Graviton localization and Newton's law for brane models with a nonminimally coupled bulk scalar field

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Brane world models with a nonminimally coupled bulk scalar field have been studied recently. In this paper we consider metric fluctuations around an arbitrary gravity-scalar background solution, and we show that the corresponding spectrum includes a localized zero mode which strongly depends on the profile of the background scalar field. For a special class of solutions, with a warp factor of the RS form, we solve the linearized Einstein equations, for a pointlike mass source on the brane, by using the brane bending formalism. We see that general relativity on the brane is recovered only if we impose restrictions on the parameter space of the models under consideration.

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I. INTRODUCTION

The old Kaluza-Klein idea of enlarging the space-time manifold to extra dimensions has been put on a new basis recently, in an attempt to solve the hierarchy problem between the electroweak scale and the Planck scale. The new point is that there is no need to consider extra dimensions of the order of the Planck length: one may consider large compact extra dimensions of the order of a millimeter [1] or even a noncompact extra dimension [2]. An interesting feature of these models, is that they have testable predictions in the near future high energy experiments.

The localization problem is a major issue in these so-called brane world models [3,4]. The graviton propagates in all dimensions (also in the bulk), since it is the dynamics of space-time itself. The SM particles are localized in some way on the 4D submanifold (brane), which corresponds to the universe we are living in. Several localization mechanisms have been proposed; see, for example, Refs. [5,6] and references therein. For lattice simulations on the same subject see Ref. [7].

In Ref. [1] one introduces n flat extra compact spatial dimensions with large volume. This brane world model is known as ADD model (Arkani-Hamed, Dimopoulos and Dvali). An important feature of a wide class of brane world models, including the ADD model, is that they predict deviations from the 4D Newton law at submillimeter distances.

The simplest brane models with a warped extra dimension have been introduced in Ref. [2]. In the first version, which is known as the first Randall-Sundrum model, we have an orbifolded extra dimension of radius r_c. Two branes are fitted to the fixed points of the orbifold, $z = 0$ and $z_c = \pi r_c$ with tensions σ and $-\sigma$ respectively; it is assumed that the particles of the standard model are trapped on the negative tension brane. The second version of the model is constructed if we send the negative tension brane to infinity ($r_c \rightarrow +\infty$) and assume that the ordinary matter lives on the positive tension brane.

In both versions of the Randall-Sundrum model the 4D graviton is obtained by considering small gravitational fluctuations $\bar{h}_{\mu\nu}$ around the classical solution of the model. The spectrum of gravitational fluctuations $\bar{h}_{\mu\nu}$ consists of a zero mode, which gives rise to Newton's law on the brane, plus a continuum of positive energy states with no gap, giving small corrections to the 4D Newtonian potential. For a derivation of the equation obeyed by the metric fluctuations see, for example, Ref. [8].

The proper calculation of the Newtonian potential on the brane plus the correction terms has been detailed by J. Garriga and T. Tanaka [9] and by S. B. Giddings *et al.* [10]. These works use the so-called bent brane formalism. The idea is that if one puts a matter source on the brane, there is a gauge choice for which the equations for the metric perturbations decouple; however, in this gauge the brane is not located at $z = 0$ but it appears bent around the position of the matter source. In Ref. [9] it is pointed out that the role of the bending is essential in reproducing the 4D graviton propagator structure out of the 5D one, as it exactly compensates for the effects of extra polarization. Calculations of the Newton potential based on different philosophies can be found in Ref. [11–14]. For a recent review on the topic the reader may consult Ref. [15], and references therein.

There are various generalizations of the ideas of standard Randall-Sundrum scenario, such as models with more than five dimensions, models with topological defects toward the extra dimensions, multibrane models and models with higher order curvature corrections (i.e. Gauss-Bonnet gravity). Details are to be found in, for example, Refs. [6,16,17] and references therein.

Brane world models with a nonminimally coupled bulk scalar field, via an interaction term of the form $-\frac{1}{2}\xi R\phi^2$, where ξ is a dimensionless coupling, have been studied recently. Static solutions of these models have been exam-

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ined numerically in Refs. [18–20], for the simplest case of a scalar field potential $V(\phi) = \lambda\phi^4$, while in Ref. [21] the same model has been examined in the presence of Gauss-Bonnet gravity. In Ref. [22] analytical solutions have been obtained by choosing appropriately the potential for the scalar field. Cosmological implications of these models are described in Ref. [23].

A crucial question is, whether standard four-dimensional gravity can be recovered on the brane in models with a nonminimally coupled scalar field. In order to answer this question we consider metric fluctuations around an arbitrary gravity-scalar background solution of these models. We see that the corresponding spectrum does include a localized zero mode, which is necessary for Newton's law to hold on the brane. For a special class of solutions, with a warp factor of the RS form, we solve the linearized Einstein equations in the case where a pointlike mass source sits on the brane, using the bent brane formalism. We see that standard four-dimensional gravity on the brane is recovered only if we impose serious restrictions on the parameters of these models.

In Sec. II of this work we define the model; in Sec. III we give the first order (linearized) version of the Ricci and the energy-momentum tensors, concluding with an equation for the gravitational fluctuation in a source-free space-time; in Sec. IV we comment on the spectrum of the metric fluctuations. In Sec. V we present the bent brane formalism for our case, we calculate the gravitational perturbations and comment on the results. Section VI summarizes our conclusions. Finally, in an appendix we give a short discussion on tensor fluctuations in the Einstein frame.

II. THE MODEL

We consider the action of five-dimensional gravity with a nonminimally coupled bulk scalar field:

$$S = \int d^5x\sqrt{|g|}\Big(F(\phi)R - \frac{\sqrt{|g^{(\text{brane})}|}}{\sqrt{|g|}}\sigma(\phi)\delta(z) - \frac{1}{2}g^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi - V(\phi)\Big), \quad (1)$$

where

$$F(\phi) = \frac{1}{2k}(1 - k\xi\phi^2), \qquad k = 8\pi G_5. \quad (2)$$

G_5 is the five-dimensional Newton's constant, and $\sigma(\phi)$ is a ϕ-dependent brane tension. We have assumed that μ, $\nu = 0, 1, 2, 3, 5$, and $d^5x = d^4x dz$ where z describes the extra dimension. Note that in the above action we did not add an explicit cosmological constant term; if it is present, it may be included in the scalar field potential $V(\phi)$.

