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Chiral dynamics of baryons from string theory

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We study baryons in a holographic model of QCD by Sakai and Sugimoto, realized as small instantons with fundamental string hairs. We introduce an effective field theory of the baryons in the five-dimensional setting, and show that the instanton interpretation implies a particular magnetic coupling. Dimensional reduction to four dimensions reproduces the usual chiral effective action, and, in particular, we estimate the axial coupling g_A between baryons and pions and the magnetic dipole moments, both of which are proportional to N_c. We extrapolate to finite N_c and discuss subleading corrections.

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I. INTRODUCTION

Understanding baryons from the microscopic theory is a long standing problem, since it amounts to solving the low-energy QCD, which is strongly coupled and highly non-linear. Though limited successes were made such as in lattice QCD, it is far from complete. Recent discovery of string/gauge duality [1], however, enables us to address the problem, based on holographic models. One interesting and realistic model among them is the one by Sakai and Sugimoto [2] (SS model for short). The model astutely implements chiral symmetry spontaneously broken, and describes the low-energy dynamics in a manner consistent with the hidden local symmetry theory of the form developed some years ago with the ρ meson [3].

In this paper, using the fully five-dimensional picture of baryons which automatically incorporates the infinite tower of vectors in construction of the baryon, we will show that chiral dynamics arises naturally in the large 't Hooft coupling limit $\lambda = g_{\rm YM}^2 N_c \to \infty$. It has been recognized for some time that the lowest-lying vector mesons as hidden local fields could play an important role in the soliton structure [4] and dynamics [5] of baryons, which was also recently reconsidered in the context of the SS model [6]. Here we find that not just the lowest members but the *whole tower* of the vector fields participate intricately in the dynamics of baryons, and this actually simplifies and relates many of four-dimensional interactions.

We start with a brief review of the SS model. In the model, the stack of D4 branes which carries the SU(N_c) pure Yang-Mills theory is replaced by the dual geometry, assuming $\lambda \gg 1$ to suppress the stringy α' correction, with the metric

$$G = \left(\frac{U}{R}\right)^{3/2}\left(\eta_4 + f d\tau^2 + \frac{R^3}{U^3}\frac{dU^2}{f} + \frac{R^3}{U} d\Omega_4^2\right) \qquad (1)$$

where $f(U) = 1 - U_{\rm KK}^3/U^3$. The coordinate τ is periodic with the period $\delta\tau = 2\pi/M_{\rm KK} = 4\pi R^{3/2}/3U_{\rm KK}^{1/2}$, which defines the Kaluza-Klein (KK) mode scale $M_{\rm KK}$ and sets the scale for massive mesons. Note that the parameters of dual QCD are mapped to the dimensionful parameters here as $R^3 = \lambda l_s^2/2M_{\rm KK}$ and $U_{\rm KK} = 2\lambda M_{\rm KK} l_s^2/9$ with the string length scale l_s. The string coupling is related to the SU(N_c) Yang-Mills (YM) coupling as $2\pi g_s = g_{\rm YM}^2/M_{\rm KK} l_s$.

The D8 branes, which share coordinates $x^{0,1,2,3}$ with D4 branes, are treated as probes and carry U(N_F) Yang-Mills multiplets from D8-D8 open strings. The induced metric on D8 is

$$g_{8+1} = g_{4+1} + R^{3/2}U^{1/2}d\Omega_4^2, \qquad (2)$$

where the five-dimensional part is conformally equivalent to $R^{3+1} \times I$,

$$g_{4+1} = H(w)(\eta_{\mu\nu}dx^\mu dx^\nu + dw^2), \qquad (3)$$

with $w = \int dU R^{3/2}/\sqrt{U^3 - U_{\rm KK}^3}$ and $H = (U/R)^{3/2}$. This fifth coordinate is of finite range $[-w_{\rm max}, w_{\rm max}]$ with $w_{\rm max} \simeq 3.64/M_{\rm KK}$. Near the origin $w = 0$, we have the approximate relation $U^3 \simeq U_{\rm KK}^3(1 + M_{\rm KK}^2 w^2)$.

