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Probing next-to-minimal-supersymmetric models with minimal fine tuning by searching for decays of the Υ to a light *CP*-odd Higgs boson

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Completely natural electroweak symmetry breaking is easily achieved in supersymmetric models if there is a SM-like Higgs boson, h, with $m_h \lesssim 100$ GeV. In the minimal supersymmetric model, such an h decays mainly to $b\bar{b}$ and is ruled out by LEP constraints. However, if the MSSM Higgs sector is expanded so that h decays mainly to still lighter Higgs bosons, e.g. $h \to aa$, with $\mathrm{Br}(h \to aa) > 0.7$, and if $m_a < 2m_b$, then the LEP constraints are satisfied even if $m_h \lesssim 100$ GeV. In this paper, we show that in the next-to-minimal supersymmetric model the above h and a properties (for the lightest *CP*-even and *CP*-odd Higgs bosons, respectively) imply a lower bound on $\mathrm{Br}(\Upsilon \to \gamma a)$ that dedicated runs at present (and future) B factories can explore.

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Low energy supersymmetry (SUSY) remains one of the most attractive solutions to the naturalness/hierarchy problem of the standard model (SM). However, the minimal supersymmetric model (MSSM), containing exactly two Higgs doublets, suffers from the "μ problem" and requires rather special parameter choices in order that the light Higgs mass is above LEP limits without electroweak symmetry breaking (EWSB) being "fine-tuned", i.e. highly sensitive to supersymmetry-breaking parameters chosen at the grand-unification scale. Both problems are easily solved by adding Higgs (super) fields to the MSSM. For generic soft-SUSY-breaking parameters well below the TeV scale, fine-tuning is absent [1] and a SM-like h is predicted with $m_h \lesssim 100$ GeV. Such an h can avoid LEP limits on the tightly constrained $e^+e^- \to Z + b's$ channel if $\mathrm{Br}(h \to b\bar{b})$ is small by virtue of large $\mathrm{Br}(h \to aa)$, where a is a new light (typically *CP*-odd) Higgs boson, *and* $m_a < 2m_b$ so that $a \to b\bar{b}$ is forbidden [2]. The perfect place to search for such an a is in Upsilon decays, $\Upsilon \to \gamma a$. The simplest MSSM extension, the next-to-minimal supersymmetric model (NMSSM), naturally predicts that the lightest h and a, h_1 and a_1, have all the right features [1–5]. In this paper, we show that large $\mathrm{Br}(h_1 \to a_1a_1)$ implies, at fixed m_{a_1}, a lower bound on $\mathrm{Br}(\Upsilon \to \gamma a_1)$ (from now on, Υ is the $1S$ resonance unless otherwise stated) that is typically within reach of present and future B factories.

In the NMSSM, a light a_1 with substantial $\mathrm{Br}(h_1 \to a_1a_1)$ is a very natural possibility for m_Z-scale soft parameters developed by renormalization group running starting from $U(1)_R$ symmetric grand unified theory-scale soft parameters [5]. (See also [6,7] for discussions of the light a_1 scenario.) The fine-tuning-preferred $m_{h_1} \sim 100$ GeV (for $\tan\beta \gtrsim$ few) gives perfect consistency with precision electroweak data and the reduced $\mathrm{Br}(h_1 \to b\bar{b}) \sim 0.09$–$0.15$ explains the $\sim 2.3\sigma$ excess at LEP in the $Zb\bar{b}$ channel at $M_{b\bar{b}} \sim 100$ GeV. The motivation for this scenario is thus very strong.