The Einstein equations with the nonminimally coupled bulk scalar field are

$$F(\phi)G_{\mu\nu} - \nabla_\mu\nabla_\nu F(\phi) + g_{\mu\nu}\Box F(\phi) + \frac{1}{2}\sigma(\phi)\delta(z)\frac{\sqrt{|g^{(\text{brane})}|}}{\sqrt{|g|}}g_{ij}\delta^i_\mu\delta^j_\nu = \frac{1}{2}T^{(\phi)}_{\mu\nu}, \quad (3)$$

where $i, j = 0, 1, 2, 3$, $G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R$, and $T^{(\phi)}_{\mu\nu}$ is the energy-momentum tensor for the scalar field

$$T^{(\phi)}_{\mu\nu} = \nabla_\mu\phi\nabla_\nu\phi - g_{\mu\nu}[\tfrac{1}{2}g^{\rho\sigma}\nabla_\rho\phi\nabla_\sigma\phi + V(\phi)]. \quad (4)$$

The equation of motion for the scalar field reads

$$\Box\phi + F'(\phi)R - V'(\phi) - \sigma'(\phi)\delta(z) = 0, \quad (5)$$

where $F'(\phi) \equiv \frac{dF(\phi)}{d\phi}$; similar definitions are understood for $V'(\phi)$ and $\sigma'(\phi)$.

The equations of motion (3) and (5) possess static solutions of the following form:

$$ds^2 = e^{2A(z)}\eta_{ij}dx^i dx^j + dz^2, \qquad \phi = \phi(z), \quad (6)$$

which exhibits four-dimensional Poincarè symmetry. The sign convention for the Minkowski metric is $\eta_{ij} = \text{diag}(-1, 1, 1, 1)$.

Static solutions of the form of Eq. (6) have been studied both numerically [19] and analytically [22].

III. LINEARIZED EQUATIONS

In this section we determine the linearized equations for the metric fluctuations $\bar{h}_{\mu\nu}$, in the case of the brane model with the nonminimally coupled scalar field introduced in Sec. II.

It is convenient to consider a Gaussian normal coordinate system $\bar{x}^i$, $\bar{z}$, where the brane is located by definition at $\bar{z} = 0$, and the fluctuations $\bar{h}_{\mu\nu}$, around the brane background solution, satisfy the conditions $\bar{h}_{\mu 5} = \bar{h}_{55} = 0$. Note that we do not consider scalar field fluctuations in the sequel; we will explain at the end of this section that this is consistent.

The perturbed metric in this coordinate frame reads

$$ds^2 = e^{2A(\bar{z})}(\eta_{ij} + \bar{h}_{ij})d\bar{x}^i d\bar{x}^j + d\bar{z}^2. \quad (7)$$

The Ricci tensor may be expanded to read

$$R_{\mu\nu} = R^{(0)}_{\mu\nu} + R^{(1)}_{\mu\nu} + \ldots, \quad (8)$$

where the zero order term is

$$R^{(0)}_{ij} = -e^{2A}(A'' + 4A'^2)\eta_{ij}, \qquad R^{(0)}_{55} = -4(A'' + A'^2), \qquad R^{(0)}_{5i} = 0, \quad (9)$$

while the first order term is

$$R^{(1)}_{ij} = -e^{2A}(\tfrac{1}{2}\partial_z^2 + 2A'\partial_z + A'' + 4A'^2)\bar{h}_{ij} - \tfrac{1}{2}\Box^{(4)}\bar{h}_{ij} - \tfrac{1}{2}\eta_{ij}e^{2A}A'\partial_z(\eta^{kl}\bar{h}_{kl}) - \tfrac{1}{2}\eta^{kl}(\partial_i\partial_j h_{kl} - \partial_i\partial_k h_{jl} - \partial_j\partial_k h_{il}), \quad (10)$$

$$R^{(1)}_{55} = -\tfrac{1}{2}(\partial_z^2 + 2A'\partial_z)\eta^{kl}\bar{h}_{kl}, \qquad R^{(1)}_{i5} = \tfrac{1}{2}\eta^{kl}\partial_z(\partial_k\bar{h}_{il} - \partial_i\bar{h}_{kl}), \tag{11}$$

where we have used the $4D$ d' Alembertian operator: $\Box^{(4)} \equiv \eta^{ij}\partial_i\partial_j$..

In this paper we will use the alternative form of the Einstein equations, which reads

$$2F(\phi)R_{\mu\nu} = \tilde{t}_{\mu\nu}, \tag{12}$$

where $\tilde{t}_{\mu\nu} = t_{\mu\nu} - \frac{1}{3}g_{\mu\nu}t$, $t = t^{\mu}_{\mu} = g^{\mu\nu}t_{\mu\nu}$, and $t_{\mu\nu}$ is defined as

$$t_{\mu\nu} \equiv T^{(\phi)}_{\mu\nu} + 2\nabla_\mu\nabla_\nu F(\phi) - 2g_{\mu\nu}\Box F(\phi) - \sigma(\phi)\delta(z)\frac{\sqrt{|g^{(\mathrm{brane})}|}}{\sqrt{|g|}}g_{ij}\delta^i_\mu\delta^j_\nu. \tag{13}$$

The tensor $\tilde{t}_{\mu\nu}$ may also be expanded as

$$\tilde{t}_{\mu\nu} = \tilde{t}^{(0)}_{\mu\nu} + \tilde{t}^{(1)}_{\mu\nu} + \ldots, \tag{14}$$

with the zero order term:

$$\tilde{t}^{(0)}_{ij} = \frac{2}{3}\eta_{ij}e^{2A}((\phi')^2F''(\phi) + 7\phi'A'F'(\phi) + \phi''F'(\phi) + V(\phi)) - \tfrac{1}{3}\eta_{ij}\sigma(\phi)\delta(z), \tag{15}$$

$$\tilde{t}^{(0)}_{55} = (\phi')^2 + \tfrac{8}{3}(\phi')^2F''(\phi) + \tfrac{8}{3}\phi'A'F'(\phi) + \tfrac{8}{3}\phi''F'(\phi) + \tfrac{2}{3}V(\phi), \qquad \tilde{t}^{(0)}_{5i} = 0, \tag{16}$$

and the first order term:

$$\tilde{t}^{(1)}_{ij} = \tfrac{2}{3}e^{2A}((\phi')^2F''(\phi) + 7\phi'A'F'(\phi) + \phi''F'(\phi) + V(\phi))\bar{h}_{ij} - \tfrac{1}{3}\sigma(\phi)\delta(z)\bar{h}_{ij} + \tfrac{1}{3}\eta_{ij}e^{2A}F'(\phi)\phi'\partial_z(\eta^{kl}\bar{h}_{kl}) + e^{2A}F'(\phi)\phi'\partial_z\bar{h}_{ij}, \tag{17}$$

$$\tilde{t}^{(1)}_{55} = \tfrac{1}{3}e^{2A}F'(\phi)\phi'\partial_z(\eta^{kl}\bar{h}_{kl}), \qquad \tilde{t}^{(1)}_{5i} = 0. \tag{18}$$

If we use Eqs. (8), (12), and (14) we obtain the zero and first order Einstein equations:

$$2F(\phi)R^{(0)}_{\mu\nu} = \tilde{t}^{(0)}_{\mu\nu}, \tag{19}$$

$$2F(\phi)R^{(1)}_{\mu\nu} = \tilde{t}^{(1)}_{\mu\nu}. \tag{20}$$

The background solution for the metric, Eq. (6), satisfies the zero order Einstein equations:

$$F(\phi)(A'' + 4A'^2) + \tfrac{1}{3}(\phi')^2F''(\phi) + \tfrac{7}{3}\phi'A'F'(\phi) + \tfrac{1}{3}\phi''F'(\phi) + \tfrac{1}{3}V(\phi) - \tfrac{1}{6}\sigma(\phi)\delta(z) = 0, \tag{21}$$

$$F(\phi)(4A'' + 4A'^2) + \tfrac{1}{2}(\phi')^2 + \tfrac{4}{3}(\phi')^2F''(\phi) + \tfrac{4}{3}\phi'A'F'(\phi) + \tfrac{4}{3}\phi''F'(\phi) + \tfrac{1}{3}V(\phi) = 0, \tag{22}$$

and the appropriate boundary conditions on the brane [19,22].

