Nonadiabatic charged spherical gravitational collapse

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We present a complete set of the equations and matching conditions required for the description of physically meaningful charged, dissipative, spherically symmetric gravitational collapse with shear. Dissipation is described with both free-streaming and diffusion approximations. The effects of viscosity are also taken into account. The roles of different terms in the dynamical equation are analyzed in detail. The dynamical equation is coupled to a causal transport equation in the context of Israel-Stewart theory. The decrease of the inertial mass density of the fluid, by a factor which depends on its internal thermodynamic state, is reobtained, with the viscosity terms included. In accordance with the equivalence principle, the same decrease factor is obtained for the gravitational force term. The effect of the electric charge on the relation between the Weyl tensor and the inhomogeneity of the energy density is discussed.

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I. INTRODUCTION

The study of self-gravitating spherically symmetric charged fluid distributions has a long and a venerable history, starting with Rosseland and Eddington's contributions [1,2]. Since then a large number of works have been dedicated to making manifest the influence of electric charge on the structure and evolution of self-gravitating systems (see [1–42] and references therein).

Although some of these works refer to static situations ([3–5,9,19,23,24,29,30,32–34,36,37,39,41]) there have been important efforts in describing dynamical situations too ([6–8,10–14,16,17,21,22,26,40,42]). Particularly relevant for the present paper are Refs. [14,22,40].

A renewed interest in this subject emerges from the appearance of new mechanisms allowing for the presence of huge electric charge in self-gravitating systems. From simple classical considerations, it can be shown that physical objects with large amounts of charge (much larger than 100 C per solar mass) cannot exist [1,2,31]. Furthermore, as shown by Bekenstein [14] the electric charge is bounded by the fact that the resulting electric field should not exceed the critical field for pair creation, 10^{16} V cm^{-1}. However, these restrictions have been questioned by several authors [15,18,20,37]. Particularly appealing is the possibility of very high electric fields in strange stars with quark matter (see [38,39] and references therein).

All this having been said, it should be clear that the restrictions mentioned above refer to equilibrium (stable) configurations. They do not apply to phases of intense dynamical activity with time scales of the order of (or even smaller than) the hydrostatic time scale, and for which the quasistatic approximation is clearly not reliable (e.g. the collapse of very massive stars [43] or the quick collapse phase preceding neutron star formation; see for example [44] and references therein). The description of this very dynamic regime is the main purpose of this manuscript.

Besides electric charge, which will be assumed to comove with the fluid, we shall also consider dissipative phenomena. It is already an established fact that gravitational collapse is a highly dissipative process (see [45–47] and references therein), so the relevance of dissipation in its study cannot be over-emphasized. Dissipation due to the emission of massless particles, photons, and/or neutrinos is a characteristic process in the evolution of massive stars. In fact, it seems that the only plausible mechanism for carrying away the bulk of the binding energy of a star collapsing to a neutron star or black hole is neutrino emission [48].

In the diffusion approximation, it is assumed that the energy flux of radiation, like that of thermal conduction, is proportional to the gradient of temperature. This assumption is in general very sensible, since the mean free path of particles responsible for the propagation of energy in stellar interiors is normally very small compared with the typical length of the object. Thus, for a main sequence star such as the sun, the mean free path of photons at the center is of the order of 2 cm. Also, the mean free path of trapped neutrinos in compact cores of densities above about 10^{12} g cm^{-3} becomes smaller than the size of the stellar core [49,50].

Furthermore, the observational data collected from supernova 1987A indicates that the regime of radiation transport prevailing during the emission process is closer to the diffusion approximation than to the free-streaming limit [51].

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However, in many other circumstances the mean free path of particles transporting energy may be large enough to justify the free-streaming approximation. Therefore it is advisable to include simultaneously both limiting cases of radiative transport, diffusion, and free streaming, allowing us, by taking both in combination, to describe a wide range of situations.

The effects of dissipation, in both limiting cases of radiative transport, within the context of the quasistatic approximation, have been studied in [52]. Using this approximation is very sensible because the hydrostatic time scale is very small, compared with stellar lifetimes, for many phases of the life of a star. It is of the order of 27 min for the sun, 4.5 sec for a white dwarf, and 10^{-4} sec for a neutron star of one solar mass and 10 km radius [53]. However, such an approximation does not apply to the very dynamic phases mentioned before. In those cases it is mandatory to take into account terms which describe departure from equilibrium, i.e. a full dynamic description has to be used.

For the sake of generality, we have considered a locally anisotropic fluid. In fact, the assumption of local anisotropy of pressure, which seems to be very reasonable for describing the matter distribution under a variety of circumstances, has been proved to be very useful in the study of relativistic compact objects (see [46,54] and references therein).

