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Big bang nucleosynthesis with long-lived charged massive particles

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We consider big bang nucleosynthesis (BBN) with long-lived charged massive particles. Before decaying, the long-lived charged particle recombines with a light element to form a bound state like a hydrogen atom. This effect modifies the nuclear-reaction rates during the BBN epoch through the modifications of the Coulomb field and the kinematics of the captured light elements, which can change the light element abundances. It is possible for heavier nuclei abundances such as ^{7}Li and ^{7}Be to decrease sizably, while the ratios Y_p, D/H, and ^{3}He/H remain unchanged. This may solve the current discrepancy between the BBN prediction and the observed abundance of ^{7}Li. If future collider experiments find signals of a long-lived charged particle inside the detector, the information of its lifetime and decay properties could provide insights into not only the particle physics models but also the phenomena in the early Universe, in turn.

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I. INTRODUCTION

Recent cosmological observations agree remarkably with standard ΛCDM models. The one- and three-year data of the Wilkinson Microwave Anisotropy Probe (WMAP) observation determined the cosmological parameters to high precision [1,2].

In light of such recent progress of cosmological observations, it has been shown that the Universe should be close to flat, and most of the matter must be in the form of nonbaryonic dark matter, which has been originally considered as one of the best candidates to explain an anomaly in the rotational curves of galaxies.

In the extension of the standard model explaining electroweak symmetry breaking and stability of the hierarchy, several candidates of the particle dark matter have been proposed, such as the neutralino [3], the gravitino [4–8], the axino [9] in supersymmetric theory, branon dark matter [10], Kaluza Klein dark matter [11,12], little Higgs dark matter [13,14], and so on. The searches and the detailed studies of dark matter have become one of the most exciting aspects of near future collider experiments and cosmological observations.

Considering such candidates in particle physics models, we expect that a large amount of the dark-matter particle will be produced at the near future colliders [15], which will be powerful tools to understand the properties of dark matter [16]. On the other hand, cosmological observations may provide information in new particle physics models, and even some implications on undetectable theoretical parameters in the collider experiments. Thus the connection of cosmology to collider physics may provide wide possibilities to understand the properties of the dark-matter particle and check the cosmological models themselves.

At the present stage, the detailed properties of dark matter are still unknown. Therefore, even exotic properties might be allowed. Future observations/experiments may prove them and single out or constrain dark-matter candidates. Even now, some problems in cosmological observations may already show some hints to understand the unknown properties of dark matter, e.g., in the small scale structure problem [17–19] indicated in the cold dark-matter halo, the low ^{7}Li problem [20], and so on. There are several proposals to solve them by new physics [21–32]. However, considerable astrophysical uncertainties may still exist.

During the radiation dominated epoch well before the decoupling of the cosmic microwave background (CMB), it is not necessary that the dominant component of matter is neutral, and that relic is the same as the present one. For stable charged massive particles (CHAMPs) [33,34], their fate in the Universe had been discussed [35], and the searches for CHAMPs inside the sea water were performed [36], which obtained null results and got constraints on stable CHAMPs [37]. According to their results, the production of stable CHAMPs at future collider experiments is unlikely. However, such null results can be applied only for the stable CHAMPs, and still the window for long-lived CHAMPs with a mass below O(TeV) is left open. Such possibilities for the long-lived CHAMPs were well motivated in a scenario of super weakly interacting massive particle (superWIMP) dark matter [4], which may inherit the desired relic density through the long-lived CHAMP decays. The dominant component of the nonrelativistic (NR) matter during/after the BBN epoch might be charged particles. In supersymmetric theories, such a situation is naturally realized in gravitino lightest supersymmetric particle (LSP) and axino LSP scenarios. Then the candidate for the long-lived CHAMP would be a charged scalar lepton [6,7,38].

Trapping such long-lived CHAMPs, the detailed studies of long-lived charged particles will be possible in future collider experiments, which may be able to provide some nontrivial tests of underlying theories, like measurement on the gravitino spin, on the gravitational coupling in the gravitino LSP scenario [39]. The trapping method in CERN LHC and the International Linear Collider (ILC) has been performed in the context of supersymmetric theories [40]. Also, the collider phenomenology [41,42] and other possible phenomena [43,44] have been discussed.

In cosmological considerations of such long-lived particles, the effects on BBN by the late-time energy injection due to their decays have been studied in detail [45–49]. On the other hand, in the past studies of the effects on the light element abundances, the analyses were simply applied to long-lived "charged" massive particles, assuming all CHAMPs are ionized and freely propagating in the radiation dominated epoch well before the CMB decoupling. However, we show that these results are not always valid if the bound state with a CHAMP and light elements may have O(MeV) binding energy [34], and the bound state might be stable against the destruction by the scattering off the huge amount of the background photons even during the BBN epoch. Also, we show that heavier elements tend to be captured at an earlier time. Namely, the heavier light elements such as ^{7}Li or ^{7}Be form their bound states earlier than the lighter light elements, D, T, ^{3}He, and ^{4}He. Such a formation of the bound state with a heavy CHAMP may provide possible changes of the nuclear-reaction rates and the threshold energy of the reactions and so on, which might result in the change of the light element abundances.

What is the crucial difference from the case of electron captures? In the case of the electron capture, since the Bohr radius of an electron is much larger than the typical pion-exchange length $O(1/m_\pi)$, two nuclei feel the Coulomb barrier significantly before they get close to each other. On the other hand, in the case of the capture of the CHAMPs, the Bohr radius could be of the same order as the typical pion-exchange length. Then, the incident charged nuclei can penetrate the weakened Coulomb barrier, and the nuclear reaction occurs relatively rapidly. The importance of such a bound state in the nuclear reaction had been identified for cosmic muons [50,51].[1]

Concerning a discrepancy in ^{7}Li between the standard big bang nucleosynthesis (SBBN) prediction by using the CMB baryon-to-photon ratio and the observational data, as we will show in detail later, it is unlikely to attribute the discrepancy only to uncertainties in nuclear-reaction rates in SBBN [52–54]. However, as we mentioned above, if CHAMPs exist, the nuclear-reaction rates during the BBN epoch could be changed from the values known by experimental data or observations of the sun, and may potentially solve the current low $^7\mathrm{Li/H}$ problem.

If such long-lived CHAMPs existed and affected the light element abundances, the lifetime would be long ($>$ 1 sec). They may be discovered as long-lived heavily ionizing massive particles inside the detector in the collider experiments. The measurements of their lifetime and properties may provide new insights to understand not only the particle physics models but also the phenomena in the early Universe, in turn.

In this paper, we discuss the possible change due to the long-lived CHAMPs during/after the BBN epoch and consider the effects on BBN.[2]

II. SBBN AND OBSERVED LIGHT ELEMENTS

The theory in SBBN has only one theoretical parameter, the baryon-to-photon ratio η, to predict primordial light element abundances. Comparing the theoretical predictions with observational data, we can infer the value of η in SBBN. It is well known that this method had been the best evaluation to predict η before WMAP reported their first-year data of the CMB anisotropy [1].

WMAP observations have determined η at high precision. The value of η reported by the three-year WMAP observations [2] is

$$\eta = \frac{n_b}{n_\gamma} = (6.10 \pm 0.21) \times 10^{-10}, \tag{1}$$

where n_b is the number density of the baryon and n_γ is the number density of the cosmic background photon. In Fig. 1 we plot the theoretical prediction of the light element abundances with their 2σ errors. The vertical band means the value of η reported by the three-year WMAP observations at 2σ.

We briefly discuss the current status of the theory of SBBN and the observational light element abundances below, and check the consistency with the CMB anisotropy observation. Further details of the observational data are presented in a recent, nice review by G. Steigman [55]. The errors of the following observational values are at the 1σ level unless otherwise stated. Hereafter n_X denotes the number density of a particle X. (X, C) denotes the bound state of CHAMPs with an element X.

The primordial abundance of D is inferred in the high redshift QSO absorption systems. Recently, new data were obtained at redshift $z = 2.525\,659$ toward Q1243 + 3074 [56]. Combined with these data [57–60], the primordial abundance is given as $n_{\mathrm{D}}/n_{\mathrm{H}}|_{\mathrm{obs}} = (2.78^{+0.44}_{-0.38}) \times 10^{-5}$.[3] It

[1] In muon catalysis fusion, the formation of an atom containing two nuclei may be important.

[2] In this paper, we use natural units for physical quantities.

[3] Some of the observed data have larger dispersion than expected and might have systematic errors which may cause higher D/H [56,58].

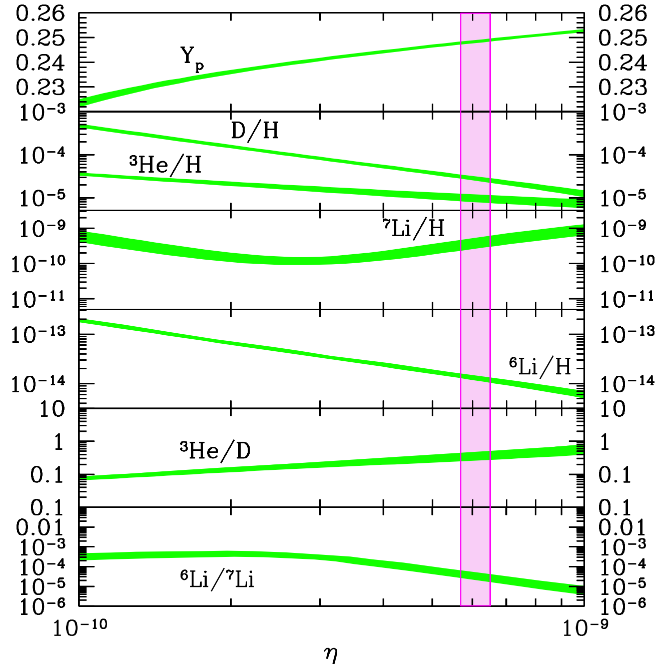

FIG. 1 (color online). Theoretical predictions of Y_p, D/H, ^{3}He/H, ^{7}Li/H ^{6}Li/H, ^{3}He/D, and ^{6}Li/^{7}Li as a function of the baryon-to-photon ratio η in standard BBN with their theoretical errors at 95% C.L. The WMAP value of η at 95% C.L. is also indicated as a vertical band. In the comparison between the BBN prediction and the central value of the observed abundances, it has been pointed out that the SBBN prediction with the WMAP value of η shows too high by a factor of a few in ^{7}Li abundance and too low by several orders of magnitude in ^{6}Li abundance if there is no late-time ^{6}Li production other than BBN [20].

agrees excellently with the value of η predicted in the CMB anisotropy observation.

The abundance of ^{3}He can increase and decrease through the chemical evolution history. However, it is known that the fraction $n_{^3\mathrm{He}}/n_{\mathrm{D}}$ is a monotonically increasing function of cosmic time [46,61]. Therefore the presolar value is an upper bound on the primordial one, $n_{^3\mathrm{He}}/n_{\mathrm{D}} < 0.59 \pm 0.54(2\sigma)$ [62]. In SBBN the theoretical prediction satisfies this constraint.

The primordial abundance of ^{4}He is obtained from the recombination lines from the low-metallicity extragalactic HII region. The mass fraction of ^{4}He is inferred by taking the zero metallicity limit as O/H $\to$ 0 for the observational data [63]. A recent analysis by Fields and Olive obtained the following value by taking into account the effect of the HeI absorption, $Y(\mathrm{FO})_{\mathrm{obs}} = 0.238 \pm (0.002)_{\mathrm{stat}} \pm (0.005)_{\mathrm{syst}}$, where the first and second errors are the statistical and systematic ones. On the other hand, Izotov and Thuan [64] reported a slightly higher value, $Y(\mathrm{IT})_{\mathrm{obs}} = 0.242 \pm (0.002)_{\mathrm{stat}}(\pm(0.005)_{\mathrm{syst}})$, where we have added the systematic errors following [65–67]. Olive and Skillman recently reanalyzed the Izotov-Thaun data [68] and obtained a much milder constraint [69], $Y(\mathrm{OS})_{\mathrm{obs}} = 0.249 \pm 0.009$. Even if we adopted the more restrictive value in Ref. [63], SBBN is consistent with CMB.

For ^{7}Li, it is widely believed that the primordial abundance is observed in Pop II old halo stars with temperature higher than ~6000 K and with low metallicity as a "Spite's plateau" value. The measurements by Bonifacio *et al.* [70] gave $\log_{10}[n_{^7\mathrm{Li}}/n_{\mathrm{H}}]|_{\mathrm{obs}} = -9.66 \pm (0.056)_{\mathrm{stat}} \pm (0.06)_{\mathrm{sys}}$. On the other hand, a significant dependence of ^{7}Li on the Fe abundance in the low-metallicity region was reported in [71]. If we take a serious attitude towards this trend, and assume that this comes from the cosmic-ray interaction [72], the primordial value is

$$\frac{n_{^7\mathrm{Li}}}{n_{\mathrm{H}}}\bigg|_{\mathrm{obs}} = (1.23^{+0.32}_{-0.25}) \times 10^{-10} \quad \text{(at 68\% C.L.)}. \quad (2)$$

Even if we adopt the higher value in Ref. [70], the theoretical prediction is excluded at 2σ outside the outskirts of observational and theoretical errors. Therefore when we adopt the lower value in (2), the discrepancy worsens. The central value of the observation is smaller than that of SBBN by a factor of about 3. This ^{7}Li problem has been pointed out by a lot of authors; e.g., see Ref. [20].

It has been thought optimistically that this discrepancy would be astrophysically resolved by some unknown systematic errors in the chemical evolution such as the uniform depletion in the convective zone in the stars.[4] So far the researchers have added large systematic errors into the observational constraint by hand [74,75].

However, recently the plateau structure of ^{6}Li in nine out of 24 Pop II old halo stars was reported by Asplund *et al.* [76]. The observed values of the isotope ratio $n_{^6\mathrm{Li}}/n_{^7\mathrm{Li}}$ uniformly scatter between $\simeq 0.01$ and 0.09 at 2σ, independently of the metallicity, and are approximately similar to the previous observational data ($= 0.05 \pm 0.02$ at 2σ [77]). Because the estimated ^{7}Li abundance in such stars is $n_{^7\mathrm{Li}}/n_{\mathrm{H}}|_{\mathrm{obs}} = (1.1\text{–}1.5) \times 10^{-10}$, the upper bound on the primordial ^{6}Li agrees with SBBN. Although so far some models of the ^{6}Li and ^{7}Li production through the cosmic-ray spallation of CNO and α-α inelastic scattering have been studied, the predicted value of $n_{^6\mathrm{Li}}/n_{^7\mathrm{Li}}$ or $n_{^6\mathrm{Li}}/n_{\mathrm{H}}$ is obviously an increasing function of metallicity [78–81].