Note, that Eq. (5) for the scalar field, is also satisfied by the background solution, since it is not independent from the Einstein equations (3); in fact it can be derived from the latter [19].

If one takes into account Eqs. (21) and (22), the first order Einstein equations (20), can be rewritten in the form:

$$2F(\phi)[-e^{2A}(\tfrac{1}{2}\partial_z^2 + 2A'\partial_z)\bar{h}_{ij} - \tfrac{1}{2}\Box^{(4)}\bar{h}_{ij} - \tfrac{1}{2}\eta_{ij}e^{2A}A'\partial_z(\eta^{kl}\bar{h}_{kl}), - \tfrac{1}{2}\eta^{kl}(\partial_i\partial_j\bar{h}_{kl} - \partial_i\partial_k\bar{h}_{jl} - \partial_j\partial_k\bar{h}_{il})] = \tfrac{1}{3}\eta_{ij}e^{2A}F'(\phi)\phi'\partial_z(\eta^{kl}\bar{h}_{kl}) + e^{2A}F'(\phi)\phi'\partial_z\bar{h}_{ij} \tag{23}$$

$$-F(\phi)(\partial_z^2 + 2A'\partial_z)\eta^{kl}\bar{h}_{kl} = \tfrac{1}{3}e^{2A}F'(\phi)\phi'\partial_z(\eta^{kl}\bar{h}_{kl}), \qquad \tfrac{1}{2}\eta^{kl}\partial_z(\partial_k\bar{h}_{il} - \partial_i\bar{h}_{kl}) = 0. \tag{24}$$

We can simplify the above equations if we perform a gauge transformation $\bar{h}_{ij} \to h_{ij}$ [see Eqs. (45), (48), and (49) below], where the metric fluctuations h_{ij} in the new coordinate system satisfies the conditions $\eta_{ij}h^{ij} = 0$ (traceless) and $\partial^i h_{ij} = 0$ (transverse). In this coordinate system the Einstein equations decouple and we get:

$$F(\phi)(-e^{2A}(\partial_z^2 + 4A'\partial_z) - \Box^{(4)})h_{ij} = e^{2A}F'(\phi)\phi'\partial_z h_{ij}, \tag{25}$$

or equivalently:

$$(\partial_z^2 + Q'(z)\partial_z + e^{-2A}\Box^{(4)})h_{ij} = 0, \tag{26}$$

where we have set

$$Q(z) \equiv 4A(z) + \ln(F(\phi(z))). \tag{27}$$

In this paper we assume that the solutions considered satisfy $F(\phi) > 0$.

Note that in this work we do not examine the case of nonzero scalar field fluctuations, that is we have assumed that the fluctuation $\tilde{\phi}$ around the background scalar field vanishes. This is consistent, since one finds that the linearized equation of motion for the scalar fluctuation reads:

$$\tilde{\phi}'' + 4A'\tilde{\phi}' + \tfrac{1}{2}\phi'\eta^{ij}h'_{ij} + F'(\phi)R^{(1)} + F''(\phi)R^{(0)}\tilde{\phi} - V''(\phi)\tilde{\phi} = 0.$$

If we impose the constraint $\tilde{\phi} = 0$, this equation becomes $\frac{1}{2}\phi'\eta^{ij}\bar{h}'_{ij} + F'(\phi)R^{(1)} = 0$. or

$$\frac{1}{2}\phi'\eta^{ij}\bar{h}'_{ij} + \frac{dF(\phi)}{d\phi}[\eta^{ij}\bar{h}''_{ij} + 5A'\eta^{ij}\bar{h}'_{ij} - e^{-2A}\Box(\eta^{ij}\bar{h}_{ij}) - e^{-2A}\partial^i\partial^j\bar{h}_{ij}] = 0. \quad (28)$$

The primes denote differentiation with respect to z. In the following we work in a gauge, in which the gravitational perturbation satisfies the constraints $\eta^{ij}h_{ij} = 0$, $\partial^i h_{ij} = 0$ [see Eq. (46) below]. It is easily seen that in this gauge Eq. (28) is manifestly satisfied.

IV. GRAVITON LOCALIZATION

In this section we study the spectrum of metric fluctuations for brane models with a nonminimally coupled scalar field. We obtain that the spectrum does include a zero mode localized on the brane (which is necessary, in order to obtain the 4D Newton's law), along with a continuum of positive energy states. It should be noted that in models with a nonminimal coupling the weight function $r(z) = F(\phi)e^{2A}$ depends on the background scalar field, which marks an important difference from the RS2-model, or models with a minimally coupled scalar field.

A. Sturm-Liouville form

It is easy to find the weight function by bringing the equation for the gravitational fluctuations in a Sturm-Liouville form. If we set

$$h_{ij}(x, z) = e^{ipx}u(m, z) \quad (29)$$

in Eq. (26) we obtain

$$(\partial_z^2 + Q'\partial_z + m^2e^{-2A})u(m, z) = 0, \quad (30)$$

where $m^2 = -p^i p_i$ is the four-dimensional mass.

We note the following boundary condition on the brane:

$$\partial_z h_{ij}(x, z)|_{z=0} = 0, \quad \text{or} \quad u'(m, 0) = 0. \quad (31)$$

Multiplying Eq. (30), with the factor e^Q, where Q is defined in Eq. (27), we get

$$-\partial_z(e^Q\partial_z u(m, z)) = m^2e^{Q-2A}u(m, z). \quad (32)$$

The above equation is of the Sturm-Liouville form:

$$-\partial_z(p(z)\partial_z u(\lambda, z)) + q(z)u(\lambda, z) = \lambda r(z)u(\lambda, z), \quad (33)$$

with the coefficients:

$$p(z) = F(\phi)e^{4A}, \quad r(z) = F(\phi)e^{2A}, \quad q(z) = 0, \quad \lambda = m^2. \quad (34)$$

The eigenvalue Eq. (33) has a constant solution $u(0, z)$ for the zero mode ($m^2 = 0$). This solution $u(0, z)$ must be square integrable with a weight function $r(z) = F(\phi)e^{2A}$, so the normalizable zero mode is

$$u(0, z) = \frac{1}{\sqrt{\int_{-\infty}^{+\infty} dzF(\phi)e^{2A}}}. \quad (35)$$

In addition, the quantity $[u(0, z)]^2F(\phi)e^{2A}dz$ may be interpreted as the probability to obtain the corresponding graviton between the positions z and $z + dz$. Note that we have considered only solutions which satisfy the restriction $F(\phi) > 0$ and thus the probability density is positive. We have also assumed that the integral $\int_{-\infty}^{+\infty} F(\phi)e^{2A(z)}dz$ exists, that is $F(\phi)e^{2A(z)}$ tends to zero quite fast for large z. From the above we conclude that the eigenvalue Eq. (30) has a *localized* zero mode which plays the role of the 4D graviton on the brane. As we show in the next section the remaining spectrum consists of continuum positive energy states.