Finally we have also included viscous effects in our study. In fact, though they are generally excluded in general relativistic models of stars, they are known to play a very important role in the structure and evolution of neutron stars. Indeed, depending on the dominant process, the coefficient of shear viscosity may be as large as $\eta \approx 10^{20}\ \mathrm{g\,cm^{-1}\,s^{-1}}$ (see [55] for a review on shear viscosity in neutron stars). Also, a theorem by Raychaudhuri and De [56], which states that in the evolution of nondissipative charged dust the shear cannot vanish, emphasizes the relevance of the shear in the evolution of charged fluids.

On the other hand the coefficient of bulk viscosity may be as large as $10^{30}\ \mathrm{g\,cm^{-1}\,s^{-1}}$ due to Urca processes in strange quark matter [57] (see also [58] for a review on bulk viscosity in nuclear and quark matter).

II. THE ENERGY-MOMENTUM TENSOR AND THE FIELD EQUATIONS

In this section we provide a full description of the matter distribution, the line element, both inside and outside the fluid boundary, and the field equations this line element must satisfy.

A. Interior spacetime

We consider a spherically symmetric distribution of collapsing charged fluid, bounded by a spherical surface Σ: we assume the fluid to be locally anisotropic and undergoing dissipation in the form of heat flow, free-streaming radiation, and shearing viscosity. For short we call this "matter." Choosing comoving coordinates inside Σ, the general interior metric can be written

$$ds^2_- = -A^2dt^2 + B^2dr^2 + (Cr)^2(d\theta^2 + \sin^2\theta d\phi^2), \quad (1)$$

where A, B, and C are functions of t and r and are assumed positive. We number the coordinates $x^0 = t$, $x^1 = r$, $x^2 = \theta$, and $x^3 = \phi$.

The assumed matter energy-momentum $T^-_{\alpha\beta}$ inside Σ has the form

$$T^-_{\alpha\beta} = (\mu + P_\perp)V_\alpha V_\beta + P_\perp g_{\alpha\beta} + (P_r - P_\perp)\chi_\alpha\chi_\beta + q_\alpha V_\beta + V_\alpha q_\beta + \epsilon l_\alpha l_\beta - 2\eta\sigma_{\alpha\beta}, \quad (2)$$

where μ is the energy density, P_r the radial pressure, $P_\perp$ the tangential pressure, q^α the heat flux, ϵ the radiation density, η the coefficient of shear viscosity, V^α the four velocity of the fluid, χ^α a unit four vector along the radial direction, and l^α a radial null four vector. These quantities satisfy

$$V^\alpha V_\alpha = -1, \qquad V^\alpha q_\alpha = 0, \qquad \chi^\alpha\chi_\alpha = 1,$$
$$\chi^\alpha V_\alpha = 0, \qquad l^\alpha V_\alpha = -1, \qquad l^\alpha l_\alpha = 0, \quad (3)$$

and the shear $\sigma_{\alpha\beta}$ is given by

$$\sigma_{\alpha\beta} = V_{(\alpha;\beta)} + a_{(\alpha}V_{\beta)} - \frac{1}{3}\Theta(g_{\alpha\beta} + V_\alpha V_\beta), \quad (4)$$

where the acceleration a_α and the expansion Θ are given by

$$a_\alpha = V_{\alpha;\beta}V^\beta, \qquad \Theta = V^\alpha{}_{;\alpha}. \quad (5)$$

We do not explicitly add bulk viscosity to the system because it can be absorbed into the radial and tangential pressures, P_r and $P_\perp$, of the collapsing fluid [59].

It should be noted that for a physically meaningful specific model we would need constitutive equations which would relate and determine the quantities μ, P_r, $P_\perp$, q^α, ϵ, and η. Without such relations we still have so many free functions that nothing useful can be said about the behavior of an individual case. However, we will show that some important general physical results follow just from assuming, for example, that dissipation carries energy radially outwards.

Since we assumed the metric (1) comoving then

$$V^\alpha = A^{-1}\delta^\alpha_0, \qquad q^\alpha = qB^{-1}\delta^\alpha_1,$$
$$l^\alpha = A^{-1}\delta^\alpha_0 + B^{-1}\delta^\alpha_1, \qquad \chi^\alpha = B^{-1}\delta^\alpha_1, \quad (6)$$

where q is a function of t and r. With (6) we obtain for (4) its non-null components

$$\sigma_{11} = \frac{2}{3}B^2\sigma, \qquad \sigma_{22} = \frac{\sigma_{33}}{\sin^2\theta} = -\frac{1}{3}(Cr)^2\sigma, \quad (7)$$

where

$$\sigma = \frac{1}{A}\left(\frac{\dot{B}}{B} - \frac{\dot{C}}{C}\right), \tag{8}$$

and the dot stands for differentiation with respect to t, which gives the scalar quantity

$$\sigma_{\alpha\beta}\sigma^{\alpha\beta} = \frac{2}{3}\sigma^2. \tag{9}$$

For (5) with (6) we have

$$a_1 = \frac{A'}{A}, \qquad \Theta = \frac{1}{A}\left(\frac{\dot{B}}{B} + 2\frac{\dot{C}}{C}\right), \tag{10}$$

where the prime stands for r differentiation.

