Proton stability in supersymmetric $SU(5)$

Zurab Tavartkiladze*

Physics Department, Theory Division, CERN, CH-1211 Geneva 23, Switzerland
and Department of Physics, Oklahoma State University, Stillwater, Oklahoma 74078, USA
(Received 31 October 2006; revised manuscript received 26 June 2007; published 28 September 2007)

Within supersymmetric $SU(5)$ grand unified theory (GUT) we suggest mechanisms for suppression of baryon number violating dimension five and six operators. The mechanism is based on the idea of split multiplets (i.e. quarks and leptons are not coming from a single GUT state) which is realized by an extension with additional vectorlike matter. The construction naturally avoids wrong asymptotic relation $\hat{M}_D = \hat{M}_E$. Thus, the long-standing problems of the minimal supersymmetric $SU(5)$ GUT can be resolved. In a particular example of flavor structure and with additional $\mathcal{U}(1) \times \mathcal{Z}_{3N}$ symmetry we demonstrate how the split multiplet mechanism works out. Namely, the considered model is compatible with successful gauge coupling unification and realistic fermion mass pattern. The nucleon decay rates are relatively suppressed and can be well compatible with current experimental bounds.

DOI: 10.1103/PhysRevD.76.055012 PACS numbers: 12.10.Dm, 12.10.Kt, 12.15.Ff

I. INTRODUCTION

Baryon number violation is one of the predictions of grand unified theories (GUT). In supersymmetric (SUSY) GUTs, usually the dimension five ($d = 5$) operator induced proton decay dominates [1]. The sources for the latter are heavy color triplets' couplings with ordinary matter supermultiplets. These couplings usually originate from the operators responsible for quark and lepton masses. Therefore, the observed Yukawa couplings and the baryon number violating operators may be closely related and this is the reason that it is not easy to satisfy the present experimental bound $\tau^{\rm exp}(p \to K\nu) \gtrsim 6.7 \cdot 10^{32}$ years [2] on proton life time [3]. On the other hand, it is not trivial to build a realistic fermion mass pattern within GUTs. Therefore, the task is two fold: (1) within the considered scenario, care must be exercised to get realistic fermion masses and mixings, and (2) within the same framework the baryon number violating processes must be suppressed up to the required level. These are two main problems and for their resolution numerous mechanisms and specific examples have been suggested [4–9]. It is a curious fact that the split multiplet mechanism, for suppressing the baryon number violation, more or less has been ignored (see, however, [6,7]). Let us note that this mechanism is naturally realized within extra dimensional constructions [10,11]. This is, most likely, the reason that within four-dimensional constructions there were not many attempts to realize and apply this possibility. However, once the multiplet splitting is achieved (i.e. quarks and leptons come from different GUT states), the baryon number can be conserved up to the needed level [11]. In this paper we suggest mechanisms for natural quark-lepton splitting within four-dimensional SUSY $SU(5)$ [12]. We show that apart from suppressing the baryon number violation this splitting enables one to build a realistic fermion mass pattern. The discussion of the mechanism and its needed ingredients are presented in the next section. In Sec. III we show how $d = 6$ nucleon decay can be suppressed. In Sec. IV, for demonstrative purposes, we present a particular example in which a split multiplet mechanism is realized. It utilizes an additional $\mathcal{U}(1) \times \mathcal{Z}_{3N}$ symmetry which plays a crucial role for adequate suppression of all unwanted baryon number violating couplings including Planck scale suppressed operators. An assumption on a particular flavor structure and simple minded SUSY spectrum near ~1 TeV is made. These give perturbative gauge coupling unification and realistic fermion masses and mixings. At the same time, nucleon's decay rate is compatible with current experimental bounds.

II. SUPPRESSION OF $d = 5$ BARYON NUMBER VIOLATION

In the minimal SUSY $SU(5)$ (MSSU5) GUT the matter sector consists of the $(10 + \bar{5})$-plets per generation with the following decomposition under $SU(3)_c \times SU(2)_L \times U(1)_Y$:

$$\begin{aligned} 10 &= q(3,2)_{-1} + u^c(\bar{3},1)_4 + e^c(1,1)_{-6}, \\ \bar{5} &= d^c(\bar{3},1)_{-2} + l(1,2)_3, \end{aligned} \tag{1}$$

where subscripts stand for the hypercharges in $1/\sqrt{60}$ units [$Y = \frac{1}{\sqrt{60}}\,{\rm Diag}(2,2,2,-3,-3)$]. The pair of scalar superfields $H(5) + \bar{H}(\bar{5})$ has the following composition:

$$\begin{aligned} H(5) &= h_u(1,2)_{-3} + T(3,1)_2, \\ \bar{H}(\bar{5}) &= h_d(1,2)_3 + \bar{T}(\bar{3},1)_{-2}, \end{aligned} \tag{2}$$

where h_u, h_d denote the MSSM Higgs doublet superfields, and T, $\bar{T}$ are their colored GUT partners. The renormalizable operators $10 \cdot 10H$ and $10 \cdot \bar{5}\bar{H}$ (the family indices are suppressed), together with ordinary Yukawa superpo-

*zurab.tavartkiladze@okstate.edu

tential couplings, generate matter-T, $\bar{T}$ interactions:

$$\lambda 10\cdot 10H = \lambda(qu^c h_u + qqT + e^c u^c T),$$
$$\lambda' 10\cdot \bar{5}\bar{H} = \lambda'(qd^c h_d + e^c l h_d + ql\bar{T} + u^c d^c \bar{T}). \tag{3}$$

Integration of T, $\bar{T}$ states (with mass M_T) generates $d = 5$ operators

$$\frac{\lambda\lambda'}{M_T}(qqql)_F, \qquad \frac{\lambda\lambda'}{M_T}(u^c u^c d^c e^c)_F, \tag{4}$$

which induce the nucleon decay. Current experimental bound on nucleon lifetime requires $\lambda\lambda' \lesssim 10^{-9}$ (for $M_T \sim 10^{16}$ GeV and all soft SUSY breaking terms ~TeV). On the other hand, in MSSU5 $\lambda\lambda'$ is directly related to the quark and lepton Yukawa couplings and is typically $\sim 10^{-6}/\sin 2\beta$. This would lead to unacceptably fast proton decay. Note that couplings in (3) also lead to the wrong asymptotic mass relations $m_\mu = m_s$, $m_e/m_\mu = m_d/m_s$ at the GUT scale. Thus, some modification should be done anyway in order to improve this situation. It is desirable to have a mechanism which simultaneously solve both—fermion mass problem and baryon number violation. Note that modification of either Yukawa sector [5,8], or sparticle spectrum [13], or increase of GUT scale [14] can improve the situation with baryon number violation. However, besides colored Higgsino mediated $d = 5$ operators (4), there exist Planck scale suppressed baryon number violating couplings which in SUSY $SU(5)$ have forms

$$\frac{\lambda_{\rm Pl}}{M_{\rm Pl}}(10\cdot 10\cdot 10\cdot \bar{5})_F \to \frac{\lambda_{\rm Pl}}{M_{\rm Pl}}(qqql + u^c u^c d^c e^c)_F. \tag{5}$$

This also mediates proton decay and in order to satisfy experimental bound one should arrange for appropriate couplings $\lambda_{\rm Pl} \lesssim 10^{-7}$. Couplings $\lambda_{\rm Pl}$ are completely independent from the Yukawa sector and therefore their smallness need separate explanation, because at SUSY $SU(5)$ level there is no symmetry argument for their suppression.

Below we present a mechanism, different from existing ones, which within SUSY $SU(5)$ GUT suppress (eliminates) baryon number violation and can solve the fermion mass problem.

A. Suppressing qqT and eliminating $e^c u^c T$ operators

In order to demonstrate how the split multiplet mechanism works, we start considerations with one family only. The generalization to three families is straightforward and will be discussed later. We extend the matter sector with vectorlike states in 15 and $\overline{15}$ representations of $SU(5)$. In terms of $SU(3)_c \times SU(2)_L \times U(1)_Y$ they decompose as

$$15 = q(3,2)_{-1} + S(6,1)_4 + \Delta(1,3)_{-6}, \tag{6}$$

and conjugate transformations for the fragments of $\overline{15} = (\bar{q}, \bar{S}, \bar{\Delta})$. The state q ($\equiv q_{15}$) from 15-plet has transformation properties of the left-handed quark doublet. The remaining S and Δ states have "exotic" quantum numbers. This feature of 15-plet can be used for the suppression of proton decay [6]. With suitable couplings we can arrange that the light left-handed quark doublet mainly comes from 15-plet. Consider the superpotential couplings

$$10\Sigma\overline{15} + M_{15}15\cdot\overline{15}, \tag{7}$$

where Σ is an adjoint 24-plet scalar superfield used for the breaking $SU(5)\to SU(3)_c\times SU(2)_L\times U(1)_Y$. Substituting in (7) the GUT vacuum expectation value (VEV) $\langle\Sigma\rangle \equiv M_G$ with $M_{15} \ll \langle\Sigma\rangle$, we see that q_{10} decouples by forming the massive state with $\bar{q}_{\overline{15}}$. Namely, for the light q and heavy q_h states we have

$$q \simeq q_{15}, \qquad q_h \simeq q_{10} + \frac{M_{15}}{M_G}q_{15} = >15 \supset q,$$
$$10 \supset \epsilon q \quad \text{with} \quad \epsilon \equiv \frac{M_{15}}{M_G}. \tag{8}$$