As we have discussed, to be consistent with the SBBN prediction and WMAP observations, we need a certain uniform depletion mechanism of ^{7}Li. Because ^{6}Li is more fragile than ^{7}Li, whenever ^{7}Li is destroyed in a star, ^{6}Li suffers from the depletion, too. If we require the primordial abundance of ^{7}Li to be uniformly depleted to a smaller value by a factor of 3, the ratio ^{6}Li/^{7}Li might have to be reduced by a factor of $\mathcal{O}(10)$ [82]. Therefore, we do not have any successful chemical evolution models at the

[4]See the recent report about spectroscopic observations of stars in the metal-poor globular cluster NGC 6397 that revealed trends of atmospheric abundance with the evolutionary stage of lithium [73].

present to consistently explain the observational value of ^{6}Li/^{7}Li by starting from the theoretical prediction of the primordial values of ^{6}Li and ^{7}Li in the framework of SBBN.

Thus, by adopting the η predicted in the CMB observations, we would now have to check SBBN itself or modified scenarios related with BBN compared with the observational light element abundances.

In recent studies, it has been pointed out that the uncertainties on nuclear-reaction rates in SBBN never solve the discrepancy of ^{7}Li between the theory and the observation. That is because the uncertainties are highly constrained by known experimental data and observations of the standard solar model. In Ref. [54], the possible nuclear uncertainties were investigated. It was shown that only a nuclear-reaction rate more than 100 times larger in ^{7}Be(n, α)^{4}He and ^{7}Be(d, p)2^{4}He might provide sizable change in the ^{7}Li abundance. Notice that ^{7}Be(n, α)^{4}He does not have an s-wave resonance due to the symmetry of the outgoing channel while ^{7}Be(n, p)^{7}Li has it. Since in the important energy region in the SBBN reaction $T \sim$ 50 keV, which is near threshold of the processes, the contribution to ^{7}Be from ^{7}Be(n, α)^{4}He is negligible in SBBN relative to ^{7}Be(n, p)^{7}Li because of the p-wave nature of the process. For ^{7}Be(d, p)2^{4}He, the possibility may not work in the light of the recent experimental data [83]. Also, Cyburt *et al.* [84] discussed the uncertainties on the normalization of the cross section for the process ^{3}He(α, γ)^{7}Be and found that the uncertainties are constrained in the light of a good agreement between the standard solar model and solar neutrino data.

Therefore the remaining possibilities may be uncertainties on the chemical evolution of Li from the BBN epoch to the present or effects due to new physics. Because now we do not have any successful chemical evolution models, it must be important to consider the effect of new physics.

As we mentioned before, the existence of CHAMPs might provide a possible change of nuclear-reaction rates during the BBN epoch, which may have some impact on the prediction of primordial light element abundances. In the next section, we will discuss the properties of the bound state and the recombination of CHAMPs and the possible change of nuclear-reaction rates.

III. BOUND STATE WITH A CHAMP AND A LIGHT ELEMENT

Evaluation of binding energy

We evaluate the binding energy for the bound state of a negatively charged massive particle and a light element. We simply consider the case that the charged particle is a scalar. The extension to a fermion or the other higher spin cases would be straightforward, although there exist little differences. Here we follow the way to evaluate the binding energy assuming uniform charge distribution inside the light element according to Ref. [34]. Then the Hamiltonian is represented by

$$H = \frac{p^2}{2m_X} - \frac{Z_X Z_C \alpha}{2r_X} + \frac{Z_X Z_C \alpha}{2r_X}\left(\frac{r}{r_X}\right)^2, \tag{3}$$

for short distances $r < r_X$, and

$$H = \frac{p^2}{2m_X} - \frac{Z_X Z_C \alpha}{r}, \tag{4}$$

for long distances $r > r_X$, where α is the fine structure constant, $r_X \sim 1.2A^{1/3}/200\ \mathrm{MeV}^{-1}$ is the nuclear radius, Z_X is the electric charge of the light element, and Z_C is the electric charge of the negatively charged massive particle. A is the atomic number, and m_X is the mass of the light element X. Here we assumed $m_X \ll m_C \sim \mathcal{O}(100\ \mathrm{GeV})$, which means the reduced mass $1/\mu = 1/m_C + 1/m_X \sim 1/m_X$.

For large nuclei, the exotic charged particle may be inside the nuclear radius. The binding energy may be estimated under the harmonic oscillator approximation by

$$E_{\mathrm{bin}} = \frac{3}{2}\left[\frac{Z_X Z_C \alpha}{r_X} - \frac{1}{r_X}\left(\frac{Z_X Z_C \alpha}{m_X r_X}\right)\right]. \tag{5}$$

For small nuclei, the binding energy may be estimated well as a Coulomb bound state like a hydrogen atom,

$$E_{\mathrm{bin}} \sim \tfrac{1}{2} Z_X^2 Z_C^2 \alpha^2 m_X. \tag{6}$$

For intermediate regions in between the above cases, by using a trial wave function, we can express

$$E_{\mathrm{bin}} \sim \frac{1}{r_X}\left(\frac{1}{m_X r_X} F(Z_X Z_C \alpha m_X r_X)\right), \tag{7}$$

where $F(x)$ is variationally determined [34]. For $0 < Z_X Z_C \alpha m_X r_X < 1$, the Coulomb model gives a good approximation. On the other hand, the harmonic oscillator

TABLE I. Table of the binding energies for the various nuclei in the case of $Z_C = 1$ given in Ref. [34]. For elements heavier than ^{8}Be, the binding energies are given by the harmonic oscillator approximation.

Nucleus (X)	Binding energy (MeV)	Atomic number
p	0.025	$Z = 1$
D	0.050	$Z = 1$
T	0.075	$Z = 1$
^{3}He	0.270	$Z = 2$
^{4}He	0.311	$Z = 2$
^{5}He	0.431	$Z = 2$
^{5}Li	0.842	$Z = 3$
^{6}Li	0.914	$Z = 3$
^{7}Li	0.952	$Z = 3$
^{7}Be	1.490	$Z = 4$
^{8}Be	1.550	$Z = 4$
^{10}B	2.210	$Z = 5$

approximation gives a better approximation for $2 < Z_X Z_C \alpha m_X r_X < \infty$.

The binding energies are shown in Table I. For a CHAMP with $Z_C = 1$ and lighter elements (p, D, and T), typically $Z_X Z_C \alpha m_X r_X < 1$. Thus the Coulomb approximation works well. However, for heavier elements such as Li or Be, there may exist deviations which are more than $\mathcal{O}(10)$ percent. For elements lighter than ^{8}B, the binding energy is still below the threshold energy of any nuclear reactions. If the atomic number is not large like Li and Be, we can ignore the effects due to finite size and the internal structure (excitations to higher levels and so on) as a good approximation to calculate the capture cross section and the nuclear-reaction rates.

IV. CAPTURE OF CHAMPS IN THE EARLY UNIVERSE

A. Recombination cross section

We evaluate the recombination cross section from the free state to the 1S bound state assuming a hydrogen-type bound state through a dipole photon emission [85] and a pointlike particle for the captured light element. Then the cross section is

$$\begin{aligned}\sigma_r v &= \frac{2^9 \pi^2 \alpha Z_X^2}{3} \frac{E_{\rm bin}}{m_X^3 v} \left(\frac{E_{\rm bin}}{E_{\rm bin} + \frac{1}{2} m_X v^2}\right)^2 \\ &\quad \times \frac{e^{-4\sqrt{(2E_{\rm bin}/m_X v^2)}\tan^{-1}(\sqrt{(m_X v^2/2E_{\rm bin})})}}{1 - e^{-2\pi\sqrt{(2E_{\rm bin}/m_X v^2)}}} \\ &\simeq \frac{2^9 \pi^2 \alpha Z_X^2}{3e^4} \frac{E_{\rm bin}}{m_X^3 v},\end{aligned} \tag{8}$$

where v is the relative velocity of a CHAMP and a light element. Note that we have $m_X v^2/2 \simeq 3T/2 \ll E_{\rm bin}$ for NR particles in kinetic equilibrium. Here we use the Coulomb model (hydrogen type) to evaluate the capture rate [33], where the binding energy $E_{\rm bin} = \alpha^2 Z_C^2 Z_X^2 m_X/2$ and the Bohr radius $r_B^{-1} \simeq \alpha Z_C Z_X m_X$.

The thermal-averaged cross section is written as

$$\begin{aligned}\langle \sigma_r v \rangle &= \frac{1}{n_1 n_2}\left(\frac{g}{(2\pi)^3}\right)^2 \int d^3p_1 d^3p_2 e^{-(E_1+E_2)/T} \sigma_r v \\ &= \frac{1}{n_G n_r}\left(\frac{g}{(2\pi)^3}\right)^2 \int d^3p_G e^{-m_G/T} e^{-p_G^2/2m_G T} \\ &\quad \times \int d^3p_r \sigma_r v e^{-p_r^2/2\mu T} \\ &= \frac{2^9 \pi \alpha Z_X^2 \sqrt{2\pi}}{3e^4} \frac{E_{\rm bin}}{m_X^2 \sqrt{m_X T}},\end{aligned} \tag{9}$$

where $m_G = m_1 + m_2$ and $\mu = m_X m_C/(m_X + m_C) \simeq m_X$ with

$$\begin{aligned}n_G &= \frac{g}{(2\pi)^3} \int d^3p_G e^{-m_G/T} e^{-p_G^2/2m_G T}, \\ n_r &= \frac{g}{(2\pi)^3} \int d^3p_r e^{-p_r^2/2\mu T}.\end{aligned}$$

Here we have assumed that only one CHAMP is captured by a nucleus. Since the photon emission from a CHAMP is suppressed, the recombination cross section for the further capture of an additional CHAMP by the bound state would be much smaller. Therefore, as a first step, it would be reasonable to ignore the multiple capture of CHAMPs by a nucleus.

Here we have estimated only the direct transition from the free state into the 1S bound state. However, if the transition from higher levels into the 1S state is sufficiently rapid against the destruction due to scatterings off the thermal photons, even the capture into the higher levels might contribute to the recombination of a CHAMP. The typical time scale of the transition from the nth level into the 1S state is $1/(E_{\rm bin:1S} - E_{{\rm bin}:n}) \sim O(1/E_{\rm bin:1S})$ where $E_{{\rm bin}:n}$ is the binding energy of the nth level. Up to some levels, this time scale might be shorter than the destruction rate after the 1S state becomes stable. However, such higher-level captures would not significantly enhance the recombination cross section because the capture rate into higher levels is relatively suppressed and small.

For highly charged massive nuclei or elements heavier than boron, the binding energies with CHAMPs can become of the order of magnitude of the excitation energies of nucleons inside the nuclei, or even of the same order of magnitude of the nuclear binding energies. In such cases, the capture process of light elements by CHAMPs may be nontrivial. In addition, to correctly calculate the capture rates, we would have to understand the modification by the effects due to not only the finite size but also the internal structure of the light element. In this paper, we ignore these effects because they are unimportant since we consider lighter nuclei up to Li and Be.

B. Case in kinetic and chemical equilibrium

To evaluate the number density of the captured CHAMPs, we would be able to use the thermal relation among chemical potentials if the capture reactions establish well the chemical equilibrium between the CHAMPs and the light elements. The number density is determined by the following Saha equation,

$$n_{(X,C)} = \frac{2}{\pi^2} \zeta(3) \frac{n_X}{n_\gamma} n_C \left(\frac{2\pi T}{m_X}\right)^{3/2} e^{E_{\rm bin}/T} \tag{10}$$

where n_X and n_γ are number densities of a light element X and thermal photons, and $E_{\rm bin}$ is the binding energy of the light element.

C. General cases

However, the question of whether such kinetic and chemical equilibriums are well established among all light elements and CHAMPs is nontrivial. Here we consider the Boltzmann equations for CHAMPs, a light element X, and the bound state (X, C). For CHAMPs,

$$\frac{\partial}{\partial t}n_C + 3Hn_C = \left[\frac{\partial}{\partial t}n_C\right]_{\rm capture}, \tag{11}$$

where H is the Hubble expansion rate. For a light element X,

$$\frac{\partial}{\partial t}n_X + 3Hn_X = \left[\frac{\partial}{\partial t}n_X\right]_{\rm fusion} + \left[\frac{\partial}{\partial t}n_X\right]_{\rm capture}. \tag{12}$$

For the bound state,

$$\frac{\partial}{\partial t}n_{(C,X)} + 3Hn_{(C,X)} = \left[\frac{\partial}{\partial t}n_{(C,X)}\right]_{\rm fusion} - \left[\frac{\partial}{\partial t}n_X\right]_{\rm capture}. \tag{13}$$

By using the detailed balance relation between the forward process $X + C \rightarrow \gamma + (X, C)$ and the reverse process $(X, C) + \gamma \rightarrow X + C$, the capture reaction may be written by

$$\left[\frac{\partial}{\partial t}n_X\right]_{\rm capture} = \left[\frac{\partial}{\partial t}n_C\right]_{\rm capture} \simeq -\langle\sigma_r v\rangle[n_C n_X - n_{(C,X)}n_\gamma(E > E_{\rm bin})], \tag{14}$$

where

$$n_\gamma(E > E_{\rm bin}) \equiv n_\gamma \frac{\pi^2}{2\zeta(3)}\left(\frac{m_X}{2\pi T}\right)^{3/2} e^{-E_{\rm bin}/T} \tag{15}$$

and

$$n_\gamma = \frac{2\zeta(3)}{\pi^2}T^3. \tag{16}$$

For a light element, if $\langle\sigma_r v\rangle n_C/H \gg 1$ is satisfied and the kinetic equilibrium is well established, we can get the Saha equation by requiring an equilibrium condition $[\frac{\partial}{\partial t}n_X]_{\rm capture} = 0$ in this equation. Since we are interested in the time evolution of not only CHAMPs but also light elements, we carefully study the case of $\langle\sigma_r v\rangle n_C/H > 1$ even in the case of $\langle\sigma_r v\rangle n_X/H \ll 1$.