B. The electromagnetic energy tensor and the Maxwell equations

The electromagnetic energy tensor $E^-_{\alpha\beta}$ is given by

$$E^-_{\alpha\beta} = \frac{1}{4\pi}\left(F_\alpha{}^\gamma F_{\beta\gamma} - \frac{1}{4}F^{\gamma\delta}F_{\gamma\delta}g_{\alpha\beta}\right), \tag{11}$$

where $F_{\alpha\beta}$ is the electromagnetic field tensor. Maxwell's equations can be written

$$F_{\alpha\beta} = \phi_{\beta,\alpha} - \phi_{\alpha,\beta}, \tag{12}$$

$$F^{\alpha\beta}{}_{;\beta} = 4\pi J^\alpha, \tag{13}$$

where ϕ_α is the four potential and J_α is the four current. Since the charge is assumed to be at rest with respect to the coordinate system used in (1), there is no magnetic field present in this local coordinate system, and therefore we can write

$$\phi_\alpha = \Phi\delta^0_\alpha, \qquad J^\alpha = \varsigma V^\alpha, \tag{14}$$

where ς, the charge density, and Φ are both functions of t and r. Charge conservation implies that

$$s(r) = 4\pi\int_0^r \varsigma B(Cr)^2 dr, \tag{15}$$

which is the electric charge interior to radius r, is time independent.

With (1) and (6) we obtain for the Maxwell Eqs. (12) and (13)

$$\Phi'' - \left(\frac{A'}{A} + \frac{B'}{B} - 2\frac{C'}{C} - \frac{2}{r}\right)\Phi' = 4\pi\varsigma AB^2, \tag{16}$$

$$\dot{\Phi}' - \left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} - 2\frac{\dot{C}}{C}\right)\Phi' = 0. \tag{17}$$

Integrating (16) and (17) produces

$$\Phi' = \frac{sAB}{(Cr)^2}. \tag{18}$$

C. The Einstein equations

Einstein's field equations for the interior spacetime (1) are given by

$$G^-_{\alpha\beta} = 8\pi(T^-_{\alpha\beta} + E^-_{\alpha\beta}). \tag{19}$$

The non-null components of (19) with (1), (2), (6), (11), and (18) become

$$\begin{aligned} 8\pi(T^-_{00} + E^-_{00}) &= 8\pi(\mu + \epsilon)A^2 + \frac{(sA)^2}{(Cr)^4} \\ &= \left(2\frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right)\frac{\dot{C}}{C} + \left(\frac{A}{B}\right)^2\left\{-2\frac{C''}{C}\right. \\ &\quad + \left(2\frac{B'}{B} - \frac{C'}{C}\right)\frac{C'}{C} + \frac{2}{r}\left(\frac{B'}{B} - 3\frac{C'}{C}\right) \\ &\quad \left. - \left[1 - \left(\frac{B}{C}\right)^2\right]\frac{1}{r^2}\right\}, \end{aligned} \tag{20}$$

$$\begin{aligned} 8\pi(T^-_{01} + E^-_{01}) &= -8\pi(q + \epsilon)AB \\ &= -2\left(\frac{\dot{C}'}{C} - \frac{\dot{B}}{B}\frac{C'}{C} - \frac{\dot{C}}{C}\frac{A'}{A}\right) + \frac{2}{r}\left(\frac{\dot{B}}{B} - \frac{\dot{C}}{C}\right), \end{aligned} \tag{21}$$

$$\begin{aligned} 8\pi(T^-_{11} + E^-_{11}) &= 8\pi\left(P_r + \epsilon - \frac{4}{3}\eta\sigma\right)B^2 - \frac{(sB)^2}{(Cr)^4} \\ &= -\left(\frac{B}{A}\right)^2\left[2\frac{\ddot{C}}{C} + \left(\frac{\dot{C}}{C}\right)^2 - 2\frac{\dot{A}}{A}\frac{\dot{C}}{C}\right] \\ &\quad + \left(\frac{C'}{C}\right)^2 + 2\frac{A'}{A}\frac{C'}{C} + \frac{2}{r}\left(\frac{A'}{A} + \frac{C'}{C}\right) \\ &\quad + \left[1 - \left(\frac{B}{C}\right)^2\right]\frac{1}{r^2}, \end{aligned} \tag{22}$$

$$\begin{aligned} 8\pi(T^-_{22} + E^-_{22}) &= \frac{8\pi}{\sin^2\theta}(T^-_{33} + E^-_{33}) \\ &= 8\pi\left(P_\perp + \frac{2}{3}\eta\sigma\right)(Cr)^2 + \left(\frac{s}{Cr}\right)^2 \\ &= -\left(\frac{Cr}{A}\right)^2\left[\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} - \frac{\dot{A}}{A}\left(\frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right) + \frac{\dot{B}}{B}\frac{\dot{C}}{C}\right] \\ &\quad + \left(\frac{Cr}{B}\right)^2\left[\frac{A''}{A} + \frac{C''}{C} - \frac{A'}{A}\left(\frac{B'}{B} - \frac{C'}{C}\right)\right. \\ &\quad \left. - \frac{B'}{B}\frac{C'}{C} + \frac{1}{r}\left(\frac{A'}{A} - \frac{B'}{B} + 2\frac{C'}{C}\right)\right]. \end{aligned} \tag{23}$$

