Bounds on unparticle couplings to electrons: From electron $g - 2$ to positronium decays

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Unparticles as suggested recently by Georgi have surprising phenomenological implications, distinctive from any other new physics that we know of. But they must interact very feebly with ordinary matter to have avoided detection thus far. We determine how feebly they can interact with the electron, using the precisely measured quantities in QED: the electron $g - 2$ and the bounds on invisible and exotic positronium decays. The most stringent bound comes from invisible orthopositronium decays: the effective energy scale entering the vector unparticle-electron interaction must exceed 4×10^5 TeV for a scaling dimension $\frac{3}{2}$ of the vector unparticle. The lower bounds on scales for other unparticles range from a few tens to a few hundreds TeV. This makes the detection of unparticles challenging in low energy electron systems.

We are so accustomed to describe physical processes in terms of particles that it is even hard to imagine what other conception we can perceive beyond that of particles. By particles we mean identities that have a definite energy-momentum relation, i.e., a mass, among other intrinsic properties. Recently, Georgi has suggested a fascinating idea of what this beyond-particle identity, dubbed unparticle, might look like [1,2]. He has also provided a scenario in which the unparticle could appear and couple to ordinary matter from certain high energy theory with a nontrivial scale invariant infrared fixed point, for instance, theories studied in Ref. [3]. Although not much is known about the details of such a high energy theory that might be relevant to the real world, its remnants at low energies, unparticles, can be well described in effective field theories and experimentally explored through their couplings to ordinary matter. As Georgi argued and demonstrated [1,2], these unparticles enjoy very funny kinematic behavior, far removed from any new physics that we know of so far. This makes the idea phenomenologically attractive.

However peculiar these unparticles might be, they must interact very feebly with ordinary matter to evade detection so far. It is the aim of the current work to determine how feeble those interactions might be from two of the most precisely measured quantities in QED: the electron $g - 2$ and the invisible and exotic decays of positronium. For an unparticle of scaling dimension $\frac{3}{2}$, we find that the former restricts the effective energy scale responsible for the unparticle-electron interactions to be higher than tens to 150 TeV, depending on the Lorentz properties of the unparticle. The constraint from positronium decays is more stringent: the lower bound ranges from 500 to 4×10^5 TeV.

Prior to this work, three papers on unparticles phenomenology appeared, but no systematic analysis has been attempted so far on experimental constraints on unparticle-electron interactions. In Ref. [2], unparticle effects at the Z resonance were elucidated where unusual patterns of interference occur due to the phases in the unparticle propagator in the timelike region. In Ref. [4], based on effective operators suggested in Ref. [1], collider signals of unparticles are studied together with effects of a vector unparticle on the muon $g - 2$. The idea of bosonic unparticles was generalized to the fermionic case in Ref. [5], where corrections of fermionic and scalar unparticles to the muon $g - 2$ are computed as well as potential flavor-changing neutral current effects mediated by a vector unparticle.

Our working Lagrangian for effective unparticle-electron interactions is

$$\mathcal{L}_{\text{int}} = C_S \bar{\psi}\psi \mathcal{U}_S + C_P \bar{\psi} i \gamma_5 \psi \mathcal{U}_P + C_V \bar{\psi}\gamma_\mu \psi \mathcal{U}_V^\mu \\ + C_A \bar{\psi}\gamma_\mu\gamma_5\psi \mathcal{U}_A^\mu, \tag{1}$$

where $\mathcal{U}_{S,P,V,A}$ are fields for scalar, pseudoscalar, vector, and axial vector unparticles. They have standard C and P parities as their particle counterparts to preserve C and P symmetries. Although these fields may have different scaling dimensions, we assign a common one to them for simplicity, d. The real couplings $C_{S,P,V,A}$ then have the dimension $1 - d$ and can be parametrized by $C_{S,P,V,A} = \pm \Lambda_{S,P,V,A}^{1-d}$, where Λ_i are effective energy scales determined by some underlying high energy theory. Our goal is to constrain these energy scales using the precisely measured electron $g - 2$ and the upper limits on invisible and exotic decays of the positronium.

