Dispersion relations and subtractions in hard exclusive processes

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We study analytical properties of the hard exclusive process amplitudes. We found that QCD factorization for deeply virtual Compton scattering and hard exclusive vector meson production results in the subtracted dispersion relation with the subtraction constant determined by the Polyakov-Weiss D-term. The relation of this constant to the fixed pole contribution found by Brodsky, Close, and Gunion and defined by parton distributions is studied and proved for momentum transfers exceeding the typical hadronic scale. The continuation to the real photon limit is considered, and the numerical correspondence between lattice simulations of the D-term and low energy Thomson amplitude is found. For sufficiently large t the subtraction may be expressed in a form similar to that suggested earlier for real Compton scattering.

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I. INTRODUCTION

Hard exclusive reactions described by the generalized parton distributions (GPDs) [1–6] are the subject of extensive theoretical and experimental studies. The analytical properties of deeply virtual Compton scattering (DVCS) and hard exclusive vector meson production (VMP) amplitudes [7–10] constitute an important aspect of these studies. They also play a major role in hadronic processes such as the nucleon-nucleon scattering at very high energies to be studied at CERN LHC and in nonaccelerator experiments [11].

The crucial point in the application of the relevant dispersion relations is a possible ambiguity due to the subtraction constants which are the counterparts of the normalization constants implied by the ultraviolet renormalization procedure. An attractive possibility is represented by cases where such constants are defined by the imaginary part of the amplitudes. This situation was explored long ago in the case of the forward Compton amplitude [12–14] and was recently reconsidered for DVCS [10].

In this paper we address the problem of dispersion relations and subtractions in the framework of the leading order QCD factorization. We find that it leads to subtracted dispersion relations with the subtraction constant defined by the Polyakov-Weiss D-term [15]. At the same time, for t exceeding the typical hadronic scale we relate the subtraction constant to the integrals of parton distribution at zero skewness.

II. DISPERSION RELATION IN THE SKEWNESS PLANE

We restrict our study to the case of large s and Q^2 and small $t \ll s$, Q^2, where QCD factorization is applicable. At the leading order, this results in the following expressions for DVCS and vector (ρ^0) meson production amplitudes:

$$T^{\mu\nu}_{\rm DVCS} = \frac{g_\perp^{\mu\nu}}{2}\,\bar{u}(p_2)\hat{n}u(p_1)\sum_{f=u,d,s,\dots} e_f^2\mathcal{A}_f(\xi,t) \quad (1)$$

and

$$T^{\mu}_{\rm VMP} = \frac{\alpha_s f_\rho C_F e_L^\mu}{\sqrt{2}N_c Q}\,\bar{u}(p_2)\hat{n}u(p_1)\mathcal{V}[e_u\mathcal{A}_u(\xi,t) - e_d\mathcal{A}_d(\xi,t)], \quad (2)$$

with the GPD part (which may be interpreted as a weighted handbag diagram, i.e. the coupling of local quark currents to two photons)

$$\mathcal{A}_f(\xi,t) = \int_{-1}^{1} dx\,\frac{H_f^{(+)}(x,\xi,t)}{x-\xi+i\epsilon} \quad (3)$$

and the meson part for the VMP case,

$$\mathcal{V} = \int_0^1 dy\,\frac{\phi_1(y)}{y}. \quad (4)$$

In (3), $H^{(+)}(x,\xi,t)$ denotes the singlet ($C=+1$) combination of GPDs, summing the contributions of quarks and antiquarks and of s and u channels:

$$H_f^{(+)}(x,\xi,t) = H_f(x,\xi,t) - H_f(-x,\xi,t). \quad (5)$$

For the sake of brevity, we will keep only the dependence of GPDs on the skewness $\xi = Q^2/(2s+Q^2)$. For $\xi \to 0$, (3) may acquire divergencies at $x=0$, which will be one of the objects of our analysis.