The states u^c and e^c (from 10-plet) and fragments (S, Δ), $(\bar{S}, \bar{\Delta})$ (from 15, $\overline{15}$) are not affected with this procedure. Therefore,

$$u^c, e^c \subset 10, \tag{9}$$

and masses of the decoupled states are given by

$$M(q_{10}, \bar{q}_{\overline{15}}) \simeq M_G, \qquad M_S = M_\Delta = M_{15}. \tag{10}$$

Now it is clear that the up quark mass will be generated through the Yukawa coupling of the 15-plet with 10. Since 15-plet is the two index symmetric representation of $SU(5)$, Σ should participate in this coupling. Namely,

$$Y\frac{\Sigma}{M_*}15\cdot 10H \to Y_U(qu^c h_u + \epsilon qqT), \quad \text{with}$$
$$Y_U = \frac{\langle\Sigma\rangle}{M_*}Y. \tag{11}$$

We see that the term qqT is suppressed by the factor ϵ in comparison to the up type quark Yukawa coupling. This occurred thanks to the splitting of the q-states living in 15- and 10-plet superfields, respectively. Note that no $e^c u^c T$ coupling arises from (11). The coupling $10\cdot 10H$ is not needed at all and can be suppressed or completely eliminated in a concrete scenario (discussed in Sec. IV).

The scale M_* in (11) is a cut off and one expects that it is much larger than the GUT scale $M_* \gg \langle\Sigma\rangle$ (in most conservative approach $M_* \sim M_{\rm Pl} \simeq 2.4\cdot 10^{18}$ GeV—the reduced Planck mass). Thus, we can use this type of coupling for the first two light families (i.e. for generation of up and charm quark masses). For the top quark mass we need to have an unsuppressed Yukawa coupling. If we do not apply this mechanism of qqT coupling suppression for the third generation, the top Yukawa can be due to the coupling $10_3 10_3 H$. However, the same coupling also generates an unsuppressed $q_3 q_3 T$ term. This would give sizable contribution to the nucleon decay [5,15] through the mixings with light families. Thus, for suppressing $q_3 q_3 T$

and generating the top Yukawa coupling at a renormalizable level we suggest a slight modification by introducing additional $10' + \overline{10}'$ states and the couplings

$$10\Sigma\overline{15} + 15\Sigma\overline{10}' + M_{15}15\cdot\overline{15} + M_{10}10'\cdot\overline{10}'. \qquad (12)$$

From these terms we can write down the mass matrices for appropriate fragments

$$\begin{array}{c} \\ q_{10'} \\ q_{10} \\ q_{15} \end{array} \begin{array}{c} \bar{q}_{\overline{10}'} \quad \bar{q}_{\overline{15}} \\ \begin{pmatrix} M_{10} & 0 \\ 0 & M_G \\ M_G & M_{15} \end{pmatrix} \end{array}, \qquad \begin{array}{c} \\ u^c_{10} \\ u^c_{10'} \end{array} \begin{array}{c} \bar{u}^c_{\overline{10}'} \\ \begin{pmatrix} 0 \\ M_{10} \end{pmatrix} \end{array}, \qquad \begin{array}{c} \\ e^c_{10} \\ e^c_{10'} \end{array} \begin{array}{c} \bar{e}^c_{\overline{10}'} \\ \begin{pmatrix} 0 \\ M_{10} \end{pmatrix} \end{array}. \qquad (13)$$

The masses of the fragments S and Δ are still given by (10). With $M_{15} \ll M_G$, $M_{10} \lesssim M_G$ the masses of remaining decoupled states are

$$\begin{aligned} &M(q_{10}, \bar{q}_{\overline{15}}) \sim M(q_{15}, \bar{q}_{\overline{10}'}) \simeq M_G, \\ &M(u^c_{10'}, \bar{u}^c_{\overline{10}'}) \simeq M(e^c_{10'}, \bar{e}^c_{\overline{10}'}) \simeq M_{10}, \end{aligned} \qquad (14)$$

and distribution of light q, u^c, and e^c fragments will be as follows:

$$10' \supset q, \quad 10 \supset u^c, e^c, \epsilon' q, \quad \text{with} \quad \epsilon' \equiv \frac{M_{10}M_{15}}{M_G^2}. \qquad (15)$$

We will identify the q state from $10'$ and u^c from 10 with the third generation matter. Therefore, the up quark mass is generated through the coupling $10'\cdot 10H$, while the qqT coupling will be suppressed. In more detail, taking into account (15) we will have

$$Y_U 10'\cdot 10H \to Y_U(qu^c h_u + \epsilon' qqT). \qquad (16)$$

Note that the $e^c u^c T$ coupling is still not generated from (16).

With these simple mechanisms we will be able to suppress $d = 5$ proton decay up to the needed level. If for ith generation ($i = 1, 2, 3$) the suppression factor of the corresponding qqT operator is ϵ_i [see Eqs. (8) and (15) for definition of these factors], and the up quark Yukawa matrix (involved in the coupling $qY_U u^c h_u$) in a family space has the form

$$Y_U = \begin{array}{c} \\ q_1 \\ q_2 \\ q_3 \end{array} \begin{array}{c} u^c_1 \quad u^c_2 \quad u^c_3 \\ \begin{pmatrix} a_1 & a_{12} & a_{13} \\ a_{21} & a_2 & a_{23} \\ a_{31} & a_{32} & a_3 \end{pmatrix} \end{array}, \qquad (17)$$

then the coupling Y_{qq} (involved in $qY_{qq}qT$) will be

$$Y_{qq} \simeq \begin{array}{c} \\ q_1 \\ q_2 \\ q_3 \end{array} \begin{array}{c} q_1 \quad q_2 \quad q_3 \\ \begin{pmatrix} \epsilon_1 a_1 & \epsilon_{12}\bar{a}_{12} & \epsilon_{13}\bar{a}_{13} \\ \epsilon_{12}\bar{a}_{12} & \epsilon_2 a_2 & \epsilon_{23}\bar{a}_{23} \\ \epsilon_{13}\bar{a}_{13} & \epsilon_{23}\bar{a}_{23} & \epsilon_3 a_3 \end{pmatrix} \end{array},$$
$$\text{with} \quad \epsilon_{12}\bar{a}_{12} = \frac{1}{2}(a_{12}\epsilon_2 + a_{21}\epsilon_1), \qquad (18)$$
$$\epsilon_{13}\bar{a}_{13} = \frac{1}{2}(a_{13}\epsilon_3 + a_{31}\epsilon_1),$$
$$\epsilon_{23}\bar{a}_{23} = \frac{1}{2}(a_{23}\epsilon_3 + a_{32}\epsilon_2).$$

Note that since q and e^c states come from different $SU(5)$ states, we can also avoid the asymptotic relation $\hat{M}_D = \hat{M}_E$ common for minimal $SU(5)$ GUT. This will be discussed in more detail later on.

B. Suppressing $ql\bar{T}$ and $u^c d^c \bar{T}$ operators

Now we will present the mechanism for suppressing $ql\bar{T}$ couplings. Recall that in $SU(5)$ this type of term originates from the couplings responsible for generation of down quark and charged lepton masses [see Eq. (3)]. The suppression of $ql\bar{T}$ can occur if the light l and d^c are coming from different $SU(5)$ states. To realize such a splitting in a natural way we introduce an additional vectorlike $SU(5)$ matter $\bar{5}' + 5'$, $\Psi(50) + \bar{\Psi}(\overline{50})$ and the following interaction terms:

$$M_5\bar{5}\cdot 5' + \rho\frac{\Sigma^2}{M_*}\bar{5}'\Psi + \bar{\rho}\frac{\Sigma^2}{M_*}5'\bar{\Psi} + M_\Psi\bar{\Psi}\Psi, \qquad (19)$$