D. Critical temperature at which a bound state is formed

When the temperature is higher than the binding energy of light elements, the destruction rate of bound states by scatterings off the thermal photons with $E > E_{\rm bin}$ is rapid. Then only a small fraction of bound states can be formed, $n_{(C,X)} \sim n_C n_X/n_\gamma(E > E_{\rm bin}) \ll n_X$. Once the temperature becomes lower than the binding energy, the capture starts, and the bound state becomes stable if the other destruction processes among the nuclei are inefficient.[5] The critical temperature at which the capture becomes efficient is estimated as follows. In the case of $n_X > n_C$, taking $n_C \sim n_{(C,X)}$, we get the relation

$$\left(\frac{m_X}{T}\right)^{3/2} e^{-E_{\rm bin}/T} \sim \frac{n_X}{n_\gamma} = \mathcal{O}(10^{-10}). \tag{17}$$

On the other hand, in the case of $n_X < n_C$, taking $n_X \sim n_{(C,X)}$, we have

$$\left(\frac{m_X}{T}\right)^{3/2} e^{-E_{\rm bin}/T} \sim \frac{n_C}{n_\gamma} \sim \mathcal{O}(10^{-10})\left(\frac{100\,{\rm GeV}}{m_C}\right)\left(\frac{\Omega_C}{0.23}\right). \tag{18}$$

This analysis shows that the critical temperature is approximately

$$T_c \simeq \frac{E_{\rm bin}}{40}. \tag{19}$$

In the case of $Z_C = 1$, we find $T_c \sim E_{\rm bin}/40 \sim 8$ keV for ^{4}He.

Here we consider the temperature where some fraction of X is captured by CHAMPs. For example, taking $n_{(C,X)}/n_X \simeq 10^{-5}$, we get

$$\left(\frac{m_X}{T}\right)^{3/2} e^{-E_{\rm bin}/T} \sim \frac{n_C}{n_\gamma}\frac{n_X}{n_{(C,X)}} = O(10^{-6}). \tag{20}$$

This condition is satisfied at $T_c^{(2)} \sim E_{\rm bin}/30$. Since the abundance of ^{4}He is large below 0.1 MeV, even though the only small fraction of ^{4}He is trapped by CHAMPs, there might be relevant effects caused by the captures.

For protons, the efficient captures start at a temperature lower than 1 keV (at cosmic time longer than 10^6 sec). Since the bound state is neutral for single-charged CHAMPs $Z_C = 1$, and might be negatively charged for multicharged CHAMPs $Z_C > 1$, there is no Coulomb repulsion anymore. Thus, even the bound states can collide with each other. If the number density of CHAMPs is not too small, and most CHAMPs are captured by protons, the change could be sizable for longer-lived CHAMPs ($\tau > 10^6$ sec).[6]

E. Capture rate

Since the capture process competes with the expansion of the Universe, we have to check if the following relation holds during the meaningful time, which ensures that the capture by CHAMPs is efficient compared to the expansion rate of the Universe,

[5]Note that the abundances of heavier elements such as Li and Be are smaller than those of lighter elements (p, D, T, and He). As we will see later, considering the relic density of relevant candidates of CHAMPs, their capture can only affect the abundance of the heavier elements. Our scenarios would not significantly change the lighter element abundances.

[6]Since the CHAMPs with a long lifetime of more than $\gg 10^6$ sec may induce the other effects on cosmology [24].

$$H \ll \langle \sigma_r v \rangle n_C. \tag{21}$$

That is, the capture rate of a light element is controlled by the following κ,

$$\kappa \equiv \frac{\langle \sigma_r v \rangle n_C}{H} = 2.6\sqrt{\frac{3.2}{g_*}}\sqrt{\frac{T}{24\ \mathrm{keV}}}\left(\frac{Z_X}{3}\right)^4\left(\frac{7\ \mathrm{GeV}}{m_X}\right)^{3/2}\frac{\Omega_C}{0.23}\frac{100\ \mathrm{GeV}}{m_C}. \tag{22}$$

κ is approximately 2.6 and 0.43 for ^{7}Li and ^{4}He at their critical temperatures, respectively. Here we assumed that $\Omega_C \simeq 0.23$ and $m_C = 100$ GeV.

In the evaluation of the capture rates for light elements, we considered relatively large number densities of CHAMPs, which are approximately similar to that of ^{4}He or even more because here we assumed that a CHAMP can decay into much lighter dark-matter or almost massless SM particles later. Under these circumstances, we naturally expect a larger value of the capture rates than the upper limit in the case of the stable CHAMP scenario. Of course, we have to check that the decay never disturbs the successful concordance of cold dark matter (CDM) with large scale structure formation in the Universe and so on. Later we will discuss this problem.

Next let us estimate the time evolution of X itself and the capture fraction of X by a CHAMP. Below the critical temperature T_c, the destruction term of (X, C) becomes negligible due to the Boltzmann suppression.[7] Then the number densities of the light element X and the bound state of X with a CHAMP, (X, C), are obtained by solving the following equations. Here any destruction reactions of X would be negligible close to the end of the BBN epoch ($\lesssim$ 50 keV),

$$\frac{d}{dT}\left(\frac{\eta_X}{\eta_X(T_c)}\right) \simeq \frac{\langle \sigma v \rangle n_C}{HT}\frac{\eta_X}{\eta_X(T_c)},$$
$$\frac{d}{dT}\left(\frac{\eta_{(C,X)}}{\eta_X(T_c)}\right) = -\frac{\langle \sigma_r v \rangle n_C}{HT}\frac{\eta_X}{\eta_X(T_c)} + (\text{fusion part}), \tag{23}$$

where $\eta_i = n_i/s$ and $\eta_X(T_c)$ is the initial number density per entropy density when the capture starts, assuming that the standard processes of the light elements are (almost) frozen out. We also assumed $n_C \gg n_X$ which is correct except for ^{4}He. We find that, if κ is larger than unity at the critical temperature, the capture will be efficient.

Ignoring the fusion part of the standard processes in Eq. (23), we find the following analytical solution of $\eta_X(T)$,

$$\eta_X(T) = \eta_X(T_c)e^{-2\kappa_i(1-\sqrt{T/T_c})}, \tag{24}$$

where $\kappa_i = \langle \sigma_r v \rangle n_C/H|_{T=T_c}$.[8] For the numerical solution of Eq. (24), see Fig. 2.

For a more precise analysis, especially for the Boltzmann equation of the CHAMP bound state, we may have to take into account the nuclear-reaction processes simultaneously. Therefore we will need to do the numerical calculations to solve the Boltzmann equations including both the capture and the BBN processes in the future [86]. However, to qualitatively understand how large changes would be possible, for simplicity we assume only the instantaneous captures in the current work.

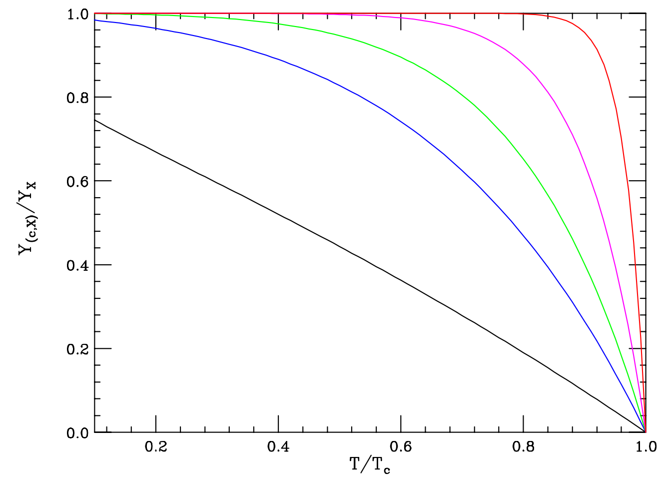

FIG. 2 (color online). $\eta_{(C,X)}/\eta_X^0$ as a function of T/T_c for $\langle \sigma_r v \rangle n_C/H|_{T=T_c} = 1$, 3, 5, 10, and 30 from left to right, respectively. Here we have ignored the standard BBN processes. Also, we have taken the initial condition as $\eta_X(T_c) \sim 0$. If $\langle \sigma_r v \rangle n_C/H|_{T=T_c} \gg 1$, the Saha equation will be a good approximation and the capture will immediately occur at $T \sim T_c$. On the other hand, if $\langle \sigma_r v \rangle n_C/H|_{T=T_c} \sim 1$ or less than 1, the approximation by the Saha equation may fail.

V. CHANGE OF NUCLEAR-REACTION RATES IN BBN BY THE CAPTURE OF CHAMPS

The capture of light elements by CHAMPs weakens the Coulomb barrier in the nuclear reactions during/after the BBN epoch. The change of nuclear-reaction rates could become large because the Coulomb factor exponentially suppresses the reaction rates. In general, the reaction rates among charged nuclei during the BBN epoch are determined by the competition between the Coulomb suppression and the Boltzmann suppression, which play important

[7] The ignorance of the destruction term at T_c may be valid if the recombination cross section is not too large. If the cross section is large enough, the number density of the bound state may be well described by the Saha equation.

[8] For ^{7}Be, ^{7}Li, and lighter elements, the above approximation works well if Z_c is close to 1. As we will see later, the change of nuclear-reaction rates does not modify the fusion part of the noncaptured light elements so much because most reverse processes have already been decoupled even after the other elements are captured by CHAMPs.

roles to determine the freeze-out of light element abundances at the end of the BBN epoch. Considering the corrections on these two exponential suppressions, we will next consider the possible changes of nuclear-reaction rates.

A. Coulomb potential and scattering problem

If there are Coulomb expulsion forces, the wave function of an incident particle would be exponentially suppressed at the target. Since we use a plane wave for the wave function to evaluate the incident flux at a sufficiently far place from the target, the real flux which is associated with the reaction would be evaluated by renormalizing the wave function. Since the change of the wave-function normalization from the plane wave is associated with the state before the nuclear reaction, it is independent of the short-distance nuclear reaction by nuclei. We can expect that the Coulomb factor is factorized as follows,[9]

$$F_{ab}(v) = \frac{2\pi Z_a Z_b \alpha/v}{e^{2\pi Z_a Z_b \alpha/v} - 1} \simeq \frac{2\pi Z_a Z_b \alpha}{v} e^{-2\pi Z_a Z_b \alpha/v}. \quad (25)$$

After a CHAMP is trapped by a light element a, for a collision between a bound state (CHAMP + the light element a) and a light element b,

$$F_{(aC)b}(\beta) \simeq \frac{2\pi\alpha Z_{(aC)} Z_b}{\beta} e^{-2\pi\alpha Z_{(aC)} Z_b/\beta}, \quad (26)$$

where $Z_{(aC)} = Z_a - Z_C$. Note that β is the relative velocity between the bound state (aC) and the b element, not the a and the b element. Hence β could be slightly different from v which is the normal relative velocity between the thermal a and the thermal b. Here we assumed that a light element can capture only one CHAMP (with the charge Z_C).

For the case of nuclear reactions through a collision between charged bound states (CHAMP + light element a and CHAMP + light element b), the Coulomb-penetration ability is determined by the relative velocity between the bound states. That is,

$$F_{(aC)(bC)}(\beta_2) = \frac{2\pi Z_{(aC)} Z_{(bC)} \alpha}{\beta_2} e^{-2\pi Z_{(aC)} Z_{(bC)} \alpha/\beta_2}, \quad (27)$$

where $\beta_2 = p_r/\mu_{(aC)(bC)} \simeq O(T/m_C) \ll \beta = O(T/m_X)$. Under these circumstances, the collision between charged bound states may be highly suppressed relative to the standard BBN reactions because $Z_{(aC)} Z_{(bC)}/\beta_2 > Z_a Z_b/v$ if $Z_X > Z_C$. This bound state–bound state collision might become important if a huge number of CHAMPs are captured by ^{4}He. However, the typical temperature to start capture is below O(10) keV, and the Coulomb factors for the normal nuclear reactions in SBBN are highly suppressed and have already been decoupled by that time. Thus this type of collision will not contribute to any sizable changes of the light element abundances.

For $Z_X = 1$ ($Z_C = 1$) cases like protons, since there is no Coulomb suppression because the bound state is neutral, the collision between two bound states may be important.

B. SBBN and thermal-averaged fusion rates

First, we discuss nuclear-reaction rates in SBBN, and next we will extend the discussions to the cases with CHAMPs.

For simplicity, we consider the case of $2 \to 2$ nonresonant reactions among charged light nuclei. The other cases may be straightforward through similar discussions. In a SBBN process $a + b \to c + d$, the forward process and the reverse process are defined by the difference between the total masses in the initial and the final state. If $Q_{ab,cd} = m_a + m_b - m_c - m_d = Q_{\rm SBBN} > 0$, the process $a + b \to c + d$ has no threshold and is called the forward process. On the other hand, the process $c + d \to a + b$ has a threshold (Q value $Q_{cd,ab} = -Q_{\rm SBBN} < 0$) and is called the reverse process of $a + b \to c + d$. Usually the reverse process $c + d \to a + b$ has a strong Boltzmann suppression by $e^{-Q_{\rm SBBN}/T}$ if the Q value is larger than the Gamow peak energy of the process.

1. SBBN reaction rates with no threshold

Naively, the nuclear reactions of SBBN occur at almost the threshold region. Thus the cross section may be well described by the lower partial wave modes. Taking into account the discussion of the wave-function normalization in the previous section, the reaction cross section is written as follow:

$$\sigma_{\rm fusion} v = (\sigma_S + \sigma_P v^2 + \ldots) F_{ab}(v) = \sigma_0 v(v) \frac{2\pi Z_a Z_b \alpha}{v} e^{-2\pi Z_a Z_b \alpha/v} \quad (28)$$

where $\sigma_0 v(v) = \sigma_S + \sigma_P v^2 + \ldots$.

Here we introduce a new variable, the "astrophysical S factor" which astrophysicists have used in the calculation of nucleosynthesis,

$$S(E_r) = \sigma_{\rm fusion} E_r e^{\sqrt{E_G/E_r}} = \sigma_0 v(v) \pi Z_a Z_b \alpha \mu_{ab} \quad (29)$$

where $E_G = 2\pi^2 Z_a^2 Z_b^2 \alpha^2 \mu_{ab}$ and $E_r = p_r^2/2\mu_{ab} = \mu_{ab} v^2/2$. Notice that this S factor is a function of the center-of-mass (CM) energy and is inferred by the mea-

[9]This factorization may be valid only if the Bohr radius of the bound state is not too large relative to the radius of the bound and incident nuclei. If the Bohr radius is large, which may be expected in $Z_X = 1$ nuclei cases, we have to understand how the bound state is disturbed by the incident nucleus. In such large Bohr radius cases, for example, to proceed nuclear fusion, the hydrogen-type bound state of the nucleus and a CHAMP may have to constitute a molecule before the nuclear fusion. Then we will have to evaluate the capture reaction rate of the molecule.

surements of $\sigma_{\rm fusion}v$ in experiments and observations. The recent fitting functions are given in Refs. [52,53].