The main point of this model is that the D8 comes with two asymptotic regions (corresponding to UV) at $w \to \pm w_{\rm max}$, where the U(N_F) gauge symmetry of D8 can be each interpreted as U$(N_F)_{L,R}$ chiral symmetry, respectively, of fermions from D4-D8 strings. As D4's are replaced by the geometry, these D4-D8 strings are connected and become D8-D8 strings representing the U(N_F) gauge field in five dimensions. The pion field appears in terms of $\xi(x) = e^{i\pi(x)/f_\pi}$ as

$$A(x; w) = i\alpha(x)\psi_0(w) + i\beta(x) + \sum_n a^{(n)}(x)\psi_{(n)}(w) \qquad (4)$$

in the $A_w = 0$ gauge, with $\alpha(x) \equiv \{\xi^{-1}, d\xi\}$ and

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$\beta(x) \equiv \frac{1}{2}[\xi^{-1}, d\xi]$. The zero mode ψ_0 approaches $\pm 1/2$ at the two boundaries. This mode expansion shows how massive (axial-)vector mesons, $a_\mu^{(n)}$, also arise from this picture. Keeping the pions only results in the Skyrme Lagrangian for $U = \xi^2$, whereas in using the gauge field picture, we are automatically working with the entire tower of (axial-)vector mesons in addition to pions.

II. BARYONS AS SMALL AND HAIRY INSTANTONS

A baryon in this model corresponds to a D4 brane wrapping the compact S^4 [7], which is dissolved into D8 as an U(N_F) instanton. The topological relation between this and the usual Skyrmion picture was clarified in [2,8]. In the present background, the compact D4 must also carry N_c fundamental strings attached, whose other endpoints can only go to D8's, pulling the wrapped D4 brane toward D8 and making it a finite-size instanton.

The 4 + 1 dimensional effective action of U(N_F) Yang-Mills fields in the conformal coordinate system is

$$\frac{1}{4}\int d^4xdw \frac{8\pi^2R^3U(w)}{3(2\pi l_s)^5(2\pi g_s)} \mathrm{tr}F_{mn}F^{mn}, \tag{5}$$

from which we find the effective electric coupling

$$\frac{1}{e^2(w)} \equiv \frac{8\pi^2R^3U_{\mathrm{KK}}}{3(2\pi l_s)^5(2\pi g_s)}\frac{U(w)}{U_{\mathrm{KK}}} = \frac{\lambda N_c M_{\mathrm{KK}}}{108\pi^3}\frac{U(w)}{U_{\mathrm{KK}}}. \tag{6}$$

A pointlike instanton that is localized at $w = 0$ would have the mass $m_B^{(0)} \equiv 4\pi^2/e^2(0) = (\lambda N_c/27\pi)M_{\mathrm{KK}}$ which is also the mass of D4 wrapping S^4 at $w = 0$.

If the instanton gets bigger, the configuration costs more energy, since $1/e^2(w)$ is an increasing function of $|w|$. For a very small instanton of size ρ, this additive correction to the instanton mass is found to be $\simeq m_B^{(0)}M_{\mathrm{KK}}^2\rho^2/6$, using the spread of the instanton density $D(x^i, w) \sim \rho^4/(x^2 + w^2 + \rho^2)^4$ [9]. The competing effects come from the energy cost related to the N_c fundamental strings, which manifests as U(N_F) electric charges. The electric charge density is proportional to $D(x^i, w)$ [2], and the five-dimensional Coulomb energy is readily estimated as [9]

$$\simeq \frac{e(0)^2N_c^2}{20\pi^2\rho^2}. \tag{7}$$

The size of the instanton localized at $w = 0$ is then determined by minimizing the sum. This gives

$$\rho_{\mathrm{baryon}} \simeq \frac{(2\cdot 3^7\cdot \pi^2/5)^{1/4}}{M_{\mathrm{KK}}\sqrt{\lambda}} \sim \frac{9.6}{M_{\mathrm{KK}}\sqrt{\lambda}}. \tag{8}$$

as the size of the baryon. For a large 't Hooft coupling λ, which is needed anyway for controlling stringy corrections, the baryon size is then significantly smaller than the scale of the dual QCD and incidentally also much smaller than the would-be size of the Skyrmion which scales as $\sim 1/M_{\mathrm{KK}}$. The same large λ controls corrections to our five-dimensional estimate such as the mass correction and the backreaction of the soliton to the position-dependent coupling $e(w)$.