The component (21) can be rewritten with (8) and (10) as

$$4\pi(q + \epsilon)B = \frac{1}{3}(\Theta - \sigma)' - \sigma\frac{(Cr)'}{Cr}. \tag{24}$$

Next, the mass function $m(t, r)$ introduced by Misner and Sharp [60] (see also [61]) can be generalized to include the electromagnetic contribution by

$$m = \frac{(Cr)^3}{2} R_{23}{}^{23} + \frac{s^2}{2Cr} = \frac{Cr}{2}\left\{\left(\frac{r\dot{C}}{A}\right)^2 - \left[\frac{(Cr)'}{B}\right]^2 + 1\right\} + \frac{s^2}{2Cr}, \qquad (25)$$

which is the same mass function used in [14,22].

D. The exterior spacetime and junction conditions

Outside Σ we assume we have the Reissner-Nordström-Vaidya spacetime (i.e. we assume all outgoing radiation is massless), described by

$$ds^2 = -\left[1 - \frac{2M(v)}{r} + \frac{Q^2}{r^2}\right]dv^2 - 2drdv + r^2(d\theta^2 + r^2 \sin\theta^2 d\phi^2), \qquad (26)$$

where $M(v)$ and Q denote the total mass and charge, respectively, and v is the retarded time.

The junction conditions for the smooth matching of an adiabatic charged sphere to the Reissner-Nordström spacetime were discussed in detail in [22,40], whereas the matching of the full nonadiabatic sphere (including viscosity) to the Vaidya spacetime was discussed in [62]. The result is that the matching of (1) and (26) on Σ implies

$$P_r + 4\eta\sigma \overset{\Sigma}{=} q, \qquad m(t, r) \overset{\Sigma}{=} M(v), \qquad s \overset{\Sigma}{=} Q, \qquad (27)$$

where $\overset{\Sigma}{=}$ means that both sides of the equation are evaluated on Σ.

III. DYNAMICAL EQUATIONS

The nontrivial components of the Bianchi identities, $(T^{-\alpha\beta} + E^{-\alpha\beta})_{;\beta} = 0$, from (2) and (11) yield

$$(T^{-\alpha\beta} + E^{-\alpha\beta})_{;\beta} V_\alpha = -\frac{1}{A}(\dot{\mu} + \dot{\epsilon}) - \left(\mu + P_r + 2\epsilon - \frac{4}{3}\eta\sigma\right)\frac{\dot{B}}{AB} - 2\left(\mu + P_\perp + \epsilon + \frac{2}{3}\eta\sigma\right)\frac{\dot{C}}{AC} - \frac{1}{B}(q + \epsilon)' - 2(q + \epsilon)\frac{(ACr)'}{ABCr} = 0, \qquad (28)$$

$$(T^{-\alpha\beta} + E^{-\alpha\beta})_{;\beta} \chi_\alpha = \frac{1}{A}(\dot{q} + \dot{\epsilon}) + \frac{1}{B}\left(P_r + \epsilon - \frac{4}{3}\eta\sigma\right)' + 2(q + \epsilon)\frac{\dot{B}}{AB} + 2(q + \epsilon)\frac{\dot{C}}{AC} + \left(\mu + P_r + 2\epsilon - \frac{4}{3}\eta\sigma\right)\frac{A'}{AB} + 2(P_r - P_\perp + \epsilon - 2\eta\sigma)\frac{(Cr)'}{BCr} - \frac{ss'}{4\pi B(Cr)^4} = 0, \qquad (29)$$

where we used (15) and (18).

To study the dynamical properties of the system, let us introduce, following Misner and Sharp [60], the proper time derivative D_T given by

$$D_T = \frac{1}{A}\frac{\partial}{\partial t}, \qquad (30)$$

and the proper radial derivative D_R,

$$D_R = \frac{1}{R'}\frac{\partial}{\partial r}, \qquad (31)$$

where

$$R = Cr \qquad (32)$$

defines the proper radius of a spherical surface inside Σ, as measured from its area.

