Magnetic phases in three-flavor color superconductivity

Efrain J. Ferrer and Vivian de la Incera
Department of Physics, Western Illinois University, Macomb, Illinois 61455, USA
(Received 21 March 2007; published 23 August 2007)

The best natural candidates for the realization of color superconductivity are quark stars—not yet confirmed by observation—and the extremely dense cores of compact stars, many of which have very large magnetic fields. To reliably predict astrophysical signatures of color superconductivity, a better understanding of the role of the star's magnetic field in the color-superconducting phase that is realized in the core is required. This paper is an initial step in that direction. The field scales at which the different magnetic phases of a color superconductor with three quark flavors can be realized are investigated. Going from weak to strong fields, the system first undergoes a symmetry transmutation from a color-flavor-locked (CFL) phase to a magnetic-CFL (MCFL) phase, and then a phase transition from the MCFL phase to the paramagnetic-CFL (PCFL) phase. The low-energy effective theory for the excitations of the diquark condensate in the presence of a magnetic field is derived using a covariant representation that takes into account all the Lorentz structures contributing at low energy. The field-induced masses of the charged mesons and the threshold field at which the CFL → MCFL symmetry transmutation occurs are obtained in the framework of this low-energy effective theory. The relevance of the different magnetic phases for the physics of compact stars is discussed.

DOI: 10.1103/PhysRevD.76.045011 PACS numbers: 12.38.Aw, 24.85.+p, 26.60.+c

I. INTRODUCTION

At present, the physics community is actively trying to find ways to differentiate a neutron star made up entirely of nuclear matter from one with a color-superconducting [1] quark matter core. Compact stars typically have very large magnetic fields. Hence any predicted signature of a color-superconducting core should take into account the presence of the star's magnetic field and its effects in the superconducting state. Given that magnetars can have surface fields as large as 10^{14}–10^{16} G [2], it is reasonable to expect that the star's interior fields can reach even higher values. Maximum strengths of 10^{18}–10^{19} G are allowed by a simple application of the virial theorem [3].

Although a color superconductor (CS) is, in principle, an electric superconductor, because the diquark condensate carries nonzero electric charge, in the color-flavor-locked (CFL) phase [4] (CFL is the color-superconducting phase that is realized in a system of three-flavor massless quarks at high densities) there is no Meissner effect for a new in-medium electromagnetic field $\tilde{A}_\mu$. This in-medium electromagnetic field—called, in the literature, a "rotated" electromagnetic field, where the "rotation" takes place in an inner space—is a combination of the regular electromagnetic field and the 8th gluon [4,5]. As the quark pairs are all neutral with respect to the rotated electromagnetic charge $\tilde{Q}$, the rotated electromagnetic field $\tilde{A}_\mu$ remains long range within the superconductor.

In this paper we are interested in the color-superconducting magnetic phases that are realized in a very dense system of three-flavor massless quarks interacting in the background of a rotated magnetic field $\tilde{B}$. As shown in Ref. [6], the color-superconducting properties of such a system are substantially affected by the penetrating $\tilde{B}$ field and, as a consequence, a new phase, called the magnetic color-flavor-locked (MCFL) phase [6], takes place. In the MCFL phase the pairing of (rotated) electrically charged quarks is reinforced by the field. Pairs of this kind have bounding energies which depend on the magnetic-field strength and are bigger than the ones existing at zero field. At field strengths of the order of the baryon chemical potential, the pairing reinforcement is sufficient to produce a distinguishable splitting of the gap in two pieces: one that only gets contributions from pairs of neutral quarks and one that gets contributions from both pairs of neutral and pairs of charged quarks.

Although the symmetry breaking patterns of the MCFL and CFL phases are different, the two phases are hardly distinguishable at weak magnetic fields. In the CFL phase the symmetry breaking is given by

$$\begin{aligned}\mathcal{G} &= SU(3)_C \times SU(3)_L \times SU(3)_R \times U(1)_B \times U(1)_{\rm e.m.} \\ &\rightarrow SU(3)_{C+L+R} \times \tilde{U}(1)_{\rm e.m.}. \end{aligned} \tag{1}$$

This symmetry reduction leaves nine Goldstone bosons: a singlet associated with the breaking of the baryonic symmetry $U(1)_B$ and an octet associated with the axial $SU(3)_A$ group.

Once a magnetic field is switched on, the difference between the electric charge of the u quark and that of the d and s quarks reduces the original flavor symmetry of the theory and, consequently, also the symmetry group remaining after the diquark condensate is formed. Then, the breaking pattern for the MCFL phase [6] becomes

$$\mathcal{G}_{\mathcal{B}} = SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_A^{(1)} \times U(1)_B \times U(1)_{\text{e.m.}} \rightarrow SU(2)_{C+L+R} \times \tilde{U}(1)_{\text{e.m.}}. \tag{2}$$

The group $U(1)_A^{(1)}$ [not to be confused with the usual anomaly $U(1)_A$] is related to the current which is an anomaly-free linear combination of s, d, and u axial currents [7]. In this case only five Goldstone bosons remain. Three of them correspond to the breaking of $SU(2)_A$, one to the breaking of $U(1)_A^{(1)}$, and one to the breaking of $U(1)_B$. Thus, an applied magnetic field reduces the number of Goldstone bosons in the superconducting phase, from nine to five.

The MCFL phase is not just characterized by a smaller number of Goldstone fields, but by the fact that all these bosons are neutral with respect to the rotated electric charge. Hence, no charged low-energy excitation can be produced in the MCFL phase. This effect can be relevant for the low-energy physics of a color-superconducting star's core and hence for its transport properties. In particular, the cooling of a compact star is determined by the particles with the lowest energy; so a star with a core of quark matter and sufficiently large magnetic field can have a distinctive cooling process.

More recently, we have found that the magnetic field can also influence the gluon dynamics [8]. At field strengths comparable to the charged gluon Meissner mass, a new phase can be realized giving rise to an inhomogeneous condensate of $\tilde{Q}$-charged gluons [8]. The gluon condensate antiscreens the magnetic field due to the anomalous magnetic moment of these spin-1 particles. Because of the antiscreening, this condensate does not give a mass to the $\tilde{Q}$ photon, but instead amplifies the applied rotated magnetic field. This means that at such applied fields the CS behaves as a paramagnet; thus we named this phase the paramagnetic-CFL (PCFL) phase [9]. This last effect is also of interest for astrophysics. Compact stars with color-superconducting cores could have larger magnetic fields than neutron stars made up entirely of nuclear matter, thanks to the gluon vortex antiscreening mechanism.