(ρ, $\bar{\rho}$ are dimensionless couplings). The 50-plet does not contain the state with the quantum number of the lepton doublet [16]; however, it includes the state with quantum numbers of d^c. Therefore, after substituting appropriate VEVs in (19), for the mass couplings of the corresponding fragments we will have

$$\begin{array}{c} \\ d^c_{\bar{5}} \\ d^c_{\bar{5}'} \\ d^c_{\bar{\Psi}} \end{array} \begin{array}{c} \bar{d}^c_{5'} \quad \bar{d}^c_{\Psi} \\ \begin{pmatrix} M_5 & 0 \\ 0 & \rho M_G\epsilon_G \\ \bar{\rho}M_G\epsilon_G & M_\Psi \end{pmatrix} \end{array}, \qquad \begin{array}{c} \\ l_{\bar{5}'} \\ l_{\bar{5}} \end{array} \begin{array}{c} \bar{l}_{5'} \\ \begin{pmatrix} 0 \\ M_5 \end{pmatrix} \end{array}, \qquad (20)$$

where $\epsilon_G \equiv M_G/M_*$. As we see, $l_{\bar{5}}$ forms a massive state with $\bar{l}_{5'}$ and therefore the light lepton doublet emerges from $\bar{5}'$. However, the situation is different for d^c. After integrating out $d^c_{\bar{\Psi}}$, $\bar{d}^c_\Psi$ states, the (2, 1) element in the first matrix of (20) receives the correction $\tilde{M} = M_G^2\epsilon_G^2/M_\Psi$. Assuming that $\tilde{M} \gg M_5$, the light d^c state mostly remains in $\bar{5}$, while the light lepton doublet l purely in $\bar{5}'$. Therefore, we have

$$\bar{5} \supset d^c, \qquad \bar{5}' \supset l, \epsilon'' d^c,$$
$$\epsilon'' = \frac{M_5}{\tilde{M}} \ll 1, \qquad \tilde{M} \sim \rho\bar{\rho}\frac{M_G^2}{M_\Psi}\epsilon_G^2. \qquad (21)$$

The masses of the decoupled states are

$$M(d^c_{\bar{5}'}, \bar{d}^c_{5'}) = \tilde{M}, \qquad M(l_{\bar{5}}, \bar{l}_{5'}) = M_5, \tag{22}$$

and all states from Ψ, $\bar{\Psi}$ have mass M_Ψ. From (21) we see that the light lepton doublet and $SU(2)_L$ singlet down quark are coming from different $SU(5)$ multiplets. This splitting will be crucial for suppression of the $ql\bar{T}$ coupling. To see this, we should discuss the mass generation of the down quarks and charged leptons. Thus, it is important to know where the light-left handed quark doublet q comes from. If the light q state emerges from 15-plet and e^c state from 10 [the mechanism ensuring suppression of qqT coupling for 1st and/or 2nd family; see Eq. (11)], then the operators responsible for down quark and charged lepton masses are $15 \cdot \bar{5}\bar{H}$ and $10 \cdot \bar{5}'\bar{H}$, respectively. Namely, taking into account (8), (9), and (21), we have

$$Y_D 15 \cdot \bar{5}\bar{H} \rightarrow Y_D q d^c h_d, \tag{23}$$

$$Y_E 10 \cdot \bar{5}'\bar{H} \rightarrow Y_E(e^c l h_d + \epsilon q l \bar{T} + \epsilon'' u^c d^c \bar{T}). \tag{24}$$

As we see the $ql\bar{T}$ term emerges from the coupling responsible for the charged lepton mass and is suppressed by factor ϵ. At the same time, the $u^c d^c \bar{T}$ coupling is also suppressed. Since the $e^c u^c T$ coupling can be absent (see the discussion in the previous subsection) the corresponding right- handed $d = 5$ operator $u^c u^c d^c e^c$ would not emerge at all. As far as the left-handed operator is concerned, taking into account (11) and (24), it will have the form

$$\epsilon^2 \frac{Y_U Y_E}{M_T} qqql. \tag{25}$$

Note that together with suppression of $ql\bar{T}$, also the relation $\hat{M}_D = \hat{M}_E$ is avoided. The reason is simple: the Yukawa matrices Y_D and Y_E arise from completely independent interaction terms of (26) and (24), respectively.

Now let us show how the suppression of $ql\bar{T}$ coupling works for the case corresponding to Eq. (15) (suppression of qqT operator involving third family). In this case the terms $10' \cdot \bar{5}\bar{H}$ and $10 \cdot \bar{5}'\bar{H}$ are responsible for down type quark and charged lepton masses, respectively. In particular, taking into account (15) and (21) we have

$$\begin{aligned} &Y_D 10' \cdot \bar{5}\bar{H} \rightarrow Y_D q d^c h_d, \\ &Y_E 10 \cdot \bar{5}'\bar{H} \rightarrow Y_E(e^c l h_d + \epsilon' q l \bar{T} + \epsilon'' u^c d^c \bar{T}). \end{aligned} \tag{26}$$

Therefore, the corresponding $d = 5$ operator emerging from (16) and (26)

$$(\epsilon')^2 \frac{Y_U Y_E}{M_T} qqql, \tag{27}$$

is suppressed by factor $(\epsilon')^2$, while the $u^c u^c d^c e^c$-type operator is still absent.

As we see, in both cases [corresponding to (25) and (27)] the $ql\bar{T}$ term emerges from the Yukawa couplings responsible for charged lepton masses. Thus, if Y_E in a family space has the structure

$$Y_E = \begin{matrix} \\ e^c_1 \\ e^c_2 \\ e^c_3 \end{matrix} \begin{matrix} \begin{matrix} l_1 & l_2 & l_3 \end{matrix} \\ \begin{pmatrix} b_1 & b_{12} & b_{13} \\ b_{21} & b_2 & b_{23} \\ b_{31} & b_{32} & b_3 \end{pmatrix} \end{matrix}, \tag{28}$$

then the matrix Y_{ql} (involved in $qY_{ql}l\bar{T}$ coupling) will be

$$Y_{ql} \simeq \begin{matrix} \\ q_1 \\ q_2 \\ q_3 \end{matrix} \begin{matrix} \begin{matrix} l_1 & l_2 & l_3 \end{matrix} \\ \begin{pmatrix} \epsilon_1 b_1 & \epsilon_1 b_{12} & \epsilon_1 b_{13} \\ \epsilon_2 b_{21} & \epsilon_2 b_2 & \epsilon_2 b_{23} \\ \epsilon_3 b_{31} & \epsilon_3 b_{32} & \epsilon_3 b_3 \end{pmatrix} \end{matrix}. \tag{29}$$

Here, the factors ϵ_i are the same as those that appeared in (18).

As we see, the split multiplet mechanisms we have discussed give a good chance for the suppression of nucleon decay. Of course, one should make sure that all couplings which may lead to fast proton decay are absent. For example, the term $u^c e^c T$ can originate from the operator $10 \cdot 10H$. Therefore, some care should be taken to suppress such a coupling. Also, the Planck (cutoff) scale suppressed $d = 5$ baryon number violating operators must be adequately suppressed. In a concrete model, presented in Sec. IV, we will show that all this can be achieved and justified by symmetry arguments.

III. NATURALLY SUPPRESSED $d = 6$ PROTON DECAY

In SUSY $SU(5)$ the exchange of superheavy V_X, V_Y gauge superfields induce dimension six baryon number violating operators. The corresponding D-terms are $(qqu^{c\dagger}e^{c\dagger})_D$ and $(qlu^{c\dagger}d^{c\dagger})_D$. Dimension six operators also emerge in non-SUSY GUTs and may be more problematic if the GUT scale is lower than one in SUSY GUT ($\sim 10^{16}$ GeV with MSSM spectrum below M_G scale).

Thanks to the mechanism discussed in the previous section, these kind of operators can be also suppressed. A crucial role is played by the splitting of appropriate matter. We will discuss the $d = 6$ operator suppression on an example of SUSY $SU(5)$. Let us start consideration with $\bar{5}$-plet superfields which include states with the quantum numbers of d^c and l. The D terms including $\bar{5}$-plets are

$$(\bar{5}^\dagger e^{gV}\bar{5} + \bar{5}'^\dagger e^{gV}\bar{5}')_D, \tag{30}$$

where V and g are $SU(5)$ gauge superfield and the gauge coupling at scale M_G, respectively. According to (21), the $\bar{5}$ states do not include light lepton doublets l at all and therefore the first term in (30) is irrelevant for us. However, from the second term of (30) we get

$$(\bar{5}'^\dagger e^{-gV}\bar{5}')_D \rightarrow \epsilon'' g(l^\dagger V_X d^c + d^{c\dagger} V_Y l)_D. \tag{31}$$

As we see, the couplings of the heavy $V_{X,Y}$ gauge superfields with the matter are suppressed by factor ϵ''.

The kinetic D-term of 15-plet is irrelevant for the baryon number violation because from light states 15-plet includes only q. For the case corresponding to Eqs. (8) and (9) only the 10-plet's D term is relevant:

$$(10^\dagger e^{gV} 10)_D \to \epsilon g(V_X(q^\dagger e^c + qu^{c\dagger}) + V_Y(qe^{c\dagger} + q^\dagger u^c))_D, \tag{32}$$

producing couplings with the suppression factor ϵ.

Upon integration of the V_X, V_Y states with mass $\simeq M_G$ from (31) and (32) we get the following baryon number violating $d = 6$ operators:

$$\frac{g^2}{M_G^2}[\epsilon^2 qqu^{c\dagger}e^{c\dagger} + \epsilon\epsilon'' qlu^{c\dagger}d^{c\dagger} + \text{H.c.}]_D. \tag{33}$$

As we see, two $d = 6$ operators in (33) are naturally suppressed by factors ϵ^2 and $\epsilon\epsilon''$, respectively. Note that if we are dealing with a case corresponding to Eq. (15), then the factor ϵ in (33) must be replaced by ϵ'. Once more, this mechanism for the suppression of $d = 6$ nucleon decay also can be applied within non=SUSY $SU(5)$.

