has been done in the flat case [12,18], by using the relation

$$G_2 = \tfrac{3}{8}G_1. \quad (36)$$

For numerical estimates, let us further choose the constant G_1 such that the chiral and/or color symmetries were completely broken. For comparison, we shall consider two cases of the choice of the constant: (1) $G_1 = 10$ (this value formally corresponds to that of [33]) and (2) $G_1 = 20$. It should be noted that different choices of the value of G_1 do not change the results in principle, but the scales of parameters are changed significantly (see Fig. 5 below). Moreover, let us now limit ourselves to the investigation of the case $D = 4$ only.

Let us first consider phase transitions in the case of zero temperature. Then, the thermodynamic potential takes the form

$$\Omega^{\rm reg}(\sigma,\Delta) = 3\left(\frac{\sigma^2}{2G_1}+\frac{|\Delta|^2}{G_2}\right) - \frac{N_f}{V}(N_c-2) \times \sum_{l=0}^{\infty} e^{-\omega_l} d_l\{E_l + (\mu-E_l)\theta(\mu-E_l)\} - \frac{N_f}{V}\sum_{l=0}^{\infty} e^{-\omega_l} d_l\Big\{\sqrt{(E_l-\mu)^2+4|\Delta|^2} + \sqrt{(E_l+\mu)^2+4|\Delta|^2}\Big\}. \quad (37)$$

Figures 1–3 show the behavior of the point of the global minimum (σ,Δ), determined from the gap equations (35) for the potential (37), as a function of μ at different values of $R = 3$, $R = 12$, and $T = 0$, for $G_1 = 10$ and $G_1 = 20$.

Using the formulas for the thermodynamic potential $\Omega^{\rm reg}$, we can calculate the fermion number density in the system

$$n = -\frac{\partial\Omega^{\rm reg}}{\partial\mu} \quad \text{at fixed } T. \quad (38)$$

The function $n(\mu)$ at fixed R is depicted in Fig. 4. The stepwise character of the curve is explained by the discreteness of the energy levels and by the presence of the step function in (37). It is seen from these figures that, when the chemical potential exceeds the critical value μ_c, the chiral symmetry is restored and the color symmetry is broken. Notice that for $R = 3$, $G_1 = 10$ our value for $\mu_c \approx 1.75$ is close to that from [33].

One may ask why the diquark condensate Δ vanishes for large chemical potential μ, i.e., why CSC will be destroyed at large densities. (An analogous behavior is seen also in Fig. 6 of [19].) In fact, since in the weak coupling QCD of flat space-time the nonvanishing CSC phase is present for arbitrary large values of μ, the NJL results for large values

[3] In flat space-time, the regularization cutoff constant Λ can be determined from the experimental results. However, in the case of a curved space-time, in order to fix the cutoff Λ we need new theoretical/experimental inputs for chiral QCD in (strong) gravitational background fields, concerning, for instance, the effective gluon mass, known values of the quark condensate, or even experimentally measured pion properties. Our aim here is to perform only a qualitative study of gravitation effects on chiral and color symmetry breaking and its restoration, focusing only on the qualitative dependence of the phase structure on gravity. To this end, due to the present lack of experimental knowledge, we scale the thermodynamic potential and all relevant quantities like condensates, curvature, chemical potential, and temperature by the unknown cutoff Λ.

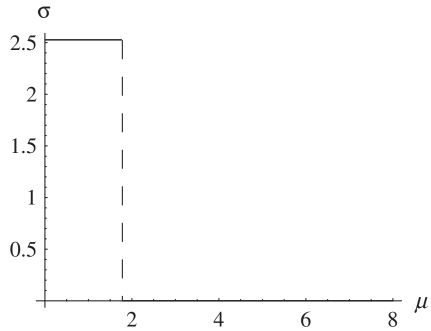

FIG. 1. Condensates σ and Δ as functions of μ for $R = 3$, $G_1 = 10$, and $T = 0$ (all quantities are given in units of Λ).

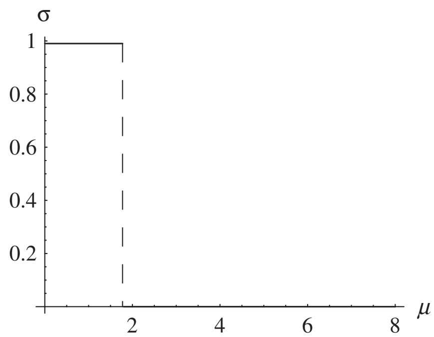

FIG. 2. Condensates σ and Δ as functions of μ for $R = 12$, $G_1 = 10$, and $T = 0$ (all quantities are given in units of Λ).