The above state of affairs underlines the need to discern the scales and field strengths at which one or another magnetic phase is physically relevant. If one ignores the quark masses, the main scales of the color superconductor are the baryon chemical potential μ, the dynamically generated gluon mass $m_g \sim g\mu$, and the gap parameter $\Delta \sim \frac{\mu}{g^5} e^{-\alpha/g}$, with α a constant that is dominated by magnetic gluon exchanges [10]. We can assume that, at sufficiently high μ, the running strong coupling g becomes $g(\mu) \ll 1$, so the hierarchy of the scales is $\Delta \ll m_g \ll \mu$. The main purpose of this paper is to elucidate how the different magnetic phases are related to the fundamental scales of the CS.

The plan of the paper is as follows. In Sec. II we develop the CFL low-energy effective theory in a magnetic background using a Lorentz covariant formalism. In this derivation the rotated magnetic field is introduced only through covariant derivatives, thus preserving the $\tilde{U}(1)_{\text{e.m.}}$ gauge invariance. The threshold rotated magnetic field that decouples the charged mesons from the low-energy theory is found in Sec. III. In Sec. IV we discuss how the different magnetic phases are realized in a hierarchical order determined by the main energy scales of the CS. In the concluding remarks we state the major outcomes of the paper and discuss possible astrophysical implications of the realization of a PCFL-like phase at moderate densities.

II. LOW-ENERGY EFFECTIVE THEORY IN A MAGNETIC BACKGROUND

The physics at energies below the lower scale Δ can be explored by constructing the effective low-energy theory in the presence of a rotated magnetic field. Since in the CFL phase all the fermions are gapped and all the gluons have dynamically generated masses, the low-energy theory of the CS is governed by the Goldstone modes arising from the breaking of the global symmetries in the presence of a magnetic field.

As discussed in the Introduction, once a magnetic field is present, the original symmetry group $\mathcal{G}$ is reduced, due to the different electric charges of the quarks, to $\mathcal{G}_{\mathcal{B}}$. One would think that the low-energy theory should correspond to the breaking pattern (2), and hence be described by five neutral Goldstone bosons. However, it is clear that at very weak magnetic fields the symmetry of the CFL phase can be treated as a good approximated symmetry, meaning that at weak fields the low-energy excitations are essentially governed by nine approximately massless scalars [those of the breaking pattern (1)] instead of five.

A question of order here is the following: what do we exactly understand as a very weak magnetic field? In other words, what is the threshold-field strength that effectively separates the CFL low-energy behavior from the MCFL one? A fundamental clue in this direction will come from the determination of the term in the low-energy CFL Lagrangian that can generate a field-induced mass for the charged Goldstone fields, disconnecting them from the low-energy dynamics at some field strength and thereby effectively reducing the number of Goldstone bosons from the nine of the CFL phase to the five of the MCFL phase. A similar approach was previously followed in Ref. [11]. Nevertheless, as it will become clear below, our treatment and results will differ from those previously obtained.

Our strategy will consist of writing the effective low-energy Lagrangian for the Goldstone bosons corresponding to the CFL breaking pattern (1), but in the presence of an external $\tilde{B}$ field, that is, ignoring the explicit breaking introduced by the electromagnetic interaction. To ensure that this Lagrangian is "invariant" with respect to the original group of the CFL case, one treats the charge operator as a spurion field [11] and assume it transforms conveniently under left/right flavor symmetries, as well as under the color symmetries, depending on whether the operator appears with flavor or color indices, as it will be shown below.

There are two important points that separate our treatment from previous works. One is that we give the general form of the low-energy Lagrangian in an arbitrary reference frame; that is, we introduce all the possible covariant structures that can be formed at finite density and in the presence of an external magnetic field. When taken in the rest frame, the Lagrangian naturally reproduces the different Lorentz structures that characterize the problem with an external magnetic field at finite density. The other is that, when proposing the allowed terms of the low-energy Lagrangian, we take into account that the coupling of the charged mesons with a rotated electromagnetic field can be traced back to the coupling of the fermions with the field. This coupling only occurs within a covariant derivative to preserve the gauge invariance of the theory under the $\tilde{U}(1)$ group. Then, the coupling between the Goldstone bosons and the rotated electromagnetic field always occurs within a covariant derivative too. We will show that, in this $\tilde{U}(1)$ gauge invariant approach, the charged Goldstone bosons acquire field-induced masses that appear at the leading order of the low-energy theory.

To find the effective low-energy theory, we can follow a similar method to that used at zero field in Refs. [12,13]. We start by introducing two scalar fields which describe the fluctuations of the diquark condensates, and are associated with left and right order parameters:

$$X^{ai} \sim \epsilon^{abc}\epsilon^{ijk}\langle\psi_L^{bj}\psi_L^{ck}\rangle^\dagger, \qquad Y^{ai} \sim \epsilon^{abc}\epsilon^{ijk}\langle\psi_R^{bj}\psi_R^{ck}\rangle^\dagger \tag{3}$$

where i, j, k denote flavor indices, a, b, c denote color indices, and L/R denote left/right chirality, respectively. Under an $SU(3)_C \times SU(3)_L \times SU(3)_R$ rotation, the above fields transform as

$$X \to g_C X g_L^\top, \qquad Y \to g_C Y g_R^\top, \tag{4}$$

with $g_C \in SU(3)_C$ and $g_{R/L} \in SU(3)_{R/L}$. The expectation values of the X and Y fields define the ground state of the CFL phase which produces the symmetry breaking $SU(3)_C \times SU(3)_L \times SU(3)_R \to SU(3)_{C+L+R}$.

We are interested in the fluctuations of the phases of the order parameters. Therefore, we can factor out the norm of the order parameters in (4) and work with unitary X and Y scalar fields. Although the axial group $U(1)_A$ is anomalous at low densities, it becomes an approximate symmetry at high densities due to the suppression of the instanton interactions that produce the anomaly. Nevertheless, in our derivations neither the pseudo Goldstone mode associated with the breaking of this group, nor the Goldstone mode associated with the $U(1)$-baryon symmetry breaking will be considered, as these bosons are both neutral with respect to the rotated electromagnetic charge, and therefore irrelevant for the analysis of the present paper (we refer the interested reader to Ref. [13] to find out the contributions of these modes to the low-energy theory; see also Ref. [14] for the diquark excitations of the CFL ground state at zero magnetic field in the framework of a Nambu-Jona-Lasinio model).

We can introduce the Goldstone canonical fields

$$X = e^{i\Pi_X^a T_a}, \qquad Y = e^{i\Pi_Y^a T_a}, \qquad a = 1, \ldots, 8 \tag{5}$$

with T_a being the $SU(3)$ generators normalized to satisfy

$$\mathrm{Tr}[T_a, T_b] = \tfrac{1}{2}\delta_{ab}. \tag{6}$$

Thus, X and Y together contain a total of 16 scalar fields. Strictly speaking, only eight of them are genuine Goldstone bosons, that is, massless scalar fields associated with the breaking of global symmetries. The other eight are Higgs fields related to the spontaneous breaking of the color gauge group $SU(3)$ that gives mass to the gluons and therefore can always be eliminated from the theory by choosing a convenient (unitary) gauge [13]. Since these considerations remain valid in the presence of the external magnetic field, we will work in the unitary gauge and concentrate our analysis on the derivation of the low-energy theory for the genuine Goldstone bosons.