By using this S factor, we calculate the thermal-averaged cross section:

$$\begin{aligned}\langle\sigma_{\rm fusion}v\rangle &= \frac{g}{(2\pi)^3 n_r}\int d^3p_r\sigma_{\rm fusion}ve^{-p_r^2/2\mu_{ab}T} = \frac{8\pi gT\mu_{ab}}{(2\pi)^3 n_r}\int dxS(xT)e^{-(x+\sqrt{x_G/x})}\\ &= \frac{8\pi gT\mu_{ab}}{(2\pi)^3 n_r}\int dxS(xT)e^{-((3/4^{1/3})x_G^{1/3}+(3/4x_0)(x-x_0)^2+\ldots.)}\\ &\simeq \frac{8\pi gT\mu_{ab}}{(2\pi)^3 n_r}\sqrt{\frac{\pi x_0}{3}}\left(1+Erfc\left(\frac{\sqrt{3x_0}}{2}\right)\right)S(x_0T)e^{-(3/4^{1/3})x_G^{1/3}}\\ &\simeq \frac{8\pi gT\mu_{ab}}{(2\pi)^3 n_r}\sqrt{\frac{4\pi x_0}{3}}S(x_0T)e^{-(3/4^{1/3})x_G^{1/3}} \end{aligned}\tag{30}$$

where $x = E_r/T$, $x_G = E_G/T$, and $x_0 = (x_G/4)^{1/3}$. Since the main contribution of this integral comes from the stationary point of the exponent, we expanded the exponent around the stationary point $x_0 = (x_G/4)^{1/3}$.

Finally, we can evaluate the thermal-averaged nuclear-reaction rate among charged light elements:

$$\langle\sigma_{\rm fusion}v\rangle(T) = \sqrt{\frac{32}{4^{1/3}}\frac{E_G^{1/3}}{3\mu_{ab}}}\frac{S(x_0T)}{T^{2/3}}e^{-(3/4^{1/3})(E_G/T)^{1/3}},\tag{31}$$

where $1/\mu_{ab} = 1/m_a + 1/m_b$.

2. SBBN reaction rates with threshold

We often evaluate reverse reaction rates from the experimental data of forward reaction rates by using the detailed balance relation. For example, in a $2\to 2$ nonresonant reaction $a+b\to c+d$,

$$\frac{\langle\sigma_{\rm fusion}v\rangle_{cd}}{\langle\sigma_{\rm fusion}v\rangle_{ab}} = \left(\frac{\mu_{ab}}{\mu_{cd}}\right)^{3/2}\left(\frac{m_a+m_b}{m_c+m_d}\right)^{3/2}\frac{g_ag_b}{g_cg_d}e^{-Q/T},\tag{32}$$

where Q is the Q value of the forward reaction and g_a is the number of degrees of freedom of the light element a. Notice that the factor $e^{-Q/T}$ arises from the Boltzmann suppression for the high-energy component with $E_r > Q$ in thermal distribution.

C. Extension to BBN with the captured CHAMP

We have shown that the collision among charged CHAMP bound states will not result in any changes to SBBN. Here we focus on the nuclear-reaction rate for the collision between a bound state (CHAMP + light element) and an unbound light element.[10]

1. Forward and backward processes

Here we discuss the modifications of the short-distance nuclear-reaction rates mainly governed by the strong interaction. In CHAMP BBN (CBBN), the corresponding dominant process for the SBBN forward process $a+b\to c+d$ may be $(a,C)+b\to(c,C)+d$ or $c+d+C$, assuming (b,C) does not have a sufficiently large binding energy against scattering of background photons, i.e., $E_{\rm bin}/T \ll 40$. Here (c,C) has a larger binding energy than that of (d,C). If the following condition is satisfied,

$$Q_{\rm SBBN} - E_{{\rm bin},aC} > 0,\tag{33}$$

the final state is given by $(a,C)+b\to c+d+C$.

On the other hand, even if the above condition is not satisfied, but if the following condition is satisfied,

$$Q_{\rm CBBN} = Q_{\rm SBBN} + E_{{\rm bin},cC} - E_{{\rm bin},aC} > 0,\tag{34}$$

$(a,C)+b\to(c,C)+d$ is kinematically allowed, and the CHAMP in the final state will be trapped again. However, if the bound state (c,C) does not have enough binding energy against the destruction due to thermal photons, the (c,C) state will be destroyed soon after the process, and the element c and the CHAMP will become free.

For the $Z_C = 1$ case and the relevant nuclei, because most of the $Q_{\rm SBBN}$ values are sufficiently large, the case that $Q_{\rm SBBN} > 0$ but $Q_{\rm CBBN} < 0$ would be rare. However, in general, it might be possible. In such cases, even though the SBBN process does not have any threshold, the CBBN can have it. But the sign flip in the Q value occurs when the binding energy of a bound state with a CHAMP exceeds the nuclear binding energy of the process, which may mean that the bound CHAMP is not a spectator in the nuclear reaction any more. In the following analysis, we do not consider these kinds of special cases.

Next we simply assume that $Q_{\rm SBBN}Q_{\rm CBBN} > 0$. Let us consider the reverse processes of $a+b\to c+d$ in CBBN, which has a threshold characterized by $Q_{\rm CBBN}$. Then, the possible dominant process would be the SBBN process $c+d\to a+b$ if (c,C) and (d,C) are not stable against scattering off the background photons. In addition, $(c,C)+d\to(a,C)+b$ can also be another dominant

[10]For the case of scatterings among neutral bound states, the collision can easily occur. In such cases the calculation is straightforward.

process if (d, C) is not stable in the thermal bath, assuming, for simplicity, that (a, C) has a larger binding energy than (b, C).[11] In these processes, we may expect a Boltzmann suppression factor in the reaction rate $e^{-Q_{\rm CBBN}/T}$, not $e^{-Q_{\rm SBBN}/T}$ in a similar fashion in SBBN.

If the SBBN strong interaction $a + b \to c + d$ occurs at a shorter time scale than the typical time scale of electromagnetic (EM) interactions of bound states, we may expect that such a short-distance reaction rate should not be deviated from the SBBN rate. For D, T, He, Li, Be, etc., this condition can be realized easily.

2. Flux

In general, the velocity $V_{\rm flux}$ which controls the flux might be different from the velocity $V_{\rm reac}$ which controls the short-distance nuclear reaction. The $\langle \sigma_{\rm fusion} V_{\rm flux} \rangle$ would be given by

$$\langle \sigma_{\rm fusion} V_{\rm flux} \rangle = \left(\sigma_S \frac{V_{\rm flux}}{V_{\rm reac}} + \sigma_P (V_{\rm flux} V_{\rm reac}) \ldots \right). \qquad (35)$$

Here, we assume that, for the short-distance reactions, the coefficients, σ_S, σ_P ..., in CBBN are the same as in SBBN.[12] Using this approximation, we evaluate the flux.

First, we consider collisions between a bound state and a free light element. Then, once we focus on the $2 \to 2$ collision between the bound and the free light element, the relative velocity V_1 may be dominated by the speed of the bound light element. If we assume that the free light element is distributed uniformly in the thermal bath, the flux is controlled by V_1. On the other hand, in the case that the radius of the bound state is smaller than the impact parameter of nuclear reactions [which is $O(1/m_\pi)$], the flux has to be estimated by the relative velocity between the bound state and the free light element, which is controlled by the relative velocity. But, even in such cases, $V_{\rm flux} \sim V_{\rm reac}$ due to the following consideration. Taking $V_{\rm reac} = V_1$, while the free element goes through the target volume, the bound light element rotates with the speed $V_1 = \sqrt{2E_{\rm bin}/m_X}$. Then the number of rotations would be $\sim V_1 \Delta t / 2\pi r_B \sim O(V_1/V_2)$, where $\Delta t \sim 2 r_B / V_2$ is the time for the free light element to go through the bound light element, V_2 is the velocity of the free light element, and r_B is the radius of the bound state.[13] Then, for the nuclear reaction due to pion exchange, if we take $V_{\rm reac} = V_1$, the flux is the relative velocity V_2 times $O(V_1/V_2)$, which would be $\sim V_1$.

Next, we consider collisions between a neutral bound state a and a neutral (or charged) bound state b. In this case, since the target is not a freely propagating particle, the speed which controls the flux is not the bound light element's $V_1 \sim V_a + V_b$ but the relative velocity V_2 between the bound states. V_2 is of order of the thermal velocity of the bound state, which is smaller than V_1. Then $V_2/V_1 \sim O(0.1)$ at around $T = 1$ keV, where neutral bound states can be formed. However, while the bound states collide with each other, the bound element would rotate around a CHAMP $\sim (V_1/V_2)$ times. Therefore, even in this case, we could estimate $V_{\rm flux} \sim V_1$.

These considerations imply that we can simply assume that the CHAMP in the bound states is a spectator and $V_{\rm reac} \simeq V_{\rm flux}$.[14]

3. Corrections for BBN nuclear-reaction rates with no threshold

Here we consider the nuclear reactions containing light elements captured by CHAMPs. In this case, as we mentioned before, the crucial differences from SBBN are in the Coulomb factor and the Boltzmann suppression. Since the radius of the bound state is very small $O(1/m_\pi)$, a simple replacement $Z_X \to Z_{(X,C)} = Z_X - Z_C$ in the Coulomb factor would be a good approximation. However, the short-distance part is also changed because the light element captured by a CHAMP has the kinetic energy $E_{\rm bin}$ not $O(T)$. As we mentioned before, we assume that the short-distance cross section $\sigma^C_{\rm fusion}$ takes the same functional form of the CM energy as those of SBBN, $\sigma_{\rm fusion}$. Thus the CM energy of the short-distance nuclear reaction may be $O(\max(E_{\rm bin}, E_0))$. We introduce these two changes in the estimation of nuclear-reaction rates. That is,

$$\begin{aligned} \sigma^C_{\rm fusion} V &= (\sigma^C_S + \sigma^C_P V^2 + \ldots) F_{(aC)b}(\beta) \\ &\simeq \sigma_0 v(V) \frac{2\pi Z_{(aC)} Z_b \alpha}{\beta} e^{-2\pi Z_{(aC)} Z_b \alpha / \beta}, \end{aligned} \qquad (36)$$

where β is the relative velocity between the bound state and the incident thermal light element, and V is the relative velocity between the bound light element and the incident thermal light element with $E = E_0$. β controls the amount of penetration in the Coulomb potential. V appears in the flux and the short-distance cross section.

[11] $(c, C) + d \to a + b + C$ is also possible if it is kinematically allowed.

[12] If the phase space is modified by the release of a CHAMP after the reaction, the difference from the SBBN case would also be small if the Q value is large.

[13] This discussion relies on an assumption that the factorization of Coulomb factor and short-distance nuclear fusion is valid. That is, we assumed that, in the collision, the bound state is not destroyed before the collision. This would be valid if $r_B \sim 1/m_\pi$. If the bound state is unstable against the incident nucleus, the effective $V_{\rm flux}/V_{\rm reac}$ may become smaller than unity.

[14] Our consideration is based on our approximation that the short-distance reaction is the same as that of the SBBN $2 \to 2$ process between light elements. In the case that the De-Broglie wavelength of an incoming nucleus is longer than the Bohr radius of the bound state, we may have to solve quantum mechanical many-body problems including a bound CHAMP to obtain a more reliable result.

Since the short-distance cross section would be governed by the kinetic energy of the bound light element which does not depend on the condition of the thermal bath much, the thermal average should be taken only for the Coulomb part which implies the evaluation of the wave function for an incident thermal light element at the position of a bound state. Then, the thermal average may be taken for the thermal light elements and the thermal bound state because the incident thermal light element approaches inside the Coulomb field of the bound state, not that of bound light elements. We assume that the short-distance reaction is faster than the EM interaction of the bound state. The thermal-averaged cross section is calculated as follows:

$$\begin{aligned}\langle\sigma^C_{\rm fusion}V\rangle &= \frac{g^2}{(2\pi)^6 n_b n_{(aC)}}\int d^3p_{(aC)}d^3p_b\sigma_0 v(V)\\ &\quad\times\frac{2\pi Z_b Z_{(aC)}\alpha}{\beta}e^{-2\pi Z_b Z_{(aC)}\alpha/\beta}e^{-(E_{(aC)}+E_b)/T}\\ &= \frac{g}{(2\pi)^3 n_r}\int d^3p_r\sigma_0 v(V)\frac{2\pi Z_b Z_{(aC)}\alpha\mu_{(aC)b}}{p_r}\\ &\quad\times e^{-2\pi Z_b Z_{(aC)}\alpha\mu_{(aC)b}/p_r}e^{-E_r/T}\\ &= \frac{8\pi g T\mu_{(aC)b}}{(2\pi)^3 n_r}\int dy S(yT)_{\rm New}e^{-(y+\sqrt{y_G/y})}, \quad (37)\end{aligned}$$

where $S(y_X T)_{\rm New} = \sigma_0 v(V)\pi Z_b Z_{(aC)}\alpha\mu_{(aC)b}$, $y_G = 2\pi^2 Z_b^2 Z_{(aC)}^2\mu_{(aC)b}\alpha^2/T$, $y_0 = (y_G/4)^{1/3}$, and $\mu_{(aC)b} = m_{(aC)}m_b/(m_{(aC)}+m_b)\simeq m_b$. Notice that we are assuming that the short-distance nuclear cross sections have the same functional forms of the CM energy as those of SBBN.

Here, in the case of $Z_{(aC)}\neq 0$, we relate the new S factor above to the SBBN S factor which could be measured by experiments,

$$S(y_X T)_{\rm New} = S(y_X T)\frac{Z_{(aC)}}{Z_a}\frac{\mu_{(aC)b}}{\mu_{ab}}. \quad (38)$$

Then we find

$$\begin{aligned}\langle\sigma^C_{\rm fusion}V\rangle(T) &= \sqrt{\frac{32}{4^{1/3}}\frac{\tilde{E}_G^{1/3}}{3\mu_{(aC)b}}}\frac{S(y_X T)_{\rm New}}{T^{2/3}}\\ &\quad\times e^{-(3/4^{1/3})(\tilde{E}_G/T)^{1/3}}, \quad (39)\end{aligned}$$

where $y_X \simeq (((\mu_{ab}/m_a)E_{\rm bin})+\tilde{E}_0)/T \sim (E_{\rm bin}+\tilde{E}_0)/T)$, $\tilde{E}_G = 2\pi^2 Z_b^2 Z_{(aC)}^2\mu_{(aC)b}\alpha^2$, and $\tilde{E}_0 = Ty_0$.