IV. EXAMPLE OF REALISTIC SUSY $SU(5)$

The possibilities for suppressing the proton decay in SUSY $SU(5)$ GUT discussed above can be successfully applied for the realistic model building. By proper selection of the appropriate mass scales we can get suppression [ϵ, ϵ' in Eqs. (8) and (15)] as strong as we wish. This requires the scales M_{15} and M_5 to be below M_G. However, this introduces an additional state below the GUT scale, and the running of gauge couplings will be altered. In order to maintain successful gauge coupling unification, an additional constraint on these scales should be imposed. Suppression of qqT coupling brings the states $(S + \bar{S})_{15}$ and $(\Delta + \bar{\Delta})_{15}$ below M_G. With their masses $M_S = M_\Delta = M_{15}$, one can see that the states S, $\bar{S}$ contribute stronger to the running of α_3 in comparison of Δ, $\bar{\Delta}$'s contribution into the α_2 running. To compensate this disbalance, additional $SU(2)_L$ states are required. This occurs naturally if the mechanism for $ql\bar{T}$ suppression is invoked. In this case below M_G we also have additional $SU(2)_L$ doublets [see Eq. (22)]. This offers the possibility for successful gauge coupling unification.

Now we present an example of SUSY $SU(5)$ realizing ideas discussed above. Considering three families of quarks and leptons, the appropriate couplings [such as of Eqs. (7), (11), (19), and (24)] should be promoted to the matrices in a family space.

Thus, we introduce three pairs of 15-plets: $(15 + \overline{15})_i$ ($i = 1, 2, 3$) and the pair $10' + \overline{10}'$ (needed for renormalizable top Yukawa couping), and also $(\bar{5}' + 5')_i$, $(\Psi + \bar{\Psi})_i$.

In addition, we introduce $\mathcal{U}(1) \times \mathcal{Z}_{3N}$ symmetry, where as will turn out $\mathcal{U}(1)$ is an anomalous and $\mathcal{Z}_{3N}$ is a discrete symmetry. The importance of these symmetries will become obvious soon. The anomalous $U(1)$ factors can appear in effective field theories from strings, and cancellation of its anomalies occurs through the Green-Schwarz mechanism [17]. Because of the anomaly, the Fayet-Iliopoulos term $-\xi\int d^4\theta V_A$ is always generated [18] and the corresponding D_A term has the form [19]

$$\frac{g_A^2}{8}D_A^2 = \frac{g_A^2}{8}\Big(-\xi + \sum Q_i|\phi_i|^2\Big)^2, \qquad \xi = \frac{g_A^2 M_P^2}{192\pi^2}\,\text{Tr}Q, \tag{34}$$

where Q_i is the $\mathcal{U}(1)$ charge of superfield ϕ_i. The transformations under $\mathcal{U}(1)$ and $\mathcal{Z}_{3N}$ are, respectively,

$$\mathcal{U}(1)\colon \phi_i \to e^{iQ_i}\phi_i, \qquad \mathcal{Z}_{3N}\colon \phi_i \to e^{iq_i\omega}\phi_i, \quad \text{with} \quad \omega = \frac{2\pi}{3N}. \tag{35}$$

The anomalous $\mathcal{U}(1)$ can be very useful for building models with realistic phenomenology [20], and we also take advantage of it here for avoiding unwanted couplings. The symmetry $\mathcal{Z}_{3N}$ also will play a crucial role. We introduce two $SU(5)$ singlet superfields X and Z which will be used for $\mathcal{U}(1) \times \mathcal{Z}_{3N}$ breaking. The Q_i and q_i charges of scalar superfields are given in Table I. Let us first discuss the VEV generation for scalar components of X and Z superfields. The lowest superpotential coupling for these superfields, allowed by $\mathcal{U}(1) \times \mathcal{Z}_{3N}$ symmetry, is

$$W(X, Z) = \sigma M_*^3\left(\frac{XZ}{M_*^2}\right)^N, \tag{36}$$

where σ is a dimensionless coupling. With $\xi > 0$, in unbroken SUSY limit the conditions $D_A = 0$, $F_X = F_Z = 0$ give $\langle X\rangle = \sqrt{\xi}$ and $\langle Z\rangle = 0$. However, the nonzero VEV for Z can be generated after including the soft SUSY breaking potential terms

$$V_{\text{SB}} = m_{3/2}^2(|X|^2 + |Z|^2) - Am_{3/2}(W + W^\dagger). \tag{37}$$

Thus, we should minimize the whole potential

$$V = \frac{g_A^2}{8}D_A^2 + |F_X|^2 + |F_Z|^2 + V_{\text{SB}}, \tag{38}$$

where

$$D_A = -\xi + |X|^2 - |Z|^2, \qquad F_X = \frac{\partial W}{\partial X}, \qquad F_Z = \frac{\partial W}{\partial Z}. \tag{39}$$

Considering soft breaking contribution as a perturbation to the potential's leading part, it is natural that by proper selection of N we will get $\langle Z\rangle \ll \langle X\rangle$. Therefore, we parameterize the vacuum as

TABLE I. $\mathcal{U}(1) \times \mathcal{Z}_{3N}$ charges Q, q of the scalar superfields.

	X	Z	$\Sigma(24)$	$H(5)$	$\bar{H}(\bar{5})$
Q	1	-1	0	$-1/3$	$-2/3$
q	0	3	0	-1	1

$$\langle |X|^2\rangle = \xi(1-\kappa^2) + \alpha m_{3/2}^2, \qquad \langle |Y|^2\rangle = \kappa^2\xi, \quad (40)$$

where

$$\alpha \sim 1, \qquad \kappa \ll 1 \quad (41)$$

should be found from the minimization. Note that with this parameterization the D_A term is shifted by the SUSY scale $\sim m_{3/2}^2$. Minimizing the potential (38) with real A and conditions in (41) we find analytically

$$\alpha = -\frac{4}{g_A^2} + \mathcal{O}(\kappa^2),$$
$$\kappa = \bar{\alpha}\left(\frac{m_{3/2}}{M_*}\right)^{1/(N-2)}\left(\frac{\sqrt{\xi}}{M_*}\right)^{-2(N-1)/(N-2)}, \quad (42)$$
$$\text{with} \quad \bar{\alpha} = \left(\frac{A \pm \sqrt{A^2 - 8(N-1)}}{2\sigma N(N-1)}\right)^{1/(N-2)}.$$

With $m_{3/2} \sim 1$ TeV, $M_* \sim 10^{17}$ GeV (we will comment on this value of the cutoff scale below), $\sqrt{\xi} \sim 10^{16}$ GeV, and $N = 6$ we have $\kappa \sim 0.1$ and therefore initial assumption (41) is justified. Thus, finally we have

$$\frac{\langle X\rangle}{M_*} \simeq \frac{\sqrt{\xi}}{M_*} = 0.1, \qquad \frac{\langle Z\rangle}{M_*} \simeq \kappa\frac{\sqrt{\xi}}{M_*} \sim 10^{-2}. \quad (43)$$

Below we will use these values obtained by $\mathcal{U}(1) \times \mathcal{Z}_{3N}$ ($N = 6$) symmetry.

One may wonder whether with charge assignments given in Table I, desirable GUT symmetry breaking can occur or not. Also, the color triplets from H, $\bar{H}$ should be superheavy, while doublets should remain massless [doublet-triplet (DT) splitting]. Since the adjoint Σ is not transformed under $\mathcal{U}(1) \times \mathcal{Z}_{3N}$ symmetry, the renormalizable superpotential includes couplings

$$W(\Sigma) = M_\Sigma \,\mathrm{Tr}\Sigma^2 + \lambda_\Sigma \,\mathrm{Tr}\Sigma^3, \quad (44)$$

and the nonzero VEV $\langle\Sigma\rangle = V_\Sigma \cdot \mathrm{Diag}(2, 2, 2, -3, -3)$ with $V_\Sigma = \frac{2M_\Sigma}{3\lambda_\Sigma}$ is obtained. This insures the breaking $SU(5) \to SU(3)_c \times SU(2)_L \times U(1)_Y$. As far as the DT splitting is concerned, without invoking some particular mechanism it can be achieved by fine-tuning [21] if couplings $(M_H + \lambda_H\Sigma)\bar{H}H$ exist. However, these couplings are forbidden by $\mathcal{U}(1) \times \mathcal{Z}_{3N}$ symmetry. Instead, the operators $\lambda' X\bar{H}H + X\Sigma\bar{H}H/M'$ are allowed. They lead to the DT splitting with $M' \sim \langle X\rangle$ and $\lambda' \sim V_\Sigma/\langle X\rangle$. The operator $X\Sigma\bar{H}H/M'$ can be generated by decoupling of additional states with mass M'. For example, introducing $H'(5)$, $\bar{H}'(\bar{5})$ states with (Q, q) charges $(-1/3, -1)$ and $(1/3, 1)$ respectively, the relevant couplings are

$$\lambda' X\bar{H}H + \sigma_1\Sigma\bar{H}'H + \sigma_2 X\bar{H}H' + M'\bar{H}'H'. \quad (45)$$

After integrating out the heavy H', $\bar{H}'$ states, we remain with effective superpotential couplings

$$\lambda' X\bar{H}H - \sigma_1\sigma_2\frac{X}{M'}\Sigma\bar{H}H. \quad (46)$$

With a selection $\lambda' = -3\sigma_1\sigma_2 V_\Sigma/M'$ the MSSM Higgs doublets remain massless, while the color triplets occur with mass $M_T = 5\lambda'\langle X\rangle/3$ ($\sim M_G$ with $\lambda' \sim V_\Sigma/\langle X\rangle$). Therefore, we see that within SUSY $SU(5)$ GUT, augmented with $\mathcal{U}(1) \times \mathcal{Z}_{3N}$ symmetry, it is possible to build a self-consistent scalar sector.