At zero magnetic field, the low-energy theory of the Goldstone bosons associated with the global symmetries can be written [12] as

$$\mathcal{L} = -\frac{f_\pi^2}{4}\mathrm{Tr}[(J_X^\mu - J_Y^\mu)^2] \tag{7}$$

with currents J_X^μ and J_Y^μ defined by

$$J_X^\mu = X\partial^\mu X^\dagger, \qquad J_Y^\mu = Y\partial^\mu Y^\dagger. \tag{8}$$

Notice that (7) does not take into account the breaking of the Lorentz invariance due to the finite density. The lack of Lorentz invariance was later incorporated in Ref. [12] by manually assigning different coefficients in front of the temporal and spatial derivative terms.

Using (4), we can verify that the currents J_X and J_Y transform as

$$J_X^\mu \to g_C J_X^\mu g_C^\dagger, \qquad J_Y^\mu \to g_C J_Y^\mu g_C^\dagger. \tag{9}$$

The Lagrangian density (7) contains the leading (second) order in derivative terms invariant under the $SU(3)_C \times SU(3)_L \times SU(3)_R$ rotations.

Two important changes occur when a $\tilde{B}$ field is switch on. First, the derivatives should be replaced by covariant derivatives containing the rotated electromagnetic potential $\tilde{A}_\mu$ associated with the magnetic field $\tilde{B}$. Second, the number of fundamental tensors available in the theory increases, because of the extra tensor $\tilde{F}_{\mu\nu}$. As a consequence, we can construct an effective theory containing a larger number of independent terms which are quadratic in the (covariant) derivative.

The covariant derivative should be consistent with the fact that the vacuum expectation values of the diquark fields, which only get contribution from the diagonal elements of the order parameter matrix, are all neutral in the rotated charge. We can generically denote the 3×3 order parameter matrix by Δ. Taking into account that a rotated electromagnetic field is a combination of the 8th-gluon field and the conventional electromagnetic field and that each element of the order parameter matrix carries an electric charge equal to the sum of the electric charges of the quarks forming the corresponding pair, we can write the covariant derivative as

$$[\partial_\mu + igG_\mu^8(T^8 \times \mathbf{1}) + ieA_\mu(\mathbf{1} \times Q_\Delta)]\Delta = \left[\partial_\mu - \frac{i\sqrt{3}}{2} gG_\mu^8(Q \times \mathbf{1}) - ieA_\mu(\mathbf{1} \times Q)\right]\Delta \quad (10)$$

where the direct products denote (color $\times$ flavor). Q_Δ is the conventional electromagnetic charge operator of the quark pairs in the quark representation (s, d, u). In this representation the first column of Δ will have (d, u) pairs, the second (s, u) pairs, and the third (s, d) pairs. Hence $Q_\Delta = \mathrm{diag}(1/3, 1/3, -2/3)$. Both the T^8 generator of the $SU(3)$ group and the usual quark charge operator $Q = \mathrm{diag}(-1/3, -1/3, 2/3)$ are connected to Q_Δ through the relations $T^8 = (\sqrt{3}/2)Q_\Delta = -(\sqrt{3}/2)Q$.

Taking into account that $\tilde{A}_\mu = A_\mu \cos\theta - G_\mu^8 \sin\theta$, where θ is the mixing angle [4], Eq. (10) can be rewritten as

$$(\partial_\mu - i\tilde{e}\tilde{A}_\mu\tilde{Q})\Delta \quad (11)$$

where $\tilde{e} = e\cos\theta = (\sqrt{3}/2)g\sin\theta$, and $\tilde{Q} = (\mathbf{1} \times Q - Q \times \mathbf{1})$ is the rotated electric charge operator. As expected, the charge operator $\tilde{Q}$ assigns zero rotated charge to the diagonal elements of Δ. Notice that we do not keep the orthogonal combination $\tilde{G}_\mu^8 = A_\mu \sin\theta + G_\mu^8 \cos\theta$, as the field $\tilde{G}_\mu^8$ acquires a mass larger than Δ_{CFL}, so it decouples from the low-energy theory.

The straightforward generalization of the currents (8) to the case with nonzero rotated magnetic field should be done by substituting the derivative in (8) by the covariant derivative (11). Taking into account that $XX^\dagger = 1$, one can write the left current as

$$(\tilde{J}_X^\mu)_{ab} = X_{ai}(\partial^\mu\delta_{ij} + i\tilde{e}\tilde{A}^\mu Q_{ij})X_{jb}^\dagger - i\tilde{e}\tilde{A}^\mu Q_{ab} = X(\partial^\mu X^\dagger + i\tilde{e}\tilde{A}^\mu Q^L X^\dagger) - i\tilde{e}\tilde{A}^\mu Q^C \quad (12)$$

where we introduced the notations Q^C and Q^L to keep track of whether the operator Q is an operator in color or (left) flavor space. A similar expression can be found for the right current $\tilde{J}_Y^\mu$, with the obvious substitution $X \to Y$ and $Q^L \to Q^R$.

As mentioned above, once a magnetic field is present one has an extra tensor in the system that allows one to create new structures in Lorentz space. Working in a covariant way, the leading order low-energy Lagrangian density can be written as

$$\mathcal{L} = -\frac{f_\pi^2}{4}\mathrm{Tr}[(\tilde{J}_X^\mu - \tilde{J}_Y^\mu)(\tilde{J}_X^\nu - \tilde{J}_Y^\nu)\Theta_{\mu\nu}] \quad (13)$$

with

$$\Theta_{\mu\nu} = C_1 g_{\mu\nu} + C_2 u_\mu u_\nu + C_3 \hat{F}_{\mu\rho}\hat{F}_{\rho\nu} \quad (14)$$

being the most general Lorentz structure that can be formed with the vectors and tensors available at low energies. In (14) we considered the normalized electromagnetic tensor $\hat{F}_{\mu\nu} = \frac{1}{|\tilde{B}|}\tilde{F}_{\mu\nu}$, in addition to the usual metric tensor $g_{\mu\nu}$ and the vector four-velocity of the center of mass of the dense medium u_μ. The C_i's are just constant coefficients. In this covariant representation the magnetic field can be expressed as $\tilde{B}_\mu = \frac{1}{2}\varepsilon_{\mu\nu\rho\lambda}u^\nu\tilde{F}^{\rho\lambda}$. In the rest frame, $u^\nu = (1, 0, 0, 0)$, and the electromagnetic tensor becomes $\tilde{F}_{\mu\nu} = \tilde{B}\delta_{\mu 1}\delta_{\nu 2}$ if we assume a magnetic field pointing along the third spatial direction. In the absence of a magnetic field, the coefficient C_3 is taken equal to zero and the Lagrangian density (13) reduces in the rest frame to that introduced in Refs. [12,13], where the system was not Lorentz invariant due to the finite density ($u_\mu \neq 0$), but it kept the rotational symmetry. In the presence of a magnetic field the structure associated with C_3 naturally separates between modes longitudinal and transverse to the field. A linear term in $\hat{F}_{\mu\nu}$ is forbidden, since it would violate the theory CP invariance. Notice that in the leading (second) order in derivatives we do not need to introduce the momentum k_μ in the structures contributing to (14).