For nuclear-reaction rates with neutrons like ^{7}Be(n, p)^{7}Li, since there is no Coulomb suppression or Boltzmann suppression if there is no threshold in the process $[(aC)+n\to(cC)+d]$, we replace CM energy by $E_{\rm CM} = (\mu_{ab}/\mu_{(aC)b})E_{\rm bin}+3T/2\sim E_{\rm bin}+3T/2$ in the cross sections because of the change of the kinematics of the bound light elements. In addition, if the bound state is neutral ($Z_{(aC)}=0$), the Coulomb factor may disappear if the bound state is not destroyed before the collision. Then the treatments may be similar to the neutron case above. Such neutral bound states will be formed in the case of $Z_C = 1$ (proton, D, and T).

In the above discussions, we have taken the approximations that the light element is pointlike and does not have an internal structure, and the selection rules in the nuclear reactions are not changed by the trapped CHAMP.[15]

4. Corrections for BBN nuclear-reaction rates with threshold

Let us consider a case where the SBBN reverse process $c+d\to a+b$ has a threshold and the SBBN cross section of $a+b\to c+d$ can be measured by collider experiments.

First, assuming the condition $E_{{\rm bin}(aC):1{\rm S}} < Q_{{\rm SBBN};ab,cd} < E_{{\rm bin}(aC):1{\rm S}} + E_{\rm bin:1E}$ is satisfied where $E_{{\rm bin}(aC):1{\rm S}}$, $E_{\rm bin:1E}$ are the binding energies of the 1S state of the (a, C) system and of the first excited level of the (c, C) system, we can estimate the cross section of $(c, C)+d\to(a, C)+b$ by using the information of the SBBN forward process $a+b\to c+d$. Under the above conditions, we may use the detailed balance relation on $(a, C)+b\to(c, C)+d$ in a fashion similar to the previous discussion. The thermal-averaged cross section of $(c, C)+d\to(a, C)+b$ may be written as follows. Applying the detailed balance relation and the modifications for the forward process which was previously discussed,

$$\begin{aligned}\langle\sigma^C_{{\rm fusion},(cC)d}V\rangle &\simeq \frac{g_{(aC)}g_b}{g_{(cC)}g_d}\left(\frac{m_b}{m_d}\right)^{3/2}\\ &\quad\times\langle\sigma_{{\rm fusion},(aC)b}V\rangle e^{-(Q_{{\rm CBBN}ab,cd}/T)} \quad (40)\end{aligned}$$

where $Q_{{\rm CBBN}ab,cd} = Q_{(aC)b,(cC)d}$. If $Q_{{\rm CBBN}ab,cd}$ is small, the Boltzmann suppression might disappear even though SBBN has a large Boltzmann suppression.

For $Q_{{\rm CBBN}:ab,cd} > E_{{\rm bin}(cC):2{\rm E}}$ where $E_{{\rm bin}(cC):2{\rm E}}$ is the binding energy of the second excited level of the bound state, we would not be able to simply apply the detailed balance relation for the forward process. But, in any case, since the crucial point for the processes with a threshold is the Boltzmann suppression which comes from the requirement that the kinetic energy of the incident particles overcomes the threshold, if the Q value is smaller than that of

[15] We have also assumed the hierarchy between the SBBN strong reactions and the EM interactions of the bound states. The Bohr radius of the CHAMP-light element system and the typical pion-exchange radius would be the same order of magnitude in our case, but we can still expect a hierarchy in the coupling strengths between the EM and the strong interaction, which may still allow us to factorize the short-distance nuclear reaction from the effects caused by binding a CHAMP. But if the incoming nucleus is very slow, this factorization may break down due to the long range nature of the EM force.

SBBN, we may expect the milder Boltzmann suppression in the process, compared to that of SBBN.[16]

In the $Z_C = 1$ case, at a relevant time when capture becomes efficient, the Boltzmann suppression is huge if the Q value is O(MeV), and then most of the BBN processes are completely decoupled. Hence we ignore the change of nuclear-reaction rates for the SBBN reverse processes if $Q_{\rm CBBN}$ is $\sim\mathcal{O}(1)$ MeV, which is a reasonable assumption.

Next, we consider the reverse process in CBBN, which corresponds to SBBN $a + b \rightarrow c + \gamma$, i.e., $(c, C) + \gamma \rightarrow (a, C) + b$ assuming that the binding energy of (a, C) is smaller than that of (b, C). It is well known that the reaction rate of this forward process is small. Notice that the incident photon with the threshold energy of the process does not have Coulomb suppression. Thus the main origin of suppression is the low abundance of the higher energy components of thermal photons.

$$\begin{aligned}\langle\sigma^C_{{\rm fusion},c\gamma}V\rangle &= \frac{8\pi g_\gamma}{(2\pi)^3 n_r}\\ &\quad\times\int dE_{r,(cC)\gamma}E^2_{r,(cC)\gamma}\sigma_{0,c\gamma}v(V)e^{-E_{r,(cC)\gamma}/T}\\ &\simeq \frac{8\pi g_\gamma}{(2\pi)^3 n_\gamma}\int_{Q_{\rm CBBN}}^{\infty} dp_\gamma p_\gamma^2\sigma_{0,c\gamma}v(V)e^{-p_\gamma/T}\\ &\simeq \frac{1}{n_\gamma}\left(\frac{\mu_{(aC)b}T}{2\pi}\right)^{3/2}O(\langle\sigma_{{\rm fusion},(aC)b}v\rangle)e^{-Q_{\rm CBBN}/T},\end{aligned} \tag{41}$$

where $E_{r,(cC)\gamma} = p_\gamma$. Although the Q value for the process $^3{\rm He}(\alpha,\gamma)^7{\rm Be}$ might be smaller than 1 MeV, the process is negligible at the capture time of ^{7}Be, and this reverse process does not seem to provide a significant change from SBBN. The change in the threshold energy of these photodissociation processes might be important when we consider the late-decay effects that the injected high-energy EM energy is thermalized and produces a huge number of soft photons, which may destroy primordial light elements.

VI. BBN WITH LONG-LIVED CHAMPS

Recently, WMAP has reported the updated values of cosmological parameters under the standard ΛCDM models. We can now check the internal consistency of SBBN in the light of WMAP3. It has been pointed out that the predicted ^{7}Li abundance seems too high to agree with observed abundances. Also, for ^{6}Li, we have to expect an additional production after the BBN epoch, like cosmic-ray nucleosynthesis. These tensions or discrepancies may be tantalizing clues to find new physics. Under these circumstances, it is interesting to study the effects of new physics.

In previous sections, we considered the possible changes of nuclear-reaction rates due to long-lived CHAMPs. Here we consider the application for the BBN in the case of $Z_C = 1$.

A. Charged massive-particle BBN

We consider the thermal freeze-out of light element abundances in CBBN, and here we simply ignore the effects of possible high-energy injections due to the late decay of CHAMPs, which may provide the initial condition to consider such late-decay phenomenon if the decay occurs long enough after the decoupling of the BBN processes. We will later discuss the case where the decays occur before the freeze-out. In our estimation, we also assume the instantaneous captures for each light element at $T_c = E_{\rm bin}/40$.[17]

In SBBN, abundances of all light elements are completely frozen until $T \sim 30$ keV. Since T_c is 24 keV for ^{7}Li and 38 keV for ^{8}Be, which is almost the end of SBBN, the formations of bound states may change their abundances. For elements lighter than ^{6}Li, since the efficient captures occur only at below 10 keV, we found that the change of nuclear reactions cannot recover the processes at such a low temperature. This conclusion will hold if the difference from our estimation of $\langle\sigma_{\rm fusion}V\rangle$ is not large. Also, in most of the reverse processes, the Boltzmann suppressions are huge at that time, even though we use the new Q value $Q_{\rm CBBN}$. They do not provide any significant change from SBBN.

Under these circumstances, if the CHAMPs decay before the captures of $Z_X = 1$ nuclei, we may expect that the sizable change due to the captures occurs in elements heavier than ^{7}Li. On the other hand, once the capture of the proton, D, and T starts, since the bound states are neutral and have no Coulomb suppressions in the nuclear reactions, the BBN processes may not freeze out. In the next subsection, first of all, we consider the case that CHAMPs decay before the captures of $Z_X = 1$ elements such as protons, D, or T, which start at below $T \lesssim 1$–2 keV ($t \gtrsim 10^6$ sec). Later we consider the possible effects due to their captures.

Since the abundances of the light elements differ by orders of magnitude, often we can identify the relevant processes and neglect the others. For example, when we are considering a process $a(b, c)d$, if n_a is much smaller than

[16]Here we naively assumed $Q_{{\rm CBBN}:ab,cd} > E_0$ where E_0 is the Gamow peak energy of the reverse processes. If this condition is not satisfied, we may need more careful treatments.

[17]As we mentioned before, if the number density of CHAMPs is low, $n_{\rm CHAMP}/n_\gamma \ll 10^{-11}$, the recombination rate might not be sufficiently large compared with the expansion rate of the Universe, and we may expect poor captures of CHAMPs. Then most of the CHAMPs and light elements will be left as freely propagating ionized particles. Because CHAMPs are supposed to decay soon, in this case we can apply the known results in decaying particle scenarios in the literature.

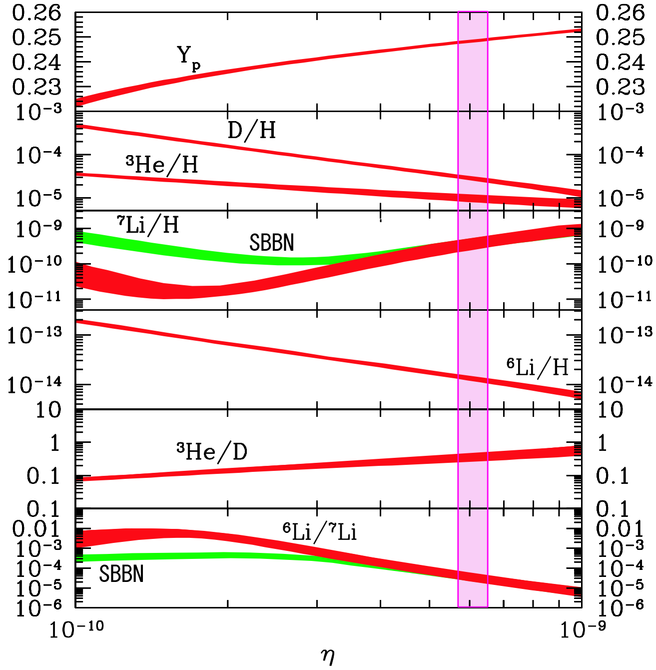

FIG. 3 (color online). Theoretical predictions of Y_p, D/H, ^{3}He/H, ^{7}Li/H ^{6}Li/H, ^{3}He/D, and ^{6}Li/^{7}Li as a function of η in standard BBN (green) and CHAMP BBN in case A (red). Here we have assumed the instantaneous capture of CHAMPs and $n_C/n_\gamma = 3.0 \times 10^{-11}$.

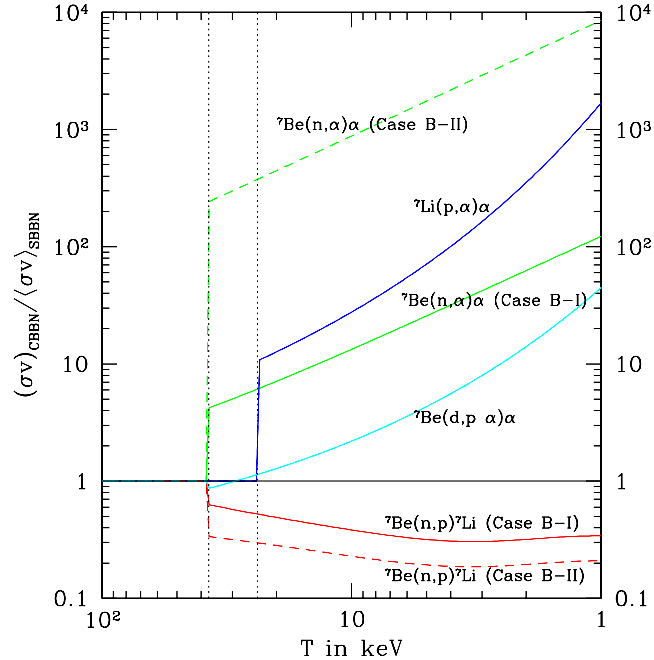

FIG. 4 (color online). Ratio of nuclear-reaction rates of SBBN and CBBN in cases B-I and B-II as a function of the cosmic temperature for the relevant processes. Here we assumed the instantaneous capture of CHAMPs by the nuclei. Case B-I means that $E_{CM} = (\mu_{ab}/\mu_{(aC)b})E_{bin} + E_0$ in a process $(a, C) + b \to (c, C) + d$, where we take E_0 to be the Gamow peak energy for collisions between two charged elements, and to be $3T/2$ for collisions between a nucleus and a neutron. Case B-II means that we take $E_{CM} = E_{bin} + E_0$ as the CM energy of processes and a 10 times larger value of the p-wave part of the cross section of ^{7}Be(n, α)^{4}He than that in the standard BBN code [52,87].

the others (n_b, n_c, and n_d), this process is negligible for the evolutions of n_b, n_c, and n_d, but important only for n_a. Therefore elements heavier than ^{7}Be do not significantly affect lighter element abundances.

1. CBBN with $Z_X > 1$

Here we consider the CBBN with captures of $Z_X > 1$ nuclei (with $Z_C = 1$). This case will be realized if the CHAMP lifetime is shorter than $\sim 10^6$ sec. In Fig. 3, we show a plot of the light element abundances as a function of η, including the corrections only in processes among charged light elements (case A). We can find that the ^{7}Li abundance could decrease much from the SBBN value for η. The decrease is induced by the enhancement of the ^{7}Li(p, α)^{4}He reaction rate due to the capture of ^{7}Li by CHAMPs. As we can see in Fig. 4, the CBBN reaction rate of ^{7}Li(p, α)^{4}He slowly decreases as a function of the energy, compared to that of SBBN at the temperature where the Coulomb suppression becomes important, which results in later-time decoupling of the process than in SBBN.

We also added processes ^{7}Be(n, p)^{7}Li and ^{7}Be(n, α)^{4}He, which are associated with neutron capture (case B). In these types of processes, the important change from SBBN is the kinetic energy to be used in the nuclear reaction. In SBBN, the typical energy is $\sim 3T/2$. However, in CBBN, the energy could be $O(E_{bin})$. If the s-wave partial wave mode dominates the process, then the difference might be small. However, if higher partial modes such as p waves dominate, we expect significant enhancements of the processes.