Now we are ready to discuss the fermion sector. The $\mathcal{U}(1) \times \mathcal{Z}_{3N}$ charge assignments for matter states are displayed in Table II. Note that with this prescription all matter parity violating operators are forbidden. Therefore, thanks to the $\mathcal{U}(1) \times \mathcal{Z}_{3N}$ symmetry the R-parity is automatic. The reason for this is the fact that by VEVs $\langle X\rangle$, $\langle Z\rangle$ the $\mathcal{U}(1) \times \mathcal{Z}_{3N}$ is not completely broken. Namely, the subgroup $\mathcal{Z}_3^A \times \mathcal{Z}_3$ remains unbroken. The transformations under $\mathcal{Z}_3^A$ and $\mathcal{Z}_3$ are, respectively,

$$\mathcal{Z}_3^A\colon\ \phi_i \to e^{i2\pi Q_i}\phi_i,$$
$$\mathcal{Z}_3\colon\ \phi_i \to e^{iq_i\bar{\omega}}\phi_i, \quad \text{with} \quad \bar{\omega} = \frac{2\pi}{3}. \quad (47)$$

The superfields X, Z are neutral under $\mathcal{Z}_3^A$ and $\mathcal{Z}_3$.

In understanding observed hierarchies between charged fermion masses and mixings, the flavor structure of the Yukawa sector plays a crucial role. The same is true in connection of the color Higgsino mediated and Planck scale suppressed $d = 5$ operator induced nucleon decay. Their structures determine the signature of the nucleon decay. Definite structures as well as predictions can be obtained by flavor symmetries. Indeed, symmetry principle is very powerful for a predictive power. We will not introduce here generation symmetries, and instead we consider one particular example demonstrating realization of the split multiplet mechanism.

Thus we promote the couplings of (7) and (12) in the flavor space as

$$\lambda_i 10_i\Sigma\overline{15}_i + \bar{\lambda}15_3\Sigma\overline{10}' + \lambda_i^Z Z\overline{15}_i 15_i + M_{10}10'\overline{10}', \quad (48)$$

where for simplicity we have assumed that the matrices λ, λ^Z are diagonal and only 15_3 couples with $\overline{10}'$. Moreover, we take

$$\lambda_i \sim \bar{\lambda} \sim 1, \qquad M_{10} \sim M_G,$$
$$M_{15_1}, M_{15_2} = M_{15_3} \equiv M_{15} \ll M_G, \quad (49)$$

where $M_{15_i} = \lambda_i^Z\langle Z\rangle$. Thus, with

TABLE II. $\mathcal{U}(1) \times \mathcal{Z}_{3N}$ charges Q, q of matter superfields.

	10_i	$\overline{15}_i$	$\bar{5}_i$, $5'_i$	$\bar{5}'_i$, $\bar{\Psi}_i$	15_i, $10'$	$\overline{10}'$	Ψ_i
Q	$-1/3$	$1/3$	0	1	$2/3$	$-2/3$	-1
q	2	-2	0	-3	-1	1	3

$$\epsilon_1 = \frac{M_{15_1}}{\lambda_1 M_G}, \qquad \epsilon_2 = \frac{M_{15_2}}{\lambda_2 M_G}, \qquad \epsilon_3 = \frac{M_{10} M_{15_3}}{\lambda_3 \bar{\lambda} M_G^2}, \qquad (\epsilon_2 \sim \epsilon_3 \equiv \epsilon), \tag{50}$$

and carrying out analysis analogous to that done in Sec. II A, we will have

$$q_{1,2} \subset 15_{1,2}, \qquad q_3 \subset 10', \qquad (u^c, e^c)_{1,2,3} \subset 10_{1,2,3}, \qquad 10_1 \supset \epsilon_1 q_1, \qquad 10_2 \supset \epsilon q_2, \qquad 10_3 \supset \epsilon q_3. \tag{51}$$

The couplings $\frac{Z}{M_*} 10' \Sigma \overline{15}_i$ because of the suppression factor $\sim\langle Z\rangle/M_* \sim 10^{-2}$ do not change these relations. They cause $15_i \supsetneq 10^{-3} q_3$, $10_i \supsetneq 10^{-2} q_3$ which will not have any impact on our studies. The mass spectrum of the decoupled states is

$$M(q_{10_i}, \bar{q}_{\overline{15}_i}) \sim M(q_{15_3}, \bar{q}_{\overline{10'}}) \sim M(u^c_{10'}, \bar{u}^c_{\overline{10'}}) \simeq M(e^c_{10'}, \bar{e}^c_{\overline{10'}}) \simeq M_G, \quad M_{S_1} = M_{\Delta_1} = M_{15_1} \simeq \epsilon_1 M_G, \quad M_{S_{2,3}} = M_{\Delta_{2,3}} = M_{15} \simeq \epsilon M_G. \tag{52}$$

Moreover, the couplings in (19) will be replaced by $\mathcal{U}(1) \times \mathcal{Z}_{3N}$ invariant terms

$$\begin{array}{c} \\ \bar{5}_i \\ \bar{5}'_i \\ \bar{\Psi}_i \end{array} \begin{array}{c} \begin{array}{cc} 5_i' & \Psi_i \end{array} \\ \begin{pmatrix} M_5 & 0 \\ M_5' Z/M_* & \rho \Sigma^2/M_* \\ \bar{\rho}\Sigma^2 Z/M_*^2 & M_\Psi \end{pmatrix} \end{array}. \tag{53}$$

Here we still assumed that the appropriate entries are diagonal and universal. We assume that $M_5 \sim M_5'$ (the smallness of both these scales, with respect to M_G or M_*, may have the same origin; however, this is not explained here). Therefore, carrying out a similar analysis as that presented in the previous section, with

$$\epsilon'' \equiv \frac{M_5}{\tilde{M}} \ll 1, \qquad \tilde{M} \sim \rho \bar{\rho} \frac{M_G^2 \langle Z\rangle}{M_\Psi M_*} \epsilon_G^2, \tag{54}$$

we have

$$\bar{5}_i \supset d_i^c, 10^{-2} l_i \qquad \bar{5}'_i \supset l_i, \epsilon'' d_i^c. \tag{55}$$

The decoupled states will have the masses

$$M(l_{\bar{5}_i}, l_{5'_i}) = M_5, \qquad M(d^c_{\bar{5}_i'}, \bar{d}^c_{5_i'}) = \tilde{M}, \qquad M_{\Psi_i} = M_\Psi. \tag{56}$$

Note that in (1, 2) entry of (53) the operator $\Sigma^2 (XZ)^5/M_*^{11}$ is allowed. However, it would induce a strongly suppressed correction $\sim 10^{-17} M_G$ and is not relevant. Now we can discuss the gauge coupling unification. The latter suggests the particular selection for appropriate mass scales.

A. Gauge coupling unification

We assume that the masses of the matter 50_i-plets are close to the cutoff scale $M_\Psi \simeq M_*$—much higher than the GUT scale. Thus, they do not affect the gauge coupling running. Moreover, with $M_\Sigma \sim M_G$ and $\lambda_\Sigma \sim 1$ in (44) for colored octet and $SU(2)_L$ triplet (from adjoint Σ) masses we get $m_8 = m_3 \sim M_G$ and with $M_G \ll M_*$ higher order operators will not affect this relation. Also, with color triplets' mass (from H, $\bar{H}$) near $\sim M_G$, these states will not contribute to the gauge coupling running and at the leading order will not play a role in determination of the GUT scale (unlike the proposals of [14]). However, for the masses of the three vectorlike pairs we have $M(d^c_{\bar{5}'}, \bar{d}^c_{5'}) = \tilde{M}$ [see Eq. (56)]. Apart from these states, below M_G we have $3 \times (l_{\bar{5}} + \bar{l}_{5'})$ and $3 \times (S + \bar{S} + \Delta + \bar{\Delta})_{15}$ states with masses given in (52) and (56). Thus, for the strong gauge coupling constant at M_Z scale in one-loop approximation we get:

$$\alpha_3^{-1} = (\alpha_3^0)^{-1} - \frac{3}{2\pi} \ln(\epsilon^2 \epsilon_1) - \frac{27}{14\pi} \ln \frac{\tilde{M}}{M_5}, \tag{57}$$

where α_3^0 is the value of the strong coupling constant within MSSM and is $\alpha_3^0(M_Z) \simeq 0.126$ [22]. The additional terms in (57) allow one to obtain the value compatible with experiments $\alpha_3^{\exp}(M_Z) \simeq 0.1176$ [3]. This can be achieved with $\frac{M_5}{\tilde{M}} \approx e^{14/15} (\epsilon^2 \epsilon_1)^{7/9}$. In order to have a more accurate estimate, we have performed calculations in two loop approximation. The picture of gauge coupling unification is given in Fig. 1.