Notice that the flavor symmetry $SU(3)_L \times SU(3)_R$ is explicitly broken by the electromagnetic coupling in (13). So, strictly speaking, (13) is invariant under $\mathcal{G}_{\mathcal{B}}$ but not under $\mathcal{G}$. However, one can make the theory invariant under the original group $\mathcal{G}$ if we treat the charge operators as spurion fields and assume that they transform as

$$Q^L \to g_L^* Q^L g_L^\top, \qquad Q^R \to g_R^* Q^R g_R^\top, \qquad Q^C \to g_C Q^C g_C^\dagger. \quad (15)$$

Using these transformations, one can show that the new currents transform under $\mathcal{G}$ in the same way as the currents at zero field:

$$\tilde{J}_X^\mu \to g_C \tilde{J}_X^\mu g_C^\dagger, \qquad \tilde{J}_Y^\mu \to g_C \tilde{J}_Y^\mu g_C^\dagger \quad (16)$$

yielding to the invariance of (13) under $\mathcal{G}$.

Following Ref. [12], we now introduce the color singlet

$$\Sigma = Y^\dagger X, \quad (17)$$

which transforms under $SU(3)_C \times SU(3)_L \times SU(3)_R$ as $\Sigma \to g_R^* \Sigma g_L^\top$. In terms of Σ the Lagrangian density (13) can be written as

$$\mathcal{L} = \frac{f_\pi^2}{4}\,\mathrm{Tr}[(D_\mu\Sigma)(D_\nu\Sigma)^\dagger\Theta_{\mu\nu}] \tag{18}$$

where the covariant derivative acting on Σ is

$$D_\mu\Sigma = \partial_\mu\Sigma + i\tilde{e}\tilde{A}_\mu(Q^R\Sigma - \Sigma Q^L). \tag{19}$$

In the rest frame, (18) becomes

$$\mathcal{L} = \frac{f_\pi^2}{4}[\mathrm{Tr}(D_0\Sigma)(D_0\Sigma^\dagger) + v_\perp^2\,\mathrm{Tr}(D_\perp\Sigma)(D_\perp\Sigma^\dagger) + v_\parallel^2\,\mathrm{Tr}(D_\parallel\Sigma)(D_\parallel\Sigma^\dagger)], \tag{20}$$

showing the expected separation between longitudinal $D_\parallel$ and transverse $D_\perp$ (to the field) components of the covariant derivatives, a direct consequence of the partial breaking of the rotational symmetry in the presence of the magnetic field. The decay constant f_π and the meson maximum velocities $v_\perp$ and $v_\parallel$ are parameters to be computed from the microscopic theory. They can, in general, depend on the baryonic chemical potential and the applied magnetic field. In the weak-field limit ($eB \ll \mu^2$), at asymptotically large values of the chemical potential μ, they can be approximated by their zero-field values [13]

$$f_\pi^2 \approx \frac{21 - 8\ln 2}{18}\frac{\mu^2}{2\pi^2}, \qquad v_\perp \approx v_\parallel \approx \frac{1}{\sqrt{3}}. \tag{21}$$

As we shall see in the next section, the low-energy theory (20) supports the generation of field-dependent masses for the charged mesons at fields larger than a threshold value, even though no quadratic-in-$\tilde{B}$ term, like the one proposed in Ref. [11] [Eq. (2.16)], is present at the leading order. To understand this, one should keep in mind that the interaction between Σ and the electromagnetic field always occurs through the covariant derivative. To generate a quadratic-in-$\tilde{B}$ (and quadratic-in-Q) term, one needs to consider a higher order contribution involving four covariant derivatives,

$$\int d^4x\,\mathrm{Tr}[(D_\mu\Sigma)(D_\nu\Sigma)^\dagger(D_\rho\Sigma)(D_\lambda\Sigma)^\dagger\hat{F}_{\mu\nu}\hat{F}_{\rho\lambda}]. \tag{22}$$

Using (we dropped total derivatives)

$$D_\mu\Sigma(D_\nu\Sigma)^\dagger\hat{F}_{\mu\nu} = \frac{1}{2}\hat{F}_{\mu\nu}[i\tilde{e}\tilde{F}_{\mu\nu}(Q^R - \Sigma Q^L\Sigma^\dagger) + 2i\tilde{e}\tilde{A}_\mu(Q^R\Sigma\partial_\nu\Sigma^\dagger - \Sigma\partial_\nu\Sigma^\dagger Q^R) - 2\Sigma(\partial_\nu\Sigma^\dagger)\partial_\mu], \tag{23}$$

it can be straightforwardly shown that the term proposed in [11],

$$\int d^4x\tilde{e}^2\tilde{B}^2\,\mathrm{Tr}[Q^R\Sigma Q^L\Sigma^\dagger], \tag{24}$$

is just one of the several terms coming out of (22). However, as proved below, the threshold field for the decoupling of the charged mesons will be determined by a contribution more relevant than (24).