By exploiting scale invariance of the unparticle field, Georgi found that the state density in phase space of an unparticle of momentum p is proportional to $\theta(p^0)\theta(p^2) \times (p^2)^{d-2}$. Since these are the same factors for the density of a system of d massless particles, he suggested that the state density of an unparticle is similarly normalized,

$$d\Phi_{\mathcal{U}}(p) = A_d \theta(p^0)\theta(p^2)(p^2)^{d-2} \frac{d^4 p}{(2\pi)^4}, \tag{2}$$

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where

$$A_d = \frac{16\pi^{5/2}}{(2\pi)^{2d}} \frac{\Gamma(d + \frac{1}{2})}{\Gamma(d - 1)\Gamma(2d)}, \quad (3)$$

though d is now generally nonintegral. This should be contrasted to that of a particle of mass m:

$$d\Phi(p) = 2\pi\theta(p^0)\delta(p^2 - m^2)\frac{d^4 p}{(2\pi)^4}. \quad (4)$$

Note that there is no mass-shell constraint to an unparticle, as is the case for a particle. This will have interesting phenomenological implications. From unitarity considerations, the above state density implies the following propagator for a spin-zero unparticle [2] (see also [4]):

$$\frac{A_d}{2\sin(\pi d)} \frac{i}{(-p^2 - i\epsilon)^{2-d}}. \quad (5)$$

For a vector or axial vector unparticle that has only transverse polarizations, a standard projector should be attached, $P_{\mu\nu}^T = -g_{\mu\nu} + p_\mu p_\nu/p^2$.

It is straightforward to work out corrections to the anomalous magnetic moment of the electron, $a = \frac{1}{2}(g - 2)$, from Fig. 1(a):

$$a_S = -\frac{A_d}{2\sin(\pi d)} \frac{(C_S m^{d-1})^2}{8\pi^2} \frac{3\Gamma(2d - 1)\Gamma(2 - d)}{\Gamma(2 + d)}, \quad (6)$$

$$a_P = +\frac{A_d}{2\sin(\pi d)} \frac{(C_P m^{d-1})^2}{8\pi^2} \frac{\Gamma(2 - d)\Gamma(2d)}{\Gamma(2 + d)}, \quad (7)$$

$$a_V = -\frac{A_d}{2\sin(\pi d)} \frac{(C_V m^{d-1})^2}{4\pi^2} \frac{\Gamma(3 - d)\Gamma(2d - 1)}{\Gamma(d + 2)}, \quad (8)$$

$$a_A = +\frac{A_d}{2\sin(\pi d)} \frac{(C_A m^{d-1})^2}{\pi^2} \frac{\Gamma(2d - 2)\Gamma(3 - d)}{\Gamma(2 + d)}, \quad (9)$$

where the subscripts denote the contributions from corresponding unparticles. Note that a_A is computed for a transverse $\mathcal{U}_A$ while a_V does not rely on the transversality assumption for $\mathcal{U}_V$. For the relevant loop integrals to converge it is necessary that $d < 2$. As argued in Ref. [1], theoretical consistency may demand that $d > 1$. Our later numerical analysis will thus focus on the narrow range of the scaling dimension, $1 < d < 2$. It is then clear

that $a_{S,V} > 0$ while $a_{P,A} < 0$. Our result on a_V coincides with that in Ref. [4], while a_S differs in sign from Ref. [5]. In the limit $d \to 1$, we have $a_S \to \frac{3C_S^2}{16\pi^2}$, $a_P \to -\frac{C_S^2}{16\pi^2}$, $a_V \to \frac{C_V^2}{8\pi^2}$ while a_A has no appropriate limit due to infrared divergence. The conventional one-loop QED result is recovered from a_V by setting further $C_V \to -e$.

The electron $g - 2$ has been recently measured in Ref. [6] (denoted as H06) with an uncertainty about 6 times smaller than in the past. Using as input the fine structure constant measured in independent experiments with Cs [7] (Cs06) and Rb [8] (Rb06) atoms, in the new theoretical evaluation of $g - 2$ [9], yields the following deviations [10] between the theoretical and measured numbers:

$$a(\text{Cs06}) - a(\text{H06}) = -2.5(9.3) \times 10^{-12}, \quad (10)$$

$$a(\text{Rb06}) - a(\text{H06}) = +7.9(7.7) \times 10^{-12}, \quad (11)$$

which are summarized in Ref. [6] as

$$|\delta a| < 15 \times 10^{-12}. \quad (12)$$

This last bound will be used below to constrain the unparticle-electron couplings.