In fact, we found that the change in ^{7}Be + ^{7}Li by the modification of the process ^{7}Be(n, p)^{7}Li is negligible. On the other hand, since ^{7}Be(n, α)^{4}He is a p-wave dominant process [87], the modification of this process should be important to predict the primordial abundance of ^{7}Be + ^{7}Li in CBBN. Unfortunately, we currently only have poor experimental data sets for ^{7}Be(n, α)^{4}He. However, since there is experimental data for the reverse process ^{4}He(α, n)^{7}Be [88], we might be able to theoretically infer the cross section of the forward process of ^{7}Be(n, α)^{4}He approximately by using detailed balance relations. For the moment, however, the experimental data do not have sufficient resolutions in the relevant energy region, because of the significant Coulomb suppression and the threshold suppression, to correctly calculate the forward rate. Therefore, according to Serpico *et al.* [52], as a conservative error we also take a factor of 10 on the process in this paper, which does not change the SBBN predictions at all and is still consistent with available experimental data of the reverse rate [88].

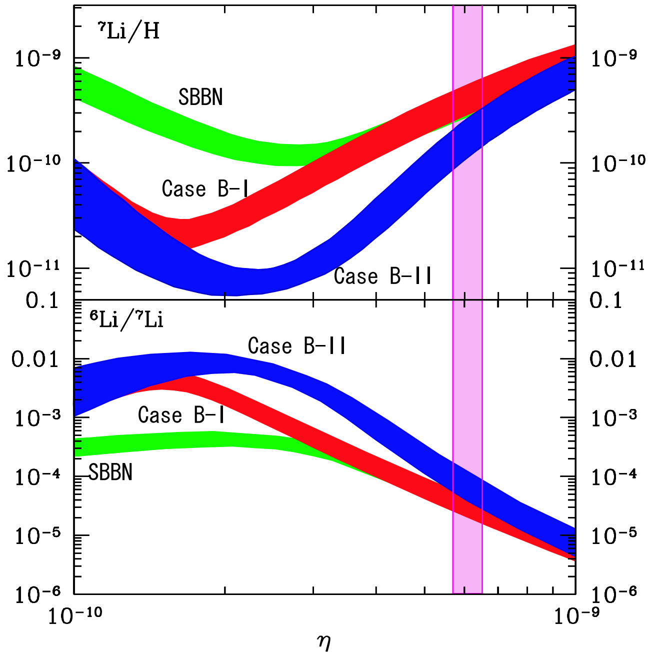

FIG. 5 (color online). Theoretical prediction of ^{7}Li/H (upper panel) and ^{6}Li/^{7}Li (lower panel) as a function of the baryon-to-photon ratio. The SBBN predictions are marked by the green bands. The red (blue) band is for case B-I (case B-II) in CBBN. Here we assumed $n_C/n_\gamma = 3.0 \times 10^{-11}$ and the instantaneous capture of CHAMPs. The definitions of case B-I and case B-II are the same as those in Fig. 4.

In Fig. 5, we plot the theoretical prediction of ^{7}Li/H (upper panel) and ^{6}Li/^{7}Li (lower panel) as a function of η. The SBBN predictions are marked by the green bands. The red (blue) band is for case B-I (case B-II) in CBBN. Here we assumed $n_C/n_\gamma = 3.0 \times 10^{-11}$ and the instantaneous capture of CHAMPs. Case B-I means that $E_{CM} = (\mu_{ab}/\mu_{(aC)b})E_{bin} + E_0$ in a process $(a, C) + b \to (c, C) + d$ where we take E_0 to be the Gamow peak energy for collisions between two charged elements, and to be $3T/2$ for collisions between a nucleus and a neutron. Case B-II means that we take $E_{CM} = E_{bin} + E_0$ as the CM energy of processes and a 10 times larger value of the p-wave part of the cross section of ^{7}Be(n, α)^{4}He than that in the standard BBN code [52,87]. In Fig. 5, it is shown that the modification by a factor of 10 on the p-wave partial cross section of ^{7}Be(n, α)^{4}He does not change the SBBN prediction (case B-I) but must be important in CBBN (case B-II).

We have also checked the reverse process of ^{10}B(p, α)^{7}Be. The threshold in this process can become smaller, which may induce milder Boltzmann suppression than that of SBBN. However, we found that this rate is simultaneously suppressed strongly by the Coulomb factor, and therefore this effect is irrelevant.

Finally we warn the readers again that our results rely on the assumption that the short-distance nuclear-reaction rates have the same functional form of the CM energy as those of SBBN. In addition, we assume that, by relevant elements, the energy to excite nucleons into higher levels and the binding energy by a CHAMP are of the same order of magnitude. To obtain a quantitative conclusion, further efforts to estimate the errors in the short-distance nuclear-reaction rates must be important. For example, in ^{7}Be(n, p)^{7}Li, the change of the nuclear-reaction rate can directly affect the final abundance of ^{7}Li(= ^{7}Li + ^{7}Be). However, notice that well before the elements lighter than Li are captured by CHAMPs, the SBBN processes are completely decoupled. Even though the errors induce a larger reaction rate, if it were within an order of magnitude level, nuclear reaction would not overcome the expansion rate again and our conclusion would not be changed, because the Coulomb suppression is significant and the neutron abundance is very small.

TABLE II. Table of typical values for nuclear-reaction rates with a captured proton, D, and T. Here, we ignored nuclear-reaction rates related to the neutron because they are negligibly smaller for $T < T_c$. Here, we also ignored processes which include heavier elements than ^{7}Be in the initial state.

Process (p, C)	Reaction rate (cm^3/ sec /mol)
^{7}Be((p, C), γ)(^{8}B, C)	(4–24) $\times 10^2$
^{7}Li((p, C), α)(^{4}He + C)	3×10^6
^{6}Li((p, C), α)(^{3}He + C)	1×10^8
^{6}Li((p, C), γ)(^{7}Be + C)	3×10^3
T((p, C), γ)(^{4}He + C)	2×10^2
D((p, C), γ)(^{3}He + C)	40
Process (D, C)	Reaction rate (cm^3/ sec /mol)
(D, C)(p, γ)(^{3}He + C)	40
(D, C)(α, γ)(^{6}Li + C)	0.6
(D, C)(d, n)(^{3}He + C)	7×10^6
(D, C)(d, p)(T + C)	4×10^6
T((d, C), n)(^{4}He + C)	1×10^9
^{3}He((d, C), p)(^{4}He + C)	4×10^8
^{7}Li((d, C), nα)(^{4}He + C)	1×10^8
^{7}Be((d, C), pα)(^{4}He + C)	3×10^8
Process (T, C)	Reaction rate (cm^3/ sec /mol)
(T, C)(p, γ)(^{3}He + C)	2×10^2
(T, C)(d, n)(^{4}He + C)	1×10^9
(T, C)(α, γ)(^{7}Li + C)	2×10^3

2. CBBN with $Z_X = 1$

Next we discuss the possible effects due to captures of $Z_X = 1$ nuclei. Since the bound states are neutral, the nuclear reactions in Table II may not have Coulomb suppression and might be significantly changed from those of SBBN.[18]

[18]Notice that here we simply assumed that the bound state is not significantly disturbed before the nuclear fusion reactions.

We consider the case that $Z_C = 1$ nuclei are captured instantaneously at temperatures below each $T_C(\sim O(1)$ keV). The captures of T only provide a significant change in T itself and ^{7}Li even if we assume instantaneous captures because of their poor abundances. The captures of D result in large enhancements for the processes listed in Table II. In particular, since T(d, n)^{4}He, ^{3}He(d, p)^{4}He, ^{7}Li(d, nα)^{4}He, and ^{7}Be(d, pα)^{4}He have large cross sections, the reaction rates may be able to become larger than the expansion rate again at a later time. Their decoupling does not occur soon because of the absence of the Coulomb suppressions. If the captured D abundance is larger than ^{3}He, D and ^{3}He mainly burn into ^{4}He through ^{3}He(d, p)^{4}He. Then the abundance of ^{3}He can decrease, and the abundance of D becomes close to $n_{\rm D} - n_{^3{\rm He}}$. On the other hand, however, D(d,p)T and D(d, n)^{3}He do not change the abundance of D so much. In addition, T(d, n)^{4}He, ^{7}Li(d, nα)^{4}He, and ^{7}Be(d, pα)^{4}He do not a change the abundance of D either, but might decrease the abundances of T, ^{7}Li, and ^{7}Be because of the small abundances compared with that of D.

Although the process D(α, γ)^{6}Li has a very small reaction rate in SBBN since the abundances of the incident particles (D and ^{4}He) are sufficiently large, this process can produce a large amount of ^{6}Li.[19] The capture of protons can reduce the ^{6}Li and ^{7}Li abundances through ^{7}Li(p, α)^{4}He and ^{6}Li(p, α)^{3}He. On the other hand, there are no significant changes in D, T, and ^{7}Be abundances because the associated processes are radiative ones, which are relatively suppressed.

In the relevant epoch for the captures of p, D, and T [$T \lesssim \mathcal{O}(1)$ keV], the condition to overcome the expansion rate is that the reaction rates are larger than that of 10^4 cm^3/sec multiplied by the captured number density of n_p. Then, in processes T(d, n)^{4}He, ^{7}Li(d, nα)^{4}He, and ^{7}Be(d, pα)^{4}He, even if the decrease of reaction rates were within a factor of $O(10)$ due to some ambiguities such as the capture rate of D, we could still expect the decrease in ^{7}Li and ^{7}Be abundances. As we showed before, since the changes in light element abundances by the captures of $Z_X > 1$ nuclei might be small, the initial condition of light element abundances for such a later-time CBBN by captured $Z_X = 1$ nuclei might be the same as those of SBBN. However, notice that the above conclusions rely on the number density of the captured $Z_X = 1$ nuclei very much. If the number density of CHAMPs is not large, the captures weaken, and the changes become milder. For example, taking possible capture fractions, $O(10^{-5})$, $O(0.1)$, and $O(10^{-2})$ for the proton, D, and T, respectively, we show the results in Fig. 6. In such cases, the nuclear-reaction rates for ^{7}Li, ^{6}Li, and ^{7}Be become more rapid than the expansion of the Universe, and we expect that ^{7}Li and ^{7}Be decrease without changing D, ^{3}He, and ^{4}He abundances. The ^{7}Li abundance is determined by the competition between two processes, ^{7}Li(p, α)^{4}He for proton capture and T(α, γ)^{7}Li for T capture. The ^{6}Li is controlled by the production reaction D(α, γ)^{6}Li and the destruction reaction ^{6}Li(p, α)^{3}He. In the case of Fig. 6, a sizable amount of ^{6}Li is produced, and the predicted primordial value of ^{6}Li/^{7}Li approximately agrees with the observational data without assuming any chemical evolution scenarios.

On the other hand, notice that some ambiguities might still exist in the nuclear-reaction rates. For $Z_X = 1$ nuclei, because the bound state with the $Z_C = 1$ CHAMP has a larger Bohr radius than those of $Z_X > 2$ nuclei, the electromagnetic disturbance on the bound state before the nuclear fusion reactions occur would have to be more carefully considered. If the bound state is electromagnetically destroyed by an incident heavier nucleus, the factorization of the Coulomb part and the short-distance nuclear-reaction part does not work well, and the nuclear-reaction rate may be changed from the value of

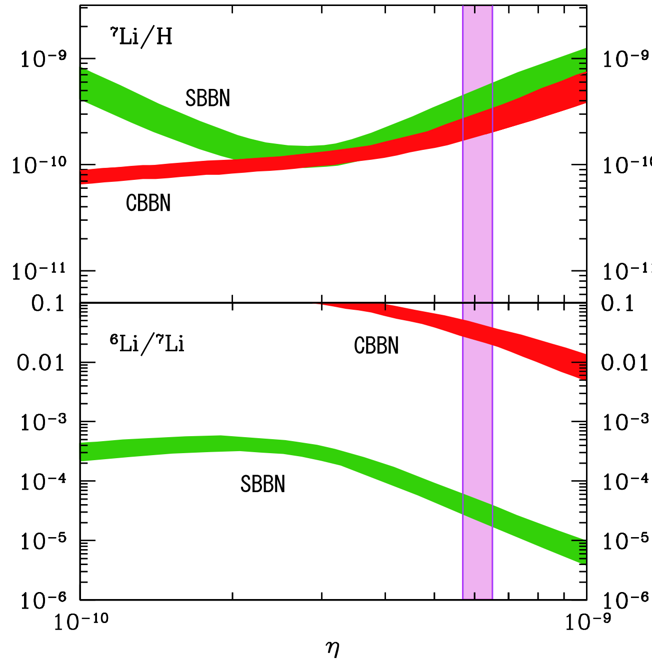

FIG. 6 (color online). Theoretical predictions of ^{7}Li/H and ^{6}Li/^{7}Li as a function of η in SBBN and CBBN with their theoretical errors at 95% C.L. Here we took the fractions of the captured proton, D, and T to be 10^{-5}, 10^{-1}, and 10^{-2}, respectively. The WMAP value of η at 95% C.L. is also indicated as a vertical band. We can find that the primordial values of ^{6}Li and ^{7}Li in the CBBN may be in the range of the observed abundances, which may simultaneously solve current ^{6}Li and ^{7}Li problems pointed out in SBBN.

[19]Recently, Pospelov pointed out that the cross section of D(α, γ)^{6}Li might be significantly enhanced by considering the virtual photon absorption due to a bound CHAMP [89], which might significantly overproduce ^{6}Li. Since his paper appeared after the completion of this work, we have not included this effect in this paper.

our calculations.[20] Also, if a large amount of CHAMPs survive until such late times ($> 10^6$ sec), we would have to simultaneously consider both effects due to the captures by the $Z_X = 1$ nuclei, and subsequently the EM energy injections by the decaying CHAMPs at a later time.

B. Late decays of long-lived CHAMPs

We have discussed the change of light element abundances before the decay of CHAMPs. On the other hand, the decaying CHAMPs might induce additional changes of primordial light element abundances, which have been studied by several groups. The effects highly depend on the decay products, i.e., electromagnetic or hadronic cascades [45,46,48,49].

