Induction of the four-dimensional Lorentz-breaking non-Abelian Chern-Simons action

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A four-dimensional Lorentz-breaking non-Abelian Chern-Simons-like action is generated as a one-loop perturbative correction via an appropriate Lorentz-breaking coupling of the non-Abelian gauge field to a spinor field. This term is shown to be regularization dependent, but nevertheless it can be found unambiguously in different regularization schemes at zero and at finite temperature.

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During the last years, different aspects of the Lorentz symmetry breaking have been intensively studied [1]. One of the theoretical consequences of this effect is the birefringence of light in vacuum. After the formulation of the concept of noncommutativity of the space-time [2], which implies in Lorentz symmetry breaking (see the discussion in Ref. [3]). The interest in this subject has greatly increased. One of the implications of the Lorentz symmetry breaking is the possibility of introducing a lot of new couplings in the standard model [4]. These terms may arise from radiative corrections to some Lorentz-breaking field theories at zero [5–10] and at finite temperature [11–14]. Alternatively, they may be induced from the deformation of the canonical commutation relation through the use of the noncommutative fields method [15,16].

Recently, the renormalizability of the Yang-Mills (YM) theory with a four-dimensional non-Abelian Lorentz-breaking Chern-Simons (CS) term was studied in Ref. [17]. The induction of such Lorentz-breaking CS term starting from a pure YM was investigated within the noncommutative fields method in Ref. [16]. In the present work we show how the same CS term can be induced through radiative corrections starting from a YM theory coupled with fermions in the presence of an interaction of the fermions with a constant external field at zero and at finite temperture.

We start with the following model which represents a non-Abelian generalization of the spinor electrodynamics with the Lorentz-breaking coupling

$$\mathcal{L}_f = \bar{\psi}^i[(i\not{\partial} - m - \gamma_5\not{b})\delta^{ij} - g\gamma^\mu A^a_\mu(\Omega^a)^{ij}]\psi^j. \quad (1)$$

Here b_ρ is a constant four-vector. The $A_\mu = A^a_\mu\Omega^a$ is a Yang-Mills field coupled to spinors ψ which carry the isotopic indices, $\psi = (\psi^i)$, with i taking values from one to N with N being the dimension of the chosen representation of the Lie algebra. The $\Omega^a = (\Omega^a)^{ij}$ are the Lie group generators in this representation satisfying the relations: $[\Omega^a, \Omega^b] = if^{abc}\Omega^c$ and $\mathrm{Tr}[\Omega^a\Omega^b] = \delta^{ab}$.

The one-loop effective action of the gauge field A^a_μ is $S_{\rm YM} + S_f[b, A]$ where $S_{\rm YM}$ is the Yang-Mills action and $S_f[b, A]$ can be expressed in the form of the following functional trace:

$$S_f[b, A] = -i\,\mathrm{Tr}\ln(i\not{\partial} - m - \gamma_5\not{b} - g\gamma^\mu A^a_\mu\Omega^a). \quad (2)$$

This functional trace can be rewritten as $S_f[b, A] = S_f[b] + S'_f[b, A]$, with the first term being $S_f[b] = -i\,\mathrm{Tr}\ln(i\not{\partial} - m - \gamma_5\not{b})$. The nontrivial dynamics is concentrated in the second term $S'_f[b, A]$, which is given by the power series:

$$S'_f[b, A] = i\,\mathrm{Tr}\sum_{n=1}^{\infty}\frac{1}{n}\left[\frac{1}{i\not{\partial} - m - \gamma_5\not{b}}g\gamma^\mu A^a_\mu\Omega^a\right]^n. \quad (3)$$

To make explicit the non-Abelian Chern-Simons term we should expand this expression up to the third order in the gauge field:

$$S'_f[b, A] = S^{(2)}_f[b, A] + S^{(3)}_f[b, A] + \ldots, \quad (4)$$

where

$$S^{(2)}_f[b, A] = \frac{ig^2}{2}\mathrm{Tr}\left[\frac{1}{i\not{\partial} - m - \gamma_5\not{b}}\gamma^\mu A^a_\mu\Omega^a\frac{1}{i\not{\partial} - m - \gamma_5\not{b}} \times \gamma^\nu A^b_\nu\Omega^b\right], \quad (5)$$

$$S^{(3)}_f[b, A] = \frac{ig^3}{2}\mathrm{Tr}\left[\frac{1}{i\not{\partial} - m - \gamma_5\not{b}}\gamma^\mu A^a_\mu\Omega^a\frac{1}{i\not{\partial} - m - \gamma_5\not{b}} \times \gamma^\lambda A^b_\lambda\Omega^b\frac{1}{i\not{\partial} - m - \gamma_5\not{b}}\gamma^\nu A^c_\nu\Omega^c\right], \quad (6)$$

and the ellipsis stands for higher order terms in the gauge field.