Using (30) we can define the velocity U of the collapsing fluid as the variation of the proper radius with respect to proper time, i.e.

$$U = rD_T C < 0 \qquad \text{(in the case of collapse)}. \qquad (33)$$

Then (25) can be rewritten as

$$E \equiv \frac{(Cr)'}{B} = \left[1 + U^2 - \frac{2m(t, r)}{Cr} + \left(\frac{s}{Cr}\right)^2\right]^{1/2}. \qquad (34)$$

With (31) and (32) we can express (24) as

$$4\pi(q + \epsilon) = E\left[\frac{1}{3}D_R(\Theta - \sigma) - \frac{\sigma}{R}\right]. \qquad (35)$$

Using (20)–(23) and (30)–(32) we obtain from (25)

$$D_T m = -4\pi\left[\left(P_r + \epsilon - \frac{4}{3}\eta\sigma\right)U + (q + \epsilon)E\right]R^2, \qquad (36)$$

and

$$D_R m = 4\pi\left[\mu + \epsilon + (q + \epsilon)\frac{U}{E}\right]R^2 + \frac{s}{R}D_R s. \qquad (37)$$

Expression (36) describes the rate of variation of the total energy inside a surface of radius Cr. On the right-hand side of (36), $(P_r + \epsilon - 4\eta\sigma/3)U$ (in the case of collapse $U < 0$) increases the energy inside Cr through the rate of work being done by the "effective" radial pressure $P_r - 4\eta\sigma/3$ and the radiation pressure ϵ. Clearly here the heat flux q

does not appear since there is no pressure associated with the diffusion process. The second term $-(q+\epsilon)E$ is the matter energy leaving the spherical surface.

Equation (37) shows how the total energy enclosed varies between neighboring spherical surfaces inside the fluid distribution. The first term on the right hand side of (37), $\mu+\epsilon$, is due to the energy density of the fluid element plus the energy density of the null fluid describing dissipation in the free-streaming approximation. The second term, $(q+\epsilon)U/E$, is negative (in the case of collapse) and measures the outflow of heat and radiation. Finally the last term is the electrostatic contribution.

Equation (37) may be integrated to obtain

$$m = \int_0^R 4\pi R^2\left[\mu+\epsilon+(q+\epsilon)\frac{U}{E}\right]dR + \frac{s^2}{2R} + \frac{1}{2}\int_0^R \frac{s^2}{R^2}dR \tag{38}$$

(assuming a regular center to the distribution, so $m(0)=0$).

The acceleration $D_T U$ of an infalling particle inside Σ can be obtained by using (22), (25), (30), and (34), producing

$$D_T U = -\frac{m}{R^2} - 4\pi\left(P_r+\epsilon-\frac{4}{3}\eta\sigma\right)R + \frac{s^2}{R^3} + \frac{EA'}{AB}, \tag{39}$$

and then, substituting A'/A from (29) and (39), we obtain

$$\left(\mu+P_r+2\epsilon-\frac{4}{3}\eta\sigma\right)D_T U = -\left(\mu+P_r+2\epsilon-\frac{4}{3}\eta\sigma\right)\left[\frac{m}{R^2}+4\pi\left(P_r+\epsilon-\frac{4}{3}\eta\sigma\right)R-\frac{s^2}{R^3}\right] - E^2\left[D_R\left(P_r+\epsilon-\frac{4}{3}\eta\sigma\right) + 2(P_r-P_\perp+\epsilon-2\eta\sigma)\frac{1}{R} - \frac{s}{4\pi R^4}D_R s\right] - E\left[D_T q + D_T\epsilon + 4(q+\epsilon)\frac{U}{R} + 2(q+\epsilon)\sigma\right], \tag{40}$$

which in the nondissipative locally isotropic case coincides with Eq. (43) in [14]. Let us now analyze in some detail the three terms on the right of (40).

The first term on the right-hand side of (40) represents the gravitational force. The factor within the round bracket [the same factor as on the left of (40)] defines the inertial mass density (“passive” gravitational mass density) and shows how it is affected by dissipative terms. Observe that it is not affected by the electric charge.

The factor within the first square bracket shows how dissipation and the electric charge affect the “active” gravitational mass term. Using (38) in (40) we see that the charge will increase the “active gravitational mass” only if

$$\int_0^R \frac{s^2}{R^2}dR > \frac{s^2}{R} \tag{41}$$

or, equivalently

$$\frac{s}{R} > D_R s; \tag{42}$$

otherwise it will decrease it. This strange effect was already noticed by Bekenstein [14], and enhances the possibility that Coulomb repulsion might prevent the gravitational collapse of the sphere.

There are three different contributions in the second square bracket. The first one is just the gradient of the total effective radial pressure (which includes the radiation pressure and the influence of shear viscosity on P_r). The second contribution comes from the local anisotropy of pressure, including the contributions from the radiation pressure and shear viscosity. Finally the last term describes Coulomb repulsion, which is always positive (always opposing gravitation).

The last square bracket contains different contributions due to dissipative processes. The third term within this bracket is positive ($U<0$) showing that the outflow of $q>0$ and $\epsilon>0$ diminish the total energy inside the collapsing sphere, thereby reducing the rate of collapse. The last term describes an effect resulting from the coupling of the dissipative flux with the shear of the fluid. The effects of $D_T\epsilon$ have been discussed in detail in [63]. Thus it only remains to analyze the effects of $D_T q$; this depends on the transport equation adopted, and we will proceed to study one case in the next section.