Hadron collider probes of the NMSSM Higgs sector are problematical. The $h_1 \to a_1a_1 \to 4\tau$ ($2m_\tau < m_{a_1} < 2m_b$) or 4 *jets* ($m_{a_1} < 2m_\tau$) signal is a very difficult one at the Tevatron and very possibly at the LHC [8–11]. Higgs discovery or, at the very least, certification of a marginal LHC Higgs signal, will require a linear e^+e^- collider (ILC). Direct production and detection of the a_1 may be impossible at both the LHC and ILC because it is rather singlet in nature. We show that by increasing sensitivity to $\mathrm{Br}(\Upsilon \to \gamma a_1)$ by one to 3 orders of magnitude (the exact requirement depends on m_{a_1} and $\tan\beta$), there is a good chance of detecting the a_1. This constitutes a significant opportunity for current B factories and a major motivation for new super-B factories. Even if ILC $h_1 \to a_1a_1$ data is available, measurement of $\mathrm{Br}(\Upsilon \to \gamma a_1)$ and a_1 decays would provide extremely valuable complementary information.

As compared to the three independent parameters needed in the MSSM context (often chosen as μ, $\tan\beta$, and M_A), the Higgs sector of the NMSSM is described by the six parameters

$$\lambda, \quad \kappa, \quad A_\lambda, \quad A_\kappa, \quad \tan\beta, \quad \mu_{\mathrm{eff}}, \tag{1}$$

where $\mu_{\mathrm{eff}} = \lambda\langle S\rangle \equiv \lambda s$ is the effective μ term generated from the $\lambda\hat{S}\hat{H}_u\hat{H}_d$ part of the superpotential, $\lambda A_\lambda SH_uH_d$ is the associated soft-SUSY-breaking scalar potential component, and κ and κA_κ appear in the $\frac{1}{3}\kappa\hat{S}^3$ and $\frac{1}{3}\kappa A_\kappa S^3$ terms in the superpotential and associated soft-supersymmetry-breaking potential. In addition, values must be input for the soft SUSY-breaking masses that contribute to the radiative corrections in the Higgs sector and to the Higgs decay widths. Our computations for branching ratios and so forth employ NMHDECAY [12]. An important ingredient for the results of this paper is the nonsinglet fraction of the a_1 defined by $\cos\theta_A$ in

$$a_1 = \cos\theta_A A_{\mathrm{MSSM}} + \sin\theta_A A_S, \tag{2}$$

where A_S is the CP-odd Higgs boson contained in the unmixed S complex scalar field. The coupling of a_1 to $\tau^+\tau^-$ and $b\bar{b}$ is then $\propto \tan\beta \cos\theta_A$; $\cos\theta_A$ itself has some $\tan\beta$ dependence with the net result that $\tan\beta\cos\theta_A$ increases modestly with increasing $\tan\beta$.

In [1,3,4], we scanned over the NMSSM parameter space holding $\tan\beta$ and the gaugino masses $M_{1,2,3}(m_Z)$ fixed, searching for choices that minimized a numerical measure, F, of EWSB fine-tuning, i.e. of how precisely the grand unified theory-scale soft-SUSY-breaking parameters must be chosen to obtain the observed value of m_Z after renormalization group evolution. For $F < 15$, fine-tuning is no worse than 7%, and we regard this as equivalent to absence of significant fine-tuning. For the sample values of $\tan\beta = 10$ and $M_{1,2,3} = 100$, 200, 300 GeV (F only depends significantly on M_3), to achieve the lowest F values ($F \sim 5$–6), the h_1 must be fairly SM-like and $m_{h_1} \sim 100$ GeV is required; this is only consistent with LEP constraints for scenarios in which $\mathrm{Br}(h_1 \to a_1 a_1)$ is large and $m_{a_1} < 2m_b$.[1] Crucially, requiring both large $\mathrm{Br}(h_1 \to a_1 a_1)$ and $m_{a_1} < 2m_b$ implies a lower bound on $|\cos\theta_A|$, e.g. $|\cos\theta_A| \gtrsim 0.04$ at $\tan\beta = 10$ [5] (*independent of the EWSB F value*).[2] And, it is this lower bound on $|\cos\theta_A|$ that leads to a lower bound on $\mathrm{Br}(\Upsilon \to \gamma a_1)$.