B. Schrödinger form

It is instructive to use an alternative form of the equation for the gravitational fluctuations, called the Schrödinger form. It involves a potential and it offers intuition about the spectrum.

If one performs the transformation $w = w(z)$, where $w'(z) = e^{-A(z)}$, Eq. (6) is put into the conformally flat form:

$$ds^2 = e^{2\tilde{A}(w)}(\eta_{ij}dx^idx^j + dw^2), \quad \tilde{\phi} = \tilde{\phi}(w), \quad (36)$$

and Eq. (30) yields:

$$(\partial_w^2 + \tilde{Q}'(w)\partial_w + m^2)\tilde{u}(m, w) = 0, \quad (37)$$

where

$$\tilde{Q}(w) = 3\tilde{A}(w) + \ln(F(\tilde{\phi}(w))). \quad (38)$$

The transformation

$$\tilde{u}(m, w) = e^{-(\tilde{Q}/2)}\tilde{v}(m, w) \quad (39)$$

brings Eq. (37) into the Schrödinger form:

$$-\partial_w^2\tilde{v}(m, w) + (\tilde{V}(w) - m^2)\tilde{v}(m, w) = 0 \quad (40)$$

with the potential:

$$\tilde{V}(w) = \tfrac{1}{2}\tilde{Q}''(w) + \tfrac{1}{4}(\tilde{Q}'(w))^2. \quad (41)$$

Equation (40) can be written alternatively as

$$L^\dagger L\tilde{v}(m, w) = m^2\tilde{v}(m, w), \quad (42)$$

where the operators L and $L^\dagger$ are defined through

$$L^\dagger \equiv -\partial_w - \tfrac{1}{2}Q'(w), \qquad L \equiv \partial_w - \tfrac{1}{2}Q'(w). \tag{43}$$

As the operator $L^\dagger L$ is hermitian and positive definite, it will have a complete system of eigenstates with non-negative eigenvalues, or $m^2 \geq 0$. In addition there is a *normalizable* zero mode, which obeys the equation

$$L\tilde{v}(0, w) = 0 \Leftrightarrow \tilde{v}(0, w) \sim e^{Q(w)/2} = e^{(3/2)\tilde{A}(w)}F(\tilde{\phi}(w)). \tag{44}$$

It is readily seen from Eqs. (38) and (41) that the potential $\tilde{V}(w)$ strongly depends on $F(\tilde{\phi}(w))$.

If the potential in Eq. (41) vanishes for $|w| \longrightarrow +\infty$, there exists a continuum spectrum of positive energy states starting from zero; for extensive discussion on this point see Ref. [24]. We have checked that the analytical solutions in Ref. [22], with the RS warp factor, give rise to a potential of the "volcano" form, in agreement with the above assertions about the spectrum.

V. GRAVITY IN THE RS BRANE WORLD WITH A NON-MINIMALLY COUPLED BULK SCALAR FIELD

In general, brane world models succeed in reproducing the Newtonian potential on the brane, as they exhibit a localized zero energy state which mimics the four-dimensional graviton. In addition, due to Kaluza-Klein-like excitations, modifications of Newton's law are predicted at distances smaller than the length scale of the model.

We would like to examine whether brane models with a nonminimally coupled bulk scalar field reproduce the 4D gravity on the brane. The proper approach for the derivation of Newton law has been presented in Ref. [9] and it is known as the bent brane formalism; this approach is adopted in the sequel. However, the bent brane formalism cannot be applied to metrics which are not of the Randall-Sundrum type in a straightforward way. For this reason in this paper we examine only single brane solutions with a warp factor $e^{A(z)}$ of the Randall-Sundrum form $A(z) = -|z|/l$, where l is the length scale of the model. As already mentioned, analytical solutions with a Randall-Sundrum type warp factor have been obtained in [22].

A. Bent brane formalism

In the previous section we considered a Gaussian normal coordinate system $(\bar{x}^i, \bar{z})$, which is defined by the hyper-surface $\bar{z} = 0$, and we assumed that the brane is exactly located on the hyper-surface $\bar{z} = 0$. In order to decouple the linearized equations of motion for the metric fluctuations, we went to another Gaussian normal coordinate system (x^i, z), where:

$$x^i = \bar{x}^i + \xi^i, \qquad z = \bar{z} + \xi^5. \tag{45}$$

In these new coordinates the gauge conditions:

$$\eta_{ij}h^{ij} = 0, \quad \text{and} \quad \partial^i h_{ij} = 0 \tag{46}$$

are satisfied. In the case of a source-free brane and bulk, the hyper-surface which defines the new Gaussian normal coordinate system is also described by the equation $z = 0$. However, in the presence of a point mass source on the brane, with energy-momentum tensor:

$$T^{\text{brane}}_{\mu\nu} = S_{\mu\nu}(\bar{x})\delta(\bar{z}), \qquad S_{\mu\nu}(\bar{x}) = M\delta^0_\mu\delta^0_\nu\delta(\bar{\mathbf{x}}), \tag{47}$$

the hyper-surface which defines the new Gaussian normal coordinate system appears to be bent, and it is described by the equation $z = -\hat{\xi}^5(x)$. We emphasize that the choice $\hat{\xi}^5(x) = 0$, when a mass source is present on the brane, is not compatible with the gauge conditions of Eq. (46), so that the bending of the brane is unavoidable in the new coordinate system.

The most general transformations between these two coordinate systems, which obey the conditions $\bar{h}_{\mu5} = \bar{h}_{55} = h_{\mu5} = h_{55} = 0$, are

$$\xi^i = -\eta^{ij}\int d\bar{z}e^{-2A(\bar{z})}\partial_j\hat{\xi}^5(\bar{x}) + \hat{\xi}^j(\bar{x}), \qquad \xi^5 = \hat{\xi}^5(\bar{x}), \tag{48}$$

where the functions $\hat{\xi}^i$ and $\hat{\xi}^5$ are independent from the bulk coordinate $\bar{z}$.