III. CFL-MCFL THRESHOLD FIELD

The unitary matrix Σ can be parametrized in terms of the elementary Goldstone bosons ϕ^A as

$$\Sigma = \exp\left(i\frac{\phi^A T^A}{f_\pi}\right), \qquad A = 1, \ldots, 8, \tag{25}$$

where T^A are the $SU(3)$ generators. Expanding (25) up to linear terms in the fields, we can write (19) as a sum of covariant derivatives for the ϕ^A fields,

$$D_\mu\Sigma = \frac{i}{f_\pi}\left[\sum_{A=1}^{3}T^A\partial_\mu\phi^A + T^8\partial_\mu\phi^8 + \sum_\pm\tau^\pm(\partial_\mu \pm i\tilde{e}\tilde{A}_\mu)\Pi^\pm + \sum_\pm\Lambda^\pm(\partial_\mu \pm i\tilde{e}\tilde{A}_\mu)\kappa^\pm\right] \tag{26}$$

where

$$\Pi^\pm = \frac{1}{\sqrt{2}}[\phi^4 \mp i\phi^5], \qquad \kappa^\pm = \frac{1}{\sqrt{2}}[\phi^6 \mp i\phi^7], \tag{27}$$

$$\tau^\pm = \frac{1}{\sqrt{2}}[T^4 \pm iT^5], \qquad \Lambda^\pm = \frac{1}{\sqrt{2}}[T^6 \pm iT^7] \tag{28}$$

and the rotated electromagnetic potential is taken in the Landau gauge

$$\tilde{A}_\mu = (0, 0, \tilde{B}x, 0). \tag{29}$$

From (20), the low-energy Lagrangian for the Goldstone bosons can be written

$$L = \int d^4x\left\{\frac{1}{4}\left[\sum_{A=1}^{3,8}|\partial_0\phi^A|^2 + |\partial_0\Upsilon|^2\right] + \frac{v_\parallel^2}{4}\left[\sum_{A=1}^{3,8}|\partial_\parallel\phi^A|^2 + |\partial_\parallel\Upsilon|^2\right] + \frac{v_\perp^2}{4}\left[\sum_{A=1}^{3,8}|\partial_\perp\phi^A|^2 + |(\partial_\perp + i\tilde{e}\tilde{A}_\perp)\Upsilon|^2\right]\right\} \tag{30}$$

where we introduced the charged meson doublet

$$\Upsilon \equiv \begin{pmatrix}\Pi^+\\ \kappa^+\end{pmatrix}. \tag{31}$$

The Lagrangian (29) represents the CFL low-energy theory in the presence of a weak ($k^2 \sim \tilde{e}\tilde{B} \ll \mu^2$) constant magnetic field. Now we are ready to determine the strength of the threshold field for which the effective symmetry transmutation from CFL to MCFL occurs.

For that aim, it is convenient to work in momentum space. Transforming to momentum space in the presence of the magnetic field can be done by applying to the scalar fields the same method originally developed for fermions in [15] and later extended to vector fields in [16]. In this approach the transformation to momentum space can be carried out using the wave functions $S_k(x)$ of the asymptotic states of the charged mesons in a uniform magnetic field. These functions play the role in the magnetized medium of the usual plane-wave (Fourier) functions e^{ipx} at zero field. Then, for the charged field Υ we have

$$\Upsilon(k) = \int d^4 x S_k(x)\Upsilon(x) \tag{32}$$

where

$$S_k(x) = \mathcal{N}\exp(ik_0x^0 + ik_2x^2 + ik_3x^3)D_n(\rho) \tag{33}$$

with $D_n(\rho)$ being the parabolic cylinder functions with the argument $\varrho = \sqrt{2|\tilde{e}\tilde{B}|}(x_1 - k_2/\tilde{e}\tilde{B})$, $\mathcal{N}$ the normalization constant $[\mathcal{N} = (4\pi|\tilde{e}\tilde{B}|)^{1/4}/\sqrt{n!}]$, and $n = 0, 1, \ldots$ denoting the Landau levels. It is easy to check that the transformation functions (32) satisfy the orthonormality condition

$$\int_{-\infty}^{\infty} d^4 x S_k(x) S_{k'}(x) = (2\pi)^4\hat{\delta}(k' - k) \tag{34}$$

where $\hat{\delta}(k' - k) = \delta(k'_0 - k_0)\delta(k'_2 - k_2)\delta(k'_3 - k_3)\delta_{n'n}$.

Using this transformation in (30) we can derive the Klein-Gordon equation for the charged mesons in momentum space,

$$[k_0^2 - \tilde{e}\tilde{B}(2n+1)v_\perp^2 - k_3^2 v_\parallel^2]\Upsilon(k) = 0, \tag{35}$$

and from it the corresponding dispersion relation

$$E^2 = \tilde{e}\tilde{B}(2n+1)v_\perp^2 + k_3^2 v_\parallel^2. \tag{36}$$

We see that at zero momentum ($k_3 = 0$, $n = 0$) the rest energy of the charged mesons is $M_{\tilde{B}}^2 = \tilde{e}\tilde{B}v_\perp^2$, meaning they acquire a mass induced by the magnetic field. For a meson to be stable, its mass should be less than twice the gap; otherwise it will decay into a quasiparticle-quasihole pair. Here we are assuming that the interactions between the Goldstone bosons and the quasiparticles are described by a Yukawa term that originates within a NJL-type model. Some authors (see [17] for details) have argued that the microscopic structure of the NG bosons in the CFL phase should be that of a quartic-quark state. In the quartic-quark picture the threshold would presumably be 4 times the gap energy. Within the NJL approach, the number of Goldstone bosons effectively changes when the magnetic field reaches the threshold value

$$\tilde{e}\tilde{B}_{\rm MCFL} = \frac{4}{v_\perp^2}\Delta_{\rm CFL}^2 \simeq 12\Delta_{\rm CFL}^2. \tag{37}$$

Here we used the weak-field approximation $v_\perp \simeq 1/\sqrt{3}$ [13]. At fields equal to or larger than this threshold field, the CFL symmetry can no longer be treated as a good approximated symmetry. Contrary to the result found in [11], our threshold field does not depend on the decay constant f_π; therefore it depends on μ only through $\Delta_{\rm CFL}$. The f_π dependence found in [11] is a direct consequence of considering a subleading contribution in the derivation of the threshold-field value, as previously shown.

For $\Delta_{\rm CFL} \sim 15$ MeV we get $\tilde{e}\tilde{B}_{\rm MCFL} \sim 10^{16}$ G. At these field strengths, the charged mesons decouple from the low-energy theory. When this decoupling occurs, the five neutral Goldstone bosons (including the one associated with the baryon symmetry breaking) that characterize the MCFL phase will drive the low-energy physics of the system. Therefore, going from low to higher fields, the first magnetic phase that effectively shows up in the magnetized system will be the MCFL, even though at fields near the threshold field the splitting of the gaps found at much stronger fields [6] may still be negligible.

We underline that the phenomenon occurring at the threshold field (37) is not a phase transition, as no symmetry is broken there. At any nonzero magnetic-field strength, below or above the threshold field (37), the symmetry of the system is, strictly speaking, that of the MCFL. However, at $\tilde{B} < \tilde{B}_{\rm MCFL}$ the charged Goldstone bosons are so light that the observable impact of the smaller symmetry of the MCFL phase, compared to that of the CFL phase, is irrelevant and the nine Goldstone bosons of the CFL phase provide a good approximated description of the low-energy physics. Based on these considerations we choose to call the CFL-MCFL transition a symmetry transmutation.