III. FIVE-DIMENSIONAL EFFECTIVE ACTION OF BARYONS

Our first task is to understand the effective action of the instanton soliton in five dimensions. The soliton by itself is a classical object. In order to treat it quantum mechanically, one first needs to quantize their collective coordinates and classify the resulting particles according to their spin content and the representations under other symmetries. Having in mind an extrapolation to the real QCD, we restrict ourselves to the case of fermionic baryons with fundamental representation under U($N_F = 2$), denoted as $\mathcal{B}$.

After a suitable rescaling of the $\mathcal{B}$ field in the conformally flat coordinates (x^μ, w), we have

$$-i\int d^4xdw[\bar{\mathcal{B}}\gamma^\mu D_\mu\mathcal{B} + \bar{\mathcal{B}}\gamma^5\partial_w\mathcal{B} + m_b(w)\bar{\mathcal{B}}\mathcal{B}] \tag{9}$$

in the $A_w = 0$ gauge and with $D_\mu = \partial_\mu - iA_\mu$. The gauge field A here is that of U(N_F) on D8, as before, which encodes the pions and the entire tower of massive (axial-)vector mesons. $m(w)$ reflects the fact that the instanton costs more energy if it moves away from the $w = 0$ structure.

However, since the baryon is represented by a small instanton soliton with a long-range tail of self-dual gauge field $F \sim \rho_{\mathrm{baryon}}^2/r^4$, there should be an additional coupling between a $\mathcal{B}$ bilinear and a SU(N_F) part of A such that each $\mathcal{B}$-particle generates this self-dual tail on F. There is a unique vertex that does the job, i.e.,

$$\int d^4xdw\left[g_5(w)\frac{\rho_{\mathrm{baryon}}^2}{e(w)^2}\bar{\mathcal{B}}\gamma^{mn}F_{mn}\mathcal{B}\right]. \tag{10}$$

Writing the upper 2-component part of $\mathcal{B}$ as $\mathcal{U}e^{-iEt}$, and approximating m_b by its central value, we find the on shell condition is solved by

$$\mathcal{B} = \begin{pmatrix}\mathcal{U}\\ \pm i\mathcal{U}\end{pmatrix}e^{\mp im_bt}, \tag{11}$$

where the two signs originate from the sign of E/m_b and thus correspond to the baryon and the antibaryon, respectively.

A static and localized spinor quantum acts as a source to the Yang-Mills field via

$$\bar{\mathcal{B}}\gamma^{mn}F_{mn}\mathcal{B} = \pm\frac{1}{2}F_{jk}^a\epsilon^{jki}\langle\sigma_i\tau^a\rangle_{\mathcal{B}} + F_{5i}^a\langle\sigma_i\tau^a\rangle_{\mathcal{B}}, \tag{12}$$

where $\langle\sigma_i\tau^a\rangle_{\mathcal{B}} \equiv 2[\mathcal{U}^\dagger\sigma_i\tau^a\mathcal{U}]$. In order to generate self-dual or anti-self-dual long-range fields, the spin index and the gauge index must be locked. For instance, one choice that gives a long-range self-dual field is $\mathcal{U}_{\alpha A} = \frac{i}{2}\epsilon_{\alpha A}$, in which case $\langle\sigma_i\tau^a\rangle_{\mathcal{B}} = -\delta_i^a$ so that the source term (with the upper sign) is $-F^a_{mn}\bar{\eta}^a_{mn}/2$ with the anti-self-dual 't Hooft symbol $\bar{\eta}$ ($m, n = 1, 2, 3, 5$ and $a = 1, 2, 3$).