The metric fluctuations in the new coordinate system h_{ij} (around the background metric $(\eta_{ij}e^{2A(z)}, 1)$), and the metric fluctuations in the old coordinate system $\bar{h}_{ij}$ (around the metric $(\eta_{ij}e^{2A(\bar{z})}, 1)$), are related via the equation

$$h_{ij} = \bar{h}_{ij} - \partial_i\hat{\xi}_j - \partial_j\hat{\xi}_i + 2\int d\bar{z}e^{2A(\bar{z})}\partial_i\partial_j\hat{\xi}_5 - 2\eta_{ij}A'(\bar{z})\hat{\xi}^5. \tag{49}$$

In the coordinate system, where the position of the brane is at $\bar{z} = 0$, the junction condition reads:

$$2F(\phi(0))\partial_{\bar{z}}\bar{h}_{ij}|_{\bar{z}=0+} = -\tilde{S}_{ij}(\bar{x}) \quad (\tilde{S}_{\mu\nu} \equiv S_{\mu\nu} - \tfrac{1}{3}g_{\mu\nu}S, \qquad S \equiv S^\mu_\mu). \tag{50}$$

The above equation is obtained from the linearized Einstein Eq. (23) if we include the source term of Eq. (47).

The junction condition for the RS metric in the new coordinate system obtains by combining Eqs. (49) and (50):

$$2F(\phi(0))\partial_z h_{ij}|_{z=0+} = -\tilde{S}_{ij}(x) + 4F(\phi(0))\partial_i\partial_j\hat{\xi}_5. \tag{51}$$

If we had an arbitrary warp factor $\tilde{A}(\bar{z})$, rather than the RS-type warp factor $A(\bar{z}) = -|\bar{z}|/l$, an extra term of the form $-4F(\phi(0))\eta_{ij}\tilde{A}''(0^+)\hat{\xi}^5$ would appear in Eq. (51), as in general the second derivative of $\tilde{A}(\bar{z})$ would not be zero. However, in this case it would be impossible to satisfy simultaneously the "transverse and traceless" conditions of Eq. (46), and as a result the bent brane formalism can not be applied in this case in a straightforward way.

Thus, in the new coordinate system, the linearized equation of motion for the metric fluctuations h_{ij}:

$$e^{2A}F(\phi)\partial_z^2 h_{ij} + e^{2A}(4A'F(\phi) + F'(\phi)\phi')\partial_z h_{ij} + F(\phi)\Box^{(4)}h_{ij} = -\Sigma_{ij}(x)\delta(z) \quad (52)$$

may be rewritten in the form:

$$F(\phi)(\partial_z^2 + Q'\partial_z + e^{-2A}\Box^{(4)})h_{ij} = -\Sigma_{ij}(x)\delta(z), \quad (53)$$

where

$$\Sigma_{ij}(x) = \tilde{S}_{ij}(x) - 4F(\phi(0))\partial_i\partial_j\hat{\xi}_5. \quad (54)$$

If one defines the retarded five-dimensional Green function through the equation:

$$F(\phi)(\partial_z^2 + Q'\partial_z + e^{-2A}\Box^{(4)})\mathcal{G}_5^{(R)}(x, z; x', z') = \delta^{(4)}(x - x')\delta(z - z'), \quad (55)$$

the solution of Eq. (53) can be expressed as

$$h_{ij}(x, z) = -\int d^4x'\mathcal{G}_5^{(R)}(x, z; x', 0)\Sigma_{ij}(x'). \quad (56)$$

The Green function can be expressed in terms of the complete set of eigenstates $e^{ipx}u(m, z)$:

$$\mathcal{G}_5^{(R)}(x, z; x', z') = -\int \frac{d^4p}{(2\pi)^4}e^{ip(x-x')}\left[\frac{u(0, z)u(0, z')}{p^2} + \sum_{m>0}\frac{u(m, z)u(m, z')}{p^2 + m^2}\right], \quad (57)$$

where $u(m, z)$ satisfies the eigenvalue Eq. (30). These eigenfunctions should be normalized according to the equation:

$$\int_{-\infty}^{+\infty} dzu(m, z)^2F(\phi)e^{2A} = 1. \quad (58)$$

In order to precisely define the summation over states in Eq. (57), it is necessary to consider a regulator brane in finite proper distance L, and then send L to infinity.

Since we will concentrate on static solutions, it is convenient to define the five-dimensional Green function for the Laplacian operator:

$$\mathcal{G}_5(\mathbf{x}, z; \mathbf{x}', z') = \int_{-\infty}^{+\infty} dt'\mathcal{G}_5^{(R)}(x, z; x', z'). \quad (59)$$

If we perform the integration over p in Eq. (57) and set $r = |\mathbf{x} - \mathbf{x}'|$, we obtain

$$\mathcal{G}_5(\mathbf{x}, z; \mathbf{x}', z') = -\frac{1}{4\pi r}\left[u(0, z)u(0, z') + \sum_{m>0}u(m, z)u(m, z')e^{-mr}\right]. \quad (60)$$

The metric fluctuation $\bar{h}_{ij}$ on the brane at $\bar{z} = 0$ can be obtained from Eq. (49):

$$\bar{h}_{ij}(x, 0) = h_{ij}(x, 0) + \partial_i\hat{\xi}_j + \partial_j\hat{\xi}_i - 2\left[\int d\bar{z}e^{2A(\bar{z})}\right]_{\bar{z}=0}\partial_i\partial_j\hat{\xi}_5 + 2\eta_{ij}A'(0)\hat{\xi}^5. \quad (61)$$

If we use the remaining gauge freedom and choose $\hat{\xi}^i(x)$ according to

$$\hat{\xi}_i = \partial_i\left(\left[\int d\bar{z}e^{2A(\bar{z})}\right]_{\bar{z}=0}\hat{\xi}_5 - 2F(\phi(0)) \times \int d^3\mathbf{x}'\mathcal{G}_5(\mathbf{x}, 0; \mathbf{x}', 0)\hat{\xi}_5\right) \quad (62)$$

(see Ref. [25]), we obtain a simple expression for the fluctuation $\bar{h}_{ij}(x, 0)$ on the brane:

$$\bar{h}_{ij} = -\int d^3\mathbf{x}'\mathcal{G}_5(\mathbf{x}, 0; \mathbf{x}', 0)\tilde{S}_{ij}(\mathbf{x}') + 2\eta_{ij}A'(0)\hat{\xi}^5. \quad (63)$$

The function $\hat{\xi}^5$ can be determined if take into account that in the new coordinate system the condition $h_i^i = 0$ must be satisfied. Then from Eq. (51) for h_{ij} we obtain that $\hat{\xi}^5$ is a solution of the equation

$$\Box^{(4)}\hat{\xi}^5 = -\frac{1}{12}F(\phi(0))^{-1}S, \quad (64)$$

where $S = S_i^i$. It is not difficult to show that the condition $\partial^i h_{ij} = 0$ is also satisfied.