On the other hand, it is worth calling attention to the analogy between the CFL-MCFL transmutation and what could be called a "field-induced" Mott transition. Mott transitions were originally considered in condensed matter in the context of metal-insulator transitions in strongly correlated systems [18]. Later on, Mott transitions were also discussed in QCD to describe delocalization of bound states into their constituents at a temperature defined as the Mott temperature [19]. By definition, the Mott temperature T_M is the temperature at which the mass of the bound state equals the mass of its constituents, so the bound state becomes a resonance at $T > T_M$. In the present work, the role of the Mott temperature is played by the threshold field $\tilde{B}_{\rm MCFL}$. Mott transitions typically lead to the appearance of singularities at $T = T_M$ in a number of physically relevant observables. It is an open question worth investigating whether similar singularities are or are not present in the CFL-MCFL transmutation at $\tilde{B}_{\rm MCFL}$.

IV. $\tilde{B}$ *VS* μ PHASES IN A COLOR SUPERCONDUCTOR WITH THREE QUARK FLAVORS

What will happen if we keep increasing the magnetic field until it reaches the next energy scale $g\mu$? As is known [8], due to the interaction of the applied magnetic field with the charged gluon anomalous magnetic moment $(i\tilde{e}\tilde{f}_{\mu\nu}G_\mu^+G_\nu^-)$, once $\tilde{B} \geq \tilde{B}_{\rm PCFL} = m_M^2$, with $m_M \sim g\mu$

being the magnetic mass of the charged gluons, one of the modes of the charged gauge field becomes tachyonic (this is the well-known "zero-mode problem" for spin-1 charged fields in the presence of a magnetic field found for Yang-Mills fields [20], for the $W^{\pm}_{\mu}$ bosons in the electroweak theory [21,22], and even for higher-spin fields in the context of string theory [23]). Similarly to other spin-1 theories with magnetic instabilities [20–22], the solution of the zero-mode problem leads to the restructuring of the ground state through the formation of an inhomogeneous gauge-field condensate G, as well as an induced magnetic field due to the backreaction of the G condensate on the rotated electromagnetic field. The magnitude of the G condensate plays the role of the order parameter for the phase transition occurring at $\tilde{B} = \tilde{B}_{\rm PCFL}$. Near the transition point, the amplitude of the condensate G is very small [8]. Then, the condensate solution can be found using a Ginzburg-Landau (GL) approach similar to Abrikosov's treatment of type II metal superconductivity near the critical field H_{c2} [24]. As in Abrikosov's case, the order parameter $|G|$ continuously increases from zero with the applied magnetic field, signalizing a second-order phase transition towards a gluon crystalline vortex state characterized by the formation of flux tubes. Both spatial symmetries—the rotational symmetry in the plane perpendicular to the applied magnetic field and the translational symmetry—are broken by the vortex state.

It turns out that, contrary to what occurs in conventional type-II superconductors, where the applied magnetic field only penetrates through flux tubes and with a smaller strength than that of the applied field, the gluon vortex state exhibits a paramagnetic behavior. That is, outside the flux tube the applied field $\tilde{B}$ totally penetrates the sample, while inside the tubes the magnetic field becomes larger than $\tilde{B}$. This antiscreening behavior is similar to that found in the electroweak system at a high magnetic field [22]. Hence, since the $\tilde{Q}$ photons remain long range in the presence of the condensate G, the $\tilde{U}(1)_{\rm e.m.}$ symmetry remains unbroken. At asymptotically large densities, because $\Delta_{\rm CFL} \ll m_M$, we have $\tilde{B}_{\rm MCFL} \ll \tilde{B}_{\rm PCFL}$ for each μ value.

At fields $\tilde{B} \gtrsim \mu^2$ the density of states on the Fermi surface of the charged quarks will be larger than that of neutral quarks [6]. Because of the different density of states, the magnitude of the gap receiving contributions from pairs of rotated charged quarks will split from the magnitude of the gap receiving contributions only from pairs of rotated neutral quarks, as is shown to happen, without taking into account the vortex state, in Ref. [6]. The splitting of the gaps at this scale, however, would not break any new symmetry that has not already been broken at much lower scales. Hence, no new phase transition occurs at fields of order μ^2.

For fields much larger than μ^2, i.e. sufficiently strong as to surpass all the energy scales of the system, the quark infrared dynamics will become predominant, and the phenomenon of magnetic catalysis of chiral symmetry breaking [25] will be activated, producing a phase that favors quark-antiquark condensates over quark-quark condensates. However, the exploration of this region goes beyond the scope of this paper.

Figure 1 provides a qualitative sketch of the magnetic phases that exist at asymptotically high densities in the framework of a three-flavor color superconductor. In that region, at very weak magnetic fields, the color-superconducting state is practically described by the CFL phase, because the charged mesons, although massive, are so light that they cannot decay in pairs of quasiparticle-quasihole. When the field strength is of the order of the quarks' energy gap, the charged mesons become heavy enough to decouple and the low-energy physics is indeed that of the MCFL phase, where five neutral massless bosons drive the low-energy behavior. At fields comparable to the magnetic masses of the charged gluons, a chromomagnetic instability is developed for these gluons, leading to the formation of a vortex state and the antiscreening of the magnetic field [8,9]. The vortex state breaks the translational symmetry, as well as the remaining rotational symmetry in the plane perpendicular to the applied magnetic field; hence the vortex formation corresponds to a phase transition from the MCFL to a PCFL phase.

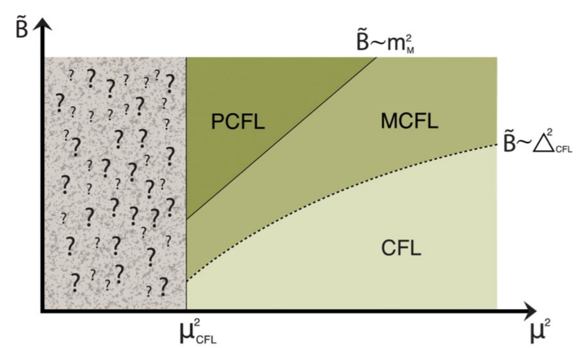

FIG. 1 (color online). Qualitative sketch in the $\tilde{B}$ vs μ^2 plane of the different phases of a color superconductor with three quark flavors in the presence of an external magnetic field at asymptotically high densities. The CFL phase appears here as an approximate symmetry at the weak field. Thus, the line between the CFL and MCFL phases does not denote a real phase transition, but the boundary separating the approximated CFL phase from the MCFL phase. This symmetry-transmutation line is reached at field values of the order of the CFL gap square. The line between the MCFL and PCFL phases indicates a second-order phase transition curve occurring at field strengths of the order of the square of the magnetic mass of the charged gluons. The rectangular region to the left corresponds to moderately high densities in the presence of a magnetic field. Since the ground state at moderately high densities has not yet been investigated in the presence of a magnetic field, this region is indicated by question marks.