However before doing that it is instructive to recover a known result for the static case.

Static charged dust

In the limit of hydrostatic equilibrium when $U=\sigma=q=\epsilon=0$, we have from (29)

$$P_r' + (\mu+P_r)\frac{A'}{A} + 2(P_r-P_\perp)\frac{(Cr)'}{Cr} - \frac{ss'}{4\pi(Cr)^4} = 0, \tag{43}$$

which is just the generalization of the Tolman-Oppenheimer-Volkov equation for anisotropic charged fluids obtained in [64] while studying dynamical instability for radiating anisotropic collapse.

When the static fluid reduces to charged dust, with $P_r = P_\perp = 0$, then, by using (15), (43)becomes

$$\mu\frac{A'}{A} - \frac{s\varsigma B}{(Cr)^2} = 0. \tag{44}$$

Since B and C depend only on r, we can transform r so that $B=C$. Eliminating s from the field equations (22) and (23), we can solve for AB, and imposing regularity conditions and rescaling t we have

$$C = B, \qquad AB = 1, \qquad s^2 = r^4 B'^2. \tag{45}$$

Substituting (45) into (44) we obtain

$$\mu^2 = s^2, \tag{46}$$

which is the well-known result originally obtained by Bonnor [5] for arbitrary symmetry.

IV. THE TRANSPORT EQUATION

We shall use a transport equation derived from the Müller-Israel-Stewart second order phenomenological theory for dissipative fluids [65,66].

Indeed, it is well known that the Maxwell-Fourier law for radiation flux leads to a parabolic equation (diffusion equation) which predicts propagation of perturbations with infinite speed (see [67–70] and references therein). This simple fact is at the origin of the pathologies [71] found in the approaches of Eckart [72] and Landau [73] for relativistic dissipative processes. To overcome such difficulties, various relativistic theories with nonvanishing relaxation times have been proposed in the past [65,66,74,75]. The important point is that all these theories provide a heat transport equation which is not of Maxwell-Fourier type but of Cattaneo type [76], leading thereby to a hyperbolic equation for the propagation of thermal perturbations.

The corresponding transport equation for the heat flux reads

$$\tau h^{\alpha\beta} V^{\gamma} q_{\beta;\gamma} + q^{\alpha} = -\kappa h^{\alpha\beta}(T_{,\beta} + T a_{\beta}) - \frac{1}{2}\kappa T^2 \left(\frac{\tau V^{\beta}}{\kappa T^2}\right)_{;\beta} q^{\alpha}, \tag{47}$$

where $h^{\mu\nu}$ is the projector onto the three space orthogonal to V^{μ}, κ denotes the thermal conductivity, and T and τ denote temperature and relaxation time, respectively. Observe that, due to the symmetry of the problem, Eq. (47) only has one independent component, which may be written after using (1), (6), and (10) as

$$\tau \dot{q} = -\frac{1}{2}\kappa q T^2 \left(\frac{\tau}{\kappa T^2}\right)^{\cdot} - \tau q \left(\frac{\dot{B}}{2B} + \frac{\dot{C}}{C}\right) - \frac{\kappa}{B}(TA)' - qA. \tag{48}$$

Now using (30)–(34) we can rewrite (48) as

$$D_T q = -\frac{\kappa T^2 q}{2\tau} D_T\left(\frac{\tau}{\kappa T^2}\right) - q\left(\frac{3}{2}\frac{U}{R} + \frac{1}{2}\sigma + \frac{1}{\tau}\right) - \frac{\kappa E}{\tau} D_R T - \frac{\kappa T}{\tau E} D_T U - \frac{\kappa T}{\tau E}\left[m + 4\pi\left(P_r + \epsilon - \frac{4}{3}\eta\sigma\right)R^3 - \frac{s^2}{R}\right]\frac{1}{R^2}. \tag{49}$$

We can couple the transport equation in the form above, (49), to the dynamical Eq. (40), in order to bring out the effects of dissipation on the dynamics of the collapsing sphere. For that purpose, let us substitute (49) into (40): then we obtain, after some rearrangements,

$$\left(\mu + P_r + 2\epsilon - \frac{4}{3}\sigma\eta\right)(1-\alpha) D_T U = (1-\alpha)F_{\rm grav} + F_{\rm hyd} + \frac{\kappa E^2}{\tau} D_R T + E\left[\frac{\kappa T^2 q}{2\tau} D_T\left(\frac{\tau}{\kappa T^2}\right) - D_T \epsilon\right] - Eq\left(\frac{5}{2}\frac{U}{R} + \frac{3}{2}\sigma - \frac{1}{\tau}\right) - 2E\epsilon\left(2\frac{U}{R} + \sigma\right), \tag{50}$$

where $F_{\rm grav}$ and $F_{\rm hyd}$ are defined by

$$F_{\rm grav} = -\left(\mu + P_r + 2\epsilon - \frac{4}{3}\eta\sigma\right) \times \left[m + 4\pi\left(P_r + \epsilon - \frac{4}{3}\eta\sigma\right)R^3 - \frac{s^2}{R}\right]\frac{1}{R^2}, \tag{51}$$

$$F_{\rm hyd} = -E^2\left[D_R\left(P_r + \epsilon - \frac{4}{3}\eta\sigma\right) + 2(P_r - P_{\perp} + \epsilon - 2\eta\sigma)\frac{1}{R} - \frac{s}{4\pi R^4} D_R s\right], \tag{52}$$

and α is given by

$$\alpha = \frac{\kappa T}{\tau}\left(\mu + P_r + 2\epsilon - \frac{4}{3}\sigma\eta\right)^{-1}. \tag{53}$$