Using the above expressions, it is easy now to verify that the one-loop effective action expanded up to first order in b_μ may be written as

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$$S'_f[b, A] = \int d^4x k_\rho \epsilon^{\rho\mu\nu\lambda}\left(\partial_\lambda A^a_\mu A^a_\nu - \frac{2}{3} ig A^a_\mu A^b_\nu A^c_\lambda f^{abc}\right), \tag{7}$$

where k_ρ is

$$k_\rho = 2ig^2 \int \frac{d^4p}{(2\pi)^4} \frac{b_\rho(p^2 + 3m^2) - 4p_\rho(b \cdot p)}{(p^2 - m^2)^3}. \tag{8}$$

This result exactly reproduces the structure of the non-Abelian Lorentz-breaking Chern-Simons term described in Ref. [17]. One can observe that the expressions (7) and (8), after reduction to the Abelian case, coincide with the known Abelian results [14,18]. Apparently, there is a relation between the induced Lorentz-breaking Chern-Simons term and Adler-Bell-Jackiw anomaly as both situation are observed for the well known triangle graph. This issue has been discursed in Ref. [6]. Also, the interesting discussion of the problem of ambiguities in the Lorentz-breaking theories is presented in Ref. [7]. By power counting, the momentum integral in expression (8) involves terms with logarithmic divergence so that different regularization prescriptions will produce diverse outcomes. Lorentz preserving regularizations, more precisely any regularization in which we can make: $p_\mu p_\nu \to \frac{g_{\mu\nu}}{D} p^2$, will produce finite results. By adopting the method of dimensional regularization [19], the above integral is promoted to D dimensions and a straightforward calculation yields

$$\begin{aligned} k_\rho &= 2ig^2 b_\rho \int \frac{d^Dp}{(2\pi)^D} \frac{1}{(p^2 - m^2)^3}\left[\left(1 - \frac{4}{D}\right)p^2 + 3m^2\right] \\ &= \frac{4g^2(4 - D)\Gamma((4 - D)/2)}{\Gamma(3)(4\pi)^{D/2}} b_\rho = \frac{g^2}{4\pi^2} b_\rho, \end{aligned} \tag{9}$$

which coincides with the result found in Ref. [18] for the Abelian situation. If, instead of dimensional regularization, the integral in Eq. (8) is kept in four dimensions the regularization enforced the replacement

$$k_\rho = 6ig^2m^2 b_\rho \int \frac{d^4p}{(2\pi)^4} \frac{1}{(p^2 - m^2)^3} = \frac{3g^2}{16\pi^2} b_\rho, \tag{10}$$

which now agrees with the Abelian result obtained in Ref. [20].

To develop calculations in the finite temperature case, let us now assume that the system is in the state of thermal equilibrium at a temperature $T = 1/\beta$. In this case, we can use the Matsubara formalism for fermions, which consists in taking $p_0 \equiv i\omega_n = (n + 1/2)\frac{2\pi i}{\beta}$ and replacing the integration over the zeroth component of the momentum by a discrete sum $(1/2\pi) \int dp_0 \to \frac{i}{\beta}\sum_n$. Thus, the Eq. (7) can be written as

$$\begin{aligned} S'_f[b, A] = \int d^4x k_\rho(\beta) \epsilon^{\rho\mu\nu\lambda}\Big(&\partial_\lambda A^a_\mu A^a_\nu \\ &- \frac{2}{3} ig A^a_\mu A^b_\nu A^c_\lambda f^{abc}\Big). \end{aligned} \tag{11}$$

Hereafter all expressions are in the Euclidean space (all greek indices run from 1 to 4). The vector $k_\rho(\beta)$ is given by

$$k_\rho(\beta) = \frac{2g^2}{\beta} \sum_{n=-\infty}^{\infty} \int \frac{d^3\vec{p}}{(2\pi)^3} \frac{b_\rho(3m^2 - p^2) + 4p_\rho(b \cdot p)}{(p^2 + m^2)^3}. \tag{12}$$