The chemical potential $\mu_{\rm CFL}$ in Fig. 1 is used to schematically separate the regions where the effects of an s-quark mass M_s can (right of $\mu_{\rm CFL}$) or cannot (left of $\mu_{\rm CFL}$) be neglected. The region of moderately high densities to the left of $\mu_{\rm CFL}$ along the zero-magnetic-field line has been subjected to intense scrutiny in the literature [26–32], as this is the most important region for applications of CS in realistic systems such as compact stars. A main problem has been to find the stable phase at these moderately high densities. Despite many clever propositions that include a modified CFL phase with a condensate of kaons [29], a LOFF phase on which the quarks pair with nonzero total momentum [30], as well as homogeneous [31] and inhomogeneous [32] gluon condensate phases, it is still unclear which of these phases produces the lowest free energy. Given that nobody has yet studied this question at a finite magnetic field, we choose to indicate it in the figure with question marks.

V. CONCLUDING REMARKS

Summarizing, in a color superconductor with three-flavor quarks at very high densities, an increasing magnetic field produces a phase transmutation from CFL to MCFL first, and then a phase transition from MCFL to PCFL. During the phase transmutation no symmetry breaking occurs, since, in principle, once a magnetic field is present the symmetry is theoretically that of the MCFL, as discussed above. However, in practice, for $\tilde{B} < \tilde{B}_{\rm MCFL} \sim \Delta^2_{\rm CFL}$ the MCFL phase is almost indistinguishable from the CFL. Only at fields comparable to $\Delta^2_{\rm CFL}$ do the main features of the MCFL phase emerge through the low-energy behavior of the system. At the threshold field $\tilde{B}_{\rm MCFL}$, only five neutral Goldstone bosons remain out of the original nine characterizing the low-energy behavior of the CFL phase. These are precisely the five Goldstone bosons determining the new low-energy behavior of the genuinely realized MCFL phase. Going from the MCFL to PCFL phase is, on the other hand, a real phase transition [8], as the translational symmetry as well as the remaining rotational symmetry in the plane perpendicular to the applied magnetic field are broken by the vortex state.

Throughout this paper we have ignored the effects due to quark masses because we assumed very large baryon density. However, the densities of interest for most astrophysical applications are just moderately high. At moderate densities the s-quark mass can and does play an important role [26]. In this case, the color superconductor develops chromomagnetic instabilities even in the absence of an external magnetic field [27]. At even lower densities, where the s quark decouples due to its larger mass, a two-flavor color superconductivity—the so-called 2CS phase—is realized. In this phase, when color neutrality and β equilibrium conditions are imposed, some chromomagnetic instabilities can also develop at certain density values [28]. Finding the stable superconducting ground state at moderate densities is one of the main objectives in the field at the present moment [29–32]. The understanding of this problem in the presence of a magnetic field, which no doubt is another crucial player in the physical scenario of a compact star, is also an important open question. In this regard, as discussed in Ref. [32], the removal of the chromomagnetic instabilities found at moderate densities in the two-flavor system may be related to the *spontaneous generation* of an inhomogeneous gluon condensate with a corresponding induced magnetic field. If this proposition is proved to be correct also for the three-flavor case, the star's core could be in a PCFL-like magnetic phase, even if the core's original magnetic field is zero or relatively low. That is, the PCFL-like phase will not be triggered by instabilities produced at a critical value of some preexisting inner magnetic field, but by instabilities connected to the interplay of the neutrality conditions and the s-quark mass at some baryon density. An interesting consequence of a PCFL core is that a star's core in this phase can generate and/or boost its inner magnetic field. Exploring the magnetic phases that are realized at realistic densities is an important pending task.

It is plausible that, if compact stars are the natural playground for color superconductivity, the magnetic phases described in this paper, or more precisely, the version of these phases at more realistic densities, may be relevant for the physics of the core of highly magnetized compact objects like magnetars [2,33], and the so-called central compact objects (CCO) [34]. CCO are pointlike sources located near the center of supernova remnants that cannot be identified as active radio pulsars or magnetars [34]. Some of them may have magnetar's strength B fields and much smaller radii. For typical neutron star masses $\sim 1.4 M_\odot$, a smaller radii means a denser star core; the denser the core, the greater the chance it can be in a color-superconducting phase. Magnetars' surface magnetic fields are typically in the range of 10^{14} G–10^{16} G, and the fields at their much denser cores can probably reach even larger values. Any of these compact objects could be a candidate for the realization of a color-superconducting phase in a highly magnetized background.

Moreover, the standard explanation of the origin of the magnetars' large magnetic fields cannot explain all the features of the supernova remnants surrounding these objects [35,36]. Magnetars are supposed to be created by a magnetohydrodynamic-dynamo mechanism that amplifies a seed magnetic field due to a rapidly rotating (spin period <3 ms) protoneutron star. Part of this rotating energy is supposed to power the supernova through rapid magnetic braking, implying that the supernova remnants associated with magnetars should be an order of magnitude more energetic than typical supernova remnants. However, recent calculations [35] indicate that their energies are similar. In addition, one would expect that, when a magnetar spins down, the rotational energy output should go into a magnetized particle wind of ultrarelativistic electrons and positrons that radiate via synchrotron emission across the electromagnetic spectrum. Nevertheless, so far no one has detected the expected luminous pulsar wind nebulae around magnetars [37].

Although more observations are needed to confirm the above, current observations indicate that alternative models to the standard magnetar model [2] need to be considered. For example, some authors [35] have suggested that magnetars could be the outcome of a stellar progenitor with highly magnetized cores. A progenitor star with a PCFL-like core would be capable of inducing and/or enhancing the star's magnetic field due to the antiscreening mechanism inherent to this color-superconducting phase [8,9], and as such provides an alternative to the observational conundrum of the standard magnetar paradigm [2].

Only after the stable color-superconducting phase at moderate densities is well established with and without an external magnetic field will we be in a well-grounded position to investigate and reliably predict observable signatures of color superconductivity in compact stars.

ACKNOWLEDGMENTS

We are grateful to C. Manuel for comments and to S. Safi-Harb and I. A. Shovkovy for insightful discussions and comments. We would also like to thank the referee for helpful comments and for calling our attention to the analogy with a Mott transition. This work has been supported in part by DOE Nuclear Theory Grant No. DE-FG02-07ER41458.

[1] S. Frautschi, in *Proceedings of the Workshop on Hadronic Matter at Extreme Energy Density, Erice, 1978*, edited by N. Cabibbo (Plenum, New York, 1978); B. C. Barrois, Nucl. Phys. **B129**, 390 (1977); D. Bailin and A. Love, Phys. Rep. **107**, 325 (1984); M. Alford, K. Rajagopal, and F. Wilczek, Phys. Lett. B **422**, 247 (1998); R. Rapp, T. Schafer, E. V. Shuryak, and M. Velkovsky, Phys. Rev. Lett. **81**, 53 (1998).