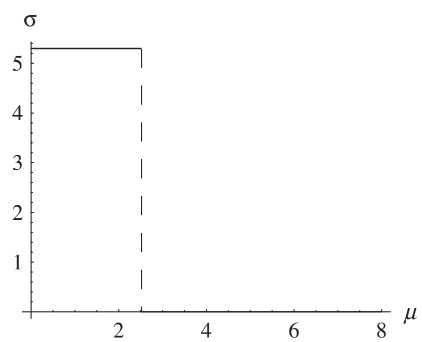

FIG. 3. Condensates σ and Δ as functions of μ for $R = 12$, $G_1 = 20$, and $T = 0$ (all quantities are given in units of Λ).

of the chemical potential could be an artifact of the model and should be considered with care.

As is clear from Figs. 1–4, a first order phase transition takes place at the point $\mu = \mu_c$ (the first derivative of the thermodynamical potential is discontinuous).

In Fig. 5, the $\mu - R$-phase portrait of the system at zero temperature is depicted for two choices of $G_1 = 10$ and $G_1 = 20$ (1, 2, and 3 denote the symmetric, chiral, and superconducting phases, respectively).

For points in the symmetric phase 1, the global minimum of the thermodynamic potential is at $\sigma = 0$, $\Delta = 0$ (chiral and color symmetries are unbroken). In the region of phase 2, only chiral symmetry is broken and $\sigma \neq 0$, $\Delta = 0$. The points in phase 3 correspond to the formation of the diquark condensate (color superconductivity) and

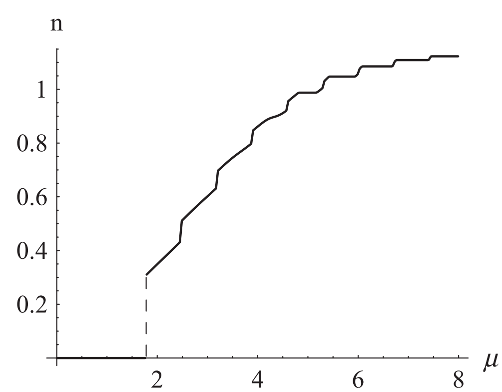

FIG. 4. The fermion number density at $R = 3$, $T = 0$, and $G_1 = 10$.

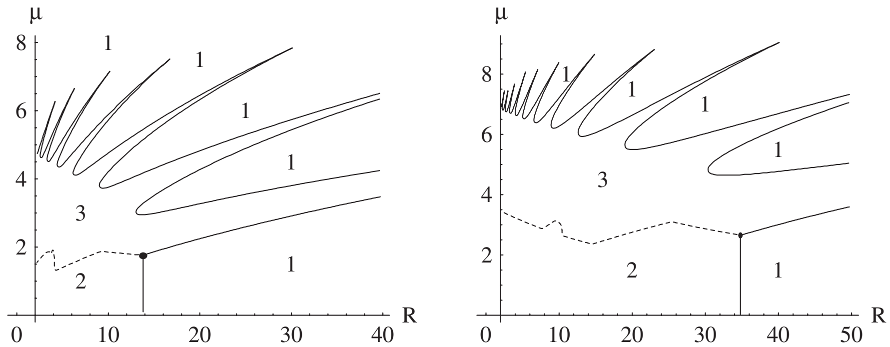

FIG. 5. The phase portrait at $T = 0$ for $G_1 = 10$ (left picture) and $G_1 = 20$ (right picture). Dashed (solid) lines denote first (second) order phase transitions. The bold point denotes a tricritical point. The numbers 1, 2, and 3 designate the symmetric phase, the phase with chiral symmetry breaking, and the superconducting phase, respectively.

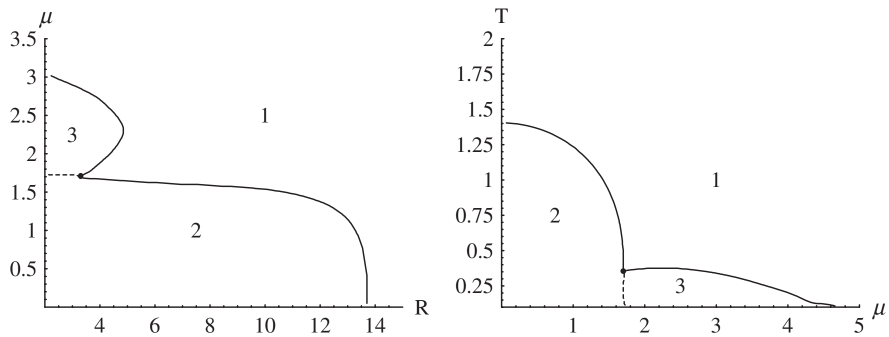

FIG. 6. The phase portraits at $T = 0.35$ (left picture) and at $R = 3$ (right picture), $G_1 = 10$.

the minimum takes place at $\sigma = 0$, $\Delta \neq 0$. The dashed line in Fig. 5 denotes the first order phase transition, and by the solid line the second order phase transition is depicted. The bold point denotes the tricritical point between transitions of the first and second orders. As was pointed out in [33], the vertical line at $T = 0$ between phases 1 and 2 implies that the lowest eigenvalue of ω_0 is nonvanishing. In the vicinity of this line, a second order phase transition takes place, i.e., in this case, the curvature R behaves exactly in the same way as the temperature T does in the flat space. Let us mention that, as in the flat space (see [18]), at $T = 0$, no mixed phase with both condensates different from zero takes place. Comparison of the plots for values $G_1 = 10$ and $G_1 = 20$ demonstrates no significant modification of the form of the phase portrait made by the different choices of the coupling constant. However, with growing value of G_1, the critical values of R and μ increase.