Contrary to the forward case, expression (3) does not have a form of the dispersion relation because of the appearance of ξ in the numerator. Nevertheless, the amplitude (3) as a function of ξ manifests the analyticity properties in the unphysical region $|\xi|>1$ [9]. This region is associated with the contribution of generalized distribution amplitudes (GDAs) [16] related to GPDs by crossing [17]. To prove the analyticity of the amplitude for $|\xi|>1$, one represents the denominator of (3) as the geometric series:

$$\mathcal{A}(\xi) = -\sum_{n=0}^{\infty} \xi^{-n-1} \int_{-1}^{1} dx H^{(+)}(x, \xi) x^n. \quad (6)$$

This series is convergent [9] thanks to the polynomiality condition (see e.g. [5,6]):

$$\int_{-1}^{1} dx x^n H(x, \xi) = \sum_{k=0,2...}^{n} \xi^k A_k + \frac{1-(-1)^n}{2} \xi^{n+1} C.$$

One may now easily calculate the discontinuity across the cut $-1 < \xi < 1$ and write the fixed-t dispersion relation [9] for the leading order amplitude (3) in the skewness plane:

$$\mathrm{Re}\, \mathcal{A}(\xi) = \frac{\mathcal{P}}{2\pi i} \int_{-1}^{1} dx \frac{\mathrm{Disc}\, \mathcal{A}(x)}{x - \xi} + \Delta(\xi), \quad (7)$$

or, using (3),

$$\mathcal{P} \int_{-1}^{1} dx \frac{H^{(+)}(x, \xi)}{x - \xi} = \mathcal{P} \int_{-1}^{1} dx \frac{H^{(+)}(x, x)}{x - \xi} + \Delta(\xi), \quad (8)$$

where $\Delta(\xi)$ is a possible subtraction. This expression represents the holographic property of GPD: the relevant information about hard exclusive amplitudes in the considered leading approximation is contained in the one-dimensional sections $x = \pm\xi$ of the two-dimensional space of x and ξ. These holographic as well as tomographic [17] properties in momentum space are complementary to the often-discussed holography and tomography in coordinate space [6].

We are now going to prove that $\Delta(\xi)$ is finite and independent of ξ, i.e. $\Delta(\xi) = \mathrm{const}$. To do this, one considers the following representation:

$$\Delta(\xi) = \mathcal{P} \int_{-1}^{1} dx \frac{H^{(+)}(x, \xi) - H^{(+)}(x, x)}{x - \xi} = -\mathcal{P} \int_{-1}^{1} dx \sum_{n=1}^{\infty} \frac{1}{n!} \frac{\partial^n}{\partial \xi^n} H^{(+)}(x, \xi) \bigg|_{\xi = x} (\xi - x)^{n-1}. \quad (9)$$

Because of the polynomiality condition the only surviving highest power term in this series is equal to a finite subtraction constant. This can also be derived with the use of the double distributions (DDs) formalism. Namely, $H^{(+)}(x, \xi)$ is expressed through the corresponding DDs as

$$H^{(+)}(x, \xi) = \int_{-1}^{1} d\alpha \int_{-1+|\alpha|}^{1-|\alpha|} d\beta [f(\alpha, \beta) + \xi g(\alpha, \beta)] \times [\delta(x - \alpha - \xi\beta) - \delta(-x - \alpha - \xi\beta)]. \quad (10)$$

Substituting this expression into (9), one gets that the $f(\alpha, \beta)$-terms which depend on ξ are canceled and Eq. (9) becomes ξ independent (cf. [9]):

$$\Delta(\xi) = -2 \int_{-1}^{1} d\alpha \int_{-1+|\alpha|}^{1-|\alpha|} d\beta \frac{g(\alpha, \beta)}{1 - \beta} \equiv \Delta. \quad (11)$$

Notice that the cancellation of $f(\alpha, \beta)$ and the validity of (11) are not spoiled (provided $\xi \neq 0$), even if the singularity corresponding to $f(\alpha, \beta) \sim \alpha^{-a}$ is present.

In (11), one can choose

$$g(\alpha, \beta) = \delta(\alpha) D(\beta), \quad (12)$$

where the function $D(\beta)$ is the D-term [15]. The assumption (12) is a result of the corresponding "gauge" [17] as discussed also in [6]. With (12), the Δ term takes the following form:

$$\Delta = 2 \int_{-1}^{1} d\beta \frac{D(\beta)}{\beta - 1}. \quad (13)$$

It should be emphasized that both integrals in (8) are divergent at $\xi = t = 0$, and these divergencies do not cancel for $\xi \to 0$. This means that Δ is not defined for $\xi = t = 0$ [18]. On the other hand, for an arbitrarily small ξ the integrals in (8) are finite and, therefore, Δ is well defined. Note that there is some similarity between (3) and (13) so that Δ may also be interpreted as the contribution of a local two-photon coupling to the quark currents which is independent on ξ.