Now assume that such a source appears in a localized form at the origin. The gauge field far away from the source obeys, after a gauge choice and ignoring w-dependence of the electric coupling,

$$\nabla^2 A^a_m = 2g_5(0)\rho^2_{\rm baryon}\bar{\eta}^a_{mn}\partial_n\delta^{(4)}(x), \tag{13}$$

whose solution is

$$A^a_m = -\frac{g_5(0)\rho^2_{\rm baryon}}{2\pi^2}\bar{\eta}^a_{mn}\partial_n\frac{1}{r^2+w^2}. \tag{14}$$

The general shape of the long-range field is consistent with the identification of the baryon as the instanton. In order to fix $g_5(0)$, we need to match the states in $\mathcal{B}$ with *quantized* instanton. This means that the long-range field of the instanton should be modified due to quantum fluctuation of the instanton along different global gauge directions. How to implement this quantum effect is explained in detail in [9]. Here we briefly sketch the reasoning.

Representing the global gauge rotation in (10) as

$$S^\dagger A^a_M(\tau^a/2)S = A^a_M(\tau^b/2)({\rm tr}[S^\dagger(\tau^a/2)S\tau^b]), \tag{15}$$

the quantization replaces the quantity in the parenthesis by its expectation value. Following a reasoning mathematically identical to that used by Adkins *et al.* [10] for the Skyrme model, we obtain

$$\langle{\rm tr}[S^\dagger(\tau^a/2)S\tau^b]\rangle_{\mathcal{B}} = -\langle\sigma_b\tau^a\rangle_{\mathcal{B}}/3 = \delta^b_a/3. \tag{16}$$

This allows us to arrive at

$$g_5(0) = 2\pi^2/3. \tag{17}$$

We have ignored the w-dependence of $g_5(w)$ and $e(w)$. This we believe is harmless for the very small size baryon/instanton for the usual reason.

IV. FOUR-DIMENSIONAL EFFECTIVE ACTION OF BARYONS

After identifying the relevant five-dimensional action, we perform the KK expansion for the five-dimensional bulk fields along w to derive a four-dimensional Lagrangian. The four-dimensional nucleon arises as the lowest eigenmode of the five-dimensional bulk baryon along the w coordinate, which should be a mode localized near $w = 0$. We mode-expand $\mathcal{B}_{L,R}(x^\mu, w) = B_{L,R}(x^\mu)f_{L,R}(w)$, where $\gamma^5 B_{L,R} = \pm B_{L,R}$ are four-dimensional chiral components, with the profile functions $f_{L,R}(w)$ satisfying

$$\begin{aligned}\partial_w f_L(w) + m_b(w)f_L(w) &= m_B f_R(w),\\ -\partial_w f_R(w) + m_b(w)f_R(w) &= m_B f_L(w).\end{aligned} \tag{18}$$

The four-dimensional Dirac field for the baryon is then reconstructed as $B = (B_L, B_R)^T$. See Ref. [11] for a similar model.

We will use the mode expansion in Eq. (4) to obtain the baryon couplings to mesons. The eigenmode analysis done in [2] shows that $\psi_{(2k+1)}(w)$ is even and $\psi_{(2k)}(w)$ is odd under $w \to -w$, corresponding to vector and axial-vector mesons, respectively. The resulting effective Lagrangian density for four-dimensional baryons coupled to pions and (axial-)vector mesons is

$$\mathcal{L}_4 = -i\bar{B}(\gamma^\mu\partial_\mu + m_B)B + \mathcal{L}_{\rm vector} + \mathcal{L}_{\rm axial} + \cdots, \tag{19}$$

with the infinite tower of vectorlike couplings

$$\mathcal{L}_{\rm vector} = -i\bar{B}\gamma^\mu\beta_\mu B - \sum_{k\geq 0} g_V^{(k)}\bar{B}\gamma^\mu a_\mu^{(2k+1)}B, \tag{20}$$

and another infinite tower of axial couplings

$$\mathcal{L}_{\rm axial} = -\frac{ig_A}{2}\bar{B}\gamma^\mu\gamma^5\alpha_\mu B - \sum_{k>0} g_A^{(k)}\bar{B}\gamma^\mu\gamma^5 a_\mu^{(2k)}B. \tag{21}$$