For simplicity we have taken all squark, slepton, Higgsino, and gaugino masses all near the TeV scale. In particular,

$$m_{\tilde{q}} = m_{\tilde{l}} = M_{\tilde{h}} = M_{\tilde{g}} = m_{\rm susy} = 10^{2.9}\ {\rm GeV}, \qquad M_{\tilde{W}} = M_{\tilde{g}} \frac{\alpha_2}{\alpha_3}\bigg|_{\mu = m_{\rm susy}} \simeq 287\ {\rm GeV}. \tag{58}$$

Also, we have taken

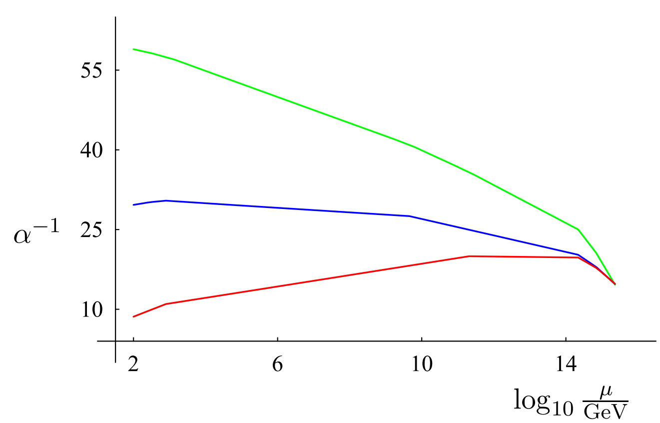

FIG. 1 (color online). Gauge coupling unification. $\alpha_3(M_Z) \simeq 0.1176$, $M_G \simeq 2.2 \cdot 10^{15}$ GeV.

$$\epsilon_1 = 1/3, \qquad \epsilon = 0.1, \qquad \frac{M_5}{\tilde{M}} \simeq 2.2 \cdot 10^{-2}, \quad \text{with} \quad \tilde{M} \simeq 2.1 \cdot 10^{11} \text{ GeV}. \tag{59}$$

(For stronger suppression of nucleon decay smaller values of ϵ_1, ϵ are required. However, this would make new states lighter and the constraint from the coupling unification does not give much flexibility.) All this and input values $\alpha_1^{-1}(M_Z) = 59.0$, $\alpha_2^{-1}(M_Z) = 29.6$ provides the successful unification with

$$\alpha_3(M_Z) = 0.1176, \qquad M_G \simeq 2.2 \cdot 10^{15} \text{ GeV}, \qquad \alpha_G^{-1}(M_G) \simeq 14.91. \tag{60}$$

As we see, due to the new states the unification scale M_G is reduced, while the unified gauge coupling α_G is enhanced:

$$\frac{M_G}{M_G^0} \simeq \frac{1}{9.1}, \qquad \frac{\alpha_G}{\alpha_G^0} \simeq 1.6, \tag{61}$$

[superscript "0" indicates the values obtained within minimal SUSY $SU(5)$]. These values will be useful for estimating the proton decay rates in this model.

One can see that near the 10^{17} GeV scale the unified gauge coupling becomes strong $\frac{\alpha_G}{4\pi} \simeq 0.26$. Thus, for the cutoff scale one should take $M_* \simeq 10^{17}$ GeV. With this, the perturbative regime is kept in quite a wide range above the GUT scale. As we will see shortly, the values of ϵ_1 and ϵ selected here provide an adequate suppression of the proton decay.

Finally, we calculate short-range renormalization factors which will be used in the next subsection. The appropriate baryon number violating $d = 5$ and $d = 6$ operators, generated at GUT scale, should be defined at scale $\mu = 1$ GeV. Thus two ranges are relevant for renormalization. Because of running from M_G down to M_Z (or SUSY scale) the appropriate factor A^S is called short-range renormalization factor. From scale M_Z down to 1 GeV the appearing factor A_L is the long-range factor which is mainly due to QCD running.

Let us start with the calculation of the short-range factor corresponding to $d = 5$ operators. Note that in our model the $ql\bar{T}$ coupling is related to the charged lepton Yukawa matrix. Therefore, generalizing the expression given in [23], we will have

$$A_{d=5}^S = A_{d=5,ue}^S = A(\lambda_t) \prod_{i;a>b} \left(\frac{\alpha_i(\mu_a)}{\alpha_i(\mu_b)}\right)^{c_i/(b_i(\mu_{a-b}))}, \qquad c_i = \left(-\frac{17}{15}, 0, \frac{4}{3}\right), \tag{62}$$

where $b_i(\mu_{a-b})$ denotes gauge coupling one-loop b factors in the mass interval $\mu_b - \mu_a$, and $A(\lambda_t)$ includes the renormalization effect due to the top Yukawa coupling. We have evaluated $A_{d=5}^S$ for our scenario (more details of the Yukawa sector are given in Sec. IV B) in 2-loop approximation for $\lambda_t(M_Z) \simeq 1$ and obtained

$$A_{d=5}^S \simeq 2.03, \tag{63}$$

[to be compared with the factor obtained in MSSU5 $(A_{d=5}^S)^0 = (A_{d=5,ud}^S)^0 \simeq 0.92$].

As far as the $d = 6$ operators are concerned, as it will turn out, the first type operator of Eq. (33) will be relevant. Its one-loop short-range renormalization factor is given by

$$A_{d=6}^S = \prod_{i;a>b} \left(\frac{\alpha_i(\mu_a)}{\alpha_i(\mu_b)}\right)^{\bar{c}_i/(b_i(\mu_{a-b}))}, \qquad \bar{c}_i = \left(\frac{23}{30}, \frac{3}{2}, \frac{4}{3}\right). \tag{64}$$

In our model numerically we get $A_{d=6}^S \simeq 2.23$. Also long-range renormalization factor A_L should be taken into account. The latter is $A_L \simeq 1.34$ [24], and finally for the $d = 6$ operator renormalization factor we have

$$A_R^{d=6} = A_L A_{d=6}^S \simeq 2.99. \tag{65}$$

B. Proton life time

The nucleon decay via $d = 5$ operators crucially depends on the Yukawa sector. Therefore, first we briefly discuss how a desirable fermion pattern can be obtained. Since we have arranged the multiplet splitting, displayed in Eqs. (51) and (55), it will not be difficult to get realistic fermion masses. Once more we stress that we consider one particular example with simple flavor structure. Starting with up type quarks, we will write appropriate couplings in such a way that the up quark mass matrix will be diagonal. Relevant terms consistent with $\mathcal{U}(1) \times \mathcal{Z}_{3N}$ symmetry are

$$\frac{\Sigma}{M_*}(\gamma_1 15_1 10_1 + \gamma_2 15_2 10_2)H + \gamma_3 10' 10_3 H, \tag{66}$$

where $\gamma_{1,2,3}$ are dimensionless constants. Taking into account (51) we will have

$$Y_U = \text{Diag}(\lambda_u, \lambda_c, \lambda_t), \qquad \lambda_{u,c} \sim \gamma_{1,2}\frac{\langle\Sigma\rangle}{M_*}, \qquad \lambda_t = \gamma_3. \tag{67}$$

Thus, the Cabibbo-Kobayashi-Maskawa (CKM) mixings (V_{ij}) should come from the down quark sector. The relevant couplings for the latter are

$$(15_1, \quad 15_2, \quad 10')Y_D \begin{pmatrix} \bar{5}_1 \\ \bar{5}_2 \\ \bar{5}_3 \end{pmatrix}, \tag{68}$$

where the nondiagonal Yukawa matrix Y_D is responsible for CKM mixings.