Aside from EWSB fine-tuning, there is a question of whether fine-tuning is needed to achieve large $\mathrm{Br}(h_1 \to a_1 a_1)$ and $m_{a_1} < 2m_b$ when $F < 15$. This was discussed in [5]. The level of such fine-tuning is determined mostly by whether A_λ and A_κ need to be fine-tuned. (For given s and $\tan\beta$, $\mathrm{Br}(h_1 \to a_1 a_1)$ and m_{a_1} depend significantly only on λ, κ, A_λ, and A_κ; all other SUSY parameters have only a tiny influence.) Since specific soft-SUSY-breaking scenarios can evade the issue of tuning A_κ and A_λ altogether, in this study we do not impose a limit on the measures of A_λ, A_κ fine-tuning discussed in [5]. However, it is worth noting that we find that A_λ, A_κ fine-tuning can easily be avoided if $m_{a_1} \gtrsim 2m_\tau$ and $\cos\theta_A$ is small and negative, e.g. near $\cos\theta_A \sim -0.1$ if $\tan\beta = 10$. In some models, the simplest measures of A_λ, A_κ fine-tuning are much larger away from the preferred $\cos\theta_A$ region and/or at substantially lower m_{a_1} values.

We now turn to $\Upsilon \to \gamma a_1$. We have computed the branching ratio for this decay based on Eqs. (3.54), (3.58), and (3.60) of [13] (which gives all appropriate references). Equation (3.54) gives the result based on the nonrelativistic quarkonium model; Eqs. (3.58) and (3.60) give the procedures for including QCD corrections and relativistic corrections, respectively. Both cause significant suppression with respect to the nonrelativistic quarkonium result. In addition, there are bound state corrections. These give a modest enhancement, rising from a small percentage at small m_{a_1} to about 20% at $m_{a_1} = 9.2$ GeV (see the references in [13]).[3] For $m_{a_1} \in [m_{\eta_b} - 2\Gamma_{\eta_b}, m_{\eta_b} + 2\Gamma_{\eta_b}]$, where $m_{\eta_b} \sim M_\Upsilon - 50$ MeV and $\Gamma_{\eta_b} \sim 50$ MeV, the a_1 mixes significantly with the η_b, giving rise to a huge enhancement of $\mathrm{Br}(\Upsilon \to \gamma a_1)$. We have chosen not to plot results for $m_{a_1} > 9.2$ GeV since we think that the old theoretical results in this region require further refinement. In Fig. 1, we present results for $\mathrm{Br}(\Upsilon \to \gamma a_1)$ that are consistent with existing experimental limits[4] in two cases: (a) using a scan over A_λ, A_κ values holding $\mu_{\mathrm{eff}}(m_Z) = 150$ GeV and $M_{1,2,3}(m_Z) = 100$, 200, 300 GeV fixed (in this scan, identical to that described in Ref. [5], λ and κ are also scanned over and all other SUSY-breaking parameters are fixed at 300 GeV—results are insensitive to this choice and, therefore, representative of the whole parameter