Some comments are in order at this point:

(i) Once the transport equation has been taken into account, then the inertial energy density and the "passive gravitational mass density," i.e. the factor multiplying $D_T U$ and the first factor at the right of (40) respectively (which of course are the same, as expected from the equivalence principle), appear diminished by the factor $1 - \alpha$, a result already obtained in [45], but here generalized by the inclusion of the viscosity and radiative phenomena.

(ii) Observe that the charge does not enter into the definition of α. However it does affect the "active gravitational mass" [the factor within the square bracket in (51)].

(iii) The repulsive Coulomb term [the last term in (52)] depends on $D_R s$ and always opposes gravitation. Its effect is reinforced if $D_R s$ is large enough to violate (42), in which case the charge will decrease the "active gravitational mass" term in (51).

V. THE WEYL TENSOR

In this section we shall find some interesting relationships linking the Weyl tensor with matter variables, from which we shall extract some conclusions about the arrow of time.

From the Weyl tensor we may construct the Weyl scalar $\mathcal{C}^2 = C^{\alpha\beta\gamma\delta}C_{\alpha\beta\gamma\delta}$ which can be given in terms of the

Kretchman scalar $\mathcal{R} = R^{\alpha\beta\gamma\delta}R_{\alpha\beta\gamma\delta}$, the Ricci tensor $R_{\alpha\beta}$, and the curvature scalar R by

$$\mathcal{C}^2 = \mathcal{R} - 2R^{\alpha\beta}R_{\alpha\beta} + \frac{1}{3}\mathrm{R}^2. \tag{54}$$

Substituting (A13) from the Appendix with the field equations (20)–(23) into (54) we obtain

$$\mathcal{E} = m - \frac{4\pi}{3}(\mu - P_r + P_\perp + 2\eta\sigma)R^3 - \frac{s^2}{R}, \tag{55}$$

where $\mathcal{E}$ is given by

$$\mathcal{E} = \frac{\mathcal{C}}{48^{1/2}}R^3. \tag{56}$$

From (55) with (36) and (37) we have

$$D_T\mathcal{E} = -4\pi\left[\frac{1}{3}R^3 D_T(\mu - P_r + P_\perp + 2\eta\sigma) + \left(\mu + P_\perp + \epsilon + \frac{2}{3}\eta\sigma\right)R^2 U + (q+\epsilon)ER^2\right] + \frac{s^2 U}{R^2}, \tag{57}$$

and

$$D_R\mathcal{E} = 4\pi\left[(q+\epsilon)\frac{R^2 U}{E} - \frac{1}{3}R^3 D_R(\mu - P_r + P_\perp + 2\eta\sigma) + (\epsilon + P_r - P_\perp - 2\eta\sigma)R^2\right] - \frac{sD_R s}{R} + \left(\frac{s}{R}\right)^2. \tag{58}$$

From (58) we obtain at once for the noncharged, nondissipative, perfect fluid case

$$D_R\mathcal{E} + \frac{4\pi}{3}R^3 D_R\mu = 0, \tag{59}$$

implying that $D_R\mu = 0$ produces $\mathcal{C} = 0$ (using the regular axis condition), and conversely the conformally flat condition implies homogeneity in the energy density.

This particularly simple relation between the Weyl tensor and density inhomogeneity, for perfect fluids, is at the origin of Penrose's proposal to provide a gravitational arrow of time in terms of the Weyl tensor [77]. The rationale behind this idea is that tidal forces tend to make the gravitating fluid more inhomogeneous as the evolution proceeds, thereby indicating the sense of time.

However the fact that such a relationship is no longer valid in the presence of local anisotropy of the pressure and/or dissipative processes, already discussed in [46], explains its failure in scenarios where the above-mentioned factors are present. Here we see how the electric charge distribution affects the link between the Weyl tensor and density inhomogeneity, suggesting that electric charge (whenever present) should enter into any definition of a gravitational arrow of time.

VI. CONCLUSIONS

We have provided a full set of the equations required for a description of physically meaningful models of collapsing charged spheres. We have included dissipative phenomena as well as anisotropic pressure; the justification for doing so was given in the Introduction.