By extending the $\vec{p}$ integration to d dimensions it follows that the timelike component of $k_\rho(\beta)$ is

$$k_4(\beta) = \frac{2g^2}{\beta} b_4 \sum_{n=-\infty}^{\infty} \int \frac{d^d\vec{p}}{(2\pi)^d} \frac{3M_n^2 - \vec{p}^2}{(\vec{p}^2 + M_n^2)^3}, \tag{13}$$

where $M_n^2 = \omega_n^2 + m^2$. Using the prescription of dimensional regularization [19], we have

$$\begin{aligned} k_4(\beta) &= \frac{g^2}{2\beta} \frac{b_4}{(4\pi)^{d/2}}[d\Gamma(2 - d/2) - 6\Gamma(3 - d/2)] \\ &\quad \times \sum_{n=-\infty}^{\infty} \frac{1}{(M_n^2)^{2-d/2}} \\ &= \frac{g^2 b_4}{m^3\pi}\left(\frac{m}{2\sqrt{\pi}}\right)^d (a^2)^{\lambda - 1/2}(3 - d)\Gamma(\lambda) \\ &\quad \times \sum_{n=-\infty}^{\infty} \frac{1}{[(n + 1/2)^2 + a^2]^\lambda}, \end{aligned} \tag{14}$$

where $a = m\beta/2\pi$ and $\lambda = 2 - d/2$. At this point the following identity [21]:

$$\begin{aligned} \sum_n \frac{1}{[(n + b)^2 + a^2]\lambda} = \frac{\sqrt{\pi}\Gamma(\lambda - 1/2)}{\Gamma(\lambda)(a^2)^{\lambda - 1/2}} &+ 4\sin(\pi\lambda) \\ &\times \int_{|a|}^{\infty} \frac{dz}{(z^2 - a^2)^\lambda} \\ &\times \mathrm{Re}\left(\frac{1}{\exp 2\pi(z + ib) - 1}\right), \end{aligned} \tag{15}$$

valid for $1/2 < \lambda < 1$ can be used to get

$$\begin{aligned} k_4(\beta) = \frac{g^2 b_4}{m^3\pi}\left(\frac{m}{2\sqrt{\pi}}\right)^d \Big\{&2\sqrt{\pi} + 4(3 - d)(a^2)^{(3-d)/2}\Gamma(\lambda) \\ &\times \sin(\pi\lambda) \int_{|a|}^{\infty} \frac{dz}{(z^2 - a^2)^\lambda} \\ &\times \mathrm{Re}\left(\frac{1}{\exp 2\pi(z + ib) - 1}\right)\Big\}. \end{aligned} \tag{16}$$

In $d = 3$ this gives

$$k_4(\beta) = \frac{g^2}{4\pi^2} b_4, \tag{17}$$

i.e., the same result (9) without any dependence on the temperature, which agrees with the one obtained in Ref. [22] for the Abelian situation. If instead of Eq. (12) we use Eq. (10) as the starting point for the computation of finite temperature effects we get

$$k_4(\beta) = b_4\left(\frac{3}{32\pi^2} + \frac{3}{16}F(a)\right), \tag{18}$$

where

$$F(a) = \int_{|a|}^{\infty} dz (z^2 - a^2)^{1/2} \frac{\tanh(\pi z)}{\cosh^2(\pi z)} \tag{19}$$

has the following asymptotics: $F(a \to \infty) \to 0$ $(T \to 0)$ and $F(a \to 0) \to 1/2\pi^2$ $(T \to \infty)$, see Fig. 1.

Let us now consider the space part, $k_i(\beta)$, of the vector $k_\rho(\beta)$. In this case, the expression (12) can be rewritten as

$$k_i(\beta) = \frac{2g^2}{\beta} \sum_{n=-\infty}^{\infty} \frac{d^3\vec{p}}{(2\pi)^3} \frac{b_i(3m^2 - p^2) + 4p_i(b \cdot p)}{(p^2 + m^2)^3}, \tag{20}$$