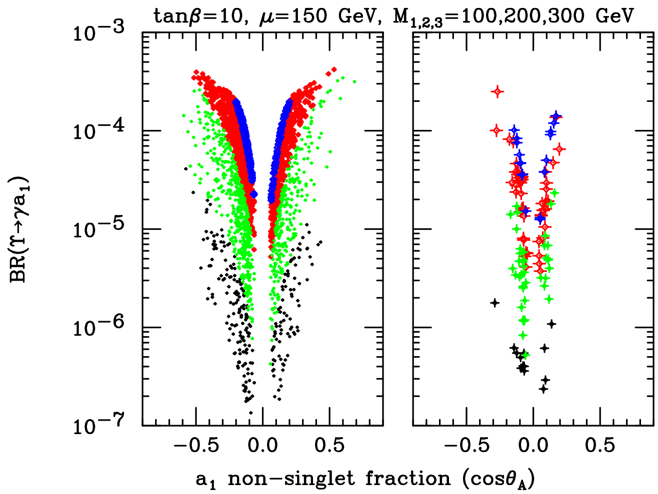

FIG. 1 (color online). $\mathrm{Br}(\Upsilon \to \gamma a_1)$ for NMSSM scenarios with various ranges for m_{a_1}: dark grey (blue) $= m_{a_1} < 2m_\tau$; medium grey (red) $= 2m_\tau < m_{a_1} < 7.5$ GeV; light grey (green) $= 7.5$ GeV $< m_{a_1} < 8.8$ GeV; and black $= 8.8$ GeV $< m_{a_1} < 9.2$ GeV. The plots are for $\tan\beta = 10$ and $M_{1,2,3}(m_Z) = 100$, 200, 300 GeV. The left plot shows the A_λ, A_κ scan described in the text, holding $\mu_{\mathrm{eff}}(m_Z) = 150$ GeV fixed, allowing any value of the EWSB fine-tuning measure, F. The right plot additionally scans over μ_{eff} and shows only points with low fine-tuning, $F < 15$.

[1] We should note that the precise location of the minimum in F shifts slightly as $\tan\beta$ is varied. For example, at $\tan\beta = 3$ ($\tan\beta = 50$) the minimum is at roughly 92 GeV (102 GeV). However, for these cases the minimum value of F is only very modestly higher at $m_{h_1} \sim 100$ GeV, the LEP excess location.

[2] Also, as one approaches the $U(1)_R$, A_κ, $A_\lambda \to 0$ symmetry limit, large $\mathrm{Br}(h_1 \to a_1 a_1)$ is not possible.

[3] In contrast, for a scalar Higgs, bound state corrections give a very large suppression at higher Higgs masses near M_Υ.

[4] We impose the limits of Fig. 3 of [14], Fig. 4 of [15], and Fig. 7b of [16]. The first two limit $\mathrm{Br}(\Upsilon \to \gamma X)$, where X is any visible state. The first provides the only strong constraint on the $m_{a_1} < 2m_\tau$ region. The third gives limits on $\mathrm{Br}(\Upsilon \to \gamma X)\mathrm{Br}(X \to \tau^+\tau^-)$ that eliminate $2m_\tau < m_{a_1} < 8.8$ GeV points with too high $\mathrm{Br}(\Upsilon \to \gamma a_1)$ (for $m_{a_1} > 2m_\tau$, $\mathrm{Br}(a_1 \to \tau^+\tau^-) \sim 0.9$). Since the inclusive photon spectrum from Υ decays falls as E_γ increases, the strongest constraints are obtained for small m_{a_1}.

space); (b) for the $F < 15$ points found in the NMSSM parameter scan described earlier. In both cases, all points plotted pass all NMHDECAY constraints—all points have $m_{h_1} \sim 100$ GeV, but avoid LEP constraints by virtue of $\mathrm{Br}(h_1 \to a_1 a_1) > 0.7$ and $m_{a_1} < 2m_b$. For both plots, we divide results into four m_{a_1} regions: $m_{a_1} < 2m_\tau$, $2m_\tau < m_{a_1} < 7.5$ GeV, $7.5\ \mathrm{GeV} < m_{a_1} < 8.8$ GeV, and $8.8\ \mathrm{GeV} < m_{a_1} < 9.2$ GeV. Figure 1 makes clear that $\mathrm{Br}(\Upsilon \to \gamma a_1)$ is mainly controlled by the nonsinglet fraction of the a_1 and by m_{a_1}. The only difference between the (a) and (b) plots is that $F < 15$ restricts the range of $\cos\theta_A$ to smaller magnitudes [implying smaller $\mathrm{Br}(\Upsilon \to \gamma a_1)$] and narrows the m_{a_1} bands. As seen in the figure, the $\cos\theta_A \sim -0.1$, $m_{a_1} > 2m_\tau$ scenarios (which are those that can have no A_λ, A_κ tuning [5]) have $\mathrm{Br}(\Upsilon \to \gamma a_1) \lesssim$ few $\times 10^{-5}$. For general $\cos\theta_A$ and m_{a_1}, values of $\mathrm{Br}(\Upsilon \to \gamma a_1)$ up to $\sim 10^{-3}$ (5×10^{-3}) are possible for $F < 15$ points (in the general A_λ, A_κ scan). In Fig. 1, points with $\mathrm{Br}(\Upsilon \to \gamma a_1) \gtrsim$ few $\times 10^{-4}$ (depending on m_{a_1}) are not present, having been eliminated by 90% C.L. limits from existing experiments. The surviving points with $m_{a_1} < 9.2$ GeV can be mostly probed if future running, upgrades, and facilities are designed so that $\mathrm{Br}(\Upsilon \to \gamma a_1) \sim 10^{-7}$ can be probed. As stated earlier, predictions at higher m_{a_1} are rather uncertain, but obviously $\mathrm{Br}(\Upsilon \to \gamma a_1) \to 0$ for $m_{a_1} \to M_\Upsilon$. To access higher m_{a_1} (but $m_{a_1} < 2m_b$), $\Upsilon(2S) \to \gamma a_1$ and $\Upsilon(3S) \to \gamma a_1$ can be employed; computation of the branching ratios requires careful attention to $a_1 - \eta_b$ mixing, which can lead to even larger branching ratios than for the Υ if $m_{a_1} \sim m_{\eta_b}$.