The above results are in agreement with the conclusions of [18,19] made for a flat space-time that in two-flavor QCD a second order phase transition takes place at high temperatures and a first order transition at high densities, and this leads to the presence of a tricritical point.

Moreover, the oscillation effect clearly visible in the phase curves of Fig. 5 should be mentioned. This may be explained by the discreteness of the fermion energy levels (26) in the compact space. This effect may be compared to the similar effect in the magnetic field H, where fermion levels are also discrete (the Landau levels). The corresponding magnetic oscillations of the $\mu - \sqrt{e}H$-phase portrait in dense cold quark matter with four-fermion interactions were found in paper [10], where it was demonstrated that in the massless case, such a phase structure leads unavoidably to the standard van Alphen–de Haas magnetic oscillations of some thermodynamical quantities, including magnetization, pressure and particle density.[4] In addition, we considered also phase transitions at finite temperatures. In Fig. 6, $\mu - R$- and $T - \mu$-phase portraits are depicted. No mixed phase with simultaneous nonzero values of $\sigma \neq 0$, $\Delta \neq 0$ can be seen in these plots.

It is clear from Fig. 6 that with growing temperature both the chiral and color symmetries are restored. The similarity of plots in $R - \mu$ and $\mu - T$ axes leads one to the conclusion that the parameters of curvature R and temperature T play similar roles in restoring the symmetries of the system. For comparison, we also studied the phase portrait for the case of vanishing curvature, $R = 0$. The obtained result turns out to be very similar to the right picture in Fig. 6. In the case $R = 0$, with chemical potential growing

[4]On the other hand, since these oscillations are related to the restoration of color symmetry for large μ, they seem to be here rather an artifact of the NJL model.

from zero, the critical temperature for the transition between phases 1 and 2 decreases from a value slightly higher than 1.5.

VII. CONCLUSION

By using a static model of compact space-time (the Einstein universe), we have derived a nonperturbative expression for the thermodynamical potential of the Nambu–Jona-Lasinio model in the mean field approximation. The corresponding effects of gravitation were exactly taken into account. This allowed us to consider phase transitions in the vicinity of the critical points. In particular, we have studied the influence of the chemical potential, the temperature, and the curvature on the chiral and color superconducting phase transitions. Moreover, an oscillation effect of the phase curves was found, which may be explained by the discreteness of the fermion energy levels in the compact space.

The analysis made in this paper demonstrates that the space curvature R in the case of compact space plays an analogous role as the temperature T does in the flat space, leading to a second order phase transitions and the restoration of chiral symmetry. On the other hand, at certain values of the chemical potential (and a sufficiently large coupling constant), a superconducting phase is formed, and the color symmetry is broken. However, as might be expected, the chiral and color symmetries become restored at high temperatures and high curvatures.[5]

Clearly, the simple schematic model of gravitation considered in this work may represent, at best, only qualitative features of the dependence of quark and diquark condensates on a strong gravitational field. Much further work has to be done towards a more quantitative and realistic description of the dynamical breaking and restoration of chiral and color symmetries in curved space-time. This concerns, in particular, the interesting question, whether the predicted CSC in cold compact objects like neutron stars (see, e.g., [40] and a more recent paper [41]) might eventually be influenced by the associated strong gravitational fields. Concerning the cosmological situation, it should be mentioned that the temperature in the cosmological phase transition after the big bang is very high. Thus, the hadronic phase transitions, considered in this paper, are not related to this cosmological phase and could only occur later at lower temperatures. Moreover, a straightforward cosmological interpretation of our model is not adequate, since real space-time at the present stage of cosmological evolution seems to be flat and not curved or closed. Despite the fact that our model space-time is in this sense just a theoretical model, we believe that the problem considered here represents an interesting new example of a quantum field theory model with interacting fields which is studied nonperturbatively in curved space-time. In particular, this framework allowed us to study the interplay of quantum effects and gravitational background (space-time curvature) in the restoration of dynamically broken chiral and color symmetries. Hopefully, such kind of studies might initiate other investigations, which are yet closer to realistic cosmological or astrophysical situations.

[5]Notice that we have primarily concentrated ourselves to the study of chiral and color symmetry breaking in the 4-dimensional Einstein universe. It should, however, be mentioned that the phase curves plotted for space dimensions different from 3 demonstrate a considerable similarity with the case studied in this work.

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