Taking into account the parametrization [15]

$$D(\beta) = (1 - \beta^2) \sum_{n=0}^{\infty} d_n C_{2n+1}^{(3/2)}(\beta), \quad (14)$$

and keeping only the lowest term, one gets

$$\Delta = -4 d_0. \quad (15)$$

This lowest term d_0 was estimated within the framework of different models. We focus on the results of the chiral quark-soliton model [19]: $d_0^{\mathrm{CQM}}(N_f) = d_0^u = d_0^d = -\frac{4.0}{N_f}$, where N_f is the number of active flavors, and lattice simulations [20]: $d_0^{\mathrm{latt}} = d_0^u \approx d_0^d = -0.5$. The subtraction constant varies as

$$\Delta^p_{\mathrm{CQM}}(2) = \Delta^n_{\mathrm{CQM}}(2) \approx 4.4, \qquad \Delta^p_{\mathrm{latt}} \approx \Delta^n_{\mathrm{latt}} \approx 1.1 \quad (16)$$

for the DVCS on both the proton and neutron targets.

III. DISPERSION RELATION IN THE ν PLANE

We have seen that the D-term determines the finite subtraction in the dispersion relation in the skewness plane. Let us now compare the dispersion relation (8) with the dispersion relation written in the ν plane [8] where $\nu = (s - u)/4m_N$. In terms of the new variables ν', ν related to x, ξ as

$$x^{-1} = \frac{4m_N \nu'}{Q^2}, \qquad \xi^{-1} = \frac{4m_N \nu}{Q^2}, \quad (17)$$

the fixed-t dispersion relation becomes the subtracted one:

$$\begin{aligned}\mathrm{Re}\mathcal{A}(\nu, Q^2) &= \frac{\nu^2}{\pi}\mathcal{P}\int_{\nu_0}^{\infty}\frac{d\nu'^2}{\nu'^2}\frac{\mathrm{Im}\mathcal{A}(\nu', Q^2)}{(\nu'^2-\nu^2)}+\Delta \\ &= \frac{\mathcal{P}}{\pi}\int_{\nu_0}^{\infty} d\nu'^2\,\mathrm{Im}\mathcal{A}(\nu', Q^2)\left[\frac{1}{\nu'^2-\nu^2}-\frac{1}{\nu'^2}\right] \\ &\quad +\Delta. \end{aligned}\tag{18}$$

Here, $\nu_0 = Q^2/4m_N$ (the Q^2 dependence here is shown explicitly) and the nucleon pole term residing in this point may be considered separately [8].

This subtracted (in the symmetric unphysical point $\nu = 0$) dispersion relation is the principal result of our paper. It is applicable for both DVCS (cf. [7,8]) and VMP (cf. [21]) amplitudes.

It can be considerably simplified provided $\mathrm{Im}\mathcal{A}(\nu)$ decreases fast enough so that both terms in the square brackets can be integrated separately:

$$\mathrm{Re}\,\mathcal{A}(\nu) = \frac{\mathcal{P}}{\pi}\int_{\nu_0}^{\infty} d\nu'^2\frac{\mathrm{Im}\mathcal{A}(\nu')}{\nu'^2-\nu^2}+\mathbf{C}_0, \tag{19}$$

where

$$\begin{aligned}\mathbf{C}_0 &= \Delta - \frac{\mathcal{P}}{\pi}\int_{\nu_0}^{\infty} d\nu'^2\frac{\mathrm{Im}\mathcal{A}(\nu')}{\nu'^2} \\ &= \Delta + \mathcal{P}\int_{-1}^{1} dx\frac{H^{(+)}(x,x)}{x}. \end{aligned}\tag{20}$$

Now, using (9) with $\xi = 0$, one gets

$$\Delta = \mathcal{P}\int_{-1}^{1} dx\frac{H^{(+)}(x,0)-H^{(+)}(x,x)}{x} \tag{21}$$

$$= 2\mathcal{P}\int_{-1}^{1} dx\frac{H(x,0)-H(x,x)}{x}, \tag{22}$$

where the symmetry property arising from the T invariance, $H(x,-x) = H(x,x)$, is used. The relation (21) can also be obtained from the "sum rules" [4]:

$$\int_{-1}^{1} dx\frac{H(x,\xi+xz)-H(x,\xi)}{x} = \sum_{n=1}^{\infty} z^n\int_{-1}^{1} dx x^{n-1}D(x)$$

for $\xi = 0$ and $z = 1$.