Results from the A_λ, A_κ scan with $\mu_{\mathrm{eff}} = 150$ GeV and $M_{1,2,3} = 100$, 200, 300 GeV are given in the cases of

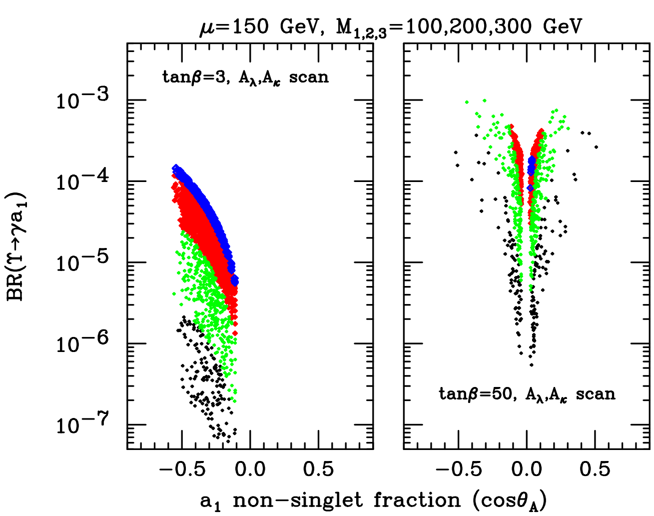

FIG. 2 (color online). We plot $\mathrm{Br}(\Upsilon \to \gamma a_1)$ as a function of $\cos\theta_A$ for the A_λ, A_κ scan allowing any value for fine-tuning F, taking $M_{1,2,3}(m_Z) = 100$, 200, 300 GeV, $\mu_{\mathrm{eff}}(m_Z) = 150$ GeV with $\tan\beta = 3$ (left) and $\tan\beta = 50$ (right). The point notation is as in Fig. 1.

$\tan\beta = 3$ and $\tan\beta = 50$ in Fig. 2. Note that almost all $\tan\beta = 3$ points that pass NMHDECAY and LEP constraints are consistent with existing limits on $\mathrm{Br}(\Upsilon \to \gamma a_1)$. To probe the full set of $m_{a_1} < 9.2$ GeV points shown, sensitivity to $\mathrm{Br}(\Upsilon \to \gamma a_1) \lesssim$ few $\times 10^{-8}$ is needed. Conversely, for $\tan\beta = 50$ a lot of the scan points consistent with NMHDECAY and LEP constraints are already absent because of existing limits and one need only probe down to $\mathrm{Br}(\Upsilon \to \gamma a_1) \sim 10^{-6}$ to cover the $m_{a_1} < 9.2$ GeV points.

We note that the points with small negative $\cos\theta_A$ (e.g. $\cos\theta_A \sim -0.1$ for $\tan\beta = 10$) that are most likely to escape A_λ, A_κ tuning issues are well below the existing limits from [14–16] for all m_{a_1} values for all three $\tan\beta$ choices.[5] However, none of the above analyses [14–16] have been repeated with the larger data sets available from CLEO-III, *BABAR*, or Belle. Presumably, much stronger constraints than those we included can be obtained. Or perhaps a γa_1 signal will be found.