In the case of a point mass source on the brane, with the energy-momentum tensor given in Eq. (47), we obtain

$$\hat{\xi}^5 = -\frac{M}{48\pi F(\phi(0))}\frac{1}{r}. \quad (65)$$

Using Eqs. (60), (63), and (65) we get

$$\bar{h}_{00} = \frac{Mu(0, 0)^2}{6\pi r} + \frac{M}{6\pi r}\sum_{m>0}u(m, 0)^2e^{-mr} - \frac{M}{24\pi lF(\phi(0))}\frac{1}{r}. \quad (66)$$

Notice that the Newton potential is given by

$$V(r) = \frac{\bar{h}_{00}}{2}.$$

Taking into account that the four-dimensional Newton's constant G_4 is defined by the dimensional reduction equation [26]

$$\int dz d^4x\sqrt{|g|}F(\phi)R + \ldots = \int_{-\infty}^{+\infty} dz F(\phi)e^{2A}\int d^4x\sqrt{|g^{(4)}|}R^{(4)} + \ldots \equiv \frac{1}{16\pi G_4}\int d^4x\sqrt{|g^{(4)}|}R^{(4)} + \ldots \quad (67)$$

we obtain

$$\frac{1}{16\pi G_4} = \int_{-\infty}^{+\infty} dz e^{2A}F(\phi) = \frac{1}{u(0,0)^2}, \quad (68)$$

where use has been made of Eq. (35).

Equation (66) gives

$$\bar{h}_{00} = \frac{2MG_4(1+\alpha_F)}{r} + \frac{8MG_4}{3r}\sum_{m>0}\frac{u(m,0)^2}{u(0,0)^2}e^{-mr}, \quad (69)$$

where

$$\alpha_F = \frac{1}{3}\left(1 - \frac{\int_{-\infty}^{+\infty} dz e^{2A}F(\phi)}{lF(\phi(0))}\right) = \frac{\xi k\phi(0)^2(-1 + \int_{-\infty}^{+\infty} d\hat{z}e^{2A}\frac{\phi(\hat{z})^2}{\phi(0)^2})}{3(1-\xi k\phi(0)^2)}, \quad \text{for } \phi(0)\neq 0, \quad (70)$$

$$\alpha_F = \frac{\xi k}{3}\int_{-\infty}^{+\infty} d\hat{z}e^{2A}\phi(\hat{z})^2, \quad \text{for } \phi(0) = 0. \quad (71)$$

We have used the following notation: $\hat{z}\equiv\frac{z}{l}$. We point out that the first term in Eq. (69), namely, the Newtonian potential, arises as a combination of two contributions, the first one is that of the zero mode, while the second one is due to the brane bending term of Eq. (65).

The second term, on the other hand, involving the tower of the massive states, gives rise to corrections to Newton's law. We can use a second regulator brane in order to express the summation over states as an integral. Now, use of dimensional analysis indicates that this term tends to zero for $r \gg l$ (where l is the AdS_5 radius), while for small r ($r \ll l$) the corrections become very important;, for example, for $r \ll l$ they should modify the potential to its five-dimensional version, namely $\frac{1}{r^2}$. However, precise knowledge of the corrections presupposes knowledge of the eigenfunctions. This can be done analytically for the RS2-model, but in our case only numerical calculations are possible.

We would like to emphasize that in this paper we will severely narrow the acceptable models; as a result the corrections of Newton's law are expected to be quite similar to those of the RS2-model, which have already been found analytically [12]. For this reason we will not attempt a numerical computation of the summation over the massive states.

B. Zero mode truncation

For large distances the dominant part of the five-dimensional Green function, for $|\mathbf{x}-\mathbf{x}'| \gg l$, is due to the contribution of the zero mode part, as the remaining part is suppressed by an $O(l)$ factor. Thus, if we neglect the contribution of the continuous modes, which is of the order of $O(l)$, we obtain

$$\mathcal{G}_5(\mathbf{x},0;\mathbf{x}',0) = u(0,0)^2\mathcal{G}_4(\mathbf{x},\mathbf{x}'), \qquad \Box^{(4)}_{\mathbf{x}}\mathcal{G}_4(\mathbf{x},\mathbf{x}') = \delta^{(4)}(\mathbf{x}-\mathbf{x}'). \quad (72)$$

From Eqs. (63), (64), and (72) we derive the equation that should be obeyed by the gravitational perturbation $\bar{h}_{ij}$:

$$\Box^{(4)}\bar{h}_{ij} = -16\pi G_4(S_{ij} - \tfrac{1}{2}\eta_{ij}S) - 8\pi G_4\alpha_F\eta_{ij}S. \quad (73)$$

We observe that the above equation is somewhat different from the linearized equation of standard 4D general relativity:

$$\Box^{(4)}\bar{h}_{ij} = -16\pi G_4(S_{ij} - \tfrac{1}{2}\eta_{ij}S). \quad (74)$$

Note that this situation, of modified linearized equations on the brane, is quite similar with the case of RS1-model; see Ref. [9].

For a pointlike source, the solutions of Eq. (73) read:

$$\bar{h}_{00} = \frac{2(1+\alpha_F)MG_4}{r}, \qquad \bar{h}_{ij} = \frac{2(1-\alpha_F)MG_4}{r}\eta_{ij}. \quad (75)$$

Absorbing the factor $(1+\alpha_F)$ into the mass M (see Ref. [27]), the solution can take the form:

$$\bar{h}_{00} = \frac{2MG_4}{r}, \qquad \bar{h}_{ij} = \gamma_F\frac{2MG_4}{r}\eta_{ij}, \quad (76)$$

where

$$\gamma_F = \frac{1-\alpha_F}{1+\alpha_F}. \quad (77)$$

We recall that the standard isotropic form of the metric can be expanded to yield the expressions:

$$\bar{h}_{00} = \left(-1 + \frac{2MG_4}{r} - \beta\frac{2M^2G_4^2}{r^2} + \ldots\right), \qquad \bar{h}_{ij} = \left(1 + \gamma\frac{2MG_4}{r} + \ldots\right)\eta_{ij}, \quad i,j = 1,2,3. \quad (78)$$

giving a fairly general version of the metric [27]. Recalling the Schwarzschild solution, one may check that the standard Einstein equations predict $\beta = 1$, $\gamma = 1$.