The role of charge distribution in the dynamics of such configurations is clearly exhibited in Eqs. (36), (37), (40), and (50). In particular it is worth stressing the fact that electric charge, unlike pressure, does not always produce a "regeneration effect" (does not always increase the "active gravitational mass"). This fact together with the presence of the Coulomb term in (40) [or (50)] indicates the relevance of the electric charge in the process of collapse.

Finally we have obtained a relation (58) exhibiting the way in which electric charge affects the link between the Weyl tensor and density inhomogeneity. The consequences of this for a definition of a gravitational arrow of time have been discussed.

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APPENDIX

The spacetime (1) has the following non-null Riemann tensor components:

$$R_{0101} = -B\ddot{B} + \frac{B}{A}\dot{A}\dot{B} + AA'' - \frac{A}{B}A'B', \tag{A1}$$

$$R_{0202} = (Cr)^2\left[-\frac{\ddot{C}}{C} + \frac{\dot{A}}{A}\frac{\dot{C}}{C} + \left(\frac{A}{B}\right)^2\frac{A'}{A}\left(\frac{C'}{C} + \frac{1}{r}\right)\right], \tag{A2}$$

$$R_{0212} = (Cr)^2\left[-\frac{\dot{C}'}{C} + \frac{\dot{B}}{B}\frac{C'}{C} + \frac{\dot{C}}{C}\frac{A'}{A} + \frac{1}{r}\left(\frac{\dot{B}}{B} - \frac{\dot{C}}{C}\right)\right], \tag{A3}$$

$$R_{1212} = (Cr)^2\left[\left(\frac{B}{A}\right)^2\frac{\dot{B}}{B}\frac{\dot{C}}{C} - \frac{C''}{C} + \frac{B'}{B}\frac{C'}{C} + \frac{1}{r}\left(\frac{B'}{B} - 2\frac{C'}{C}\right)\right], \tag{A4}$$

$$R_{2323} = (Cr)^2\sin^2\theta\left[\left(\frac{r\dot{C}}{C}\right)^2 - \left(\frac{rC'}{B}\right)^2 - 2\frac{CrC'}{B^2} - \left(\frac{C}{B}\right)^2 + 1\right], \tag{A5}$$

and

$$R_{0303} = R_{0202}\sin^2\theta, \qquad R_{0313} = R_{0212}\sin^2\theta, \qquad R_{1313} = R_{1212}\sin^2\theta, \tag{A6}$$

hence it has 5 independent components and the Kretchman scalar becomes

$$\mathcal{R} = 4\left[\frac{1}{(AB)^4}(R_{0101})^2 + \frac{2}{(ACr)^4}(R_{0202})^2 - \frac{4}{(AB)^2(Cr)^4}(R_{0212})^2 + \frac{2}{(BCr)^4}(R_{1212})^2 + \frac{1}{(Cr)^8\sin^4\theta}(R_{2323})^2\right]. \tag{A7}$$

The components (A1)–(A5) can be written in terms of the Einstein tensor $G_{\alpha\beta} = R_{\alpha\beta} - g_{\alpha\beta}\mathrm{R}/2$ and the mass function (25) producing

$$R_{0101} = (AB)^2\left[\frac{1}{2A^2}G_{00} - \frac{1}{2B^2}G_{11} + \frac{1}{(Cr)^2}G_{22} - \frac{2}{(Cr)^3}\left(m - \frac{s^2}{2Cr}\right)\right], \tag{A8}$$

$$R_{0202} = (ACr)^2\left[\frac{1}{2B^2}G_{11} + \frac{1}{(Cr)^3}\left(m - \frac{s^2}{2Cr}\right)\right], \tag{A9}$$

$$R_{0212} = \frac{(Cr)^2}{2}G_{01}, \tag{A10}$$

$$R_{1212} = (BCr)^2\left[\frac{1}{2A^2}G_{00} - \frac{1}{(Cr)^3}\left(m - \frac{s^2}{2Cr}\right)\right], \tag{A11}$$

$$R_{2323} = 2Cr\sin^2\theta\left(m - \frac{s^2}{2Cr}\right). \tag{A12}$$

Substituting (A8)–(A12) into (A7) we obtain

$$\mathcal{R} = \frac{48}{(Cr)^6}\left(m - \frac{s^2}{2Cr}\right)^2 - \frac{16}{(Cr)^3}\left(m - \frac{s^2}{2Cr}\right)\left[\frac{G_{00}}{A^2} - \frac{G_{11}}{B^2} + \frac{G_{22}}{(Cr)^2}\right] - 4\left(\frac{G_{01}}{AB}\right)^2 + 3\left[\left(\frac{G_{00}}{A^2}\right)^2 + \left(\frac{G_{11}}{B^2}\right)^2\right] + 4\left[\frac{G_{22}}{(Cr)^2}\right]^2 - 2\frac{G_{00}}{A^2}\frac{G_{11}}{B^2} + 4\left(\frac{G_{00}}{A^2} - \frac{G_{11}}{B^2}\right)\frac{G_{22}}{(Cr)^2}. \tag{A13}$$

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