We expect that the best way to search for the NMSSM light a_1 is to use its exclusive decay modes, as this reduces backgrounds, especially those important when the photon is soft. For $m_{a_1} > 3.6$ GeV and $\tan\beta \gtrsim 1$, the dominant decay mode is $a_1 \to \tau^+\tau^-$. For example, Ref. [19] has proposed looking for nonuniversality in $\Upsilon \to \gamma\tau^+\tau^-$ vs $\Upsilon \to \gamma e^+e^-$, $\gamma\mu^+\mu^-$ decays. This would fit nicely with the low-F scenarios. For $m_{a_1} < 2m_\tau$, $2m_c$ the decay mode $a_1 \to gg$ is generally in the range 20%–30%, giving a contribution to $\Upsilon \to \gamma gg$ at the 10^{-4}–10^{-6} level; the $s\bar{s}$ mode is typically larger.

In the $\gamma\tau^+\tau^-$ final state, the direct $\gamma\tau^+\tau^-$ production cross section is 61 pb. Using signal = background as the criterion, this becomes the limiting factor for branching ratios below the 4×10^{-5} level when running on the $\Upsilon(1S)$, and below the 2×10^{-4} level when running on the $\Upsilon(3S)$. To improve upon the latter, one can select a sample of known $\Upsilon(1S)$ events by looking for dipion transitions from the higher resonances. The dipion transition gives a strong kinematic constraint on the mass difference between the two Υ's. When running on the $\Upsilon(3S)$, the effective cross section in $\Upsilon(3S) \to \pi^+\pi^-\Upsilon(1S)$ is 179 pb [20].[6] To limit $\mathrm{Br}(\Upsilon \to \gamma a_1) \lesssim 10^{-6}$, 5.6 $\mathrm{fb}^{-1}/\epsilon$ would need to be collected on the $\Upsilon(3S)$, where ϵ is the experimental efficiency for isolating the relevant events.

[5]For a CP-odd a that decays into noninteracting states, there are further constraints available from Crystal Ball and CLEO [17]; these only apply to the scenarios considered here if M_1 is reduced to a very small value (as possible without affecting EWSB fine-tuning) so that $a_1 \to \tilde{\chi}_1^0\tilde{\chi}_1^0$ decays are significant. For example, at $\tan\beta = 10$, our low fine-tuning scenarios with M_1 decreased to 3 GeV can yield $m_{\tilde{\chi}_1^0} \lesssim 2$ GeV and $\mathrm{Br}(a_1 \to \tilde{\chi}_1^0\tilde{\chi}_1^0) \in [0.15, 0.35]$. (Generic scenarios with substantial $\mathrm{Br}(\Upsilon \to \gamma a_1)\mathrm{Br}(a_1 \to \tilde{\chi}_1^0\tilde{\chi}_1^0)$ were considered in [18].)

[6]This can also be done on the $\Upsilon(2S)$ but the pions are softer, implying much lower efficiency. On the $\Upsilon(4S)$ this transition has a very small branching ratio $\lesssim 10^{-4}$.

This analysis can also be done on the $\Upsilon(4S)$, where the $\Upsilon(3S)$ is produced via initial state radiation. The effective $\gamma_{\rm ISR}\Upsilon(3S) \to \gamma_{\rm ISR}\pi^+\pi^-\Upsilon(1S)$ cross section is 0.78 fb. To limit $\mathrm{Br}(\Upsilon \to \gamma a_1) \lesssim 10^{-6}$, 1.3 $\mathrm{ab}^{-1}/\epsilon$ would need to be collected. These integrated luminosities needed to probe $\mathrm{Br}(\Upsilon \to \gamma a_1) \sim 10^{-6}$ would appear to be within reach at existing facilities and would allow discovery of the a_1 for many of the favored NMSSM scenarios.