Thanks to the mechanism discussed in Sec. II, the charged lepton Yukawa matrix elements are independent from Y_D. For simplicity we take diagonal couplings

$$Y_i 10_i \bar{5}'_i \bar{H}, \tag{69}$$

which with (51) and (55), give

$$Y_E = \mathrm{Diag}(\lambda_e, \lambda_\mu, \lambda_\tau), \qquad \lambda_{e,\mu,\tau} = Y_{1,2,3}. \tag{70}$$

Note, that the $\mathcal{U}(1) \times \mathcal{Z}_{3N}$ symmetry provides the suppression of the coupling $15 \cdot \bar{5}' \bar{H}$ by a factor $\sim\langle Z\rangle/M_* \sim 10^{-2}$ in comparison of (68) and (69) operators, and therefore can be ignored. As far as the $10 \cdot \bar{5}\,\bar{H}$ type couplings are concerned, they are strongly suppressed ($\sim \langle\frac{X(XZ)^5}{M_*^{11}}\rangle \sim 10^{-16}$). From all this and Eqs. (18) and (29), Y_{qq} and Y_{ql} matrices will be

$$\begin{aligned} Y_{qq} &= \mathrm{Diag}(\epsilon_1 \lambda_u, \epsilon\lambda_c, \epsilon\lambda_t), \\ Y_{ql} &= \mathrm{Diag}(\epsilon_1 \lambda_e, \epsilon\lambda_\mu, \epsilon\lambda_\tau). \end{aligned} \tag{71}$$

These couplings induce $qqql$ type $d = 5$ left-handed operators.

Before estimating the proton life time, let us note that the couplings $10_i 10_j H$ are forbidden by $\mathcal{U}(1) \times \mathcal{Z}_{3N}$ symmetry. Only the higher order operators $\frac{Z^5 X^6}{M_*^{11}} 10_i 10_i H$ are allowed. For the VEVs given in (43), induced operator $u^c e^c T$ is suppressed by factor $\sim 10^{-16}$. This makes color triplet induced $d = 5$ right-handed baryon number violating operators completely irrelevant. As far as the cutoff scale suppressed $d = 5$ baryon number violating operators are concerned, one can easily see that they also involve high powers of $\langle Z\rangle/M_*$ and $\langle X\rangle/M_*$. Namely, the allowed couplings are $X(XZ)^4 10_i 10_j 10_k \bar{5}$, $(XZ)^5 10_i 10_j 10_k \bar{5}'$, etc. Thus the suppression by factors $\lesssim 10^{-13}$ is guaranteed. Therefore, we conclude that in our model *only* sources for the proton decay are the couplings given in (71) and X, Y boson induced decay which we discuss afterwards.

The appropriate $d = 5$ left-handed operator is converted to four fermion operators through the W-ino dressings. Those, relevant for nucleon decay, have forms

$$-\frac{1}{M_T} \mathcal{F} \alpha_{ijk} (ud^i)(d^j \nu^k), \tag{72}$$

$$\frac{1}{M_T} \mathcal{F} \alpha'_{ij} (ud^i)(ue^j), \tag{73}$$

where, together with other family independent factors, $\mathcal{F}$ includes the loop integral, and for simplicity we have assumed that the squarks and sleptons of all families have universal mass. The flavor dependent couplings α_{ijk} and $\alpha_{ij}{}'$ are given by [5,25]

$$\begin{aligned} \alpha_{ijk} = {} & (L_d^\dagger Y_{ql} L_e)_{jk} (V^T L_u^\dagger Y_{qq} L_d^* V^\dagger)_{i1} \\ & + (L_d^\dagger Y_{qq} L_u^* V)_{ji} (V^* L_d^\dagger Y_{ql} L_e)_{1k} \\ & - (L_u^\dagger Y_{qq} L_d^*)_{1i} (V^T L_u^\dagger Y_{ql} L_e)_{jk} \\ & + (L_d^\dagger Y_{qq} L_u^* V)_{ij} (L_u^\dagger Y_{ql} L_e)_{1k}, \end{aligned} \tag{74}$$

$$\begin{aligned} \alpha'_{ij} = {} & -(L_u^\dagger Y_{qq} L_d^*)_{1i} (V^* L_d^\dagger Y_{ql} L_e)_{1j} \\ & + (L_u^\dagger Y_{qq} L_d^* V^\dagger)_{11} (L_d^\dagger Y_{ql} L_e)_{ij} \\ & + (L_u^\dagger Y_{ql} L_e)_{1j} (V^T L_u^\dagger Y_{qq} L_d^* V^\dagger)_{1i} \\ & + (L_u^\dagger Y_{qq} L_d^* V^\dagger)_{11} (L_e^T Y_{ql}^T L_u^* V)_{j1}. \end{aligned} \tag{75}$$

$L_{u,d,e}$ are unitary matrices transforming the left-handed fermion states in order to diagonalize corresponding mass matrices.

For the considered case here we have $L_u = L_e = \mathbf{1}$, $L_d = V^*$. Therefore, the only nondiagonal matrix is the CKM matrix. In particular, using (71), we have $\alpha_{ijk} = 2\delta_{1k}\lambda_e\epsilon_1 (V^T Y_{qq} V)_{ij}$. These factors are responsible for the decays with neutrino emission. The dominant decay mode is $p \to K^+ \nu_e$ and the corresponding amplitude is proportional to $\frac{1}{M_G} 2\lambda_e\lambda_c\theta_c\epsilon\epsilon_1$ ($\theta_c = 0.22$ is a Cabibbo angle). Note that in MSSU5 the amplitude of the dominant decay mode $p \to K\nu_\mu$ is $\sim\frac{1}{M_G^0} 2\lambda_s\lambda_c\theta_c^2$. Thus, in our model for the corresponding partial life time we expect

$$\begin{aligned} \tau_{d=5}(p \to K^+ \nu_e) = {} & \left(\frac{\lambda_s\theta_c}{\lambda_e\epsilon\epsilon_1}\right)^2 \left(\frac{M_G}{M_G^0}\right)^2 \left(\frac{(A^S_{d=5})^0}{A^S_{d=5}}\right)^2 \\ & \times \tau_0(p \to K^+ \nu_\mu), \end{aligned} \tag{76}$$

where τ_0 is proton life time in MSSU5. Taking all SUSY breaking soft terms near the TeV scale, we have

$$\begin{aligned} \tau_{d=5}(p \to K^+ \nu_e) \simeq {} & 3.8 \cdot 10^3 \cdot \left(\frac{0.1}{\epsilon}\right)^2 \left(\frac{1/3}{\epsilon_1}\right)^2 \\ & \times \left(\frac{M_G}{2.2 \cdot 10^{15}\ \mathrm{GeV}}\right)^2 \tau_0. \end{aligned} \tag{77}$$

In (77) we used $A^S_{d=5} = 2.03$ calculated for our model [see Eq. (63)]. The decays with emission of the charged leptons are due to α' factors. The dominant mode is $p \to K^0 \mu^+$ (with corresponding factor $\alpha'_{22} \simeq 2\lambda_u\lambda_\mu\epsilon\epsilon_1$) with the life time

$$\begin{aligned} \tau_{d=5}(p \to K^0 \mu^+) \simeq {} & 5.4 \cdot 10^3 \cdot \left(\frac{0.1}{\epsilon}\right)^2 \left(\frac{1/3}{\epsilon_1}\right)^2 \\ & \times \left(\frac{M_G}{2.2 \cdot 10^{15}\ \mathrm{GeV}}\right)^2 \tau_0. \end{aligned} \tag{78}$$

As we see, both decay modes of Eqs. (77) and (78) are suppressed in comparison to the dominant decay mode of MSSU5. In order to make an estimate of proton life time one should make selection of sparticle spectrum. With soft terms near TeV, given in (58), we will have [26] $\tau_0 \simeq 3.5 \cdot 10^{30}$ years [5]. Thus, we will have

$$\begin{aligned} \tau_{d=5}(p \to K^+ \nu_e) & \simeq 0.7 \cdot \tau_{d=5}(p \to K^0 \mu^+) \\ & \simeq 1.3 \cdot 10^{34}\ \text{years} \times \left(\frac{\sin 2\beta}{0.50}\right)^2. \end{aligned} \tag{79}$$

These are above current experimental bounds $\tau^{\rm exp}(p \to K^+\nu) \gtrsim 6.7\cdot 10^{32}$ years and $\tau^{\rm exp}(p\to K^0\mu^+) \gtrsim 1.2\cdot 10^{32}$ years [3]. Ongoing and planned experiments give promise to probe partial lifetimes given in (79) (these life times decrease with the increase of $\tan\beta$).

Since in our model the GUT scale is reduced nearly by factor 10 and the unified gauge coupling is stronger, the $d=6$ operators become relevant. However, due to multiplet splitting, the suppression still occurs [see Eq. (33)]. The dominant $d=6$ operator is $\epsilon_1^2 \frac{g^2}{M_G^2}(q_1q_1u_1^{c\dagger}e_1^{c\dagger})_D$, where subscripts label the flavor indices. This operator induces the process $p\to\pi^0e^+$ with a decay width:

$$\Gamma_{d=6}(p\to\pi^0e^+) = \frac{m_p}{16\pi f_\pi^2}\bar{\alpha}^2(1+D+F)^2 \times\left(\frac{g^2}{M_G^2}\epsilon_1^2A_R^{d=6}\right)^2. \quad (80)$$

With $f_\pi = 0.13$ GeV, $\bar{\alpha}=0.015\ {\rm GeV}^3$, $D=0.8$, $F=0.47$, $A_R^{d=6}=2.99$, and $\epsilon_1 = 1/3$ we get

$$\tau_{d=6}(p\to\pi^0e^+) = \frac{1}{\Gamma_{d=6}(p\to\pi^0e^+)} \simeq 5\cdot 10^{33}\ \text{years}, \quad (81)$$

which is slightly above the experimental limit [$\tau^{\rm exp}(p\to\pi^0e^+)\gtrsim 1.6\cdot 10^{33}$ yrs.] This (possibly) dominant decay mode is a characteristic signature of our model. Future experiments will probe such decays and test viability of the particular model presented here.