[2] C. Thompson and R. C. Duncan, Astrophys. J. **392**, L9 (1992); **473**, 322 (1996).

[3] L. Dong and S. L. Shapiro, Astrophys. J. **383**, 745 (1991).

[4] M. Alford, K. Rajagopal, and F. Wilczek, Nucl. Phys. **B537**, 443 (1999).

[5] E. V. Gorbar, Phys. Rev. D **62**, 014007 (2000).

[6] E. J. Ferrer, V. de la Incera, and C. Manuel, Phys. Rev. Lett. **95**, 152002 (2005); Nucl. Phys. **B747**, 88 (2006); Proc. Sci., JHW2005 (2006) 022; J. Phys. A **39**, 6349 (2006).

[7] V. A. Miransky, and I. A. Shovkovy, Phys. Rev. D **66**, 045006 (2002).

[8] E. J. Ferrer and V. de la Incera, Phys. Rev. Lett. **97**, 122301 (2006).

[9] E. J. Ferrer and V. de la Incera, J. Phys. A **40**, 6913 (2007).

[10] D. T. Son, Phys. Rev. D **59**, 094019 (1999).

[11] C. Manuel, Proc. Sci., JHW2005 (2006) 011.

[12] R. Casalbuoni and R. Gatto, Phys. Lett. B **464**, 111 (1999).

[13] D. T. Son and M. A. Stephanov, Phys. Rev. D **61**, 074012 (2000).

[14] D. Ebert and K. G. Klimenko, Phys. Rev. D **75**, 045005 (2007).

[15] V. I. Ritus, Ann. Phys. (N.Y.) **69**, 555 (1972); Zh. Eksp. Teor. Fiz. **75**, 1560 (1978) [Sov. Phys. JETP **48**, 788 (1978)].

[16] E. Elizalde, E. J. Ferrer, and V. de la Incera, Ann. Phys. (N.Y.) **295**, 33 (2002); Phys. Rev. D **70**, 043012 (2004).

[17] K. Fukushima, Phys. Rev. D **70**, 094014 (2004); N. Yamamoto, M. Tachibana, T. Hatsuda, and G. Baym, arXiv:0704.2654.

[18] N. F. Mott, Rev. Mod. Phys. **40**, 677 (1968); *Metal-Insulator Transitions* (Taylor and Francis, London, 1974).

[19] J. Hüfner, S. P. Klevansky, and P. Rehberg, Nucl. Phys. **A606**, 260 (1996).

[20] V. V. Skalozub, Sov. J. Nucl. Phys. **28**, 113 (1978); N. K. Nielsen and P. Olesen, Nucl. Phys. **B144**, 376 (1978).

[21] V. V. Skalozub, Sov. J. Nucl. Phys. **43**, 665 (1986); **45**, 1058 (1987).

[22] J. Ambjorn and P. Olesen, Nucl. Phys. **B315**, 606 (1989); Phys. Lett. B **218**, 67 (1989).

[23] S. Ferrara and M. Porrati, Mod. Phys. Lett. A **8**, 2497 (1993); E. J. Ferrer and V. de la Incera, Int. J. Mod. Phys. A **11**, 3875 (1996).

[24] A. A. Abrikosov, Sov. Phys. JETP **5**, 1174 (1957).

[25] V. P. Gusynin, V. A. Miransky, and I. A. Shovkovy, Phys. Rev. Lett. **73**, 3499 (1994); Phys. Lett. B **349**, 477 (1995); Phys. Rev. D **52**, 4747 (1995); Nucl. Phys. **B462**, 249 (1996); K. G. Klimenko, Z. Phys. C **54**, 323 (1992); Theor. Math. Phys. (Engl. Transl.) **90**, 1 (1992); D.-S. Lee, C. N. Leung, and Y. J. Ng, Phys. Rev. D **55**, 6504 (1997); C. N. Leung, Y. J. Ng, and A. W. Ackley, Phys. Rev. D **54**, 4181 (1996); E. J. Ferrer and V. de la Incera, Phys. Rev. D **58**, 065008 (1998); Phys. Lett. B **481**, 287 (2000); E. Elizalde, E. J. Ferrer, and V. de la Incera, Phys. Rev. D **68**, 096004 (2003).

[26] M. Alford, C. Kouvaris, and K. Rajagopal, Phys. Rev. Lett. **92**, 222001 (2004).

[27] R. Casalbuoni, R. Gatto, M. Mannarelli, G. Nardulli, and M. Ruggieri, Phys. Lett. B **605**, 362 (2005); **615**, 297(E) (2005); M. Alford and Q. H. Wang, J. Phys. G **31**, 719 (2005); K. Fukushima, Phys. Rev. D **72**, 074002 (2005).

[28] M. Huang and I. A. Shovkovy, Phys. Rev. D **70**, 051501 (2004); **70**, 094030 (2004).

[29] P. F. Bedaque and T. Schafer, Nucl. Phys. **A697**, 802 (2002); A. Kryjevski and T. Schafer, Phys. Lett. B **606**, 52 (2005); A. Gerhold and T. Schafer, Phys. Rev. D **73**,

125022 (2006).

[30] M. Alford, J. A. Bowers, and K. Rajagopal, Phys. Rev. D **63**, 074016 (2001); I. Giannakis and H. C. Ren, Phys. Lett. B **611**, 137 (2005); Nucl. Phys. **B723**, 255 (2005); R. Casalbuoni *et al.*, Phys. Lett. B **627**, 89 (2005); M. Ciminale, G. Nardulli, M. Ruggieri, and R. Gatto, Phys. Lett. B **636**, 317 (2006); K. Rajagopal and R. Sharma, Phys. Rev. D **74**, 094019 (2006).

[31] E. V. Gorbar, M. Hashimoto, and V. A. Miransky, Phys. Lett. B **632**, 305 (2006); Phys. Rev. D **75**, 085012 (2007).

[32] E. J. Ferrer and V. de la Incera, arXiv:0705.2403.

[33] A. I. Ibrahim *et al.*, Astrophys. J. **609**, L21 (2004).

[34] G. G. Pavlov, D. Sanwal, and M. A. Teter, arXiv:astro-ph/0311526; R. A. Fesen, G. G. Pavlov, and D. Sanwal, Astrophys. J. **636**, 848 (2006).

[35] J. Vink and L. Kuiper, Mon. Not. R. Astron. Soc. **370**, L14 (2006).

[36] R-X. Xu, arXiv:astro-ph/0611608.

[37] S. Safi-Harb (private communication).