At an earlier epoch before $t = 10^4$ sec, only the hadronic energy injection is important, and there is almost no constraint from EM energy injections. Therefore, at such an epoch, even though the injected energy is not small, if the branching ratio into a hadronic cascade is sufficiently suppressed, there are no significant effects on the primordial light element abundances. Such a case is well known if CHAMPs decay into leptons with a branching ratio into hadrons of the order of $\mathcal{O}(10^{-3})$–$\mathcal{O}(10^{-6})$. For late decays after 10^4 sec, the amount of energy released may be highly constrained by the EM energy injections.

In the following sections, we consider possible new changes by taking into account the capture of CHAMPs.

1. Are there corrections on the evaluation for the primary energy injection by CHAMP decays?

Since the binding energy of CHAMP bound states is below the nuclear binding energy of the light elements, the recoil of nucleons inside the captured light element due to CHAMP decays would not destroy the light element. On the other hand, the decay products of the bound CHAMP might directly hit the bound light element and destroy it. Let us consider the case that the primary decay product is a charged lepton as an example. Of course, if the lepton is a tau, the tau lepton soon decays into hadronic particles. However, the lifetime of a tau lepton is long enough to go through the Bohr radius of the bound state. Thus we will deal with all kinds of leptons in a similar fashion.

Naively, we may speculate that the light elements are distributed inside the radius $r_X^{-1} \simeq A^{-1/3} m_\pi$, and the CHAMP stays somewhere inside the radius. Then the number density of quarks (or nucleons/nuclei) inside a bound nucleus is roughly

$$n_{\mathrm{bound}\,X} \sim A\left(\frac{1}{r_X}\right)^3 \simeq m_\pi^3. \tag{42}$$

The mean free path is roughly estimated by

$$\lambda_{\mathrm{mfp}} \simeq \frac{1}{\sigma \times n_{\mathrm{bound}\,X}}, \tag{43}$$

where we have chosen $\sigma \simeq 2\pi\alpha^2/t$ where t is the Mandelstam variable t for the momentum transfer from a primary decay product (a charged lepton) to a bound light element. Then the naive probability of the primary decay product (a charged lepton) scattering off a quark (or nucleon/nucleus) inside the bound nucleus is

$$\mathrm{Prob} \simeq \frac{2r_X}{\lambda_{\mathrm{mfp}}} \sim O(10^{-9})\left[\frac{(10\ \mathrm{GeV})^2}{t}\right]\left[\frac{A^{1/3}}{2}\right]. \tag{44}$$

Among light elements, ^{4}He destruction would be most dangerous. If we assume that all ^{4}He's are completely captured by CHAMPs, if such a probability is below 10^{-4}, the change in D/H, ^{3}He/H abundances due to the direct collision will be below the $O(10^{-5})$ level, which may not disagree with observed abundances. Elements heavier than the destroyed parent nuclei should not be directly produced significantly.[21]

We can find that, if the momentum transfer from the primary decay product is hard [$t > (100\ \mathrm{MeV})^2$],[22] the light element bound by a CHAMP is sufficiently transparent and may not disturb the SBBN prediction for elements lighter than ^{4}He. In this case, for the evaluation of the primary energy injection, past studies in the literature will be a good approximation, which considered that CHAMPs are freely propagating in a thermal bath. For the secondary products through the hadronization of a recoiled quark or direct production of nucleons/nuclei, the above probability will be identified as the hadronic branching ratio for a CHAMP decay, which may provide only negligible effects on elements lighter than ^{4}He. On the other hand, we can consider another extreme case where the momentum transfer is sufficiently soft. For example, if the energy is smaller than the nuclear binding energy, the charged lepton of decay products could not inelastically scatter off the bound light element. In the middle range between them, we may have to simultaneously consider the direct collision and EM/hadronic cascade induced by the late decay which was considered before.

In the case of hadronic decays, we may replace α by the strong coupling α_s in the above estimation. Then, we find that, for a sufficiently hard momentum transfer $t > (1\ \mathrm{GeV})^2$, the bound light element is still transparent.

[20]For the collision of a neutral bound state with the $Z_X = 1$ nucleus, the formation of a molecule may be important to evaluate the nuclear-reaction rate.

[21]However, in a recent work [90], they have pointed out the possibility that energetic T and ^{3}He, which are produced from the destruction of the bound ^{4}He, can nonthermally produce sizable amount of ^{6}Li.

[22]The energy transfer due to the momentum transfer $<(100\ \mathrm{MeV})^2$ may be below the typical threshold $\sim\mathcal{O}(10)$ MeV to destroy a bound light element by NR nucleon/nuclei scattering inside the light element.

2. Other new possible corrections to BBN with late-time energy injection

There are three types of other possible effects on light element abundances by the late-time decaying CHAMPs when some fraction of such light elements are captured by CHAMPs.

The first type originates from the change in the Coulomb barrier and the kinematics of background light elements as targets for nonthermal processes by their own bound states. This change could be important for the hadronic-decay scenario. The injected high-energy hadrons eventually lose their energy due to the thermal interactions and become nonrelativistic and collide with the background light elements. If the target nuclei are captured by CHAMPs, the reaction rates for various hadronic processes might be different from experimental values.

The second type is related to the change of the Q value. This might be important for both high-energy hadronic and EM energy injections, especially in high-energy photon injections. The injected high-energy photons produce many soft photons through the EM cascade before the scattering off background light elements. Then the spectrum of the soft photons has a cutoff at the energy above the threshold of electron-positron pair creation, which depends on the cosmic time or the cosmic temperature. Only when the cutoff energy is higher than the threshold energy of the photodissociation are the target nuclei destroyed. The change in the Q value may modify the epoch when the light element is destroyed by the photodissociation processes.[23]

The third type is related to neutron injection from late decays. If the decay occurs while some of the SBBN processes are still active, the high-energy hadronic injections might produce many neutrons. At late time ($t > 100$ sec), since neutrons have an extremely low abundance due to the β decay, the produced neutrons can significantly affect the light element abundances because the related nuclear reactions do not have a Coulomb suppression. The neutron injection at around 10^3 sec was discussed as a solution to obtain low ^{7}Be abundance by the destruction of SBBN ^{7}Be through ^{7}Be(n, p)^{7}Li and subsequently ^{7}Li(p, α)^{4}He [28,46,91]. In our scenario, there may exist some differences from the previous studies. As we mentioned before, in CBBN, ^{7}Be(n, α)^{4}He could be more important for the ^{7}Be abundance than ^{7}Be(n, p)^{7}Li because the center-of-mass energy in the process can be completely different from that of SBBN. Thus the neutron injections from the late decaying CHAMPs may enhance the destruction ability of ^{7}Be, and the effects could be different from the noncaptured case. Notice also that there may still exist unknown errors even on the reaction rate of ^{7}Be(n, p)^{7}Li with captured ^{7}Be, as was discussed before.

[23] On the other hand, we may also have to take care of the destruction of the bound state due to huge soft photons from high-energy photon injection.

The modification of the reaction rate of ^{3}He(d, p)^{4}He might be interesting with the decaying CHAMP scenario below 1 keV. If a sizable fraction of D is captured and the reaction rate of the process is enhanced due to the capture of a CHAMP, the destruction rate of ^{3}He might become more rapid than the production due to late decays of CHAMPs, which may weaken the bound on ^{3}He production due to the decay. This new possibility may relax the ^{3}He/D bound, which is generally the most severe constraint on radiatively or hadronically decaying massive-particle scenarios at $t > 10^7$ sec.

Considering the above possibilities, it is important to reanalyze the effect of the late-time decaying CHAMPs in their bound states with the light elements [86].

VII. DISCUSSIONS

If the CBBN prediction is not significantly disturbed by late decays, the superWIMP dark-matter scenario would be interesting. Here we discuss how much relic of CHAMPs might be allowed in this scenario. Since the number density of CHAMPs is important to evaluate the capture rate of the light element, we consider the possible constraints on the number density of CHAMPs, assuming that the whole dark matter originates from the two body decay of CHAMPs into a dark-matter particle and a SM particle. We consider the free streaming by the whole dark matter produced from CHAMP decays. The relic density of CHAMP is

$$\Omega_C = \frac{m_C}{m_{\rm DM}} \Omega_{\rm DM}. \tag{45}$$

As we found before, the capture rate is governed by the number density of the CHAMP.

$$\frac{n_C}{n_\gamma} = \frac{n_{\rm DM}}{n_\gamma} = 3 \times 10^{-11} \frac{100\ {\rm GeV}}{m_{\rm DM}} \frac{\Omega_{\rm DM}}{0.23}. \tag{46}$$

Hence the lighter mass of the dark matter allows larger CHAMP abundance. Since keV warm dark matter is still allowed from Lyα data [92], we naively require that the dark matter is nonrelativistic at $T =$ keV. Then we find the following condition:

$$u < 1.0 \sqrt{\frac{10^6\ {\rm sec}}{t}}, \tag{47}$$

where $u = \sqrt{|p^i p_i|}/m$ and p_i is the three-momentum of dark matter. Assuming the two body decay, the four-velocity at the decay time is $u = (m_{\rm CHAMP}^2 - m_{\rm DM}^2)/2m_{\rm DM}m_{\rm CHAMP}$. Then we find that, for a lifetime $\sim 10^4$ sec, $u \sim 20$ may be allowed. Then it is possible to take $n_{\rm CHAMP}/n_\gamma \sim O(10^{-9})$, which will lead to a considerable capture rate.

For the case that the decaying CHAMPs contribute only to part of the dark matter, or their contribution is negligible, the above constraint may not be applicable.

VIII. CONCLUSION

In this paper, we have discussed the role of long-lived charged particles during/after the BBN epoch. We found that the existence of CHAMPs during the BBN epoch can change the light element abundances if the capture rate of CHAMPs by light elements is sufficiently large. Since the bound state for heavier elements tends to be more stable against the destruction by the background photon, the abundances are modified only for heavier elements such as Li and Be, thanks to the capture at an earlier time before the nuclear reactions decouple. On the other hand, the abundances of lighter elements such as D, T, ^{3}He, and ^{4}He are unchanged. In fact, even though more work needs to be done to find quantitative results, we have shown that the capture of CHAMPs may possibly have some impact on the BBN prediction of the primordial ^{7}Li abundance. Our approach to consider the cosmological effects of the formation of the CHAMP bound states should also be attractive in some particle physics models [93,94].

To understand CBBN more correctly, we need to understand the nuclear fusion rates and the capture rates more precisely. However, unfortunately there are still some uncertainties in the experimental data of the reaction rates at present. We expect that the future nuclear experiments will clarify these points. If future collider experiments find a signal of long-lived charged particles inside the detector, the measurement of the lifetime and decay properties of the charged particles will provide new insights to understand the phenomena in the early Universe, in turn.

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[1] D. N. Spergel *et al.* (WMAP Collaboration), Astrophys. J. Suppl. Ser. **148**, 175 (2003).
[2] http://lambda.gsfc.nasa.gov/product/map/.
[3] H. Goldberg, Phys. Rev. Lett. **50**, 1419 (1983); J. R. Ellis, J. S. Hagelin, D. V. Nanopoulos, K. A. Olive, and M. Srednicki, Nucl. Phys. **B238**, 453 (1984).
[4] J. L. Feng, A. Rajaraman, and F. Takayama, Phys. Rev. Lett. **91**, 011302 (2003); Phys. Rev. D **68**, 085018 (2003); J. L. Feng, S. f. Su, and F. Takayama, Phys. Rev. D **70**, 063514 (2004).
[5] H. Pagels and J. R. Primack, Phys. Rev. Lett. **48**, 223 (1982); S. Weinberg, Phys. Rev. Lett. **48**, 1303 (1982); L. M. Krauss, Nucl. Phys. **B227**, 556 (1983); D. V. Nanopoulos, K. A. Olive, and M. Srednicki, Phys. Lett. **127B**, 30 (1983); M. Y. Khlopov and A. D. Linde, Phys. Lett. **138B**, 265 (1984); J. R. Ellis, J. E. Kim, and D. V. Nanopoulos, Phys. Lett. **145B**, 181 (1984); R. Juszkiewicz, J. Silk, and A. Stebbins, Phys. Lett. **158B**, 463 (1985); T. Moroi, H. Murayama, and M. Yamaguchi, Phys. Lett. B **303**, 289 (1993).
[6] J. L. Feng, S. Su, and F. Takayama, Phys. Rev. D **70**, 075019 (2004); J. R. Ellis, K. A. Olive, Y. Santoso, and V. C. Spanos, Phys. Lett. B **588**, 7 (2004).
[7] M. Fujii, M. Ibe, and T. Yanagida, Phys. Lett. B **579**, 6 (2004); L. Roszkowski, R. Ruiz de Austri, and K. Y. Choi, J. High Energy Phys. 08 (2005) 080; D. G. Cerdeno, K. Y. Choi, K. Jedamzik, L. Roszkowski, and R. Ruiz de Austri, J. Cosmol. Astropart. Phys. 06 (2006) 005.
[8] M. Bolz, W. Buchmuller, and M. Plumacher, Phys. Lett. B **443**, 209 (1998); M. Bolz, A. Brandenburg, and W. Buchmuller, Nucl. Phys. **B606**, 518 (2001).
[9] L. Covi, J. E. Kim, and L. Roszkowski, Phys. Rev. Lett. **82**, 4180 (1999); L. Covi, H. B. Kim, J. E. Kim, and L. Roszkowski, J. High Energy Phys. 05 (2001) 033.
[10] J. A. R. Cembranos, A. Dobado, and A. L. Maroto, Phys. Rev. Lett. **90**, 241301 (2003); Phys. Rev. D **68**, 103505 (2003).
[11] G. Servant and T. M. P. Tait, Nucl. Phys. **B650**, 391 (2003).
[12] H. C. Cheng, J. L. Feng, and K. T. Matchev, Phys. Rev. Lett. **89**, 211301 (2002).
[13] A. Birkedal, A. Noble, M. Perelstein, and A. Spray, Phys. Rev. D **74**, 035002 (2006).
[14] J. Hubisz and P. Meade, Phys. Rev. D **71**, 035016 (2005).
[15] J. L. Feng, S. Su, and F. Takayama, Phys. Rev. Lett. **96**, 151802 (2006).
[16] For example, E. A. Baltz, M. Battaglia, M. E. Peskin, and T. Wizansky, Phys. Rev. D **74**, 103521 (2006).
[17] B. Moore, Nature (London) **370**, 629 (1994); R. A. Flores and J. R. Primack, Astrophys. J. **427**, L1 (1994); J. J. Binney and N. W. Evans, Mon. Not. R. Astron. Soc. **327**, L27 (2001); A. R. Zentner and J. S. Bullock, Phys. Rev. D **66**, 043003 (2002); J. D. Simon *et al.*, Astrophys. J. **621**, 757 (2005).
[18] A. A. Klypin, A. V. Kravtsov, O. Valenzuela, and F. Prada, Astrophys. J. **522**, 82 (1999); A. R. Zentner and J. S. Bullock, Astrophys. J. **598**, 49 (2003).
[19] J. F. Navarro and M. Steinmetz, Astrophys. J. **528**, 607 (2000).
[20] R. H. Cyburt, B. D. Fields, and K. A. Olive, Phys. Lett. B **567**, 227 (2003).
[21] W. B. Lin, D. H. Huang, X. Zhang, and R. H. Brandenberger, Phys. Rev. Lett. **86**, 954 (2001); J. Hisano, K. Kohri, and M. M. Nojiri, Phys. Lett. B **505**, 169 (2001); M. Fujii and K. Hamaguchi, Phys. Lett. B **525**, 143 (2002); R. Kitano and I. Low, arXiv:hep-ph/0503112.
[22] S. Borgani, A. Masiero, and M. Yamaguchi, Phys. Lett. B