Then, considering this expression formally in d space dimensions, we can replace $p_i p_j$ by $\frac{\vec{p}^2}{d}\delta_{ij}$, hence we get

$$k_i(\beta) = \frac{2g^2}{\beta} b_i \sum_{n=-\infty}^{\infty} \frac{d^d\vec{p}}{(2\pi)^d} \frac{4m^2 - (\frac{d-4}{d})\vec{p}^2 - M_n^2}{(\vec{p}^2 + M_n^2)^3}, \tag{21}$$

which now furnishes

$$\begin{aligned} k_i(\beta) &= \frac{4m^2 g^2}{\beta} b_i \frac{\Gamma(3 - \frac{d}{2})}{(4\pi)^{d/2}} \sum_{n=-\infty}^{\infty} \frac{1}{(m^2 + \omega_n^2)^{3-(d/2)}} \\ &= \frac{g^2 b_i}{2m\pi^3}\left(\frac{m}{2}\right)^{d/2} (a^2)^{\lambda - 1/2}\Gamma(\lambda) \\ &\quad \times \sum_{n=-\infty}^{\infty} \frac{1}{[(n + \frac{1}{2})^2 + a^2]^\lambda}, \end{aligned} \tag{22}$$

where we have introduced $\lambda = 3 - \frac{d}{2}$. We cannot apply the relation (15) for $d = 3$, because the integral in that expression does not converge. Thus, let us perform the analytic continuation of that relation; we obtain [13]

$$\begin{aligned} &\int_{|a|}^{\infty} \frac{dz}{(z^2 - a^2)^\lambda} \mathrm{Re}\left(\frac{1}{\exp 2\pi(z + ib) - 1}\right) \\ &= \frac{1}{2a^2} \frac{3 - 2\lambda}{1 - \lambda} \int_{|a|}^{\infty} \frac{dz}{(z^2 - a^2)^{\lambda - 1}} \mathrm{Re}\left(\frac{1}{\exp 2\pi(z + ib) - 1}\right) \\ &\quad - \frac{1}{4a^2} \frac{1}{(2 - \lambda)(1 - \lambda)} \int_{|a|}^{\infty} \frac{dz}{(z^2 - a^2)^{\lambda - 2}} \\ &\quad \times \frac{d^2}{dz^2} \mathrm{Re}\left(\frac{1}{\exp 2\pi(z + ib) - 1}\right). \end{aligned} \tag{23}$$

Thus for $d = 3$ the Eq. (22) takes the form

$$k_i(\beta) = b_i\left(\frac{1}{4\pi^2} + \frac{1}{2}F(a)\right), \tag{24}$$

where $F(a)$ was defined in Eq. (19). Thus, we see that at high temperature the Chern-Simons coefficient is twice its value at zero temperature, i.e., $k_i(\beta \to 0) = \frac{1}{2\pi^2}$. On the other hand, at zero temperature, one recovers the result $k_i(\beta \to \infty) = \frac{1}{4\pi^2}$.

We have generated the non-Abelian Lorentz-breaking Chern-Simons term via the Lorentz-breaking coupling of the Yang-Mills field with the spinor field at zero and at finite temperature. The essential property of the result is that within the framework of dimensional regularization this term turns out to be finite. We note that the derivative expansion approach naturally allows to preserve the gauge invariance for the quantum corrections. It is natural to expect that at least some of the other Lorentz-breaking terms whose existence was predicted in Ref. [4] also can be generated via appropriate couplings of the gauge or gravity fields with some matter fields.

We have also obtained the coefficient k_ρ for the non-Abelian Lorentz-breaking Chern-Simons term at the finite temperature. We found that the results for this term turn out to be dependent on the regularization scheme both at zero and at finite temperature (in a particular regularization scheme the timelike component was found to be temperature independent). Considering the dependence on the regularization scheme, one should note that the momentum integral determining the value of the vector k_ρ is formally superficially divergent, thus dependence of its finite part on the renormalization procedure is very natural. However, in the regularization schemes suggested in the paper, the divergent part identically disappears as a consequence of the rotational invariance of the relevant integrands.

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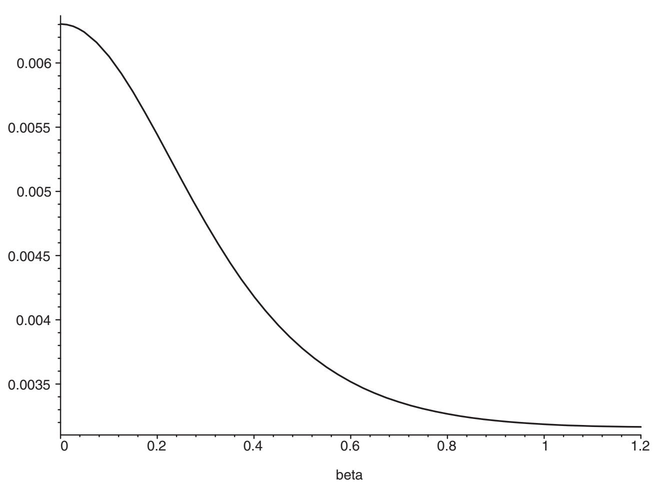

FIG. 1. The function $F(a)$ is different from zero everywhere. At zero temperature ($\beta \to \infty$), the function tends to a nonzero value $\frac{1}{2\pi^2}$.