For numerical illustration, we assume $d = \frac{3}{2}$, then

$$a_S = \frac{1}{10\pi^3}\frac{m}{\Lambda_S}, \qquad a_P = -\frac{1}{15\pi^3}\frac{m}{\Lambda_P},$$
$$a_V = \frac{1}{30\pi^3}\frac{m}{\Lambda_V}, \qquad a_A = -\frac{2}{15\pi^3}\frac{m}{\Lambda_A}. \quad (13)$$

We will not attempt here a sophisticated data fitting; instead, we assume that only one of the four unparticles exists at a time. The separate bounds are found to be

$$\Lambda_S > 110 \text{ TeV}, \qquad \Lambda_P > 73 \text{ TeV},$$
$$\Lambda_V > 37 \text{ TeV}, \qquad \Lambda_A > 146 \text{ TeV}. \quad (14)$$

If all unparticles are accommodated simultaneously, only a bound on the combination of Λ's can be set which is generally weaker due to cancellations. We mention in passing that the bounds become weakened when d increases.

Now we move to the positronium decays. A positronium of orbital and spin angular momenta ℓ, s has parities $C = (-1)^{\ell+s}$, $P = (-1)^{\ell+1}$. Thus, the ground states have, re-
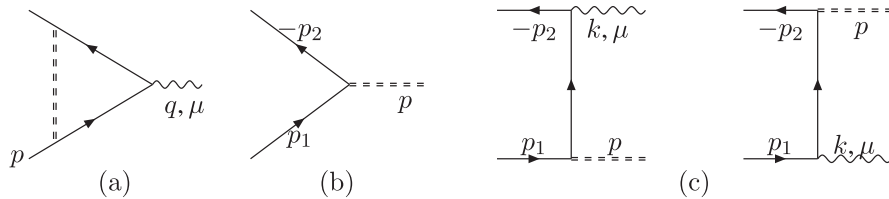

FIG. 1.   Diagrams that contribute to electron $g - 2$ (a), invisible (b), and exotic (c) positronium decays. Double-dashed, solid, and wavy lines stand for unparticle, electron, and photon fields, respectively.

spectively, $C = P = -1$ for an orthopositronium (o-Ps with $s = 1$) and $-C = P = -1$ for a parapositronium (p-Ps with $s = 0$). While a p-Ps must decay into an even number of photons, an o-Ps has to decay into an odd number of photons, at least three. This makes the latter a particularly sensitive probe for new physics effects. For obvious reasons, we restrict ourselves to decays involving a single unparticle in the final state. Then, only the following invisible one-body transitions are allowed:

$$\text{o-Ps} \rightarrow \mathcal{U}_V; \qquad \text{p-Ps} \rightarrow \mathcal{U}_P; \qquad (15)$$

while for exotic two-body decays, the following ones are possible:

$$\text{o-Ps} \rightarrow \mathcal{U}_S \gamma, \; \mathcal{U}_P \gamma, \; \mathcal{U}_A \gamma; \qquad \text{p-Ps} \rightarrow \mathcal{U}_V \gamma; \quad (16)$$

where the last one cannot compete with the dominant two-photon decay and thus will not be considered below. These symmetry arguments have been checked against explicit calculations.

The amplitudes for the constituent processes shown in Fig. 1(b) and 1(c) are found in the nonrelativistic limit:

$$
\begin{aligned}
i\mathcal{A}(\mathcal{U}_P) &= -2mC_P\zeta^\dagger\xi, \\
i\mathcal{A}(\mathcal{U}_V) &= +i2mC_V\zeta^\dagger\sigma^i\xi\epsilon^{i*}(p), \\
i\mathcal{A}(\gamma\mathcal{U}_S) &= -i2eC_S\zeta^\dagger\sigma^i\xi\epsilon^{i*}(k), \\
i\mathcal{A}(\gamma\mathcal{U}_P) &= -i2eC_P\zeta^\dagger\sigma^k\xi\epsilon^{ijk}\hat{k}^j\epsilon^{i*}(k), \\
i\mathcal{A}(\gamma\mathcal{U}_A) &= -2eC_A\zeta^\dagger\sigma^k\xi\epsilon^{ijk}\epsilon^{i*}(k)\epsilon^{j*}(p),
\end{aligned}
\qquad (17)
$$