The constants g_A and $g^{(k)}_{V,A}$ are all calculable by the overlap of wave functions [9]. Generically, both the minimal coupling and the magnetic coupling in the five dimensions contribute to each four-dimensional vertex.

However, in the large N_c limit, the contribution from the magnetic term is dominant for all axial couplings. For the leading axial coupling with the pion, the main contribution to g_A is then

$$g_A \simeq C\int dw\left[\left(\frac{4U(w)g_5(w)}{g_5(0)U_{\rm KK}}\right)\frac{\partial_w\psi_0}{M_{\rm KK}}|f_L|^2\right], \tag{22}$$

where the coefficient C is, with (8) and (17),

$$C \equiv g_5(0)\rho^2_{\rm baryon}M_{\rm KK}/e^2(0) \simeq 0.18N_c. \tag{23}$$

For $\lambda N_c \gg 1$, furthermore, $|f_L|^2$ will have a vanishing width as $1/\lambda N_c \to 0$ and be localized at $w = 0$. Thus, the integral is given by the value of the rest of the integrand at $w = 0$. With the function $\psi_0(w)$ taken from Ref. [2] and after a coordinate transformation, we find

$$g_A \simeq \frac{4C\partial_w\psi_0(0)}{M_{\rm KK}} \simeq 0.18N_c \times (4/\pi) \simeq 0.7(N_c/3). \tag{24}$$

This result is a consequence of including the infinite tower of the massive (axial-)vector mesons, $a_\mu^{(n)}$, since the effective action was derived for a soliton which solves the five-dimensional gauge theory of (4). If we had truncated them, we would have neither a small soliton nor the simple five-dimensional effective theory leading to this.

Let us now consider electromagnetic responses of the baryons. The simplest way to obtain the coupling is to include the electromagnetic field as a non-normalizable mode of the gauge fields on D8 branes,

$$A_\mu(x;w) = \mathcal{A}_\mu(x) + i\alpha_\mu(x)\psi_0(w) + \cdots. \quad (25)$$

The five-dimensional gauge interaction of such a non-normalizable mode gives the vertex $\int d^4x \mathcal{A}_\mu J^\mu$, into which we embed the electromagnetic interaction. Here we are interested in isolating the magnetic moment of the nucleon from the magnetic vertex we found above.

For $N_F = 2$, for example, we find the isovector magnetic moment of the nucleon, $\Delta\mu \equiv \mu^p - \mu^n$, to be

$$\left(\frac{4C}{M_{KK}}\right)\int d^4x[\mathcal{U}^\dagger \mathbf{B}\cdot\sigma\mathcal{U}] \quad (26)$$

using $\int dw|f_L|^2 = 1$, where $\mathbf{B}$ is the magnetic field strength, embedded into $\mathcal{F} \equiv d\mathcal{A} + \mathcal{A}^2$. With the normalization $\mathrm{tr}\mathcal{U}^\dagger\mathcal{U} = 1/2$, one identifies $\mathrm{tr}\mathcal{U}^\dagger\sigma\mathcal{U}$ as the spin operator $\mathbf{S}$ of the baryon. Recall that the minimal coupling of the Dirac field to a vector field produces a universal magnetic moment $e_{EM}\mathbf{S}/m_B$. It is easy to show that this latter contribution, smaller by relative factor of $1/N_c^2$, *adds* to the above leading contribution. For $N_F = 2$, the isovector part of the electromagnetic charge is given as diag$(1/2, -1/2)$. Thus we find the "anomalous magnetic moment" of the nucleon is

$$\frac{\mu_{an}^p}{e_{EM}} = -\frac{\mu_{an}^n}{e_{EM}} = \frac{C}{M_{KK}} \simeq \frac{0.18N_c}{M_{KK}} \quad (27)$$

with the electromagnetic coupling constant e_{EM}.