[1] V. Kostelecky and S. Samuel, Phys. Rev. D **39**, 683 (1989); S. Coleman and S. Glashow, Phys. Rev. D **59**, 116008 (1999); J. Magueijo and L. Smolin, Phys. Rev. D **67**, 044017 (2003); Classical Quantum Gravity **21**, 1725 (2004); O. Bertolami and C. Carvalho, Phys. Rev. D **74**, 084020 (2006).

[2] N. Seiberg and E. Witten, J. High Energy Phys. 09 (1999) 032.

[3] S. Carroll, J. Harvey, V. Kostelecky, C. Lane, and T. Okamoto, Phys. Rev. Lett. **87**, 141601 (2001); C. E. Carlson, C. D. Carone, and R. F. Lebed, Phys. Lett. B **518**, 201 (2001); **549**, 337 (2002); O. Bertolami and L. Guisado, J. High Energy Phys. 12 (2003) 013; J. Collins, A. Perez, D. Sudarsky, L. Urrutia, and H. Vucetich, Phys. Rev. Lett. **93**, 191301 (2004); A. F. Ferrari, H. O. Girotti, and M. Gomes, Phys. Rev. D **73**, 047703 (2006).

[4] V. A. Kostelecky, Phys. Rev. D **69**, 105009 (2004); arXiv:hep-ph/0412406.

[5] J. M. Chung and P. Oh, Phys. Rev. D **60**, 067702 (1999).

[6] M. Perez-Victoria, Phys. Rev. Lett. **83**, 2518 (1999).

[7] R. Jackiw, Int. J. Mod. Phys. B **14**, 2011 (2000).

[8] A. A. Andrianov, P. Giacconi, and R. Soldati, J. High Energy Phys. 02 (2002) 030.

[9] M. Chaichian, W. F. Chen, and R. Gonzalez Felipe, Phys. Lett. B **503**, 215 (2001).

[10] B. Altschul, Phys. Rev. D **69**, 125009 (2004); **70**, 101701 (2004).

[11] D. Bazeia, T. Mariz, J. R. Nascimento, E. Passos, and R. F. Ribeiro, J. Phys. A **36**, 4937 (2003).

[12] T. Mariz, J. R. Nascimento, E. Passos, and R. F. Ribeiro, Phys. Rev. D **70**, 024014 (2004).

[13] L. Cervi, L. Griguolo, and D. Seminara, Phys. Rev. D **64**, 105003 (2001).

[14] T. Mariz, J. R. Nascimento, E. Passos, and R. F. Ribeiro, J. High Energy Phys. 10 (2005) 019.

[15] J. M. Carmona, J. L. Cortes, J. Gamboa, and F. Mendez, Phys. Lett. B **565**, 222 (2003); J. High Energy Phys. 03 (2003) 058; J. Gamboa and J. Lopez-Sarrion, Phys. Rev. D **71**, 067702 (2005); H. Falomir, J. Gamboa, J. Lopez-Sarrion, F. Mendez, and A. J. da Silva, Phys. Rev. D **74**, 047701 (2006).

[16] H. Falomir, J. Gamboa, J. Lopez-Sarrion, F. Mendez, and A. J. da Silva, Phys. Lett. B **632**, 740 (2006).

[17] D. Colladay and P. McDonald, Phys. Rev. D **75**, 105002 (2007).

[18] A. A. Andrianov, P. Giacconi, and R. Soldati, J. High Energy Phys. 02 (2002) 030.

[19] G. 't Hooft and M. Veltman, Nucl. Phys. **B44**, 189 (1972).

[20] R. Jackiw and V. A. Kostelecky, Phys. Rev. Lett. **82**, 3572 (1999).

[21] L. H. Ford, Phys. Rev. D **21**, 933 (1980).

[22] D. Ebert, V. Ch. Zhukovsky, and A. S. Razumovsky, Phys. Rev. D **70**, 025003 (2004).