Let us stress that for the valence ($C = -1$) contributions to the amplitudes of the hard exclusive production of, say, pions [4] and exotic hybrid mesons [22], $\Delta = 0$ because of the mentioned symmetry in x and the ξ independence.

Substituting (21) into (20), one can see that the D-term is canceled from the expression for the subtraction constant

$$\mathbf{C}_0(t) = 2\mathcal{P}\int_{-1}^{1} dx\frac{H(x,0,t)}{x}, \tag{23}$$

where we restored the dependence on t which was omitted for brevity. This constant is similar to the result obtained in the studies of the fixed pole contribution to the forward Compton amplitude [12]. At the same time, at ξ, $t = 0$, GPDs are expressed in terms of standard parton distributions $H(x,0) = q(x)\theta(x) - \bar{q}(-x)\theta(-x)$. Formally one has

$$\mathbf{C}_0(0) = 2\int_0^1 dx\frac{q(x)+\bar{q}(x)}{x} = 2\int_0^1 dx\frac{q_v(x)+2\bar{q}(x)}{x}. \tag{24}$$

However, the integral defining $\mathbf{C}_0(0)$ diverges at low x in both the valence and sea quark contributions. Therefore, for $t = 0$ we should consider (18) as a correct general form of the dispersion relation which includes infinite subtraction at the point $\nu = 0$ and the subtraction constant associated with the D-term.

For $t \neq 0$, the integral in (23) converges for sufficiently large t. In the case of the Regge inspired parametrization [4] $H(x,0,-t) \sim x^{-\alpha(0)+\alpha' t}$, this condition reads as $t > \alpha(0)/\alpha'$, resulting in $t \gtrsim 1(10)\ \mathrm{GeV}^2$ for the valence (sea) quark distributions.

The divergence of (23) was originally discussed within the framework of the parton model and its modifications [12], while we address this problem in the framework of the leading order QCD factorization. Therefore, our result (18), although being formally ξ independent, cannot, generally speaking, be continued to the forward limit $\xi \sim Q^2 \to 0\,(s = \mathrm{const})$. The limit $\xi \to 0$, $s \to \infty$, $Q^2 = \mathrm{const}$ still corresponds to the highly nonforward kinematics with the masses of initial and final photons being rather different. At the same time, further exploration of the possible manifestation of the D-term in forward Compton scattering seems very interesting.

The formal continuation of (18) would result in the following subtracted dispersion relation for the forward Compton scattering amplitude:

$$\mathrm{Re}\,\mathcal{A}(\nu) = \frac{\nu^2}{\pi}\mathcal{P}\int_0^{\infty}\frac{d\nu'^2}{\nu'^2}\frac{\mathrm{Im}\mathcal{A}(\nu')}{(\nu'^2-\nu^2)}+\Delta. \tag{25}$$

Comparing this expression with the dispersion relation for the forward Compton scattering amplitude [23], one can observe an interesting numerical coincidence. For the proton target, our subtraction combined with the lattice simulations (16) is rather close to the low energy Thomson term (note that $\Delta_{\mathrm{Thomson}} = 1$ for our normalization of the Compton amplitude). As a result, the mysterious occurrence of the Thomson term at large energies [23] can now be supplemented with its possible appearance also at large Q^2. This does not hold in the case of the neutron target where the DVCS subtraction term is the same, while the Thomson term is zero. This may be because the sum of squares of valence quark charges is equal to the square of the proton charge, i.e. to the square of their sum (cf. [24]), whereas for the neutron these quantities differ.

IV. CONCLUSIONS

In this paper we show that the fixed-t dispersion relations for the DVCS and VMP amplitudes require infinite subtractions at the unphysical point $\nu = 0$ with the subtraction constants associated with the D-terms. However, for the production of the mesons defined by valence ($C = -1$) GPDs, the finite subtraction is absent.

We also show that the appearance of the subtraction expressed in terms of (forward) parton distributions [12] may be investigated in the framework of the leading order QCD factorization. We consider the possibility of the continuation of our results to the real photon limit. The surprising similarity between the lattice simulations of the D-term and the low energy Thomson amplitude in the proton target case is found.

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Note added.—After this work was completed, the papers [25,26] appeared, confirming our results and generalizing them to the next-to-leading order.

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