The expansion (78) can also represent alternative gravity theories. For instance, the Brans-Dicke theory would yield $\alpha = 1$, $\beta = 1$, $\gamma = \frac{\omega_{BD}+1}{\omega_{BD}+2}$, where ω_{BD} is the Brans-Dicke parameter. In our case we have not yet calculated the parameter β, since that would require second order perturbation theory [28] and is deferred to a future publication. If we assume that $\beta = 1$, our result (77) is compatible with a four-dimensional Brans-Dicke theory, hence we obtain that the Brans-Dicke parameter is

$$\omega_{\rm BD} = \frac{1}{2\alpha_F} - \frac{3}{2}. \tag{79}$$

A lower bound on $\omega_{\rm BD}$, necessary for consistency with solar system measurements, which appears in the literature [29] reads: $\omega_{\rm BD} \geq 500$. Then, with the help of Eq. (79) this restriction becomes

$$0 \leq \alpha_F \leq 10^{-3}, \tag{80}$$

or, equivalently,

$$0 \leq \frac{\xi k \phi(0)^2(-1 + \int_{-\infty}^{+\infty} d\hat{z} e^{2A} \frac{\phi(\hat{z})^2}{\phi(0)^2})}{3(1 - \xi k \phi(0)^2)} \leq 10^{-3}, \quad \text{for } \phi(0) \neq 0, \tag{81}$$

$$0 \leq \frac{\xi k}{3} \int_{-\infty}^{+\infty} d\hat{z} e^{2A} \phi(\hat{z})^2 \leq 10^{-3}, \quad \text{for } \phi(0) = 0. \tag{82}$$

We conclude that static solutions of brane models with nonminimal coupling, with a warp factor of the RS type $A(z) = -|z|/l$, *are acceptable only if they satisfy the conditions* (81) or (82), otherwise we cannot recover general relativity on the brane. In other publications [30], the lower bound for $\omega_{\rm BD}$ is set to even larger numbers. If these numbers are adopted, the restrictions for our model will become even stronger.

A complete investigation of the restriction (80) is beyond the scope of this paper, as the analytical static solutions of the model depend on a large number of free parameters [22]. However, one may describe two circumstances, in which Eq. (81) is satisfied: (a) If $|-1 + \int_{-\infty}^{+\infty} d\hat{z} e^{2A} \frac{\phi(\hat{z})^2}{\phi(0)^2}| \sim 1$, then it is necessary that $|\xi| k \phi(0)^2 \sim 10^{-3}$. (b) If Eq. (81) is to be satisfied for $|\xi| k \phi(0)^2$ of order one, then another type of fine tuning appears, namely, one should have $|-1 + \int_{-\infty}^{+\infty} d\hat{z} e^{2A} \frac{\phi(\hat{z})^2}{\phi(0)^2}| \sim 10^{-3}$. We observe that Eq. (81) is satisfied only if the quantities $(-1 + \int_{-\infty}^{+\infty} d\hat{z} e^{2A} \frac{\phi(\hat{z})^2}{\phi(0)^2})$ and $\xi k \phi(0)^2$ have the same sign. Numerical study of the analytical solutions contained in Ref. [22], indicates that these models fall into case (a).

We would like to emphasize that in other publications [30], the lower bound for $\omega_{\rm BD}$ is set to even larger numbers, for example $\omega_{\rm BD} > 10^5$. If these numbers are adopted, the restrictions for our model will become even stronger, and as a result the class of acceptable models will be severely narrow, and quite closely to RS2-model.

VI. CONCLUSIONS

We have studied gravitational perturbations for a category of brane models involving a scalar field nonminimally coupled with gravity. The focus of our work has been on the study of the resulting gravitational potential, in particular its comparison against the Newtonian one: we considered the effect of a point mass at rest on the brane and used the bent brane formalism. It turned out that the perturbation $\bar{h}_{ij}$ belongs to a more general class of metrics, also including the Brans-Dicke theory. Observational constraints yield restrictions on the model, which deserve further investigation to determine the region of the parameter space which is relevant for model building. These restrictions constitute a test, which should be passed before any theory belonging to this category is further considered.

As we mentioned in previous sections, in the case of the bent brane formalism the gauge is fixed from the beginning. In particular, we have chosen a specific gauge ($h_{5\mu} = 0$), which holds in Gaussian normal coordinates. The remaining gauge freedom [see Eq. (49)] can be used for imposing the transverse and traceless conditions, where the equations of motion decouple. However, one could work in a gauge invariant formalism as is done in Ref. [31]. In this case graviphotons, graviscalar and the perturbation of the bulk scalar field are mixed in complicated equations of motion. Of course, the gauge choice in our paper is not enough to cover completely this general case. New solutions may exist, where the contribution of these additional fields is significant.

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APPENDIX: EINSTEIN FRAME

In this appendix we derive the linearized equation for metric fluctuations by working in the Einstein Frame, rather than the Jordan frame which has mainly been used in this paper. In particular, we show that the descriptions of the metric fluctuations in the two frames are equivalent.

Using the conformal transformation

$$\tilde{g}_{\mu\nu} = \omega^2(x) g_{\mu\nu}, \tag{A1}$$

one may transform the action of Eq. (2) to a minimally coupled form. We refer to $g_{\mu\nu}$ as the Jordan frame metric, while $\tilde{g}_{\mu\nu}$ is the corresponding metric in the Einstein frame.

In this paper we have studied tensor fluctuations in the Jordan frame, and we determined the corresponding linearized Eq. (37). However, one might transform Eq. (A1) to the Einstein frame, where the action is reduced to a minimally coupled form. The derivation of the linearized Einstein equations has already been done previously (see, for example, Ref. [8]).

In the Einstein frame we assume the conformal background metric with fluctuations of the form:

$$ds_{\rm EF}^2 = e^{2\tilde{B}(w)}((\eta_{ij} + h_{ij})dx^i dx^j + dw^2). \tag{A2}$$

If we set

$$h_{ij}(x, w) = e^{ipx}\tilde{u}(m, w), \tag{A3}$$

we obtain the linearized equation for the metric fluctuations (see, for example, Ref. [8], where a minimally coupled model is examined)

$$(\partial_w^2 + 3\tilde{B}'(w)\partial_w + m^2)\tilde{u}(m, w) = 0. \tag{A4}$$

The question that arises is whether our results in the Jordan frame are compatible with the corresponding results in the Einstein frame (A4).

If we just consider the gravity part of the action and impose the condition

$$S = \int d^5x\sqrt{|g|}(F(\phi)R + \ldots) = \int d^5x\sqrt{|\tilde{g}|}(\tilde{R} + \ldots), \tag{A5}$$

taking into account that

$$\tilde{g}_{\mu\nu} = \omega^2(x)g_{\mu\nu}, \qquad \tilde{R} = \omega^{-2}(x)R, \qquad \sqrt{|\tilde{g}|} = \omega^5(x)\sqrt{|g|}, \tag{A6}$$

we obtain

$$\omega^2(x) = (F(\phi))^{2/3}. \tag{A7}$$

Assuming that the conformal background metric in the Jordan frame has the form

$$ds_{\rm JF}^2 = e^{2\tilde{A}(w)}((\eta_{ij} + h_{ij})dx^i dx^j + dw^2) \tag{A8}$$

then in the Einstein frame, with the help of Eqs. (A1) and (A7), we obtain:

$$ds_{\rm EF}^2 = (F(\phi))^{2/3}e^{2\tilde{A}(w)}((\eta_{ij} + h_{ij})dx^i dx^j + dw^2), \tag{A9}$$

or, equivalently,

$$\tilde{B}(w) = \tilde{A}(w) + \tfrac{1}{3}\ln(F(\phi)) \tag{A10}$$

If we substitute the above equation in (A4), we obtain

$$(\partial_w^2 + \tilde{Q}'(w)\partial_w + m^2)\tilde{u}(m, w) = 0, \tag{A11}$$

where

$$\tilde{Q}(w) = 3\tilde{A}(w) + \ln(F(\tilde{\phi}(w))). \tag{A12}$$

which coincides with Eq. (37), proving that the descriptions of the tensor fluctuations in the Jordan and the Einstein frames are equivalent.