V. CONCLUSIONS

In this paper we have suggested the mechanism for suppressing the nucleon decay within SUSY $SU(5)$ GUT. The mechanism is based on the idea of split multiplets and also helps to build a realistic fermion pattern. For transparent demonstration of the presented mechanism we have considered a simple example consistent with gauge coupling unification, realistic fermion mass pattern, and the proton life time compatible with experiments.

The suggested possibilities can be applied for building various realistic $SU(5)$ scenarios with interesting phenomenological implications. In particular, it would be interesting, in this context, to address the problem of flavor and try to gain a natural understanding of observed hierarchies between fermion masses and mixings. Also, it is desirable to understand the origin of hierarchies between various mass scales appearing in the construction. For all this additional symmetries, such as flavor symmetry, may play a crucial role guaranteeing the robustness of predictions. In a concrete model, for realizing suggested mechanisms and for suppression of unwanted baryon number violation, we have applied $\mathcal{U}(1)\times\mathcal{Z}_{3N}$ symmetry (also providing an automatic R-parity). It will be interesting to use such a symmetry as flavor symmetry.

Finally, here we have not attempted to have a natural solution of the doublet-triplet splitting problem. For the latter GUTs, such as $SO(10)$ [27] and $SU(6)$ [7,28], are more motivated. One can attempt to realize the split multiplet mechanism within these constructions and also study other phenomenology. These and related issues will be discussed elsewhere.

ACKNOWLEDGMENTS

I thank K. S. Babu for useful discussions.

[1] N. Sakai and T. Yanagida, Nucl. Phys. **B197**, 533 (1982); S. Weinberg, Phys. Rev. D **26**, 287 (1982).

[2] This decay mode dominates in simple minded SUSY $SU(5)$ and $SO(10)$ GUTs.

[3] W. M. Yao *et al.* (Particle Data Group), J. Phys. G **33**, 1 (2006).

[4] (a) G. R. Dvali, Phys. Lett. B **287**, 101 (1992); (b) K. S. Babu and S. M. Barr, Phys. Rev. D **48**, 5354 (1993); (c) J. Hisano, T. Moroi, K. Tobe, and T. Yanagida, Phys. Lett. B **342**, 138 (1995); (d) Z. Berezhiani, arXiv:hep-ph/9602325; (e) I. Gogoladze and A. Kobakhidze, Phys. At. Nucl. **60**, 126 (1997); (f) Z. Chacko and R. N. Mohapatra, Phys. Rev. D **59**, 011702 (1998).

[5] Z. Berezhiani, Z. Tavartkiladze, and M. Vysotsky, arXiv:hep-ph/9809301.

[6] Q. Shafi and Z. Tavartkiladze, Phys. Lett. B **451**, 129 (1999); **459**, 563 (1999).

[7] Q. Shafi and Z. Tavartkiladze, Nucl. Phys. **B573**, 40 (2000).

[8] B. Bajc, P. Fileviez Perez, and G. Senjanovic, Phys. Rev. D **66**, 075005 (2002); D. Emmanuel-Costa and S. Wiesenfeldt, Nucl. Phys. **B661**, 62 (2003); S. Wiesenfeldt, Mod. Phys. Lett. A **19**, 2155 (2004); N. Haba and T. Ota, arXiv:hep-ph/0608244; I. Dorsner, P. F. Perez, and G. Rodrigo, Phys. Lett. B **649**, 197 (2007).

[9] For a review and extended list of references see P. Nath and P. F. Perez, Phys. Rep. **441**, 191 (2007).

[10] Y. Kawamura, Prog. Theor. Phys. **105**, 999 (2001); G. Altarelli and F. Feruglio, Phys. Lett. B **511**, 257 (2001); A. B. Kobakhidze, Phys. Lett. B **514**, 131 (2001); L. J. Hall and Y. Nomura, Phys. Rev. D **64**, 055003 (2001); A. Hebecker and J. March-Russell, Nucl. Phys. **B613**, 3 (2001).

[11] R. Dermisek and A. Mafi, Phys. Rev. D **65**, 055002

(2002); A. Hebecker and J. March-Russell, Phys. Lett. B **539**, 119 (2002); L. J. Hall and Y. Nomura, Phys. Rev. D **66**, 075004 (2002); Q. Shafi and Z. Tavartkiladze, Phys. Rev. D **67**, 075007 (2003); Nucl. Phys. **B665**, 469 (2003); B. Kyae, C. A. Lee, and Q. Shafi, Nucl. Phys. **B683**, 105 (2004); W. Buchmuller, L. Covi, D. Emmanuel-Costa, and S. Wiesenfeldt, J. High Energy Phys. 09 (2004) 004; M. L. Alciati, F. Feruglio, Y. Lin, and A. Varagnolo, J. High Energy Phys. 03 (2005) 054; 11 (2006) 039.

[12] Although we will discuss the mechanism in an example of SUSY $SU(5)$, it can be applied also for non-SUSY $SU(5)$ in order to suppress GUT gauge boson mediated $d = 6$ baryon number violation.

[13] Q. Shafi and Z. Tavartkiladze, Phys. Lett. B **473**, 272 (2000); arXiv:hep-ph/0207231.

[14] C. Bachas, C. Fabre, and T. Yanagida, Phys. Lett. B **370**, 49 (1996); B. Bajc, P. Fileviez Perez, and G. Senjanovic, arXiv:hep-ph/0210374.

[15] J. Hisano, H. Murayama, and T. Yanagida, Nucl. Phys. **B402**, 46 (1993).

[16] This property of 50-plet can be used to realize natural doublet-triplet splitting in the scalar sector by missing partner mechanism: H. Georgi, Phys. Lett. **108B**, 283 (1982); B. Grinstein, Nucl. Phys. **B206**, 387 (1982); A. Masiero, D. V. Nanopoulos, K. Tamvakis, and T. Yanagida, Phys. Lett. **115B**, 380 (1982).

[17] M. B. Green and J. H. Schwarz, Phys. Lett. **149B**, 117 (1984).

[18] E. Witten, Nucl. Phys. **B188**, 513 (1981); W. Fischler, H. P. Nilles, J. Polchinski, S. Raby, and L. Susskind, Phys. Rev. Lett. **47**, 757 (1981).

[19] M. Dine, N. Seiberg, and E. Witten, Nucl. Phys. **B289**, 589 (1987); J. J. Atick, L. J. Dixon, and A. Sen, Nucl. Phys. **B292**, 109 (1987); M. Dine, I. Ichinose, and N. Seiberg, Nucl. Phys. **B293**, 253 (1987).

[20] G. R. Dvali and S. Pokorski, Phys. Rev. Lett. **78**, 807 (1997).

[21] Of course, it is desirable to have a more appealing mechanism for the DT splitting, but this goes beyond the scope of this work.

[22] P. Langacker and N. Polonsky, Phys. Rev. D **52**, 3081 (1995); J. Bagger, K. T. Matchev, and D. Pierce, Phys. Lett. B **348**, 443 (1995).

[23] L. E. Ibanez and C. Munoz, Nucl. Phys. **B245**, 425 (1984).

[24] T. Nihei and J. Arafune, Prog. Theor. Phys. **93**, 665 (1995).

[25] P. Nath and R. Arnowitt, Phys. Rev. D **38**, 1479 (1988); N. Sakai and T. Yanagida, Nucl. Phys. **B197**, 533 (1982).

[26] For $M_G^0 \simeq 2 \cdot 10^{16}$ GeV, $m_s(M_Z) = 93$ MeV, and $\tan\beta \simeq 3.7$.

[27] S. Dimopoulos and F. Wilczek, Print-81-0600 (Santa Barbara); See [4(b)]; L. J. Hall and S. Raby, Phys. Rev. D **51**, 6524 (1995); Z. Berezhiani and Z. Tavartkiladze, Phys. Lett. B **409**, 220 (1997); N. Maekawa, Prog. Theor. Phys. **106**, 401 (2001); B. Kyae and Q. Shafi, Phys. Rev. D **72**, 063515 (2005); and references therein.

[28] Z. G. Berezhiani and G. R. Dvali, Bull. Lebedev Phys. Inst. **5**, 55 (1989); R. Barbieri, G. R. Dvali, and A. Strumia, Nucl. Phys. **B391**, 487 (1993); Z. Berezhiani, C. Csaki, and L. Randall, Nucl. Phys. **B444**, 61 (1995); G. R. Dvali and S. Pokorski, Phys. Rev. Lett. **78**, 807 (1997).