where p, $\epsilon(k)$ $(k, \epsilon(k))$ are the momentum and polarization of the photon (unparticle), and ξ, ζ are the spin wave functions for the electron and positron of mass m. The decay amplitudes for the positronium are

$$
\begin{aligned}
i\mathcal{A}(\text{p-Ps} \rightarrow \mathcal{U}_P) &= -2\sqrt{2m}C_P\psi(0), \\
i\mathcal{A}(\text{o-Ps} \rightarrow \mathcal{U}_V) &= +i2\sqrt{2m}C_V\psi(0)\mathbf{n}\cdot\epsilon^*(p), \\
i\mathcal{A}(\text{o-Ps} \rightarrow \gamma\mathcal{U}_S) &= -i2\sqrt{\frac{2}{m}}eC_S\psi(0)\mathbf{n}\cdot\epsilon^*(k), \\
i\mathcal{A}(\text{o-Ps} \rightarrow \gamma\mathcal{U}_P) &= -i2\sqrt{\frac{2}{m}}eC_P\psi(0)(\epsilon^*(k)\times\hat{\mathbf{k}})\cdot\mathbf{n}, \\
i\mathcal{A}(\text{o-Ps} \rightarrow \gamma\mathcal{U}_A) &= -2\sqrt{\frac{2}{m}}eC_A\psi(0)(\epsilon^*(k)\times\epsilon^*(p))\cdot\mathbf{n},
\end{aligned}
\qquad (18)
$$

where $\mathbf{n}$ is the o-Ps polarization vector and $\psi(0)$ is the wave function of the positronium bound state evaluated at the origin.

Since the polarization dependence is standard, we will study directly the unpolarized decay rates. Again for simplicity, we will assume that the vector and axial vector unparticles have only transverse polarizations. The decay rate to a single unparticle is

$$dΓ = \frac{1}{4m}A_d\theta(p^0)\theta(p^2)(p^2)^{d-2}\frac{d^4p}{(2\pi)^4}$$
$$\times (2\pi)^4\delta^4(p - p_1 - p_2)|\mathcal{A}|^2, \qquad (19)$$

which can be integrated to

$$Γ = 4^{d-3}m^{2d-5}A_d|\mathcal{A}|^2. \qquad (20)$$

Note that in contrast to the particle case there is no delta function remaining because unparticles have no mass-shell constraints. The invisible decay rates are

$$Γ(\text{p-Ps} \rightarrow \mathcal{U}_P) = 2^{2d-3}A_d m(m^{d-1}C_P)^2|m^{-3/2}\psi(0)|^2, \qquad (21)$$

$$\barΓ(\text{o-Ps} \rightarrow \mathcal{U}_V) = 3^{-1}2^{2d-2}A_d m(m^{d-1}C_V)^2|m^{-3/2}\psi(0)|^2, \qquad (22)$$

with the corresponding branching ratios being

$$\text{Br}(\text{p-Ps} \rightarrow \mathcal{U}_P) = \frac{2^{2d-5}}{\pi\alpha^2}A_d(m^{d-1}C_P)^2, \qquad (23)$$

$$\text{Br}(\text{o-Ps} \rightarrow \mathcal{U}_V) = \frac{3\cdot 2^{2d-6}}{(\pi^2 - 9)\alpha^3}A_d(m^{d-1}C_V)^2. \qquad (24)$$

The decay rate to a photon plus an unparticle is

$$dΓ = \frac{1}{4m}\left[A_d\theta(p^0)\theta(p^2)(p^2)^{d-2}\frac{d^4p}{(2\pi)^4}\right]\left[\frac{d^3\mathbf{k}}{(2\pi)^3 2\omega}\right]$$
$$\times (2\pi)^4\delta^4(p + k - p_1 - p_2)|\mathcal{A}|^2.$$

Upon finishing p integration and for $\mathcal{A}$ independent of $|\mathbf{k}|$ which is the case here, the differential rate in fractional photon energy and solid angles is

$$\frac{dΓ}{dxd\Omega} = A_d 2^{2d-10}\pi^{-3}m^{2d-3}|\mathcal{A}|^2 x(1 - x)^{d-2}, \quad (25)$$

where $p^2 \approx 4m(m - \omega)$ is used and the integration region is fixed by the step functions to be $0 \le x \le 1$ with $x = \omega/m$. Contrary to the particle case where the photon in a two-body final state is monochromatic, the photon accompanying the unparticle follows a continuous spectrum. This is again due to the lack of a mass-shell constraint and the like for unparticles. This is more than a mere missing energy or momentum that could be used to separate unparticle signals from "normal" new physics.