[1] N. Arkani-Hamed, S. Dimopoulos, and G. Dvali, Phys. Lett. B **429**, 263 (1998); I. Antoniadis, N. Arkani-Hamed, S. Dimopoulos, and G. Dvali, Phys. Lett. B **436**, 257 (1998).

[2] L. Randall and R. Sundrum, Phys. Rev. Lett. **83**, 3370 (1999); **83**, 4690 (1999).

[3] K. Akama, in *Proceedings on Gauge Theory and Gravitation, Nara, 1982*, Lect. Notes Phys. vol. 176, edited by K. Kikkawa, N. Nakanishi, and H. Nariai (Springer-Verlag, Berlin, 1983), p. 267–271; Prog. Theor. Phys. **60**, 1900 (1978); **78**, 184 (1987); **79**, 1299 (1988); **80**, 935 (1988).

[4] V. A. Rubakov and M. E. Shaposhnikov, Phys. Lett. B **125**, 136 (1983); **125**, 139 (1983).

[5] G. Dvali and M. Shifman, Phys. Lett. B **396**, 64 (1997).

[6] V. A. Rubakov, Phys. Usp. **44**, 871 (2001).

[7] P. Dimopoulos, K. Farakos, A. Kehagias, and G. Koutsoumbas, Nucl. Phys. **B617**, 237 (2001); P. Dimopoulos, K. Farakos, and G. Koutsoumbas, Phys. Rev. D **65**, 074505 (2002); P. Dimopoulos and K. Farakos, Phys. Rev. D **70**, 045005 (2004); A. Hulsebos, C. P. Korthals-Altes, and S. Nicolis, Nucl. Phys. **B450**, 437 (1995); M. Laine, H. B. Meyer, K. Rummukainen, and M. Shaposnikov, J. High Energy Phys. 04 (2004) 027; P. Dimopoulos, K. Farakos, and S. Vrentzos, Phys. Rev. D **74**, 094506 (2006).

[8] O. DeWolfe, D. Freedman, S. Gubser, and A. Karch, Phys. Rev. D **62**, 046008 (2000).

[9] J. Garriga and T. Tanaka, Phys. Rev. Lett. **84**, 2778 (2000).

[10] S. B. Giddings, E. Katz, and L. Randall, J. High Energy Phys. 03 (2000) 023.

[11] R. Arnowitt and J. Dent, talk given at the 10th International Symposium on Particles, Strings and Cosmology (PASCOS 04 and Pran Nath Fest), Boston, Massachusetts, 2004; Phys. Rev. D **75**, 064001 (2007).

[12] P. Callin and F. Ravndal, Phys. Rev. D **70**, 104009 (2004).

[13] W. Mueck, K. S. Viswanathan, and I. V. Volovich, Phys. Rev. D **62**, 105019 (2000).

[14] Mikhail N. Smolyakov and Igor P. Volobuev, arXiv:hep-th/0208025.

[15] K. A. Bronnikov, S. A. Kononogov, and V. N. Melnikov, Gen. Relativ. Gravit. **38**, 1215 (2006).

[16] Abdel Perez-Lorenzana, J. Phys. Conf. Ser. **18**, 224 (2005); Roy Maartens, Living Rev. Relativity **7**, 7 (2004).

[17] N. Arkani-Hamed, S. Dimopoulos, N. Kaloper, and R. Sundrum, Phys. Lett. B **480**, 193 (2000); N. M. Mavromatos and J. Rizos, Phys. Rev. D **62**, 124004 (2000); A. Kehagias and K. Tamvakis, Phys. Lett. B **504**, 38 (2001); M. Giovannini, Phys. Rev. D **64**, 124004 (2001); **75**, 064023 (2007); **74**, 087505 (2006); Pierre Binetruy, Christos Charmousis, Stephen C Davis, and Jean-Francois Dufaux, Phys. Lett. B **544**, 183 (2002); Christos Charmousis, Stephen C. Davis, and Jean-

Francois Dufaux, J. High Energy Phys. 12 (**2003**) 029; A. Papazoglou, arXiv:hep-ph/0112159; Nandinii Barbosa-Cendejas and Alfredo Herrera-Aguilar, J. High Energy Phys. 10 (2005) 101; Phys. Rev. D **73**, 084022 (2006); Alexey S. Mikhailov, Yuri S. Mikhailov, Mikhail N. Smolyakov, and Igor P. Volobuev, Classical Quantum Gravity **24**, 231 (2007).

[18] K. Farakos and P. Pasipoularides, Phys. Lett. B **621**, 224 (2005).

[19] K. Farakos and P. Pasipoularides, Phys. Rev. D **73**, 084012 (2006).

[20] K. Farakos and P. Pasipoularides, in Proceedings of Brane world scenario in the presence of a non-minimally coupled bulk scalar field,Nafplio NEB XII, 2006 (unpublished).

[21] K. Farakos and P. Pasipoularides, Phys. Rev. D **75**, 024018 (2007).

[22] C. Bogdanos, A. Dimitriadis, and K. Tamvakis, Phys. Rev. D **74**, 045003 (2006).

[23] C. Bogdanos, A. Dimitriadis, and K. Tamvakis, arXiv:hep-th/0611181.

[24] Csaba Csaki, Joshua Erlich, Timothy J. Hollowood, and Yuri Shirman, Nucl. Phys. **B581**, 309 (2000).

[25] Csaba Csaki, Joshua Erlich, and Timothy J. Hollowood, Phys. Lett. B **481**, 107 (2000).

[26] N. M. Mavromatos and J. Rizos, Int. J. Mod. Phys. A **18**, 57 (2003).

[27] S. Weinberg, *Gravitation and Cosmology: Principles and Aplications of the General Theory of Relativity* (John Wiley and Sons, New York, 1972).

[28] Hideaki Kudoh and Takahiro Tanaka, Phys. Rev. D **64**, 084022 (2001); **65**, 104034 (2002).

[29] S. M. Carroll, *Spacetime and Geometry* (Addison Wesley, Reading, MA, 2004).

[30] C. M.Will, arXiv:gr-qc/9811036; C. M. Will and N. Yunes, Classical Quantum Gravity **21**, 4367 (2004).

[31] M. Giovannini, Phys. Rev. D **64**, 064023 (2001); **65**, 064008 (2002).