386, 189 (1996); E. Pierpaoli, S. Borgani, A. Masiero, and M. Yamaguchi, Phys. Rev. D **57**, 2089 (1998).
[23] J. A. R. Cembranos, J. L. Feng, A. Rajaraman, and F. Takayama, Phys. Rev. Lett. **95**, 181301 (2005); M. Kaplinghat, Phys. Rev. D **72**, 063510 (2005); X. J. Bi, M. z. Li, and X. m. Zhang, Phys. Rev. D **69**, 123521 (2004); L. E. Strigari, M. Kaplinghat, and J. S. Bullock, Phys. Rev. D **75**, 061303 (2007).
[24] K. Sigurdson and M. Kamionkowski, Phys. Rev. Lett. **92**, 171302 (2004); S. Profumo, K. Sigurdson, P. Ullio, and M. Kamionkowski, Phys. Rev. D **71**, 023518 (2005).
[25] F. Wang and J. M. Yang, Nucl. Phys. **B709**, 409 (2005).
[26] J. L. Feng, A. Rajaraman, and F. Takayama, Phys. Rev. D **68**, 063504 (2003); K. Jedamzik, K. Y. Choi, L. Roszkowski, and R. Ruiz de Austri, J. Cosmol. Astropart. Phys. 07 (2006) 007; J. R. Ellis, K. A. Olive, and E. Vangioni, arXiv:astro-ph/0503023.
[27] D. N. Spergel and P. J. Steinhardt, Phys. Rev. Lett. **84**, 3760 (2000); A. Kusenko and P. J. Steinhardt, Phys. Rev. Lett. **87**, 141301 (2001).
[28] K. Jedamzik, Phys. Rev. D **70**, 063524 (2004).
[29] B. Moore *et al.*, Astrophys. J. **535**, L21 (2000); N. Yoshida, V. Springel, S. D. M. White, and G. Tormen, Astrophys. J. **535**, L103 (2000).
[30] P. Colin, V. Avila-Reese, and O. Valenzuela, Astrophys. J. **542**, 622 (2000); P. Bode, J. P. Ostriker, and N. Turok, Astrophys. J. **556**, 93 (2001).
[31] M. Kaplinghat, L. Knox, and M. S. Turner, Phys. Rev. Lett. **85**, 3335 (2000).
[32] K. Ichikawa and M. Kawasaki, Phys. Rev. D **69**, 123506 (2004); K. Ichikawa, M. Kawasaki, and F. Takahashi, Phys. Lett. B **597**, 1 (2004).
[33] A. De Rujula, S. L. Glashow, and U. Sarid, Nucl. Phys. **B333**, 173 (1990); S. Dimopoulos, D. Eichler, R. Esmailzadeh, and G. D. Starkman, Phys. Rev. D **41**, 2388 (1990).
[34] R. N. Cahn and S. L. Glashow, Science **213**, 607 (1981).
[35] R. N. Boyd, K. Takahashi, R. J. Perry, and T. A. Miller, Science **244**, 1450 (1989).
[36] P. F. Smith and J. R. J. Bennett, Nucl. Phys. **B149**, 525 (1979); P. F. Smith, J. R. J. Bennett, G. J. Homer, J. D. Lewin, H. E. Walford, and W. A. Smith, Nucl. Phys. **B206**, 333 (1982); T. K. Hemmick *et al.*, Phys. Rev. D **41**, 2074 (1990); P. Verkerk, G. Grynberg, B. Pichard, M. Spiro, S. Zylberajch, M. E. Goldberg, and P. Fayet, Phys. Rev. Lett. **68**, 1116 (1992); T. Yamagata, Y. Takamori, and H. Utsunomiya, Phys. Rev. D **47**, 1231 (1993).
[37] A. Kudo and M. Yamaguchi, Phys. Lett. B **516**, 151 (2001).
[38] J. L. Feng, A. Rajaraman, and B. T. Smith, arXiv:hep-ph/0512172.
[39] W. Buchmuller, K. Hamaguchi, M. Ratz, and T. Yanagida, Phys. Lett. B **588**, 90 (2004); J. L. Feng, A. Rajaraman, and F. Takayama, Int. J. Mod. Phys. D **13**, 2355 (2004); T. Okui, Phys. Rev. D **73**, 075012 (2006).
[40] J. L. Feng and B. T. Smith, Phys. Rev. D **71**, 015004 (2005); **71**, 019904(E) (2005); K. Hamaguchi, Y. Kuno, T. Nakaya, and M. M. Nojiri, Phys. Rev. D **70**, 115007 (2004).
[41] J. L. Goity, W. J. Kossler, and M. Sher, Phys. Rev. D **48**, 5437 (1993); M. Drees and X. Tata, Phys. Lett. B **252**, 695 (1990).
[42] J. L. Feng and T. Moroi, Phys. Rev. D **58**, 035001 (1998); A. Brandenburg, L. Covi, K. Hamaguchi, L. Roszkowski, and F. D. Steffen, Phys. Lett. B **617**, 99 (2005).
[43] I. Albuquerque, G. Burdman, and Z. Chacko, Phys. Rev. Lett. **92**, 221802 (2004); X. J. Bi, J. X. Wang, C. Zhang, and X. m. Zhang, Phys. Rev. D **70**, 123512 (2004); M. H. Reno, I. Sarcevic, and S. Su, Astropart. Phys. **24**, 107 (2005); I. F. M. Albuquerque, G. Burdman, and Z. Chacko, Phys. Rev. D **75**, 035006 (2007); M. Ahlers, J. Kersten, and A. Ringwald, arXiv:hep-ph/0604188.
[44] B. L. Ioffe, L. B. Okun, M. A. Shifman, and M. B. Voloshin, Acta Phys. Pol. B **12**, 229 (1981); K. Hamaguchi, T. Hatsuda, and T. T. Yanagida, arXiv:hep-ph/0607256.
[45] M. Kawasaki, K. Kohri, and T. Moroi, Phys. Lett. B **625**, 7 (2005).
[46] M. Kawasaki, K. Kohri, and T. Moroi, Phys. Rev. D **71**, 083502 (2005).
[47] K. Jedamzik, Phys. Rev. D **74**, 103509 (2006).
[48] S. Dimopoulos, R. Esmailzadeh, L. J. Hall, and G. D. Starkman, Astrophys. J. **330**, 545 (1988); M. H. Reno and D. Seckel, Phys. Rev. D **37**, 3441 (1988); K. Kohri, Phys. Rev. D **64**, 043515 (2001).
[49] M. Kawasaki and T. Moroi, Prog. Theor. Phys. **93**, 879 (1995); E. Holtmann, M. Kawasaki, K. Kohri, and T. Moroi, Phys. Rev. D **60**, 023506 (1999); K. Jedamzik, Phys. Rev. Lett. **84**, 3248 (2000); M. Kawasaki, K. Kohri, and T. Moroi, Phys. Rev. D **63**, 103502 (2001); R. H. Cyburt, J. R. Ellis, B. D. Fields, and K. A. Olive, Phys. Rev. D **67**, 103521 (2003).
[50] J. D. Jackson, Phys. Rev. **106**, 330 (1957); J. Rafelski, M. Sawicki, M. Gajda, and D. Harley, Phys. Rev. A **44**, 4345 (1991).
[51] As a recent application, see K. Ishida, K. Nagamine, T. Matsuzaki, and N. Kawamura, J. Phys. G **29**, 2043 (2003).
[52] P. D. Serpico, S. Esposito, F. Iocco, G. Mangano, G. Miele, and O. Pisanti, J. Cosmol. Astropart. Phys. 12 (2004) 010.
[53] R. H. Cyburt, Phys. Rev. D **70**, 023505 (2004).
[54] A. Coc, E. Vangioni-Flam, P. Descouvemont, A. Adahchour, and C. Angulo, Astrophys. J. **600**, 544 (2004).
[55] G. Steigman, Int. J. Mod. Phys. E **15**, 1 (2006).
[56] D. Kirkman *et al.*, Astrophys. J. Suppl. Ser. **149**, 1 (2003).
[57] D. Tytler, X. m. Fan, and S. Burles, Nature (London) **381**, 207 (1996); S. Burles and D. Tytler, Astrophys. J. **499**, 699 (1998).
[58] S. Burles and D. Tytler, Astrophys. J. **507**, 732 (1998).
[59] J. M. O'Meara *et al.*, Astrophys. J. **552**, 718 (2001).
[60] M. Pettini and D. V. Bowen, Astrophys. J. **560**, 41 (2001).
[61] G. Sigl, K. Jedamzik, D. N. Schramm, and V. S. Berezinsky, Phys. Rev. D **52**, 6682 (1995).
[62] J. Geiss, in *Origin and Evolution of the Elements*, edited by N. Prantzos, E. Vangioni-Flam, and M. Cassé (Cambridge University Press, Cambridge, 1993), p. 89.
[63] B. D. Fields and K. A. Olive, Astrophys. J. **506**, 177 (1998).
[64] Y. I. Izotov and T. X. Thuan, Astrophys. J. **602**, 200 (2004).
[65] K. A. Olive and G. Steigman, Astrophys. J. Suppl. Ser. **97**, 49 (1995).
[66] K. A. Olive, E. Skillman, and G. Steigman, Astrophys. J.

483, 788 (1997).
[67] Y. I. Izotov, T. X. Thuan, and V. A. Lipovetsky, Astrophys. J. Suppl. Ser. **108**, 1 (1997).
[68] Y. I. Izotov and T. X. Thuan, Astrophys. J. **500**, 188 (1998).
[69] K. A. Olive and E. D. Skillman, Astrophys. J. **617**, 29 (2004).
[70] P. Bonifacio *et al.*, Astron. Astrophys. **390**, 91 (2002).
[71] S. G. Ryan, J. Norris, and T. C. Beers, Astrophys. J. **523**, 654 (1999).
[72] S. G. Ryan *et al.*, Astrophys. J. Lett. **530**, L57 (2000).
[73] A. J. Korn *et al.*, Nature (London) **442**, 657 (2006).
[74] B. D. Fields, K. Kainulainen, K. A. Olive, and D. Thomas, New Astron. Rev. **1**, 77 (1996).
[75] S. Eidelman *et al.* (Particle Data Group), Phys. Lett. B **592**, 1 (2004).
[76] M. Asplund, D. L. Lambert, P. E. Nissen, F. Primas, and V. V. Smith, Astrophys. J. **644**, 229 (2006).
[77] V. V. Smith, D. L. Lambert, and P. E. Nissen, Astrophys. J. **408**, 262 (1993); L. M. Hobbs and J. A. Thorburn, Astrophys. J. **491**, 772 (1997); V. V. Smith, D. L. Lambert, and P. E. Nissen, Astrophys. J. **506**, 405 (1998); R. Cayrel *et al.*, Astron. Astrophys. **343**, 923 (1999).
[78] D. K. Duncan, D. L. Lambert, and M. Lemke, Astrophys. J. **401**, 584 (1992); M. Cassé, R. Lehoucq, and E. Vangioni-Flam, Nature (London) **373**, 318 (1995); R. Ramaty, B. Kozlovsky, and R. E. Lingenfelter, Astrophys. J. Lett. **438**, L21 (1995).
[79] M. Lemoine, D. N. Schramm, J. W. Truran, and C. J. Copi, Astrophys. J. **478**, 554 (1997).
[80] B. D. Fields and K. A. Olive, New Astron. Rev. **4**, 255 (1999).
[81] T. K. Suzuki and S. Inoue, Astrophys. J. **573**, 168 (2002).
[82] M. H. Pinsonneault, T. P. Walker, G. Steigman, and V. K. Narayanan, Astrophys. J. **527**, 180 (2002).
[83] C. Angulo *et al.*, Astrophys. J. **630**, L105 (2005).
[84] R. H. Cyburt, B. D. Fields, and K. A. Olive, Phys. Rev. D **69**, 123519 (2004).
[85] H. A. Bethe and E. E. Salpeter, *Quantum Mechanics of One- and Two-Electron Atoms* (Springer-Verlag OHG, Berlin, Gottingen, Heidelberg, 1957).
[86] K. Kohri and F. Takayama (unpublished).
[87] R. Wagoner, Astrophys. J. **18**, 247 (1969).
[88] Experimental Nuclear Reaction Data (EXFOR/CSISRS) homepage, http://www.nndc.bnl.gov/exfor/index.html; B. G. Glagola *et al.*, Phys. Rev. Lett. **41**, 1698 (1978); S. M. Read and V. E. Viola, Jr., At. Data Nucl. Data Tables **31**, 359 (1984).
[89] M. Pospelov, Phys. Rev. Lett. **98**, 231301 (2007).
[90] M. Kaplinghat and A. Rajaraman, Phys. Rev. D **74**, 103004 (2006).
[91] K. Kohri, T. Moroi, and A. Yotsuyanagi, Phys. Rev. D **73**, 123511 (2006).
[92] M. Viel, J. Lesgourgues, M. G. Haehnelt, S. Matarrese, and A. Riotto, Phys. Rev. D **71**, 063534 (2005).
[93] A. Arvanitaki, C. Davis, P. W. Graham, A. Pierce, and J. G. Wacker, Phys. Rev. D **72**, 075011 (2005); A. Arvanitaki, S. Dimopoulos, A. Pierce, S. Rajendran, and J. G. Wacker, arXiv:hep-ph/0506242.
[94] D. Fargion and M. Khlopov, arXiv:hep-ph/0507087; D. Fargion, M. Khlopov, and C. A. Stephan, arXiv:astro-ph/0511789; K. M. Belotsky, M. Y. Khlopov, and K. I. Shibaev, arXiv:astro-ph/0604518.