V. DISCUSSIONS

The main result of this work is the simple closed form of the five-dimensional effective action of the general form (9) and (10) where the baryon carries a direct coupling to the SU($N_F = 2$) field strength with (17). Despite this simple form, it promises definite and computable couplings (19)–(21) between nucleons and the infinite variety of mesons. Finally, the electromagnetic interaction is also holographically encoded in the same formalism. The full implication of this effective action will be discussed in a separate paper [9], where we will present both qualitative features, such as complete vector dominance of nucleon form factors, and quantitative predictions for various meson-nucleon couplings.

In trying to see how our model fares with nature, the strategy we will adhere to here and also in Ref. [9] is to perform *all* calculations in this well-controlled limit, after which we extrapolate to a realistic regime. This has been fairly successful for such quantities as the ratios of masses, meson form factors, and the viscosity-entropy ratio. We expect that quantities independent of λ in the large λ limit compare relatively well with nature. In this letter, we computed two such quantities, (24) and (27), the experimental values of which are $g_A^{\exp} \simeq 1.26$ and $(\mu_{an}^p)^{\exp} \simeq 1.79\mu_N$ and $(\mu_{an}^n)^{\exp} \simeq -1.91\mu_N$ (where μ_N is the nuclear magneton).

To proceed, we first fix the only mass parameter as $M_{KK} \simeq 0.94$ GeV from the meson sector, following [2]. We also note that $f_\pi \simeq M_{KK}/10$ gives $\lambda N_c \simeq 50$, although it does not enter most of our comparisons. Let us then consider the N_c-independent ratio of g_A against the anomalous magnetic moment measured in the nuclear magneton. Since M_{KK} is roughly the physical nucleon mass, our result is $(|\mu_{an}^n|/\mu_N)/g_A \simeq \pi/2 \simeq 1.57$, to be compared with the experimental ratio $1.91/1.26 = 1.51$. Using μ_{an}^p leads to a slightly worse agreement.

Individual comparisons may demand a better knowledge of what is the subleading correction in $1/N_c$. For this, there is a tantalizing suggestion coming from conventional quark models and from their equivalence to the Skyrme model in the large N_c expansion [12]. All models based on the constituent quark idea are known to admit a shift of $N_c \to N_c + 2$ for both g_A and μ in the large N_c expansion, the reasoning for which is anchored solely on a group theoretical bookkeeping. On the other hand, an extended quantization of the spin-flavor operator involved in the Skyrme model (which is inherited in our 5D model) also leads to the same shift without involving fermion loops [13].

We are thus encouraged to employ the shift in the extrapolation. Whether it comes from within the current setup or after some unknown improvement of the model toward real QCD would be a separate issue. The goal here is to test whether the coefficients of N_c are sensible ones or not in the $1/N_c$ expansion for the bona fide QCD. With $N_c = 3$, the shift produces $g_A \simeq 1.17$ and $\mu_{an}^p \simeq -\mu_{an}^n \simeq 1.8\mu_N$. We have also isolated a "dynamical" $O(1/N_c)$ correction to g_A from the minimal coupling within the model, which is of the same sign and about 10% of this leading term for $\lambda N_c \simeq 50$, bringing us even closer to the experimental values [9].

Clearly much more study is needed to understand the model, implications, and corrections thereof. What is intriguing is that even at this "crude" leading order, the chiral Lagrangian, derived from the string/gauge duality, is found to describe baryons remarkably well, thus indicating that it captures some important aspects of strong interactions. Further evidence of this will be given in Ref. [9], which also contains more careful discussions of subleading corrections and the extrapolation procedure.

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Note Added.—After this work was completed and has appeared, a paper has appeared [14] with an overlap on the instanton size estimate.

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