The unpolarized differential decay rates are, upon finishing the angular integration,

$$\frac{d\barΓ}{dx}(\text{o-Ps} \rightarrow \gamma\mathcal{U}_{S,P,A}) = \frac{A_d 2^{2d-2}}{3\pi}m|m^{-3/2}\psi(0)|^2$$
$$\times \alpha(m^{d-1}C_{S,P,A})^2 x(1 - x)^{d-2}, \qquad (26)$$

with the total rates and branching ratios being

$$\bar{\Gamma}(\text{o-Ps} \to \gamma \mathcal{U}_{S,P,A}) = \frac{4\Gamma(d + \frac{1}{2})}{3\Gamma(d+1)\Gamma(2d)}$$
$$\times \pi^{(3/2)-2d} m |m^{-3/2}\psi(0)|^2$$
$$\times \alpha(m^{d-1}C_{S,P,A})^2, \qquad (27)$$

$$\text{Br}(\text{o-Ps} \to \gamma \mathcal{U}_{S,P,A}) = \frac{\Gamma(d + \frac{1}{2})}{\Gamma(2d)\Gamma(d+1)} \frac{3}{4(\pi^2 - 9)\alpha^2}$$
$$\times \pi^{(3/2)-2d}(m^{d-1}C_{S,P,A})^2. \qquad (28)$$

Now we confront our results with data. The most recent measurement on invisible positronium decays is reported in Ref. [11]:

$$\text{Br}(\text{p-Ps} \to \text{invisible}) \leq 4.3 \times 10^{-7} \qquad (90\% \text{ C.L.}), \qquad (29)$$

$$\text{Br}(\text{o-Ps} \to \text{invisible}) \leq 4.2 \times 10^{-7} \qquad (90\% \text{ C.L.}), \qquad (30)$$

while the most stringent bounds on exotic two-body decays were set some years ago [12]:

$$\text{Br}(\text{o-Ps} \to \gamma X^0) \leq 1.1 \times 10^{-6} \qquad (90\% \text{ C.L.}), \qquad (31)$$

where X^0 is an unknown neutral boson interacting weakly with ordinary matter. We take $d = \frac{3}{2}$ as previously. Then Eqs. (29) and (30) imply respectively

$$\Lambda_P \geq 5.6 \times 10^2 \text{ TeV}, \qquad \Lambda_V \geq 4.3 \times 10^5 \text{ TeV}. \qquad (32)$$

Since several channels contribute to the exotic decays, we consider one unparticle at a time for simplicity, then Eq. (31) gives

$$\Lambda_{S,P,A} \geq 5.1 \times 10^2 \text{ TeV}. \qquad (33)$$

If we combine the bounds in Eqs. (32) and (33), the latter is mainly a bound on $\Lambda_{S,A}$. These are more stringent bounds than those from the electron $g - 2$.

Unparticles descending from some high energy scale invariant theory behave very differently from familiar particles due to the lack of a mass-shell constraint and a nonintegral scaling dimension. This makes them phenomenologically very distinctive. But whether this is observable depends on how feebly they interact with ordinary matter. We have considered the general effective interactions of unparticles with the electron, and investigated their implications on two of the most precisely measured quantities in QED: the electron $g - 2$ and the invisible and exotic decays of the positronium. We found that the most stringent constraint is from invisible orthopositronium decays. For a scaling dimension of $\frac{3}{2}$, the effective energy scale responsible for the vector unparticle-electron interaction exceeds 4×10^5 TeV. The bounds on the energy scales of other unparticles range from a few tens to a few hundreds TeV. This result makes the experimental observation of unparticles rather challenging in low energy electron systems. It remains to be seen whether they are detectable in high energy processes.

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