Are there other modes that would allow direct a_1 detection? Reference [21] advocates $e^+e^- \to \tilde{\chi}_1^+\tilde{\chi}_1^- a_1$ with $a_1 \to \gamma\gamma$. This works if the a_1 is very singlet, in which case $\mathrm{Br}(a_1 \to \gamma\gamma)$ can be large. However, see [5] and earlier discussion, a minimum value of $|\cos\theta_A|$ (e.g. $|\cos\theta_A| > 0.04$ if $\tan\beta = 10$) is required in order that $\mathrm{Br}(h_1 \to a_1a_1) > 0.7$ and $m_{a_1} < 2m_b$. For the general A_λ, A_κ scans with $\mathrm{Br}(h_1 \to a_1a_1) > 0.7$ and $m_{a_1} < 2m_b$ imposed, $\mathrm{Br}(a_1 \to \gamma\gamma) < 4 \times 10^{-4}$ with values near few $\times$ 10^{-5} being very common. It is conceivable that a super-B factory could detect a signal for $\Upsilon \to \gamma a_1 \to \gamma\gamma\gamma$.

Flavor changing decays based on $b \to sa_1$ or $s \to da_1$, in particular $B \to X_s a_1$, have been examined in [7]. All penguin diagrams containing SM particles give contributions to the $b \to sa_1$ amplitude that are suppressed by $\cos\theta_A/\tan\beta$ or $\cos\theta_A/\tan^3\beta$ (since up-type quarks couple to the $A_{\rm MSSM}$ with a factor of $1/\tan\beta$). Reference [7] identifies two diagrams involving loops containing up-type squarks and charginos that give $b \to sa_1$ amplitudes that are proportional to $\cos\theta_A \tan\beta$. However, the sum of these diagrams vanishes in the super GIM limit (e.g. equal up-type squark masses), yielding a tiny $B \to X_s a_1$ transition rate. Away from this limit, results are highly model-dependent. In contrast, the predictions for $\Upsilon \to \gamma a_1$ depend essentially only on $\cos\theta_A \tan\beta$ and m_{a_1}, both of which are fairly constrained by the need to escape LEP limits for $m_{h_1} < 110$ GeV.

If $m_{a_1} < 2m_c$, $J/\psi \to \gamma a_1$ decay will be possible. However, $\mathrm{Br}(J/\psi \to \gamma a_1)$ is $\sim 10^{-9}$ ($\sim 10^{-7}$) for the smallest (largest) $|\cos\theta_A|$ values in the standard A_λ, A_κ scan for $\tan\beta = 10$, increasing modestly as $\tan\beta$ increases.

Before concluding, we note that a light, not-too-singlet a_1 could allow consistency with the observed amount of dark matter if the $\tilde{\chi}_1^0$ is largely b-ino and $2m_{\tilde{\chi}_1^0} \sim m_{a_1}$. This is explored in [18]. We report here that this can be coincident with the $F < 15$ scenarios (as well as the small negative $\cos\theta_A$, $m_{a_1} > 2m_\tau$ scenarios that are the most likely to have small A_λ, A_κ fine-tuning). Reducing M_1 from the nominal 100 GeV employed here to $\frac{1}{2}m_{a_1}$ has no effect on either type of fine-tuning.

In summary, aside from discovering the a_1 in $h_1 \to a_1a_1$ decays at the LHC or maybe not until the ILC, it seems that the most promising near-term possibility for testing the NMSSM scenarios for which EWSB fine-tuning is absent, or more generally any scenario with large $\mathrm{Br}(h_1 \to a_1a_1)$ and $m_{a_1} < 2m_b$, is to employ the $\Upsilon \to \gamma a_1$ decay at either existing B factories or future factories. From a more model-independent point of view, there are many generalizations of the NMSSM with many largely singlet light Higgs bosons, some of which might not even be observable in decays of heavier Higgs bosons. These might only be discoverable in Υ decays (in particular, not at an ILC) and might play a role in dark-matter annihilation. We might never unravel the full theory without B factory data.

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